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NAMANGAN DAVLAT UNIVERSITETI

MATEMATIK ANALIZ KAFEDRASI

“MATEMATIKADAN MISOL VA MASALALAR YECHISH”

fanidan

O’QUV – USLUBIY MAJMUA

Namangan

O‘quv uslubiy majmua Namangan davlat universiteti Kengashininig 2023 yil "…" ________dagi "..." - son yig‘ilishida ko‘rib chiqilgan va foydalanishga tavsiya etilgan.

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Ma’ruza № 1

Mavzu: Sanoq sistemalar. Turli sanoq sistemalarida arifmetik amallar bajarish.

REJA

1. Matematikadan misol va masalalar yechish metodikasini o‘qitishning maqsad va vazifalari.
2. Pozitsion, nopozitsion sanoq sistemalari.
3. Ixtiyoriy pozitsion sanoq sistemalarida arifmetik amallar.

Matematikadan misol va masalalar yechish metodikasi matematikaning bir qismi bo‘lib, unda algebra va sonlar nazariyasi hamda geometriyaning asosiy tushunchalari, aksiomalari, shuning bilan birga matematika nazariyasi hamda amaliyoti orasidagi bog‘lanishlar o‘rganiladi. Kursning vazifasi – talabalarni matematikaning qadimgi zamonda yaratilishi va hozirgi zamondagi axvoli bilan tanishtirish, o‘rta maktab, kasb-hunar kollejlarida o‘qiladigan matematika kursidagi asosiy va prinsipial masalalarga ongli va tanqidiy munosabatda bo‘lish mahoratini tug‘dirish va shu bilan birga matematikadan dars berishda unga yordam berishdan iboratdir. Matematikaning mohiyatini tushunish va ularning yuzaga kelish sabablarini fahmlash uchun qisqacha bo‘lsada , tarixga nazar tashlash lozim.

Barcha mavjud tillar kabi sonlar tili ham mavjud bo‘lib, u ham o‘z alifbosiga ega. Mazkur alifbo hozir jahonda qo‘llanilayotgan 0 dan 9 gacha bo‘lgan o‘nta arab raqamlaridir, ya’ni: 0,1,2,3,4,5,6,7,8,9. Bu tilda o‘nta belgi (raqam) bo‘lganligi uchun ham, bu til o‘nlik sanoq sistemasi deb ataladi.

Bizning kundalik hayotimizda qo‘llanilayotgan o‘nlik sanoq sistemasi hozirgidek yuqori ko‘rsatkichni tez egallamagan. Turli davrlarda turli xalqlar bir–biridan keskin farqlanuvchan sanoq sistemalaridan foydalanganlar.

Masalan, 12 lik sanoq sistemasi juda keng qo‘llanilgan.Uning kelib chiqishida albatta tabiiy hisoblash vositasi – qo‘limizning ahamiyati katta. Bosh barmog‘imizdan farqli qolgan to‘rttala barmog‘imizning har biri 3 tadan, ya’ni hammasi bo‘lib 12 ta bo‘g‘indan iboratdir. Mazkur sanoq sistema izlari hanuzgacha saqlanib qolgan. Masalan, inglizlarda

uzunlikni o‘lchash birligi: 1 fut = 12 dyum=30 sm,

pul birligi: 1 shilling = 12 pens.

Qadimgi Bobilda ancha murakkab bo‘lgan sanoq sistemasi – 60lik sanoq sistemasi qo‘llanilgan. Bu sanoq sistemasining qoldiqlari hozir ham bor. Masalan:

1 soat = 60 minut

1 minut = 60 sekund

XVI – XVII asrlargacha Amerika qit’asining katta qismini egallagan atstek va mayyalarda 20 lik sanoq sistemasi qo‘llanilgan. Bunday misollarni ko‘plab keltirish mumkin.

Biz asosan o‘nlik sanoq sistemasidan foydalanamiz. Lekin, o‘nlik sanoq sistemasidan kichik sanoq sistemalarida sonlarni belgilash uchun arab raqami belgilaridan foydalaniladi. Masalan, beshlik sanoq sistemasida 0, 1, 2, 3, 4 raqamlari, yettilik sanoq sistemasida esa 0, 1, 2, 3, 4, 5, 6 raqamlaridan foydalaniladi.

Hisoblash texnikasida va dasturlashda asosi 2, 8 va 16 ga teng bo‘lgan sanoq sistemalari qo‘llaniladi.

O‘n ikkilik, o‘n oltilik sanoq sistemalarida qanday belgilardan foydalaniladi?− degan savolga javob aniq: raqamlardan keyin lotin alifbosidagi bosh harflardan foydalaniladi.

Shunday qilib, o‘n ikkilik sanoq sistemasida raqamlar 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B kabi; o‘n oltilik sanoq sistemasida esa 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F kabi yoziladi.

Sanoq sistemasi bu – sonlarni o‘qish va arifmetik amallarni bajarish uchun qulay ko‘rinishda yozish usuli.

Qadimda hisob ishlarida ko‘proq barmoqlardan foydalanilgan. Shu sababli narsalarni 5 yoki 10 tadan taqsimlashgan. Keyinchalik o‘nta o‘nlik maxsus nom – yuzlik, o‘nta yuzlik – minglik nomini olgan va h.k. Yozuv qulay bo‘lishi uchun bu muhim sonlar maxsus belgilar bilan ifodalana boshlagan. Agar hisoblashda 2 ta yuzlik, 7 ta o‘nlik, yana 4 ta birlik bo‘lsa, u holda yuzlikning belgisini ikki marta, o‘nlik belgisini yetti marta, birlik belgisini to‘rt marta takrorlashgan. Birlik, o‘nlik va yuzliklarning belgisi bir–biriga o‘xshash bo‘lmagan. Sonlarni bunday yozganda belgilarni ixtiyoriy tartibda joylashtirish mumkin bo‘lgan, chunki yozilgan sonning qiymati tartibga bog‘liq emas. Bunday yozuvda belgi holatining ahamiyati bo‘lmaganidan, mos sanoq sistemasi nopozitsion sistema deb ataladi. Qadimgi misrliklar, yunonlar va rimliklarning sanoq sistemasi nopozitsion edi. Nopozitsion sanoq sistemasi qo‘shish va ayirish amallari uchun ozgina yarasada, ko‘paytirish va bo‘lish uchun butunlay yaroqsiz edi. Ishni osonlashtirish maqsadida hisob taxtalari – abaklar ishlatilar edi. Hozirgi zamon cho‘tlari abakning o‘zgargan ko‘rinishidir.

Qadimgi bobilliklarning sanoq sistemasi dastlab nopozitsion edi, keyinchalik ular belgilarni yozish tartibida ham informatsiya borligini sezishib, undan foydalanishga o‘rganishdi va pozitsion sanoq sistemasiga o‘tishdi. Bunda biz hozir qo‘llayotgan sistemadan (raqamning o‘rni bir xonaga siljitilganda uning qiymati 10 martaga o‘zgaradigan o‘nli sanoq sistemadan) farqli, bobilliklarda belgi bir xonaga siljitilganda sonning qiymati 60 marta o‘zgarar edi (bunday sanoq sistemasi oltmishli sistema deb ataladi). Uzoq vaqtgacha Bobilning sanoq sistemasida nol belgisi, ya’ni bo‘sh qolgan xonaning belgisi yo‘q edi. Odatda, sonlarning tartibi ma’lum bo‘lganidan bu noqulay emas edi. Ammo keng ko‘lamli matematik va astronomik jadvallar tuzish boshlanganda, ana shunday belgiga ehtiyoj tug‘ildi. Bu belgi keyinchalik mixxat yozuvlarda va eramizning boshida Iskandariyada tuzilgan jadvallarda uchraydi. IX asrda nol uchun maxsus belgi paydo boldi. O‘nli sanoq sistemasida sonlar ustida amallar bajarish qoidasi ishlab chiqildi. Muhammad ibn Muso al–Xorazmiy tomonidan yozilgan “Hind hisobi” nomli risola tufayli o‘nli sanoq sistemasi Yevropaga, keyin esa butun dunyoga tarqaldi.

Sanoq sistemasining asosi uchun na faqat 10 va 60 ni, balki birdan katta ihtiyoriy p natural sonni olish mumkin.

Sanoq sistemalarini tashkil etilishi deyarli bir xil. Biror *p* soni – sanoq sistemasi asosi sifatida qabul qilinib, ixtiyoriy N soni quyidagi ko‘rinishda ifodalanadi:

$$N = a_n p^n + a_{n-1} p^{n-1} + \ldots + a_1 p^1 + a_0 p^0 + a_{-1} p^{-1} + \ldots + a_{-m} p^{-m}$$

Ko‘phad ko‘rinishida ifodalangan shu sonni

$$(a_n\ a_{n-1}\ \ldots\ a_1\ a_0\ a_{-1}\ \ldots\ a_{-m})_p$$

kabi yozish ham mumkin (*n* va *m* – sonning butun va kasr qismi honalari (razryadlari) soni).

Sonning bu kabi ifodalanishida har bir raqam qiymati o‘z o‘rniga qarab turli xil bo‘ladi. Masalan, o‘nlik sanoq sistemasida 98327 sonida 7 – raqami birlikni, 2 – o‘nlikni, 3 – yuzlikni, 8 – minglikni, 9 – o‘n minglikni ifodalaydi (bu hol faqat o‘nlik sanoq sistemasida):

98327 = 9 □□□ 10^4 + 8□□□10^3 + 3□□□10^2 + 2□□□10^1 + 7□□□10^0.

Biror boshqa *p* – asosli sanoq sistemasida a_0, a_1, a_2 ...raqamlar $a_0, a_1 p$, $a_2 p^2$,... qiymatlarni bildiradi.

Bunday ko‘rinishda tuzilgan sanoq sistemalari pozitsion sanoq sistemalari deyiladi.

Pozitsiyali sanoq sistemasida butun sonlarni quyidagi qonuniyat asosida hosil qilinadi: keyingi son oldingi sonning o‘ngdagi oxirgi raqamini surish orqali hosil qilinadi; agar surishda biror raqam 0ga aylansa, u holda bu raqamdan chapda turgan raqam suriladi.

Shu qonuniyatdan foydalanib, birinchi 10 ta butun sonni hosil qilamiz:

- Ikkilik sanoq sistemasida : 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001;
- Uchlik sanoq sistemasida : 0, 1, 2, 10, 11, 12, 20, 21, 22, 100;
- Beshlik sanoq sistemasida : 0, 1, 2, 3, 4, 10, 11, 12, 13, 14;
- Sakkizlik sanoq sistemasida : 0, 1, 2, 3, 4, 5, 6, 7, 10, 11.

Pozitsion sanoq sistemasi o‘zining qulayligi bilan hayotda keng qo‘llanilmoqda.

Boshqa usulda tuziladigan sanoq sistemalari ham mavjud. Ular pozitsiyaga bog‘liq bo‘lmagan sanoq sistemalari deyiladi. Masalan rim raqamlari. Mazkur sistemada maxsus belgilar to‘plami kiritilgan bo‘lib, ixtiyoriy son shu belgilar ketma−ketligidan iborat bo‘ladi.

Rim sanoq sistemasida

Bir (1)	–	I belgi bilan;
Besh (5)	–	V belgi bilan;
O‘n (10)	–	X belgi bilan;
Ellik (50)	–	L belgi bilan;
Yuz (100)	–	C belgi bilan;
Besh yuz (500)	–	D belgi bilan;
Ming (1000)	–	M bilan belgilanadi.

Bu belgilar va ularning kombinatsiyasi yordamida turli sonlarni hosil qilinadi. Masalan, 1 dan 3 gacha − I, II, III kabi, to‘rt (4) – IV , 5 – V tarzida ifodalanadi. Bu yerda 4 sonini yozish uchun 5 sonidan 1 sonini ayirib yoziladi, ya’ni I belgi V dan oldinga qo‘yilsa ayirish ma’nosini, agar keyinga qo‘yilsa qo‘shishni anglatadi. Umumiy holda: 6 – VI, 7 – VII, 400 – CD, 600 – DC ko‘rinishda ifodalanadi.

Rim sanoq sistemasida yozilgan sonlarni o‘nlik sanoq sistemasiga quyidagicha o‘tkazish mumkin:

VI $\rightarrow$ V $\geq$ I $\rightarrow$ 5 + 1 = 6

IV $\rightarrow$ (I $\geq$ V)?$\rightarrow$ 5 − 1 = 4

XIX $\rightarrow$ X + (I $\geq$ X)? $\rightarrow$ 10 + (10−1) =19

XCIX $\rightarrow$ (X $\geq$ C)? + (I $\geq$ X)? $\rightarrow$ (100−10) + (10−1) =99

MCMLXIII $\rightarrow$ M+(C $\geq$ M)?+L+X+I+I+I$\rightarrow$1000+(1000−100)+50+1+1+1 =1963.

Demak, bu sistemada har bir belgining ma’nosi va qiymati uning turgan pozitsiyasiga bog‘liq emas. Shuning uchun rim raqamlarini hayotda keng qo‘llash imkoniyati bo‘lmagan. Ammo ularni kitoblar bobini qo‘yishda, soatlarni yozuvida va boshqalarda qo‘llab turamiz.

Asosiy darsliklar va o’quv qo’llanmalar

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Amaliy mashg'ulot: Sanoq sistemalari. Turli sanoq sistemalarida arifmetik amallar bajarish

Bir sanoq sistemasidan boshqa sanoq sistemasiga o‘tish uchun avval o‘ngli sanoq sistemasiga o‘tib, so‘ngra, aytilgan sistemaga o‘tiladi. 10 lik sanoq sistemasidan boshqa sanoq sistemasigao‘tish uchun ana shu sistema asosida berilgan sonni bo‘linadi, so‘ngra bo‘linmani ana shu asosga bo‘linsada bo‘linma asosdan kichik bo‘lguncha davom ettiriladi.

Masalan $2895_{10}=231033_4$

1. $2895=X_4$

```
2895 | 4
28     723 | 4
  9    4     180 | 4
8       32   16    45 | 4
  15    32     20  4    11 | 4
12       3     20  5    8    2
   3              0  4    3
                     1
```

$2895=23\ 10\ 33_4$

2. $1234=X_6$

```
194 | 6
18    32 | 6
  14  30   5
12     2
```

$194_{10}=522_6$

3. $1234_5=194_{10}$

$1 \cdot 5^3+2 \cdot 5^2+3 \cdot 5+4=125+50+15+4=194$

1) $234_5 + 343_5 = 1132_5$

```
  2345
+ 3435
-------
 11325
```

2) $343_5 - 244_5 = 44_5$

```
  3435
- 2445
-------
    445
```

3) $224_5 \times 13_5 = 4022_5$

```
   2245
x   135
--------
   1232
   224
--------
  40225
```

4) $4022_5 : 13_5 = 224_5$

```
4022 | 13
31     224
 42
 31
 112
 112
```

0

5) $4203_5+2132_5-24_5 \cdot 13_5 x 11340_5$

1) 420_5 2) 24_5 3) 11340_5

2132_5 13_5 422_5

$11340_5 132$ 10413_5

24

422_5

MUSTAQIL ISH

1) $23432_5=X_6$
2) $34212_6=X_3$
3) $42134_5=X_5$
4) $231456_7=X_3$
5) $432167_8=X_4$
6) $41213_5=X_2$
8) $213413_5=X_8$
9) $41235_6=X_9$
10) $312142_5=X_4$
11) $423189=X_2$
12) $14325_6=X_7$
13) $32454_6=X_3$

XISOBLANG

1) $34532_6+24535_6$
2) $67817_9-34769_9$
3) $434567_8 \cdot 34_8$
4) $453454_6:14_6$
5) 6235_7+3463_7
6) $7062_8 \cdot 504_8$
7) $1432013_5:433_5$
8) $42401_5-13432_5$
9) 321024-10334
10) $((331324_5:4_5+33214_5)-43124_5).243_5$
12) $306_x+124_x=225$ *x-ни топинг*

Ma'ruza№2

Mavzu 2 : Bo'linish belgilari. Qoldiqli bo'lish. Tub va murakkab sonlar. EKUB va EKUK.

Berilgan sonning raqamlar yig'indisi 3 ga bo'linsa, berilgan son 3 ga bo'linadi. Berilgan sonning ohirgi raqami 0 yoki 5 bo'lsa, 5 ga bo'linadi. Ohirgi ikkita raqami 4 ga bo'linsa, berilgan son 4 ga bo'linadi. Berilgan son t ga yoki p ga bo'linsa, u holda berilgan son $t \cdot p$ ga bo'linadi.
546+174+390 da amallarni bajarmasdan oldin 6 ga bo'linishini aniqlang.

546	5+4+6=15	15:3	6:2
174	1+7+4=12	12:3	4:2
340	3+9+0=12	12:3	0:2

Demak $2 \cdot 3=6$ 6 ga bo'linadi

Berilgan sonlarni bo'ladigan sonlarni eng kattasi eng katta umumiy bo'luvchi deyiladi. Berilgan sonlarga bo'linadigan sonlarni eng kichigi eng kichkina umumiy bo'linuvchi deyiladi. Bularni topish ikki usulda bajariladi.
I-usul. Tub ko'paytuvchilarga ajratish yordamida.
72, 48 ga EKUB va EKUK topilsin

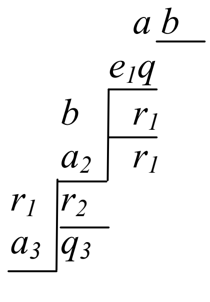
72	**2**	**48**	**2**
36	**2**	**24**	**2**
18	**2**	**12**	**2**
9	**3**	**6**	**2**
3	**3**	**3**	**3**
1		1	

$72=2^3 \cdot 3^2$
$48=2^4 \cdot 3$

Д (72,48)= $2^3 \cdot 3=8 \cdot 3=24$
К (72,48)= $2^4 \cdot 3^2=16 \cdot 9=144$

(EKUBni olishda кщpaytuvchiga ajralganlarni borlarini kichik darajalarini olib ko'paytiriladi)
(EKUK olishda bir hil qatnashgan sonlarning katta darajaga olindi)

II-usul. yevklid algoritmi yordamida topiladi. Ya'ni berilgan sonlarni kattasini kichigiga bo'linadi. Agar 0 qoldiq chiqsa, ikkinchi son birinchi sonning eng katta umumiy bo'luvchi bo'ladi. Agar qoldiq chiqsa, ikkinchi sonni qoldiqqa bo'linadi, nol chiqsa birinchi qoldiq berilgan sonlarga eng katta umumiy bo'luvchi bo'ladi.
Qoldiq qolsa, birinchi qoldiqni ikkinchi qoldiqqa, ikkinchi qoldiqni uchinchi qoldiqqa va hokazo nol qoldiq chiqquncha davom ettiriladi. Nol qoldiqdan oddingi qoldiq berilgan sonlarga eng katta umumiy bo'luvchi bo'ladi.
Д $(a,в)=r_n$



$r_{n-1}|r_n$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $r_2 | r_3$

$\underline{r_{n-1}}|q_n$

0

Масалан 1. *Д (72;49)=24*

$$\begin{array}{r|l} 72 & 48 \\ \underline{48} & 1 \end{array}$$

$$\begin{array}{r|l} 48 & 24 \\ \underline{48} & 2 \\ 0 & \end{array}$$

К (а,в)= $\frac{a \cdot b}{Д(a \cdot b)}$ *ga asosan,*

K(72,48)= $\frac{72 \cdot 48}{24} = 144$

K(72,48)=144

Masalan 2.

46362 va 41034 sonlarning eng kichik umumiy karralisini va eng katta umumiy bo'luvchisini toping

$$\begin{array}{r|l} 46362 & 41034 \\ \underline{41034} & 1 \end{array}$$

$$\begin{array}{r|l} 41034 & 5328 \\ \underline{37296} & 7 \end{array}$$

$$\begin{array}{r|l} 5328 & 3738 \\ \underline{3738} & 1 \\ 1590 & \end{array}$$

$$\begin{array}{r|l} 3738 & 1590 \\ \underline{3180} & 2 \end{array}$$

$$\begin{array}{r|l} 1590 & 558 \\ \underline{1116} & 2 \end{array}$$

$$\begin{array}{r|l} 558 & 474 \\ 474 & 1 \end{array}$$

$$\begin{array}{r|l} 474 & 84 \\ \underline{420} & 5 \end{array}$$

$$\begin{array}{r|l} 84 & 54 \\ \underline{54} & 1 \end{array}$$

$$\begin{array}{r|l} 54 & 30 \\ 30 & 1 \end{array}$$

Д (46362,41034)=6

$$\begin{array}{r|l} 30 & 24 \\ \underline{24} & 1 \end{array}$$

$$\begin{array}{r|l} 24 & 6 \\ 24 & 4 \end{array}$$

K(43362,41034)= $\frac{46362 \cdot 41034}{6} = 46362 \cdot 684 = 31711608$

Amaliy mashg'ulot

1. Quyidagi sonlarning qaysi biri 3 ga qoldiqsiz bo‘linadi.
413, 535, 1275, 5748, 5710, 20145, 3120, 201450, 4356782

2. Qo‘shish amalini bajarmasdan turib
180+144, 720+308, 3240+7560 yig‘indilarning 2ga, 4 ga, 3 ga, 5 ga, 9 ga bo‘linish-bo‘linmasligini ayting.

3. Qo‘shish va ayirish amalini bajarmasdan turib, quyidagi yig‘indi va ayirmalarning 4, 9, 5 sonlarga bo‘linishini aniqlang.
a) 3456+10116, b) 6375-3025, 648+1071+80424, v) 5625+1584

4. Ko‘paytirish amalini bajarmasdan turib, quyidagi ko‘paytmalarni 2ga, 4ga, 3ga bo‘linish-bo‘linmasligini aniqlang.
a) $144 \cdot 75$, b) $123 \cdot 280 \cdot 50$ v) $97 \cdot 504 \cdot 225$

5. Quyidagi sonlarning eng katta umumiy bo‘luvchisini va eng kichik umumiy karralisini toping. Tub ko‘paytuvchilarga ajratish usuli bilan toping.
(1200, 960); (2400, 1920); (1920, 1260);
(12870, 7650); (3600, 1920); (30295, 36354);

yevklid algoritmi yordamida toping
(42595, 20145); (2585, 2975)
(2760765, 11864145); (420135, 455565)
(7651563, 14456712); (457566, 400551)

6. Quyidagi sonlarni eng katta umumiy bo‘luvchisini va eng kichik umumiy karralisini toping.
D (2551665, 10664145)
K (8740, 2430)
D (775845, 304005)
K (4970, 1330)

Ma’ruza 3 : Sonlarni kiritish yo’llari. Taqribiy hisoblashlar va ularning tatbiqi

Butun sonlar to’plamida har doim qo’shish, ayirish, ko’paytirish amallarini bajarish o’rinlidir, lekin bo’lish amali har doim bajarilavermaydi. Chunki bir butun sonni ikkinchi butun songa bo’lganda har doim bo’linmada butun son hosil bo’lavermayda.

Masalan, 7:2 = 3.5, 9:4 = $2\frac{1}{4}$, ... Bu erda hosil qilingan bo’linmadagi 3,5; $2\frac{1}{4}$, ... sonlari butun sonlar to’plamida mavjud emas. Umuman olganda $m\cdot x=n$, $m\neq 0$ ko’rinishdagi tenglamaning yechimi butun sonlar to’plamida har doim mavjud emas, bu tenglama har doim $x=\frac{n}{m}$ ko’rinishdagi yechimga ega bo’lishi uchun kasr tushunchasini kiritish orqali butun sonlar to’plamini kengaytirib, unga barcha manfiy va musbat kasr sonlarni qo’shish kerak. Bu degan so’z $\left\{-\frac{p}{q},0,\frac{p}{q}\right\}$ ko’rinishdagi ratsional sonlar to’plamini hosil qilish kerak deganidir. Shundagina $mx=n$ ko’rinishdagi tenglamalar har doim yechimga ega bo’ladi. Bu erda r va q lar natural sonlardir. Yuqoridagi mulohazalarga ko’ra ratsional songa quyidagicha ta’rif berish mumkin: $\frac{p}{q}$ *ko’rinishdagi qisqarmas kasrga ratsional son deyiladi.*

Endi kasr tushunchasini kiritish uchun foydalaniladigan misollarni ko’rib o’taylik.

Agar bir metr uzunlikdagi yog’ochni o’zaro teng ikki bo’lakga bo’linsa, u holda bo’laklarning har birining uzunligi ana shu yog’och uzunligining yarmiga teng bo’ladi va uni $\frac{1}{2}$ kabi yoziladi. Agar ana shu bir metr uzunlikdagi yog’ochni o’zaro teng uch bo’lakka bo’linsa, u holda bo’laklardan har birining uzunlngi shu yog’och uzunligining uchdan biriga teng bo’ladi va uni $\frac{1}{3}$ kabi yoziladi. Xuddi shuningdek, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ ...

Agar bir metr uzunlikdagi yog’ochni teng uch bo’lakka bo’lib, undan ikki qismini oladigan bo’lsak, olingan uzunligini $\frac{2}{3}$ kabi yoziladi.

Agar ana shu yog’ochni to’rt bo’lakga bo’lib, undan uch qismini olsak, olingan qism uzunligini $\frac{3}{4}$ kabi ifodalanadi. Yuqorida qilingan mulohazalarga asoslanib kasr tushunchasining ta’rifini quyidagicha berish mumkin.

T a ‘ r i f. *Butun sonning o’zaro teng bo’lgan ma’lum bir ulushi, shu sonning kasri deyiladi.*

Yuqorida $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ kasr sonlarni hosil qildik. Berilgan narsalarni yoki butun sonni qancha teng qismga bo’linganligini ko’rsatuvchi sonni kasrning *maxraji*, shunday qismdan nechtasi olinganligini ko'rsatuvchi sonni kasrning *surati* deyiladi. Maxraj kasr chizig'ining ostida, surat esa kasr chizig'ining ustiga yoziladi.

Umumiy holda kasrni $\frac{p}{q}$ ko'rinishda ifodalanadi. Bunda *r* - kasrning surati, *q* - kasrning maxraji deb yuritiladi. $\frac{p}{q}$ ko'rinishdagi kasrlarga qarama-qarshi kasrlarni -$\frac{p}{q}$ ko'rinishda ifodalanadi.

Koordinata o'qida -$\frac{p}{q}$ ko'rinishdagi kasrlar nol sonidan chapda joylashgan bo'ladi. Biz butun sonlar to'plamini kengaytirish orqali -$\frac{p}{q}$ va $\frac{p}{q}$ ko'rinishdagi kasrlarni hosil qildik. Natijada koordinata o'qida $\{-\frac{p}{q}, 0, \frac{p}{q}\}$ ko'rinishdagi sonlar to'plami hosil bo'ldi.

Bunday to'plam ***ratsional sonlar to'plami*** deb ataladi. Agar ratsional sonlar to'plamidagi -$\frac{p}{q}$ va $\frac{p}{q}$ kasrlarning maxrajlari q = 1 desak, bizga ma'lum bo'lgan butun sonlar to'plami hosil bo'ladi. Bundan ko'rinadiki, butun sonlar ratsional sonlar to'plamining xususiy bir holi ekan. Ratsional sonlar to'plami bilan koordinata to'g'ri chizig'i nuqtalari orasida o'zaro bir qiymatli moslik o'rnatish mumkinmi, degan savol tug'ilishi tabiiydir. Bu savolga quyidagicha javob berishimiz mumkin, aksincha, har bir nuqtaga bittadan ratsional soni mos keltirish mumkin emas.

Kasrlar uch xil bo'ladi:

1. To'g'ri kasrlar. 2. Noto'g'ri kasrlar. 3. O'nli kasrlar.

1. Agar kasrning surati uning maxrajidan kichik bo'lsa, bunday kasrlarni ***to'g'ri kasrlar*** deyiladi.

Masalan: $\frac{1}{2}, \frac{3}{4}, \frac{1}{6}$ ....

2. Agar kasrning surati uning maxrajidan katta bo'lsa, bunday kasrlarni ***noto'g'ri kasrlar*** deyiladi. Masalan, $\frac{5}{2}, \frac{7}{4}, \frac{17}{5}$ ... .

3. Agar kasrning maxraji bir va nol sonlaridan iborat bo'lsa, bunday kasrlarni ***o'nli kasrlar*** deyiladi. Masalan, $\frac{1}{10}$=0,1; $\frac{1}{100}$=0,01; ... .

Kasr tushunchasi kiritilganidan keyin kasrlarning tengligi tushunchasi kiritiladi. Bu tushunchani o'quvchilarga quyidagicha tushuntirish mumkin.

Faraz qilaylik, bizga bir metr uzunlikdagi kesma be-rilgan bo'lsin. Agar shu kesmani teng ikkiga bo'lsak, har bir kesmaning uzunligi $\frac{1}{2}$ kabi kasr bilan ifodalanadi. Endi bo'lingan hap bir kesmani yana ikkiga bo'lsak har bir kesmaning uzunligi $\frac{1}{4}$ kasr bilan ifodalanadi. Ana shu teng to'rtga bo'lingan kesmalardan ikkitasining uzunligi $\frac{2}{4}$ kasr bilan ifodalanadi. Bu esa butun kesma uzunligining teng ikkiga bo'lgandagi $\frac{1}{2}$ kasr bilan ifodalangan qiymatiga tengdir. Shuning uchun $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\ldots$. Bundan ko'rinadiki, $\frac{1}{2}$ va $\frac{2}{4}$ kasrlarning qiymatlari teng bo'lib, ularni ifoda qilish har xildir.

O'quvchilarga kasrlarning tengligi tushunchasini tushuntirilganidan so'ng kasrning quyidagi xossalarini ifoda qilish mumkin.

1 - x o s s a. Agar kasrning surat va maxrajini bir xil songa ko'paytirilsa, kasrning qiymati o'zgarmaydi. $\frac{p}{q}=\frac{p}{q}\cdot\frac{n}{n}$. 1) $\frac{2}{5}=\frac{2}{5}\cdot\frac{2}{2}=\frac{4}{10}$;

2) $\frac{3}{7}=\frac{3}{7}\cdot\frac{4}{4}=\frac{12}{28}$; 3) $1=\frac{1}{1}=\frac{1}{1}\cdot\frac{4}{4}=\frac{4}{4}=\frac{4\cdot 25}{4\cdot 25}=\frac{100}{100}$.

II - x o s s a. Agar kasrning surat va maxrajini bir xil songa bo'linsa, kasrning qiymati o'zgarmaydi. $\frac{p:n}{q:n}=\frac{p}{q}$ Bu erda *n* > *1* bo'lishi kerak.

M i s o l l a r 1) $\frac{4}{8}=\frac{4}{4\cdot 2}=\frac{1}{2}$ 2) $\frac{15}{3}=\frac{3\cdot 5}{3}=\frac{5}{1}=5$

III - x o s s a. Agar kasrning surat va maxrajidagi sonlar umumiy bo'luvchilarga ega bo'lmasa, u holda bunday kasr qisqarmas kasr bo'ladi. Masalan $\frac{5}{7},\frac{4}{5},\frac{17}{19},\ldots$. qisqarmas kasrlardir, chunki 5 va 7, 4 va 5, 17 va 19 sonlari o'zaro umumiy bo'luvchilarga ega emas.

Mavzu 4: Bir o`zgaruvchili va birjinsli ko`phad. Ko`phadlar ustida amallar.

REJA

1.Biro`zgaruvchili va birjinsli ko`phad.

2.Ko`phadning kanonik ko'rinishi, ko`phadlar ustida amallar.

3. Ko`phadning bo`linishi. Ko`phadlarni ko`paytuvchilarga ajratish.

4. Ratsional ifodalarni ayniy almashtirishlar.

Birhadlar yig'indisi *ko'phad* deyiladi. Masalan, $3a^2b + 7b^2c$, $9x^2y + xy^2$ ifodalarning bar biriko'phaddir.Ko'phadtarkibidagiengkatta darajali birhadnin darajasi shu *ko'phadning darajasi* deyiladi. Masalan,

$$P(x) = c + ax^2 + bx,\ R(x, y) = 3xy + z$$

ikkinchidarajaliko'phaddir.

$$P(x) = c + ax^2 + bx \text{ va } P(x) = ax^2 + bx + c$$

ko'phadlarni qaraylik, ular bitta ko'phadning ikki ko'rinishli yozuvi. Ulardan ikkinchisi *x* o'zgaruvchi daraja ko'rsatkichlarining kamayib borishi tartibida, ya'ni *standart* ko'rinishdagi yozuvdir. Ko'p argumentli ko'phadlar ham standart ko'-rinishda yozilishi mumkin. *x, y, ..., z* ~ o'zgaruvchilar, *a, b* lar noldan farqli sonlar bo'lsin. $ax^{k_1} y^{k_2} \ldots z^{k_n}$ va $bx^{m_1} y^{m_2} \ldots z^{m_n}$birhadlarni solishtiraylik. $k_1 = m_1$, $k_2 = m_2$, ..., $k_i = m_i$,lekin$k_{i+1} > m_{i+1}$ bo'lsa, birinchi birhad ikkinchisidank atta, chunki ulardagi *x* va *y* lar daraja ko'rsatkichlari birxil bo'lsada, *z* ning ko'rsatkichi birinchi bir-hadda katta. Agar ko'p o'zgaruvchili ko'phadda har qaysi qo'shi-luvchi o'zidan o'ngda turgan barcha qo'shiluvchilardan katta bo'lsa, qo'shiluvchilar *lug'aviy (leksikografik)* tartibda joylashtirilgan deyiladi. Masalan,$P(x, y, z) = 8x^5y^6z^2 - 5x^4y^8z + 16x^4y^5z^4$ko'phadning qo'shiluvchilari lug'aviy tartibda joylashtirilgan. Agar ko'phadning barcha hadlarida *x, y,..., z* o'zgaruvchilarning ko'rsatkichlari yig'indisi *m* ga teng bo'lsa, uni *m- darajali birjinsli ko'phad* deyiladi. Masalan, $8x - 5y + z$— birinchi darajali birjinsli (bunda m=l), $x^3 + y^3 + z^3 - 7xy^2 - 5xyz$ — uchinchi darajali *(m* = 3) birjinsli ko'phad. Agar$ax^{k_1} \ldots z^{k_n}$ birhad $m = k_1 + \ldots + k_n$darajali bo'lsa, ixtiyoriy umumiy λ ko'paytuvchi uchun *a(λx)* ga ega bo'lamiz. Agar ixtiyoriy λ soni uchun $f(\lambda x, \ldots, \lambda z) = \lambda^m f(x, \ldots, z)$ tenglik bajarilsa, $f(x, \ldots, z)$ko'phad funksiya) *m*- darajali bir jinsli ko'phad (funksiya) bo'ladi. Masalan, $f(x, y) = =y^3 + x^2 \sqrt{xy + \frac{x^3}{y}}$ ftinksiya 3-darajali birjinsli funksiyadir, chunki

$$f(2x, 2y) = 8y^3 + 4x^2 \cdot \sqrt{4\left(xy + \frac{x^3}{y}\right)} = 2^3 f(x; y).$$

Shukabi, $f(x, y) = x^3 + 2x^2y - y^3 + x^2 \sqrt{xy + \frac{x^3}{y}}$ −uchinchidarajali $(m = 3)$, $f(x, y, z) = \frac{y+z}{3x+y}$ nolinchi darajali$(m = 0)$, $f(x, y, z) = z \cdot \frac{y+z}{3x+y}$ birinchi darajali *(m = 1) birjinslifunksiyalardiт.* Agar$x^3y + xy^3$ko'phadda*x* o'rniga *y, y* o'rniga *x* yozilsa (ya'ni *x* va*y* lar o'rin almashtirilsa), oldingi ko'phadning o'zi hosil bo'ladi. Agar$P(x,y,...,z)$ko'phad tarkibidagi harflarning har qanday o'rin almashtirilishida unga aynan teng ko'phad hosil bo'lsa, *P* ko'phad *simmetrik ko'phad* deyiladi. Simmetrik ko'phadda qo'shiluvchilar o'rina almashtirilganda yig'indi, ko'paytuvchilar o'rin almashtirilganda ko'paytma o'zgarmaydi. Agar $(\lambda + x)(\lambda + y)...(\lambda + z)$ ifodadagi qavslar ochilsa, λ darajalarining koeffitsientlari sifatida$x, y, ..., z$o'zgaruvchilarning simmetrik ko'phadlari turgan bo'ladi. Ular *asosiy simmetrik ko 'phadlar* deyiladi. Masalan, o'zgaruvchilar soni *n - 2* bo'lsa, $(\lambda + x)(\lambda + y) = \lambda^2 + (x + y)\lambda + xy$ bo'lib, asosiy simmetrik ko'phadlar *x + y* va *xy* bo'ladi. Ularni$\sigma_1 = x + y$, $\sigma_2 = xy$ orqali ifodalaymiz. Shu kabi, $n = 3$ da $\sigma_1 = x + y + z$, $\sigma_2 = xy + xz + yz$, $\sigma_3 = xyz$ bo'ladi. Bulardan tashqari, quyidagi ko'rinishdagi$\sigma_1 = x + + y + ... + z$ *(n* ta qo'shiluvchi), $\sigma_2 = x^2 + y^2 + ... + z^2$, ..., $\sigma_k = x^k + y^k + ... + z^k$ *darajali yig'indilar ham* simmetrik ko'phadlardir.

Ko'phadlarni bo'lish. Bir o'zgaruvchili *A(x)* va *B(x)* ko'phadlar uchun

$$A(x) = B(x) \cdot Q(x) \qquad (1)$$

Tenglik o'rinli bo'ladigan *Q(x}* ko'phad mavjud bo'lsa, *A(x) ko'phad B(x) ko'phadga bo'linadi* (yoki qoldiqsiz bo'linadi) deyiladi. Bunda *A(x)* ko'phad *bo'linuvchi, B(x)* ko'phad *bo'luvchi, Q(x)* ko'phad esa *bo'linma* deyiladi.

$x^3 - 1 = (x^2 + x + 1)(x - 1)$ ayniyatdan, $A(x) = x^3 - 1$ ko'phadning $B(x) = x^2 + x + 1$ ko'phadga (qoldiqsiz) bo'linishini va bo'linma $Q(x) = x - 1$ko'phadga tengligini ko'ramiz.

Ayniy almashtirishlarda arifmetik amallarning xossalaridan foydalaniladi Quyidagi ayniyatlar o'rinli:

1) $(AB)^n = A^n B^n$;

2) $A^m A^n = A^{m+n}$;

3) $(A^m)^n = A^{mn}$;

4) $\frac{A}{B} + \frac{C}{D} = \frac{AD+BC}{BD}$, $B \neq 0$, $D \neq 0$;

5) $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$, $B \neq 0$, $D \neq 0$;

6) $\frac{A}{B} : \frac{C}{D} = \frac{AD}{BC}$, $B \neq 0$, $C \neq 0$, $D \neq 0$;

7) $\frac{AC}{BD} = \frac{A}{B}$, $B \neq 0$, $C \neq 0$;

8) $\frac{A^m}{A^n} = \begin{cases} A^{m-n}, & m > n \\ 1, & m = n, \ A \neq 0 \ \text{da}; \end{cases}$

9) $|AB| = |A| \cdot |B|$;

10) $|A^n| = |A|^n$.

Ratsional ifodalarning kanonik shakli qisqarmas $\frac{P(x)}{Q(x)}$ kasrdan iborat bo'ladi. Bu yerda *P(x)* va *Q(x)* lar ko'phadlar bo'lib, $Q(x)$ ko'phadning bosh koeffitsienti esa 1 ga teng.

Miso1. $\frac{16-x^2}{2x^4+9} : \left(\frac{1}{x-3} - \frac{1}{x-3} \cdot \frac{x-3}{2x+1} \right)$ ratsional ifodani kanonik ko'rinishga keltiring.

Yechish. $\frac{1}{x-3} - \frac{1}{x-3} \cdot \frac{x-3}{2x+1} = \frac{x+4}{(x-3)(2x+1)}, \ldots.$

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Mavzu 5: Tenglama va uning turlari. Tenglamalarni yechish tasnifi.

Tenglamalar sistemasi. Tenglamalar sistemasini yechish.

REJA

1.Tenglama. Tenglamalar tasnifi.

2. Teng kuchli tenglamalar.

3. Birinchi va ikkinchi darajali tenglamalar.

4. Qaytma va yuqori darajali tenglamalar.

5. Kasr-ratsional tenglamalar.

Tayanch iboralar: Tenglama, tengsizlik, noma'lum, tenglik, tenglamani yechish, tenglamaning yechimi, chizikli tenglama, kvadrat tenglama, irratsional tenglama, modulli tenglama, kursatkichli tenglama, logarifmik tenglama, trigonometrik tenglama.

Tenglama – matematikaning eng muxim tushunchalaridan biri. Kupgina Ma'ruza va ilmiy masalalarda biror kattalikni bevosita ulchash yoki tayyor formula buyicha xisoblash mumkin bulmasa, bu mikdor qanoatlantiradigan munosabat (yoki bir necha munosabat) tuzishga erishiladi. Noma'lum kattalikni aniqlash uchun tenglama (yoki tenglamalar sistemasi) ana shunday xosil qilinadi.

Matematikaning fan sifatida vujudga kelganidan boshlab uzoq vaqtgacha tenglamalar yechish metodlarini rivojlantirish algebraning asosiy tadqiqot predmeti bo`ldi. Tenglamalarni bizga odat bo'lib qolgan xarfiy yozilishi XVI asrda uzil-kesil shakllandi; noma'lumlarni lotin alifbosining oxirgi x, y, z,… xarflari, ma'lum mikdorlar (parametrlar)ni lotinalif bosining dastlabki *a,b,c,* … harflari orqali belgilash an'anasi frantsuz olimi R. Dekartdan boshlangan.

Matematikaning maktab kursidagi masalalari ichida tenglamalar xakidagi ta'limot eng muxim urin tutadi. Haqiqatdan ham, tenglamalar xaqidagi ta'limot – funktsiyalar xakidagi ta'limotga bog'langandir, u, real voqelikdagi xar xil xodisalarni tasvirlovchi mikdorlar orasidagi bog'lanishlarni va buboglanishlarning ifodalanishlarini tushunib olishda o'quvchilarga yordam beradi.

Tenglamalar yangi sonlar kiritish manbalaridan biridir. Tenglamalar yechish ayniy shakl almashtirishlarning konkret tadbiq etilishini o'quvchilarga kursatishga imkon beradi; tenglamalar konkret mazmundagi masalalarni yechish uchun o'quvchilarga arifmetikadan ko`pincha sodda metodlarni beradi va tipik masalalardan bir qanchasini yechish usullarini umumlashtirishga imkon beradi.

Tenglama deb noma'lum son qatnashgan tenglikka aytiladi. Noma'lumning berilgan tenglamani to'g'ri tenglikka aylantiradigan qiymati tenglamaning ildizi (yechimi) deyiladi. Tenglamani yechish deganda tenglamaning xamma ildizlarini topish yoki ildizlari yo'qligini ko'rsatish tushuniladi.

Maktab matematika kursida chizikli tenglama tushunchasiga ta'rif berilmaydi. Konkret misollar keltirilib, ularni chiziqli tenglamalar deb o'rgatiladi.

Chiziqli tenglamalarni yechish xaqida dastlab 6-sinf matematika kursida, so'ngra 7-sinf algebra kursida tushuncha beriladi. Bunda quyidagi xossalar o'rgatiladi:

1-xossa. Tenglamaning istagan xadi ishorasini qarama-qarshisiga o'zgartirib, uning bir qismidan ikkinchi qismiga o'tkazish mumkin.

2-xossa. Tenglamaning ikkala qismini nolga teng bo'lmagan birxil songa ko'paytirish yoki bo'ish mumkin.

Buxossalaristaganbirnoma'lumlibirinchidarajalitenglamani yechishimkoniniberadi. Buninguchun:

I. Noma'lum katnashgan xadlarni tenglikning chap kismiga, noma'lum katnashmagan xadlarni esa o`ng kismiga o`tkazish lozim.

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Ma'ruza 7: Irratsional tenglama va tenglamalar sistemasi. Irratsional tengsizliklar va tengsizliklar sistemasi

1 - misol. Tenglamalar sistemasi yechilsin:

$$\begin{cases} \sqrt{x}+\sqrt{y}=\frac{5}{6}\sqrt{xy}, \\ x+y=13 \end{cases}$$

Yechish. Sistemadagi $\sqrt{x}+\sqrt{y}=\frac{5}{6}\sqrt{xy}$ tenlamaning har ikki tomonini kvadrat ko'tarib, ayniy almashtirishlarni bajarish orqali ratsional tenglamalar sistemasini hosil qilamiz;

$$x+y+2\sqrt{xy}=\frac{25}{36}xy$$

$x+y=13$ bo'lgani uchun $13+2\sqrt{xy}=\frac{25}{36}xy$ bo'ladi. $25xy-72\sqrt{xy}-468=0.$ Agar $\sqrt{xy}=t$ desak, $25t^2-72t-468=0$ bo'ladi, bundan $t_1=6$ va $t_2=-\frac{78}{25}$ ildizlarni hosil qilamiz. $\sqrt{xy}=6$ bo'lsa, $xy=36$ bo'ladi:

$$\begin{cases} x+y=13 \\ xy=36 \end{cases}$$

bu sistemani yechamiz: $x=13-y$, $(13-y)y=36;$ $y^2-3y=36;$

$$y_{1,2}=\frac{13}{2}\pm\sqrt{\frac{169}{4}-36}=\frac{13}{2}\pm\frac{5}{2}.$$

J: $y_1=9$, $y_2=4$, $x_1=4$, $x_2=9$.

2 - m i s o l. $\begin{cases} x+y-\sqrt{\frac{x+y}{x-y}}=\frac{12}{x-y} \\ xy=15 \end{cases}$ tenglama yechilsin.

Yechish. $\frac{x+y}{x-y}\geq 0$ bo'lsa, ikki hol bo'lishi mumkin:

a) $x>0, y>0, x>y$ u holda $x+y-\sqrt{\frac{x^2+y^2}{(x-y)^2}}-\frac{12}{x-y}=0$

yoki $x^2-y^2-\sqrt{x^2-y^2}-12=0.$ Endi $\sqrt{x^2-y^2}=t$ desak,

$$t^2-t-12=0; \quad t_{1,2}=\frac{1}{2}\pm\sqrt{\frac{1}{4}+12}=\frac{1}{2}\pm\frac{7}{2}$$

$t_1=4$, $t_2=-3$, shuning uchun $\sqrt{x^2-y^2}=4$, bundan $x^2-y^2=16$ bo'ladi. Shuning uchun

$$\begin{cases} x^2+y^2=16, \\ xy=15 \end{cases}$$

ratsional tenglama sistemasi hosil bo'ladi. Bu tenglamani echib, $x=\pm 5$, $y=\pm 3$ yechimlarni hosil qilamiz.

b) $x<0, y<0$ va $x<y$ bo'lsa, tenglama quyidagi ko'rinishda yoziladi:

$$x+y-\frac{\sqrt{x^2-y^2}}{-(x-y)}-\frac{12}{x-y}=0,$$
$$x^2-y^2+\sqrt{x^2-y^2}-12=0,$$
$$\sqrt{x^2-y^2}=t,\quad t^2+t-12=0,$$
$$t_{1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+12}=-\frac{1}{2}\pm\frac{7}{2};\quad t_1=3,\quad t_2=-4.$$
$$\sqrt{x^2-y^2}=3 \text{ yoki } x^2-y^2=9.$$

Natijada

$$\begin{cases} x^2-y^2=9, \\ xy=15. \end{cases}$$

sistemani hosil qilamiz. Bu sistemani echib,

$$x_{1,2}=\pm\sqrt{\frac{\sqrt{981}+9}{2}},\quad y_{1,2}=\pm\sqrt{\frac{\sqrt{981}-9}{2}},$$

yechimlar hosil qilamiz.

3-misol. Tenglamalar sistemasini yeching:

$$\begin{cases} \sqrt{x^2+y^2}+\sqrt{2xy}=8\sqrt{2}, \\ \sqrt{x}+\sqrt{y}=4. \end{cases}$$

Yechish. Sistemadagi ikkinchi tenglamaning har ikkala tomonini kvadratga ko'taramiz: $x + y + 2\sqrt{xy} = 16$ (1) hosil bo'ladi. Sistemadagi birinchi tenglamaning har ikki tomonini $\sqrt{2}$ ga ko'paytiramiz:

$$\sqrt{2(x^2+y^2)}+2\sqrt{xy}=16 \qquad (2)$$

(2) dan (1) ni ayiramiz:

$$\sqrt{2(x^2+y^2)}-(x+y)=0,$$
$$\sqrt{2(x^2+y^2)}=x+y,$$
$$2(x^2+y^2)=x^2+2xy+y^2,$$
$$x^2-2xy+y^2=0,\quad (x-y)^2=0,\quad x=y.$$

Sistemadagi ikkinchi tenglamadagi x o'rniga u ni qo'ysak, $2\sqrt{y}=4$, $\sqrt{y}=2$, bundan $y=4$ bo'ladi. J: $x = y = 4$.

4 - m i s o l. Sistemani yeching:

$$\begin{cases} \sqrt[4]{1+5x}+\sqrt[4]{5-y}=3, \\ 5x-y=11. \end{cases}$$

Yechish: Agar $\sqrt[4]{1+5x}=u$ va $\sqrt[4]{5-y}=v$ desak,

$$\begin{cases} u+v=3, \\ u^4+v^4=17. \end{cases}$$ bo'ladi.

$$u^4+v^4=(u^2+v^2)^2-2u^2v^2=[(u+v)^2-2uv]^2=2u^2v^2=17,$$

$u + v = 3$ bo'lgani uchun $(uv)^2-18uv+32=0$ bundan $uv=2$ va $uv=16$ bo'ladi. Bu yechimlarga ko'ra quyidagi tenglamalar sistemasini hosil qilamiz:

a) $\begin{cases} u+v=3, \\ uv=2. \end{cases}$

Bu sistemani yechsak, quyidagi yechimlar hosil bo'ladi: $u_1=1$, $v=2$, $u_2=2$, $v_2 = 1$.

b) $\begin{cases} u+v=3, \\ uv=16; \end{cases}$

bu tenglamalar sistemasini yechsak, u haqiqiy yechimga ega emas. Bu yechimlarga ko'ra quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \sqrt[4]{1+5x} = 2, \\ \sqrt[4]{5-y} = 1. \end{cases} \Rightarrow \begin{cases} 1+5x = -16, \\ 5-y = 1 \end{cases} \Rightarrow \begin{cases} x_1 = 3, x_2 = 0, \\ y_1 = 4, y_2 = -1. \end{cases}$$

yechimlarni hosil qilamiz.

5-misol. Sistemani yeching:

$$\begin{cases} \dfrac{2xy + y\sqrt{x^2 - y^2}}{14} = \sqrt{\dfrac{x+y}{2}} + \sqrt{\dfrac{x-y}{2}}, \\ \sqrt{\left(\dfrac{x+y}{2}\right)^3} + \sqrt{\left(\dfrac{x-y}{2}\right)^3} = 9, \end{cases}$$

Yechish. $\sqrt{\dfrac{x+y}{2}} = u$, $\sqrt{\dfrac{x-y}{2}} = v$ desak, $\sqrt{x^2 - y^2} = 2uv$, $x = u^2 + v^2$, $y = u^2 - v^2$ bo'ladi, bularga ko'ra $\begin{cases} 2(u^4 - v^4) + 2uv(u^2 - v^2) = 14(u+v), \\ u^3 + v^3 = 9. \end{cases}$ bo'ladi. Sistemadagi birinchi tenglamaning har ikkala tomonini $u+v\neq 0$ ga bo'lamiz. $\begin{cases} u^3 + v^3 = 9 \\ u^3 - v^3 = 7 \end{cases}$ Buni yechsak, $u^3=8$ va $v^2=1$ yechimlar hosil bo'ladi.

U holda $\begin{cases} \sqrt{\dfrac{x+y}{2}} = 2, \\ \sqrt{\dfrac{x-y}{2}} = 1 \end{cases}$ ёки $\begin{cases} x+y=8, \\ x-y=2. \end{cases}$

bundan $x=5$ va $u=3$ yechimlar hosil bo'ladi.

MUSTAQIL YECHISH UCHUN MISOLLAR.

1. Sistemani yeching:

$$\begin{cases} x^2 + x\sqrt[3]{xy^2} = 208 \\ y^2 + y\sqrt[3]{yx^2} = -1053, \end{cases}$$

J: $x_1 = 8$, $y_1 = 27$, $x_2 = -8$, $y_2 = 27$,

$x_3 = 8$, $y_3 = -27$, $x_4 = -8$, $y_4 = -27$.

2. Sistemani yeching:

$$\begin{cases} 8\sqrt{x^2 - y^2} = x + 9y, \\ x^4 + 2x^2y + y^2 + x = 2x^3 + 2xy + y + 506. \end{cases}$$

$$J: x_1 = 5, y_1 = 3;$$

$$x_2 = \frac{25 - 48\sqrt{6}}{29}, \quad y_2 = \frac{21(5 - 48\sqrt{6})}{29^2}.$$

3. Sistemani yeching:

$$\begin{cases} \dfrac{x+\sqrt{x^2 - y^2}}{x-\sqrt{x^2 - y^2}} + \dfrac{x-\sqrt{x^2 - y^2}}{x+\sqrt{x^2 - y^2}} = \dfrac{17}{4}, \\ x(x+y) + \sqrt{x^2 + xy + 4} = 52. \end{cases} \quad j: \begin{matrix} x_1 = 5, & y_1 = 4; \\ x_2 = -5, & y_2 = -4; \\ x_3 = 15, & y_3 = -12; \\ x_4 = -15, & y_4 = 12; \end{matrix}$$

4. Sistemani yeching:

$$\begin{cases} \dfrac{\sqrt{x^2 + y^2} + \sqrt{x^2 - y^2}}{\sqrt{x^2 + y^2} - \sqrt{x^2 - y^2}} = \dfrac{5+\sqrt{7}}{5-\sqrt{7}} \\ x^3 + 2y^3 = 118. \end{cases} \quad \text{j.} \quad \begin{matrix} x_1 = 4, \quad y_1 = 3, \\ x_2 = \sqrt[4]{\dfrac{59}{5}}, \\ y_2 = \sqrt[4]{\dfrac{59}{5}}, \end{matrix}$$

Ma'ruza 8: Ko'rsatkichli va logarifmik tenglama, tenglamalar sistemasi. Ko'rsatkichli va logarifmik tengsizliklar, tengsizliklar sistemasi. Parametr qatnashgan tenglama va tengsizliklar, tenglama va tengsizliklar sistemasi

Ko'rsatkichli tenglama tushunchasini tushuntirishdan oldin o'qituvchi o'quvchilarga daraja, ko'rsatkichli funksiya va ularning xossalari haqidagi ma'lumotlarni takrorlashi, so'ngra ko'rsatkichli funksiyaning ta'rifini berish lozim.

T a ' r i f. *Daraja ko'rsatkichida noma'lum miqdor qatnashgan tenglamalar ko'rsatkichli tenglamalar deyiladi.* Masalan, $3^x=2^{x-1}$, $5^{x^2-6}-1=0{,}7^{x-2}-\sqrt[3]{49}$ va hokazo. $a^x=b$ tenglama maktab matematika kursidagi eng sodda ko'rsatkichli tenglamadir. Bu erda a va b berilgan musbat sonlar bo'lib, $a\neq 1$ $a>0$ bo'lishi kerak. x esa noma'lum miqdordir. $a^x=b$ tenglama bitta yechimga egadir. Har qanday ko'rsatkichli tenglama ayniy almashtirishlarni bajarish orqali algebraik yoki $a^x=b$ ko'rinishdagi sodda holga keltirib yechimlari topiladi. Ko'rsatkichli tenglamalarni yechish darajasini quyidagi xossalariga asoslanadi:

1. Agar o'zaro ikkita teng darajaning asoslari teng bo'lsa, ularning daraja ko'rsatkichlari ham o'zaro teng bo'ladi.

Masalan, agar $a^m = a^n$ bo'lsa, $m = n$ bo'ladi, albatta bu erda $a \neq 0$ va $a \neq 1$, $a>0$ bo'lishi kerak.

2. Agar o'zaro teng darajaning ko'rsatkichlari teng bo'lsa, u holda ularning asoslari ham teng bo'ladi, ya'ni $a^m=b^m$ bo'lsa, u holda $a = b$ bo'ladi. Maktab matematika kursidagi ko'rsatkichli tenglamalar asoslarini tenglash, kvadrat tenglamaga keltirish, logarifmlash, ya'ni o'zgaruvchini kiritish va gruppalash usullari bilan yechiladi. Bu usullarni quyidagi misollar orqali ko'rib chiqaylik.

1-misol. $36^x=\frac{1}{216}$ tenglamani yeching.

Yechish. Bu tenglama asoslarini tenglash yo'li orqali yechiladi:

$$(36^x = 216^{-1}),\ (6^{2x} = 6^{-3}) \Rightarrow (2x = -3) \Rightarrow (x = -\frac{3}{2})$$

Ushbu tenglamani logarifmlash usuli bilan ham yechish mumkin. Logarifm ta'rifiga ko'ra: $x=\log_{36}\left(\frac{1}{216}\right)$ bundan $x=-\log_{36}216=-\frac{1}{2}\log_6 216=-\frac{3}{2}$, chunki $\log_6 216=3$.

2-misol. $5^{2x}-5^x-600=0$ tenglama yechilsin. Bu tenglama yangi o'zgaruvchi kiritish usuli orqali kvadrat tenglamaga keltirib yechiladi. Agar $y=5^x$ desak, berilgan tenglama $y^2-y-600=0$ ko'rinishni oladi.

$$y_{1,2}=\frac{1}{2}\pm\sqrt{\frac{1}{4}+600}=\frac{1}{2}\pm\frac{49}{2};\quad y_1=25,\quad y_2=24$$

$5^x = y$ yoki $5^x=25$, $5^x=5^2$, $x=2$

Javob: $x = 2$

3 - m i s o l. $3^{x^2+1}+3^{x^2-1}=270$ tenglamani yeching.

Yechish. $3^{x^2}\cdot 3+3^{x^2}\cdot\frac{1}{3}=270$, $3^{x^2}=y$ desak, $3y+\frac{1}{3}y=270$ yoki $\frac{10}{3}y=270$, bundan $y=81$, $3^{x^2}=81$ yoki $3^{x^2}=3^4$ bundan $x^2 = 4$ va $x_1 =2$, $x_2 =-2$

4 - m i s o l. $5^{2x}–7^{x}–5^{2x}\cdot 35+7^{x}\cdot 35=0$ tenglamani yeching. Bu tenglama gruppalash usuli bilan yechiladi:

$5^{2x}(1-35)=7^{x}(1-35),\quad 5^{2x}=7^{x},\quad x=0$

5 - m i s o l. $\left(\sqrt{5+2\sqrt{6}}\right)^{x}+\left(\sqrt{5-2\sqrt{6}}\right)^{x}=10$ tenglamani yeching.

Yechish. Bu tenglamani yechishda $\left(\sqrt{5+2\sqrt{6}}\right)^{x}\cdot\left(\sqrt{5-2\sqrt{6}}\right)^{x}=1$ ekanligidan foydalanamiz. Agar $\left(\sqrt{5+2\sqrt{6}}\right)^{x}=y$ desak, u holda $\left(\sqrt{5-2\sqrt{6}}\right)^{x}=\frac{1}{y}$ bo'ladi. bu belgilashlarga ko'ra tenglama quyidagicha ko'rinishni oladi. $y+\frac{1}{y}=10$, bundan $y^2–10y+1=0$ yoki $y_1=5–-2\sqrt{6}$ va $y_2=5+2\sqrt{6}$ ildizlarga ega bo'lamiz.

a) $\left(\sqrt{5+2\sqrt{6}}\right)^{x}=5-2\sqrt{6}$ bo'lsin, u holda

$(5+2\sqrt{6})^{\frac{x}{2}}=\frac{(5+2\sqrt{6})(5-2\sqrt{6})}{5+2\sqrt{6}}=(5+2\sqrt{6})^{-1}$, bundan $\frac{x}{2}=-1,\quad x_1=-2;$

b) $\left(\sqrt{5+2\sqrt{6}}\right)^{x}=5+2\sqrt{6}$ bo'lsin, u holda $(5+2\sqrt{6})^{\frac{x}{2}}=(5+2\sqrt{6})^{1}$, bundan $\frac{x}{2}=1,\ x_1=2;$

Javob: $x=–2$ va $x=2$

6 - m i s o l. $100^{x}=300$ tenglama yechilsin.

Yechish. Tenglikning ikkala tomonini 10 asosga ko'ra logarifmlaymiz. $x\lg100=\lg300.$

Bizga ma'lumki, $\lg100=2$. Bu erda $\lg300=\lg(100\cdot3)=\lg100+\lg3=2+\lg3$ kabi ayniy almashtirishlar bajaramiz. Bu almashtirishlarga ko'ra berilgan tenglama $x\cdot2=2+\lg3$ ko'rinishni oladi. Bundan: $x=\frac{2+\lg3}{2}=1+\frac{\lg3}{2}$

7 - m i s o l. $\left(2^{3x}-\frac{8}{2^{3x}}\right)-6\left(2^{x}-\frac{1}{2^{x-1}}\right)=1$

Yechish. Bu tenglamani quyidagi ko'rinishda yozish mumkin:

$$\left(2^{3x}-\frac{8}{2^{3x}}\right)-6\left(2^{x}-\frac{2}{2^{x}}\right)-1=0$$

$2^{x}-\frac{2}{2^{x}}=y$ deb belgilasak, u holda

$$2^{3x}-\frac{8}{2^{3x}}=\left(2^{x}-\frac{2}{2^{x}}\right)\left(2^{2x}+2+\frac{4}{2^{2x}}\right)=\left(2^{x}-\frac{2}{2^{x}}\right)\left[\left(2^{x}-\frac{2}{2^{x}}\right)^{2}+6\right]=y(y^2+6)$$

bo'ladi. Bu almashtirishlarga ko'ra berilgan tenglama o'zgaruvchi u ga nisbatan quyidagi ko'rinishni oladi: $y(y^2+6)–6y–1=0$ yoki $y^3=1,\ y=1.$ $2^{x}-\frac{2}{2^{x}}=1$, bundan $2^{2x}–2^{x}–2=0$ bo'ladi. Agar $2^{x}=t$ desak, u holda tenglama $t^2–t–2=0$ ko'rinishni oladi. Uning yechimlari $t_1=2,\ t_2=–1$ bo'ladi. U holda $2^{x}=2$ yoki $x=1,\ 2^{x}=–1$ tenglama yechimga ega emas.

Javob: $x=1$

MUSTAQIL YECHISH UCHUN MISOLLAR.

Quyidagi tenglamalarni yeching:

1. $5^{x+1}+3\cdot 5^{x-1}-6\cdot 5^{x}+10=0$ J: 2
2. $\sqrt{27^{x-1}}=\sqrt[3]{9^{2-x}}$ J: $x=\frac{17}{13}$
3. $16\sqrt{(0.25)^{5-\frac{x}{4}}}=2^{\sqrt{x+1}}$ J: $x=24$
4. $4+\frac{2}{3^{x}-1}=\frac{5}{3^{x-1}}$ J: $x=1$
5. $\left(\frac{3}{5}\right)^{x+1}+\left(\frac{3}{5}\right)^{1-x}=1{,}2$ J: $x=0$
6. $7\cdot 2^{x}=5\cdot 3^{x}$ J: $x=\frac{\lg 7-\lg 5}{\lg 3-\lg 2}$
7. $5^{2x}-7^{x}-5^{2x}\cdot 17+7^{x}\cdot 17=0$ J: $x=0$
8. $9^{x}=\left(\frac{1}{243}\right)^{5x}$ J: $x=0$
9. $(0{,}4)^{\lg^{2}x-1}=(6{,}25)^{2-\lg x^{2}}$ J: $x_1=10^{5}$, $x_2=10$
10. $x^{\lg^{2}x+\lg x+3}=\dfrac{2}{\frac{1}{\sqrt{x+1}}-\frac{1}{\sqrt{x+1}+1}}$ J: $x_1=1$, $x_2=\frac{1}{100}$
11. $2\cdot 11^{4(x-1)}-1=121^{x-1}$ J: $x=1$
12. $2\cdot 3^{x+1}-5\cdot 9^{x-2}=81$ J: $x_1=4$, $x_2=4-\frac{\lg 5}{\lg 3}$

9-§. Logarifmik tenglamalar.

Maktab matematika kursida logarifmik tenglamaga ta'rif berib, so'ngra uni yechish usullari ko'rsatiladi.

T a ' r i f. *Noma'lum miqdor logarifm belgisi ostida qatnashgan tenglamalar logarifmik tenglamalar deyiladi.*

Masalan, $lgx=3-lg5$, $lgx=lg2$, $2lg\sqrt{x}=lg(15-2x)$ va hokazo. Logarifmik tenglama ham ko'rsatkichli tenglama singari transsendent tenglama turiga kiradi. $log_a x=b$ tenglama eng sodda logarifmik tenglamadir. Bu erda *a, b* lar ma'lum sonlar, *x* noma'lum sondir. Bu ko'rinishdagi tenglama $x=a^b$ bitta yechimga ega bo'ladi.

Logarifmik tenglamaning yechish jarayonida o'qituvchi o'quvchilarga logarifmik funksiya va uning xossalari haqidagi ma'lumotlarni takrorlab berish lozim. Ayniqsa, o'qituvchi ko'paytmaning $lg(a\cdot b)=lga+lgb$, kasrning $lg\frac{a}{b}=lga-lgb$ va darajaning $lga^n=nlgb$ logarifmlari hamda logarifmlarning bir asosidan boshqa asosiga o'tish $\log_a b=\frac{\log_c b}{\log_c a}$ formulasi va qoidalarini imkoniyat boricha isboti bilan tushuntirib berishi maqsadga muvofiqdir, chunki logarifmik tenglamalarni yechish jarayonida ana shu qoidalardan foydalaniladi. Logarifmik tenglamalarni yechish jarayonida ko'pincha $lgA=lgB$ bo'lsa, A=B bo'ladi degan qoidaga amal qilamiz. Ayrim hollarda o'quvchilar $lgA+lgB=lgC$ tenglikdan ham $A+B=C$ bo'ladi degan noto'g'ri xulosaga keladilar. Mana shunday xatoliklarning oldini olish uchun o'qituvchi yuqoridagi tengliklarni aniq misollar yordamida ko'rsatib berishi lozim. Masalan. $lg5+lg9=lg\ 45$. Bu tenglikdan yuqoridagi xato mulohazaga ko'ra $5+9=\ 45$ bo'lishi kerak, bunda $14\neq45$. Bundan ko'rinadiki, $lgA+lgB=lgC$ dan $A+B=C$ deb yozish katta xatolikka olib kelar ekan. Demak, $lgA+lgB=lgC$ bo'lsa, ikki son ko'paytmasining logarifmi qoidasiga ko'ra $lg(A\cdot B)=lgC$ bo'ladi, bundan $A\cdot B=C$ ekanligi ko'rsatish kifoya. $lg5+lg9=lg45$, $lg(5\cdot9)=lg45$. $45=45$. $log_a f(x)=log_a g(x)$ tenglamani yechish uchun $f(x)=g(x)$ tenglamani yechish kerak va topilgan yechimlar ichidan $f(x)>0$, $g(x)>0$ tengsizliklarni qanoatlantiradiganlarini tanlab olinadi. $f(x)=g(x)$ tenglamaning qolgan ildizlari esa $log_a f(x)=log_a g(x)$ tenglama uchun chet ildiz bo'ladi. Har qanday logarifmik tenglama ayniy almashtirishlar yordamida uni $log_a f(x)=log_a g(x)$ ko'rinishga keltirib, $f(x)=g(x)$ tenglamani yechish orqali va yangi o'zgaruvchi kiritish orqali yechiladi. Logarifmik tenglamalarni yechishni uning aniqlanish sohasini topishdan boshlash lozim.

1 - m i s o l. $log_a x=b$ tenglama yechilsin.

Yechish. Agar $a>0$ va $a\neq1$ bo'lsa, $x=a^b$ bo'ladi.

2 - m i s o l. $\frac{\lg 2x}{\lg(4x-15)}=2$ tenglama yechilsin.

Yechish. $lg2x$ ning aniqlanish sohasi $x>0$ bo'ladi. $lg(4x-15)$ ning aniqlanish sohasi $4x-15>0$, bundan $x>\frac{15}{4}$ bo'ladi. Bundan tashqari $4x-15\neq0$ yoki $x\neq4$

bo'lishi kerak, bularga asoslanib tenglamaning aniqlanish sohasi $x>3\frac{3}{4}$ va $x\neq 4$ bo'ladi. Tenglamani yechish uchun quyidagicha ayniy almashtirish bajaramiz: $lg2x=21g(4x-15)$, $lg2x=lg(4x-15)^2$; $2x=16x^2-120x+225$ yoki $16x^2-122x+225=0$, bundan $x_1=\frac{72}{15}=\frac{9}{2}=4\frac{1}{2}$ yechim tenglamaning aniqlanish sohasida yotadi, shuning uchun $x_1=4\frac{1}{2}$ yechim bo'ladi.

3 - m i s o l. $log_2(lgx+2\sqrt{\lg x}+1)-log_2(\sqrt{\lg x}+1)=1$ tenglama yechilsin.

Yechish. Bu tenglamadagi o'zgaruvchining qabul qiladigan qiymatlari sohasi $x\geq 1$ bo'ladi. Berilgan tenglamani potensirlasak, $\frac{\lg x+2\sqrt{\lg x}+1}{\sqrt{\lg x}+1}=2$ yoki $\sqrt{\lg x}+1=2$, $\sqrt{\lg x}=1$ bundan $x=10$.

4 - m i s o l. $x^{1+lgx}=100$ tenglamani yeching.

Yechish. Bu tenglamadagi noma'lumning qabul qiladigan qiymatlar sohasi $x>0$ dir. Tenglikning har ikkala tomonini 10 asosga ko'ra logarifmlaymiz: $lgx\cdot(1+lgx)=lg100$

Agar $lgx=t$ desak, $lg100=2$ bo'ladi. U holda $(1+t)t=2$ yoki $t^2+t-2=0$, bundan $t_1=1$, $t_2=-2$. $lgx=1$, bundan $x=10$, $lgx=-2$, bundan $x=\frac{1}{100}$ Javob. $x_1=10$, $x_2=\frac{1}{100}$

5-misol. $\lg\sqrt{5x-4}+\lg\sqrt{x+1}=2+\lg 0{,}18$ tenglama yechilsin

Yechish. Bu tenglamaning aniqlanish sohasi $5x-4>0$ va $x+1>0$ bo'lishi kerak, bundan $x>\frac{4}{5}$ bo'ladi. Tenglamani potensirlasak: $\sqrt{5x-4}\cdot\sqrt{x+1}=100\cdot 0{,}18$ yoki $\sqrt{5x-4}\cdot\sqrt{x+1}=18$. Bunda $5x^2+x-328=0$, bundan $x_1=-\frac{41}{5}$ va $x_2=8$, $x_1=-\frac{41}{5}$ bo'lgani uchun yechim bo'lolmaydi. Javob. $x=8$.

6 - m i s o l. $\frac{1}{12}\lg^2 x=\frac{1}{3}-\frac{1}{4}\lg x$ tenglamani yeching.

Echish. Bu tenglamaning aniqlanish sohasi $x>0$. Agar $lgx=y$ desak, $\frac{1}{12}y^2=\frac{1}{3}-\frac{1}{4}y$, $y^2+3y-4=0$, bundan $y_1=1$ va $y_2=-4$, u holda $lgx=1$ yoki $x=10$. $lgx=-4$ yoki $x=\frac{1}{10^4}$. J a v o b. $x_1=10$, $x_2=\frac{1}{10^4}$

7 - m i s o l. $log_5x + log_x5 = 2{,}5$

Yechish. Tenglamaning aniqlanish sohasi $x>0$ va $x\neq 1$. Bu tenglamada logarifm asoslarini bir xilga keltirish kerak. Buning uchun $log_ab=\frac{\log_c b}{\log_c a}$ formuladan foydalanamiz:

$$log_x5=\frac{\log_5 5}{\log_5 x}=\frac{1}{\log_5 x}$$

Bu almashtirishlarga ko'ra tenglama quyidagi ko'rinishni oladi: $\log_5 x + \frac{1}{\log_5 x} = 2{,}5$, agar $log_5x=y$ desak, $y + \frac{1}{y} = 2{,}5$ yoki $y^2-2{,}5y+1=0$. Uni yechsak, $y_1=2$ va $y_2=\frac{1}{2}$. Bularga ko'ra $log_5x=2$, bundan $x=25$ va $log_5x=\frac{1}{2}$, bundan $x = \sqrt{5}$.

Javob: $x_1 = 25,\ x_2 = \sqrt{5}$

MUSTAQIL YECHISH UCHUN MISOLLAR.

Quyidagi tenglamalarni yeching:

1. $lgx=3 - lg5$ J: $x = 200$
2. $100^{lg(x+20)} = 10000$ J: $x = 80$
3. $lg(0{,}5 + x) = lg2 - lgx$ J: $x = \frac{\sqrt{33}-1}{4}$
4. $lg(x+6) - 2 = \frac{1}{2} lg(2x-3) - lg25$ J: $x_1 = 14,\ x_2 = 6$
5. $\frac{1}{5-4\lg x} + \frac{1}{1+\lg x} = 1$ J: $x = 100,\ x = 1000$
6. $\frac{1}{5-4\lg(x+1)} + \frac{4}{1+\lg(x+1)} = 3$ J: $x = 9,\ x = \sqrt{1041}$
7. $x^x = x$ J: $x = 1,\ x = -1$
8. $x^{lgx+2} = 1000$ J: $x = \frac{1}{1000},\ x = 10$
9. $x = 10^{1-0.25lgx}$ J: $\sqrt[5]{10000}$
10. $log_3x + log_5x = log_x15$ J: $x = 5$
11. $log_{16}x + log_4x + log_2x = 7$ J: $x = 16$
12. $log_{3x}3 = log_3(3x)^2$ J: $x = 1$
13. $\log_{\sqrt{\frac{1}{16}}} x = 4$ J: $x=\frac{1}{256}$
14. $\log_{\sqrt{x-2}}(2\sqrt{x}+6) = 2$ J: $x = 16$
15. $\log_3\left(3 + \sqrt{3+x)}\right) = \frac{2}{\log_x 3}$ J: $x = \frac{1+\sqrt{3}}{2}$
16. $\sqrt{\log_x \sqrt{2x}} \cdot \log_2 x = -1$ J: $x = \frac{1}{4}$
17. $\frac{\lg(x+4)-\lg(x-3)}{\lg 200 - \lg 25} = 1$ J: $x = 4$
18. $log_bx + \log_{b^2} x + \log_{b^4} x = 1{,}75$ J: $x = b$
19. $\frac{\lg(2x+5)-\lg x}{2+\lg 100} = \frac{1}{4}$ J: $x = \frac{5}{8}$
20. $\log_3\{1+\log_2[1+\log_4(1+\log_{\frac{1}{2}} x)]\} = 0$ J: $x = 1$
21. $log_4x + log_x4 = 2$ J: $x = 4$

22. $\log_{\sqrt[4]{x}} x + \log_3 x - \log_{\frac{1}{3}} x = 8$ J: 9

23. $\log_7[x + \log_2(9 - 2^x) + 4] = 1$ J: $x = 0$

24. $\log_7 \log_4 \log_3^2 (x-7) = 0$ J: $x = 7\frac{1}{9}$, $x=16$

25. $\log_3 x + 6\log_x 3 = 5$ J: $x=9$, $x=27$

26. $\lg x + \lg(x+3) = \lg 2 + \lg(9-2\sqrt{x^2+3x-6})$ J: $x = 2$

10-§. Parametrli logarifmik va ko'rsatkichli tenglamalarni yechish.

Parametrli logarifmik va ko'rsatkichli tenglamalarni yechish parametrsiz shunday tenglamalardan ana shu parametrni qanoatlantiruvchi tenglama yechimini uning yo'l qo'yiladigan qiymatlari ichidan izlash bilan farq qiladi.

1-m i s o l. $\log_a(a+\sqrt{a+x}) = \dfrac{2}{\log_x a}$ tenglama yechilsin.

Yechish. Bu tenglamani yechish uchun avvalo uning paramet-rini qanoatlantiruvchi yo'l qo'yiladigan qiymatlar sohanini topamiz:

$x>0,\ x\neq 1,\ a>0,\ a\neq 1.\ \ \log_a(a+\sqrt{a+x})=\log_a x^2$

Potensirlash qoidasiga ko'ra $a+\sqrt{a+x}=x^2$ $\sqrt{a+x}=x^2-a,$ bu erda $x^2>a$ tenglikning har ikki tomonini kvadratga ko'tarsak, $a+x=x^4-2ax^2+a^2,$ $a^2-(2x^2+1)a+(x^4-x)=0$ bu tenglamani yechsak, $a_{1,2}=\dfrac{2x^2+1\pm(2x+1)}{2}$ hosil bo'ladi: $a_1=x^2+x+1$ va $a^2=x^2-x.$ $a_1=x^2+x+1$ tenglamaning yechimi yo'l qo'yiladigan qiymatlar sohasida yotmaydi, $x^2-a>0,\ x>0$ bo'lgani uchun $a=x^2-x$ tenglamani yechamiz: $x^2-x-a=0$, bundan

$$x_{1,2}=\frac{1}{2}\pm\sqrt{\frac{1}{4}+a}=\frac{1}{2}\pm\sqrt{\frac{1+4a}{4}}=\frac{1\pm\sqrt{4a+1}}{2}$$

Bulardan: $x_1=\dfrac{1+\sqrt{4a+1}}{2}$, $x_2=\dfrac{1-\sqrt{4a+1}}{2}$. Bu yechimlardan $x_1=\dfrac{1+\sqrt{4a+1}}{2}$ tenglamaning yo'l qo'yiladigan qiymatlar sohasida yotadi, shuning uchun u yechim bo'ladi.

Bu berilgan tenglamaning logarifm xossalari va potensirlashga ko'ra $a=\sqrt{a+x}=x^2$ ko'rinishda yozib olamiz. Bu tenglamaning har ikki tomoniga x ni qo'shamiz.

$$a+x+\sqrt{a+x}=x^2+x$$

agar $\sqrt{a+x}=b$ desak, $b^2+b=x^2+x$ hosil bo'ladi. Bundan

$$(x^2-b^2)+(x-b)=0,$$
$$(x-b)(x+b)+(x-b)=0,$$
$$(x-b)(x+b+1)=0;$$

$x+b+1\neq 0$ bo'lgani uchun $x-b=0$ bo'ladi, b ning o'rniga $\sqrt{a+x}$ ni qo'ysak, $x-\sqrt{a+x}=0$ yoki $x^2-x-a=0$ bo'ladi. Biz bu tenglamani yechishni yuqorida ko'rib o'tdik.

2 - m i s o l. $a^{\frac{x}{2}}+b^{\frac{x}{2}}=m(ab)^{\frac{1}{x}}$ tenglama yechilsin.

Yechish. Bu tenglamadagi o'zgaruvchining yo'l qo'yiladigan qiymati $x\neq 0$

a) $a\cdot b>0$ bo'lsin, u holda tenglamaning ikkala tomonidagi ifodalarni $(a\cdot b)^{\frac{1}{x}}$ ga bo'lamiz.

$$\left(\frac{a}{b}\right)^{\frac{1}{x}}+\left(\frac{b}{a}\right)^{\frac{1}{x}}=m,\qquad (m>0).$$

Agar $\left(\frac{a}{b}\right)^{\frac{1}{x}}=t$ desak, $t+\frac{1}{t}=m$, bundan $t^2-tm+1=0$ bo'ladi. Bu tenglamani yechamiz:

$$t_{1,2}=\frac{m\pm\sqrt{m^2-4}}{2}, \quad \left(\frac{a}{b}\right)^{\frac{1}{x}}=\frac{m\pm\sqrt{m^2-4}}{2},. \qquad (1)$$

Bu erda $m\geq 2$ bo'ladi.

a) $m>2$ bo'lsin, bu holda (1) ning har ikki tomonini 10 asosga ko'ra logarifmlaymiz:

$$\frac{1}{x}\lg\frac{a}{b}=\lg(m\pm\sqrt{m^2-4})-\lg 2,$$

$$x=\frac{\lg a-\lg b}{\lg(m\pm\sqrt{m^2-4})-\lg 2}, \qquad (a\neq b).$$

b) $m=2$ bo'lsin, u holda (1) quyidagi ko'rinishni oladi:

bundan: $\left(\frac{a}{b}\right)^{\frac{1}{x}}=1, \quad a^{\frac{1}{x}}=b^{\frac{1}{x}}$; $a=b\neq 0$ bo'lishi kerak.

2) $a\cdot b=0$ bo'lsin.

a) $a=b=0$ bo'lsa, berilgan tenglamaning yechimi bo'lgan barcha sonlar.

b) $a=0,\ b\neq 0$ yoki $a\neq 0,\ b=0$ bo'lsa, tenglama yechimga ega emas.

J: 1) Agar $m>2,\ a\neq 0,\ b\neq 0,\ a\neq b$ bo'lsa,

$$x=\frac{\lg a-\lg b}{\lg(m\pm\sqrt{m^2-4})-\lg 2}$$

2) Agar a) $m=2,\ a=b\neq 0$ bo'lsa, x – ixtiyoriy son.

b) $a=b=0,\ x\neq 0$ – ixtiyoriy son

3-m i s o l. $\left(\frac{1+a^2}{2a}\right)^x+\left(\frac{1-a^2}{2a}\right)^x=1$ tenglama yechilsin.

Yechish. Bu tenglamadagi a parametrning yo'l qo'yiladigan qiymatlari sohasi $0<a<1$ bo'ladi. Tenglamaning har ikki tomonini $\left(\frac{1+a^2}{2a}\right)^x\neq 0$ ga bo'lamiz:

$$\left(\frac{1-a^2}{1+a^2}\right)^x+\left(\frac{2a}{1+a^2}\right)^x=1, \quad \frac{2a}{1+a^2}=\sin z, \quad \frac{1-a^2}{1+a^2}=\cos z,$$

$$(\cos z)^x+(\sin z)^x=1. \qquad (1)$$

(1) tenglikning chap tomonida turgan ifodaning yo'l qo'yiladigan qiymatlar sohasi $0<z<\frac{\pi}{2}$ bo'ladi, bu oraliqda $f(x)=(\sin z)^x+(\cos z)^x$ funksiya monoton kamayuvchidir. $x=2$ da $f(x)=1$ bo'ladi, shuning uchun $x=2$ bu tenglamaning yechimi bo'ladi.

4-misol. $a^x-\frac{a^{2x}-4a^x+4}{\sqrt{a^{2x}-4a^x+4}}=1$ tenglama yechilsin.

Yechish: Bu tenglama ma'noga ega bo'lishi uchun $\sqrt{a^{2x}-4a^x+4}=\sqrt{(a^x-2)^2}=|a^x-2|\neq 0$ bo'lishi kerak.

$$a^x - \frac{(a^x-2)^2}{|a^x-2|} = 1; \qquad a^x - a^x - 2 = 1$$

A) agar $a^x \geq 2$ bo'lsa, $a^x - a^x + 2 = 1$ tenglama yechimga ega emas.

B) agar $0 < a^x < 2$ bo'lsa, $2a^x = 3$; $x = \log_a \frac{3}{2}$ yechim hosil bo'ladi.

5-misol. $2\log_x^2 b - 3\log_x bx^2 + 14\log_{b^2x^2} bx = 0$ tenglama yechilsin.

Yechish. $x > 0,\ x \neq \frac{1}{b^2}$ va $b > 0$, agar $\log_x b = y$ desak, $2y^2 - 3y + 1 = 0$,

$$y_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4}; \quad y_1 = 1,\ y_2 = \frac{1}{2},\ \log_x b = 1$$

bundan $x_1 = b$, $\log_x b = \frac{1}{2}$ bundan $x_2 = \sqrt{b}$

6-misol. $\frac{2}{a}\lg x = 1 + \frac{a}{\lg x}$, $a \neq 0$ tenglama yechilsin.

Yechish.

$$\frac{2}{a}\lg^2 x = \lg x + a; \quad \frac{2}{a}\lg^2 x - \lg x - a = 0;$$

$$\lg x = y; \quad 2y^2 - ay - a^2 = 0,$$

$$y_{1,2} = \frac{a \pm \sqrt{a^2 + 8a^2}}{4} = \frac{a \pm 3a}{4},$$

$$y_1 = a, \quad y_2 = -\frac{a}{2};$$

$$\lg x = a, \quad x = 10^a; \quad \lg x = -\frac{a}{2}, \quad x = 10^{-\frac{a}{2}}.$$

MUSTAQIL YECHISH UCHUN MISOLLAR.

1-misol. $2\log_x a + \log_{ax} a + 3\log_{a^2x} a \neq 0$, $a \neq 1$, $a > 0$ tenglama yechilsin.

$$j: \quad x_1 = \frac{1}{\sqrt{a}}, \quad x_2 = \frac{1}{a\sqrt[3]{a}}.$$

2-misol. $\frac{\log_3 4 - 2}{\log_3(x+2)} = \frac{\log_a(5-x)}{\log_a(x+2)} - 1$, $a > 0$, $a \neq 1$ tenglama yechilsin.

$$j: \quad x = 2\frac{11}{13}.$$

3-misol. $\frac{\lg(4+a-x)}{\lg x} = 1 + \frac{\log_a 4 - 2}{\log_a x}$, $a > 0$, $a \neq 1$ tenglama yechilsin.

$$j: \quad x = \frac{a^2(a+4)}{a^2+4}.$$

14-§. Ko'rsatkichli va logarifmik tenglamalar sistemasini yechish.

1-misol. $\begin{cases} \sqrt[x]{x+y} = 5 \\ (x+y)\cdot 2^x = 100 \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish. Sistemadagi birinchi tenglamaning har ikki tomonini x darajaga ko'taramiz:

$$\begin{cases} x+y=5^x, \\ (x+y)\cdot 2^x=100. \end{cases}$$

$$(5^x\cdot 2^x=100)\Rightarrow(10^x=10^2)\Rightarrow(x=2).$$

$$(2+y=5^2)\Rightarrow y=25-2=23. \qquad Ж: x=2, \quad y=23$$

2-misol. $\begin{cases} x^y=243 \\ \sqrt[y]{1024}=\left(\frac{2}{3}x\right)^2 \end{cases}$ tenglamalar sistemasi yechilsin.

Echish. Sistemadagi ikkinchi tenglamaning har ikki tomonini y darajaga ko'taramiz.

$$\begin{cases} x^y=243, \\ 1024=\left(\frac{2}{3}x\right)^{2y} \end{cases}\Rightarrow\begin{cases} x^y=243, \\ 1024=\left(\frac{2}{3}\right)^{2y}\cdot\left(x^y\right)^2 \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} x^y=243, \\ 1024=\left(\frac{2}{3}\right)^{2y}\cdot(243)^2 \end{cases}\Rightarrow\begin{cases} x^y=243, \\ 2^{10}=\left(\frac{2}{3}\right)^{2y}\cdot(3^5)^2 \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} x^y=243, \\ \left(\frac{2}{3}\right)^{10}=\left(\frac{2}{3}\right)^{2y} \end{cases}\Rightarrow\begin{cases} x^y=243, \\ 2y=10 \end{cases}\Rightarrow\begin{cases} x^y=243, \\ y=5 \end{cases}\Rightarrow\begin{cases} x=3, \\ y=5. \end{cases}$$

3 - m i s o l. $\begin{cases} y^{\frac{x}{y}}=x, \\ y^3=x^2 \end{cases}$ tenglamalar sistemasi yechilsin.

Sistemadagi birinchi tenglamaning har ikki tomonini *2y* darajaga, ikkinchi tenglamani esa y darajaga ko'taramiz:

$\begin{cases} y^{2x}=x^{2y} \\ y^{3y}=x^{2y} \end{cases}\Rightarrow y^{2x}=y^{3y}; \quad y\neq 1$ былса $2x=3y, \quad x=\frac{3}{2}y,$ x ning bu topilgan qiymatini sistemadagi ikkinchi tenglamaga qo'yamiz:

$$\left(y^3=\frac{9}{4}y^2\right)\Rightarrow\left(y^3-\frac{9}{4}y^2=0\right)\Rightarrow y^2\left(y-\frac{9}{4}\right)=0;$$

$$y_{1,2}=0, \quad y_3=\frac{9}{4}, \quad x_{1,2}=0, \quad x_3=\frac{27}{8}, \quad y_3=\frac{9}{4},$$

4-misol. $\begin{cases} 9\cdot 5^x+7\cdot 2^{x+y}=457, \\ 6\cdot 5^x-14\cdot 2^{x+y}=-890 \end{cases}$ sistemani yeching.

Yechish. $5^x=a$, $2^{x+y}=b$ desak,

$$\begin{cases} 9a+7b=457, \\ 6\cdot a-14b=-890 \end{cases}\Rightarrow\begin{cases} a=1 \\ b=64, \end{cases}$$

$$(5^x=1)=(5^x=5^0)=(x=0); \quad 2^y=64=2^6; \quad y=6.$$

J: $x=0$, $y=6$.

5-misol. $\begin{cases} \sqrt[x-y]{x+y}=2\sqrt{3} \\ (x+y)\cdot 2^{y-x}=3 \end{cases}$ sistemani yeching.

Yechish. $\begin{cases} x+y=2^{x-y}\cdot 3^{\frac{x-y}{2}} \\ x+y=3\cdot 2^{x-y} \end{cases} \Rightarrow (2^{x-y}\cdot 3^{\frac{x-y}{2}}=3\cdot 2^{x-y}) \Rightarrow (3^{\frac{x-y}{2}}=3) \Rightarrow$

$$\Rightarrow \left(\frac{x-y}{2}=1\right) \Rightarrow [(x-y)=2] \Rightarrow (x=2+y).$$

$$\begin{cases} x+y=3\cdot 2^2=12, \\ x-y=2 \end{cases} \Rightarrow (2x=14) \Rightarrow (x=7);$$

$$\begin{cases} y=x-2 \Rightarrow y=5. \\ x=7. \end{cases}$$

J: $x=7$, $y=5$.

6-misol. $\begin{cases} \left(\sqrt{3}\right)^{x-y}=\left(\frac{1}{3}\right)^{x-2y}, \\ \log_2(x+y)+\log_2(x-y)=4 \end{cases}$ sistemani yeching.

Yechish.

$$\begin{cases} 3^{\frac{x-y}{2}}=3^{2y-x} \\ \log_2(x+y)(x-y)=4 \end{cases} \Rightarrow \begin{cases} \frac{x-y}{2}=2y-x, \\ (x+y)(x-y)=16 \end{cases} \Rightarrow \begin{cases} x-y=4y-2x, \\ x^2-y^2=16 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 3x=5y \\ x^2-y^2=16 \end{cases} \Rightarrow \begin{cases} x=\frac{5}{3}y, \\ x^2-y^2=16 \end{cases} \Rightarrow \begin{cases} x=\frac{5}{3}y \\ \frac{25}{9}y^2-y^2=16 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x=\frac{5}{3}y \\ 16y^2=144 \end{cases} \Rightarrow \begin{cases} x=\frac{5}{3}y \\ y^2=9 \end{cases} \Rightarrow \begin{cases} x=\frac{5}{3}y \\ y_{1,2}=\pm 3. \end{cases}$$

j: $x_1=5;\quad y_1=3;\quad x_2=-5;\quad y_2=-3.$

7-misol. $\begin{cases} 7\cdot 3^{x+1}-6\cdot 3^{y+z-x+1}=9, \\ 2\cdot 3^{x+1}+3^{y+z-x+1}=27, \\ \lg(x+y+z)-3\lg x=\lg yz+\lg 2 \end{cases}$ sistemani yeching.

Yechish. Agar $3^{x+1}=y$, $3^{y+z-x+1}=v$ desak, berilgan sistemadagi birinchi ikki tenglama quyidagi ko'rnishni oladi:

$$\begin{cases} 7u-6v=9, \\ 2u+v=27 \end{cases}$$

Bu sistemani yechsak, $y=9$, $v=9$ yechimlarga ega bo'lamiz. Bu yechimlarga ko'ra $x=1$, $y+z-x=2$ yoki $y+z=3$ tengliklarni hosil qilamiz. Bu topilganlarga ko'ra sistemadagi uchinchi tenglama quyidagi ko'rinishni oladi:

$$\lg(3+1)-3\lg 1=\lg yz+\lg 2,$$

$$\lg 4-\lg 2=\lg yz,$$

$$\lg 2=\lg yz; \qquad 2=yz$$

Bularga ko'ra quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} y+z=3, \\ yz=2 \end{cases} \quad \text{bundan} \quad \begin{cases} y_1=1, \quad y_2=2, \\ z_1=2, \quad z_2=1, \end{cases}$$

$$\text{j}: \quad \begin{cases} x_1=1, \quad y_1=1, \quad z_1=2, \\ x_2=1, \quad y_2=2, \quad z_2=1, \end{cases}$$

8-misol. $\begin{cases} \lg(x^2+y^2)=\lg 13+1, \\ \lg(x+y)-\lg(x-y)=3\lg 2 \end{cases}$ sistemani yeching.

Yechish. Bu tenglamalar sistemasidagi o'zgaruvchining yo'l quyiladigan qiymatlar sohasi $x+y>0$ va $x–y>0$ bo'ladi.

Potensirlash qoidasiga ko'ra tenglama sistemasi quyidagi ko'rinishni oladi;

$$\begin{cases} x^2+y^2=130 \\ \dfrac{x+y}{x-y}=8. \end{cases}$$

Sistemadagi ikkinchi tenglamadan y ni topamiz:

$x+y=8x–8y$ yoki $9y=7x$, $y=\dfrac{7}{9}x.$

Bu qiymatni birinchi tenglamaga qo'yamiz:

$$x^2+\frac{49}{81}x^2=130,$$

$$81x^2+49x^2=130\cdot 81,$$

$$130x^2=130\cdot 81, \quad x^2=81, \quad x_{1,2}=\pm 9;$$

$$y=\frac{7}{9}\cdot 9=7$$

$$j: x=9, \quad y=7.$$

9-misol. $\begin{cases} \lg(x-y)-2\lg 2=1-\lg(x+y) \\ \lg x-\lg 3=\lg 7-\lg y \end{cases}$ sistemani yeching.

Yechish. Tenglamalar sistemasidagi noma'lumlarning yo'l quyiladigan qiymatlar sohasi $x>0, y>0, x>y$.

$$\begin{cases} \lg(x-y)+\lg(x+y)=\lg 4+\lg 10, \\ \lg x+\lg y=\lg 7+\lg 3. \end{cases} \Rightarrow \begin{cases} (x+y)(x-y)=40 \\ xy=21 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2-y^2=40, \\ xy=21. \end{cases}$$

Bu sistemani yechsak. Quyidagi yechimni hosil qilamiz: $x=7$, $y=3$.

10-misol. $\begin{cases} \log_2 \dfrac{x^2\sqrt{y+1}}{2}=2, \\ \log_8 x\log_2(y+1)^2=\dfrac{4}{3} \end{cases}$ sitemani yeching.

Yechish. $x>0$, $y>0$ bo'lishi kerak. Ikki son ko'paytmasining logarifmi qoidasiga ko'ra sistemani quyidagicha yozish mumkin:

$$\begin{cases} 2\log_2 x+\log_2 \dfrac{\sqrt{y+1}}{2}=2, \\ \dfrac{1}{3}\log_2 x\cdot 2\log_2(y+1)=\dfrac{4}{3} \end{cases} \Rightarrow \begin{cases} 2\log_2 x+\dfrac{1}{2}\log_2(y+1)-\log_2 2=2, \\ \dfrac{1}{3}\log_2 x\cdot 2\log_2(y+1)=\dfrac{4}{3} \end{cases}$$

Agar $log_2x=u$, $log_2(y+1)=v$ deb belgilasak, u holda tenglama sistemasi quyidagi ko’rinishni oladi:

$$\begin{cases} 2u+\frac{1}{2}v=3, \\ uv=2 \end{cases} \Rightarrow \begin{cases} 4u+v=6, \\ uv=2 \end{cases} \Rightarrow \begin{cases} u_1=1, \\ v_1=2 \end{cases} \text{ ва } \begin{cases} u_2=\frac{1}{2}, \\ v_2=4 \end{cases}$$

1) $(\log_2 x=1)\Rightarrow(x=2).$

2) $[\log_2(y+1)=2]\Rightarrow(y+1=4)\Rightarrow(y=3).$

3) $\left(\log_2=\frac{1}{2}\right)\Rightarrow(x=\sqrt{2}).$

4) $[\log_2(y+1)=4]\Rightarrow(y+1=16)\Rightarrow(y=15).$

$$j:\begin{cases} x_1=2, \quad y_1=3, \\ x_2=\sqrt{2} \quad y_2=15, \end{cases}$$

11-misol. $\begin{cases} \lg^2 x+\lg^2 y=5\lg^2 a^2 \\ xy=a^2 \end{cases}$ sistemani yyeching.

Yechish. $x>0$, $y>0$ va $a>0$. Sistemadagi ikkinchi tenglamaning quyidagicha yozib olamiz: lg$x$+lg$y$=2lg[a].

Agar lg$x$=u va lg$y$=v desak, sistemani quyidagi ko’rinishni oladi:

$$\begin{cases} u^2+v^2=10\lg^2 a \\ u+v=2\lg a \end{cases}$$

Sistemadagi ikkinchi tenglamaga ko’ra quyidagi tengliklarni yoza olamiz:

$$u=3\lg|a|=\lg x, \quad v=-\lg|a|=\lg y.$$

$$j: x_1=|a|^3, \quad y_1=\frac{1}{|a|}, \quad x_2=\frac{1}{|a|} \quad y_2=|a|^3.$$

Bu tengliklardan $x=|a|^3$, $y=\left|\frac{1}{a}\right|$ bo’ladi.

12-misol. $\begin{cases} \log_2 x+\log_4 y+\log_4 z=2, \\ \log_9 x+\log_3 y+\log_9 z=2, \\ \log_{16} x+\log_{16} y+\log_4 z=2 \end{cases}$ sistemani yeching.

Yechish. $x>0$, $y>0$, $z>0$ bo’lishi kerak. Berilgan sistemaga bir asosdan boshqa asosga o’tish va potensirlash qoidalarini qo’llash orqali uni quyidagicha yoza olamiz:

$$\begin{cases} x^2yz=16, \\ xy^2z=81, \\ xyz^2=256. \end{cases}$$

Bu hosil qilingan sistemaning chap va o’ng tomonlarini o’zaro ko’paytirsak, $(xyz)^4=24^4$ yoki $xyz=24$ hosil bo’ladi. Bunga ko’ra $x=\frac{2}{3}$, $y=\frac{27}{8}$, $z=\frac{32}{3}$ yechimlarni qosil qilamiz.

13-misol. $\begin{cases} \log_3 x(1+\log_x y) = 4, \\ 3\left(2\log_{y^4} x^2 - \frac{1}{2}\log_{\frac{1}{x^2}} y^4\right) = 10 \end{cases}$ sistemani yeching.

Yechish. Berilgan tenglamadagi no'malumlarning qabul qilidigan qiymatlari to'plami $x>0$ va $y>0$ bo'lishi kerak.
Sistemadagi birinchi tenglamani quyidagicha yozish mumkin:

$$\log_3 x = \frac{\log_x x}{\log_x 3} = \frac{1}{\log_x 3}, \quad 1+\log_x y = \frac{4}{\log_3 x}$$

$$1+\log_x y = 4\cdot\log_x 3 \quad \text{ёки} \quad \log_x x + \log_x y = 4\log_x 3,$$

$$\log_x(xy) = \log_x 3^4, \quad xy = 3^4$$

Sistemadagi ikkinchi tenglamada quyidagicha $\log_{y^4} x^2 = z$ belgilashni kiritamiz:

$$6z + \frac{3}{2z} = 10 \quad \text{ёки} \quad 12z^2 - 20z + 3 = 0,$$

$$z_1 = \frac{3}{2}, \quad z_2 = \frac{1}{6}.$$

$$a) \quad \left(\log_{y^4} x^2 = \frac{3}{2}\right) \Rightarrow (x = y^2), \quad \begin{cases} x = y^2, & x_1 = 27. \\ xy = 81, & y_1 = 3; \end{cases}$$

$$b) \quad \left(\log_{y^4} x^2 = \frac{1}{6}\right) \Rightarrow (y = x^3), \quad \begin{cases} xy = 81, & x_2 = 3, \\ y = x^3, & y_2 = 27. \end{cases}$$

Ma'ruza 11: Kvadrat tenglamaga keltirib yechiladigan tenglamalar.

1. $ax^4+bx^2+c=0$ (1) tenglama bikvadrat tenglama deyiladi. Bu yerda a, b va c berilgan sonlar bo'lib, $a\neq 0$ dir. Agar (1) da $x^2=z$ desak, $az^2+bz+c=0$ (2) ko'rinishdagi kvadrat tenglama hosil bo'ladi. Bu tenglamani z ga nisbatan yechamiz: $z_1=\frac{-b+\sqrt{b^2-4ac}}{2a}$ va $z_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$ Agar $z_1>0$ va $z_2>0$ $(a>0,\ c>0,\ b^2-4ac\geq 0,\ b<0$ *yoki* $a<0,\ c<0,\ b^2-4ac\geq 0,\ b>0)$ bo'lsa, (1) ko'rinishdagi kvadrat tenglama quyidagi ko'rinishdagi to'rtta yechimga ega bo'ladi:

$$x_{1,2}=\pm\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a}},\qquad x_{3,4}=\pm\sqrt{\frac{-b+\sqrt{b^2-4ac}}{2a}}.$$

1 - m i s o l. $x^4-3x^2-4=0$ tenglama yechilsin.
Agar $x^2=z$ deb belgilasak, tenglama $z^2-3z-4=0$ ko'rinishni oladi. Bu tenglamaning yechimi $z_1=4$ va $z_2=-1$ bo'lib $x_{1,2}=\pm 2$ bo'ladi, $x_{3,4}=\pm\sqrt{-1}$ yechimi esa haqiqiy sonlar to'plamida mavjud emas.

2 - m i s o l. $2x^4-5x^2+3=0.\ a=2,\ b=-5,\ c=3.$

Echish. Agar $x^2=z$ desak, berilgan tenglama $2z^2-5z+3=0$ ko'rinishni oladi. Bunda $D=b^2-4ac=25-24=1>0$:

$$z_{1,2}=\frac{5\pm 1}{4},\qquad z_1=\frac{3}{2},\qquad z_2=1.$$

$$x_{1,2}=\pm\sqrt{z}=\pm\sqrt{\frac{3}{2}};\qquad x_{3,4}=\pm\sqrt{1}=\pm 1.$$

Yechimni formulalardan foydalanib topish mumkin:

$$x_{1,2}=\pm\sqrt{\frac{5-\sqrt{25-4\cdot 3\cdot 2}}{4}}=\pm\sqrt{\frac{5-1}{4}}=\pm\sqrt{\frac{4}{4}}=\pm 1;$$

$$x_{3,4}=\pm\sqrt{\frac{5-\sqrt{25-4\cdot 3\cdot 2}}{4}}=\pm\sqrt{\frac{5+1}{4}}=\pm\sqrt{\frac{3}{2}}.$$

Maktab matematika kursida o'zaro teskari noma'lum ifodalarni o'z ichiga olgan tenglamalar ham kvadrat tenglamaga keltirib yechiladi. Fikrimizning dalili quyidagi tenglamani yechaylik:

$$\left(\frac{x}{x+1}\right)^2-\left(\frac{x+1}{x}\right)^2=\frac{3}{2}.$$

Bu ko'rinishdagi tenglamalarni yechish jarayonida o'qituvchi eng avvalo noma'lum o'zgaruvchining yo'l qo'yiladigan qiymatlari sohasini aniqlash lozimligini o'quvchilarga tushuntirishi kerak. Bu tenglamadagi o'zgaruvchining yo'l qo'yiladigan qiymatlari sohasi $x\neq -1$ va $x\neq 0$. Agar $\left(\frac{x}{x+1}\right)^2=z$ desak, $\left(\frac{x+1}{x}\right)^2=\frac{1}{z}$ bo'lib, z o'zgaruvchiga ko'ra berilgan tenglama $z-\frac{1}{z}=\frac{3}{2}$ yoki $2z^2-3z-2=0$ ko'rinishni oladi. Bu tenglamadan: $z_1=-\frac{1}{2},\quad z_2=2$

1) $z_1=-\frac{1}{2}$ bo'lganda $\left(\frac{x}{x+1}\right)^2=-\frac{1}{2}$ bo'ladi, bundan $\frac{x}{x+1}=\pm\sqrt{-\frac{1}{2}}$ tenglama hosil bo'ladi. Bu tenglama haqiqiy sonlar to'plamida yechimga ega emas.

2) $z = 2$ bo'lganda $\left(\frac{x}{x+1}\right)^2=2$ bo'ladi, bundan yoki $\frac{x}{x+1}=\pm\sqrt{2}$ tenglamalar hosil qilamiz.

$$a)\quad \left(\frac{x}{x+1}=\sqrt{2}\right)\Rightarrow\left(x=\sqrt{2}x+\sqrt{2}\right)\Rightarrow\left(x-\sqrt{2}x=\sqrt{2}\right)\Rightarrow$$

$$\Rightarrow\left[x(1-\sqrt{2})=\sqrt{2}\right]\Rightarrow\left(x=\frac{\sqrt{2}}{1-\sqrt{2}}\right)\Rightarrow\left(x=\frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}\right)\Rightarrow$$

$$\Rightarrow\left(x=\frac{\sqrt{2}+2}{1-2}\right)\Rightarrow\left(x_1=-2-\sqrt{2}\right).$$

$$б)\quad \left(\frac{x}{x+1}=-\sqrt{2}\right)\Rightarrow\left(x=-\sqrt{2}x-\sqrt{2}\right)\Rightarrow\left(x+\sqrt{2}x=-\sqrt{2}\right)\Rightarrow$$

$$\Rightarrow\left[x(1+\sqrt{2})=-\sqrt{2}\right]\Rightarrow\left(x=\frac{-\sqrt{2}}{1+\sqrt{2}}\right)\Rightarrow\left(x=\frac{(1-\sqrt{2})(-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}\right)\Rightarrow$$

$$\Rightarrow\left(x=\frac{-\sqrt{2}+2}{1-2}\right)\Rightarrow\left(x_2=\sqrt{2}-2\right).$$

Javob: $x_1=-2-\sqrt{2}$, $x_2=\sqrt{2}-2$.

3. To'rtinchi darajali $ax^4+bx^3+cx^2+dx+c=0$ ko'rinishdagi tenglamalarni ham to'la kvadrat ajratish yo'li bilan kvadrat tenglama ko'rinishiga keltirib yechiladi.

M i s o l. $x^4+6x^3+9x^2-4x^2+12x+3=0$ tenglamani yeching.

Yechish. $x^4+6x^3+9x^2-4x^2+12x+3=(x^2+3x)^2-4(x^2+3x)+3=0$

$x^2+3x=z$ desak, tenglama $z^2-4z+3=0$ ko'rinishni oladi. Bundan $z_1 = 1$ va $z_2 = 3$.

1) $z_1=1$ bo'lganda $x^2+3x=1$ yoki $x^2+3x-1=0$ bo'ladi.

$$x_{1,2}=-\frac{3}{2}\pm\sqrt{\frac{9}{4}+1}=-\frac{3}{2}\pm\frac{\sqrt{13}}{2}=\frac{-3\pm\sqrt{13}}{2};$$

2) $z_2=3$ bo'lganda $x^2+3x=3$ yoki $x^2+3x-3=0$ bo'ladi.

$$x_{3,4}=-\frac{3}{2}\pm\sqrt{\frac{9}{4}+3}=-\frac{3}{2}\pm\frac{\sqrt{21}}{2}=\frac{-3\pm\sqrt{21}}{2}.$$

Javob: $x_{1,2}=\frac{-3\pm\sqrt{13}}{2}$; va $x_{3,4}=\frac{-3\pm\sqrt{21}}{2}$.

4. Agar $a_nx^n+a_{n-1}x^{n-1}+...+a_1x+a$ tenglamada koeffitsientlarning $a_n=a_0$, $a_{n-1}=a_1$, $a_{n-2}=a_2$, ... tengliklari o'rinli bo'lsa, bunday tenglama qaytma tenglama deyiladi. Qaytma tenglamalar ham ayniy almashtirishlar bajarish orqali kvadrat tenglama ko'rinishiga keltirib yechiladi.

M i s o l. $2x^4+3x^3-16x^2+3x+2=0$ tenglamani yeching.

Yechish. Berilgan tenglamani har ikkala tomonini $x^2 \neq 0$ ga bo'lamiz. $2x^2+3x-16+\frac{3}{x}+\frac{2}{x^2}=0$ yoki $2\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)-16=0$. Agar $x+\frac{1}{x}=z$ desak, $\left(x+\frac{1}{x}\right)^2=z^2$ bo'ladi, bunda: $x^2+\frac{1}{x^2}=z^2-2$. Bu belgilashlarga asosan berilgan tenglama quyidagi ko'rinishni oladi: $2(z^2-2)+3z-16=0$ yoki $2z^2+3z-20=0$, bundan

$$z_{1,2}=\frac{-3\pm\sqrt{9+4\cdot2\cdot20}}{4}=\frac{-3\pm13}{4},\quad z_1=\frac{5}{2},\quad z_2=-4$$

1) Agar $z_1=\frac{5}{2}$ bo'lsa, $x+\frac{1}{x}=\frac{5}{2}$ yoki $2x^2-5x+2=0$, bundan $x_{1,2}=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm3}{4},\quad x_1=2,\quad x_2=\frac{1}{2};$

2) agar $z_2=-4$ bo'lsa, $x+\frac{1}{x}=-4$ yoki $x^2+4x+1=0$ bo'ladi, bundan $x_1=-2+\sqrt{3}$ va $x_2=-2-\sqrt{3}$.

J a v o b: $x_1=2,\ x_2=\frac{1}{2},\ x_{3,4}=-2\pm\sqrt{3}$

5. $ax^4+bx^3+cx^2+dx+e=0$ $(a\neq0,\ b\neq0)$ ko'rinishdagi tenglama ham qaytma tenglama ko'rinishiga keltirib yechiladi. Berilgan tenglamani $x^2\neq0$ ga bo'lsak, $a\left(x^2+\frac{e}{ax^2}\right)+b\left(x+\frac{d}{bx}\right)+c=0$ bo'ladi. Agar $x+\frac{d}{bx}=t$ desak, $\left(x+\frac{d}{bx}\right)^2=t^2$ bo'ladi, bundan $x^2+\frac{d^2}{b^2x^2}=t^2-\frac{2d}{b}$, agar $\frac{e}{d}=\frac{d^2}{b^2}$ bo'lsa, $x^2+\frac{e}{dx^2}=x^2+\frac{d^2}{b^2x^2}=t^2-\frac{2d}{b}$ yoki $at^2+bt+c-\frac{2ad}{b}=0$ ko'rinishdagi kvadrat tenglama hosil bo'ladi. Demak, $ax^4+bx^3+cx^2+dx+e=0$ tenglamada $\frac{e}{d}=\frac{d^2}{b^2}$ tenglik bajarilsa, bu tenglama ham qaytma tenglama kabi kvadrat tenglamaga keltirib yechilar ekan.

M i s o l. $2x^4-21x^3+74x^2-105x+50=0.$

Yechish. $a=2,\ e=50,\ d=105,\ b=21$. Shartga ko'ra $\frac{e}{a}=\frac{d^2}{b^2}$ bo'lishi kerak edi, shuning uchun $\frac{50}{2}=\left(\frac{105}{21}\right)^2$ yoki 25=25 tenglik o'rinli bo'ladi. Bu tenglikning har ikkala tomonini $x^2\neq0$ ga bo'lsak, $2x^2-21x+74-\frac{105}{x}+\frac{50}{x^2}=0$. $2\left(x^2+\frac{25}{x^2}\right)-21\left(x+\frac{5}{x}\right)+74=0$ Agar $x+\frac{5}{x}=t$ desak, $x^2+\frac{25}{x^2}=t^2-10$ tenglik hosil bo'ladi, u holda tenglama $2t^2-21t+54=0$ ko'rinishni oladi.

Bundan $t_1=\frac{9}{2}$ va $t_2=6$ yechimlarni hosil qilamiz.

1) Agar $t_1=\frac{9}{2}$ bo'lsa, $x+\frac{5}{x}=\frac{9}{2}$ yoki $2x^2-9x+10=0$ bundan $x_1=2$ va $x_2=\frac{5}{2}$ yechimlarni topamiz.

2) agar $t_2=6$ bo'lsa, $x+\frac{5}{x}=6$ yoki $x^2-6x+5=0,$ bundan $x_3=1$ va $x_4=5$ yechimlarni topamiz. Javob. $x_1=2, x_2=\frac{5}{2}, x_3=1, x_4=5.$

6. $(x+a)(x+b)(x+c)(x+d)=m$ ko'rinishdagi tenglama ham ma'lum bir shart va ayniy almashtirishlarni bajarish orqali kvadrat tenglama ko'rinishiga keltirib yechiladi. Agar bu berilgan tenglamada $a+b=c+d$ yoki $a+c=b+d$ yoki $a+d=b+c$ tengliklar o'rinli bo'lsa, bu tenglama ham kvadrat tenglama ko'rinishiga keltirib yechiladi.

M i s o l. $(x+2)(x-3)(x+1)(x+6)=-96.$

$a=2,\ b=-3,\ c=1,\ d=6.$ Shartga ko'ra $a+c=b+d$ edi, shuning uchun $2+1=-3+6,$ bunga ko'ra berilgan tenglamani quyidagicha guruhlaymiz: $[(x+2)(x+1)][(x-3)[x+6)]=-96,$ $(x^2+3x+2)(x^2+3x-18)=-96.$ $x^2+3x=t$ desak, $(t+2)\cdot(t-18)=-96$ tenglik o'rinli bo'ladi, bundan tenglama hosil bo'ladi. Bu tenglamaning yechimi $t_1=6$ va $t_2=10$ bo'ladi.

1) agar $t_1=6$ bo'lsa, $x^2+3x=6$ yoki $x^2+3x-6=0,$ bundan

$$x_{1,2}=\frac{-3\pm\sqrt{9+24}}{2}=\frac{-3\pm\sqrt{33}}{2};$$

2) agar $t_2=10$ bo'lsa, $x^2+3x=10$ yoki $x^2+3x-10=0$ bo'ladi.

$$x_{3,4}=\frac{-3\pm\sqrt{9+40}}{2}=\frac{-3\pm 7}{2},\quad x_3=-5,\ x_4=2.$$

Javob: $x_{1,2}=\frac{-3\pm\sqrt{33}}{2},\qquad x_3=-5,\ x_4=2.$

7. $(x+a)^4+(x+b)^4=c$ ko'rinishdagi tenglama ham $x=t-\frac{a-b}{2}$ almashtirish orqali bikvadrat tenglama ko'rinishiga keltirib yechiladi.

Agar $\begin{cases} x+a=t+m \\ x+b=t-m \end{cases}$ desak, bu sistemadagi tenglamalarni o'zaro hadma-had ayirsak, $a-b=2m,\ m=\frac{a-b}{2}$ bo'ladi, u holda $x+a=t+\frac{a-b}{2}$ yoki $x=t-\frac{a+b}{2}$ bo'ladi. U holda berilgan tenglama quyidagi ko'rinishni oladi: $\left(t+\frac{a-b}{2}\right)^4+\left(t-\frac{a-b}{2}\right)^4=c$

Bundan:

$$t^4+4t^3\frac{a-b}{2}+6t^2\frac{(a-b)^2}{4}+4t\frac{(a-b)^3}{8}+\frac{(a-b)^4}{16}+$$

$$+t^4-4t^3\frac{a-b}{2}+6t^2\frac{(a-b)^2}{4}-4t\frac{(a-b)^3}{8}+\frac{(a-b)^4}{16}=$$

$$=2t^4+12t^2\left(\frac{a-b}{2}\right)^2+2\left(\frac{a-b}{2}\right)^4=c.$$

$$t^4+6t^2\left(\frac{a-b}{2}\right)^2+\left(\frac{a-b}{2}\right)^4=\frac{c}{2}.$$

Bu tenglamani bikvadrat tenglamani yechish usuli bo'yicha echa olamiz. Masalan, $(x+6)^4+(x+4)^4=82$ tenglama berilgan bo'lsin.

Bu tenglamada $x=t-\frac{6+4}{2}=t-5$ almashtirishni bajaramiz, u holda berilgan tenglama ko'rinishi quyidagicha bo'ladi:

$(t+1)^4+(t-1)^4=82.$

$t^4+4t^3+6t^2+4t+1+t^4-4t^3+6t^2-4t+1=82,$

$2t^4+12t^2+2=82, \qquad t^4+6t^2-40=0.$

$t^2=y$ desak, $y^2+6y-40=0, \quad y_1=4, \quad y_2=-10.$

1) agar $t^2=4$ bo'lsa, $t_{1,2}=\pm 2$;

2) agar $t^2=-10$ bo'lsa, haqiqiy sonlar to'plamida yechim mavjud emas.

$x_1=t-5=2-5=-3, \quad x_2=t-5=-2-5=-7.$

8. Agar tenglama $\frac{ax}{px^2+nx+q}+\frac{bx}{px^2+mx+q}=c$ ko'rinishda berilgan bo'lsa, unda $px+\frac{q}{x}=t$ almashtirish bajariladi. Agar tenglamada $c=0$ bo'lsa, $x_1=0$ bo'lib qolgan yechimlarni x o'zgaruvchiga nisbatan kvadrat tenglamaga keltirib topiladi. Agar $c\neq 0$ bo'lsa, $x\neq 0$ bo'lib, bu holda berilgan tenglamaning surat va maxrajini x ga bo'lamiz: ,

$$\frac{a}{px+n+\frac{q}{x}}+\frac{b}{px+m+\frac{q}{x}}=c$$

Bunda $px+\frac{q}{x}=t$ desak, t o'zgaruvchiga nisbatan kvadrat tenglama hosil bo'ladi: $\frac{a}{t+n}+\frac{b}{t+m}=c$. Bu erda $t\neq -n$, $t\neq -m$ dir. Bularga ko'ra tenglamaning ko'rinishi quyidagicha bo'ladi:

$ct^2+(mc+nc-a-b)t+mnc-am-bn=0$

M i s o l. $\frac{2x}{2x^2-5x+3}+\frac{13x}{2x^2+x+3}=6$ tenglama yechilsin.

Yechish. Tenglamaning chap tomonida turgan qo'shiluvchilarning surat va maxrajlarini x ga bo'lsak, $\frac{2}{2x-5+\frac{3}{x}}+\frac{13}{2x+1+\frac{3}{x}}=6$ tenglik hosil bo'ladi. $2x+\frac{3}{x}=t$ desak, $\frac{2}{t-5}+\frac{13}{t+1}=6$ tenglik hosil bo'ladi, (bu erda $t\neq -5$ va $t\neq -1$ bo'lishi kerak):

$2t^2-13t+11=0$, bundan $t_1=1, \quad t_2=\frac{11}{2}$.

1) agar $t_1=1$ bo'lsa, $2x+\frac{3}{x}=1$ yoki $2x^2-x+3=0$ bo'lib, uning yechimlari haqiqiy sonlar to'plamida mavjud emas.

2) agar $t_2=\frac{11}{2}$ bo'lsa, $2x+\frac{3}{x}=\frac{11}{2}$ yoki $4x^2-11x+6=0$ bo'ladi, bundan $x_1=\frac{3}{4}$ va $x_2=2$ yechimlar topiladi.

J a v o b: $x_1=\frac{3}{4}$, $x_2=2$.

Mavzu 12: Ko`pburchaklarga ichki aylana chizish shartlari. Ko`pburchakka doir masalalar. Geometrik figuralar yuzasi mavjudligining zaruriy va yetarli shartlari, ularning xossalari. Yuzalarni hisoblashga doir masalalar.

Reja

1.Styuart teoremasi.
2.Ptolomey teoremasi.
3. Ko'pburchaklarda ichki aylana chizish shart
4. Ko'pburchakka doir masalalar.

Yuza tushunchasi planimetriyaning asosiy tushunchasidir. O'quvchilar dastlab bu yerda sodda figura tushunchasi bilan tanishadi. Agar figurani chekli sondagi uchburchaklarga ajratish mumkin bo'lsa, bu figura sodda figura deb ataladi. Barcha qavariq ko'pburchaklar sodda figuralarga misol bo'la oladi. Shundan keyin sodda figuraning yuzasi tushunchasi ta'riflanadi.

Ta'rif. Sodda figuralar uchun yuza bu musbat miqdor (kattalik) bo'lib, uning son qiymati quyidagi xossalarga ega:

1) Teng figuralar teng yuzalarga ega.
2) Agar figura sodda figuralardan iborat qismlarga ajratilgan bo'lsa, u holda bu figuraning yuzi qismlari yuzalarining yig'indisiga teng.
3) Tomoni bir o'lchov birligiga teng bo'lgan kvadratning yuzi birga teng.

Albatta, yuzalarni ifodalashda 1mm^2, 1 sm^2, 1 dm^2 va hokazolar haqida so'z bormoqda. Ana shundan keyin, to'g'ri to'rtburchakning yuzasi tushunchasi kiritiladi va to'g'ri to'rtburchakning yuzasi eni bilan bo'yi ko'paytmasiga teng ekanligi isbotlanadi. To'rt burchak yuzasi tushunchasi kiritilgandan keyin parallelogramm yuzi, trapetsiya, uchburchak yuzi tushunchalari kiritiladi va eng muhim tushuncha doiraning yuzasi tushunchasi kiritiladi.

Doiraning yuzi uni chegaralovchi aylana uzunligi bilan radius ko'paytmasining yarmiga teng. Shuningdek, doiraviy sektorning yuzasini hisoblash formulasi keltirilib chiqariladi. $S_{sekt} = \frac{\pi R^2}{360^\circ} \cdot \alpha$ stereometriyada hajm tushunchasi kiritiladi. Bu planimetriyadagidek dastlab sodda jismlarning hajmlari qaraladi, ya'ni sodda jism tushunchasi kiritiladi.

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Mavzu 13: Stereometriyaning asosiy aksiomalari va ularning natijalari. Ko`pyoqli figuralar, ularning sirtlari, hajmlari.

REJA

1. Stereometriyaning asosiy aksiomalari va ularning natijalari.
2. Fazoda to`g'ri chiziq va tekisliklarning o`zaro joylashuvi.
3. Fazoda tekisliklarning o`zaro joylashuvi.

Ko`pyoqning turlari

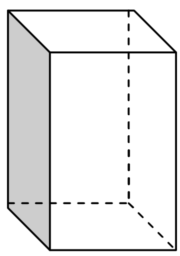
Chekli sondagi tekisliklar bilan chegaralangan jism *ko`pyoq* deyiladi. Ko`pyoqning chegarasi uning *sirti* deyiladi.

Sodda ko`pyoqlarga prizma va piramida kiradi. Biz prizma va piramidaning sirti haqidagi tushunchani to`ldirib, soda ko`pyoqlarga misollar keltiramiz.

Chekli sondagi ko`pburchaklarning quyidagi shartlarni qanoatlantiruvchi birlashmasi *soda ko`pyoqli sirt* deyiladi.

1. Bu ko`pburchaklarning ixtiyoriy ikkita uchi uchun ularning tomonlaridan tuzilgan siniq chiziq mavjud bo`lib, olingan uchlar shu siniq chiziqning uchlari bo`ladi.

2. Ko`pburchaklar birlashmasining ixtiyoriy nuqtasi berilgan ko`pburchaklardan faqat birining nuqtasi bo`ladi yoki ikkita va faqat ikkita ko`pburchakning umumiy tomoniga tegishli bo`ladi. Ko`pyoqliburchakning tekis burchaklari vazifasini o`tovchi birgina ko`pyoqli burchakning uchi bo`ladi. Bu talablarni 1 va 2-rasmlarda tasvirlangan ko`pburchaklar birlashmasi qanoatlantiradi. Bundan keyin soda sirtlar haqida so`z yuritilganda "sodda" so`zini ishlatmasdan ko`pyoq deb gapiramiz.



1-rasm

Ko`p yoqli sirtni tashkil qiluvchi ko`pburchaklar uning *yoqlari* deyiladi, bu ko`pburchaklarning tomonlari ko`pyoqli sirtning *qirralari*, uchlari esa ko`pyoqli shaklning *uchlari* deyiladi.

Agar ko`pyoqli sirtning har bir qirrasi uning ikkita yog`iga tegishli bo`lsa, u holda bu ko`pyoqli sirt *yopiq sirt* deyiladi. Prizmaning yon sirti yopiq bo`lmagan ko`p yoqli sirtga misoldir, piramidaning sirti yopiq ko`pyoqli sirtga misoldir.

Yopiq ko`pyoqli sirt fazoning shu sirtga tegishli bo`lmagan barcha nuqtalari to`plamini ikkita qism to`plamga ajratadi. Bu qism to`plamlardan biri uchun shu qism to`plamga tegishli to`g`ri chiziqlar mavjud; ikkinchisi uchun esa bunday to`g`ri chiziqlar mavjud emas. Ko`rsatilgan qism to`plamlardan birinchisi ko`pyoqli sirtning *tashqi sohasi*, ikkinchisi *ichki sohasi* deyiladi.

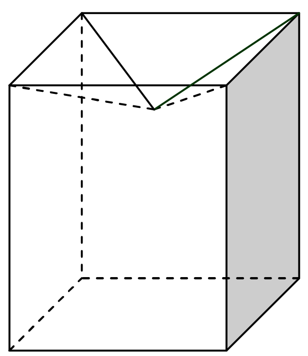
Ta`rif. Yopiq ko`pyoqli sirt bilan uning ichki sohasining birlashmasi ko`pyoq deyiladi.

Ta`rif. Ko`pyoqli sirt va uning ichki sohasi mos ravishda *ko`pyoqning sirti* va *ko`pyoqning ichki sohasi* deyiladi.

Ta`rif. Ko`pyoqli sirtning yoqlari, qirralari, uchlari mos ravishda ko`pyoqning *yoqlari, qirralari* va *uchlari* deyiladi.

Ta`rif. Ko`pyoqning bir yog`iga tegishli bo`lmagan ikki uchini birlashtiruvchi kesma ko`pyoqning diagonali deyiladi.

73-rasmda *ABCDEF* oltiyoq va uning diagonali *DF, BE* tasvirlangan. Ko`pyoqlar ko`pburchaklar singariqavariq va noqavariq bo`lishi mumkin.



2-rasm

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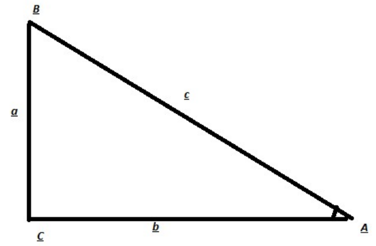
Mavzu 14: Haqiqiy argumentli trigonometrik funksiyalar va ularning xossalari. Bir xil argumentli trigonometrik funksiyalar orasidagi munosabatlar. Trigonometrik ayniyatlar. Qo'shish teoremalari va ularning natijalari.

O'tkir burchaklaridan biri α bo'lgan ABC to'g'ri burchakli uchburchakni qaraylik:AB=c - gipotenuza,



BC=a-o'tkir burchakka qarshisidagi katet va

AC=b o'tkir burchakka yopishgan katet.

ABC uchburakning a,b va c tomonlaridan foydalanib, $\frac{b}{c}$, $\frac{a}{c}$, $\frac{b}{a}$,$\frac{a}{b}$, $\frac{c}{a}$ va $\frac{c}{b}$ nisbatlarni yozishimiz mumkin. Bu nisbatlarni o'zgarishi α burchakni ham o'zgarishiga olib keladi.Bu nisbatlarning har biri bilan α burchak orasidagi bog'lanishlarni alohida nomlaymiz.

Ta'rif. α burchak qarshisida yotgan katet uzunligini gipotenuza uzunligiga nisbati α burchakning *sinusi* deyiladi va uni $\sin\alpha$ deb yoziladi. Demak, $\sin\alpha = \frac{a}{c}$.

Ta'rif. α burchakka yopishgan katet uzunligini gipotenuza uzunligiga nisbati, α burchakning *kosinusi*deyiladi va u $\cos\alpha$ deb yoziladi. Demak, $\cos\alpha = \frac{b}{c}$.

Ta'rif. α burchak qarshisida yotgan katet uzunligini, α burchakka yopishgan katet uzunligiga nisbati, α burchakning *tangensi* deyiladi va u $tg\alpha$ deb yoziladi. Demak, $tg\alpha = \frac{a}{b}$.

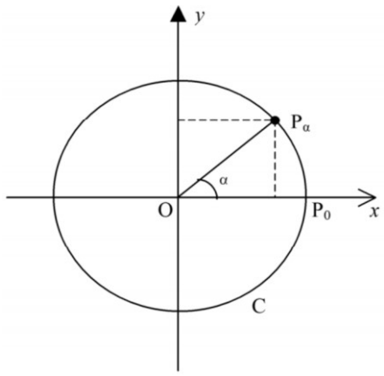
Ta'rif. α burchakka yopishgan katet uzunligini, α burchak qarshisida yotgan katet uzunligiga nisbati, α burchakning *kotangensi* deyiladi va u $ctg\alpha$ deb yoziladi. Demak, $ctg\alpha = \frac{b}{a}$.

$y = \sin x$, $y = \cos x$, $y = tgx$, va $y = ctgx$ funksiyalar *trigonometrik funksiyalar* deb ataladi.

Agar biz $a^2+b^2=c^2$ (Pifagor teoremasi) tenglikni har ikkala qismini hadma-had c^2 ga bo'lsak, u holda $\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=1$ yoki $\sin^2 x+\cos^2 x=1$ ni hosil qilamiz. Bu tenglik Pifagor teoremasiga ekvivalent bo'lib, uni asosiy *trigonometrik ayniyat* deb ataladi.

Biz o'tkir burchak trigonometrik funksiyalarining ta'riflarini keltirdik.Bu ta'riflarni ixtiyoriy burchak uchun ham umumlashtirish mumkin. Buning uchun markazi koordinata boshida va radiusi 1 ga teng bo'lgan aylanani olamiz. Aylanadagi (1;0) koordinatali nuqtani P_0 bilan belgilaymiz va uni boshlang'ich nuqta deb ataymiz. Ixtiyoriy α sonni olamiz va boshlang'ich nuqtani α burchakka buramiz.

Natijada P_α nuqtani hosil qilamiz. P_α nuqtaning koordinatalarini x_α va y_α deb belgilaymiz. OP_α=R=1 ekanligini e'tiborga olsak, $\sin\alpha=\frac{y_\alpha}{R}=y_\alpha$ va $\cos\alpha=\frac{x_\alpha}{R}=x_\alpha$ larni yoki $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$ larni hosil qilamiz. Bulardan esa sinus va kosinuslar uchun boshqacha ta'riflar berish mumkinligi kelib chiqadi.



P_α nuqtaning ordinatasiga α burchakning sinusi va abstsissasiga α burchakning kosinusi deyiladi.Demak, $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$.

$P_\alpha(x_\alpha;y_\alpha)$ nuqtaning koordinatalari uchun $x_\alpha^2+y_\alpha^2=1$ ya'ni $\sin^2 x+\cos^2 x=1$ tenglik o'rinlidir. Bu asosiy trigonometrik ayniyatdir. Undan quyidagilarni yozish mumkin: $\sin^2 x=1-\cos^2 x$ va $\cos^2 x=1-\sin^2 x$.

Agar biz $tg\alpha=\frac{y_\alpha}{x_\alpha}$ va $ctg\alpha=\frac{x_\alpha}{y_\alpha}$ hamda $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$ ekanligini e'tiborga olsak, u holda $tg\alpha=\frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha=\frac{\cos\alpha}{\sin\alpha}$ larni hosil qilamiz. Demak, α burchak sinusini uning kosinusiga nisbatiga α *burchakningtangensi* deyiladi. α

burchak kosinusini uning sinusiga nisbatiga esa α *burchakning kotangensi* deyiladi.

Demak, ta'riflarga asosan $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$.

Bu yerda tangens $\cos\alpha \neq 0$ va kotangens $\sin\alpha \neq 0$ hollarda aniqlangan.

$tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$ lardan $tg\alpha$ va $ctg\alpha$ larni o'zaro teskari sonlar ekanligini ko'ramiz. O'zaro teskari sonlar ko'paytmasi esa 1 ga teng. Demak, bundan esa $tg\alpha = \frac{1}{ctg\alpha}$ va $ctg\alpha = \frac{1}{tg\alpha}$ ifodalar kelib chiqadi.

Trigonometrik funksiyalarning ishoralari qaralayotgan burchakning qaysi chorakda yotishiga qarab aniqlanadi.

α burchakning sinusi P_α nuqtaning ordinatasidan iborat bo'lganligi uchun u I va II choraklarda musbat, III va IV choraklarda esa manfiy bo'ladi.

α burchakning kosinusi P_α nuqtaning abstsissasidan iborat bo'lgani uchun u I va IV choraklarda musbat, II va III choraklarda esa manfiy bo'ladi.

α burchakning tangensi va kotangensi P_α nuqta koordinatalarining nisbatlari bo'lganligi uchun, ular P_α nuqtaning koordinatalari bir xil ishorali bo'lgan (I va III) choraklarda musbat va har xil ishorali bo'lgan (II va IV) choraklarda manfiy bo'ladi.

Choraklar	I	II	III	IV
$\sin\alpha$	+	+	-	-
$\cos\alpha$	+	-	-	+
tgα	+	-	+	-
ctgα	+	-	+	-

Amaliyotda ko’pincha trigonometrik funksiyalarning qiymatlari bilan ish ko’riladi. α burchakning trigonometrik funksiyalarini qiymatlari P_α nuqtaning koordinatalari bilan bog’liq. Ya’ni, $\sin\alpha = y_\alpha$, $\cos\alpha = x_\alpha$ $tg\alpha = \frac{y_\alpha}{x_\alpha}$

α burchak 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ va 2π qiymatlarni qabul qilganda P_α nuqtaning koordinatalarini osongina topiladi. α burchak 0^0, 30^0, 45^0 va 60^0 qiymatlarni qabul qilganda P_α nuqtaning koordinatalarini o’tkir burchagi α bo’lgan to’g’ri burchakli uchburchakdan topiladi.

Quyidagi jadvalda trigonometrik funksiyalarning ba’zi bir burchaklardagi qiymatlari keltirilgan.

Burchak	0	30^0	45^0	60^0	90^0	180	270
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tgα	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞
ctgα	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0

O’tkirburchakning trigonometric funksiyalarini jadval yoki to’g’ri burchakli uchburchakdan foydalanib hisoblash mumkin. Ixtiyoriy burchakni trigonometric funksiyalarini qiymatlarini hisoblashni doimo o’tkir burchak trigonometric funksiyalari qiymatlarini hisoblashga keltirish mumkin. Bunday formulalarni keltirish formulalari deyiladi.

Ko'p hollarda α burchakning trigonometrik funksiyalarini bilganholda $\frac{\alpha}{2}$ burchakning trigonometric funksiyalarinianiqlashgato'g'rikeladi. Bunda quyidagi formulalardan foydalaniladi:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}; \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}; \qquad tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};$$

$$ctg\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}; \; tg\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}.$$

Bulardan dastlabki ikkitasi $\cos 2\alpha = 1-2\sin^2\alpha$ va $\cos 2\alpha = 2\cos^2\alpha - 1$ formulalardan keltirib chiqariladi. Keyingi ikkitasi esa tangens va kotangenslarni sinus va kosinuslar orqali ifodalaridan keltirib chiqariladi.Dastlabki to'rtta formulalardagi $\pm$ ishoralardan qaysi birini olinishi $\frac{\alpha}{2}$ burchakni qaysi chorakda yotishiga bog'liq bo'ladi.

Ko'p hollarda trigonometrik funksiyalardan birini qolganlari orqali ifodalash formulalaridan foydalaniladi.Bu formulalarni asosiy trigonometrik ayniyatlardan keltirib chiqariladi.$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$ va $\cos\alpha = \pm\sqrt{1-\mathrm{s}in^2\alpha}$.

$\sin\alpha$ ni tgα orqali ifodasi $tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$ ni $\sin\alpha$ ga nisbatan yechib keltirib chiqariladi. Ularni barchasini quyidagi jadvalda keltiramiz:

Funksiya	$\sin\alpha$	$\cos\alpha$	tgα	ctgα
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\frac{tg\alpha}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{1}{\sqrt{1+ctg^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\frac{1}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{ctg\alpha}{\sqrt{1+ctg^2\alpha}}$
tgα	$\pm\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	tgα	$\frac{1}{ctg\alpha}$
ctgα	$\pm\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{tg\alpha}$	ctgα

Na'munaviy misollar yechilishi

1-misol. $\sin\alpha\cdot\sin(60^o-\alpha)\cdot\sin(60^o+\alpha)=\frac{1}{4}\sin3\alpha$ ayniyatni isbotlang.

Isboti. $\sin\alpha\cdot\sin(60^0-\alpha)\cdot\sin(60^0+\alpha)=$

$$=\sin\alpha(\sin^2 60^0-\sin^2\alpha)=\sin\alpha\left(\frac{3}{4}-\sin^2\alpha\right)=$$

$$=\frac{1}{4}\left(3\sin\alpha-4\sin^3\alpha\right)=\frac{1}{4}\sin3\alpha.$$

2-misol. $\cos\alpha\cdot\cos(60^o-\alpha)\cdot\cos(60^o+\alpha)=\frac{1}{4}\cos3\alpha$ ayniyatni isbotlang.

3-misol. $tg\alpha\cdot tg(60^o-\alpha)\cdot tg(60^o+\alpha)=tg3\alpha$ ayniyatni isbotlang.

Bu ayniyatlardan foydalanib, quyidagi trigonometrik ifodalarni osonlikcha hisoblash mumkin:

a) $\sin20^0\cdot\sin40^0\cdot\sin80^0=\frac{1}{4}\sin3\cdot20^0=\frac{1}{4}\sin60^0=\frac{1}{4}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{8};$

б) $\cos10^0\cdot\cos50^0\cdot\cos70^0=\frac{1}{4}\cos30^0=\frac{\sqrt{3}}{8};$

в) $tg6^0\cdot tg54^0\cdot tg66^0=tg18^0.$

4-misol. $\sin3\alpha\cos^3\alpha+\sin^3\alpha\cos3\alpha=\frac{3}{4}\sin\alpha$ ayniyatni isbotlang.

Isboti. $\sin3\alpha\cos^3\alpha+\sin^3\alpha\cos3\alpha=\sin3\alpha\frac{\cos3\alpha+3\cos\alpha}{4}+$

$$+\cos3\alpha\frac{3\sin\alpha-\sin3\alpha}{4}=\frac{3}{4}(\sin3\alpha\cos\alpha+\sin\alpha\cos3\alpha)=\frac{3}{4}\sin4\alpha.$$

5-misol. $\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha$ ifodani soddalashtiring.

Yechish. Berilgan ifodani $\sin\alpha$ ga ko'paytiramiz hamda bo'lamiz.

$$\frac{\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha\cdot\sin\alpha}{\sin\alpha}=\frac{\frac{1}{2}\sin2\alpha\cdot\cos2\alpha\cdot\cos4\alpha}{\sin\alpha}=$$

$$=\frac{\frac{1}{2}\left(\frac{1}{2}\sin4\alpha\cdot\cos4\alpha\right)}{\sin\alpha}=\frac{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}\sin8\alpha\right)\right]}{\sin\alpha}=\frac{\sin8\alpha}{8\sin\alpha}.$$

6-misol. $tg4\alpha-sec4\alpha=\frac{\sin2a-\cos2a}{\sin2a+\cos2a}$ ayniyatni isbotlang.

V. Yarim argumentning trigonometrik funksiyalari

1) $\left|\sin\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{2}}$; 2) $\left|\cos\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{2}}$;

3) $\left|tg\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$; $[\alpha \neq \pi(2n+1), \quad n \in Z]$;

4) $\left|ctg\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$;

5) $tg\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$; $[\alpha \neq \pi n, n \in Z]$;

6) $ctg\frac{\alpha}{2} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha}$; $[\alpha \neq \pi n, n \in Z]$;

1-misol. $tg7^0 30'$ ni hisoblang.

$$tg\,7^0 30' = \frac{1-\cos 15^0}{\sin 15^0} = \frac{1-\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)}{\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)} =$$

Yechish. $= \frac{\left(4-\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)} = \frac{4\sqrt{6}+4\sqrt{2}-4\sqrt{3}-8}{4} =$

$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2.$

2-misol. $\frac{1-tg^2 15^0}{1+tg^2 15^0} = \frac{\sqrt{3}}{2}$ ni isbotlang.

Isboti. $\frac{1-tg^2 15^0}{1+tg^2 15^0} = \frac{1-\frac{1-\cos 30^0}{1+\cos 30^0}}{1+\frac{1-\cos 30^0}{1+\cos 30^0}} = \cos 30^0 = \frac{\sqrt{3}}{2}.$

VI. Trigonometrik funksiyalar ko'paytmasini yig'indiga keltirish formulalari:

1) $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$;

2) $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$;

3) $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$;

Misol. *$\cos\alpha+\cos(\alpha+2\beta)+...+\cos(\alpha+\pi\beta)$* ifodani soddalashtiring.

Yechish. Berilgan ifodani $\sin\frac{\beta}{2}$ ga ko'paytiramiz va bo'lamiz.

$$\frac{1}{\sin\frac{\beta}{2}}\left[\sin\frac{\beta}{2}\cos\alpha + \sin\frac{\beta}{2}\cos(\alpha+\beta) + \sin\frac{\beta}{2}\cos(\alpha+2\beta) + \right.$$

$$+\ldots+\sin\frac{\beta}{2}\cos(\alpha+n\beta)\Big]=\frac{1}{2\sin\frac{\beta}{2}}\left[\sin\left(\alpha+\frac{\beta}{2}\right)-\sin\left(\alpha-\frac{\beta}{2}\right)+\right.$$

$$+\sin\left(\alpha+\frac{3\beta}{2}\right)-\sin\left(\alpha+\frac{\beta}{2}\right)+\sin\left(\alpha+\frac{5\beta}{2}\right)-\sin\left(\alpha+\frac{3\beta}{2}\right)+\ldots+$$

$$+\sin\left(\alpha+\frac{3\beta}{2}\right)-\sin\left(\alpha+\frac{\beta}{2}\right)+\sin\left(\alpha+\frac{5\beta}{2}\right)-\sin\left(\alpha+\frac{3\beta}{2}\right)+\ldots+$$

$$\left.+\sin\left(\alpha+\frac{2n+1}{2}\beta\right)-\sin\left(\alpha+\frac{2n-1}{2}\beta\right)\right]=$$

$$=\frac{1}{\sin\frac{\beta}{2}}\left[\sin\left(\alpha+\frac{2n+1}{2}\beta\right)-\sin\left(\alpha-\frac{\beta}{2}\right)\right]=$$

$$=\frac{1}{\sin\frac{\beta}{2}}2\sin\frac{n+1}{2}\beta\cos\left(\alpha+\frac{n}{2}\beta\right)=\frac{\sin\frac{n+1}{2}\beta\cos\left(\alpha+\frac{n}{2}\beta\right)}{\sin\frac{\beta}{2}}.$$

VII. Trigonometrik funksiyalar yig'indisi va ayirmasining formulalari:

1) $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$;

2) $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$;

3) $\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$;

4) $\cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$;

5) $tg\alpha+tg\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$, $\left[\alpha,\beta\neq\frac{\pi}{2}(2n-1),n\in Z\right]$;

6) $tg\alpha-tg\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$, $\left[\alpha,\beta\neq\frac{\pi}{2}(2n-1),n\in Z\right]$;

7) $ctg\alpha+ctg\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$, $[\alpha,\beta\neq\pi n,n\in Z]$;

8) $ctg\alpha-ctg\beta=\frac{\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$.

1-misol. $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\gamma}{2}\cos\frac{\beta+\gamma}{2}$ ayniyatni isbotlang.

$$\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=$$

$$=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\gamma+\alpha+\beta+\gamma}{2}\cdot\cos\frac{\gamma-\alpha-\beta-\gamma}{2}=$$

Isboti. $$=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta+2\gamma}{2}\right)=$$

$$=2\cos\frac{\alpha+\beta}{2}2\cos\frac{\alpha-\beta+\alpha-\beta+2\gamma}{4}\cdot\cos\frac{\alpha-\beta-\alpha-\beta-2\gamma}{4}=$$

$$=4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\gamma}{2}\cos\frac{\beta+\gamma}{2}.$$

2-misol. Agar $\alpha+\beta+\gamma=\pi$ bo'lsa, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma-2=2\cos\alpha\cos\beta\cos\gamma$ tenglikning o'rinli ekanligini isbotlang.

Isboti. Shartga ko'ra $\gamma=\pi-\alpha-\beta$ u holda

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma-2=\sin^2\alpha+\sin^2\beta+\sin^2[\pi-(\alpha+\beta)]-2=$$
$$=\sin^2\alpha+\sin^2\beta+\sin^2(\alpha+\beta)-2=$$

$$=\frac{1-\cos 2\alpha}{2}+\frac{1-\cos 2\beta}{2}+\frac{1-\cos 2(\alpha+\beta)}{2}-2=$$
$$=-\frac{1}{2}[\cos 2\alpha+\cos 2\beta+\cos 2(\alpha+\beta)+1]=$$

$$=-\frac{1}{2}[\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos^2(\alpha+\beta)]=$$
$$=-\cos(\alpha+\beta)[\cos(\alpha-\beta)+\cos(\alpha+\beta)]=$$
$$=-2\cos(\alpha+\beta)\cos\alpha\cos\beta=$$
$$=-2\cos(\pi+\gamma)\cos\alpha\cos\beta=2\cos\alpha\cos\beta\cos\gamma.$$

MUSTAQIL YECHISH UCHUN MISOLLAR.

12. $4\sin 90^0+3\cos 720^0-3\sin 630^0+5\cos 900^0.$ $j.$ 5.
13. $5tg540^0+2\cos 1170^0+4\sin 990^0-3\cos 540^0.$ $j.-1.$
14. $100ctg^2 990^0+25tg^2 540^0-3\cos^2 900.$ $j.$ $-3.$
15. $tg900^0-\sin(-1095^0)+\cos(-1460^0).$ $j.$ $\sqrt{1{,}5}.$
16. $\sin(-1125^0)+\cos^2(-900^0)+tg1710^0$ $j.$ $\frac{2-\sqrt{2}}{2}.$
17. $cos20^0+cos40^0+cos60^0+...+cos160^0+cos180^0.$ $j.-1.$
18. $\sin\left(-\frac{14\pi}{3}\right)+\cos ec^2\frac{29\pi}{4}-tg^2\frac{3\pi}{4}.$ $j.$ $\frac{2-\sqrt{3}}{2}.$

Mavzu 15: Teskari trigonometrik funksiyalar va ularning xossalari, grafiklari. Arkfunksiyalarning trigonometrik funksiyalari

Shu vaqtga qadar biz α burchakning berilgan qiymatlariga asosan $sin\alpha$, $cos\alpha$, $tg\alpha$ va $ctg\alpha$ larning qiymatlarini topish bilan shug'ullandik. Endi bunga teskari masalani ya'ni $sin\alpha$, $cos\alpha$, $tg\alpha$ va $ctg\alpha$ larning qiymatlariga asosan α burchakning qiymatlarini aniqlash masalasini ham qo'yish mumkin. Bu masala teskari trigonometrik funksiya tushunchasini kiritishga olib keladi.Teskari trigonometrik funksiya tushunchasini kiritish uchun esa dastlab teskari funksiya tushunchasini kiritish kerak bo'ladi.

Aniqlanish sohasi D va qiymatlar sohasi E dan iborat bo'lgan $y = f(x)$ funksiya o'zining aniqlanish sohasida monoton bo'lsin. U holda x ning D dan olingan har bir qiymatiga y ning E dagi bitta qiymati mos keladi va aksincha. y ning E dan olingan har bir qiymatiga x ningdagi bitta qiymati mos keladi. Demak, bu holda E da aniqlangan shunday yangi funksiyani tuzish mumkinki, unda E dan olingan har bir y ga D da $y = f(x)$tenglamani qanoatlantiruvchi bitta x ni mos qo'yish mumkin. Hosil qilingan bu yangi funksiya $y = f(x)$funksiyaga teskari funksiya deyiladi.

$y = f(x)$funksiyaga teskari funksiyani topish uchun x ni y orqali ifodalab so'ngra x va y larni o'rinlarini o'zaro almashtirish kerak. $y = f(x)$funksiyaga teskari funksiyani$y = f^{-1}(x)$ko'rinishda yoziladi.

Agar $y = f(x)$ va $y = f^{-1}(x)$ funksiyalar o'zaro teskari funksiyalar bo'lsa, u holda $y = f(x)$ning aniqlanish sohasi $y = f^{-1}(x)$ uchun qiymatlar sohasi, qiymatlar sohasi esa $y = f^{-1}(x)$uchun aniqlanish sohasi bo'ladi.

O'zaro teskari funksiyalar grafiklari y=x to'g'ri chiziqqa nisbatan simmetrik bo'ladi. $y = sinx$ funksiyaga teskari funksiyani topish masalasi bilan shug'allanamiz. Bu funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda $y = sinx$ funksiyaga teskari funksiya mavjud emas.$y = sinx$ funksiya$[-\frac{\pi}{2};\frac{\pi}{2}]$

kesmada monoton bo'lganligi uchun, bu kesmada unga teskari bo'lgan funksiyaga o'tish mumkin.

$[-\frac{\pi}{2};\frac{\pi}{2}]$kesmada funksiya –1 dan 1 gacha o'sadi. Demak, x vay ning qiymatlari o'zaro bir qiymatli moslik orqali bog'langan. Moslik o'zaro bir qiymatli bo'lgani sababli, u ning [-1;1] kesmadagi har bir qiymatiga x ning $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmadagi bitta qiymati mos keladi. Demak, bu holda yangi funksiya tuzish mumkin.

Ta'rif: $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada qaralayotgan $y = sinx$ funksiyaga teskari bo'lgan funksiya arksinus deyiladi. Bu funksiya $y = arcsinx$ kabi yoziladi

$y = arcsinx$ ifoda$[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada olingan yoydan iborat bo'lib, uning sinusi x ga teng, ya'ni $\sin(arcsinx) = x$

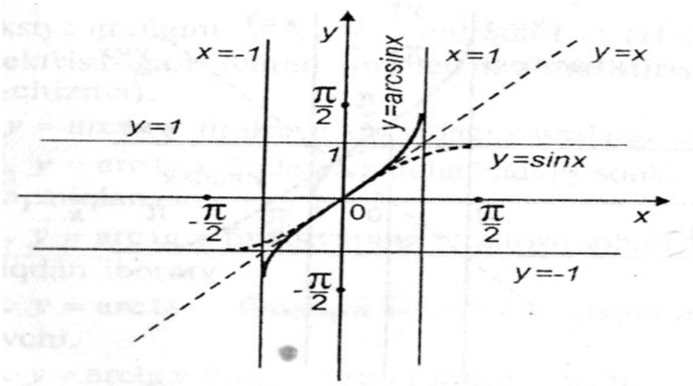
$y = arcsinx$ funksiya quyidagi xossalarga ega :

a) $y = arcsinx$ funksiya [-1 ;1] kesmada aniqlangan

b) $y = arcsinx$ funksiyaning bosh qiymati $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmada o' zgaradi

c) $y = arcsinx$ funksiya [-1 ;1] kesmada monoton o' sadi

d) bu funksiya toq funksiyadir, ya' ni $\arcsin(-x) = -arcsinx$



$y = cosx$ funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda $y = cosx$ ga teskari Funksiya mavjud emas. $y = cosx$

$[0;\pi]$kesmada monoton bo'lgani uchun bu kesmada unga teskari bo'lgan Funksiyaga o'tish mumkin.

$[0;\pi]$kesmada $y = cosx$ funksiya 1 dan –1 gacha kamayadi. Ya'ni, bu kesmada x va u ning qiymatlari o'zaro bir qiymatli moslikda. Demak, bu holda yangi Funksiya tuzish mumkin.

Ta'rif: $[0;\pi]$ kesmada qaralayotgan $y = cosx$ ga teskari bo'lgan funksiyani arkkosinus deyiladi va $y = arccosx$ kabi yoziladi.

$y = arccosx$ 0 dan π gacha bo'lgan kesmada olingan yoy ya'ni: $0 \le \arccos x \le \pi$ bo'lib, bu yoyning kosinusi x ga teng: $cos(arccosx) = x$, bunda $-1 \le x \le 1$.

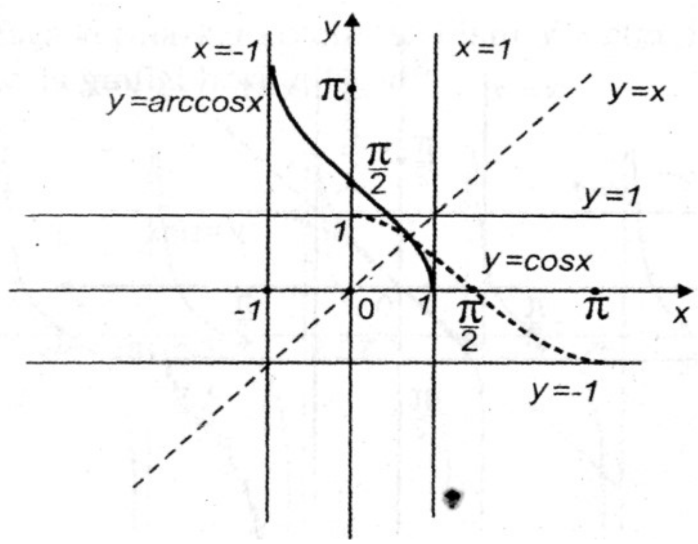
$y = arccosx$ funksiya quyidagi xossalarga ega:

a) $y = arccosx$ funksiya [-1;1] kesmada aniqlangan

b) funksiyaning bosh qiymati [0;π] kesmada o' zgaradi

c) $y = arccosx$ funksiya [-1;1] kesmada kamayadi

d) $y = arccosx$ funksiya toq ham emas, juft ham emas



Amaliy mashg'ulot

Quyidagilarni mustaqil yeching

Hisoblang. $\frac{\log_3 24}{\log_{72} 3} - \frac{\log_3 216}{\log_8 3}$

A) 1 B)0 C)3 D)2

2. Tenglamaning ildizlari yig'indisini toping. $x \cdot 2^{\log_x 5} = 10$ A)12 B)10 C)3 D)7

3. Tengsizlikni [-10:20] oraqliqda nechta butun yechimi bor.

$$\log_{\sqrt{x+1}+\sqrt{x-1}}(x^2 - 3x + 1) \geq 0$$

A)16 B)17 C)18 D)19

4. Tenglamaning eng katta manfiy ildizini toping. $\sqrt{3}cosx = sinx$ A)-120^0 B)-150^0 C)-60^0 D)-30^0

5. Hisoblang $\sin(\arccos\frac{1}{3} + \arcsin\frac{3}{4})$

A)$\frac{6\sqrt{2}+\sqrt{7}}{12}$ B) $\frac{4\sqrt{7}+3\sqrt{2}}{12}$ C) $\frac{2\sqrt{7}+2}{12}$ D) $\frac{2\sqrt{14}+3}{12}$

6. $y = cos(x\sqrt{2}) + cos\frac{x}{\sqrt{2}}$ funksiyaning eng kichik musbat davrini toping.

$A)2\pi$ $B)$ 3π $C)$ $2\pi\sqrt{2}$$D)$ $3\pi\sqrt{3}$

7. Agar $sinx + cosx = a$ bo'lsa, $\frac{\sin^3 x+\cos^3 x}{(a^2-3)a}$ ning qiymatini toping. A)1/3 B) -1/2 C) 1/2 D) 2/5

8. a=sin1; b=sin2; c=sin3; d=sin4 va e=sin5 sonlarni kamayish tartibida joylashtiring.

A) a>b>c>d>e B) e>b>a>d>c

C) b>c>a>d>eD) b>a>c>d>e

9. $tga = 2$ bo'lsa, $\frac{2}{3+4cos2a}$ =?

A)-10/3 B) -10/27 C) 10/27 D) 10/3

10. Ifodaning qiymatini toping.

$$\left(\frac{tg^2 49^0 - tg^2 11^0}{1 - tg^2 49^0 \cdot tg^2 11^0} \cdot tg52^0\right)^4$$

A) 9 B) 1/9 C) 81 D) 1/81

11. $\sqrt{2cosx + 1} = \sqrt{3}cosx$ tenglamani yeching.

A) $\pi n, n\epsilon Z$ B) $2\pi n, n\epsilon Z$ C) $\frac{\pi n}{2}, n\epsilon Z$ D) $\frac{\pi}{2} + \frac{\pi n}{4}, n\epsilon Z$

12. Tenglamani yeching. $\frac{\sqrt{1+sin}}{cosx} = 1$

A)$\pi n, n\epsilon Z$ B) $2\pi n, n\epsilon Z$ C) $\frac{\pi n}{2}, n\epsilon Z$ D) $\frac{\pi}{2}+\frac{\pi n}{4}, n\epsilon Z$

13. Tenglamaning eng kichik musbat va eng katta manfiy yechimlari yig'indisini toping.

$$sin\frac{x}{3}\left(tg\frac{x}{4}-1\right)=0$$

A) -2π B)-3π C)0   D)2π E)3π

14. Tengsizlikni yeching.

$$(cosx+2)|x-5|(x-2)\leq 0$$

A)$(-\infty;2]\cup\{5\}$ B)$(-\infty;2]$

C)$[2;5]$ D)$\{5\}$ E)$\emptyset$

15. $P(-3;0)$ nuqtani koordinata boshi atrofida 270^0ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

A)(O;3) B)(-3;0) C)(0;-3) D)(3;0)

16. $ctga+tga=p$ bo'lsa, $tg^2a+ctg^2a=?$

A) p^2-2 B)$-p^2+2$ C) p^2+2 D) p^2-1 E) p^2+1

17. Tengsizlikni yeching.

$$(e-\pi)x>7(\pi-e)$$

A)$(-\infty;-7)$B)$(-\infty;7)$C)$(7;\infty)$D)$(-7;\infty)$E)$(-\infty;\infty)$

18. Soddalashtiring. $(ctga-cosa)\cdot\left(\frac{\sin^2 a}{cosa}+tga\right)$

A) $\sin^2 a$ B) tga C) $\frac{1}{cosa}$ D) ctg^2a E) $\cos^2 a$

19. Tenglamani yeching.

$$x\cdot cos50^0+sin50^0+x=0$$

A) $sin25^0$ B) $-tg25^0$ C) $-cos25^0$ D) $ctg25^0$ E)1

20. $sinx+cosx=0{,}5$ bo'lsa, $16(\sin^3 x+\cos^3 x)$ ni toping. A)8 B)14 C)12D)16 E)11

21. Hisoblang. $tg\frac{\pi}{6}\cdot\sin\frac{\pi}{3}\cdot ctg\frac{5\pi}{4}$

A) 1,5 B) $\frac{\sqrt{3}}{4}$ C) $-\frac{1}{2}$ D) 0,5 E) 0,75

22. Soddalashtiring. $\frac{sina+cosa}{\sqrt{2}\cos\left(a-\frac{\pi}{4}\right)}$

A) 1,6 B) 1,5 C) 1 D) $ctg\left(\frac{\pi}{4}+a\right)$ E) $tg\left(\frac{\pi}{4}+a\right)$

23. Hisoblang. $(tg60^0\cdot cos15^0-sin15^0)\cdot 7\sqrt{2}$

A)16 B)12   C)18 D)14 E)10

24. Hisoblang. $cos92^0\cdot cos2^0+\frac{1}{2}\cdot sin4^0+1$

A) $\frac{1}{2}$ B) 1 C) 0 D) 2 E) $-\frac{\sqrt{2}}{2}$

Teskari trigonometrik funksiyalar. xossalari va grafigi

$y = tgx$ funksiya $(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi)$ oraliqlarning har birida $-\infty$ dan $+\infty$ gacha o'sadi. Shuning uchun bu oraliqlarning har birida $y = tgx$ ga teskari funksiyaga o'tsa bo'ladi.

Ta'rif: $(-\frac{\pi}{2}; \frac{\pi}{2})$ oraliqda $y = tgx$ ga nisbatan teskari bo'lgan funksiya *arktangens* deyiladi va $y = arctgx$ kabi yoziladi.

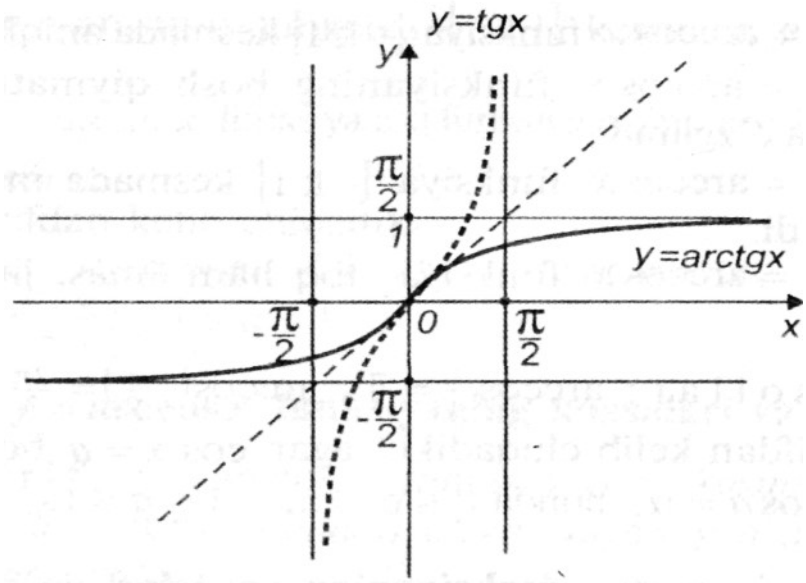
$y = arctgx$ $(-\frac{\pi}{2}; \frac{\pi}{2})$ oraliqda olingan yoy, ya'ni $-\frac{\pi}{2} < arctgx < \frac{\pi}{2}$ bo'lib, uning tangensi x ga teng. Bu yerda x-istalgan haqiqiy son.

$y = arctgx$ quyidagi xossalarga ega.

1^0. $y = arctgxx$ ning barcha qiymatlarida aniqlangan, o'suvchi funksiyadir.

2^0. $y = arctgx$ toq funksiyadir: $arctg(-x) = -arctgx$

$y = tgx$ funksiyani grafigini yasash uchun $x = tgy$ tangensoidaning tarmog'ini yasash kifoyadir.



1-chizma

Ta'rif $(0; \pi)$ oraliqda $y = ctgx$ ga nisbatan teskari bo'lgan funksiyani arkkotangens deyiladi va $y = arcctgx$ kabi yoziladi.

Mavzu 16: Trigonometrik tenglamalar. Arkfunksiyalar qatnashgan tenglamalar. Trigonometrik tenglamalar sistemasi. Trigonometrik tengsizliklar va tengsizliklar sistemasi

Trigonometrik tenglamalar va tengsizliklarni yechishda va funksiyalarni tekshirishda trigonometrik funksiyalarning xossalarini bilish muhim ahamiyatga ega. Shuning uchun $y = sinx$ funksiyaning xossalarini keltiramiz:

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat. Demak, $y = sinx$ funksiya chegaralangan.

3. Funksiya toq. Chunki $\sin(-x) = -sinx$; $(x \in R)$

4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2π ga teng. Ya'ni $x \in R$ lar uchun $\sin(x + 2\pi) = sinx$

5. $x = \pi k$, $k \in Z$ da $sinx = 0$.

6. $x \in (2\pi k; \pi + 2\pi k)$, $k \in Z$ da $sinx > 0$.

7. $x \in (\pi + 2\pi k; 2\pi + 2\pi k)$, $k \in Z$ da $sinx > 0$.

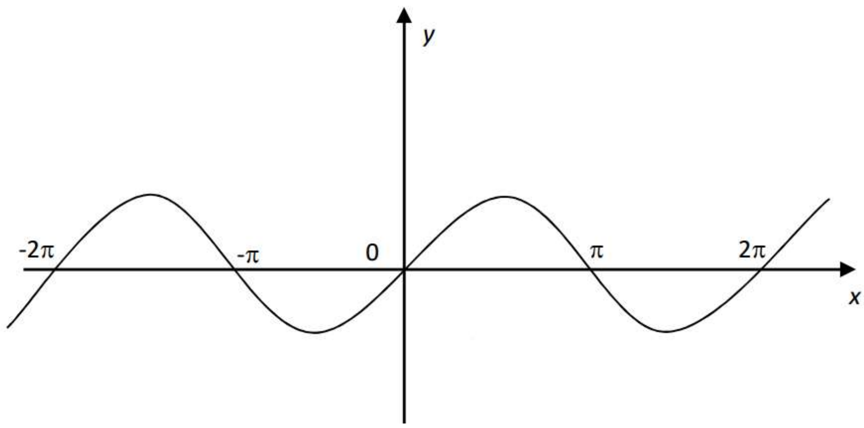
8. Funksiya $\left[-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada –1 dan 1 gacha o'sadi.

9. Funksiya $\left[\frac{\pi}{2} + 2k\pi; \frac{3\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada 1 dan -1 gacha kamayadi.

10. Funksiya $x = \frac{\pi}{2} + k\pi$, $k \in Z$ nuqtalarda 1 ga teng eng katta qiymatga erishadi.

11. Funksiya $x = \frac{3\pi}{2} + 2k\pi$, $k \in Z$ nuqtalarda -1 ga teng eng kichik qiymatga erishadi.

funksiyaning yuqoridagi xossalariga asoslanib $[-\pi;\pi]$ kesmada, ya'ni uzunligi 2π ga teng kesmada uni davriyligini e'tiborga olib esa butun sonlar to'g'ri chizig'ida grafigini yasash mumkin.

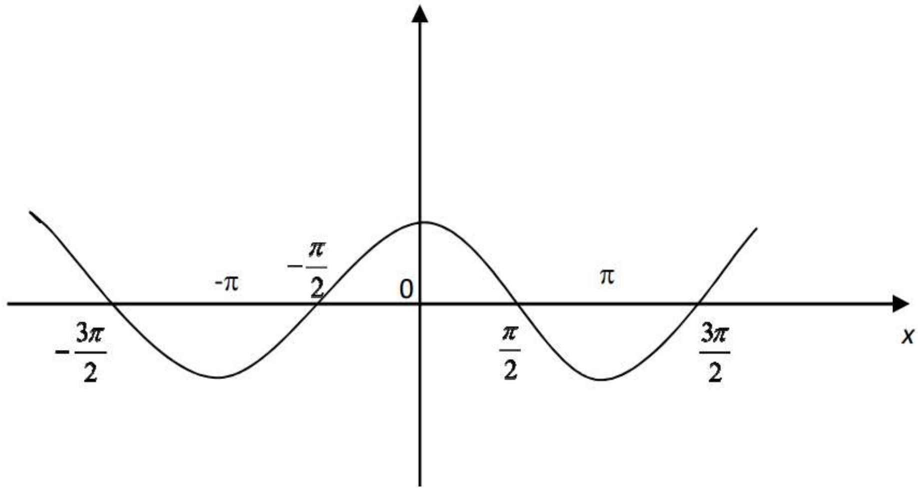


1-chizma

Xuddi shunday $y = cosx$ funksiyaning xossalarini keltirsak:

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.
2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat.
3. Funksiya juft, chunki $\cos(-x) = cosx$.
4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2π ga teng. Ya'ni $x \in R$ lar uchun $\cos(x + 2\pi) = cosx$
5. Barcha $x = \frac{\pi}{2} + k\pi$, $k \in Z$ larda $cosx = 0$.
6. xning $(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi)$, $k \in Z$ qiymatlarida musbat
7. x ning $(\frac{\pi}{2} + 2k\pi; \frac{3\pi}{2} + 2k\pi)$, $k \in Z$ qiymatlarida manfiy
8. Funksiya $(2k\pi; \pi + 2k\pi)$, $k \in Z$ da 1 dan –1 gacha kamayadi.
9. Funksiya $(-\pi + 2k\pi; 2k\pi)$, $k \in Z$ da -1 dan 1 gacha o'sadi.
10. Funksiya $x = 2k\pi$, $k \in Z$ nuqtalarda 1 ga teng
11. Funksiya $x = \pi + 2k\pi$, $k \in Z$ nuqtalarda -1 ga teng eng kichik qiymatni qabul qiladi.

ning bu xossalaridan foydalanib dastlab uni grafigini $[-\pi;\pi]$ da so'ngra butun sonlar to'g'ri chizig'ida yasash mumkin.



2-chizma

$y = tgx$ funksiyaning xossalariquyidagilar:

1. Funksiyaning aniqlanish sohasi $x = \frac{\pi}{2} + k\pi$, $k \in Z$ dan farqli barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar to'plami barcha haqiqiy sonlar to'plamidan iborat.

3. Funksiya toq, chunki aniqlanish sohasidan olingan barcha x lar uchun $tg(-x) = -tgx$.

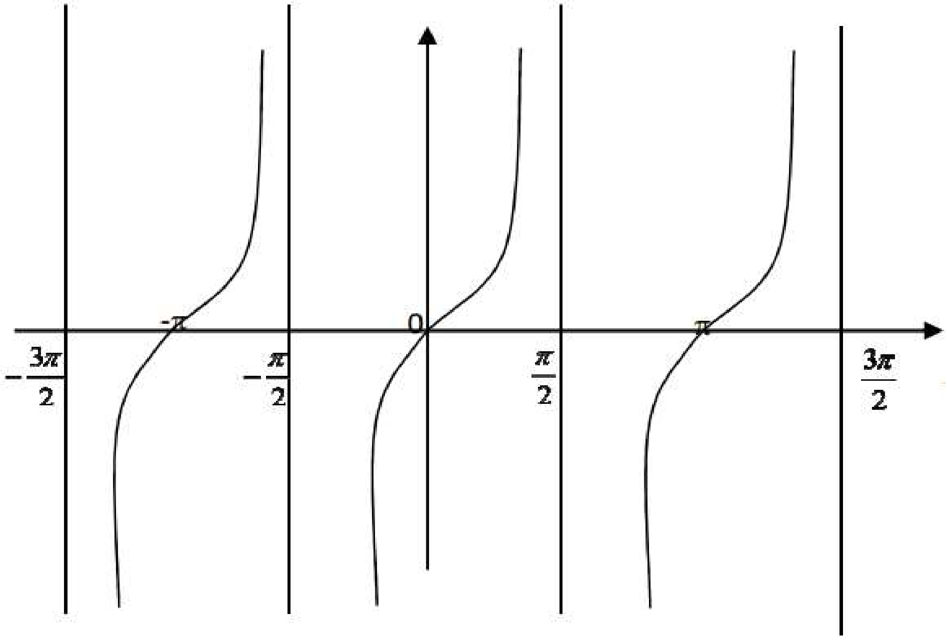
4. Funksiya davriy bo'lib, uning eng kichik musbat davri π ga teng. Ya'ni $x \in R$ lar uchun $tg(x + \pi) = tgx$

5. Barcha x=k π, $k \in Z$ nuqtalarda $tgx = 0$.

6. $(k\pi; \frac{\pi}{2} + k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda tgx >0.

7. $(-\frac{\pi}{2} + k\pi, k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda $tgx < 0$.

8. Funksiya $(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi)$, $k \in Z$ oraliqda o'suvchidir. Yuqoridagi xossalarga asoslanib dastlab, $(-\frac{\pi}{2};\frac{\pi}{2})$ oraliqda, so'ngra butun sonlar o'qida $y = tgx$ funksiyani grafigini yasash mumkin.



3-chizma

$y = ctgx$ funksiyaning xossalari:

1. Funksiyaning aniqlanish sohasi $x = k\pi$, $k \in Z$ dan farqli barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi sonlar o'qining barcha nuqtalari to'plamidan iborat. Ya'ni funksiya chegaralanmagan.

3. Funksiya toq, Chunki aniqlanish sohasidan olingan barcha x larda $ctg(-x) = -ctgx$

4. Funksiya π davrli davriy Funksiyadir. Chunki aniqlanish sohasidan olingan barcha x larda $ctg(x+\pi) = ctgx$

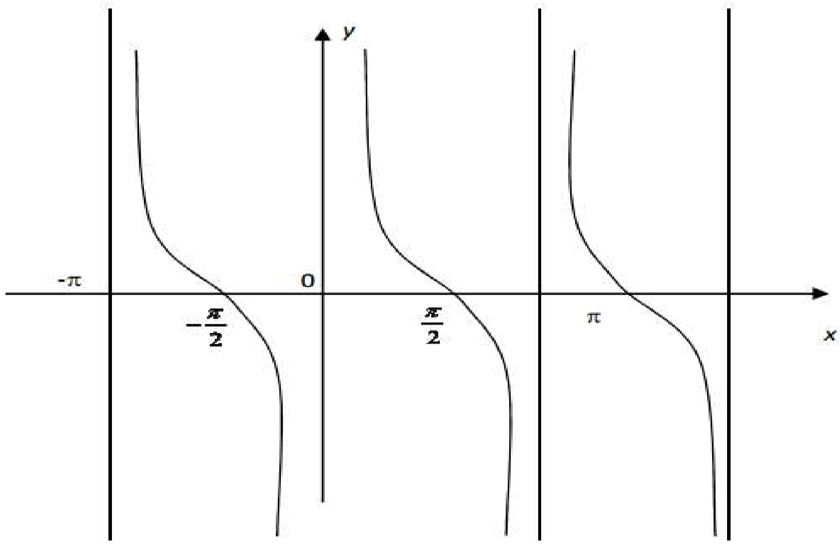
5. $x = \frac{\pi}{2}+k\pi$, $k \in Z$ nuqtalarda $ctgx = 0$.

6. $x \in (k\pi;\frac{\pi}{2}+k\pi)$, $k \in Z$ nuqtalarda $ctgx > 0$

7. $x \in (-\frac{\pi}{2} + k\pi; k\pi)$, $k \in Z$ nuqtalarda $ctgx < 0$.

8. Funksiya $(k\pi; \pi + k\pi)$ oraliqlarda kamayuvchidir.

Funksiyaning yuqoridagi xossalaridan foydalanib dastlab $(0;\pi)$ oraliqda so'ngra butun koordinatalar to'g'ri chizig'ida kotangensni grafigini yasash mumkin.



4-chizma

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Na'munaviy misol yechimlari

1-misol. $(1-\sin a)(1+\sin a)-\cos^2 a$ ifodani soddalashtiring.

I-usul. $(1-\sin a)(1+\sin a)-\cos^2 a=1-\sin^2 a-\cos^2 a=1-(1-\cos^2 a)-\cos^2 a=$
$=1-1+\cos^2 a-\cos^2 a=1-1+\cos^2 a-\cos^2 a=0.$

II- usul. $(1-\sin\alpha)(1+\sin\alpha)-\cos^2\alpha = \ =1-\sin^2\alpha-\cos^2\alpha=1-(\sin^2\alpha+\cos^2\alpha)=1-1=0.$

2 misol. $\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}$ ifodani soddalashtiring.

Yechish. $\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}=\frac{(\sin^2 x)^2+\cos^4 x-1}{(\sin^2 x)^3+\cos^6 x-1}=$

$$=\frac{(1-\cos^2 x)^2+\cos^4 x-1}{(1-\cos^2 x)^3+\cos^6 x-1}=\frac{1-2\cos^2 x+\cos^4 x+\cos^4 x-1}{1-3\cos^2 x+3\cos^4 x-\cos^6 x+\cos^6 x-1}=$$

$$=\frac{2\cos^2 x(\cos^2 x-1)}{3\cos^2 x(\cos^2-1)}=\frac{2}{3}.$$

3 misol $\frac{1-(\sin x-\cos x)^2}{1+\sin^2 x-\cos^2 x}$ ifodani soddalashtiring.

Yechish. $\frac{1-(\sin x-\cos x)^2}{1+\sin^2 x-\cos^2 x}=\frac{1-\sin^2 x+2\sin x\cos x-\cos^2 x}{\sin^2 x+\cos^2 x+\sin^2 x-\cos^2 x}=$

$$=\frac{1-(\sin^2 x+\cos^2 x)+2\sin x\cos x}{2\sin^2 x}=\frac{2\sin x\cos x}{2\sin^2 x}=\frac{\cos x}{\sin x}=ctgx.$$

4-misol. $\frac{1}{\cos^2 x}-\frac{1}{ctg^2 x}-\frac{\sin^2 x}{tg^2 x}$ ifodani soddalashtiring.

Yechish. $\frac{1}{\cos^2 x}-\frac{1}{ctg^2 x}-\frac{\sin^2 x}{tg^2 x}=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}-\frac{\sin^2 x\cos^2 x}{\sin^2 x}=$

$$=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}-\cos^2 x=\frac{1-\sin^2 x-\cos^4 x}{\cos^2 x}=\frac{\cos^2 x-\cos^4 x}{\cos^2 x}=$$

$$=\frac{\cos^2 x(1-\cos^2 x)}{\cos^2 x}=\sin^2 x.$$

5-misol. $\frac{[\cos(-a)+\sin(-a)]^2-1}{\cos^2(-a)+\sin^2(-a)-1}$ ifodani soddalashtiring.

$$\frac{[\cos(-a)+\sin(-a)]^2-1}{\cos^2(-a)+\sin^2(-a)-1}=\frac{(\cos a-\sin a)^2-1}{\cos^2 a-\sin^2 a-1}=$$

$$=\frac{\cos^2 a-2\cos a\sin a+\sin^2 a-1}{\cos^2 a-\sin^2 a-(\cos^2 a+\sin^2 a)}=\frac{-2\cos a\sin a}{-2\sin^2 a}=ctga.$$

6-misol. $1+\sin\alpha+\cos\alpha$ ifodani ko'paytma shakliga keltiring.

Yechish. $1+\sin\alpha+\cos\alpha=(1+\cos\alpha)+\sin\alpha=$

$$=2\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}=2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)=$$

$$=2\cos\frac{\alpha}{2}\left(\sin\left(90^0-\frac{\alpha}{2}\right)+\sin\frac{\alpha}{2}\right)=2\cos\frac{\alpha}{2}\cdot 2\sin 45^0\cdot$$

$$\cdot\cos\left(45^0-\frac{\alpha}{2}\right)=2\sqrt{2}\cos\frac{\alpha}{2}\cdot\cos\left(45^0-\frac{\alpha}{2}\right).$$

7-misol. $\sqrt{3}-2\sin\alpha$ ifodani ayniy almashtirish orqali ko'paytma shakliga keltiring.

Yechish. $\sqrt{3}-2\sin\alpha=2\left(\frac{\sqrt{3}}{2}-\sin\alpha\right)=2\left(\sin 60^0-\sin\alpha\right)=$

$$=4\sin\left(30^0-\frac{\alpha}{2}\right)\cdot\cos\left(30^0+\frac{\alpha}{2}\right).$$

8-misol. $\dfrac{\sin\alpha+2\sin 3\alpha+\sin 5\alpha}{\sin 3\alpha+2\sin 5\alpha+\sin 7\alpha}$ ifodani soddalashtiring.

Yechish. $\dfrac{\sin\alpha+2\sin3\alpha+\sin5\alpha}{\sin3\alpha+2\sin5\alpha+\sin7\alpha}=\dfrac{(\sin\alpha+\sin5\alpha)+2\sin3\alpha}{(\sin3\alpha+\sin7\alpha)+2\sin5\alpha}=$

$$=\frac{2\sin 3\alpha\cos 2\alpha+2\sin 3\alpha}{2\sin 5\alpha\cos 2\alpha+2\sin 5\alpha}=\frac{2\sin 3\alpha(\cos 2\alpha+1)}{2\sin 5\alpha(\cos 2\alpha+1)}=\frac{\sin 3\alpha}{\sin 5\alpha}.$$

9-misol. $1-\cos\left(\frac{\alpha}{2}-3\pi\right)-\cos^2\frac{\alpha}{4}+\sin^2\frac{\alpha}{2}$ ifodani soddalashtiring.

Yechish. $1-\cos\left(\frac{\alpha}{2}-3\pi\right)-\cos^2\frac{\alpha}{4}+\sin^2\frac{\alpha}{4}=1-\cos\left(\frac{\alpha}{2}-3\pi\right)-$

$$-\left(\cos^2\frac{\alpha}{4}-\sin^2\frac{\alpha}{4}\right)=1+\cos\frac{\alpha}{2}-\cos\frac{\alpha}{2}=1.$$

10-misol. $\sin^4\alpha+\cos^4\alpha-\sin^6\alpha-\cos^6\alpha-\sin^2\alpha\cdot\cos^2\alpha$ ifodani soddalashtiring.

Yechish. $\sin^4\alpha+\cos^4\alpha=\left(\sin^2\alpha+\cos^2\alpha\right)^2-2\sin^2\alpha\cos^2\alpha=1-2\sin^2\alpha\cdot\cos^2\alpha,$

$$\sin^6\alpha+\cos^6\alpha=\left(\sin^2\alpha+\cos^2\alpha\right)^3-3\sin^4\alpha\cdot\cos^2\alpha-3\sin^2\alpha\cdot\cos^4\alpha=$$
$$=1-3\sin^2\alpha\cos^2\alpha\left(\sin^2\alpha+\cos^2\alpha\right)=1-3\sin^2\alpha\cdot\cos^2\alpha,$$
$$\sin^4\alpha+\cos^4\alpha-\sin^6\alpha-\cos^6\alpha-\sin^2\alpha\cos^2\alpha=$$
$$=1-2\sin^2\alpha\cos^2\alpha-\left(1-3\sin^2\alpha\cos^2\alpha\right)-\sin^2\alpha\cos^2\alpha=$$
$$=1-2\sin^2\alpha\cos^2\alpha-1+3\sin^2\alpha\cos^2\alpha-\sin^2\alpha\cos^2\alpha=0.$$

11-misol. $A=\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}$ ifodani hisoblang.

Yechish. $A=\left(\sin^2\frac{\pi}{16}\right)^2+\left(\sin^2\frac{3\pi}{16}\right)^2+\left(\sin^2\frac{5\pi}{16}\right)^2+\left(\sin^2\frac{7\pi}{16}\right)^2=$

$$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{8}\right)^2+\left(1-\cos\frac{3\pi}{8}\right)^2+\left(1-\cos\frac{5\pi}{8}\right)^2+\left(1-\cos\frac{7\pi}{8}\right)^2\right]=$$

$$=\frac{1}{4}\left[4-2\left(\cos\frac{\pi}{8}+\cos\frac{3\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{7\pi}{8}\right)+\right.$$

$$\left.+\left(\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}\right)\right]=1-\frac{1}{2}\times$$

$$\times 2\left(2\cos\frac{\pi}{2}\cos\frac{3\pi}{8}+2\cos\frac{\pi}{2}\cos\frac{\pi}{2}\right)+$$

$$+\frac{1}{4}\cdot\frac{1}{2}\left(4+\cos\frac{\pi}{4}+\cos\frac{3\pi}{4}+\cos\frac{5\pi}{4}+\cos\frac{7\pi}{4}\right)=1+\frac{1}{2}=\frac{3}{2}=1\frac{1}{2}.\Big)$$

QUYIDAGI IFODALARNI HISOBLANG:

30. $cos^2\frac{\alpha}{2}co\sec^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\cdot co\sec^2\frac{\alpha}{2}.$ *j.* $co\sec^2\frac{\alpha}{2}.$

31. $\frac{\cos\alpha}{1-\cos\alpha}-\frac{\cos\alpha}{1+\cos\alpha}-2ctg^2\alpha.$ *j.* 0.

32. $\frac{1+2\sin 2x\cdot\cos 2x}{\cos 2x+\sin 2x}-\cos 2x$ *j.* $\sin 2x.$

33. $\left(tg^2\alpha-\sin^2\alpha\right)ctg^2\alpha+\cos^2\alpha.$ *j.* 1.

34. $\frac{\sin^2\alpha-\sin^2\beta}{\sin^2\alpha\cdot\sin^2\beta}+ctg^2\alpha+\cos^2\beta+\sin^2\beta.$ $j.\cos ec^2\beta.$

35. $\sqrt{\frac{\cos ec^4\alpha-ctg^4\alpha}{\sin^2\alpha+2\cos^2\alpha}}$ *j.* $\frac{1}{|\sin\alpha|}=|\cos ec\alpha|.$

36. $\frac{co\sec^2\alpha-1}{cos^2 2\alpha-\sin^2\alpha+\sin^2 2\alpha}.$ *j.* $co\sec^2\alpha$

37. $\frac{\sin^3\alpha-\cos^3\alpha}{\cos\alpha-\sin\alpha}-\sin\alpha\cdot\cos\alpha$ *j.* $-1.$

38. $\frac{\sin^4 3\alpha-\cos^4 3\alpha}{\sin^2 3\alpha-\cos^2 3\alpha}+tg^2\alpha.$ *j.* $\sec^2\alpha.$

39. $\frac{\sin^2\beta-tg^2\beta}{\cos ec^2\beta-ctg^2\beta}\cdot ctg^2\beta.$ *j.* $-\sin^2\beta.$

40. $\sin\alpha(1+tg\alpha)+\cos\alpha(1+ctg\alpha)$ $j.\cos ec\alpha+\sec\alpha.$

41. $\frac{\cos ec\alpha-\sin\alpha}{\cos\alpha}\cdot\frac{\sec\alpha-\cos\alpha}{\sin\alpha}$ $j.1.$

42. $\frac{\sqrt{2}\cos\alpha-2\cos(45^0-\alpha)}{\sqrt{3}\sin\alpha-2\sin(30^0+\alpha)}$ $j.\sqrt{2}tg\alpha.$

43. $\frac{\sin(\alpha-\beta)-2\cos\alpha\sin\beta}{\cos(\alpha-\beta)-2\cos\alpha\cos\beta}$ $j.tg(\alpha+\beta).$

44. $\frac{\cos(\alpha-\beta)\cos(\alpha+\beta)-\sin(\alpha+\beta)\sin(\alpha-\beta)}{\sin 2\alpha}$ $j.ctg2\alpha.$

45. $\dfrac{\sqrt{2}cos\alpha - 2\cos\left(\dfrac{\pi}{4}+\alpha\right)}{2\sin\left(\dfrac{\pi}{4}+\alpha\right)-\sqrt{2}\sin\alpha}$ $\quad j.\ tg\alpha.$

46. $\dfrac{\sin\left(\dfrac{\pi}{4}+\alpha\right)\cos\alpha-\cos\left(\dfrac{\pi}{4}+\alpha\right)\sin\alpha}{\cos\dfrac{\pi}{4}}$ $\quad j.\ 1.$

47. $\dfrac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\sin\alpha+\sin\beta}.$ $\quad j.\ \sin\alpha-\sin\beta.$

48. $\left(\dfrac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}\right)^2+\sin^4\alpha+\cos^4\alpha+2\sin^2\alpha\cos^2\alpha$ $\quad j.\ \sec^2(\alpha+\beta)$

49. $\dfrac{\cos^2\alpha-\sin^2\alpha+\cos^2(\alpha-\beta)}{\cos\alpha\cdot\cos\beta\cdot\cos(\alpha-\beta)}.$ $\quad j.\ 2ctg\beta.$

Trigonometrik tenglamalar.

1. $sinx=a$ tenglamada $|a|\le 1$ bo'lsa, u $x=(-1)^k arcsina+\pi k,\ k\in Z$ yechimga ega bo'ladi. Xususiy holda

a) agar $sin\,x = 0$ bo'lsa, $x = \pi k,\ k \in Z;$

b) agar $sin\,x = 1$ bo'lsa, $x = \frac{\pi}{2}+2\pi k,\ k \in Z;$

v) agar $sin\,x = -1$ bo'lsa, $x = -\frac{\pi}{2}+2\pi k,\ k \in Z;$

g) agar $sin^2x = a$ bo'lsa, $x = \pm arcsin\sqrt{a} + \pi k,\ k \in Z;$

Misol. $2\sin\left(\frac{\pi}{4}+x\right)+\sqrt{3}=0$ tenglama yechilsin.

Yechish:

$$\left[2\sin\left(\frac{\pi}{4}+x\right)+\sqrt{3}=0\right]\Leftrightarrow\left[\sin\left(\frac{\pi}{4}+x\right)=\frac{-\sqrt{3}}{2}\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\frac{\pi}{4}+x=(-1)^k\arcsin\left(-\frac{\sqrt{3}}{2}\right)+\pi k\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\frac{\pi}{4}+x=(-1)^k\left(-\frac{\pi}{3}\right)+\pi k\right]\Leftrightarrow$$

$$\Leftrightarrow\left(x=(-1)^{k+1}\cdot\frac{\pi}{3}-\frac{\pi}{4}+\pi k\right),\quad k\in Z$$

2. $cosx=a$ tenglamada $|a|\le 1$ bo'lsa, u $x=\pm arccosa+2\pi k,\ k\in Z;$ yechimga ega bo'ladi. Xususiy holda:

a) agar $cos\,x = 0$ bo'lsa, $x = \frac{\pi}{2}+\pi k,\ k \in Z;$

b) agar $cos\,x = 1$ bo'lsa, $x = 2\pi k,\ k \in Z;$

v) agar $cos\,x = -1$ bo'lsa, $x = \pi+2\pi k,\ k \in Z;$

g) agar $cos^2x = a$ bo'lsa, $x = \pm arccos\sqrt{a}+\pi k,\ k \in Z;$

Misol. $\cos\left(\frac{2}{3}x-\frac{1}{2}\right)-1=0$ tenglama yechilsin.

Yechish.

$$\left[\cos\left(\frac{2}{3}x-\frac{1}{2}\right)-1=0\right]\Leftrightarrow\left[\left(\frac{2}{3}x-\frac{1}{2}\right)=\frac{\pi k}{2}\right]\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{2}{3}x=\frac{1}{2}+\frac{\pi k}{2}\right)\Leftrightarrow\left(x=\frac{3}{4}+\frac{3\pi k}{2}\right),\quad k\in Z.$$

3. $tg\,x=a$ tenglama $x=arctg\,a+\pi k,\ k\in Z$ yechimga ega bo'ladi. Xususiy holda

a) agar $tg\,x=0$ bo'lsa, $x=\pi k,\ k\in Z;$

b) agar $tg\,x=1$ bo'lsa, $x=\frac{\pi}{4}+\pi k,\ k\in Z;$

v) agar $tg\,x=-1$ bo'lsa, $x=-\frac{\pi}{4}+\pi k,\ k\in Z;$

g) agar $tg^2x=a$ bo'lsa, $x=\pm arctg\sqrt{a}+\pi k,\ k\in Z.$

Misol. $3tg^2 3x-1=0$ tenglama yechilsin.

Yechish.

$$\left(tg^2 3x=\frac{1}{3}\right)\Leftrightarrow\left(3x=\pm arctg\frac{1}{\sqrt{3}}+k\pi\right)\Leftrightarrow$$

$$\Leftrightarrow\left(3x=\pm\frac{\pi}{6}+k\pi\right)\Leftrightarrow\left(x=\pm\frac{\pi}{18}+\frac{\pi}{3}k\right)\Leftrightarrow\left(x=\frac{\pi}{18}(6k+1)\right).$$

4. $ctg\,x=a$ tenglama $x=arcctg\,a+\pi k,\ k\in Z$ yechimga ega bo'ladi.

a) agar $ctg\,x=0$ bo'lsa, $x=\frac{\pi}{2}\pi k,\ k\in Z;$

b) agar $ctg\,x=1$ bo'lsa, $x=\frac{\pi}{4}+\pi k,\ k\in Z;$

v) agar $ctg\,x=-1$ bo'lsa, $x=-\frac{\pi}{4}+\pi k,\ k\in Z;$

g) agar $ctg^2x=a$ bo'lsa, $x=\pm arcctg\sqrt{a}+\pi k,\ k\in Z.$

Misol. $ctg^2\left[2x-\frac{\pi}{3}\right]=3$ tenglama yechilsin.

Yechish.

$$\left[ctg^2\left(2x-\frac{\pi}{3}\right)=3\right]\Leftrightarrow\left[ctg\left(2x-\frac{\pi}{3}\right)=\pm\sqrt{3}\right]\Leftrightarrow\left[\left(2x-\frac{\pi}{3}=\pm arcctg\sqrt{3}+\pi k\right)\right]\Leftrightarrow$$

$$\Leftrightarrow\left(2x=\pm\frac{\pi}{6}+\frac{\pi}{3}+\pi k\right)\Leftrightarrow\left(x=\pm\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi k}{2}\right),\qquad k\in z.$$

Matematika kursida har qanday trigonometrik tenglamalar ayniy almashtirishlarni bajarish orqali sodalashtirib, $sin\ x=a,\ cos\ x=a,\ tg\ x=a,\ ctg\ x=a$ ko'rinishdagi eng sodda trigonometrik tenglamalarga keltiriladi.

Trigonometrik tenglamalar quyidagi metodlar yordamida yechiladi.

1. Ko'paytuvchilarga keltirish usuli.

1 - m i s o l. $sin2x = cos2x\ sin2x$ tenglama yechilsin.

Yechish. $sin2x - cos2x\ sin2x = 0,\ sin2x(1-cosx) = 0$

1) Agar $1-cosx\neq 0$ bo'lib, $sin2x=0$ bo'lsa, $x=\frac{\pi}{2}n,\ n\in Z$ bo'ladi.

2) Agar $sin2x\neq 0$ bo'lib, $1-cosx=0$ bo'lsa, $cosx=1,\ x=2\pi n,\ n\in Z$ bo'ladi.

2 - m i s o l. $sin3x - sin\ x = 0$ tenglama yechilsin.

Yechish. $sin3x - sin\ x = 2sin\ x\ cos\ 2x = 0$

1) Agar $cos2x\neq 0$ bo'lib, $sinx=0$ bo'lsa, $x=\pi n,\ n\in Z$

2) Agar $sinx\neq 0$ bo'lib, $cos2x=0$ bo'lsa, $x=\frac{\pi}{4}+\frac{n\pi}{2},\ n\in Z$ bo'ladi.

3-misol. $cos^2x+cos^22x+cos^23x=1,5$ tenglama yechilsin.

Yechish. $cos^2x = \frac{1+\cos 2x}{2}$ formulaga ko'ra

$$\left(\frac{1+\cos 2x}{2}+\frac{1+\cos 4x}{2}+\frac{1+\cos 6x}{2}=\frac{3}{2}\right)\Leftrightarrow(\cos 2x+\cos 4x+\cos 6x=0)\Leftrightarrow[(\cos 2x+\cos 6x)+\cos 4x=0]\Leftrightarrow$$

$$\Leftrightarrow[2\cos 4x\cos 2x+\cos 4x=0]\Leftrightarrow\cos 4x(\cos 2x+1)=0.$$

1) Agar $2cos2x+1\neq 0$ bo'lib, $cos4x=0$ bo'lsa, $x=\frac{\pi}{8}+\frac{n\pi}{4},\ n\in Z;$

2) Agar $cos4x\neq 0$ bo'lib, $2cos2x+1=0$ bo'lsa, $cos2x=-\frac{1}{2},\quad 2x=-\frac{2\pi}{3}+2\pi n;\ x=-\frac{\pi}{3}+\pi n;\ n\in Z.$

II. O'zgaruvchilarni kiritish usuli.

1 - m i s o l. $2cos^2x=3\ sin\ x$ tenglama yechilsin.

Yechish. $(2cos^2x - 3\ sin\ x=0)\Leftrightarrow(3sin\ x - 2(1-sin^2x) = 0$

$\Leftrightarrow (3sin\ x - 2 + 2\ sin^2x = 0).$

Agar $sin\ x = y$ desak,

$$2y^2+3y-2=0,\quad y_1=\frac{1}{2},\quad y_2=-2.$$

$$\left(\sin x=\frac{1}{2}\right)\Leftrightarrow\left(x=\frac{\pi}{6}+2k\pi\right),\quad k\in z.$$

2 - m i s o l. *cos2x – 5 sin x – 3 = 0* tenglama yechilsin.

Yechish. $cos2x=1-2sin^2x$ formulaga ko'ra $(1-2sin^2x-5sinx-3=0)$ $\Leftrightarrow$ $(2sin^2x+5sinx+2=0)\Leftrightarrow sinx=y$ desak, $2y^2+5y+2=0,\ y_1=-2,\ y_2=-\frac{1}{2}$.

1) *sin x =–2* tenglama yechimga ega emas.

2) $\left(\sin x=-\frac{1}{2}\right)\Leftrightarrow\left(x=(-1)^k\arcsin\left(-\frac{1}{2}\right)+\pi k\right), k\in z;$

$$x=(-1)^{k+1}\frac{\pi}{6}+\pi k, k\in Z.$$

III. Bir jinsli tenglamalarni yechish.

1-misol. $2sin^2x-sinxcosx-cos^2x=0$ tenglamani yeching.

Yechish. Bu tenglama sinus va kosinus funksiyalariga nisbatan bir jinslidir. Tenglamalarning har ikki tomonini $cos^2x\neq0$ ga bo'lsak, $2tg^2x-tg\,x-1=0$ hosil bo'ladi. Bundan $tgx=1$ va $tg\,x=-\frac{1}{2}$.

1) Agar $tgx=1$ bo'lsa, $x=\frac{\pi}{4}+\pi k,\ k\in Z;$

2) Agar $tg\,x=-\frac{1}{2}$ bo'lsa, $x=-arctg\frac{1}{2}+\pi k,\ k\in Z$ bo'ladi.

2-m i s o l. $cos^2x+3sin^2x+2\sqrt{3}\,sinxcosx=3$ tenglamani yeching.

Yechish. Bu tenglamani ayniy almashtirishlar bajarish orqali bir jinsli ko'rinishga keltiramiz.

$$\cos^2 x+3\sin^2 x+2\sqrt{3}\sin x\cos x=3(\sin^2 x+\cos^2 x),$$
$$\cos^2 x+3\sin^2 x+2\sqrt{3}\sin x\cos x-3\sin^2 x-3\cos^2 x=0$$
$$2\cos^2 x-2\sqrt{3}\sin x\cos x=0,$$
$$2\cos x(\cos x-\sqrt{3}\sin x)=0.$$

1) Agar $cosx-\sqrt{3}\,sinx\neq0$ bo'lib, $cosx=0$ bo'lsa, $x=\frac{\pi}{2}+k\pi,\ k\in Z;$

2) Agar $cos\,x\neq0$ bo'lib, $cosx-\sqrt{3}\,sin\,x=0$ bo'lsa, $tgx=\frac{1}{\sqrt{3}},\quad x=\frac{\pi}{6}+k\pi,\ k\in Z;$

IV. $a\sin x + b\cos x = c$ ko'rinishdagi tenglamani yeching.

1-usul. Bu tenglamani yechish uchun $tg\frac{x}{2}=t$ almashtirish bajaramiz. Bizga ma'lumki, $\sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$, $\cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$, edi, shunga ko'ra berilgan tenglama quyidagi ko'rinishni oladi:

$$\frac{2at}{1+t^2}+\frac{b(1-t^2)}{1+t^2}=c,$$

$$2at+b-bt^2=c+ct^2, \qquad (b+c)t^2-2at+(c-b)=0,$$

$$t=\frac{a\pm\sqrt{a^2+b^2-c^2}}{c+b},$$

$$x=2arctg\frac{a\pm\sqrt{a^2+b^2-c^2}}{c+b}+2k\pi,\ k\in z,\quad a^2+b^2\geq c^2 \text{ ва } b\neq -c.$$

Agar $b=-c$ bo'lsa, kvadrat tenglama chiziqli tenglamaga almashadi:

$2at+2b=0, \quad t=-\frac{b}{a},\ x=-2arctg\frac{b}{a}+2k\pi,\ k\in Z.$

2-usul. Tenglamaning har ikkala tomonini $\sqrt{a^2+b^2}$ ga bo'lamiz:

$$\frac{a}{\sqrt{a^2+b^2}}\sin x+\frac{b}{\sqrt{a^2+b^2}}\cos x=\frac{c}{\sqrt{a^2+b^2}}$$

$$\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2+\left(\frac{b}{\sqrt{a^2+b^2}}\right)^2=1 \text{ ва } \left|\frac{a}{\sqrt{a^2+b^2}}\right|^2\leq 1, \qquad \left|\frac{b}{\sqrt{a^2+b^2}}\right|^2\leq 1$$

Agar $\frac{a}{\sqrt{a^2+b^2}}=\cos\varphi$ va $\frac{b}{\sqrt{a^2+b^2}}=\sin\varphi$ desak, berilgan tenglama $\sin x\cdot\cos\varphi+\cos x\cdot\sin\varphi=\frac{c}{\sqrt{a^2+b^2}}$ ko'rinishni oladi, bundan $\sin(x+\varphi)\frac{c}{\sqrt{a^2+b^2}}$ bo'ladi. $\varphi=arctg\frac{b}{a}$; agar $a^2+b^2\geq c^2$ bo'lsa,

$$x=(-1)^k\arcsin\frac{c}{\sqrt{a^2+b^2}}+\pi k-arctg\frac{b}{a}, \qquad k\in z.$$

1-misol. $3\cos x+4\sin x=5$ tenglama yechilsin.

Yechish. $\sqrt{3^2+4^2}=\sqrt{25}$ bo'lgani uchun tenglamaning har ikki tomonini 5 ga bo'lamiz: $\frac{3}{5}\cos x+\frac{4}{5}\sin x=1,$ $\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1,$ shuning uchun $\frac{3}{5}=\sin\varphi$ va $\frac{4}{5}=\cos\varphi$ bo'ladi, bundan $\sin\varphi\cdot\cos x+\cos\varphi\sin x=1$ tenglamani hosil qilamiz yoki $\sin(x+\varphi)=1$ bo'ladi:

$$x+\varphi=\frac{\pi}{2}+2k\pi,\qquad x=\frac{\pi}{2}+2k\pi-\varphi,\qquad k\in z,$$

$$\varphi=\arcsin\frac{3}{5},\qquad x=\frac{\pi}{2}-\arcsin\frac{3}{5}+2k\pi,\qquad k\in z.$$

2-usul. Agar $\sin x=\dfrac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}},$ va $\cos x=\dfrac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}},$ ekanligini nazarda tutib, $tg\frac{x}{2}=y$ desak, $3\cdot\frac{1-y^2}{1+y^2}+4\cdot\frac{2y}{1+y^2}=5$, yoki $3-3y^2+8y=5+5y^2$ yoki $4y^2-4y+1=0$ bundan $y=\frac{1}{2}$ yechim hosil bo'ladi:

$$\left(tg\frac{x}{2}=\frac{1}{2}\right)\Leftrightarrow\left(\frac{x}{2}=arctg\frac{1}{2}+\pi k\right),$$

$$x=2arctg\frac{1}{2}+2\pi k,\qquad k\in\pi.$$

MUSTAQIL YECHISH UCHUN MISOLAR.

Trigonometrik tenglamalarni yeching.

1. $tgx+\sin xtgx=0$ $\qquad j:n\pi$
2. $2\sin x-3\cos x=6.$ $\qquad j:x\in\varnothing.$
3. $\sin^2 x-(1+\sqrt{3})\sin x\cos x+\sqrt{3}\cos^2 x=0.$

$$j:\frac{\pi}{4}+n\pi,\qquad arctg3+n\pi$$

4. $\sin^2 x - 4\sin x\cos x + 3\cos^2 x = 0.$ $\quad j: \dfrac{\pi}{4} + n\pi, \quad arctg\, 3 + n\pi.$

5. $\sqrt{3}\sin^2 x - 4\sin x\cos x + \sqrt{3}\cos^2 x = 0.$ $\quad j: \dfrac{\pi}{3} + n\pi, \quad \dfrac{\pi}{6} + k\pi.$

6. $\sin^2 x + 3\cos^2 x - 2\sin x\cos x = \dfrac{-5-\sqrt{3}}{2}.$ $\quad j: \dfrac{\pi}{6} + k\pi$

7. $7\cos^2 x - 7\sin 2x = 2.$ $\quad j: x = arctg\, \dfrac{-7 \pm \sqrt{53}}{2}.$

8. $\dfrac{2}{3\sqrt{2}\sin x - 1} = 1.$ $\quad j: x = (-1)^k \cdot \dfrac{\pi}{4} + k\pi, \quad k \in z.$

9. $\dfrac{6}{tgx - 2} = 3 - tgx$ $\quad j: \varnothing$

10. $(1-2\sin x)\sin x = 2\cos 2x - 1$ $\quad j: x = 2k\pi - \dfrac{\pi}{2}$

11. $\sin^4 x + \cos^4 x - 2\sin 2x + \sin^2 2x = 0$ $\quad j: x = \dfrac{(-1)^k}{2}\arcsin(2-\sqrt{2}) + \dfrac{k\pi}{2}$

12. $\sin^3 x\cos x - \cos^3 \sin x = \cos^4 \dfrac{x}{3}$

$$j: x = \frac{1}{4}\left[(-1)^{k+1}\arcsin\frac{1}{4} + k\pi\right].$$

13. $\sin^4 x + \cos^4 x = \cos 4x$ $\quad j: \dfrac{\pi}{2}n.$

14. $tg(40^0 + x)ctg(5^0 - x) = \dfrac{2}{3}$ $\quad j: x = \dfrac{1}{2}\left[\pi k - 35^0 + (-1)^{k+1}\arcsin\dfrac{\sqrt{2}}{10}\right]$

15. $(\sin x + \cos x)\sqrt{2} = tgx + ctgx$ $\quad J: x = 2k\pi + \dfrac{\pi}{4}$

16. $\sin 3x \cdot \sin^3 x + \cos 3x \cdot \cos^3 x = \dfrac{1}{8}$ $\quad J: x = \pi k \pm \dfrac{\pi}{6}$

17. $\sin x + \sin 3x + \sin 7x = 3$ $\quad J: x = \varnothing$

18. $tg7x + tg3x = 0$ $\quad J: x = \dfrac{\pi k}{10}$

19. $1 + \sin x + \cos x = 0$ $\quad J: x_1 = 2\pi k - \dfrac{\pi}{2},\ x_2 = \pi(2\pi + 1)$

20. $\sin^{10} x + \cos^{10} x = \dfrac{29}{16}\cos^4 2x$ $\quad J: x = \dfrac{2k+1}{8}\pi$

21. $|tgx + ctgx| = \dfrac{4}{\sqrt{3}}$ $\quad J: x = \dfrac{\pi k}{2} \pm \dfrac{\pi}{6}$