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T R I G O N O M E T R I Y A

fani bo`yicha

O'QUV – USLUBIY MAJMUA

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Matematika yo’nalishi talabalarining tanlov fanlari tarkibiga kiruvchi **“Trigonometriya”** fani bo`yicha tayyorlangan mazkur o′quv – uslubiy majmua Namangan davlat universiteti o`quv-uslubiy kengashining 2023 yil ___ avgustdagi yig’ilishi (№ 1 sonli bayonnoma) da ko`rib chiqilgan va nashrga tavsiya qilingan.

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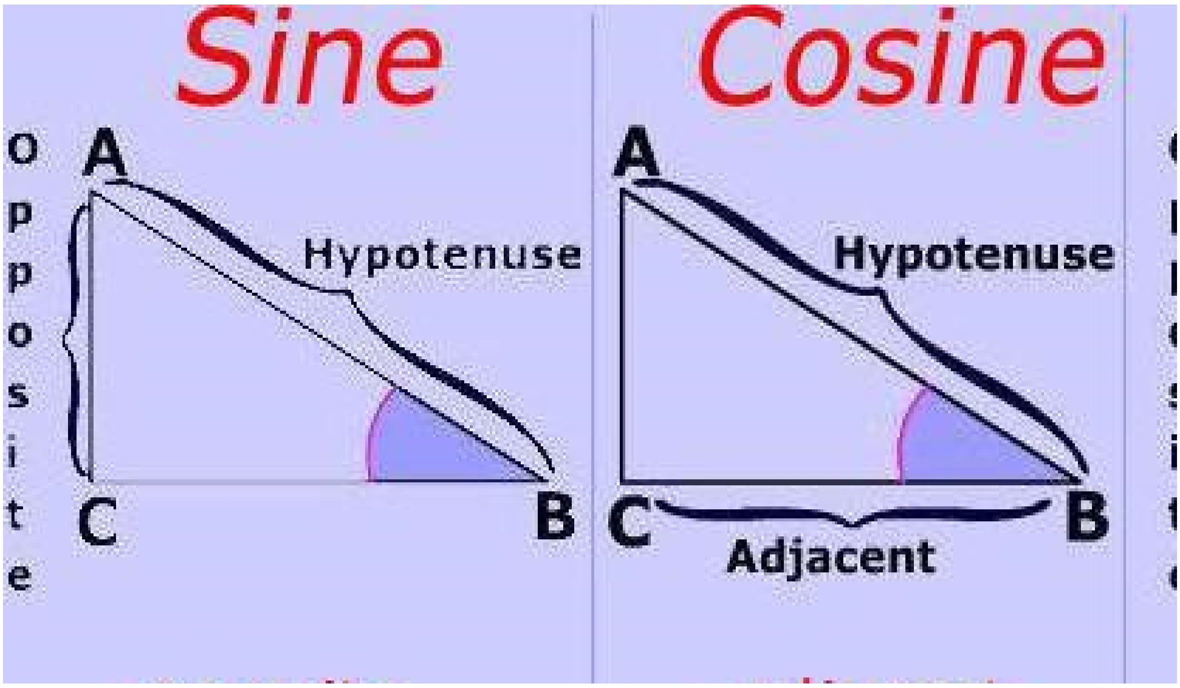
Ushbu o`quv-uslubiy majmua NamDU Matematik analiz kafedrasining 2023 yil «___» avgustdagi № 1 sonli yig′ilishida muhokama etilgan va universitet uslubiy kengashining 2023 yil «___» avgustdagi yig′ilishida № q sonli bayonnoma bilan tasdiqlangan.

MUNDARIJA

1-Mavzu: TRIGONOMETRIYA TARIXI

Trigonometriya tarixi bundan ikki ming yil oldin boshlangan. Dastlab, uning paydo bo'lishi uchburchakning burchaklari va tomonlarining nisbatlarini aniqlashtirish zarurati bilan bog'liq edi.

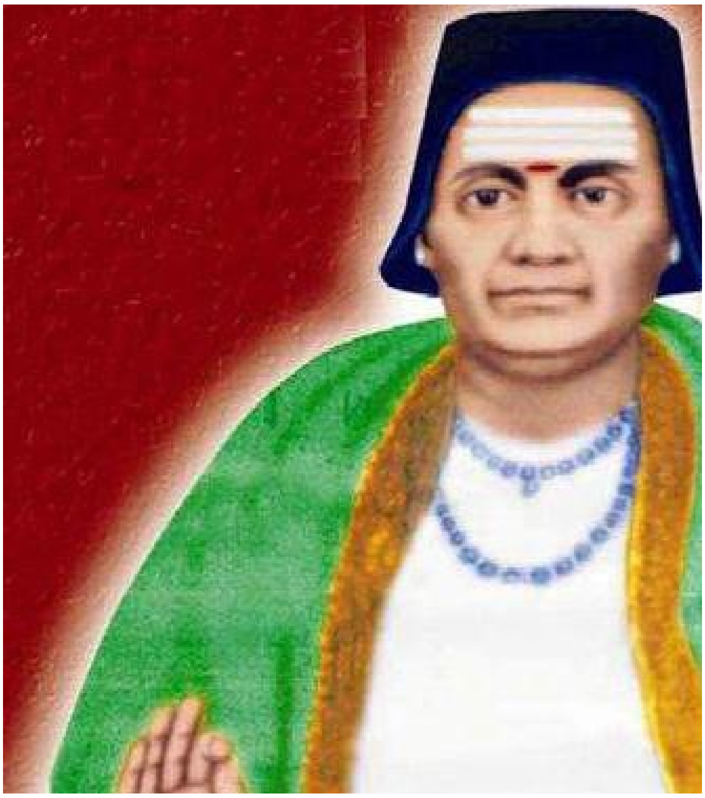
Tadqiqot jarayonida ushbu munosabatlarning matematik ifodasi dastlab raqamli jadvallar shaklida ishlab chiqilgan maxsus trigonometrik funksiyalarni joriy qilishni talab qilishi ma'lum bo'ldi.



Matematika bilan bog'liq ko'plab fanlar uchun trigonometriyani rivojlanishi turtki bo'lgan. Qadimgi Bobil olimlari izlanishlari bilan bog'liq bo'lgan burchaklarni (darajalarni) o'lchash birliklarining kelib chiqishi zamonaviy o'nliklarni keltirib chiqargan olti kasrli hisoblash tizimiga asoslangan bo'lib, ko'plab amaliy fanlarda qo'llaniladi. Dastlab trigonometriya astronomiyaning bir qismi sifatida mavjud bo'lgan deb tahmin qilinadi. Keyin u arxitekturada ishlatila boshlandi. Vaqt o'tishi bilan ushbu fanni inson faoliyatining turli sohalarida qo'llash maqsadga muvofiqligi sezildi. Bu xususan, astronomiya, dengiz va aeronavigatsiya, akustika, optika, elektronika, arxitektura va boshqalar. Ilmiy izlanishlar haqidagi ma'lumotlarga asoslanib, tadqiqotchilar trigonometriya tarixi birinchi bo'lib sferik uchburchaklar yechimini topishni o'ylagan yunon astronomi Gipparxning ishi bilan bog'liq degan xulosaga kelishdi. Gipparxning asarlari miloddan avvalgi II asrga borib taqaladi. Trigonometriyaning paydo bo'lishi va rivojlanishi tarixi bir asrdan ko'proq vaqtga to'g'ri keladi. Matematik fanning ushbu bo'limining asosini tashkil etadigan tushunchalarning kiritilishi ham bir zumda yuz bermadi.

Shunday qilib, “sinus” tushunchasi juda uzoq tarixga ega. Sinus dastlab uchburchaklar va doiralar segmentlari orasidagi turli xil aloqalar haqida miloddan avvalgi III asrga oid ilmiy ishlarda aytilgan. Evklid, Arximed, Perga Apollonius kabi buyuk qadimgi olimlarning asarlarida bu munosabatlarning dastlabki izlanishlari mavjud.

Yangi kashfiyotlar muayyan terminologik izohlarni talab qildi. Shunday qilib, hind olimi Ariabxata sinusga “jiva” deb nom beradi, bu “kamon” degan ma'noni anglatadi. Arab matematiklariga tegishli matnlar lotin tiliga tarjima qilinganida, “jiva” atama o'xshash ma'noga ega bo'lgan “sinus” bilan almashtirildi (ya'ni "egilish"). Kosinus so'zi ancha keyinroq paydo bo'lgan. Ushbu atama lotincha "qo'shimcha sinus" iborasining qisqartmasi. Tangens soyaning uzunligini aniqlash muammosini hal qilish bilan bog'liq holda paydo bo'ldi.Shuningdek, kotangens, sekans va kosekans kiritiladi. X asrda Arab matematigi Abu al-Vafo tangensva kotangeslarni qiymatini topish uchun birinchi jadvallarni tuzgan. Biroq, bu kashfiyotlar uzoq vaqt davomida yevropalik olimlar uchun noma'lum bo'lib qoldi va tangens XIV asrdaqayta kashf etildi. XIV asr avvalida ingliz olimi T.Braverdin, so'ngra nemis matematiki va astronomi Regiomontan (1467) tomonidan tangensni xisoblash jadvali yozilgan. Tangens nomi lotinchadan olingan bo'lib "tegib turish" deb tarjima qilinadi. Zamonaviy belgi "arcsin" va "arctg" 1772 yilda Vena matematigi Sherfer va taniqli fransuz olimi Lagranj asarlarida paydo bo'lgan. Garchi turli xil belgilar orqali Y. Bernulli ularni biroz oldinroq ishlatgan edi. Ammo bu ramzlar faqat XVIIIasr oxirida qabul qilindi. "arc" so'zi lotincha "kelgan" ma'nosini beradi.



Uzoq vaqt davomida trigonometriya geometriyaning bir qismi sifatida rivojlandi. Hozirda biz trigonometrik funksiyalar nuqtai nazaridan shakllantirayotgan faktlar geometrik tushunchalar va bayonotlar yordamida shakllantirildi va isbotlandi. Ehtimol, trigonometriyani rivojlantirish uchun eng katta rag'bat katta amaliy qiziqish bo'lgan astronomiya muammolarini hal qilish bilan bog'liq bo'lgan (masalan, kema joylashishini aniqlash, Oy va Quyosh tutilishni bashorat qilish va boshqalar).

"Trigonometriya" so'zi birinchi marta (1505 yil) nemis ilohiyotshunosi va matematigi Pititusning kitobida uchraydi.Bu so'zning kelib chiqishi yunoncha “uchburchak” va“o’lchov” so’zlatidan olingan. Boshqacha qilib aytganda, trigonometriya - bu uchburchaklar o’lchash fanidir. Bu nom nisbatan yaqinda paydo bo’lgan bo’lsada, hozirgi vaqtda trigonometriya bilan bog’liq bo’lgan ko’plab tushunchalar va dalillar bundan ikki ming yil oldin ma’lum edi.

Trigonometriya matematika kursining muxim bo’limlaridan biri bo’lib, u trigonometrik funksiyalar va ularning xossalari, trigonometrik funksiyalar qatnashgan tenglamalar va tengsizliklarni yechishni o’rganadi. Trigonometrik funksiyalar tabiatshunoslikda uchrab turadigan turli davriy jarayonlarni tavsiflash uchun ishlatiladi. Davriy ravishda takrorlanib turuvchi jarayonlarga insonlar har qadamda duch keladilar. Bularga misol sifatida astronomik hodisalarni keltirish mumkin. Masalan, quyoshning chiqishi va botishi, yil vaqtlarini takrorlanishi, osmondagi yulduzlarning holati, planeta harakati va hokazo.

Yurakning urishi, inson organizmining hayot faoliyati, g’ildirakning aylanishi, gripp epidemiyasining tarqalishi va hokazo jarayonlardagi umumiylik ularning davriyligidir. Bu davriy jarayonlar esa trigonometrik funksiyalar bilan ifodalanadi.

Hozirgi vaqtda trigonometrik funksiyalar yordamida yechiladigan masalalar qadim zamonlarda paydo bo’lgan. Qadimdan bunday masalalarni yecha bilishga jiddiy talablarni astronomiya qo’ygan.Astronomlarni sferada yotgan katta doiralarning yoylaridan tuzilgan sferik uchburchaklarning tomonlari bilan burchaklari orasidagi munosabatlar qiziqtirgan.Ular tekkis uchburchaklarni “yechish”ga doir masalalarga qaraganda murakkabroq masalalarni yechishni yaxshigina uddalaganlar.Bizning trigonometrik jadvallarimiz o’rnida qadimgi matematiklar berilgan uzunliklardagi yoylarni tortib turuvchi vatarlar jadvalini tuzishgan. Eramizdan avvalgi III – II asrlarda grek matematiklari tomonidan tuzilgan bunday qadimiy jadvallar bizgacha yetib kelmagan. Vatar uzunliklari haqidagi bizgacha saqlanib qolgan eng qadimiy jadval Aleksandriyalik astronom Ptolemey (eramizning II asri) tomonidan tuzilgan. Bu jadvallarda aylana vatarlarining uzunliklari 30^0 dan oralatib berilgan. $\sin x, \cos x, tgx$ va $ctgx$ trigonometrik funksiyalar aylanada o'tkazilgan kesmalar uzunliklarining nisbatlari sifatida V – X asr hind va arab matematiklarida uchraydi. Hind matematigi Ariabxata (V asrning oxiri) $\sin^2 x + \cos^2 x = 1$ formulani va hatto yarim burchak sinusi, kosinusi va tangensi formulalarini bilar edi. Bu formulalar unga shu funksiyalarning jadvallarini tuzish uchun xizmat qilgan.

G'arbiy Yevropada trigonometriya va teskari trigonometrik funsiyalarga oid nazariyalar XV – XVI asrlarda rivojlandi. Bunda bir qator natijalar fransuz matematigi F.Vietga (1540-1603) tegishlidir.Differensial hisob paydo bo'lishi bilan trigonometrik funksiyalarning hosilalari uchun formulalar topildi.Bu formulalar I.Nyutonga ma'lum edi.Bu formulalarning geometrik usul bilan chiqarilishini Kotesning (1682 - 1716) ishlaridan topish mumkin.Argument $-\infty$ dan $+\infty$ gacha o'zgarganda trigonometrik funksiyalarning qanday o'zgarishi haqidagi ochiq tasavvurlar D.Vallis (1616 - 1703) ning asarlarida uchraydi. Ammo, umuman aytganda, L.Eyler (1707 - 1783) gacha bo'lgan matematiklar bu xususida uncha katta izchillik ko'rsatmadilar va ba'zi masalalarga bog'liq ravishda trigonometrik funksiyalarning aniqlanish sohalarini turli usullar bilan cheklab qo'ydilar. Son argumentning sonli funksiyalari yoki kesma uzunliklarining burchak kattaligiga yoki yoy uzunligiga bog'liqligi deyilganda nima nazarda tutilishi ochiq emas edi.Trigonometrik funksiyalar nazariyasi hozirgi ko'rinishi L.Eyler asarlari, jumladan uning "Cheksiz kichiklar analiziga kirish" 1748 yildagi kitobidan olindi.

Umuman olganda, matematikaning, xususan trigonometriyaning rivojida nafaqat chet el olimlari, balki o'zimizning buyuk allomalarimiz ham o'zlarini hissalarini qo'shganlar. Bulardan Muhammad al-Xorazmiy, Ahmad al-Farg'oniy, Abu Rayhon Beruniy, Mirzo Ulug'bek, Ali Qushchi, G'iyosiddin Jamshid al-Koshiy kabilarni keltirishimiz mumkin.

Yulduzlarning osmon sferasidagi harakati, sayyoralarning harakatlarini kuzatish, Oy va Quyosh tutilishini oldindan aytib berish va boshqa ilmiy, amaliy ahamiyatga molik masalalar aniq hisoblarni, bu hisoblarga asoslangan jadvallar tuzishni taqozo etar edi. Ana shunday astronomik jadvallar Sharqda “Zij”lar deb atalgan.Muhammad al-Xorazmiy, Abu Rayhon Beruniy, Mirzo Ulug’bek kabi olimlarimizning matematik asarlari bilan birga “Zij”lari ham mashhur bo’lgan, ular lotin va boshqa tillarga tarjima qilingan. Yevropada matematikaning, jumladan astronomiyaning taraqqiyotiga salmoqli ta’sir o’tkazgan.Beruniyning “Qonun Ma’sudiy” asarida sinuslar jadvali 15 minut oraliq bilan, tangenslar jadvali 1^0 oraliq bilan 10^{-8} gacha aniqlikda berilgan. Nihoyatda aniq “Zij”lardan biri Mirzo Ulug’bekning “Ziji Ko’ragoniy” dir. Bunda sinuslar jadvali 1 minut oraliq bilan, tangenslar jadvali 0^0 dan 45^0 gacha 1 minut oraliq bilan, 46^0 dan 90^0 gacha esa 5 minut oraliq bilan 10^{-10} gacha aniqlikda berilgan.

G’iyosiddin Jamshid al-Koshi “Vatar va sinus” haqida risolasida $\sin 1^0$ ni 17 xona aniqligida hisoblaydi ya’ni: $\sin 1^0 = 0{,}01445246437283512$

Trigonometrik funksiyalar umumiy o’rta ta’lim maktablarining 9-sinfida o’rganila boshlanib, keyinchalik akademik litsey va kasb-xunar kollejlarida davom ettiriladi. 9-sinfda o’qitiladigan trigonometrik funksiyalar “Trigonometriya elementlari” deb ataladi. “Trigonometriya elementlari” o’rta maktab matematikasining an’anaviy boblaridan hisoblanadi. Uni o’rganish trigonometriyaning paydo bo’lishi, rivojlanishiga, ayniqsa uning tatbiqlariga xissa qo’shgan allomalarimiz Muxammad Muso al-Xorazimiy, Axmad Farghoniy, Abu Rayxon Beruniy, Mirzo Ulug’beklarni faoliyatlarini o’rganishdan boshlanadi. Keyinchalik burchakning radian o’lchovi nuqtani kordinatalar boshi atrofida burish, burchakning sinusi, kosinusi, tangensi va kotangensi ta’riflari “$sin\alpha$,$cos\alpha$,$tg\alpha$ va$ctg\alpha$ ishoralari”, “Ayni bir burchak sinusi, kosinusi, tangensi va kotangensi orasidagi munosabat”, “Trigonometrik ayniyatlar“, “α va $-\alpha$ burchaklarining sinusi, kosinusi, tangensi va kotangensi” “Qo’shish formulalari “, “Sinuslar yig’indisi va ayrmasini ko’paytmaga almashtirish” mavzulari o’rganiladi. Trigonometrik funksiyalarni o’rganish akademik litsey va kasb-hunar kollejlarida davom ettiriladi hamda trigonometrik tenglama va tengsizliklarni yechish bilan yakunlanadi. Maktab matematika kursining trigonometriya bo'limida juda ko'p ayniy munosabatlar, jumladan, quyidagi munosabatlar o'rganiladi:

1. Trigonometrik funksiyalarning birini ikkinchisi orqali ifodalaydigan ayniy almashtirishlar.

2. Trigonometrik ifodalarni soddalashtirishdagi ayniy almashtirishlar.

3. Trigonometrik ayniyatlarni isbotlashdagi ayniy almashtirishlar.

4. Trigonometrik tenglamalarni yechishdagi ayniy almashtirishlar.

Oliy ta'lim muassasalarida esa trigonometrik va teskari trigonometrik funksiyalrni uzluksizligi, differensilalash, integrallash kabi bir nechta amallar qo'shiladi.

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1-Amaliy mashg'ulot

Na'munaviy misollar

1-misol. $(1-\sin\alpha)(1+\sin\alpha)-\cos^2\alpha$ ifodani soddalashtiring.

1-usul.

$$(1-\sin\alpha)(1+\sin\alpha)-\cos^2 a = 1-\sin^2\alpha-\cos^2\alpha =$$
$$= 1-(1-\cos^2 a)-\cos^2 a = 1-1+\cos^2 a-\cos^2 a = 0.$$

2-usul.
$$(1-\sin\alpha)(1+\sin\alpha)-\cos^2 a = 1-\sin^2\alpha-\cos^2\alpha =$$
$$= 1-(\sin^2\alpha+\cos^2\alpha) = 1-1 = 0;$$

2-misol. $\dfrac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}$ ifodani soddalashtiring.

$$\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1} = \frac{(\sin^2 x)^2+\cos^4 x-1}{(\sin^2 x)^3+\cos^6 x-1} = \frac{(1-\cos^2 x)^2+\cos^4 x-1}{(1-\cos^2 x)^3+\cos^6 x-1} =$$
$$= \frac{1-2\cos^2 x+\cos^4 x+\cos^4 x-1}{1-3\cos^2 x+3\cos^4 x-\cos^6 x+\cos^6 x-1} = \frac{2\cos^2 x(\cos^2 x-1)}{3\cos^2 x(\cos^2 x-1)} = \frac{2}{3}.$$

3-misol. $\dfrac{cos(\alpha+\beta)+cos(\alpha-\beta)}{\sin(\alpha+\beta)+\sin(\alpha-\beta)} = ctg\,\alpha$ ayniyatni isbotlang.

$$\frac{cos(\alpha+\beta)+cos(\alpha-\beta)}{\sin(\alpha+\beta)+\sin(\alpha-\beta)} = \frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta} =$$
$$= \frac{2\cos\alpha\cos\beta}{2\sin\alpha\cos\beta} = \frac{\cos\alpha}{\sin\alpha} = ctg\alpha.$$

4 - misol. $\dfrac{1+\cos\beta+\cos^2\beta}{1+\sec\beta+\sec^2\beta}$ ifodani soddalashtiring.

$$\frac{1+cos\beta+\cos^2\beta}{1+\sec\beta+\sec^2\beta} = \frac{1+cos\beta+\cos^2\beta}{1+\dfrac{1}{cos\beta}+\dfrac{1}{\cos^2\beta}} = \frac{1+cos\beta+\cos^2\beta}{\dfrac{\cos^2\beta+cos\beta+1}{\cos^2\beta}} =$$
$$= \frac{(1+cos\beta+\cos^2\beta)\cos^2\beta}{\cos^2\beta+\cos\beta+1} = \cos^2\beta.$$

Yuqoridagilardan ko'rinadiki, trigonometriya kursida ayniy almashtirishlar muhim o'rin egallaydi. O'quvchilar trigonometrik ayniy shakl almashtirishlarni yaxshi o'zlashtirishlari uchun birinchidan, trigonometrik funksiyalarni birini ikkinchisi orqali ifodalovchi va asosiy ayniyat kabi formulalarni, ikkinchidan esa shu formulalarni trigonometrik ifodani berilishiga qarab tadbiq qila olish malakalariga bog'liqdir. Trigonometrik ayniy shakl almashtirishlarni bajarish uchun quyidagi formulalarni bilishlari kerak:

1. **Asosiy trigonometrik ayniyatlar:**

$$1)\ \sin^2\alpha+\cos^2\alpha=1; \qquad 2)\ tg\alpha=\frac{\sin\alpha}{\cos\alpha},\left[\alpha\neq\frac{\pi}{2}(2n+1)\right], n\in Z;$$

$$3)\ ctg\alpha=\frac{\cos\alpha}{\sin\alpha},(\alpha\neq\pi n); \qquad 4)\ \sec\alpha=\frac{1}{\cos\alpha},\left[\alpha\neq\frac{\pi}{2}(2n+1)\right], n\in Z;$$

$$5)\ \cos ec\alpha=\frac{1}{\sin\alpha}, \quad (\alpha\neq\pi n), \quad n\in Z.$$

Bu ayniyatlardan kelib chiqadigan formulalar quyidagilardir:

$$1)\quad tg\alpha\cdot ctg\alpha=1 \qquad \left(\alpha\neq\frac{\pi}{2}n\right), \quad n\in Z.$$

$$2)\quad 1+tg^2\alpha=\sec^2\alpha, \qquad \left[\alpha\neq\frac{\pi}{2}(2n+1)\right], \quad n\in Z.$$

$$3)\quad 1+ctg^2\alpha=\cos ec^2\alpha, \qquad (\alpha\neq\pi), \quad n\in Z.$$

1-misol. Ayniyatni isbotlang.

$$\cos^2\alpha(tg\alpha+2)(2tg\alpha+1)-5\sin\alpha\cos\alpha=2,\left[\alpha\neq\frac{\pi}{2}(2n+1)\right].$$

Isboti:

$$\cos^2\alpha(tg\alpha+2)(2tg\alpha+1)-5\sin\alpha\cos\alpha=$$
$$=\cos^2\alpha\left(\frac{\sin\alpha}{\cos\alpha}+2\right)\left(\frac{2\sin\alpha}{\cos\alpha}+1\right)-5\sin\alpha\cos\alpha=$$
$$=2\sin^2\alpha+4\sin\alpha\cos\alpha+2\cos^2\alpha+\sin\alpha\cos\alpha-$$
$$-5\sin\alpha\cos\alpha=2(\sin^2\alpha+\cos^2\alpha)=2.$$

2-misol. Ayniyatni isbotlang:

$$(1+\sin\alpha)(tg\alpha+ctg\alpha)(1-\sin\alpha)=ctg\alpha.\left(\alpha\neq\frac{\pi}{2},n\in Z\right).$$

Исботи $\quad (1+\sin\alpha)(tg\alpha+ctg\alpha)(1-\sin\alpha)=(1+\sin\alpha)\left(\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}\right)(1-\sin\alpha)=$

$$=\frac{(1-\sin^2\alpha)(\sin^2\alpha+\cos^2\alpha)}{\sin\alpha\cdot\cos\alpha}=\frac{\cos^2\alpha}{\sin\alpha\cdot\cos\alpha}=ctg\alpha.$$

II. Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalari.

1) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

2) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

3) $tg(\alpha \pm \beta) = \dfrac{tg\alpha \pm tg\beta}{1 \pm tg\alpha \cdot tg\beta}.$

$$\left[\alpha, \beta, \alpha \pm \beta \neq \frac{\pi}{2}(2n+1), n \in Z\right].$$

4) $ctg(\alpha \pm \beta) = \dfrac{ctg\alpha \cdot ctg\beta \mp 1}{ctg\alpha \pm ctg\beta}.$ $\quad (\alpha, \beta, \alpha + \beta \neq \pi n, n \in Z).$

1-misol. *$cos15^o$* ni hisoblang.

Hisoblash.

$$\cos 15^0 = \cos(45^0 - 30^0) = \cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0 =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right) \approx 0{,}9659.$$

2-misol. *$sin15^o$* ni hisoblang .

Hisoblash

$$\sin 15^0 = \sin(45^0 - 30^0) = \sin 45^0 \cos 30^0 - \cos 45^0 \sin 30^0 =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$$

Xuddi shuningdek, $tg15^o = 2 - \sqrt{3}$, $stg15^o = 2 + \sqrt{3}$, $sec15^o = \sqrt{6} - \sqrt{2}$ larni hisoblash mumkin.

3-misol. $\dfrac{tg^2 a - tg^2\beta}{1 - tg^2 a \cdot tg^2\beta} = tg(a + \beta)(tga - \beta)$ ayniyatni isbotlang.

Исботи. $\dfrac{tg^2\alpha - tg^2\beta}{1 - tg^2\alpha \cdot tg^2\beta} = \dfrac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta} \cdot \dfrac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta} =$

$= tg(\alpha + \beta)tg(\alpha - \beta).$

4-misol. $sin(\alpha+\beta) \cdot sin(\alpha-\beta) = sin^2\alpha - sin^2\beta$ ayniyatni isbotlang.

Isboti $\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\times$

$\times(\sin\alpha\cos\beta-\cos\alpha\sin\beta)=\sin^2\alpha\cos^2\beta-\sin^2\beta\cos^2\alpha=$

$=\sin^2\alpha(1-\sin^2\beta)-\sin^2\beta(1-\sin^2\alpha)=$

$=\sin^2\alpha-\sin^2\alpha\sin^2\beta-\sin^2\beta+\sin^2\alpha\sin^2\beta=$

$=\sin^2\alpha-\sin^2\beta.$

Keltirish formulalari:

1) $\sin\left(\frac{\pi}{2}\pm\alpha\right)=\cos\alpha,\quad \sin(\pi\pm\alpha)=\mp\sin\alpha;$

2) $\sin\left(\frac{3\pi}{2}\pm\alpha\right)=-\cos\alpha,\quad \sin(2\pi\pm\alpha)=\pm\sin\alpha;$

3) $\cos\left(\frac{\pi}{2}\pm\alpha\right)=\mp\sin\alpha,\quad \cos(\pi\pm\alpha)=-\cos\alpha;$

4) $\cos\left(\frac{3\pi}{2}\pm\alpha\right)=\pm\sin\alpha,\quad \cos(2\pi\pm\alpha)=\cos\alpha;$

5) $tg\left(\frac{\pi}{2}\pm\alpha\right)=\mp ctg\alpha,\quad tg(\pi\pm\alpha)=\pm tg\alpha;$

6) $ctg\left(\frac{\pi}{2}\pm\alpha\right)=\mp tg\alpha,\quad ctg(\pi\pm\alpha)=\pm ctg\alpha;$

IV. Ikkilangan va uchlangan burchakning trigonometrik funksiyalari:

1) $\sin 2\alpha=2\sin\alpha\cos\alpha;$ 2) $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha;$

3) $tg2\alpha=\frac{2tg\alpha}{1-tg^2\alpha}\quad\left[2a,\alpha\neq\frac{\pi}{2}(2n+1),n\in Z\right];$

4) $ctg2\alpha=\frac{ctg^2\alpha-1}{2ctg\alpha}\quad(2a,\alpha\neq\pi n,n\in Z);$

5) $\sin 3\alpha=3\sin\alpha-4\sin^3\alpha;$ 6) $\cos 3\alpha=4\cos^3\alpha-3\cos\alpha;$

7) $tg3\alpha=\frac{1-3tg^2\alpha}{3tg\alpha-tg^3\alpha}\quad\left[2a,\ \alpha\neq\frac{\pi}{2}(2n+1),\ n\in Z\right];$

8) $ctg3\alpha=\frac{3tg\alpha-tg^3\alpha}{1-3tg^2\alpha}\quad\left[\alpha\neq\frac{\pi n}{3},\ n\in Z\right];$

9) $\sin^2\alpha=\frac{1-\cos 2\alpha}{2};$ 10) $\cos^2\alpha=\frac{1+\cos 2\alpha}{2};$

11) $\sin^3\alpha=\frac{3\sin\alpha-\sin 3\alpha}{4};$ 12) $\cos^3\alpha=\frac{\cos 3\alpha+3\cos\alpha}{4};$

MUSTAQIL YECHISH UCHUN MISOLLAR.

1. $\dfrac{1+tg(-60^0)}{\sin 60^0+\sin(-30^0)}$ ifodani soddalashtiring. J: –2

2. $\dfrac{2-\sin(-45^0)\cdot\cos(-45^0)}{tg(-45^0)\cdot ctg45^0}$ ifodani soddalashtiring. J: -2,5

3. $\dfrac{\left[\cos(-\alpha)-\sin(-\alpha)\right]^2}{1-2\sin(-\alpha)\cdot\cos(-\alpha)}$ ifodani soddalashtiring. J: 1

4. $\dfrac{\cos(-\alpha)}{1+2\sin^2(-\alpha)}-\dfrac{\sin(-\alpha)}{1-2\cos^2(-\alpha)}$ ifodani soddalashtiring. $j.\dfrac{1}{\sin\alpha+\cos\alpha}$

5. $\dfrac{4-2tg45^0+tg60^0}{3\sin 90^0-4\cos 60^0+4ctg45^0}.\quad j.\dfrac{2+\sqrt{3}}{5}$

6. $\dfrac{4-tg^2\dfrac{\pi}{4}+ctg^4\dfrac{\pi}{3}}{3\sin^3\dfrac{\pi}{2}+\cos^2\dfrac{\pi}{3}+ctg\dfrac{\pi}{4}}.\quad j.\dfrac{28}{54}$

7. $\left(4\sin\dfrac{\pi}{4}\right)^2-\left(2tg\dfrac{\pi}{6}\right)^2-\left(2\cos\dfrac{\pi}{6}\right)^2-\left(2ctg\dfrac{\pi}{4}\right)^2.\quad j.-\dfrac{1}{3}.$

8. $\sin 2\pi+\cos 4\pi+tg2\pi\quad j.1$

9. $ctg\dfrac{\pi}{2}+\cos ec\dfrac{\pi}{2}+\sec 0^0.\quad j.2.$

10. $a^2\sin\dfrac{\pi}{2}+2ab\cos\pi-b^2\sin\dfrac{3}{2}\pi.\quad j.\quad (a-b)^2.$

11. $10tg2\pi+3\cos\dfrac{3}{2}\pi-4tg\pi-5\sin\dfrac{3}{2}\pi.\quad j.\quad 5.$

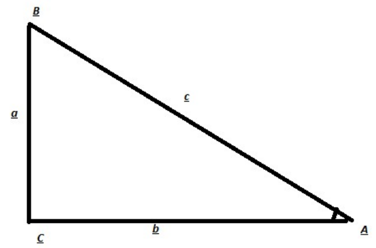
12. $4\sin 90^0+3\cos 720^0-3\sin 630^0+5\cos 900^0.\quad j.\quad 5.$

13. $5tg540^0+2\cos 1170^0+4\sin 990^0-3\cos 540^0.\quad j.-1.$

14. $100ctg^2 990^0+25tg^2 540^0-3\cos^2 900.\quad j.\quad -3.$

2-Mavzu: TRIGONOMETRIK FUNKSIYALAR

O'tkir burchaklaridan biri α bo'lgan ABC to'g'ri burchakli uchburchakni qaraylik:AB=c - gipotenuza,



BC=a-o'tkir burchakka qarshisidagi katet va

AC=b o'tkir burchakka yopishgan katet.

ABC uchburakning a,b va c tomonlaridan foydalanib, $\frac{b}{c}$, $\frac{a}{c}$, $\frac{b}{a}$,$\frac{a}{b}$, $\frac{c}{a}$ va $\frac{c}{b}$ nisbatlarni yozishimiz mumkin. Bu nisbatlarni o'zgarishi α burchakni ham o'zgarishiga olib keladi.Bu nisbatlarning har biri bilan α burchak orasidagi bog'lanishlarni alohida nomlaymiz.

Ta'rif. α burchak qarshisida yotgan katet uzunligini gipotenuza uzunligiga nisbati α burchakning *sinusi* deyiladi va uni $\sin\alpha$ deb yoziladi. Demak, $\sin\alpha = \frac{a}{c}$.

Ta'rif. α burchakka yopishgan katet uzunligini gipotenuza uzunligiga nisbati, α burchakning *kosinusi*deyiladi va u $\cos\alpha$ deb yoziladi. Demak, $\cos\alpha = \frac{b}{c}$.

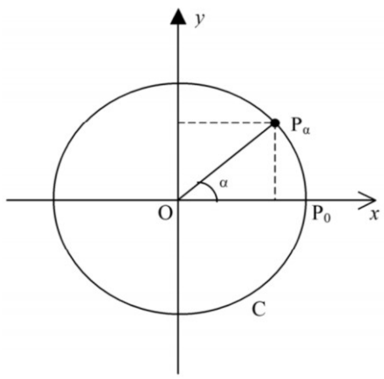
Ta'rif. α burchak qarshisida yotgan katet uzunligini, α burchakka yopishgan katet uzunligiga nisbati, α burchakning *tangensi* deyiladi va u $tg\alpha$ deb yoziladi. Demak, $tg\alpha = \frac{a}{b}$.

Ta'rif. α burchakka yopishgan katet uzunligini, α burchak qarshisida yotgan katet uzunligiga nisbati, α burchakning *kotangensi* deyiladi va u $ctg\alpha$ deb yoziladi. Demak, $ctg\alpha = \frac{b}{a}$.

$y = \sin x$, $y = \cos x$, $y = tgx$, va $y = ctgx$ funksiyalar *trigonometrik funksiyalar* deb ataladi.

Agar biz $a^2+b^2=c^2$ (Pifagor teoremasi) tenglikni har ikkala qismini hadma-had c^2 ga bo'lsak, u holda $\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=1$ yoki $\sin^2 x+\cos^2 x=1$ ni hosil qilamiz. Bu tenglik Pifagor teoremasiga ekvivalent bo'lib, uni <u>asosiy</u> *trigonometrik ayniyat* deb ataladi.

Biz o'tkir burchak trigonometrik funksiyalarining ta'riflarini keltirdik.Bu ta'riflarni ixtiyoriy burchak uchun ham umumlashtirish mumkin. Buning uchun markazi koordinata boshida va radiusi 1 ga teng bo'lgan aylanani olamiz. Aylanadagi (1;0) koordinatali nuqtani P_0 bilan belgilaymiz va uni boshlang'ich nuqta deb ataymiz. Ixtiyoriy α sonni olamiz va boshlang'ich nuqtani α burchakka buramiz.



Natijada P_α nuqtani hosil qilamiz. P_α nuqtaning koordinatalarini x_α va y_α deb belgilaymiz. OP_α=R=1 ekanligini e'tiborga olsak, $\sin\alpha=\frac{y_\alpha}{R}=y_\alpha$ va $\cos\alpha=\frac{x_\alpha}{R}=x_\alpha$ larni yoki $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$ larni hosil qilamiz. Bulardan esa sinus va kosinuslar uchun boshqacha ta'riflar berish mumkinligi kelib chiqadi.

P_α nuqtaning ordinatasiga α burchakning sinusi va abstsissasiga α burchakning kosinusi deyiladi.Demak, $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$.

$P_\alpha(x_\alpha;y_\alpha)$ nuqtaning koordinatalari uchun $x_\alpha^2+y_\alpha^2=1$ ya'ni $\sin^2 x+\cos^2 x=1$ tenglik o'rinlidir. Bu asosiy trigonometrik ayniyatdir. Undan quyidagilarni yozish mumkin: $\sin^2 x=1-\cos^2 x$ va $\cos^2 x=1-\sin^2 x$.

Agar biz $tg\alpha=\frac{y_\alpha}{x_\alpha}$ va $ctg\alpha=\frac{x_\alpha}{y_\alpha}$ hamda $\sin\alpha=y_\alpha$ va $\cos\alpha=x_\alpha$ ekanligini e'tiborga olsak, u holda $tg\alpha=\frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha=\frac{\cos\alpha}{\sin\alpha}$ larni hosil qilamiz. Demak, α burchak sinusini uning kosinusiga nisbatiga α *burchakning tangensi* deyiladi.

α burchak kosinusini uning sinusiga nisbatiga esa α *burchakning kotangensi* deyiladi.

Demak, ta'riflarga asosan $tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$.

Bu yerda tangens $\cos\alpha \neq 0$ va kotangens $\sin\alpha \neq 0$ hollarda aniqlangan.

$tg\alpha = \frac{\sin\alpha}{\cos\alpha}$ va $ctg\alpha = \frac{\cos\alpha}{\sin\alpha}$ lardan $tg\alpha$ va $ctg\alpha$ larni o'zaro teskari sonlar ekanligini ko'ramiz. O'zaro teskari sonlar ko'paytmasi esa 1 ga teng. Demak, bundan esa $tg\alpha = \frac{1}{ctg\alpha}$ va $ctg\alpha = \frac{1}{tg\alpha}$ ifodalar kelib chiqadi.

Trigonometrik funksiyalarning ishoralari qaralayotgan burchakning qaysi chorakda yotishiga qarab aniqlanadi.

α burchakning sinusi P_α nuqtaning ordinatasidan iborat bo'lganligi uchun u I va II choraklarda musbat, III va IV choraklarda esa manfiy bo'ladi.

α burchakning kosinusi P_α nuqtaning abstsissasidan iborat bo'lgani uchun u I va IV choraklarda musbat, II va III choraklarda esa manfiy bo'ladi.

α burchakning tangensi va kotangensi P_α nuqta koordinatalarining nisbatlari bo'lganligi uchun, ular P_α nuqtaning koordinatalari bir xil ishorali bo'lgan (I va III) choraklarda musbat va har xil ishorali bo'lgan (II va IV) choraklarda manfiy bo'ladi.

Choraklar	I	II	III	IV
$\sin\alpha$	+	+	-	-
$\cos\alpha$	+	-	-	+
tgα	+	-	+	-
ctgα	+	-	+	-

Amaliyotda ko'pincha trigonometrik funksiyalarning qiymatlari bilan ish ko'riladi. α burchakning trigonometrik funksiyalarini qiymatlari P_α nuqtaning koordinatalari bilan bog'liq. Ya'ni, $\sin\alpha = y_\alpha$, $\cos\alpha = x_\alpha$ $tg\alpha = \frac{y_\alpha}{x_\alpha}$

α burchak 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ va 2π qiymatlarni qabul qilganda P_α nuqtaning koordinatalarini osongina topiladi. α burchak 0^0, 30^0, 45^0 va 60^0 qiymatlarni qabul qilganda P_α nuqtaning koordinatalarini o'tkir burchagi α bo'lgan to'g'ri burchakli uchburchakdan topiladi.

Quyidagi jadvalda trigonometrik funksiyalarning ba'zi bir burchaklardagi qiymatlari keltirilgan.

Burchak	0	30^0	45^0	60^0	90^0	180	270
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tgα	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞
ctgα	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0

O'tkirburchakning trigonometric funksiyalarini jadval yoki to'g'ri burchakli uchburchakdan foydalanib hisoblash mumkin. Ixtiyoriy burchakni trigonometric funksiyalarini qiymatlarini hisoblashni doimo o'tkir burchak trigonometric funksiyalari qiymatlarini hisoblashga keltirish mumkin. Bunday formulalarni keltirish formulalari deyiladi.

Ko'p hollarda α burchakning trigonometrik funksiyalarini bilganholda $\frac{\alpha}{2}$ burchakning trigonometric funksiyalarinianiqlashgato'g'rikeladi. Bunda quyidagi formulalardan foydalaniladi:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}; \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}; \qquad tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};$$

$$ctg\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}; \; tg\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}.$$

Bulardan dastlabki ikkitasi $\cos 2\alpha = 1 - 2\sin^2\alpha$ va $\cos 2\alpha = 2\cos^2\alpha - 1$ formulalardan keltirib chiqariladi. Keyingi ikkitasi esa tangens va kotangenslarni sinus va kosinuslar orqali ifodalaridan keltirib chiqariladi.Dastlabki to'rtta formulalardagi $\pm$ ishoralardan qaysi birini olinishi $\frac{\alpha}{2}$ burchakni qaysi chorakda yotishiga bog'liq bo'ladi.

Ko'p hollarda trigonometrik funksiyalardan birini qolganlari orqali ifodalash formulalaridan foydalaniladi.Bu formulalarni asosiy trigonometrik ayniyatlardan keltirib chiqariladi.$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$ va $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$.

$\sin\alpha$ ni tgα orqali ifodasi $tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$ ni $\sin\alpha$ ga nisbatan yechib keltirib chiqariladi. Ularni barchasini quyidagi jadvalda keltiramiz:

Funksiya	$\sin\alpha$	$\cos\alpha$	$tg\alpha$	$ctg\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\pm\frac{tg\alpha}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{1}{\sqrt{1+ctg^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha$	$\pm\frac{1}{\sqrt{1+tg^2\alpha}}$	$\pm\frac{ctg\alpha}{\sqrt{1+ctg^2\alpha}}$
$tg\alpha$	$\pm\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$tg\alpha$	$\frac{1}{ctg\alpha}$

ctgα	$\pm\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{tg\alpha}$	ctgα

Foydalanilgan adabiyotlar

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2-Amaliy mashg'ulot

Na'munaviy misollar yechilishi

1-misol. $\sin\alpha\cdot\sin(60^o-\alpha)\cdot\sin(60^o+\alpha)=\frac{1}{4}\sin3\alpha$ ayniyatni isbotlang.

Isboti. $\sin\alpha\cdot\sin(60^0-\alpha)\cdot\sin(60^0+\alpha)=$

$$=\sin\alpha(\sin^2 60^0-\sin^2\alpha)=\sin\alpha\left(\frac{3}{4}-\sin^2\alpha\right)=$$

$$=\frac{1}{4}\left(3\sin\alpha-4\sin^3\alpha\right)=\frac{1}{4}\sin3\alpha.$$

2-misol. $\cos\alpha\cdot\cos(60^o—\alpha)\cdot\cos(60^o+\alpha)=\frac{1}{4}\cos3\alpha$ ayniyatni isbotlang.

3-misol. $tg\alpha\cdot tg(60^o-\alpha)\cdot tg(60^o+\alpha)=tg3\alpha$ ayniyatni isbotlang.

Bu ayniyatlardan foydalanib, quyidagi trigonometrik ifodalarni osonlikcha hisoblash mumkin:

$$a)\quad \sin20^0\cdot\sin40^0\cdot\sin80^0=\frac{1}{4}\sin3\cdot20^0=\frac{1}{4}\sin60^0=\frac{1}{4}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{8};$$

$$б)\quad \cos10^0\cdot\cos50^0\cdot\cos70^0=\frac{1}{4}\cos30^0=\frac{\sqrt{3}}{8};$$

$$в)\quad tg6^0\cdot tg54^0\cdot tg66^0=tg18^0.$$

4-misol. $\sin3\alpha\cos^3\alpha+\sin^3\alpha\cos3\alpha=\frac{3}{4}\sin\alpha$ ayniyatni isbotlang.

Isboti. $\sin3\alpha\cos^3\alpha+\sin^3\alpha\cos3\alpha=\sin3\alpha\frac{\cos3\alpha+3\cos\alpha}{4}+$

$$+\cos3\alpha\frac{3\sin\alpha-\sin3\alpha}{4}=\frac{3}{4}(\sin3\alpha\cos\alpha+\sin\alpha\cos3\alpha)=\frac{3}{4}\sin4\alpha.$$

5-misol. $\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha$ ifodani soddalashtiring.

Yechish. Berilgan ifodani $\sin\alpha$ ga ko'paytiramiz hamda bo'lamiz.

$$\frac{\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha\cdot\sin\alpha}{\sin\alpha}=\frac{\frac{1}{2}\sin2\alpha\cdot\cos2\alpha\cdot\cos4\alpha}{\sin\alpha}=$$

$$=\frac{\frac{1}{2}\left(\frac{1}{2}\sin4\alpha\cdot\cos4\alpha\right)}{\sin\alpha}=\frac{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}\sin8\alpha\right)\right]}{\sin\alpha}=\frac{\sin8\alpha}{8\sin\alpha}.$$

6-misol. $tg4\alpha - sec4\alpha = \frac{\sin 2a - \cos 2a}{\sin 2a + \cos 2a}$ ayniyatni isbotlang.

V. Yarim argumentning trigonometrik funksiyalari

1) $\left|\sin\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{2}}$; 2) $\left|\cos\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{2}}$;

3) $\left|tg\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$; $[\alpha \neq \pi(2n+1), \quad n \in Z]$;

4) $\left|ctg\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$;

5) $tg\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$; $[\alpha \neq \pi n, n \in Z]$;

6) $ctg\frac{\alpha}{2} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha}$; $[\alpha \neq \pi n, n \in Z]$;

1-misol. $tg7^0 30'$ ni hisoblang.

$$tg\,7^0 30' = \frac{1-\cos 15^0}{\sin 15^0} = \frac{1-\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)}{\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)} =$$

Yechish. $= \frac{\left(4-\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)} = \frac{4\sqrt{6}+4\sqrt{2}-4\sqrt{3}-8}{4} =$

$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2.$

2-misol. $\frac{1-tg^2 15^0}{1+tg^2 15^0} = \frac{\sqrt{3}}{2}$ ni isbotlang.

Isboti. $\frac{1-tg^2 15^0}{1+tg^2 15^0} = \frac{1-\frac{1-\cos 30^0}{1+\cos 30^0}}{1+\frac{1-\cos 30^0}{1+\cos 30^0}} = \cos 30^0 = \frac{\sqrt{3}}{2}.$

VI. Trigonometrik funksiyalar ko'paytmasini yig'indiga keltirish formulalari:

1) $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$;

2) $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$;

3) $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$;

Misol. $\cos\alpha+\cos(\alpha+2\beta)+...+\cos(\alpha+\pi\beta)$ ifodani soddalashtiring.

Yechish. Berilgan ifodani $\sin\frac{\beta}{2}$ ga ko'paytiramiz va bo'lamiz.

$$\frac{1}{\sin\frac{\beta}{2}}\left[\sin\frac{\beta}{2}\cos\alpha+\sin\frac{\beta}{2}\cos(\alpha+\beta)+\sin\frac{\beta}{2}\cos(\alpha+2\beta)+\right.$$

$$\left.+\ldots+\sin\frac{\beta}{2}\cos(\alpha+n\beta)\right]=\frac{1}{2\sin\frac{\beta}{2}}\left[\sin\left(\alpha+\frac{\beta}{2}\right)-\sin\left(\alpha-\frac{\beta}{2}\right)+\right.$$

$$+\sin\left(\alpha+\frac{3\beta}{2}\right)-\sin\left(\alpha+\frac{\beta}{2}\right)+\sin\left(\alpha+\frac{5\beta}{2}\right)-\sin\left(\alpha+\frac{3\beta}{2}\right)+\ldots+$$

$$+\sin\left(\alpha+\frac{3\beta}{2}\right)-\sin\left(\alpha+\frac{\beta}{2}\right)+\sin\left(\alpha+\frac{5\beta}{2}\right)-\sin\left(\alpha+\frac{3\beta}{2}\right)+\ldots+$$

$$\left.+\sin\left(\alpha+\frac{2n+1}{2}\beta\right)-\sin\left(\alpha+\frac{2n-1}{2}\beta\right)\right]=$$

$$=\frac{1}{\sin\frac{\beta}{2}}\left[\sin\left(\alpha+\frac{2n+1}{2}\beta\right)-\sin\left(\alpha-\frac{\beta}{2}\right)\right]=$$

$$=\frac{1}{\sin\frac{\beta}{2}}2\sin\frac{n+1}{2}\beta\cos\left(\alpha+\frac{n}{2}\beta\right)=\frac{\sin\frac{n+1}{2}\beta\cos\left(\alpha+\frac{n}{2}\beta\right)}{\sin\frac{\beta}{2}}.$$

VII. Trigonometrik funksiyalar yig'indisi va ayirmasining formulalari:

1) $\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$;

2) $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$;

3) $\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$;

4) $\cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$;

5) $tg\alpha+tg\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$, $\left[\alpha,\beta\neq\frac{\pi}{2}(2n-1),n\in Z\right]$;

6) $tg\alpha-tg\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$, $\left[\alpha,\beta\neq\frac{\pi}{2}(2n-1),n\in Z\right]$;

7) $ctg\alpha+ctg\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$, $[\alpha,\beta\neq\pi n,n\in Z]$;

8) $ctg\alpha-ctg\beta=\frac{\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$.

1-misol. $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma) = 4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\gamma}{2}\cos\frac{\beta+\gamma}{2}$

ayniyatni isbotlang.

Isboti.

$$\begin{aligned}
&\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=\\
&=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\gamma+\alpha+\beta+\gamma}{2}\cdot\cos\frac{\gamma-\alpha-\beta-\gamma}{2}=\\
&=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta+2\gamma}{2}\right)=\\
&=2\cos\frac{\alpha+\beta}{2}2\cos\frac{\alpha-\beta+\alpha-\beta+2\gamma}{4}\cdot\cos\frac{\alpha-\beta-\alpha-\beta-2\gamma}{4}=\\
&=4\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\gamma}{2}\cos\frac{\beta+\gamma}{2}.
\end{aligned}$$

2-misol. Agar $\alpha+\beta+\gamma=\pi$ bo'lsa, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma-2=2\cos\alpha\cos\beta\cos\gamma$ tenglikning o'rinli ekanligini isbotlang.

Isboti. Shartga ko'ra $\gamma=\pi-\alpha-\beta$ u holda

$$\begin{aligned}
&\sin^2\alpha+\sin^2\beta+\sin^2\gamma-2=\sin^2\alpha+\sin^2\beta+\sin^2[\pi-(\alpha+\beta)]-2=\\
&=\sin^2\alpha+\sin^2\beta+\sin^2(\alpha+\beta)-2=
\end{aligned}$$

$$\begin{aligned}
&=\frac{1-\cos 2\alpha}{2}+\frac{1-\cos 2\beta}{2}+\frac{1-\cos 2(\alpha+\beta)}{2}-2=\\
&=-\frac{1}{2}[\cos 2\alpha+\cos 2\beta+\cos 2(\alpha+\beta)+1]=
\end{aligned}$$

$$\begin{aligned}
&=-\frac{1}{2}[\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos^2(\alpha+\beta)]=\\
&=-\cos(\alpha+\beta)[\cos(\alpha-\beta)+\cos(\alpha+\beta)]=\\
&=-2\cos(\alpha+\beta)\cos\alpha\cos\beta=\\
&=-2\cos(\pi+\gamma)\cos\alpha\cos\beta=2\cos\alpha\cos\beta\cos\gamma.
\end{aligned}$$

MUSTAQIL YECHISH UCHUN MISOLLAR.

12. $4\sin 90^0 + 3\cos 720^0 - 3\sin 630^0 + 5\cos 900^0.$ $j.$ 5.

13. $5tg540^0 + 2\cos 1170^0 + 4\sin 990^0 - 3\cos 540^0.$ $j. -1.$

14. $100ctg^2 990^0 + 25tg^2 540^0 - 3\cos^2 900.$ $j.$ $-3.$

15. $tg900^0 - \sin(-1095^0) + \cos(-1460^0).$ $j.$ $\sqrt{1,5}.$

16. $\sin(-1125^0) + \cos^2(-900^0) + tg1710^0$ $j.$ $\frac{2-\sqrt{2}}{2}.$

17. $cos20^0 + cos40^0 + cos60^0 + ... + cos160^0 + cos180^0.$ $j. -1.$

18. $\sin\left(-\frac{14\pi}{3}\right) + \cos ec^2 \frac{29\pi}{4} - tg^2 \frac{3\pi}{4}.$ $j.$ $\frac{2-\sqrt{3}}{2}.$

19. $\dfrac{5 + \sin 30^0 \cos 60^0 - tg\frac{\pi}{4}}{a + b\cos 2\pi - \sin\pi}.$ $j. \frac{17}{4(a+b)}.$

20. $\dfrac{m\cos\frac{\pi}{4} + n\sin\frac{\pi}{4} - tg\pi}{mn - mtg\frac{\pi}{4} - ctg\frac{\pi}{2}}.$ $j. \frac{\sqrt{2}(m+n)}{2m(n-1)}.$

21. $(\sin\varphi + \cos\varphi)^2 + (\sin\varphi - \cos\varphi)^2.$ $j.2.$

22. $\dfrac{1 + \cos\beta + \cos^2\beta}{1 + \sec\beta + \sec^2\beta}.$ $j.\cos^2\beta.$

23. $\sin^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} + \cos^4\frac{\alpha}{2}.$ $j.1.$

24. $\dfrac{1}{\cos ec^2 2\alpha - 1} + \dfrac{\cos^2 2\alpha}{1 - \sin^2\alpha}.$ $j.\sec^2 2\alpha.$

25. $\dfrac{1 - tg^2\beta}{1 + tg^2\beta} + \sin^2\beta.$ $j.\cos^2\beta.$

26. $(1 - \cos^2 x)ctg^2 x - 1.$ $j. -\sin^2 x.$

3-MAVZU: TRIGONOMETRIK FUNKSIYALAR. XOSSALARI VA ULARNING GRAFIKLARI

Trigonometrik tenglamalar va tengsizliklarni yechishda va funksiyalarni tekshirishda trigonometrik funksiyalarning xossalarini bilish muhim ahamiyatga ega. Shuning uchun $y = sinx$ funksiyaning xossalarini keltiramiz:

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat. Demak, $y = sinx$ funksiya chegaralangan.

3. Funksiya toq. Chunki $\sin(-x) = -sinx; \quad (x \in R)$

4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2π ga teng. Ya'ni $x \in R$ lar uchun $\sin(x + 2\pi) = sinx$

5. $x = \pi k$, $k \in Z$ da $sinx = 0$.

6. $x \in (2\pi k; \pi + 2\pi k)$, $k \in Z$ da $sinx > 0$.

7. $x \in (\pi + 2\pi k; 2\pi + 2\pi k)$, $k \in Z$ da $sinx > 0$.

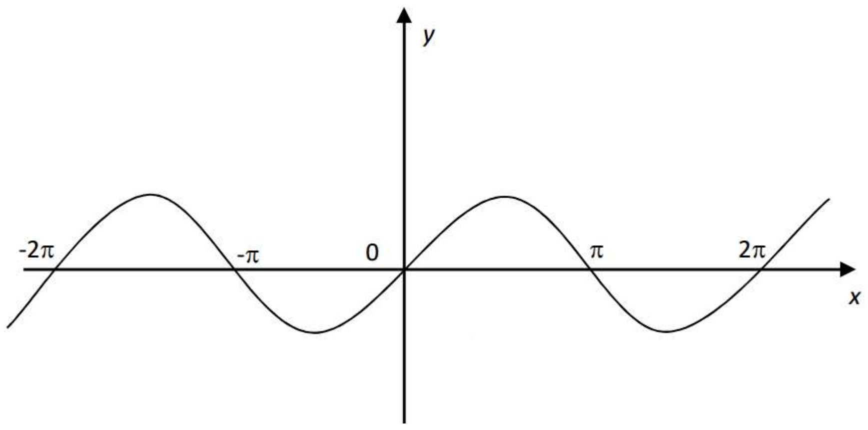
8. Funksiya $\left[-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada –1 dan 1 gacha o'sadi.

9. Funksiya $\left[\frac{\pi}{2} + 2k\pi; \frac{3\pi}{2} + 2k\pi\right]$, $k \in Z$ kesmada 1 dan -1 gacha kamayadi.

10. Funksiya $x = \frac{\pi}{2} + k\pi$, $k \in Z$ nuqtalarda 1 ga teng eng katta qiymatga erishadi.

11. Funksiya $x = \frac{3\pi}{2} + 2k\pi$, $k \in Z$ nuqtalarda -1 ga teng eng kichik qiymatga erishadi.

funksiyaning yuqoridagi xossalariga asoslanib $[-\pi;\pi]$ kesmada, ya'ni uzunligi 2π ga teng kesmada uni davriyligini e'tiborga olib esa butun sonlar to'g'ri chizig'ida grafigini yasash mumkin.



1-chizma

Xuddi shunday $y = cosx$ funksiyaning xossalarini keltirsak:

1. Funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar sohasi [-1;1] kesmadan iborat.

3. Funksiya juft, chunki $\cos(-x) = cosx$.

4. Funksiya davriy bo'lib, uning eng kichik musbat davri 2π ga teng. Ya'ni $x \in R$ lar uchun $\cos(x + 2\pi) = cosx$

5. Barcha $x = \frac{\pi}{2} + k\pi$, $k \in Z$ larda $cosx = 0$.

6. xning $(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi)$, $k \in Z$ qiymatlarida musbat

7. x ning $(\frac{\pi}{2} + 2k\pi; \frac{3\pi}{2} + 2k\pi)$, $k \in Z$ qiymatlarida manfiy

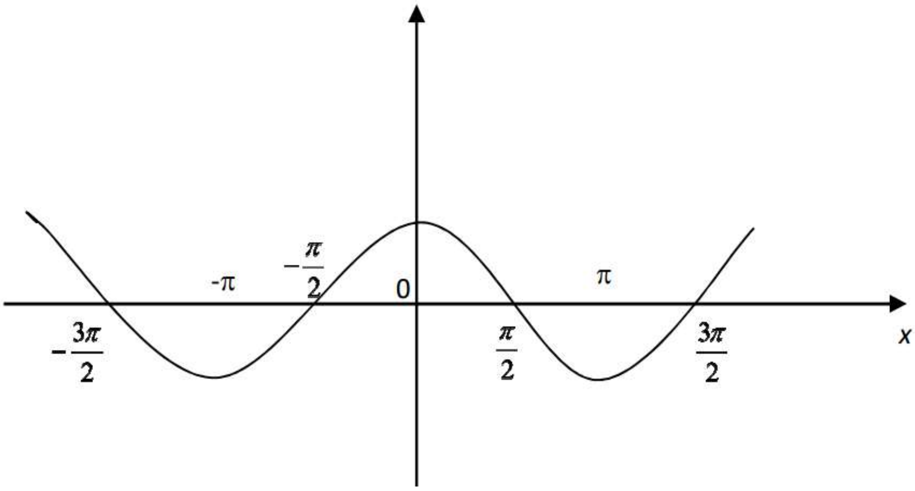
8. Funksiya $(2k\pi; \pi + 2k\pi)$, $k \in Z$ da 1 dan –1 gacha kamayadi.

9. Funksiya $(-\pi + 2k\pi; 2k\pi)$, $k \in Z$ da -1 dan 1 gacha o'sadi.

10. Funksiya $x = 2k\pi$, $k \in Z$ nuqtalarda 1 ga teng

11. Funksiya $x = \pi + 2k\pi$, $k \in Z$ nuqtalarda -1 ga teng eng kichik qiymatni qabul qiladi.

ning bu xossalaridan foydalanib dastlab uni grafigini $[-\pi;\pi]$ da so'ngra butun sonlar to'g'ri chizig'ida yasash mumkin.



2-chizma

$y = tgx$ funksiyaning xossalariquyidagilar:

1. Funksiyaning aniqlanish sohasi $x = \frac{\pi}{2} + k\pi$, $k \in Z$ dan farqli barcha haqiqiy sonlar to'plamidan iborat.

2. Funksiyaning qiymatlar to'plami barcha haqiqiy sonlar to'plamidan iborat.

3. Funksiya toq, chunki aniqlanish sohasidan olingan barcha x lar uchun $tg(-x) = -tgx$.

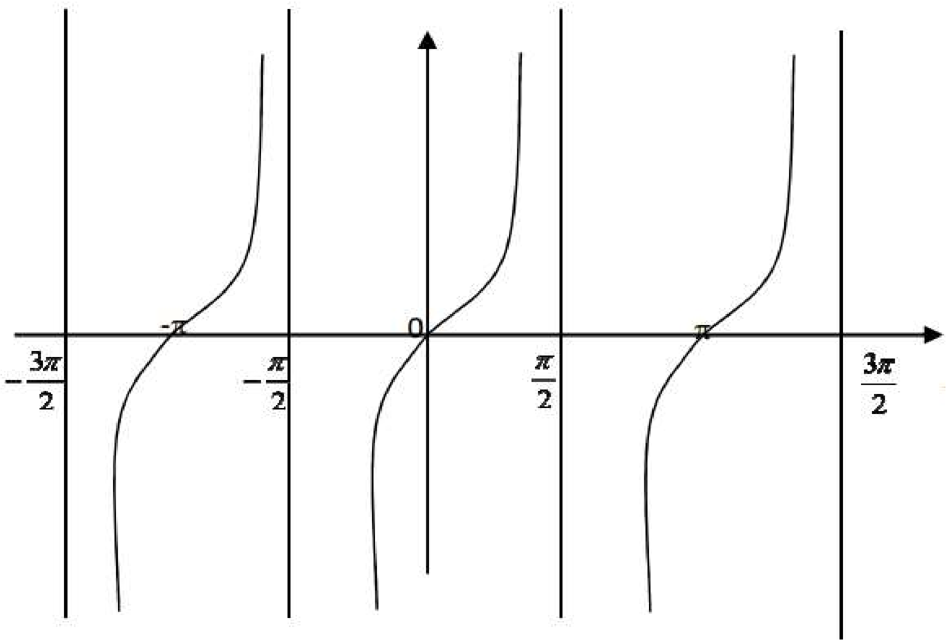
4. Funksiya davriy bo'lib, uning eng kichik musbat davri π ga teng. Ya'ni $x \in R$ lar uchun $tg(x + \pi) = tgx$

5. Barcha x=k π, $k \in Z$ nuqtalarda $tgx = 0$.

6. $(k\pi; \frac{\pi}{2} + k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda tgx >0.

7. $(-\frac{\pi}{2} + k\pi, k\pi)$, $k \in Z$ dan olingan barcha nuqtalarda $tgx < 0$.

8. Funksiya $(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi)$, $k\in Z$ oraliqda o’suvchidir. Yuqoridagi xossalarga asoslanib dastlab, $(-\frac{\pi}{2};\frac{\pi}{2})$ oraliqda, so’ngra butun sonlar o’qida $y=tgx$ funksiyani grafigini yasash mumkin.



3-chizma

$y=ctgx$ funksiyaning xossalari:

1. Funksiyaning aniqlanish sohasi $x=k\pi$, $k\in Z$ dan farqli barcha haqiqiy sonlar to’plamidan iborat.

2. Funksiyaning qiymatlar sohasi sonlar o’qining barcha nuqtalari to’plamidan iborat. Ya’ni funksiya chegaralanmagan.

3. Funksiya toq, Chunki aniqlanish sohasidan olingan barcha x larda $ctg(-x)=-ctgx$

4. Funksiya π davrli davriy Funksiyadir. Chunki aniqlanish sohasidan olingan barcha x larda $ctg(x+\pi)=ctgx$

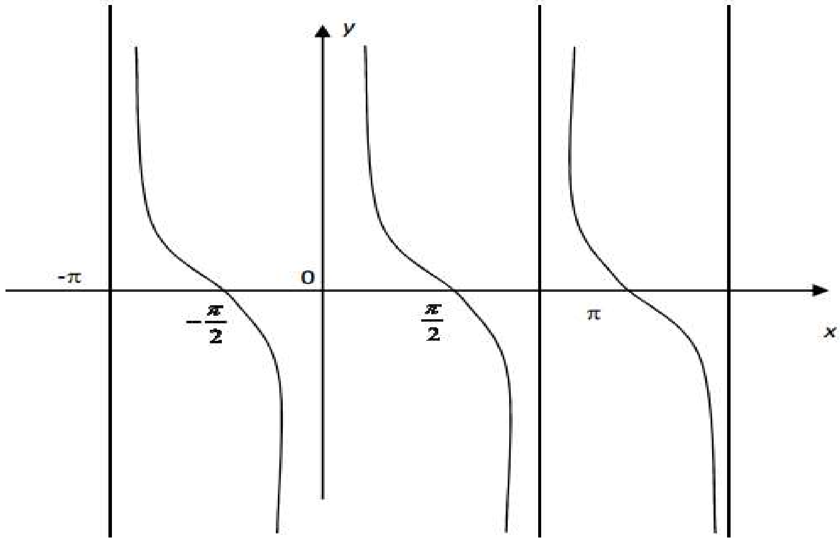
5. $x=\frac{\pi}{2}+k\pi$, $k\in Z$ nuqtalarda $ctgx=0$.

6. $x\in(k\pi;\frac{\pi}{2}+k\pi)$, $k\in Z$ nuqtalarda $ctgx>0$

7. $x \in (-\frac{\pi}{2} + k\pi; k\pi)$, $k \in Z$ nuqtalarda $ctgx < 0$.

8. Funksiya $(k\pi; \pi + k\pi)$ oraliqlarda kamayuvchidir.

Funksiyaning yuqoridagi xossalaridan foydalanib dastlab $(0;\pi)$ oraliqda so'ngra butun koordinatalar to'g'ri chizig'ida kotangensni grafigini yasash mumkin.



4-chizma

Foydalanilgan adabiyotlar

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3-AMALIY MASHG'ULOT

Na'munaviy misol yechimlari

1-misol. $(1-\sin a)(1+\sin a)-\cos^2 a$ ifodani soddalashtiring.

I-usul. $(1-\sin a)(1+\sin a)-\cos^2 a=1-\sin^2 a-\cos^2 a=1-(1-\cos^2 a)-\cos^2 a=$
$=1-1+\cos^2 a-\cos^2 a=1-1+\cos^2 a-\cos^2 a=0.$

II- usul. $(1-\sin\alpha)(1+\sin\alpha)-\cos^2\alpha=$

$=1-\sin^2\alpha-\cos^2\alpha=1-(\sin^2\alpha+\cos^2\alpha)=1-1=0.$

2 misol. $\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}$ ifodani soddalashtiring.

Yechish. $\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}=\frac{(\sin^2 x)^2+\cos^4 x-1}{(\sin^2 x)^3+\cos^6 x-1}=$

$$=\frac{(1-\cos^2 x)^2+\cos^4 x-1}{(1-\cos^2 x)^3+\cos^6 x-1}=\frac{1-2\cos^2 x+\cos^4 x+\cos^4 x-1}{1-3\cos^2 x+3\cos^4 x-\cos^6 x+\cos^6 x-1}=$$
$$=\frac{2\cos^2 x(\cos^2 x-1)}{3\cos^2 x(\cos^2-1)}=\frac{2}{3}.$$

3 misol $\frac{1-(\sin x-\cos x)^2}{1+\sin^2 x-\cos^2 x}$ ifodani soddalashtiring.

Yechish. $\frac{1-(\sin x-\cos x)^2}{1+\sin^2 x-\cos^2 x}=\frac{1-\sin^2 x+2\sin x\cos x-\cos^2 x}{\sin^2 x+\cos^2 x+\sin^2 x-\cos^2 x}=$

$$=\frac{1-(\sin^2 x+\cos^2 x)+2\sin x\cos x}{2\sin^2 x}=\frac{2\sin x\cos x}{2\sin^2 x}=\frac{\cos x}{\sin x}=ctgx.$$

4-misol. $\frac{1}{\cos^2 x}-\frac{1}{ctg^2 x}-\frac{\sin^2 x}{tg^2 x}$ ifodani soddalashtiring.

Yechish. $\frac{1}{\cos^2 x}-\frac{1}{ctg^2 x}-\frac{\sin^2 x}{tg^2 x}=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}-\frac{\sin^2 x\cos^2 x}{\sin^2 x}=$

$$=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}-\cos^2 x=\frac{1-\sin^2 x-\cos^4 x}{\cos^2 x}=\frac{\cos^2 x-\cos^4 x}{\cos^2 x}=$$
$$=\frac{\cos^2 x(1-\cos^2 x)}{\cos^2 x}=\sin^2 x.$$

5-misol. $\frac{[\cos(-a)+\sin(-a)]^2-1}{\cos^2(-a)+\sin^2(-a)-1}$ ifodani soddalashtiring.

$$\frac{\left[\cos(-a)+\sin(-a)\right]^{2}-1}{\cos^2(-a)+\sin^2(-a)-1}=\frac{(\cos a-\sin a)^2-1}{\cos^2 a-\sin^2 a-1}=$$

$$=\frac{\cos^2 a-2\cos a\sin a+\sin^2 a-1}{\cos^2 a-\sin^2 a-(\cos^2 a+\sin^2 a)}=\frac{-2\cos a\sin a}{-2\sin^2 a}=ctga.$$

6-misol. $1+\sin\alpha+\cos\alpha$ ifodani ko'paytma shakliga keltiring.

Yechish. $1+\sin\alpha+\cos\alpha=(1+\cos\alpha)+\sin\alpha=$

$$=2\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}=2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)=$$

$$=2\cos\frac{\alpha}{2}\left(\sin\left(90^0-\frac{\alpha}{2}\right)+\sin\frac{\alpha}{2}\right)=2\cos\frac{\alpha}{2}\cdot 2\sin 45^0\cdot$$

$$\cdot\cos\left(45^0-\frac{\alpha}{2}\right)=2\sqrt{2}\cos\frac{\alpha}{2}\cdot\cos\left(45^0-\frac{\alpha}{2}\right).$$

7-misol. $\sqrt{3}-2\sin\alpha$ ifodani ayniy almashtirish orqali ko'paytma shakliga keltiring.

Yechish. $\sqrt{3}-2\sin\alpha=2\left(\frac{\sqrt{3}}{2}-\sin\alpha\right)=2\left(\sin 60^0-\sin\alpha\right)=$

$$=4\sin\left(30^0-\frac{\alpha}{2}\right)\cdot\cos\left(30^0+\frac{\alpha}{2}\right).$$

8-misol. $\frac{\sin\alpha+2\sin 3\alpha+\sin 5\alpha}{\sin 3\alpha+2\sin 5\alpha+\sin 7\alpha}$ ifodani soddalashtiring.

Yechish. $\frac{\sin\alpha+2\sin 3\alpha+\sin 5\alpha}{\sin 3\alpha+2\sin 5\alpha+\sin 7\alpha}=\frac{(\sin\alpha+\sin 5\alpha)+2\sin 3\alpha}{(\sin 3\alpha+\sin 7\alpha)+2\sin 5\alpha}=$

$$=\frac{2\sin 3\alpha\cos 2\alpha+2\sin 3\alpha}{2\sin 5\alpha\cos 2\alpha+2\sin 5\alpha}=\frac{2\sin 3\alpha(\cos 2\alpha+1)}{2\sin 5\alpha(\cos 2\alpha+1)}=\frac{\sin 3\alpha}{\sin 5\alpha}.$$

9-misol. $1-\cos\left(\frac{\alpha}{2}-3\pi\right)-\cos^2\frac{\alpha}{4}+\sin^2\frac{\alpha}{2}$ ifodani soddalashtiring.

Yechish. $1-\cos\left(\frac{\alpha}{2}-3\pi\right)-\cos^2\frac{\alpha}{4}+\sin^2\frac{\alpha}{4}=1-\cos\left(\frac{\alpha}{2}-3\pi\right)-$

$$-\left(\cos^2\frac{\alpha}{4}-\sin^2\frac{\alpha}{4}\right)=1+\cos\frac{\alpha}{2}-\cos\frac{\alpha}{2}=1.$$

10-misol. $\sin^4\alpha+\cos^4\alpha-\sin^6\alpha-\cos^6\alpha-\sin^2\alpha\cdot\cos^2\alpha$ ifodani soddalashtiring.

Yechish. $\sin^4\alpha+\cos^4\alpha=\left(\sin^2\alpha+\cos^2\alpha\right)^2-2\sin^2\alpha\cos^2\alpha=1-2\sin^2\alpha\cdot\cos^2\alpha,$

$$\sin^6\alpha+\cos^6\alpha=(\sin^2\alpha+\cos^2\alpha)^3-3\sin^4\alpha\cdot\cos^2\alpha-3\sin^2\alpha\cdot\cos^4\alpha=$$
$$=1-3\sin^2\alpha\cos^2\alpha(\sin^2\alpha+\cos^2\alpha)=1-3\sin^2\alpha\cdot\cos^2\alpha,$$
$$\sin^4\alpha+\cos^4\alpha-\sin^6\alpha-\cos^6\alpha-\sin^2\alpha\cos^2\alpha=$$
$$=1-2\sin^2\alpha\cos^2\alpha-(1-3\sin^2\alpha\cos^2\alpha)-\sin^2\alpha\cos^2\alpha=$$
$$=1-2\sin^2\alpha\cos^2\alpha-1+3\sin^2\alpha\cos^2\alpha-\sin^2\alpha\cos^2\alpha=0.$$

11-misol. $A=\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}$ ifodani hisoblang.

Yechish. $A=\left(\sin^2\frac{\pi}{16}\right)^2+\left(\sin^2\frac{3\pi}{16}\right)^2+\left(\sin^2\frac{5\pi}{16}\right)^2+\left(\sin^2\frac{7\pi}{16}\right)^2=$

$$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{8}\right)^2+\left(1-\cos\frac{3\pi}{8}\right)^2+\left(1-\cos\frac{5\pi}{8}\right)^2+\left(1-\cos\frac{7\pi}{8}\right)^2\right]=$$
$$=\frac{1}{4}\left[4-2\left(\cos\frac{\pi}{8}+\cos\frac{3\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{7\pi}{8}\right)+\right.$$
$$\left.+\left(\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}\right)\right]=1-\frac{1}{2}\times$$
$$\times2\left(2\cos\frac{\pi}{2}\cos\frac{3\pi}{8}+2\cos\frac{\pi}{2}\cos\frac{\pi}{2}\right)+$$
$$+\frac{1}{4}\cdot\frac{1}{2}\left(4+\cos\frac{\pi}{4}+\cos\frac{3\pi}{4}+\cos\frac{5\pi}{4}+\cos\frac{7\pi}{4}\right)=1+\frac{1}{2}=\frac{3}{2}=1\frac{1}{2}.)$$

QUYIDAGI IFODALARNI HISOBLANG:

30. $cos^2\frac{\alpha}{2}cosec^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\cdot cosec^2\frac{\alpha}{2}.$ *j.* $cosec^2\frac{\alpha}{2}.$

31. $\frac{\cos\alpha}{1-\cos\alpha}-\frac{\cos\alpha}{1+\cos\alpha}-2ctg^2\alpha.$ *j.* $0.$

32. $\frac{1+2\sin2x\cdot\cos2x}{\cos2x+\sin2x}-\cos2x$ *j.* $\sin2x.$

33. $(tg^2\alpha-\sin^2\alpha)ctg^2\alpha+\cos^2\alpha.$ *j.* $1.$

34. $\frac{\sin^2\alpha-\sin^2\beta}{\sin^2\alpha\cdot\sin^2\beta}+ctg^2\alpha+\cos^2\beta+\sin^2\beta.$ *j.* $\operatorname{cosec}^2\beta.$

35. $\sqrt{\frac{\operatorname{cosec}^4\alpha-ctg^4\alpha}{\sin^2\alpha+2\cos^2\alpha}}$ *j.* $\frac{1}{|\sin\alpha|}=|\operatorname{cosec}\alpha|.$

36. $\frac{cosec^2\alpha-1}{cos^2 2\alpha-\sin^2\alpha+\sin^2 2\alpha}.$ *j.* $cosec^2\alpha$

37. $\frac{\sin^3\alpha-\cos^3\alpha}{\cos\alpha-\sin\alpha}-\sin\alpha\cdot\cos\alpha$ *j.* $-1.$

38. $\frac{\sin^4 3\alpha-\cos^4 3\alpha}{\sin^2 3\alpha-\cos^2 3\alpha}+tg^2\alpha.$ *j.* $\sec^2\alpha.$

39. $\frac{\sin^2\beta-tg^2\beta}{\operatorname{cosec}^2\beta-ctg^2\beta}\cdot ctg^2\beta.$ *j.* $-\sin^2\beta.$

40. $\sin\alpha(1+tg\alpha)+\cos\alpha(1+ctg\alpha)$ $\quad j.\cos ec\alpha+\sec\alpha.$

41. $\dfrac{\cos ec\alpha-\sin\alpha}{\cos\alpha}\cdot\dfrac{\sec\alpha-\cos\alpha}{\sin\alpha}$ $\quad j.1.$

42. $\dfrac{\sqrt{2}\cos\alpha-2\cos(45^0-\alpha)}{\sqrt{3}\sin\alpha-2\sin(30^0+\alpha)}$ $\quad j.\sqrt{2}tg\alpha.$

43. $\dfrac{\sin(\alpha-\beta)-2\cos\alpha\sin\beta}{\cos(\alpha-\beta)-2\cos\alpha\cos\beta}$ $\quad j.tg(\alpha+\beta).$

44. $\dfrac{\cos(\alpha-\beta)\cos(\alpha+\beta)-\sin(\alpha+\beta)\sin(\alpha-\beta)}{\sin2\alpha}$ $\quad j.ctg2\alpha.$

45. $\dfrac{\sqrt{2}cos\alpha-2\cos\left(\dfrac{\pi}{4}+\alpha\right)}{2\sin\left(\dfrac{\pi}{4}+\alpha\right)-\sqrt{2}\sin\alpha}$ $\quad j.\ tg\alpha.$

46. $\dfrac{\sin\left(\dfrac{\pi}{4}+\alpha\right)\cos\alpha-\cos\left(\dfrac{\pi}{4}+\alpha\right)\sin\alpha}{\cos\dfrac{\pi}{4}}$ $\quad j.\ 1.$

47. $\dfrac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\sin\alpha+\sin\beta}.$ $\quad j.\ \sin\alpha-\sin\beta.$

48. $\left(\dfrac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}\right)^2+\sin^4\alpha+\cos^4\alpha+2\sin^2\alpha\cos^2\alpha$ $\quad j.\ \sec^2(\alpha+\beta)$

49. $\dfrac{\cos^2\alpha-\sin^2\alpha+\cos^2(\alpha-\beta)}{\cos\alpha\cdot\cos\beta\cdot\cos(\alpha-\beta)}.$ $\quad j.\ 2ctg\beta.$

4-Mavzu: Trigonometrik tenglamalar.

1. *sinx=a* tenglamada $|a|\le 1$ bo'lsa, u $x=(-1)^k \arcsin a+\pi k,\ k\in Z$ yechimga ega bo'ladi. Xususiy holda

a) agar $\sin x = 0$ bo'lsa, $x = \pi k,\ k \in Z;$

b) agar $\sin x = 1$ bo'lsa, $x = \frac{\pi}{2}+2\pi k,\ k \in Z;$

v) agar $\sin x = -1$ bo'lsa, $x = -\frac{\pi}{2}+2\pi k,\ k \in Z;$

g) agar $\sin^2 x = a$ bo'lsa, $x = \pm\arcsin\sqrt{a} + \pi k,\ k \in Z;$

Misol. $2\sin\left(\frac{\pi}{4}+x\right)+\sqrt{3}=0$ tenglama yechilsin.

Yechish:

$$\left[2\sin\left(\frac{\pi}{4}+x\right)+\sqrt{3}=0\right]\Leftrightarrow\left[\sin\left(\frac{\pi}{4}+x\right)=\frac{-\sqrt{3}}{2}\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\frac{\pi}{4}+x=(-1)^k\arcsin\left(-\frac{\sqrt{3}}{2}\right)+\pi k\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\frac{\pi}{4}+x=(-1)^k\left(-\frac{\pi}{3}\right)+\pi k\right]\Leftrightarrow$$

$$\Leftrightarrow\left(x=(-1)^{k+1}\cdot\frac{\pi}{3}-\frac{\pi}{4}+\pi k\right),\quad k\in Z$$

2. *cosx=a* tenglamada $|a|\le 1$ bo'lsa, u $x=\pm\arccos a+2\pi k,\ k\in Z;$ yechimga ega bo'ladi. Xususiy holda:

a) agar $\cos x = 0$ bo'lsa, $x = \frac{\pi}{2}+\pi k,\ k \in Z;$

b) agar $\cos x = 1$ bo'lsa, $x = 2\pi k,\ k \in Z;$

v) agar $\cos x = -1$ bo'lsa, $x = \pi+2\pi k,\ k \in Z;$

g) agar $\cos^2 x = a$ bo'lsa, $x = \pm\arccos\sqrt{a}+\pi k,\ k \in Z;$

Misol. $\cos\left(\frac{2}{3}x-\frac{1}{2}\right)-1=0$ tenglama yechilsin.

Yechish.

$$\left[\cos\left(\frac{2}{3}x-\frac{1}{2}\right)-1=0\right]\Leftrightarrow\left[\left(\frac{2}{3}x-\frac{1}{2}\right)=\frac{\pi k}{2}\right]\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{2}{3}x=\frac{1}{2}+\frac{\pi k}{2}\right)\Leftrightarrow\left(x=\frac{3}{4}+\frac{3\pi k}{2}\right),\quad k\in Z.$$

3. *tg x=a* tenglama *x= arctg a+πk, k*∈*Z* yechimga ega bo'ladi. Xususiy holda

a) agar *tg x = 0* bo'lsa, $x = \pi k,\ k \in Z;$

b) agar *tg x = 1* bo'lsa, $x = \frac{\pi}{4} + \pi k,\ k\in Z;$

v) agar *tg x = –1* bo'lsa, $x = -\frac{\pi}{4} + \pi k,\ k\in Z;$

g) agar $tg^2x=a$ bo'lsa, $x=\pm arctg\,\sqrt{a}+\pi k,\ k \in Z.$

Misol. $3tg^2 3x-1=0$ tenglama yechilsin.

Yechish.

$$\left(tg^2 3x=\frac{1}{3}\right)\Leftrightarrow\left(3x=\pm arctg\frac{1}{\sqrt{3}}+k\pi\right)\Leftrightarrow$$

$$\Leftrightarrow\left(3x=\pm\frac{\pi}{6}+k\pi\right)\Leftrightarrow\left(x=\pm\frac{\pi}{18}+\frac{\pi}{3}k\right)\Leftrightarrow\left(x=\frac{\pi}{18}(6k+1)\right).$$

4. *ctg x = a* tenglama *x = arcctg a + πk, k* ∈ *Z* yechimga ega bo'ladi.

a) agar *ctg x = 0* bo'lsa, $x = \frac{\pi}{2}\pi k,\ k \in Z;$

b) agar *ctg x = 1* bo'lsa, $x = \frac{\pi}{4} + \pi k,\ k \in Z;$

v) agar *ctg x = -1* bo'lsa, $x = -\frac{\pi}{4} + \pi k,\ k \in Z;$

g) agar $ctg^2x = a$ bo'lsa, $x = \pm arcctg\ \sqrt{a} + \pi k,\ k \in Z.$

Misol. $ctg^2\left[2x-\frac{\pi}{3}\right]=3$ tenglama yechilsin.

Yechish.

$$\left[ctg^2\left(2x-\frac{\pi}{3}\right)=3\right]\Leftrightarrow\left[ctg\left(2x-\frac{\pi}{3}\right)=\pm\sqrt{3}\right]\Leftrightarrow\left[\left(2x-\frac{\pi}{3}=\pm arcctg\sqrt{3}+\pi k\right)\right]\Leftrightarrow$$

$$\Leftrightarrow\left(2x=\pm\frac{\pi}{6}+\frac{\pi}{3}+\pi k\right)\Leftrightarrow\left(x=\pm\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi k}{2}\right),\qquad k\in z.$$

Matematika kursida har qanday trigonometrik tenglamalar ayniy almashtirishlarni bajarish orqali sodalashtirib, *sin x=a, cos x=a, tg x=a, ctg x=a* ko'rinishdagi eng sodda trigonometrik tenglamalarga keltiriladi.

Trigonometrik tenglamalar quyidagi metodlar yordamida yechiladi.

1. Ko'paytuvchilarga keltirish usuli.

1 - m i s o l. $\sin 2x = \cos 2x \sin 2x$ tenglama yechilsin.

Yechish. $\sin 2x - \cos 2x \sin 2x = 0$, $\sin 2x(1-\cos x) = 0$

1) Agar $1-\cos x \neq 0$ bo'lib, $\sin 2x=0$ bo'lsa, $x=\frac{\pi}{2}n,\ n\in Z$ bo'ladi.

2) Agar $\sin 2x\neq 0$ bo'lib, $1-\cos x=0$ bo'lsa, $\cos x=1,\ x=2\pi n,\ n\in Z$ bo'ladi.

2 - m i s o l. $\sin 3x - \sin x = 0$ tenglama yechilsin.

Yechish. $\sin 3x - \sin x = 2\sin x \cos 2x = 0$

1) Agar $\cos 2x \neq 0$ bo'lib, $\sin x=0$ bo'lsa, $x=\pi n,\ n \in Z$

2) Agar $\sin x\neq 0$ bo'lib, $\cos 2x=0$ bo'lsa, $x=\frac{\pi}{4}+\frac{n\pi}{2},\ n\in Z$ bo'ladi.

3-misol. $\cos^2 x+\cos^2 2x+\cos^2 3x=1{,}5$ tenglama yechilsin.

Yechish. $\cos^2 x = \frac{1+\cos 2x}{2}$ formulaga ko'ra

$$\left(\frac{1+\cos 2x}{2}+\frac{1+\cos 4x}{2}+\frac{1+\cos 6x}{2}=\frac{3}{2}\right)\Leftrightarrow(\cos 2x+\cos 4x+\cos 6x=0)\Leftrightarrow[(\cos 2x+\cos 6x)+\cos 4x=0]\Leftrightarrow$$

$$\Leftrightarrow[2\cos 4x\cos 2x+\cos 4x=0]\Leftrightarrow\cos 4x(\cos 2x+1)=0.$$

1) Agar $2\cos 2x+1\neq 0$ bo'lib, $\cos 4x=0$ bo'lsa, $x=\frac{\pi}{8}+\frac{n\pi}{4},\ n\in Z;$

2) Agar $\cos 4x\neq 0$ bo'lib, $2\cos 2x+1=0$ bo'lsa, $\cos 2x=-\frac{1}{2}$, $2x=-\frac{2\pi}{3}+2\pi n;$ $x=-\frac{\pi}{3}+\pi n;\ n\in Z.$

II. O'zgaruvchilarni kiritish usuli.

1 - m i s o l. $2\cos^2 x=3\sin x$ tenglama yechilsin.

Yechish. $(2\cos^2 x - 3\sin x=0)\Leftrightarrow(3\sin x - 2(1-\sin^2 x) = 0$

$\Leftrightarrow (3\sin x - 2 + 2\sin^2 x =0)$.

Agar $\sin x = y$ desak,

$$2y^2+3y-2=0, \quad y_1=\frac{1}{2}, \quad y_2=-2.$$

$$\left(\sin x=\frac{1}{2}\right) \Leftrightarrow \left(x=\frac{\pi}{6}+2k\pi\right), \quad k\in z.$$

2 - m i s o l. *cos2x – 5 sin x – 3 = 0* tenglama yechilsin.

Yechish. $cos2x=1-2sin^2x$ formulaga ko'ra $(1-2sin^2x-5sinx-3=0) \Leftrightarrow$ $(2sin^2x+5sinx+2=0) \Leftrightarrow sinx=y$ desak, $2y^2+5y+2=0$, $y_1=-2$, $y_2=-\frac{1}{2}$.

1) $sin\ x=-2$ tenglama yechimga ega emas.

2) $\left(\sin x=-\frac{1}{2}\right) \Leftrightarrow \left(x=(-1)^k \arcsin\left(-\frac{1}{2}\right)+\pi k\right), k\in z;$

$$x=(-1)^{k+1}\frac{\pi}{6}+\pi k, k\in Z.$$

III. Bir jinsli tenglamalarni yechish.

1-misol. $2sin^2x-sinxcosx-cos^2x=0$ tenglamani yeching.

Yechish. Bu tenglama sinus va kosinus funksiyalariga nisbatan bir jinslidir. Tenglamalarning har ikki tomonini $cos^2x\neq 0$ ga bo'lsak, $2tg^2x-tg\ x-1=0$ hosil bo'ladi. Bundan $tgx=1$ va $tg\ x=-\frac{1}{2}$.

1) Agar $tgx=1$ bo'lsa, $x=\frac{\pi}{4}+\pi k,\ k\in Z;$

2) Agar $tg\ x=-\frac{1}{2}$ bo'lsa, $x=-arctg\frac{1}{2}+\pi k,\ k\in Z$ bo'ladi.

2-m i s o l. $cos^2x+3sin^2x+2\sqrt{3}\,sinxcosx=3$ tenglamani yeching.

Yechish. Bu tenglamani ayniy almashtirishlar bajarish orqali bir jinsli ko'rinishga keltiramiz.

$$\cos^2 x+3\sin^2 x+2\sqrt{3}\sin x\cos x=3(\sin^2 x+\cos^2 x),$$
$$\cos^2 x+3\sin^2 x+2\sqrt{3}\sin x\cos x-3\sin^2 x-3\cos^2 x=0$$
$$2\cos^2 x-2\sqrt{3}\sin x\cos x=0,$$
$$2\cos x(\cos x-\sqrt{3}\sin x)=0.$$

1) Agar $cosx-\sqrt{3}\,sinx\neq 0$ bo'lib, $cosx=0$ bo'lsa, $x=\frac{\pi}{2}+k\pi,\ k\in Z;$

2) Agar $cos\ x\neq 0$ bo'lib, $cosx-\sqrt{3}\,sin\ x=0$ bo'lsa, $tgx=\frac{1}{\sqrt{3}}$, $x=\frac{\pi}{6}+k\pi,\ k\in Z;$

IV. $a\sin x + b\cos x = c$ ko'rinishdagi tenglamani yeching.

1-usul. Bu tenglamani yechish uchun $tg\frac{x}{2} = t$ almashtirish bajaramiz. Bizga ma'lumki, $\sin x = \dfrac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}$, $\cos x = \dfrac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$, edi, shunga ko'ra berilgan tenglama quyidagi ko'rinishni oladi:

$$\frac{2at}{1+t^2} + \frac{b(1-t^2)}{1+t^2} = c,$$

$$2at + b - bt^2 = c + ct^2, \qquad (b+c)t^2 - 2at + (c-b) = 0,$$

$$t = \frac{a \pm \sqrt{a^2+b^2-c^2}}{c+b},$$

$$x = 2arctg\frac{a \pm \sqrt{a^2+b^2-c^2}}{c+b} + 2k\pi, \ k \in z, \quad a^2+b^2 \geq c^2 \ \text{ва} \ b \neq -c.$$

Agar $b = -c$ bo'lsa, kvadrat tenglama chiziqli tenglamaga almashadi:

$2at+2b=0, \quad t = -\frac{b}{a}, \ x = -2arctg\frac{b}{a} + 2k\pi, \ k \in Z.$

2-usul. Tenglamaning har ikkala tomonini $\sqrt{a^2+b^2}$ ga bo'lamiz:

$$\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

$$\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2}}\right)^2 = 1 \quad \text{ва} \quad \left|\frac{a}{\sqrt{a^2+b^2}}\right|^2 \leq 1, \qquad \left|\frac{b}{\sqrt{a^2+b^2}}\right|^2 \leq 1$$

Agar $\frac{a}{\sqrt{a^2+b^2}} = \cos\varphi$ va $\frac{b}{\sqrt{a^2+b^2}} = \sin\varphi$ desak, berilgan tenglama $\sin x \cdot \cos\varphi + \cos x \cdot \sin\varphi = \frac{c}{\sqrt{a^2+b^2}}$ ko'rinishni oladi, bundan $\sin(x+\varphi) \frac{c}{\sqrt{a^2+b^2}}$ bo'ladi. $\varphi = arctg\frac{b}{a}$; agar $a^2+b^2 \geq c^2$ bo'lsa,

$$x = (-1)^k \arcsin\frac{c}{\sqrt{a^2+b^2}} + \pi k - arctg\frac{b}{a}, \qquad k \in z.$$

1-misol. $3\cos x + 4\sin x = 5$ tenglama yechilsin.

Yechish. $\sqrt{3^2+4^2}=\sqrt{25}$ bo'lgani uchun tenglamaning har ikki tomonini 5 ga bo'lamiz: $\frac{3}{5}\cos x+\frac{4}{5}\sin x=1,$ $\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2=1,$ shuning uchun $\frac{3}{5}=\sin\varphi$ va $\frac{4}{5}=\cos\varphi$ bo'ladi, bundan *sin*φ *·cosx+cos*φ*sin x=1* tenglamani hosil qilamiz yoki *sin(x+*φ*)*=1 bo'ladi:

$$x+\varphi=\frac{\pi}{2}+2k\pi, \qquad x=\frac{\pi}{2}+2k\pi-\varphi, \qquad k\in z,$$
$$\varphi=\arcsin\frac{3}{5}, \qquad x=\frac{\pi}{2}-\arcsin\frac{3}{5}+2k\pi, \qquad k\in z.$$

2-usul. Agar $\sin x=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}},$ va $\cos x=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}},$ ekanligini nazarda tutib, $tg\frac{x}{2}=y$ desak, $3\cdot\frac{1-y^2}{1+y^2}+4\cdot\frac{2y}{1+y^2}=5$, yoki $3-3y^2+8y=5+5y^2$ yoki $4y^2-4y+1=0$ bundan $y=\frac{1}{2}$ yechim hosil bo'ladi:

$$\left(tg\frac{x}{2}=\frac{1}{2}\right)\Leftrightarrow\left(\frac{x}{2}=arctg\frac{1}{2}+\pi k\right),$$
$$x=2arctg\frac{1}{2}+2\pi k, \qquad k\in\pi.$$

MUSTAQIL YECHISH UCHUN MISOLAR.

Trigonometrik tenglamalarni yeching.

1. $tgx + \sin x tgx = 0$ $\qquad j: n\pi$

2. $2\sin x - 3\cos x = 6.$ $\qquad j: x \in \varnothing.$

3. $\sin^2 x - (1+\sqrt{3})\sin x\cos x + \sqrt{3}\cos^2 x = 0.$

$$j: \frac{\pi}{4} + n\pi, \quad arctg3 + n\pi$$

4. $\sin^2 x - 4\sin x\cos x + 3\cos^2 x = 0.$ $\qquad j: \frac{\pi}{4} + n\pi, \quad arctg\, 3 + n\pi.$

5. $\sqrt{3}\sin^2 x - 4\sin x\cos x + \sqrt{3}\cos^2 x = 0.$ $\qquad j: \frac{\pi}{3} + n\pi, \quad \frac{\pi}{6} + k\pi.$

6. $\sin^2 x + 3\cos^2 x - 2\sin x\cos x = \frac{-5-\sqrt{3}}{2}.$ $\qquad$ j: $\frac{\pi}{6} + k\pi$

7. $7\cos^2 x - 7\sin 2x = 2.$ $\qquad$ j: $x = arctg\,\frac{-7\pm\sqrt{53}}{2}.$

8. $\frac{2}{3\sqrt{2}\sin x - 1} = 1.$ $\qquad$ j: $x = (-1)^k \cdot \frac{\pi}{4} + k\pi, \quad k \in z.$

9. $\frac{6}{tgx - 2} = 3 - tgx$ $\qquad$ j: $\varnothing$

10. $(1-2\sin x)\sin x = 2\cos 2x - 1$ $\qquad$ j: $x = 2k\pi - \frac{\pi}{2}$

11. $\sin^4 x + \cos^4 x - 2\sin 2x + \sin^2 2x = 0$

$j: \ x = \frac{(-1)^k}{2}\arcsin\,(2-\sqrt{2}) + \frac{k\pi}{2}$

12. $\sin^3 x\cos x - \cos^3 \sin x = \cos^4 \frac{x}{3}$

$$j: \ x = \frac{1}{4}\left[(-1)^{k+1}\arcsin\,\frac{1}{4} + k\pi\right].$$

13. $\sin^4 x + \cos^4 x = \cos 4x$ $\qquad j: \ \frac{\pi}{2}n.$

14. $tg(40^0 + x)ctg(5^0 - x) = \frac{2}{3}$ $\qquad$ j: $x = \frac{1}{2}\left[\pi k - 35^0 + (-1)^{k+1}\arcsin\,\frac{\sqrt{2}}{10}\right]$

15. $(\sin x + \cos x)\sqrt{2} = tgx + ctgx$ $\qquad J: \ x = 2k\pi + \frac{\pi}{4}$

16. $\sin 3x \cdot \sin^3 x + \cos 3x \cdot \cos^3 x = \frac{1}{8}$ $\qquad J: x = \pi k \pm \frac{\pi}{6}$

17. $\sin x + \sin 3x + \sin 7x = 3$ $\qquad$ J: $x = \varnothing$

18. $tg7x+tg3x=0$ J: $x=\frac{\pi k}{10}$

19. $1+\sin x+\cos x=0$ J: $x_1=2\pi k-\frac{\pi}{2}$, $x_2=\pi(2\pi+1)$

20. $\sin^{10}x+\cos^{10}x=\frac{29}{16}\cos^4 2x$ J: $x=\frac{2k+1}{8}\pi$

21. $\left|tgx+ctgx\right|=\frac{4}{\sqrt{3}}$ J: $x=\frac{\pi k}{2}\pm\frac{\pi}{6}$

22. $13\sin x-12\cos x+13\sin 3x=0$ J: $x=\frac{\pi k}{2}$

23. $1+2\cos 2x+2\cos 4x+2\cos 6x=0$ J: $x=-\frac{\pi k}{4}+k\pi$

24. $\cos^4 x+\cos^4(x-\frac{\pi}{4})=\frac{1}{4}$ J: $x=\frac{\pi}{2}(2k+1)$.

25. $2\sin 2x+\sin x+\cos x=1$ J: $x_1=2k\pi$, $x_2=2k\pi+\frac{\pi}{2}$

26. $3\sin x+\sin(x+\frac{3\pi}{2})=1-3\sin x\cos x$ J: $(2k+1)\pi$, $k\epsilon Z$

27. $3-\cos^2 x-3\sin x=0$ J: $\frac{\pi}{2}+2k\pi$, $k\epsilon Z$

28. $\cos^3 x+4\sin^2\frac{x}{2}\cdot\cos^2\frac{x}{2}\cos x=0$ J: $\frac{\pi}{2}+2k\pi$

29. $\cos^2 2x-\sin^2 2x=-\frac{1}{2}$ J: $\pm\frac{\pi}{6}+\frac{\pi}{2}k$

30. $\sin^3 x+2\sin\frac{x}{2}\cos\frac{x}{2}=0$ J: πk

31. $\sin^2 x\cos\frac{x}{2}-\frac{1}{2}\sin 2x\sin\frac{x}{2}=0$ J: πk

32. $2\,tg2x\sin\frac{x}{2}\cos\frac{x}{2}\sin x+tg2x=0$ J: $\frac{1}{2}\pi k$

33. $\frac{4tg\frac{3x}{2}}{1-tg^2\frac{3x}{2}}-tg3x=-\frac{\sqrt{3}}{3}$ J: $-\frac{\pi}{18}+\frac{\pi}{3}k$

34. $\frac{2tgx}{1-tg^2 x}+2tg2x=3$ J: $\frac{\pi}{8}+\frac{\pi}{2}k$

35. $4\sin^4 x + \sin^2 2x = 2$ *J:* $\frac{\pi}{4}(2k+1)$

36. $tg(x+\frac{\pi}{4}) + tg(x-\frac{\pi}{4}) = 2$ *J:* $\frac{\pi}{8} + \frac{\pi}{2}k$

5-MAVZU: PARAMETRLI TRIGONOMETRIK TENGLAMALARNI YECHISH.

Parametrli trigonometrik tenglamani yechish parametrlarning mumkin bo'lgan har bir qiymatlari sistemasi uchun berilgan tenglamaning hamma yechimlari to'plamini aniqlash demakdir. Parametrik ko'rinishdagi trigonometrik tenglamalarni yechish quyidagi ketma-ketliklar asosida amalga oshiriladi.

1. Parametrlarning mumkin bo'lgan har bir qiymatlari sistemasi uchun yechimlar sonini aniqlash.
2. Hosil qilingan parametrli yechim formulalarini topish.
3. Tenglamani parametrlar asosidagi qiymatlar sistemasini aniqlash.

1-misol. $\sin(a+x)+\sin x=\cos\frac{a}{2}$ tenglama yechilsin.

Yechish. Berilgan tenglamaning chap tomonida turgan ifodaga trigonometrik funksiyalar yig'indisini ko'paytmaga keltirish formulasini tadbiq qilsak, u quyidagi ko'rinishni oladi:

$$2\sin(\frac{\alpha}{2}+x)\cos\frac{\alpha}{2}=\cos\frac{\alpha}{2}.$$

Bu tenglamada quyidagi ikki hol bo'lishi mumkin.

1. Agar $\cos\frac{\alpha}{2}\neq 0, \quad \alpha\neq(2n+1)\pi$ bo'lsa, u holda tenglamani quyidagicha yozish mumkin:

$$2\sin(\frac{\alpha}{2}+x)=1,$$

$$x=-\frac{\alpha}{2}+(-1)^k\frac{\pi}{6}+k\pi, \quad k\in Z.$$

2. Agar $\alpha=(2n+1)\pi$ bo'lsa, u holda $\cos\frac{\alpha}{2}=0$ bo'ladi. Bu holda ham tenglik o'rinli bo'ladi:

$$J: \quad x=\begin{cases} -\frac{\alpha}{2}+(-1)^k\frac{\pi}{6}+k\pi, & \alpha\neq(2n+1)\pi, \\ -\,ixtiyoriy \quad son, & \alpha=(2n+1)\pi \end{cases}$$

2-misol. $\sin^2 x + a\sin^2 2x = \sin\frac{\pi}{6}$ tenglama yechilsin.

Yechish. $\frac{1-\cos 2x}{2} + a(1-\cos^2 2x) = \frac{1}{2}$,

$1 - \cos 2x + 2a - 2a\cos^2 2x = 1$,

$2a\cos^2 2x + \cos 2x - 2a = 0$,

$(\cos 2x)_1 = \frac{-1+\sqrt{1+16a^2}}{4a}$; $\quad (\cos 2x)_2 = \frac{-1-\sqrt{1+16a^2}}{4a}$

$a)\quad \cos 2x = \frac{-1+\sqrt{1+16a^2}}{4a}$, $1+16a^2>0$, $\left|\frac{-1+\sqrt{1+16a^2}}{4a}\right| \le 1$,

$-1+\sqrt{1+16a^2} \le 4a,$

$1+16a^2 \le 4a^2+8a+1$,

$12a^2 - 8a \le 0$; $a(12a-8) \le 0$

1) $a=0$, $12a-8 \ne 0$,

2) $a \ne 0$, $12a-8=0$, $a=\frac{2}{3}$, bundan $a \le \frac{2}{3}$ bo'ladi.

b) $\cos 2x = \frac{-1-\sqrt{1+16a^2}}{4a}$; $\quad \left|\frac{-1-\sqrt{1+16a^2}}{4a}\right| \le 1$,

tengsizlik a ning hech qanday qiymatlarida bajarilmaydi. Agar $a=0$ bo'lsa $\left[\sin^2 x = \frac{1}{2}\right] \Rightarrow x_{1,2} = \pm\frac{\pi}{4} + k\pi, \quad k \in Z.$

J: $a \ne 0$ da $x_{1,2} = \pi k \pm \frac{1}{2}\arccos\left[\frac{-1+\sqrt{1+16a^2}}{4a}\right]$.

$a=0$ da $x_{3,4} = \pm\frac{\pi}{4} + k\pi, \ k \in Z.$

3-misol. $\cos x + \sin x = a$ tenglama yechilsin.

Yechish. $\cos x + \sin x = \sqrt{2}\cos[x - \frac{\pi}{4}]$; agar $|a| \le \sqrt{2}$ bo'lsa, bu tenglama yechimga ega bo'ladi. Agar $|a| \le \sqrt{2}$ bo'lsa, $x = \frac{\pi}{4} \pm \arccos\frac{a}{\sqrt{2}} + 2k\pi$, agar $|a| > \sqrt{2}$ bo'lsa, yechim yo'q.

4-misol. $\sin 2x + 3\cos 2x = a$ tenglama yechilsin.

Yechish. $2\sin x \cdot \cos x + 3\cos^2 x - 3\sin^2 x = a,$

$$2tgx+3-3tg^2x=a(1+tg^2x),$$

$$tg^2x(a+3)-2tgx+(a-3)=0,$$

$$[tgx]_{1,2}=\frac{1\pm\sqrt{10-a^2}}{a+3}.$$

Agar $|a|\le\sqrt{10}$ bo'lsa, $x_1=arctg\frac{1\pm\sqrt{10-a^2}}{a+3}+k\pi,\quad k\in Z$. Agar $a=-3$ bo'lsa, $\sin 2x+3\cos 2x=-3$ bo'ladi.

$2\sin x\cos x+3\cos^2x+3\cos^2x-3=-3,$ $2\sin x\cos x+6\cos^2x=0,$ $\cos x(\sin x+3\cos x)=0,$

1) $\cos x=0,\quad \sin x+3\cos x\ne 0,\quad x_1=\frac{\pi}{2}+k\pi,\quad k\in Z;$

2) $\cos x\ne 0,\quad \sin x+3\cos x=0,\quad x_2=\mathrm{arctg}(-3)+k\pi,\ k\in Z.$

J: Agar $|a|\le\sqrt{10}$ $(a\ne -3)$ bo'lsa, $x=arctg\frac{1\pm\sqrt{10-a^2}}{a+3}+k\pi,\quad k\in Z$, agar $(a\ne -3)$ bo'lsa, $x=\frac{\pi}{2}+k\pi,\ k\in Z;$ agar $|a|>\sqrt{10}$ bo'lsa, tenglama yechimga ega emas.

5-misol. $\sin^6x+\cos^6x=a(\sin^4x+\cos^4x)$ tenglama yechilsin.

Yechish. $\sin^4x+\cos^4x=(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x=1-2\sin^2x\cos^2x,$

$\sin^6x+\cos^6x=(\sin^2x+\cos^2x)^3-3\sin^4x\cos^2x-3\sin^2x\cdot\cos^4x)=1-3\sin^2x\cos^2x,$

$1-3\sin^2x\cos^2x=a(1-2\sin^2x\cos^2x),$

$1-3\sin^2x\cos^2x=a-2a\sin^2x\cos^2x,$

$\sin^2x\cdot\cos^2x=\frac{a-1}{2a-3},\quad \sin^22x=4\cdot\frac{a-1}{2a-3}.$

Agar bu erda $0\le 4\cdot\frac{a-1}{2a-3}\le 1$ bo'lsa, tenglama ma'noga ega bo'ladi. Bu tengsizlikni yechsak, $\frac{1}{2}\le a\le 1$ bo'ladi. Agar $a=\frac{3}{2}$ bo'lsa, $1-3\sin^2x\cos^2x=\frac{3}{2}-3\sin^2x\cos^2x$ hosil bo'ladi, bundan $1=\frac{3}{2}$ tenglik hosil bo'ladi, buning bo'lishi mumkin emas.

Agar $\frac{1}{2}\le a\le 1$ bo'lsa, $x=\pm\frac{1}{2}\arcsin\left[2\sqrt{\frac{a-1}{2a-3}}\right]+\frac{k\pi}{2}$

Agar $a<\frac{1}{2}$ va $a>1$ bo'lsa, yechim yo'q.

6-misol. $\sin\frac{3x}{2}+\sin x=m\sin\frac{x}{2}$ tenglama yechilsin.

Yechish. $\sin\left[x+\frac{x}{2}\right]=\sin x\cos\frac{x}{2}+\cos x\sin\frac{x}{2}$, bunga ko'ra

$$\sin x+\sin x\cos\frac{x}{2}+\cos x\sin\frac{x}{2}-m\sin\frac{x}{2}=0,$$

$$\sin x\left[1+\cos\frac{x}{2}\right]+\sin\frac{x}{2}(\cos x-m)=0.$$

$$\sin\frac{x}{2}\left[2\cos\frac{x}{2}+4\cos^2\frac{x}{2}-1-m\right]=0,$$

$$1)\quad \sin\frac{x}{2}=0,\quad x=2k\pi,\quad k\in Z;$$

$$2)\quad 4\cos^2\frac{x}{2}+2\cos\frac{x}{2}-(1+m)=0,$$

$$\cos\frac{x}{2}=\frac{-1\pm\sqrt{5+m\cdot 4}}{4}.$$

$a)\quad \cos\frac{x}{2}=\frac{-1\pm\sqrt{5+m\cdot 4}}{4}$, bu erda quyidagi shartlar bajarilishi kerak:

$$\begin{cases}5+4m\geq 0,\\ \left|\frac{-1\pm\sqrt{5+4m}}{4}\right|\leq 1,\end{cases}$$

Bu tengsizliklarni yechsak, $-\frac{5}{4}\leq m\leq 5$ hosil bo'ladi. Bu shartga ko'ra

$$x=\pm 2\arccos\frac{-1\pm\sqrt{5+4m}}{4}+4k\pi,\quad k\in Z;$$

$$б)\quad \cos\frac{x}{2}=\frac{-1-\sqrt{5+4m}}{4}.$$

Buni yechsak, $x=\pm 2\arccos\frac{-1\pm\sqrt{5+4m}}{4}+4k\pi,\quad k\in Z,\quad -\frac{5}{4}\leq m\leq 1.$

J: Agar $-\frac{5}{4}\leq m\leq 1.$ bo'lsa, $x_1=2k\pi,\ k\in Z$

$x_{2,3}=\pm 2\arccos\frac{-1\pm\sqrt{5+4m}}{4}+4k\pi,\quad k\in Z$, Agar $1\leq m\leq 5$ bo'lsa, $x_1=2k\pi,\ k\in Z$

$x_2 = \pm 2\arccos\dfrac{-1+\sqrt{5+4m}}{4} + 4k\pi, \quad k \in Z,$ Agar $m > 5, \quad m < -\dfrac{5}{4}$ bo'lsa, $x=2k\pi,\ k\in Z$.

7-misol. $a\sin 2x + b\sin\left[2x - \dfrac{\pi}{2}\right] + \sqrt{a^2+b^2}\sin 6x = 0$ tenglamani yeching.

Yechish. Bizga ma'lumki, $a\sin x + b\cos x = A\sin(x+\varphi)$.

Bu erda $A=\sqrt{a^2+b^2}$ burchakning qiymati esa quyidgai shartlardan kelib chiqadi: $\sin\varphi=\dfrac{b}{A},\ \cos\varphi=\dfrac{a}{A},\ \varphi=arctg\dfrac{b}{a}$. Shuning uchun $\sin(2x-\varphi)-\sin 6x=0,\ \sin\dfrac{4x+\varphi}{2}\cos\dfrac{8x-\varphi}{2}=0;$

a) $\sin\dfrac{4x+\varphi}{2}=0,\ 4x+\varphi=(-1)^k\arcsin 0^0+2\pi k,$

$4x=-\varphi+2\pi k,\ x=\dfrac{\pi k}{2}-\dfrac{1}{4}arctg\dfrac{b}{a}.$

b) $\cos\dfrac{8x-\varphi}{2}=0,\ \dfrac{8x-\varphi}{2}=\arccos 0^0+2\pi k,\quad 8x-\varphi=\pi+2\pi k.$

J: $x=\dfrac{2k+1}{8}\pi+\dfrac{1}{8}arctg\dfrac{b}{a},\ k\epsilon Z.$

MUSTAQIL YECHISH UCHUN MISOLLAR.

1-misol. *secx+cosecx+secxcosecx=a, (a≠0)* tenglama yechilsin.

J: Agar $\left|\frac{a+2}{a\sqrt{2}}\right| \leq 1$ bo'lsa, $x=-\frac{\pi}{4}+(-1)^k \arcsin\frac{a+2}{a\sqrt{2}}+\pi k$, *k*ϵ*Z.*

2-misol. *tg(a+x)tg(a–x)=1–2cos2x* tenglama yechilsin.

J: $x=\pi k\pm\frac{1}{2}\arccos\frac{-\cos a\pm\sqrt{\cos^3 2a+4\cos 2a}}{2}$

6-TRIGONOMETRIK TENGLAMALAR SISTEMASINI YECHISH

Tarkibida trigonometrik funksiyalar qatnashgan bir necha tenglama trigonometrik tenglamalar sistemasini hosil qiladi. Trigonometrik tenglamalar sistemasini yechish tenglamadagi no'malumlarning shu tenglamalar sistemasini qanoatlantiradigan qiymatlarini topish demakdir. Ikki noma'lumli ikkita tenglama sistemasining yechimi deb, noma'lumlarning ikkala tenglamani ham qanoatlantiradigan juft qiymatlariga aytiladi.

1-misol. $\begin{cases} x+y=\dfrac{\pi}{4} \\ tgx+tgy=1 \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish.

$$y=\frac{\pi}{4}-x, \quad tgx+tg\left[\frac{\pi}{4}-x\right]=1,$$

$$\frac{tgx(tgx-1)}{1+tgx}=0;$$

$$a) \quad tgx=1,\ x=\frac{\pi}{4}+k\pi, \quad y=-k\pi, \quad k\in Z;$$

$$b) \quad tgx=0,\ x=k\pi, \quad y=\frac{\pi}{4}-k\pi, \quad k\in Z.$$

$$J{:}\ \begin{cases} x_1=\pi k+\dfrac{\pi}{4}, \\ y_1=-\pi k, \end{cases} \quad \begin{cases} x_2=\pi k, \\ y_2=\dfrac{\pi}{4}-\pi k,\ k\in Z. \end{cases}$$

2- Misol. $\begin{cases} \dfrac{\cos(x+y)}{\cos(x-y)}=\dfrac{1}{9} \\ \sin x\cdot\sin y=\dfrac{1}{3} \end{cases}$ tenglamalar sistemasi yechilsin.

Yechish:

$$\left[\frac{\cos x\cos y-\sin x\sin y}{\cos x\cos y+\sin x\sin y}=\frac{1}{9}\right]\Rightarrow \qquad \Rightarrow\left[\frac{\cos x\cos y+\dfrac{1}{3}}{\cos x\cos y-\dfrac{1}{3}}\right]=\frac{1}{9},$$

$\cos x\cos y=\frac{1}{3}+\frac{1}{9}\left[\cos x\cos y+\frac{1}{3}\right],\quad \cos x\cos y=\frac{1}{3}+\frac{1}{27}+\frac{1}{9}\cos x\cos y,\quad \frac{8}{9}\cos x\cos y=\frac{10}{27}$

$\cos x\cos y=\frac{5}{12},$

$$\overset{-}{+}\begin{cases}\cos x\cos y=\frac{5}{12}\\ \sin x\sin y=\frac{1}{3}\end{cases}\Rightarrow\begin{cases}\cos x\cos y-\sin x\sin y=\frac{1}{12},\\ \cos x\cos y+\sin x\sin y=\frac{3}{4}\end{cases}\Rightarrow\begin{cases}\cos(x+y)=\frac{1}{12}\\ \cos(x-y)=\frac{3}{4}\end{cases}\Rightarrow\begin{cases}x+y=2k_1\pi\pm\arccos\frac{1}{2}\\ x-y=2k_2\pi\pm\arccos\frac{3}{4}\end{cases}$$

$$x=\pi(k_1+k_2)\pm\frac{1}{2}\left[\arccos\frac{1}{12}+\arccos\frac{3}{4}\right]=\pi(k_1+k_2)\pm\frac{1}{2}\arccos\frac{3-\sqrt{1001}}{48}.$$

$$y=\pi(k_1-k_2)\pm\frac{1}{2}\left[\arccos\frac{1}{12}-\arccos\frac{3}{4}\right]=\pi(k_1-k_2)\pm\frac{1}{2}\arccos\frac{3+\sqrt{1001}}{48}.$$

3-Misol. $\begin{cases}\sin x+\sin y=a\\ x+y=2b\end{cases}$ Tenglamalar sistemasi yechilsin.

Yechish:

$$\begin{cases}2\sin\frac{x+y}{2}\cos\frac{x-y}{2}=a\\ x+y=2b\end{cases}\Rightarrow\begin{cases}2\sin b\cos\frac{x-y}{2}=a,\\ x+y=2b\end{cases}\Rightarrow$$

$$\Rightarrow\left[\cos\frac{x-y}{2}=\frac{a}{2\sin b}\right]\Rightarrow\left[x-y=2\left[k\pi\cdot 2\pm\arccos\frac{a}{2\sin b}\right]\right].$$

$$y=\frac{b}{2}-\arccos\frac{a}{\sin b}+k\pi,\quad x=\frac{3b}{2}+\arccos\frac{a}{\sin b}+k\pi,\quad k\in Z.$$

1) agar $b=0,\ a\neq 0$ bo'lsa, sistema yechimga ega emas.

2) agar $b\neq 0,\ \left|\frac{a}{2\sin b}\right|\leq 1$ bo'lsa, sistema yechimga ega.

3) agar $b=0,\ a=0$ bo'lsa, sistema cheksiz ko'p yechimga ega.

4-Misol. $\begin{cases}\sin x+\sin y=1,\\ x+y=\frac{\pi}{3}\end{cases}$ tenglamalar sistemasi yechilsin.

Yechish. $\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$, shuning uchun

$$\begin{cases}\cos\frac{x-y}{2}=1,\\ x+y=\frac{\pi}{3}\end{cases}\Rightarrow\begin{cases}\frac{x-y}{2}=\pm\arccos 1+2k\pi,\\ x+y=\frac{\pi}{3}\end{cases}\Rightarrow$$

$$+\begin{cases}x-y=4k\pi\pm 2\arccos 1,\\ x+y=\frac{\pi}{3}\end{cases}\Rightarrow$$

$$\Rightarrow (2x = 4\pi k \pm \arccos 1 + \frac{\pi}{3}) \Rightarrow (x = 2k\pi \pm \frac{\pi}{6}), \quad k \in Z.$$

$$-\begin{cases} x - y = 4k\pi \pm 2\arccos 1, \\ x + y = \dfrac{\pi}{3} \end{cases} \Rightarrow \left[2y = \frac{\pi}{3} - 4k\pi \right] \Rightarrow$$

$$\Rightarrow \left[y = \frac{\pi}{6} - 2k\pi \right].$$

$$j: x = 2k\pi + \frac{\pi}{6}, \quad y = \frac{\pi}{6} - 2k\pi, \quad k \in Z.$$

5-misol. $\begin{cases} \sin x - \sin y = \cos ecx, \\ \cos x + \cos y = \sec x \end{cases}$ tenglamalar sistemasini yeching.

Yeching.

$$\begin{cases} \sin x - \sin y = \cos ecx = \dfrac{1}{\sin x}, \\ \cos x + \cos y = \sec x = \dfrac{1}{\cos x} \end{cases} \Rightarrow$$

$$\begin{cases} \sin^2 x - \sin y \sin x = 1, \\ \cos^2 x + \cos x \cos y = 1. \end{cases} \qquad (1)$$

(1) sistemadagi tenglamalarni o'zaro qo'shsak,

$$\sin^2 x + \cos^2 x + \cos(x + y) = 2$$

yoki

$$\cos(x + y) = -1; \qquad x + y = \pi + 2k_1\pi$$

hosil bo'ladi. (1) sistemadagi tenglamalarni o'zaro ayirsak quyidagi tenglik hosil bo'ladi.

$$\cos^2 x - \sin^2 x = \cos x \cos y - \sin x \cos y$$

yoki

$$\cos 2x = \cos(x + y), \qquad \cos 2x - \cos(x + y) = 0,$$

$$\cos 2x - \cos(x - y) = -2\sin\frac{2x + x + y}{2}\sin\frac{2x - x - y}{2} =$$

$$= -2\sin\frac{3x + y}{2}\sin\frac{x - y}{2} = 0.$$

$$1) \quad \sin\frac{3x + y}{2} = 0, \quad 3x + y = 2k_2\pi.$$

$$2) \quad \sin\frac{x - y}{2} = 0, \quad x - y = 2k_2\pi.$$

$$\begin{cases} x+y=\pi+2\pi k_1, \\ 3x+y=2k_2\pi \end{cases} \Rightarrow \begin{cases} x=\pi(k_2-k_1)-\dfrac{\pi}{2}, \\ y=\pi(3k_1-k_2)+\dfrac{3\pi}{2}, \quad k\in Z. \end{cases}$$

6-misol. $\begin{cases} \cos^2 x+\cos^2 y=\dfrac{1}{4}, \\ x+y=150^0 \end{cases}$ tenglamalar sistemasini yeching.

Yechish: $\begin{cases} \cos^2 x+\cos^2 y=\dfrac{1}{4}, \\ x+y=150^0 \end{cases} \Rightarrow \begin{cases} \dfrac{1+\cos 2x}{2}+\dfrac{1+\cos 2y}{2}=\dfrac{1}{4}, \\ x+y=150^0 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} \cos 2x+\cos 2y=\dfrac{1}{2}-2, \\ x+y=150^0 \end{cases} \Rightarrow \begin{cases} 2\cos(x+y)\cos(x-y)=-\dfrac{3}{2} \\ x+y=150^0 \end{cases} \Rightarrow$$

$$\Rightarrow \left[\cos 150^0\cos(x-y)=-\frac{3}{4}\right] \Rightarrow \left(-\frac{\sqrt{3}}{2}\cos(x-y)=-\frac{3}{4}\right) \Rightarrow$$

$$\Rightarrow \left[\cos(x-y)=\frac{\sqrt{3}}{2}\right] \Rightarrow (x-y)=\pm\arccos\frac{\sqrt{3}}{2}+2k\pi \Rightarrow$$

$$\Rightarrow \left[x-y=\pm\frac{\pi}{6}+2k\pi, \quad k\in Z\right]. \qquad \begin{cases} x+y=150^0, \\ x-y=30^0+360^0 k \end{cases} \Rightarrow \begin{cases} x=180^0 k+15^0+75^0, \\ y=75^0-180^0 k-15^0 \end{cases}$$

7-misol. $\begin{cases} \cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=\dfrac{1}{2}, \\ \cos x\cos y=\dfrac{1}{4} \end{cases}$ tenglama sistemasini yeching.

Yechish.

$$\begin{cases} \cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=\dfrac{1}{2}, \\ \cos x\cos y=\dfrac{1}{4} \end{cases} \Rightarrow \begin{cases} \cos x+\cos y=1, \\ \cos x\cos y=\dfrac{1}{4} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \cos x+\cos y=1, \\ \cos x=\dfrac{1}{4\cos y} \end{cases} \Rightarrow \left[\frac{1}{4\cos y}+\cos y=1\right] \Rightarrow$$

$$\Rightarrow \left[4\cos^2 y-4\cos y+1=0\right]$$

$\cos y=t$ desak,

$$4t^2 - 4t + 1 = 0, \qquad t_{1,2} = \frac{2 \pm \sqrt{4-4}}{4} = \frac{1}{2}, \quad t_{1,2} = \frac{1}{2}.$$

$$\left[\cos y = \frac{1}{2}\right] \Rightarrow \left[y = \pm\frac{\pi}{3} + 2k\pi\right], \quad k \in Z.$$

$$\cos x = \frac{1}{4\cos y} = \frac{1}{4\cos\frac{\pi}{3}} = \frac{1}{4\cdot\frac{1}{2}} = \frac{1}{2}.$$

$$\left[\cos x = \frac{1}{2}\right] \Rightarrow \left[x = \pm\frac{\pi}{3} + 2k\pi\right], \quad k \in Z.$$

$$J\!: x = y = \pm\frac{\pi}{3} + 2k\pi \qquad k \in Z.$$

8-misol. $\begin{cases} \sin x \sin y = \frac{3}{4} \\ tgx = 3ctgy \end{cases}$ tenglamalar sistemasini yechilsin.

Yechish. $\begin{cases} \sin x \sin y = \frac{3}{4} \\ tgx = 3ctgy \end{cases} \Rightarrow \begin{cases} \sin x \sin y = \frac{3}{4}, \\ \frac{\sin x}{\cos x} = 3\cdot\frac{\cos y}{\sin y} \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} \sin x \sin y = \frac{3}{4}, \\ \frac{\sin x \sin y}{\cos x \cos y} = 3 \end{cases} \Rightarrow \begin{cases} \sin x \sin y = \frac{3}{4}, \\ \cos x \cos y = \frac{1}{4}. \end{cases}$$

Bu sistemadagi tenglamalarni o’zaro hadlab qo’shamiz:

$$(\sin x \sin y + \cos x \cos y = 1) \Rightarrow (\cos(x-y)) = 1, \quad (x-y) = 0$$

$$\begin{cases} \sin x \sin y = \frac{3}{4}, \\ x - y = 0 \end{cases} \Rightarrow \begin{cases} \sin x \sin y = \frac{3}{4}, \\ x = y \end{cases} \Rightarrow \left[\sin^2 x = \frac{3}{4}\right] \Rightarrow$$

$$\Rightarrow \left[\sin x = \pm\frac{\sqrt{3}}{2}\right] \Rightarrow \left[x = \pm\frac{\pi}{3} + 2k\pi\right], \quad k \in Z.$$

$$Ж: \quad x = y = \pm\frac{\pi}{3} + 2k\pi, \quad k \in Z.$$

MUSTAQIL YECHISH UCHUN MISOLLAR.

Tenglamalar sistemasini yeching.

1. $\begin{cases} \sin x \sin y = \dfrac{\sqrt{3}-1}{4}, \\ \cos x \cos y = \dfrac{\sqrt{3}+1}{4}. \end{cases}$ $\quad j: \quad x = 45^0, \; y = 15^0.$

2. $\begin{cases} \sin x : \sin y = \sqrt{1{,}5}, \\ \cos x : \cos y = \sqrt{0{,}5}. \end{cases}$ $\quad j: \quad x = 60^0, \; y = 45^0.$

3. $\begin{cases} \sin x = \sqrt{2} \sin y, \\ tgx = \sqrt{3} tgy. \end{cases}$ $\quad j: \quad x = 45^0, \; y = 30^0.$

4. $\begin{cases} \sin x + \cos y = \dfrac{1}{2}, \\ \sin^2 x + \cos^2 y = \dfrac{1}{4}. \end{cases}$ $\quad j: \quad x = 0^0, \; y = 60^0.$

5. $\begin{cases} x + y = \dfrac{\pi}{2}, \\ \sin x + \cos y = 1. \end{cases}$ $\quad j: \quad x = \dfrac{\pi}{6}, \; y = \dfrac{\pi}{3}.$

6. $\begin{cases} x - y = \dfrac{\pi}{6}, \\ \sin x \cos y = 0{,}75 \end{cases}$ $\quad j: \quad x = \dfrac{\pi}{3}, \; y = \dfrac{\pi}{6}.$

7. $\begin{cases} x + y = \dfrac{2}{3}\pi, \\ \sin x \sin y = \dfrac{1}{2}. \end{cases}$ $\quad j: \quad x = \dfrac{5\pi}{12}, \; y = \dfrac{\pi}{4}.$

7-MAVZU: TESKARI TRIGONOMETRIK FUNKSIYALAR

Shu vaqtga qadar biz α burchakning berilgan qiymatlariga asosan $sin\alpha$, $cos\alpha$, $tg\alpha$ va $ctg\alpha$ larning qiymatlarini topish bilan shug'ullandik. Endi bunga teskari masalani ya'ni $sin\alpha$, $cos\alpha$, $tg\alpha$ va $ctg\alpha$ larning qiymatlariga asosan α burchakning qiymatlarini aniqlash masalasini ham qo'yish mumkin. Bu masala teskari trigonometrik funksiya tushunchasini kiritishga olib keladi.Teskari trigonometrik funksiya tushunchasini kiritish uchun esa dastlab teskari funksiya tushunchasini kiritish kerak bo'ladi.

Aniqlanish sohasi D va qiymatlar sohasi E dan iborat bo'lgan $y = f(x)$ funksiya o'zining aniqlanish sohasida monoton bo'lsin. U holda x ning D dan olingan har bir qiymatiga y ning E dagi bitta qiymati mos keladi va aksincha. y ning E dan olingan har bir qiymatiga x ningdagi bitta qiymati mos keladi. Demak, bu holda E da aniqlangan shunday yangi funksiyani tuzish mumkinki, unda E dan olingan har bir y ga D da $y = f(x)$tenglamani qanoatlantiruvchi bitta x ni mos qo'yish mumkin. Hosil qilingan bu yangi funksiya $y = f(x)$ funksiyaga teskari funksiya deyiladi.

$y = f(x)$funksiyaga teskari funksiyani topish uchun x ni y orqali ifodalab so'ngra x va y larni o'rinlarini o'zaro almashtirish kerak. $y = f(x)$funksiyaga teskari funksiyani$y = f^{-1}(x)$ko'rinishda yoziladi.

Agar $y = f(x)$ va $y = f^{-1}(x)$ funksiyalar o'zaro teskari funksiyalar bo'lsa, u holda $y = f(x)$ning aniqlanish sohasi $y = f^{-1}(x)$ uchun qiymatlar sohasi, qiymatlar sohasi esa $y = f^{-1}(x)$uchun aniqlanish sohasi bo'ladi.

O'zaro teskari funksiyalar grafiklari y=x to'g'ri chiziqqa nisbatan simmetrik bo'ladi. $y = sinx$ funksiyaga teskari funksiyani topish masalasi bilan shug'allanamiz. Bu funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda $y = sinx$ funksiyaga teskari funksiya mavjud emas.$y = sinx$ funksiya

$[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada monoton bo'lganligi uchun, bu kesmada unga teskari bo'lgan funksiyaga o'tish mumkin.

$[-\frac{\pi}{2};\frac{\pi}{2}]$kesmada funksiya –1 dan 1 gacha o'sadi. Demak, x va y ning qiymatlari o'zaro bir qiymatli moslik orqali bog'langan. Moslik o'zaro bir qiymatli bo'lgani sababli, u ning [-1;1] kesmadagi har bir qiymatiga x ning $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmadagi bitta qiymati mos keladi. Demak, bu holda yangi funksiya tuzish mumkin.

Ta'rif: $[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada qaralayotgan $y = sinx$ funksiyaga teskari bo'lgan funksiya arksinus deyiladi. Bu funksiya $y = arcsinx$ kabi yoziladi

$y = arcsinx$ ifoda$[-\frac{\pi}{2};\frac{\pi}{2}]$ kesmada olingan yoydan iborat bo'lib, uning sinusi x ga teng, ya'ni $\sin(arcsinx) = x$

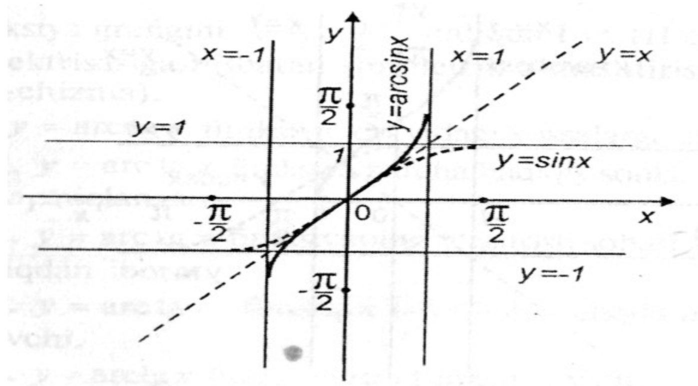
$y = arcsinx$ funksiya quyidagi xossalarga ega :

a) $y = arcsinx$ funksiya [-1 ;1] kesmada aniqlangan

b) $y = arcsinx$ funksiyaning bosh qiymati $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesmada o' zgaradi

c) $y = arcsinx$ funksiya [-1 ;1] kesmada monoton o' sadi

d) bu funksiya toq funksiyadir, ya' ni $\arcsin(-x) = -arcsinx$



$y = cosx$ funksiya $(-\infty;+\infty)$ oraliqda monoton emas. Demak, bu oraliqda $y = cosx$ ga teskari Funksiya mavjud emas. $y = cosx$ $[0;\pi]$ kesmada monoton bo'lgani uchun bu kesmada unga teskari bo'lgan Funksiyaga o'tish mumkin.

$[0;\pi]$ kesmada $y = cosx$ funksiya 1 dan –1 gacha kamayadi. Ya'ni, bu kesmada x va u ning qiymatlari o'zaro bir qiymatli moslikda. Demak, bu holda yangi Funksiya tuzish mumkin.

Ta'rif: $[0;\pi]$ kesmada qaralayotgan $y = cosx$ ga teskari bo'lgan funksiyani arkkosinus deyiladi va $y = arccosx$ kabi yoziladi.

$y = arccosx$ 0 dan π gacha bo'lgan kesmada olingan yoy ya'ni: $0 \le \arccos x \le \pi$ bo'lib, bu yoyning kosinusi x ga teng: $cos(arccosx) = x$, bunda $-1 \le x \le 1$.

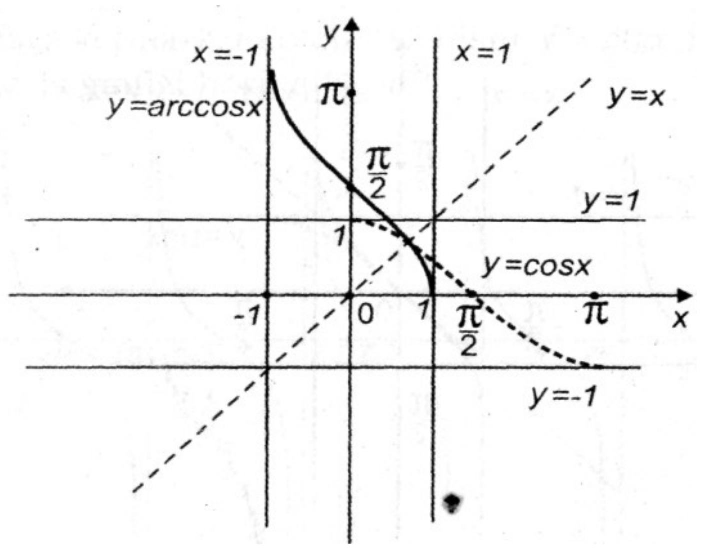
$y = arccosx$ funksiya quyidagi xossalarga ega:

a) $y = arccosx$ funksiya [-1;1] kesmada aniqlangan

b) funksiyaning bosh qiymati [0;π] kesmada o' zgaradi

c) $y = arccosx$ funksiya [-1;1] kesmada kamayadi

d) $y = arccosx$ funksiya toq ham emas, juft ham emas



7-Amaliy mashg'ulot

Quyidagilarni mustaqil yeching

Hisoblang. $\frac{\log_3 24}{\log_{72} 3} - \frac{\log_3 216}{\log_8 3}$

A) 1 B)0 C)3 D)2

2. Tenglamaning ildizlari yig'indisini toping. $x \cdot 2^{\log_x 5} = 10$ A)12 B)10 C)3 D)7

3. Tengsizlikni [-10:20] oraqliqda nechta butun yechimi bor.

$$\log_{\sqrt{x+1}+\sqrt{x-1}}(x^2 - 3x + 1) \geq 0$$

A)16 B)17 C)18 D)19

4. Tenglamaning eng katta manfiy ildizini toping. $\sqrt{3}cosx = sinx$ A)-120^0 B)-150^0 C)-60^0 D)-30^0

5. Hisoblang $\sin(\arccos\frac{1}{3} + \arcsin\frac{3}{4})$

A)$\frac{6\sqrt{2}+\sqrt{7}}{12}$ B) $\frac{4\sqrt{7}+3\sqrt{2}}{12}$ C) $\frac{2\sqrt{7}+2}{12}$ D) $\frac{2\sqrt{14}+3}{12}$

6. $y = cos(x\sqrt{2}) + cos\frac{x}{\sqrt{2}}$ funksiyaning eng kichik musbat davrini toping.

$A)2\pi$ $B)$ 3π $C)$ $2\pi\sqrt{2}D)$ $3\pi\sqrt{3}$

7. Agar $sinx + cosx = a$ bo'lsa, $\frac{\sin^3 x+\cos^3 x}{(a^2-3)a}$ ning qiymatini toping. A)1/3 B) -1/2 C) 1/2 D) 2/5

8. a=sin1; b=sin2; c=sin3; d=sin4 va e=sin5 sonlarni kamayish tartibida joylashtiring.

A) a>b>c>d>e B) e>b>a>d>c

C) b>c>a>d>eD) b>a>c>d>e

9. $tga = 2$ bo'lsa, $\frac{2}{3+4cos2a}$ =?

A)-10/3 B) -10/27 C) 10/27 D) 10/3

10. Ifodaning qiymatini toping.

$$\left(\frac{tg^2 49^0 - tg^2 11^0}{1 - tg^2 49^0 \cdot tg^2 11^0} \cdot tg52^0\right)^4$$

A) 9 B) 1/9 C) 81 D) 1/81

11. $\sqrt{2cosx+1}=\sqrt{3}cosx$ tenglamani yeching.

A) $\pi n, n\epsilon Z$ B) $2\pi n, n\epsilon Z$ C) $\frac{\pi n}{2}, n\epsilon Z$ D) $\frac{\pi}{2}+\frac{\pi n}{4}, n\epsilon Z$

12. Tenglamani yeching. $\frac{\sqrt{1+sinx}}{cosx}=1$

A)$\pi n, n\epsilon Z$ B) $2\pi n, n\epsilon Z$ C) $\frac{\pi n}{2}, n\epsilon Z$ D) $\frac{\pi}{2}+\frac{\pi n}{4}, n\epsilon Z$

13. Tenglamaning eng kichik musbat va eng katta manfiy yechimlari yig'indisini toping.

$$sin\frac{x}{3}\left(tg\frac{x}{4}-1\right)=0$$

A) -2π B)-3π C)0 D)2π E)3π

14. Tengsizlikni yeching.

$$(cosx+2)|x-5|(x-2)\leq 0$$

A)$(-\infty;2]\cup\{5\}$ B)$(-\infty;2]$

C)$[2;5]$ D)$\{5\}$ E)$\emptyset$

15. $P(-3;0)$ nuqtani koordinata boshi atrofida 270^0ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

A)(O;3) B)(-3;0) C)(0;-3) D)(3;0)

16. $ctga+tga=p$ bo'lsa, $tg^2a+ctg^2a=$?

A) p^2-2 B)$-p^2+2$ C) p^2+2 D) p^2-1 E) p^2+1

17. Tengsizlikni yeching.

$$(e-\pi)x>7(\pi-e)$$

A)$(-\infty;-7)$B)$(-\infty;7)$C)$(7;\infty)$D)$(-7;\infty)$E)$(-\infty;\infty)$

18. Soddalashtiring. $(ctga-cosa)\cdot\left(\frac{\sin^2 a}{cosa}+tga\right)$

A) $\sin^2 a$ B) tga C) $\frac{1}{cosa}$ D) ctg^2a E) $\cos^2 a$

19. Tenglamani yeching.

$$x\cdot cos50^0+sin50^0+x=0$$

A) $sin25^0$ B) $-tg25^0$ C) $-cos25^0$ D) $ctg25^0$ E)1

20. $sinx + cosx = 0{,}5$ bo'lsa, $16(\sin^3 x + \cos^3 x)$ ni toping. A)8 B)14 C)12D)16 E)11

21. Hisoblang. $tg\frac{\pi}{6} \cdot \sin\frac{\pi}{3} \cdot ctg\frac{5\pi}{4}$

A) 1,5 B) $\frac{\sqrt{3}}{4}$ C) $-\frac{1}{2}$ D) 0,5 E) 0,75

22. Soddalashtiring. $\frac{sina+cosa}{\sqrt{2}\cos\left(a-\frac{\pi}{4}\right)}$

A) 1,6 B) 1,5 C) 1 D) $ctg\left(\frac{\pi}{4} + a\right)$ E) $tg\left(\frac{\pi}{4} + a\right)$

23. Hisoblang. $(tg60^0 \cdot cos15^0 - sin15^0) \cdot 7\sqrt{2}$

A)16 B)12 C)18 D)14 E)10

24. Hisoblang. $cos92^0 \cdot cos2^0 + \frac{1}{2} \cdot sin4^0 + 1$

A) $\frac{1}{2}$ B) 1 C) 0 D) 2 E) $-\frac{\sqrt{2}}{2}$

8-MAVZU: TESKARI TRIGONOMETRIK FUNKSIYALAR. XOSSALARI VA GRAFIGI

$y = tgx$ funksiya $(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi)$ oraliqlarning har birida $-\infty$ dan $+\infty$ gacha o'sadi. Shuning uchun bu oraliqlarning har birida $y = tgx$ ga teskari funksiyaga o'tsa bo'ladi.

Ta'rif: $(-\frac{\pi}{2}; \frac{\pi}{2})$ oraliqda $y = tgx$ ga nisbatan teskari bo'lgan funksiya *arktangens* deyiladi va $y = arctgx$ kabi yoziladi.

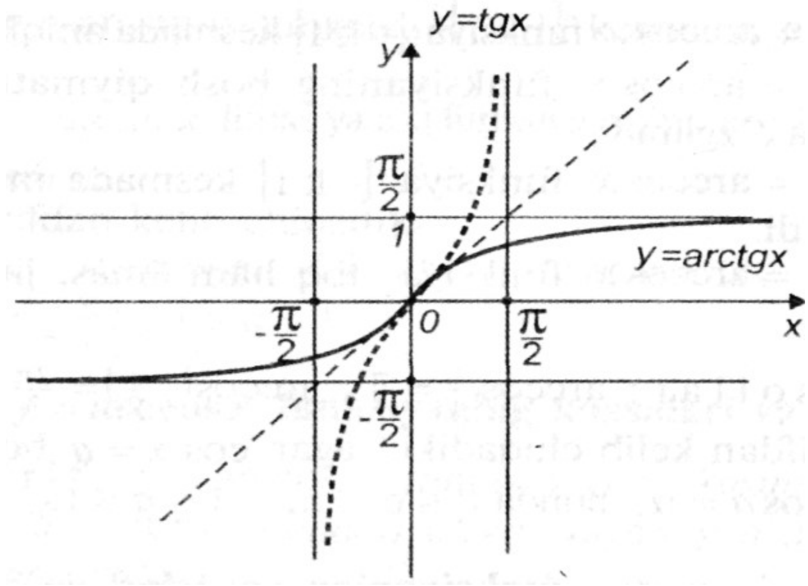
$y = arctgx$ $(-\frac{\pi}{2}; \frac{\pi}{2})$ oraliqda olingan yoy, ya'ni $-\frac{\pi}{2} < arctgx < \frac{\pi}{2}$ bo'lib, uning tangensi x ga teng. Bu yerda x-istalgan haqiqiy son.

$y = arctgx$ quyidagi xossalarga ega.

1^0. $y = arctgxx$ ning barcha qiymatlarida aniqlangan, o'suvchi funksiyadir.

2^0. $y = arctgx$ toq funksiyadir: $arctg(-x) = -arctgx$

$y = tgx$ funksiyani grafigini yasash uchun $x = tgy$ tangensoidaning tarmog'ini yasash kifoyadir.



1-chizma

Ta'rif $(0;\pi)$ oraliqda $y = ctgx$ ga nisbatan teskari bo'lgan funksiyani arkkotangens deyiladi va $y = arcctgx$ kabi yoziladi.

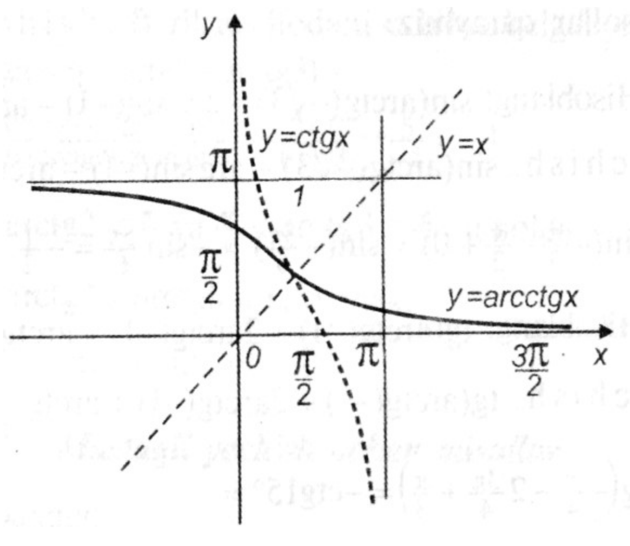
$y = arcctgx$, $(0;\pi)$ oraliqda olingan yoy, yani $0 < arcctgx < \pi$ bo' lib, uning kotangensi x ga tengdir: ya' ni $ctg(arcctgx) = x$. Bu yerda x -istalgan haqiqiy sondir. y=arcctgx funksiya quyidagi xossalarga ega:

a) $y = arcctgx$ funksiyaning aniqlanish sohasi $(-\infty;\infty)$

b) funksiyaning qiymatlar sohasi $(0;\pi)$ intervaldan iborat

c) $y = arcctgx$ funksiya $(-\infty;\infty)$ intervalda monoton kamayuvchi

d) $y = arcctgx$ funksiya toq ham emas, juft ham emas



2-chizma

Amaliy mashg'ulot.

Quyidagilarni mustaqil yeching

26. Soddalashtiring. $\frac{\sqrt{3}}{sin100^0}+\frac{1}{cos260^0}$

A)2B)-4 C)-3 D)-1 E)-2

27. Ifodaning qiymatini toping.

$sin50^0 + sin40^0 \cdot tg20^0$ A) 0,5 B) 1 C) $\sin^2 20^0$

D)$\cos^2 20^0$ E)1,5

28. Soddalashtiring. $\frac{cos6a-cos4}{sin5a}$

A) $2sina$ B) $2cosa$ C)$-2cosa$ D)$-sina$ E) $-2sina$

29. Hisoblang. $\frac{sin73^0-cos17^0}{2cos60^0}$

A) 0 B) 1 C) 2 D) $\sqrt{3}$ E) -1

30. Ifodani soddalashtiring.

$$\frac{sin2a + 2sina \cdot cos2a}{1 + cosa + cos2a + cos3a}$$

A) $2tga$ B) $2sina$ C) $4tga$ D) $ctga$ E) tga

31. Hisoblang. $ctg35^0 - tg35^0 - 2tg20^0$

A) $\frac{1}{2}$ B) 1 C) 0 D) $\frac{\sqrt{3}}{2}$ E) $\sqrt{3}$

32. $\sin 70^0 - (\sin 10^0 + \sin 50^0) = ?$

A)0 B)2 C)1 D) -1

33. $tg68^0 = m$ bo'lsa, $\cos 224^0 = ?$

A) $2m^2 - 1$ B) $\frac{1-m^2}{1+m^2}$ C) $-2m^2 - 1$ D) $2m^2 + 1$

34. $\cos 12^0 \cos 24^0 \cos 48^0 \cos 96^0 - ?$

A)1/8 B)-1/8 C)0 D)-1/16

35. Agar $\lg 7 \approx 0{,}8451$ bo'lsa, 7^{70} soni necha xonali?

A) 65 B)59 C)60 D)57

36. Ifodaning qiymatini toping.

$$\left(\frac{\log_5^2 15-\log_5^2 3+2\cdot\log_5 15+2\cdot\log_5 3}{\log_5 15+\log_5 3}\right)^{-\frac{2}{\log_5 3}}$$

A)0,04 B)3 C)0,02 D)1

37. Nechta butun son $3^{\sqrt{5-x}}$ ≤(x−4)ln(x−4) tengsizlikni qanoatlantiradi?

A) 0 B) 1 C) 2 D) 3

38. y = 2 + 3cos(8x − 7) funksiyaning eng kichik musbat davrini toping.

A) 2π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$ E) π

39. $2^{3-\frac{x}{2}}=3$ tenglamani yeching.

A) $\log_2\sqrt{3}$ B) $\log_3(2\frac{2}{3})$ C) $\log_2(3\frac{3}{5})$

D) $\log_2(2\frac{2}{3})$ E) $\log_2(7\frac{1}{9})$

40. Agar |cosx|=2+cosx bo`lsa,

$2^{\cos x}+3^{\sin x}$ ning qiymatini toping.

A) 1 B) 0,5 C) 0,75 D) 1,25 E) 1,5

41. $\log_x 3 < 2$ tengsizlikni yeching. A)($\sqrt{3}$;∞)

B)(3; ∞)C) (0; 1)U($\sqrt{3}$; ∞) D) (0; 1) E) (0; 1)U(3; ∞)

42. Agar tgx = 0,5 bo`lsa, $\cos^8 x-\sin^8 x$

ning qiymatini toping. A)0,52 B)0,408 C) 0,392 D)0,416 E) 0,625

43. $\log_x(5x-4)=2$ tenglamaning ildizlari yig`indisini toping. A) 5 B) 4 C) 3 D) 2 E) 4,5

44. $sin\frac{x}{2}$= 0,6 bo'lsa sinx=?

A) 4/5 B) 24/25 C)6/25 D) 12/25

45. sinx= $\frac{\sqrt{5}}{5}$ bo'lsa, tg2x ni hisoblang.

A) 4/3 B) 3/2 C) 3/4 D) 2/3

46. $\frac{sin15^0}{sin5^0}+\frac{cos15^0}{cos5^0}$ soddalashtiring?

A) $2\sin10^0$ B) $4\cos10^0$ C) $4\sin10^0$ D) $2tg10^0$

47. tg(arcsin0,8 + π) ni hisoblang?

A) -4/5 B) -4/3 C) 3/4 D) 4/3

48. sinx=s va cosx=c bo'lsa, $3(s^4+c^4)-2(s^6+c^6)$ ni hisoblang. A) s B) c C) 4 D) 1

49. $\log_a(\sin 19^0) - \log_a(\cos 71^0)$ ni hisoblang. Bunda a> 1. A) -1 B)1 C) 0 D)1/2

50. $4^{\frac{x}{2}-y} = 2$ va $5^{x-3y} = 3$ bo'lsa, 5^y ni toping.

A)5/3 B)3/5 C) 2/5 D)5/2

51. tgx-ctgx=m va $tg^2 x + ctg^2 x$ =n

bo'lsa, quyidagilarning qaysi biri to'g'ri?

A) $m^2 - n + 2 = 0$ B) $m - n + 2 = 0$

C) $m^2 + n - 2 = 0$ D) $m + n + 2 = 0$

52. $\log_{5x}(25 \cdot x) = m$ bo'lsa, $\log_5 x$ ni toping

A) $\frac{m-1}{m-2}$ B) $\frac{m-2}{1-m}$ C) $\frac{1-m}{m-2}$ D) $\frac{2m}{1-m}$

53. $\log_4(sinx) = -0{,}25$ bo'lsa, x qancha bo'lishi mumkin? A) $\frac{\pi}{2}$ B) $\frac{3\pi}{2}$ C) $\frac{3\pi}{4}$ D) $\frac{4\pi}{3}$

54. (2x-1)cos(2arcsin0,6)+(x-3)sin(2arccos0,6)=0,6

A)7/24 B) 47/19 C) 23/15 D) 25/7

55. $\log_p 15 < \log_p 10$ va $\log_{5p} 8 > \log_{5p} 6$ bo\`lsa, p ni toping. A)0$< p < 1$ B)p$> 0{,}2$ C) p> 1 D) 0,2$< p < 1$

56. $a = \frac{\pi}{18}$ bo'lsa $\frac{\cos 5a - \cos a}{\sin 8a \sin a} = ?$

A)1B)2C)-1D)-2E)-4

57. $\frac{\cos^2 5x - \cos^2 x}{\sin^2 5x - \sin^2 x} = ?$ A)1 B)-1 C)cos3x D)-tgx E) tg2x

58. $\cos\frac{\pi}{12} + \sin\frac{\pi}{12} = a/(4\cos\frac{\pi}{12})$ bo'lsa a=?

A) $\sqrt{3}$ B) $\sqrt{3}+1$ C) $\sqrt{3}+2$ D) $\sqrt{3}+3$ E) $\sqrt{3}+4$

59. $\cos^2\frac{\pi}{10} - \sin^2\frac{2\pi}{5} = ?$ A)1B)1/2C) $\sqrt{2}/2$ D) $\sqrt{3}/2$ E)0

60. $\cos 4\cdot\cos x \ge \sqrt{\frac{\cos x}{1+ctg^2 x}}$ tengsizlikni yeching $(n\in Z)$ $A)\left[-\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi n\right]$ $B)\left(\pi n;\frac{\pi}{2}+\pi n\right)$

$C)\pi n$ $D)\frac{\pi}{2}+\pi n$

61. arcsinx<arcsin(1-x) tengsizlikni eching.

A)[-1;1] B)(-1;$\frac{1}{2}$] C)0 D)[0;2] E)[0;$\frac{1}{2}$)

62. Soddalashtiring: $\frac{2\sin 2\alpha - \sin 4\alpha}{2\sin 2\alpha + \sin 4\alpha}$.