O‘ZBEKISTON RESPUBLIKASI

OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

FARG‘ONA POLITEXNIKA INSTITUTI
OLIY MATEMATIKA
KAFEDRASI

E.M. MIRZAKARIMOV , X.TILAVOLDIYEV

OLIY MATEMATIKA
fanidan
L A B O R A T O R I Y A
ISHLARINI
MAPLE DASTURIDA BAJARISH

USLUBIY QO‘LLANMA

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Ushbu uslubiy qo’llanma texnika yo’nalishi talabalari uchun 1 kurs 2-semestrda o’qitiladigan “Oliy matematika” fanidan 18 soatli laboratoriya ishlarini bajarishda foydalanish uchun mo’ljallangan.

Uslubiy qo’llanma MAPLE dasturidan foydalanib masalalarni kompyuterda tez va sifatli yechishga bag’ishlangan.

Qo’llanmada sonli usullarning amaliyotda ko’p qo’llaniladigan bo’limlari- algebraik va transtsendent tenglamalarni taqribiy yechish, chiziqli tenglamalar sistemalarini yechish, aniq integrallarni taqribiy hisoblash, differentsial tenglamalar hamda funltsiyani iterpolyatsiyalash va kichik kvadratlar usullarida amaliy masalalarni yechish usullari ko’rsatilgan. Laboratoriya ishlarida ko’rsatilgan masalalarni yechishdagi usullar uchun Maple tizimida dasturlar tuzilgan. Mustaqil ishlar uchun har bir mavzuga mos topshiriqlar berilgan.

Institut uslubiy kengash tomonidan tasdiqlangan (№ 2 , 28 .12 . 2010)

“Oliy matematika” kafedrasining uslubiy semenarida muxokama qilingan (№ 5 , 22 .12. 2010)

Iqtisod va menejment fakultetining uslubiy komissiyasi tomonidan maqullangan. (№ 2 , 23 . 12. 2010)

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So’z boshi

O’zbekiston mustaqillikka erishgandan so’ng, o’z taraqqiyotining muhim shartlaridan biri bo’lgan xalqning boy maonaviy salohiyati va umuminsoniy qadriyatlariga hamda hozirgi zamon madaniyati, iqtisodiyoti, ilmi, texnikasi va texnologiyasining so’nggi yutuqlariga asoslangan mukammal taolim tizimi barpo etilmoqda.

“Ta’lim to’g’risida” gi qonun va “Kadrlar tayyorlash milliy dasturi”ning qabul qilinishi natijasida ilmiy-texnika taraqqiyoti yutuqlarini xalq xo’jaligiga tadbiq qilish ijtimoiy-iqtisodiy rivojlanish bilan uzviy bog’liq ekanligining ahamiyati tobora ortib bormoqda.

Oliy o’quv yurtlarining texnika yo’nalishi bo’yicha bakalavrlar tayyorlashning yangi o’quv rejasi va dasturlarida kompyuter va axborot texnologiyalari bilan ishlash, axborotlarga zamonaviy texnik vositalar yordamida ishlov berish va uni tahlil qilish, sonli usullarni amaliy masalalarni yechishda tadbiq qilinishiga katta ehtibor qaratilgan.

Oliy o’quv yurtlarida tayyorlanayotgan mutaxassislarga matematik usullarni o’rganishdan maqsad shuki, ular bu usullarni o’zlashtiribgina qolmay, balki ularni xalq xo’jaligining turli amaliy masalalarini yechishga qo’llay olishlari, olingan yechimni tahlil qila bilishlari hamda yechimga asoslanib to’g’ri qaror qabul qilishga qodir malakali mutaxassis bo’lib yetishishlari kerak.

Ushbu uslubiy qo’llanma masalalarni kompyuterda tez va sifatli yechishda MAPLE dasturidan foydalanish qulay hamda samarali ekanligini ko’rsatadi.

KIRISH

Ushbu uslubiy qo’llanma sonli yechish usullariga bag’ishlangan bo’lib, uni bayon qilishda qathiy matematik asoslashni maqsad qilib qo’yilmagan holda sonli usul algoritmi, uni misol va masalalar yechishga tadbiqlari hamda xatoligini baholashga mo’ljallangan. Qo’llanma 5 ta laboratoriya ishidan iborat bo’lib, 1- laboratoriya ishlarida chiziqli tenglamalar sistemasini yechimini, determinantning qiymatini va teskart matritsani Gauss usulida topishga bag’ishlangan. 2- laboratoriya ishlarida algebraik va trantsendent tenglamalarni taqribiy yechish usullari, 3- laboratoriya ishlari Lagranj usulida interpolyatsiya ko’phadini topish va kichik kvadratlar usuli yordamida tajriba natijalarinig chiziqli va parabolik bog’laninshini aniqlash usullariga, 4- laboratoriya ishida aniq integrallarni taqribiy hisoblash usullari, 5-laboratoriya ishida oddiy differentsial tnglama va sistemasini yechimini hamda ikkinchi tartibli differentsial tnglamani tenglamalar sistemasiga keltirish bilan taqribiy hisoblashda Eyler usuli keltirilgan. **M**ustaqil ishlash uchun topshiriqlar bo’limida har bir laboratoriya ishi uchun variantlar berilgan.

1-LABORATORIYA ISHI

Chiziqli tenglamalar sistemasini yechish

Maqsad: Gauss usulida ko'p noma'lumli chiziqli tenglamalar sistemasini, yuqori tartibli determinantlarni hisoblash va matritsaga teskari matritsa topishni o'rganish.

Reja:

1.1. Chiziqli tenglamalar sistemasini Gauss usulida yechish
1.2.Gauss usulida determinantni hisoblash
1.3. Jardan-Gauss usulida matritsaga teskari matritsa topish

1.1. Chiziqli tenglamalar sistemasini Gauss usulida yechish

CHiziqli algebraik tenglamalar sistemasini yechishda keng tarqalgan Gausc usuli aniq yechish usullari guruhiga mansub bo'lib, uning mohiyati shundan iboratki, nomahlumlarni ketma – ket yo'qotish yo'li bilan berilgan sistema o'ziga ekvivalent bo'lgan pog'onali (uch burchakli) sistemaga keltiriladi. Bu kompg'yuter xotirasidan samarali ravishda foydalanish imkonini beradi .

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \\ \qquad a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \qquad\qquad a_{kk}x_k + \ldots + a_{kn}x_n = b_k \end{cases} \qquad (1.1)$$

ko'rinishdagi chiziqli tenglamalar sistemasi *pog'onali sistema* deyiladi, buyerda k≤n, $a_{ii} \neq 0$, i=1, 2, …, k.

Agar k=n bo'lsa, u holda (1.1) sistema *uch burchakli* deyiladi.

Nomahlumlarni ketma – ket yo'qotib borish, asosan, sistemada elementar almashtirishlar qilish yordamida amalga oshiriladi. Bu elementar almashtirishlarga quyidagilar kiradi:

1) sistemaga tegishli istalgan ikkita tenglamaning o'rnini almashtirish;
2) tenglamalardan birining har ikkala qismini noldan farqli istalgan songa ko'paytirish;
3) biror tenglamaning har ikkala qismiga, biror songa ko'paytirilgan ikkinchi tenglamaning mos qismlarini qo'shish.

Elementar almashtirishlar berilgan tenglamalar sistemasini unga ekvivalent sistemaga o'tkazishini isbotlash mumkin.

Oddiylik uchun quyidagi chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = a_{15}, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = a_{25}, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = a_{35}, \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = a_{45}. \end{cases}$$

Aytaylik, berilgan sistemada $a_{11} \neq 0$ (etakchi element) bo'lsin, aks holda tenglamalarning o'rinlarini almashtirib, x_1 oldidagi koeffitsienti noldan farqli bo'lgan tenglamani birinchi o'ringa ko'chiramiz.

Sistemaning birinchi tenglamasining barcha koeffitsientlarini a_{11} ga bo'lib,

$$x_1+b_{12}x_2+b_{13}x_3+b_{14}x_4=b_{15} \qquad (1.2)$$

tenglamani hosil qilamiz, bu yerda.

$$b_{ij} = \frac{a_{ij}}{a_{11}}, (j = 2,3,4,5)]$$

Bu topilgan (1.2) tenglamadan foydalanib, yuqoridagi sistemaning qolgan tenglamalaridagi x_1 qatnashgan hadni yo'qotish mumkin. Buning uchun (1.2) tenglamani ketma-ket a_{21}, a_{31} va a_{41} larga ko'paytirib, mos ravishda sistemaning ikkinchi, uchinchi va to'rtinchi tenglamalaridan ayiramiz.

Natijada quyidagi uchta tenglamalar sistemasini hosil qilamiz.

$$\begin{cases} a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 + a_{24}^{(1)}x_4 = a_{25}^{(1)} \\ a_{32}^{(1)}x_2 + a_{33}^{(1)}x_3 + a_{34}^{(1)}x_4 = a_{35}^{(1)} \\ a_{42}^{(1)}x_2 + a_{43}^{(1)}x_3 + a_{44}^{(1)}x_4 = a_{45}^{(1)} \end{cases} \qquad (1.3)$$

bu sistemadagi $a_{ij}^{(1)}$ koeffitsientlar

$$a_{ij}^{(1)} = a_{ij} - a_{i1}b_{1j} \text{(i=2,3,4; j=2,3,4,5)} \qquad (1.4)$$

formula yordamida hisoblanadi. Endi (1.3) sistemaning birinchi tenglamasini $a_{22}^{(1)}$ ga bo'lib,

$$x_2 + b_{23}^{(1)}x_3 + b_{24}^{(1)}x_4 = b_{25}^{(1)} \qquad (1.5)$$

tenglamani hosil qilamiz, bu yerda

$$b_{2j}^{(1)} = \frac{a_{2j}^{(1)}}{a_{22}}, \quad (j = 3,4,5)$$

(1.5) tenglama yordamida (1.3) sistemaning keyingi tenglamalaridan x_2 ni, yuqoridagidek qoida asosida, yo'qotamiz va quyidagi tenglamalar sistemasini topamiz:

$$\begin{cases} a_{33}^{(2)}x_3 + a_{34}^{(2)}x_4 = a_{35}^{(2)} \\ a_{43}^{(2)}x_3 + a_{44}^{(2)}x_4 = a_{35}^{(2)} \end{cases} \qquad (1.6)$$

bu yerda

$$a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)}b_{2j}^{(1)} \quad (i = 3,4; \quad j = 3,4,5) \qquad (1.7)$$

(1.6) sistemaning birinchi tenglamasini $a_{33}^{(2)}$ ga bo'lib,

$$x_3 + b_{34}^{(2)}x_4 = b_{35}^{(2)} \qquad (1.8)$$

tenglamani hosil qilamiz, bu yerda

$$b_{3j}^{(2)} = \frac{a_{3j}^{(2)}}{a_{33}^{(2)}}, \qquad (j = 4,5)$$

Bu (1.8) tenglama yordamida (3.6) sistemaning ikkinchi tenglamasidan x_3 ni yo'qotamiz. Natijada

$$a_{44}^{(3)} x_4 = a_{45}^{(3)}$$

tenglamani hosil qilamiz, bu yerda

$$a_{4j}^{(3)} = a_{4j}^{(2)} - a_{43}^{(2)} b_{3j}^{(2)} \qquad (j = 4,5) \tag{1.9}$$

SHunday qilib biz qaralayotgan sistemasini unga ekvivalent bo'lgan quyidagi uchburchakli chiziqli tenglamalar sistemasiga olib keldik.

$$\left.\begin{array}{rl} x_1 + b_{12}x_2 + b_{13}x_3 + b_{14}x_4 & = b_{25} \\ x_2 + b_{23}^{(1)}x_3 + b_{24}^{(1)}x_4 & = b_{25}^{(1)} \\ x_3 + b_{34}^{(2)}x_4 & = b_{35}^{(2)} \\ a_{44}^{(3)}x_4 & = b_{45}^{(3)} \end{array}\right\} \tag{1.10}$$

Bu (3.10) sistemadan foydalanib nomhlumlarni, ketma-ket quyidagicha topamiz:

$$\begin{cases} x_4 = \dfrac{a_{45}^{(3)}}{a_{44}^{(3)}} \\ x_3 = b_{35}^{(2)} - b_{34}^{(2)}x_4 \\ x_2 = b_{25}^{(1)} - b_{24}^{(1)}x_4 - b_{23}^{(1)}x_3 \\ x_1 = b_{15} - b_{14}x_4 - b_{13}x_3 - b_{12}x_2 \end{cases} \tag{1.11}$$

Demak, yuqorida keltirilgan Gauss usulida sistemaning yechimini topish 2 qismdan iborat bo'lar ekan.

Olg'a borish – (1.1) sistemani uchburchakli (1.10) sistemaga keltirish

Orqaga qaytish- (1.11) formulalar yordamida nomahlumlarni topish.

Gauss usuli bilan noma'lumli n ta chiziqli algebraik tenglamalar sistemasini yechish uchun bajariladigan arifmetik amallarning miqdori quyidagidan iborat:

$(n^3+3n^2-n)/3$ ta ko'paytirish va bo'lish,

$(2n^3+3n^2-5n)/6$ ta qo'shish.

Xususan:

n=2 da $(2^3+3*2^2-2)/3=6$ ko'paytirish va bo'lish

$(2*2^3+3*2^2-5*2)/6=3$ qo'shish,

n=3 da $(3^3+3*3^2-3)/3=17$ ko'paytirish va bo'lish

$(2*3^3+3*3^2-5*3)/6=11$ qo'shish.

1**.1-masala.** Berilgan quyidagi sistemani Gauss usilida yechamiz. Buning uchun nomahlumlarni ketma-ket yo'qotamiz. Yetakchi satr uchun birinchi tenglamani tanlasak bo'ladi, chunki $a_{11} = 2 \neq 0$.

$$\begin{cases} 2x_1 + \ \ 7x_2 \ + 13x_3 = \ 0 \\ 3x_1 + 14x_2 \ + 12x_3 = 18 \\ 5x_1 + 25x_2 \ + 16x_3 = 39 \end{cases} \qquad (1.12)$$

Gauss usili yordamida yechish uchun sistema satr koeffitsientlarini quyidagicha belgilaymiz:

$a_{11}=2,$	$a_{12}= 7,$	$a_{13}=13$	$b_1 = 0$	[1]	
$a_{21}=3,$	$a_{22}= 14,$	$a_{23}=12$	$b_2=18$	[2]	(1.13)
$a_{31}=5,$	$a_{32}= 25,$	$a_{33}=16$	$b_3=39$	[3]	

Hisoblash jarayoni quyidagicha bo'ladi.

Olg'a b o r i sh

1) (1.6) dagi tenglama koeffitsientlari [1] ni $a_{11}= 2$ ga bo'lamiz:

$$(1,\ a_{12}/a_{11}\ ,\ a_{13}/a_{11}\ ,\ b_1/a_{11}) = (1,\ 7/2\ ,\ 13/2\ ,\ 0/2) \qquad (1.14)$$

2) (1.12) ning 2- tenglamasidagi x_1 ni yo'qatish uchun (1.14) ni a_{21}=3 ga ko'paytirib, [2] satrdan mos ravishda ayiramiz, yahni [2] –(3.14) a_{21}:

$$a^{(1)}_{21}= a_{21} - a_{21}= 0$$
$$a^{(1)}_{22}= a_{22} - a_{21}a_{12}/a_{11} = 14 - 3(2/2) = 7/2$$
$$a^{(1)}_{23}= a_{23} - a_{21}a_{13}/a_{11} = 12 - 3(6/2) = - 15/2$$
$$b^{(1)}_{1} = b_1 - a_{21}b_1/a_{11} = 18 - 3(0/2) = 18$$

Demak, 2- tenglama koeffitsentlari:

$$(\ 0,\ 7/2\ ,\ -15/2\ ,\ 18) \qquad (1.15)$$

bo'ladi.

3) (1.12) ning 3- tenglamasidagi x_1 ni yo'qatish uchun (3.14) ni a_{31}=5 ga ko'paytirib, [3] satrdan mos ravishda ayiramiz, yahni [3] – (1.14) a_{31} :

$$a^{(1)}_{31}= a_{31} - a_{31}= 0$$
$$a^{(1)}_{32}= a_{32} - a_{31}a_{12}/a_{11} = 25 - 5(\ 7/2) = 15/2$$
$$a^{(1)}_{33}= a_{32} - a_{31}a_{13}/a_{11} = 16 - 5(6/2) = - 33/2$$
$$b^{(1)}_{3} = b_3 - a_{31}b_1/a_{11} = 39 - 5(\ 0/2\ ) = 39$$

Demak, 3- tenglama koefitsentlari:

$$(\ 0,\ 15/2\ ,\ - 33/2\ ,\ 39) \qquad (1.16)$$

bo'ladi.

Natijada topilgan yangi koeffitsientlar asosida quyidagi sistemani hosil qilamiz:

$$\begin{cases} x_1 + \ (2/2)x_2 \ \ + (13/2)x_3 = \ 0 \\ \qquad (7/2)x_2 \ \ - \ (15/2)x_3 = 18 \\ \qquad (15/2)x_2 \ - \ (33/2)x_3 = 39 \end{cases} \qquad (1.17)$$

bu sistemaning 2 va 3-tenglamalaridan x_2 nomahlumni yo'qotish uchun 2- tenglamani $a^{(1)}_{22}$ = 7/2 ga bo'lamiz. Bu tenglama koeffitsentlari:

$$(0, 1, -15/7 , 36/7) \qquad (1.11)$$

bo'ladi. Bu (1.11) koefitsentlardan foydalanib (1.17) sistemaning 3- tenglamasidagi x_2 ni yo'qotaimz. Buning uchun (1.11) ni 15/2 ga ko'paytirib 3-tenglama koefitsentlardan mos ravishda ayirib quyidagi koefitsentlar topamiz:

$$(0, 0, -3/7 , 3/7) \qquad (1.12)$$

Natijada berilgan sistemani quyidagicha yozamiz:

$$\begin{cases} x_1 + (2/2)x_2 + (13/2)\, x_3 = 0 \\ x_2 - (15/7)x_3 = 36/7 \\ -\ (3/7)\, x_3 = 3/7 \end{cases}$$

Orqaga qaytish

Bu oxirgi sistemadagi 3- tenglamadan x_3 qiymatini topib bu asosida 2-tenglamadan x_2 ni topamiz. Topilgan x_2 va x_3 asosida 1- tenglamadan x_1 ni topamiz:

$x_3 = -1$

$x_2 = 36/7 + (15/7)(-1) = 21/7 = 3$

$x_1 = (-7/2)(3) - (6/2)(-1) = -8/2 = -4$

Berilgan chiziqli tenglamalar sistemasining yechimi:

$$x_1 = -4, \quad x_2 = 3, \quad x_3 = -1$$

1.1-MAPLE dasturi

1) *Gauss usilida yechamiz*

```
> with(LinearAlgebra):
A := <<2,3,5>|<7,14,25>|<13,12,16>>;
```

$$A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$$

```
> b := <0,18,39>;
```

$$b := \begin{bmatrix} 0 \\ 18 \\ 39 \end{bmatrix}$$

```
> GaussianElimination(A);
```

$$\begin{bmatrix} 2 & 7 & 13 \\ 0 & \frac{7}{2} & \frac{-15}{2} \\ 0 & 0 & \frac{-3}{7} \end{bmatrix}$$

```
> GaussianElimination(A,'method'='FractionFree');
```

$$\begin{bmatrix} 2 & 7 & 13 \\ 0 & 7 & -15 \\ 0 & 0 & -3 \end{bmatrix}$$

```
> ReducedRowEchelonForm(`<|>`(A, b));
```

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

2) *KENGAYTIRILGAN matritsa yordamida yechimni topish*

```
> restart; with(Student[LinearAlgebra]):
> A := <<2,3,5>|<7,14,25>|<13,12,16>|<0,18,39>>;
```

$$A := \begin{bmatrix} 2 & 7 & 13 & 0 \\ 3 & 14 & 12 & 18 \\ 5 & 25 & 16 & 39 \end{bmatrix}$$

```
> LinearSolve( A );
```

$$\begin{bmatrix} -4 \\ 3 \\ -1 \end{bmatrix}$$

```
> LinearSolveTutor( A );
```

```
4 REM----- 3.4 – BYEYSIK TILI DASTURI -------------------
5 REM SAVE"gauss.bas",a
10 INPUT" nomalumlar soni N=";n
20 DIM A(N+1,N+1),B(N,N+1),X(N)
30 FOR I=1 TO N
40 FOR J=1 TO N+1 :READ A(I,J)
50 NEXT J:NEXT I
60 FOR  I=1 TO N:FOR J=I+1 TO N+1
70 B(I,J)=A(I,J)/A(I,I)
72 FOR K=I+1 TO N
80 A(K,J)=A(K,J)-B(I,J)*A(K,I)
90 NEXT K:NEXT J:NEXT I
100 X(N)=A(N,N+1)/A(N,N)
110 FOR I=N-1 TO 1 STEP -1:
112 X(i)=B(I,N+1)
120 FOR J=I+1 TO N
130 X(I)=X(I)-X(J)*B(I,J)
140 NEXT J,I
150 FOR I=1 TO N:PRINT  "x("I")=";X(I)
160 NEXT I
170 DATA  2,7,13,0
180 DATA  3,14,12,18
190 DATA  5,25,16,39
200 END
RUN
nomalumlar soni N=? 3
x(1)=  -4
x(2)=   3
x(3)=  -1
```

1.2. Gauss usulida determinantni hisoblash

Determinantlarning tartibi(satr va ustunlar soni) katta bo’lganda yuqoridagi determinantlarni hisoblash qiyin bo’ladi. SHuning uchun bu determinantlarni kompg’terda hisoblash dasturini GAUSS usuli asosida tuzamiz. Bu amalni namuna sifatida quyidagi determinant uchun bajaramiz.

1.2-masala. Ushbu

$$d=\begin{vmatrix}2 & 7 & 13\\ 3 & 14 & 12\\ 5 & 25 & 16\end{vmatrix}$$

asosiy determinantdagi birinchi satrning yetakchi birinchi $a_{11}=2\neq 0$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d=2\begin{vmatrix}1 & 7/2 & 13/2\\ 3 & 14 & 12\\ 5 & 25 & 16\end{vmatrix}$$

hosil bo'lgan determinantda birinchi satr elementlarini ketma-ket 3 va 5 larga ko'paytirib, mos ravishda 2- va 3- satrlardan ayiramiz:

$$d=2\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 7/2 & -15/2\\ 0 & 15/2 & -33/2\end{vmatrix}$$

Bu determinantning ikkinchi satridagi yetakchi $a^{(1)}_{22}=7/2$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d=2\cdot(7/2)\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 1 & -15/7\\ 0 & 15/2 & -33/2\end{vmatrix}$$

hosil bo'lgan determinantda ikkinchi satr elementlarini 15/2 ga ko'paytirib, mosravishda 3- satrdan ayiramiz:

$$d=2\cdot(7/2)\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 1 & -15/7\\ 0 & 0 & -3/7\end{vmatrix}$$

hosil bo'lgan determinantning oxirgi satridagi yetakchi $a^{(2)}_{33}=-3/7$ elementini determinant belgisidan tashqariga chiqaramiz:

$$d=2\cdot(7/2)\cdot(-3/7)\begin{vmatrix}1 & 7/2 & 13/2\\ 0 & 1 & -15/7\\ 0 & 0 & 1\end{vmatrix}$$

hosil bo'lgan determinant diagonalp elementlari 1 sonidan va diagonalg' ostidagi elementlari 0 dan iborat bo'lgani uchun uning qiymati 1 ga teng Natijada asosiy determinant qiymati yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$d=a_{11}\,a^{(1)}_{22}\,a^{(2)}_{33}\cdot 1=2\cdot(2/2)\cdot(-3/7)\cdot 1=-3$$

Huddi shuningdek Gauss usuli bilan qolgan determinantlarni ham hisoblash mumkin.

2. Yuqoridagi Gauss usulini NxN tartibli determinant uchun xisoblash formulasini beramiz:

$$d = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Bu determinant qiymati yetakchi elementlar ko'paytmasidan iborat bo'ladi:

$$\mathrm{d} = \det \mathrm{A} = a_{11} a_{22}^{(1)} a_{33}^{(2)} \cdots a_{nn}^{(n-1)}$$

Bu yetakchi elementlarni quyidagi formulalar asosida topamiz:

i=1,

$$b_{1i} = a_{1\iota} / a_{11}, \qquad j = 2,3,\dots,n$$

$$a_{ii}^{(1)} = a_{ii} - a_{i1} b_{1j}, \quad \iota = 2,3,\dots,n$$

i=2,

$$b_{2i}^{(1)} = a_{2i}^{(1)} / a_{22}^{(1)}, \qquad j = 2,3,\dots,n$$

$$a_{ii}^{(2)} = a_{ii}^{(1)} - a_{i2}^{(1)} b_{2i}^{(1)}, \qquad \iota = 2,3,\dots,n$$

. . . .

Agar berilgan determinant yetakchi satridagi yetakchi element $a_{11}=0$ bo'lsa, bu satrni yetakchi elementi noldan farqli bo'lga satr bilan almashtiramiz.

1.2-MAPLE dasturi

```
> GAUSS usulida DETERMINANTNI hisoblash :
> restart; with(LinearAlgebra):
A := <<2,3,5>|<7,14,25>|<13,12,16>>;
```

$$A := \begin{bmatrix} 2 & 7 & 13 \\ 3 & 14 & 12 \\ 5 & 25 & 16 \end{bmatrix}$$

```
> A:=GaussianElimination(A);
```

$$A := \begin{bmatrix} 2 & 7 & 13 \\ 0 & \frac{7}{2} & -\frac{15}{2} \\ 0 & 0 & -\frac{3}{7} \end{bmatrix}$$

```
> d:=Determinant(A);
```

$$d := -3$$

```
4 REM---------------- 1.2 BYEYSIK TILI DASTURI -------------------
10 REM SAVE"DITER.bas",a
20 INPUT "Determenant tartibi N=";;N
30 DIM A(N,N):D=1
32 REM Determinant elementlarini  o'qish
40 FOR I=1 TO N
50 FOR J=1 TO N : READ A(I,J)
60 NEXT J : NEXT I:PRINT"d=";
62  REM Gauss usulida hisoblash
70 FOR  I=1 TO N-1 : FOR J=I+1 TO N
80 B(I,J)=A(I,J)/A(I,I)
90 FOR  K=I+1 TO N
100  A(K,J)=A(K,J)-B(I,J)*A(K,I)
110 NEXT K : NEXT J
120 D=D*A(I,I) : PRINT "("A(I,I)")";:NEXT I
```

```
130 D=D*A(N,N) :PRINT "("A(N,N)")";:PRINT "="D
132 PRINT" Determinantning qiymati  D="D
134 REM Determinant elementlari:
140 DATA 2,7,13
150 DATA 3,14,12
160 DATA 5,25,16
170 END
 RUN
   Determenant tartibi N=? 3
  d=( 2 )( 3.5 )(-.4285717 )= -3.000002
  Determinantning qiymati  D= -3
  Ok
```

1.3. Jardan-Gauss usulida matritsaga teskari matritsa topish

Berilgan A matritsaga asosan teskari $B=A^{-1}$ matritsani Jordan-Gauss usulida topish uchun quyidagicha kengaytirilgan matritsani tuzamiz.

$$\begin{pmatrix} a_{11}\ a_{12}\ \dots\ a_{1n} & \vdots & b_{11}\ b_{12}\ \dots b_{1n} \\ a_{21}\ a_{22}\ \dots\ a_{2n} & \vdots & b_{21}\ b_{22}\ \dots b_{2n} \\ \dots\dots\dots\dots & & \dots\dots\dots \\ a_{n1}\ a_{n2}\ \dots\ a_{nn} & \vdots & b_{n1}\ b_{n2}\ \dots\ b_{nn} \end{pmatrix}$$

Bu matritsadagi b_{ij} - $i,j=1,2,3,..,n$ elementlar boshlang'ich holatda birlik matritsa o'rnida bo'lib, A matritsani birlik matritsaga aylantirish bilan teskari matritsa elementlariga aylanadi.

$$\begin{pmatrix} 1\ \ 0 \dots 0\ \ a^{(\kappa)}_{1,\kappa+1} \dots a^{(\kappa)}_{1n} & \vdots & b^{(k)}_{11}\ b^{(k)}_{12} \dots b^{(k)}_{1n} \\ 0\ \ 1\ \dots 0\ \ a^{(\kappa)}_{2,\kappa+1} \dots a^{(\kappa)}_{2n} & \vdots & b^{(k)}_{21}\ b^{(k)}_{22} \dots b^{(k)}_{2n} \\ \dots\dots\dots\dots & & \dots\dots\dots \\ 0\ \ 0 \dots 1\ \ a^{(\kappa)}_{n,\kappa+1} \dots a^{(\kappa)}_{nn} & \vdots & b^{(k)}_{n1}\ b^{(k)}_{n2} \dots b^{(k)}_{nn} \end{pmatrix} \Rightarrow \dots \Rightarrow \begin{pmatrix} 1\ \ 0 \dots 0 & \vdots & b^{(n)}_{11}\ b^{(n)}_{12} \dots b^{(n)}_{1n} \\ 0\ \ 1\ \dots 0 & \vdots & b^{(n)}_{21}\ b^{(n)}_{22} \dots b^{(n)}_{2n} \\ \dots\dots\dots & & \dots\dots\dots \\ 0\ \ 0 \dots 1 & \vdots & b^{(b)}_{n1}\ b^{(n)}_{n2} \dots b^{(n)}_{nn} \end{pmatrix}$$

Bu almashtirish elementlarini quyidagicha bog'lash mumkin:

$$a^{(k)}_{ij} = a^{(k-1}_{kj} / a^{(k-1}_{kk}, \qquad \kappa = 1,2,..,n;\quad j = \kappa+1,\dots,n$$

$$b^{(k)}_{k} = b^{(k-1)}_{kj} / b^{(k-1)}_{kk}, \qquad \kappa = 1,2,..,n;\quad j = 1,2,\dots,n$$

$$a^{(k)}_{ij} = a^{(k-1)}_{ij} - a^{(k-1)}_{kj}\, a^{(k-1)}_{ik} / a^{(k-1)}_{kk}, \qquad \iota = 1,\dots,\kappa-1.\kappa+1,\dots,n;\quad j = \kappa+1,\dots,n$$

$$a^{(0)}_{ik} = a_{ik}$$

$$b^{(k)}_{ij} = b^{(k-1)}_{ij} - b^{(k)}_{kj}\, a^{(k-1)}_{ik} / a^{(k-1)}_{kk}, \qquad \iota = 1,\dots,\kappa-1.\kappa+1,\dots,n;\quad j = \kappa+1,\dots,n$$

$$b^{(0)}_{ii} = a_{ii}$$

1.3-masala. Quyidagi 3-tartibli matritsaga Jardano- Gauss usuli bilan teskari matritsni toping.

$$A=\begin{pmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{pmatrix}$$

Echish. Teskari matritsa topish jarayonini matrits yonida ko'rsatib boramiz.
Berilgan matritsaga teskari matritsani Jardano-Gauss usulida topish:

$$AE=\begin{pmatrix} 4 & 3 & 1 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

Bu matritsadagi AE matritsaning satrlarini mosravishda [1], [2], [3] kabi belgilab, A matritsani birlik matritsaga, Ye matritsani teskari matritsaga aylantirish uchun quyidagi amallarni **Jardano-Gauss usulida** bajaramiz.

[1]/4

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 7 & 1 & -3 & 0 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2]- [1]*2
[3]- [1] *7

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & -5/2 & -3/2 & -1/2 & 1 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2] *(-2/5)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & -17/2 & -19/4 & -7/4 & 0 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2]
[3]+ [2]*(17/4)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & -11/5 & -9/10 & -17/10 & 1 \end{pmatrix} \Rightarrow$$

[1]
[2]
[3]*(-5/11)

$$\begin{pmatrix} 1 & 3/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [2]*(-3/4)
[2]
[3]

$$\begin{pmatrix} 1 & 0 & -1/5 & 1/10 & 3/10 & 0 \\ 0 & 1 & 3/5 & 1/5 & -2/5 & 0 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix} \Rightarrow$$

[1]+ [2]*(-3/4)
[2]+[3]*(-3/4)
[3]

$$\begin{pmatrix} 1 & 0 & 0 & 2/11 & 5/11 & -1/11 \\ 0 & 1 & 0 & -1/22 & -19/22 & 3/11 \\ 0 & 0 & 1 & 9/22 & 17/22 & -5/11 \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix} 2/11 & 5/11 & -1/11 \\ -1/22 & -19/22 & 3/11 \\ 9/22 & 17/22 & -5/11 \end{pmatrix}$$

1.3-MAPLE dasturi

1) *GAUSS usulida teskari matritsa topish:*

```
> restart; with(Student[LinearAlgebra]):
> A := <<4,2,7>|<3,-1,1>|<1,-1,-3>>;
```

$$A := \begin{bmatrix} 4 & 3 & 1 \\ 2 & -1 & -1 \\ 7 & 1 & -3 \end{bmatrix}$$

```
> A^(-1);
```

$$\begin{bmatrix} \frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{22} & -\frac{19}{22} & \frac{3}{11} \\ \frac{9}{22} & \frac{17}{22} & -\frac{5}{11} \end{bmatrix}$$

2) Tutor *oynasida GAUSS usulida teskari matritsa topish:*

```
> InverseTutor( A );
```

```
4 REM--------1.3.- BYEYSIK TILI DASTURI ---------
8 REM SAVE"TECMAT2.BAS",a
10 INPUT N:DIM A(N,N),B(N),E(N,N)
20 FOR  I=1 TO N:FOR  J=1 TO N
40 READ A(I,J) : NEXT J:NEXT I
60 FOR  I=1 TO N: FOR  J=1 TO N
80 READ E(I,J) :NEXT J:NEXT I
90 FOR  L=1 TO N
110 FOR  I=1 TO N : B(I) = E(I,L) : NEXT I
130 FOR K=1 TO N-1:  FOR I=K+1 TO N
150  G(I,K)= A(I,K)/A(K,K)
170 FOR J=K+1 TO N
180  A(J,I)=A(J,I)-G(I,K)*A(K,J)
190 NEXT J
200 B(I)=B(I)-G(I,K)*B(K)
210 NEXT I,K
220 X(N)=B(N)/A(N,N)
230 FOR I= N-1 TO 1 STEP -1
232 S=0
240 'X(I)=B(I)
250 FOR J=I+1 TO N
252 S=S+ A(I,J)*X(J)
254 NEXT J
246 X(I)=(B(I)-S)/A(I,I)
260 'X(I)=X(I)-A(I,J)*X(J)
270 NEXT I
280 FOR I=1 TO N
290 A1(I,L)=X(I)
300 NEXT I,L
310 FOR I=1 TO N
320 FOR J=1 TO N
330 PRINT USING "###.####";A1(I,J);
340 NEXT J:PRINT :NEXT I
342 DATA 4,3,1
343 DATA 2,-1,-1
344 DATA 7,1,-3
346 DATA 1,0,0
347 DATA 0,1,0
348 DATA 0,0,1
350 END
RUN
ТЕСКАРИ МАТРИЦА:
A1( 1 , 1 )= .1818182          A1( 1 , 2 )= .4545455      A1( 1 , 3 )=-9.090911E-02
A1( 2 , 1 )=-4.545456E-02      A1( 2 , 2 )=-.8636364      A1( 2 , 3 )= .2727273
A1( 3 , 1 )= .4090909          A1( 3 , 2 )= .7727274      A1( 3 , 3 )=-.4545455
```

Ok

1-laboratoriya ishl bo'yica
mustaqil ishlash uchun topshiriqlar

Quyidagi berilgan jadval chiziqli tenglamalar sisitemasining koeffitsentlaridan tuzilgan bo'lib, bu jadval asosida **:**

1)hosilbo'ladigan chiziqli tenglamalar sisitemasini Gauss usulida yeching;

2) noma'lumlarning koeffitsentidan tuizilgan determinantni Gauss usulida yeching;

3) noma'lumlarning koeffitsentidan tuizilgan determinantni Gauss usulida yeching;

Masalan 1-variant quyidagi chiziqli tenglamalar sisitemasining koeffitsentlaridan tuzilgan:

$$\begin{cases} 5x_1 + 8x_2 - x_3 = -7 \\ x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 - 3x_2 + 2x_3 = 9 \end{cases}$$

V=1

a_{ij}	1	2	3	Oz.h
1	**5**	**8**	**-1**	**-7**
2	**1**	**2**	**3**	**1**
3	**2**	**-3**	**2**	**9**

V=2

a_{ij}	1	2	3	Oz.h
1	**1**	**2**	**1**	**4**
2	**3**	**-5**	**3**	**1**
3	**2**	**7**	**-1**	**8**

V=3

a_{ij}	1	2	3	Oz.h
1	**3**	**2**	**1**	**5**
2	**2**	**3**	**2**	**1**
3	**2**	**1**	**3**	**11**

V=4

a_{ij}	1	2	3	Oz.h
1	**1**	**2**	**4**	**31**
2	**5**	**1**	**2**	**29**
3	**3**	**-1**	**1**	**10**

V=5

a_{ij}	1	2	3	Oz.h
1	**4**	**-3**	**2**	**9**
2	**2**	**5**	**-3**	**4**
3	**5**	**6**	**-2**	**18**

V=6

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-1**	**4**
2	**3**	**4**	**-2**	**11**
3	**3**	**-2**	**4**	**11**

V=7

a_{ij}	1	2	3	Oz.h
1	**1**	**1**	**2**	**1**
2	**2**	**-1**	**2**	**4**
3	**4**	**1**	**4**	**-2**

V=8

a_{ij}	1	2	3	Oz.h
1	**3**	**-1**	**0**	**5**
2	**-2**	**1**	**1**	**0**
3	**2**	**-1**	**4**	**15**

V=9

a_{ij}	1	2	3	Oz.h
1	**3**	**-1**	**1**	**4**
2	**2**	**-5**	**-3**	**-17**
3	**1**	**1**	**-1**	**0**

V=10

a_{ij}	1	2	3	Oz.h
1	**1**	**1**	**1**	**2**

V=11

a_{ij}	1	2	3	Oz.h
1	**2**	**1**	**-1**	**1**

V=12

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-3**	**3**

2	**2**	**-1**	**-6**	**-1**
3	**3**	**-2**	**0**	**8**

2	**1**	**1**	**1**	**6**
3	**3**	**-1**	**1**	**4**

2	**3**	**4**	**-5**	**8**
3	**0**	**2**	**7**	**17**

V=13

a_{ij}	1	2	3	Oz.h
1	**1**	**5**	**1**	**-7**
2	**2**	**-1**	**-1**	**0**
3	**1**	**-2**	**-1**	**2**

V=14

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**3**	**6**
2	**2**	**3**	**-4**	**16**
3	**3**	**-2**	**-5**	**12**

V=15

a_{ij}	1	2	3	Oz.h
1	**3**	**4**	**2**	**8**
2	**2**	**-1**	**-3**	**-1**
3	**1**	**5**	**1**	**0**

V=16

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**3**	**7**
2	**1**	**3**	**-2**	**0**
3	**0**	**2**	**-1**	**2**

V=17

a_{ij}	1	2	3	Oz.h
1	**2**	**1**	**4**	**20**
2	**2**	**-1**	**-3**	**3**
3	**3**	**4**	**-5**	**-8**

V=18

a_{ij}	1	2	3	Oz.h
1	**1**	**-1**	**0**	**4**
2	**2**	**3**	**1**	**1**
3	**2**	**1**	**3**	**11**

V=19

a_{ij}	1	2	3	Oz.h
1	**1**	**5**	**-1**	**7**
2	**2**	**-1**	**-1**	**4**
3	**3**	**-2**	**4**	**11**

V=20

a_{ij}	1	2	3	Oz.h
1	**11**	**3**	**-1**	**2**
2	**2**	**5**	**-5**	**0**
3	**1**	**1**	**1**	**2**

V=21

a_{ij}	1	2	3	Oz.h
1	**7**	**5**	**2**	**18**
2	**1**	**-1**	**-1**	**3**
3	**1**	**1**	**2**	**-2**

V=22

a_{ij}	1	2	3	Oz.h
1	**2**	**3**	**1**	**1**
2	**1**	**0**	**1**	**0**
3	**1**	**-1**	**-1**	**2**

V=23

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**-2**	**3**
2	**1**	**1**	**-2**	**0**
3	**1**	**-1**	**-1**	**1**

V=24

a_{ij}	1	2	3	Oz.h
1	**3**	**1**	**-5**	**-7**
2	**2**	**-3**	**4**	**-1**
3	**5**	**-1**	**3**	**0**

V=25

a_{ij}	1	2	3	Oz.h
1	**1**	**-2**	**1**	**15**
2	**2**	**1**	**3**	**9**
3	**2**	**3**	**2**	**-2**

V=26

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-2**	**1**
2	**3**	**2**	**1**	**1**
3	**2**	**3**	**3**	**0**

V=27

a_{ij}	1	2	3	Oz.h
1	**2**	**3**	**4**	**5**
2	**3**	**4**	**-1**	**3**
3	**4**	**5**	**-2**	**3**

V=28

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**-3**	**-9**
2	**1**	**2**	**1**	**3**
3	**3**	**1**	**-1**	**-1**

V=29

a_{ij}	1	2	3	Oz.h
1	**3**	**1**	**-2**	**4**
2	**2**	**-3**	**1**	**9**
3	**5**	**1**	**3**	**-4**

V=30

a_{ij}	1	2	3	Oz.h
1	**2**	**-1**	**3**	**-4**
2	**1**	**3**	**-1**	**2**
3	**5**	**2**	**1**	**5**

2-LABORATORIYA ISHI

Trantsendent va algebraik tenglama ildizlari aniqlash.

Maqsad: Ciziqsiz bo'lgan murakkab transtsendent tenglama va ko'phadning ildizi yotgan oraliqni aniqlash usullarini o'rganish.
Reja:
2.1. Tenglama ildizini ajratish qoidalari
2.2. Transtsendent tenglama ildizini ajratish
2.3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash.
2.4. Tenglama ildizini urinmalar (ng'yuton) usulida hisoblash.

2.1. Tenglama ildizini ajratish

Amaliyotda, baozi masalalarda

$$f(x)=0 \qquad (2.1)$$

ko'rinishdagi tenglamalarni yechishga to'g'ri keladi. Bunda *f(x)* [*a*,*b*] oraliqda aniqlangan funktsiya bo'lib, *f*(t)=0 bo'lsa, x=t ni (2.1) tenglamaning yechimi deyiladi. Tenglamaning aniq yechimini topish qiyin bo'lgan hollarda uning taqribiy yechimini topish ikki bosqichga bo'linadi.

1) Yechimni ajratish(yakkalash), yahni yagona yechim yotgan intervalni aniqlash;
2) Taqribiy yechimni berilgan aniqlikda topish.

Tenglamaning yagona yechimi yotgan oraliqni aniqlash uchun quyidagi teoremadan foydalaniladi.

2.1-teorema . ***Aytaylik,***

1) f(x) funktsiya [a,v] kesmada uzluksiz va (a,v) irtervalda hosilaga ega bo'lsin;

2) f(a)·f(b)<0, yahni f(x) funktsiya kesmaning chetlarida har xil ishoraga ega bo'lsin;

3) f′(x) hosila (a,v) irtervalda o'z ishorasini saqlasin.

U holda, (2.1) tenglama [a,v] oraliqda yagona yechimga ega bo'ladi.

2.2. Transtsendent tenglama ildizini ajratish

Teoremadagi [***a,b***] kesmani topishda, baozan grafik usuldan foydalaniladi. Bu usulga asosan (2.1) tenglamaning ildizini ajratish uchun y=*f(x)* funktsiyaning [*a*,*b*] oraliqdagi grafigini chizamiz. Bu grafikning OX o'qi bilan kesishish nuqtasining abstsissasi (2.1) tenglamaning yechimi bo'ladi. u=*f(x)* funktsiyani grafigini chizish qiyin bo'lsa, *f(x)*=0 tenglamani grafigini chizish mumkin bo'lgan funktsiyalarga fjratamiz va

$$f_1(x)=f_2(x) \qquad (2.2)$$

ko'rinishga keltiramiz va y=f_1(x), y=f_2(x) funktsiyalarning grafiklarini chizamiz. Bu grafiklar kesishish nuqtasining abstsissasi *f(x)*=0 tenglamaning yechimi bo'ladi. Bu yechimni o'z ichiga oluvchi [***a,b***] oraliqni yuqoridagi teorema shartlarini tekshirish asosida tanlaymiz.

2.1-Misol. ye^x -10x -2=0 tenglamaning yagona ildizi yotgan oraliq topilsin.

Echish. Tenglamani

$$e^x = 10x+2$$

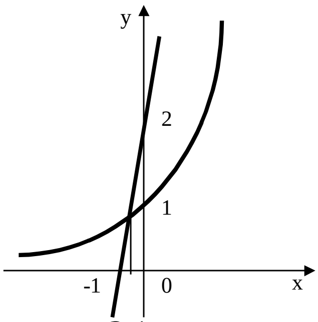
ko'rinishda yozamiz. So'ngra,

$$y=e^x, \ y=10x+2$$

funktsiyalarning grafiklarini chizamiz. 2.1-rasm dan ko'rinadiki,

$$e^x-10x-2=0$$

tenglama yagona yechimini o'z ichiga olgan oraliq [-1,0] bo'ladi. [-1,0] oraliqda teorema shartlarini tekshiramiz.



2.1-расм

1) $f(x)=e^x-10x-2$ funktsiya [-1,0] oraliqda uzluksiz, (-1,0) intervalda $f(x)=e^x-10$ hosilaga ega.

2) $f(-1)=e^{-1}-10(-1)-2\approx 3.368>0$, $f(0)=e^0-10\cdot 0-2=-1<0$

bundan: $f(-1)\cdot f(0)<0$

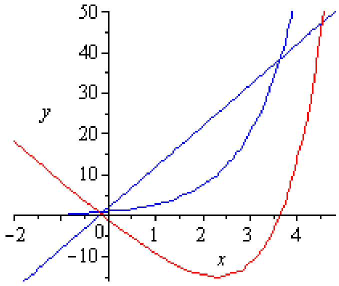
3) $x\in(-1,0)$ bo'lganda $f'(x)=e^x-10<0$

Demak, 2.1-teoremaning barcha shartlari [-1,0] oraliqda bajariladi. Bu [-1,0] oraliqda tenglama yagona yechimga ega ekanligini bildiradi.

2.1-MAPLE dasturi

Berilgan funktsiyalarning grafiklarini qurish:

```
> with(plots):
implicitplot([y=exp(x),y=10*x+2,y=exp(x)-10*x-2],
x=-2..10,y=-16..50, color=[blue,blue,red]);
```



2.1.2-rasm

```
5 ' -----------2.2- BYEYSIK TILI DASTURI --------
10 ' --- Транцендент тенглама илдизини ажратиш ----
20 DEF FNF(X)= EXP(X)-10*X-2
30 INPUT "Илдизлар ётган оралиқ ва кадамни киритинг
a,b,h=";A,B,H
40 X1=A
50 X2=X1+H
60 IF X2>B THEN   100
70 IF FNF(X1)*FNF(X2)>0 THEN   90
80 PRINT "Илдиз ётган оралиқ:      ("X1″,″X2″)″
90 X1=X2 : GOTO 50
100 END
RUN
a,b,h? -2,4,0.1
(-0.200, -0.100)
(3.600 , 3.700)
Ok
```

2.3. Algebraik tenglama ildizlari yotgan oraliqlarni aniqlash.

Aytaylik, bizga

$$f(x)= \boldsymbol{a}_0 x^n+ \boldsymbol{a}_1 x^{n-1} +...+ \boldsymbol{a}_{n-1} x+ \boldsymbol{a}_n=0 \qquad (2.3)$$

n-darajali algebraik tenglama berilgan bo'lsin.

1. Algebraik tenglama ildizlarini ajratishda quyidagi teorema va qoidalardan foydalanamiz.

2.2-teorema. ***Agar***

$$A=\max\{|a_1/a_0|, |a_2/a_0|,..., |a_n/a_0|\}$$
$$A1=\max\{|a_0/a_n|, |a_1/a_n|,..., |a_{n-1}/a_n|\}$$

bo'lsa, (7.3) tenglamaning barcha ildizlari

$$r=1/(1+A1)<|x|<1+A=R$$

halqada yotadi.

Musbat ildizlar chegarasi: $r< x^+ <R$

Manfiy ildizlar chegarasi: $-R< x^- < -r$

Agar (2.3) tenglamani

$$f_1(x)=x^n f(1/x)=0$$
$$f_2(x)= f(- x)=0$$
$$f_3(x)=x^n f(- 1/x)=0$$

ko'rinishga keltirib, mos ravishda topilgan musbat ildizlarining yuqori chegaralari R_1, R_2, R_3 bo'lsa, (2.3) tenglama ildizlarining chegaralari:

$$1/R_1< x^+ <R_2 \quad \text{va} \quad -R_2< x^- < - 1/R_3$$

2. Ishorasi almashinuvchi algebraik tenglamalarning musbat ildizlarining yuqori chegarasini topishda quyidagi Lagranj teoremasidan foydalanamiz:

2.3- teorema. ***(2.3) tenglamada $a_0>0$ va a_k ($k\geq 1$ -tartib raqami) - birinchi manfiy koeffitsient bo'lib, B manfiy koeffitsientlar ichida modul bo'yicha eng kattasi bo'lsa, musbat ildizla rining yuqori chegarasi***

$$R = 1+\sqrt[k]{\frac{B}{a_0}} \qquad (2.4)$$

formula bilan topiladi.

Berilgan (2.3) tenglamaning manfiy ildizlarining quyi chegarasini aniqlash uchun tenglamani

$$f(-x)=0 \qquad (2.5)$$

ko'rinishga keltirib, hosil bo'lgan (2.5) tenglamaga Lagranj teoremasini qo'llab, topilgan musbat ildizlarining yuqori chegarasi ***R1*** bo'lsa, (2.3) tenglama manfiy ildizlarining quyi chegarasi uchun ***-R1*** bo'lishi ayondir. Demak, berilgan (2.3) tenglamaning barcha haqiqiy ildizlarining chegarasi:

$$R1<x< R\,.$$

2.3-Misol. $2x^3-9x^2-60x+1=0$ tenglama ildizlari yotgan oralikning chegarasini aniqlang.

Echish.

1) Teorema bo'yicha:

$$A=\max\left\{\left|\frac{a_1}{a_0}\right|,\left|\frac{a_2}{a_0}\right|,\left|\frac{a_3}{a_0}\right|,\ldots,\left|\frac{a_n}{a_0}\right|\right\}=\max\left\{\left|\frac{-9}{2}\right|,\left|\frac{-60}{2}\right|,\left|\frac{1}{2}\right|\right\}=30$$

$$A_1=\max\left\{\left|\frac{a_0}{a_n}\right|,\left|\frac{a_1}{a_n}\right|,\left|\frac{a_2}{a_n}\right|,\ldots,\left|\frac{a_{n-1}}{a_n}\right|\right\}=\max\left\{\left|\frac{2}{1}\right|,\left|\frac{-9}{1}\right|,\left|\frac{-60}{1}\right|\right\}=60$$

$$r=\frac{1}{1+60}<|x|<1+30=R,\quad r=0.016,\ R=31$$

2.2-MAPLE dasturi

2.2-teorema asosida berilgan ko'phadning ildizlarining chegarasini aniqlash :

```
> C := proc(p,x)
local i; # declare index variable to be local
   [seq(coeff(p,x,i), i=0..degree(p,x) )];
end proc:
C( 2*x^3-9*x^2-60*x+1, x );
```

$$[1, -60, -9, 2]$$

```
> A := proc(p,x)
local i; # declare index variable to be local
   [max(seq(abs(coeff(p,x,i)/lcoeff(p)), i=0..degree(p,x)) )];
end proc:
A:=A( 2*x^3-9*x^2-60*x+1, x );
```

$$A := [30]$$

```
> A1 := proc(p,x)
local i; # declare index variable to be local
  [max(seq(abs(coeff(p,x,i)/tcoeff(p)), i=0..degree(p,x)) )];
end proc:
A1:=A1( 2*x^3-9*x^2-60*x+1, x );
```

$$A1 := [60]$$

```
> A:=30:A1:=60:R1:=1/(1+A1);R2:=1+A;
```

$$R1 := \frac{1}{61} \qquad R2 := 31$$

2) Lagranj formulasiga asosan a_0=2, B=60, $k_1(a_1=-9)=1$ dan

$$R=1+(B/a_0)^{\wedge}(1/k)=1+(60/2)^{\wedge}(1/1)=31$$

Bu musbat ildizlarining yuqori chegarasi R=31.

Manfiy ildizlarining quyi chegarasini topamiz.

$$f(-x)=2(-x)^3-9(-x)^2-60(-x)+1=0$$

$$f(-x)=2x^3+9x^2-60x-1=0$$

tenglamadan:

$$a_0=2,\ B2=60,\ k2=2$$

$$R1=1+(B2/a_0)^{\wedge}(1/k2)=1+(60/2)^{\wedge}(1/2)\approx 6.77$$

Bundan manfiy ildizlar quyi chegarasini R1=-6.77 bo'ladi

2.3-MAPLE dasturi

2.3-teorema asosida berilgan ko'phadning musbat ildizlarining yuqori chegarasini aniqlash :

Musbat ildizlarining yuqori chegarasi:

```
> p:=2*x^3 - 9*x^2 - 60*x + 1;
```
$p := 2\,x^3 \text{ K } 9\,x^2 \text{ K } 60\,x \text{ C } 1$

```
> coeffs(p,x);
```
$1,\ 2,\ \text{K}\ 9,\ \text{K}\ 60$

```
> M1:=max(coeffs(p,x));
```
$M1 := 2$

```
> B:=min(coeffs(p,x));
```
$B := \text{K}\ 60$

```
> R:=1+(abs(B)/a0)^1;
```
$R := 31$

2.3-MAPLE dasturi

Musbat ildizlarining yuqori chegarasi:

```
> restart;
> f:= 2*x^3-9.*x^2-60*x+1=0;
```
$$f := 2\,x^3 - 9.\,x^2 - 60\,x + 1 = 0$$

```
> solve(f,x);
```
$$0.01662535946,\ 8.166187279,\ -3.68281263$$

```
> sols := [solve(f,x)];
```
$$sols := [0.01662535946,\ 8.166187279,\ -3.682812638]$$

```
> sols[1];
```
$$0.0166253594$$

Manfiy ildizlarining quyi chegarasi:

```
> p1:=2*(-x)^3-9*(-x)^2-60*(-x)+1;
```
$p1 := \text{K}\ 2\,x^3 \text{ K } 9\,x^2 \text{ C } 60\,x \text{ C } 1$

```
> p:=(-1)^3*p1;
```
$p := 2\,x^3 \text{ C } 9\,x^2 \text{ K } 60\,x \text{ K } 1$

```
> coeffs(p,x); a0:=lcoeff(p);
```
$\text{K}\ 1,\ 2,\ 9,\ \text{K}\ 60 \qquad a0 := 2$

```
> B1:=min(coeffs(p,x));
```
$B1 := \text{K}\ 60$

```
> R1:= -1-(abs(B1)/a0)^(1/2); evalf(R1);
```
$$R1 := \text{K}\ 1 \text{ K } \sqrt{30} \qquad -6.477225575$$

3. Agar berilgan (2.3) tenglamaning barcha koeffitsientlari musbat bo'lsa, ildizlarining chegarasini

$$m < |x| < M$$

tengsizlikka asosan aniqlaymiz, bunda

$$\mathbf{m=min(}\boldsymbol{a}_{k} / \boldsymbol{a}_{k-1}\mathbf{),\ \ M=max(}\boldsymbol{a}_{k} / \boldsymbol{a}_{k-1}\mathbf{),\ \ 1<k<n}$$

SHuningdek, (7.3) tenglamaning barcha koeffitsientlari musbat bo'lganda:

a) $\boldsymbol{a}_0 > \boldsymbol{a}_1 > ... > \boldsymbol{a}_n$ bo'lsa, ildizlar $|x|>1$ doiradan tashqarida yotadi;

b) $\boldsymbol{a}_0 < \boldsymbol{a}_1 < ... < \boldsymbol{a}_n$ bo'lsa, ildizlar $|x|<1$ doira ichida yotadi.

4. Toq darajali algebraik tenglama hech bo'lmaganda bitta ildizga ega bo'ladi.

5 ' ------2.3-DASTUR-----------------

6 ' --- Algebraik tenglama ildizini ajratish ----

```
10 REM SAVE"altil1.BAS",A
20 DEF FNF(X)=2*X^3-9*X^2-60*X+1
22 PRINT"2*X^3-9*X^2-60*X+1=0"
30 INPUT "Alg.teng.tartibi N=";N
40 DIM A(N+1):H=.01
42 REM  Tenglama  koefitsentlari
50 FOR I=0 TO N
60 PRINT"a("I")=";:INPUT A(I)
70 NEXT I
80 A1=ABS(A(1)/A(0))
90 A2=ABS(A(0)/A(N))
100 FOR I=2 TO N
110 IF A1<ABS(A(I)/A(0)) THEN A1=ABS(A(I)/A(0))
120 NEXT I
122 FOR I=0 TO N-1
124 IF A2<ABS(A(I)/A(N)) THEN A2=ABS(A(I)/A(N))
126 NEXT I
130 PRINT"A1="A1,"  a2="A2
140 R1=1/(1+A2):'RINT"R1=-";USING "###.##";R1
150 R2=1+A1:'RINT"R2=";USING "###.##";R2
180 X1=-R2
190 X2=X1+H
200 IF X2>R2 THEN END
210 IF FNF(X1)*FNF(X2)>0 THEN 220
212 PRINT "(";USING "##.###";X1;
214 PRINT ",";USING "##.###";X2;:'RINT")"
220 X1=X2:GOTO 190
230 END
 RUN
```

```
2*X^3-9*X^2-60*X+1=0
Alg.teng.tartibi N=? 3
a( 0 )=? 2
a( 1 )=? -9
a( 2 )=? -60
a( 3 )=? 1
A1= 30
A2= 60
R1=- 0.02
R2= 31.00
( 0.014,  0.024)
( 8.164,  8.174)
Ok
```

2.4. Tenglama ildizini urinmalar(Nyuton) usulida hisoblash.

Maqsad: Transtsendent tenglama ildizi yotgan oraliqda ildizini hisoblashda urinmalar usulini o'rganish.

Reja:

2.4.1. Tenglama ildizini urinmalar usulida hisoblash.

2.4.**2. Tenglama ildizini hisoblashda urinmalar usulini qo'llash shartlari.**

2.4.**3. Urinmalar usulida ildizga yaqinlashish shartlari**

Aytaylik, $f(x)$ funktsiya $[a,b]$ oraliqda 2.1-teoremaning barcha shartlarini bajarsin. Bu holda, $f(x)=0$ tenglama $[a,b]$ oraliqda yagona x=t yechimga ega bo'ladi. Bu teorema asosida ildizni hisoblash uchun urinmalar usulini $f(x)f'(x)>0$ shart oraliqning qaysi chetida bajarilsa, shu tarafdan qo'llash kerakligini ko'ramiz. Bundan:

$f(a)f''(a)>0$ bo'lganda, boshlang'ich yaqinlashishni chapdan $a_0=a$, aks holda o'ngdan $b_0=b$ deb olinadi.

$f(a)f'(a)>0$ bo'lganda x=t yechimning taqribiy qiymatlaridan tuzilgan $\{a_n\}$ ketma- ketlik quyidagicha topiladi.

y=$f(x)$ funktsiya grafigining A(a, $f(a)$) nuqtasiga urinma o'tkazamiz (2.2-rasm), so'ngra bu urinmaning tenglamasini tuzamiz.

$$y-f(a)=f'(a)(x-a)$$

Urinmaning OX o'qi bilan kesishish nuqtasi x=a_1 -desak, bu nuqtada y=0 ekanligidan

$$0-f(a)=f'(a)(a_1-a)$$

ni olamiz. Oxirgidan esa

$$a_1=a-f(a)/f'(a)$$

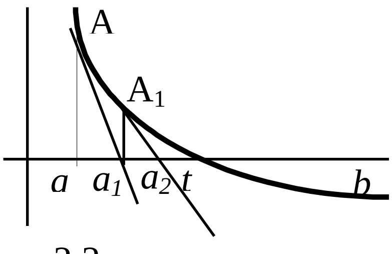
formula topiladi. So'ngra $[a_1,b]$ oraliq uchun yuqoridagi jarayonni takrorlab,

$$a_2=a_1-f(a_1)/f'(a_1)$$

formulani olamiz va hokazo, jarayonning n- takrorlanishida (n- qadamda)

$$a_n=a_{n-1}-f(a_{n-1})/f'(a_{n-1}) \qquad (2.6)$$

formulaga ega bo'lamiz. Bu jarayonni cheksiz takrorlash (davom ettirish) natijasida $\{a_n\}$ ketma-ketlikni tuzamiz. Bu urinmalar usulining mohiyatidan iboratdir.



2.2-rasm

Olingan $\{a_n\}$ ketma-ketlik 2.1-teoremaning shartlari bajarilganda aniq yechim x=t ga yaqinlashadi.(2.6) Jarayon $|a_n-a_{n-1}|<\varepsilon$ shart bajarilguncha davom ettiriladi va taqribiy ildiz uchun $x\approx a_n$ ni qabul qilinadi.

Agar $f(b)f''(b)>0$ bo'lsa, $b_0=b$ deb olib,

$$b_n=b_{n-1}-\frac{f(b_{n-1})}{f'(b_{n-1})}$$

formula asosida $\{b_n\}$ ketma-ketlikni olamiz.

2.4-**Misol.** $e^x-10x-2=0$ tenglama taqribiy yechimini $\varepsilon=0.01$ aniqlik bilan toping.

Ye**chish.** $f(x)=e^x-10x-2$ funktsiya [-1;0] oraliqda 2.1-teoremaning barcha shartlarini qanoatlantiradi.

$$f'(x)=e^x>0,\ x\in[-1;0] \quad \text{va} \quad f(-1)=8.386>0$$

dan

$$f(-1)\,f'(-1)>0$$

bo'lgani uchun $a_0=-1$ deb olinadi. $f'(-1)=e^{-1}-10=-9.632$ ni ehtiborga olib, birinchi yaqinlashish a_1 ni hisoblaymiz:

$$a_1=a-f(a)/f'(a)=a-f(-1)/f'(-1)=-1-8.386/(-9.632)=-0.131.$$

Yaqinlashish shartini tekshiramiz:

$$|\,a_1-a_0\,|=|-0.131+1|=0.869>\varepsilon=0.01$$

bo'lgani uchun ikkinchi yaqinlashish a_2 ni

$$a_2=a_1-f(a_1)/f'(a_1)$$

formula bilan topamiz.

$$f(a_1)=e^{-0.131}+10(0.131)-2=0.1895,\ f'(a_1)=e^{-0.131}-10=-9.123$$

lar asosida: $a_2=-0.131-0.1895/(-9.123)=-0.1104$.

Yana $|a_2-a_1|=0.0214>\varepsilon$ bo'lgani uchun a_3 ni topamiz.

$$a_2=-0.1104,\ f(a_2)=0.0006,\ f'(a_2)=-9.1046$$

lar asosida: $a_3=a_2-f(a_2)/f'(a_2)=-0.1104-0.0006/(-0.1046)=-0.1104;$

yaqinlashish sharti $|a_3-a_2|<\varepsilon=0.01$ bajarilganligi uchun tenglamaning $\varepsilon=0.01$ aniqlikdagi taqribiy yechimi:

$$x\approx a_3=-0.11$$

bo'ladi.

2.4-MAPLE dasturi

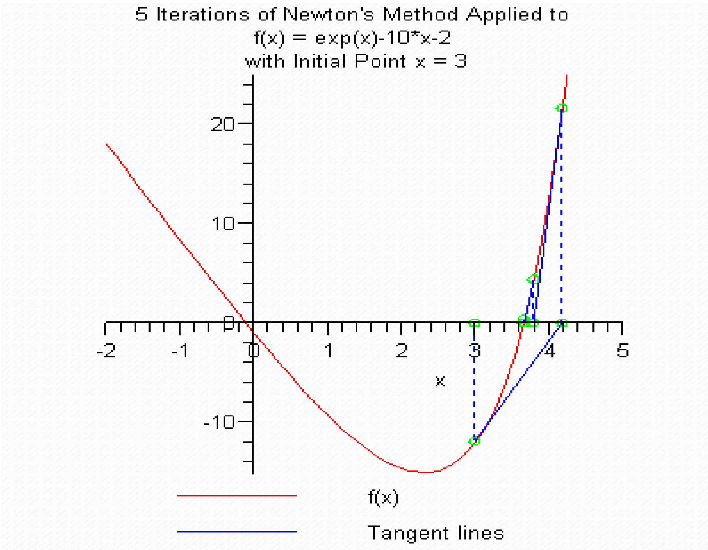
Urinmalar (Nyuton) usulida e^x-10x-2=0 tenglama ildizini aniqlash

1-ildiz: x =-1 dan o'ngdagi

```
> with(Student[Calculus1]):
NewtonsMethod(exp(x)-10*x-2, x =-1);
                                        -.1104575675
> NewtonsMethod(exp(x)-10*x-2, x =-1,
output = sequence);
                K 1, K .1312526261 , K .1104784974 , K .1104575675
```

2-ildiz: x =3 dan o'ngdagi

```
> with(Student[Calculus1]):  NewtonsMethod(exp(x)-10*x-2, x =3);
                                   3.650889174
> NewtonsMethod(exp(x)-10*x-2, x =3, output = sequence);
          3, 4.181341477, 3.791101988, 3.663011271, 3.650987596, 3.650889174
> NewtonsMethod(exp(x)-10*x-2, x =3, view = [-2..5, DEFAULT], output =plot);
```



2.2.1-rasm

```
4 ‘ ---------- 2.4- BYESIK TILI DASTURI ------------------
5 ‘Urinmalar usulida trantsendent tenglama
6 ‘--------- ildizini aniqlash-------------
10 REM SAVE”kas-1.bas”,a
20 DEF FNF(X)=EXP(X)-10*X-2
30 DEF FNF1(X)=EXP(X)-10
40 DEF FNF2(X)=EXP(X)
50 INPUT” ildiz chegarasi a,b=”; A,B
52 H=.1:E=.001
60 X1=A
70 X2=X1+H:X=X1:A=X2
80 IF X2>B THEN 180
90 IF FNF(X1)*FNF(X2)>0 THEN 170
100 IF FNF(X1)*FNF2(X1)>0 THEN 120
110 X=X2:A=X1
120 X=X-FNF(X)/FNF1(X)
130 IF FNF(X)>=E THEN  120
140 PRINT “(“;USING “##.###”;X1;
150 PRINT “,”;USING “##.###”;X2;
160 PRINT “)   x= “;USING “##.######”;X
170 X1=X2:GOTO 70
180 END
Ok
RUN
        ? -2,5
        (-0.200,-0.100)   x= -0.110458
        ( 3.600, 3.700)   x=  3.650891
        Ok
```

2-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Quyidagi tenglamalarning:

1) Ildizlarning qisqa atrofini analitik yoki grafik usulda aniqlang;
2) Aniqlangan oraliqda ildizni urinmalar usulida hisoblang;

1	1) $2^x+5x-3=0$ 2) $3x^4-4x^3-12x^2-5=0$ 3) $0.5^x+1=(x-2)^2$ 4) $(x-3)Cosx=1$, $(-2\pi\le x\le 2\pi)$	2	1) $arctgx-1/(3x^3)=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $[\log_2(-x)](x+2)=-1$ 4) $Sin(x+\pi/3)-0.5x=0$
3	1) $5^x+3x=0$ 2) $x^4-x-1=0$ 3) $0.5^x+x^2= 2$ 4) $(x-1)^2Ln(x+1)=1$	4	1) $2e^x=2+5x$ 2) $2x^4-x^2-10=0$ 3) $x\ Log_3(x+1)=1$ 4) $Cos(x+0.5)=x^3$
5.	1) $3^{x-1}-2-x=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $(x-4)^2\log_{0.5}(x-3) =-1$ 4) $5Sinx=x$	6.	1) $arctgx-1/(2x^3)=0$ 2) $x^4-18x^2+6=0$ 3) $x^2 2^x=1$ 4) $tgx=x+1$, $(-\pi/2\le x\le\pi/2)$
7.	1) $e^{-2x}-2x+1=0$ 2) $x^4+4x^3-8x^2-17=0$ 3) $0.5^x-1=(x+2)^2$ 4) $x^2Cos2=-1$	8.	1) $5^x-6x-3=0$ 2) $x^4-x^3-2x^2+3x-3=0$ 3) $0.5^x -2x^2- 3=0$ 4) $x\ Log(x+1)=1$
9.	1) $arctg(x-1)+2x=0$ 2) $3x^4+ 4x^3-12x^2+1=0$ 3) $(x-2)^2 2^x=1$ 4) $x^2-20Sinx=0$	10.	1) $2arctgx-x+3=0$ 2) $3x^4-8x^3-18x^2+3=0$ 3) $2Sin(x+\pi/3)=0.5x^2-1$ 4) $2Logx-x/2+1=0$
11	1) $3^x+2x-2=0$ 2) $2x^4-8x^3+8x^2-1=0$ 3) $[(x-2)^2-1]2^x =1$ 4) $(x-2)Cosx=1$	12.	1) $2arctgx-3x+2=0$ 2) $2x^4+8x^3+8x^2 -1=0$ 3) $Sin(x-0.5)-x+0.8=0$ 4) $(x-1)Log_2(x+2)=1$
13.	1) $3^x+2x-5=0$ 2) $x^4-4x^3-8x^2+1=0$ 3) $0.5^x +x^2- 3=0$ 4) $(x-2)^2Lg(x+1)=1$	14.	1) $2e^x+3x+3x+1=0$ 2) $3x^4+4x^3-12x^2-5=0$ 3) $Cos\ (x+0.3)=x^2$ 4) $x\ Log_3(x+1)=2$
15.	1) $3^{x-1}-4-x=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $(x-3)^2Log_{0.5}(x-2)=-1$ 4) $Sinx=x-1$	16.	1) $arctgx-1/(3x^3)=0$ 2) $x^4-x--1=0$ 3) $(x-1)^2 2^x=1$ 4) $tg^3x=x-1$
17.	1) $e^x+x+1=0$ 2) $2x^4-x^2-1=0$ 3) $0.5^x –3=(x+2)^2$ 4) $x^2Cos2x=-1$, $(-2\pi\le x\le 2\pi)$	18.	1) $3^x-2x+5=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $2x^2-0.5^x=0$ 4) $x\ lg(x+1)=1$
19	1) $arctg(x-1)+3x-2=0$ 2) $x^4-18x^2+6=0$ 3) $x^2-20Sinx=0$ 4) $(x-2)^2 2^x=1$	20.	1) $2arctgx-x+3=0$ 2) $x^4+4x^3-8x^2 -17=0$ 3) $2\ Sin(x+\pi/2)=x^2-0.8$ 4) $2\ lgx-x/2+1=0$
21	1) $2^x-3x-2=0$. 2)$x^4-x^3-2x^2 +3x- 3=0$; 3)$(0.5)^x+ 1=(x-2)^2$	22	1) $arctgx+2x-1=0$ 2) $3x^4+4x^3-12x^2 +1=0$ 3)$(x+2)Log_2(x)=1$

	4)(x-3)sosx = -1, -2p<x<2p.		4) Sin(x+1)=0.5x
23	1) $3^x+2x-3=0$. 2) $3x^4-8x^3-18x^2+2=0$; 3) $(0.5)^x=4-x^2$ 4) $(x+2)^2Lg(x+11)=1$	24	1) $2e^x-2x-3=0$. 2) $3x^4+4x^3-12x^2-5=0$; 3) $xLog_2(x+1)=1$ 4) $cos(x+0.5)=x^3$
25	1) $3^x+2+x=0$. 2) $2x^3-9x^2-60x+1=0$; 3) $(x-4)^2Log_{0.5}(x-3)=-1$ 4) 5Sinx=x-0.5	26	1) arctg(x-1)+2x-3=0 2) $x^4x-1=0$; 3) $(x-1)^2 2^x=1$ 4) $tg^3x=x-1$, $(-\pi/2\leq x\leq\pi/2)$
27	1) $2e^x-2x-3=0$. 2) $2x^4-x^2-10=0$; 3) $(0.5)^x-3=-(x+1)^2$ 4) $x^2sos2x=1$	28	1) $3^x-2x-5=0$. 2) $3x^4+8x^3+6x^2-10=0$; 3) $2x^2-0.5^x-3=0$ 4) xLg(x+1)=1
29	1) arctg(x-1)+2x=0 2) $x^4-18x^2+6=0$ 3) $(x-2)^2 2^x=1$ 4) $x^2-10sinx=0$	30	1) $3^x+5x-2=0$. 2) $3x^4+4x^3-12x^2+1=0$; 3) $(x-2)^2=0.5^x+1$ 4)(x+3)sosx = 1, $-2\pi<x<2\pi$.

3-LABORATORIYA ISHI

3.1. Lagranj interpolyatsiya ko'phadini topish

Maqsad: Tajriba natijalarida topilgan qiymatlarning o'zgaruvchilari orasidagi bog'lanishni Lagranj interpolyatsiya ko'phadini yordamida topishni o'rganish.

Reja:

3.1.1. Interpolyatsiya masalasini qo'yilishi.

3.1.2. Lagranj interpolyatsiya ko'phadini topish.

3.1.1. Interpolyatsiya masalasini kuyilishi.

Agar y=f(x) funktsiya [a,b] kesmaning x_k, k=0,1,2......,n nuktalarda $f(x_k)=y_k$ qiymatlarga ega bo'lsa, kuyidagi jadvalni tuzish mumkin.

$$\begin{cases} x & x_0\ \ x_1\ \ x_2 \ldots x_n \\ y & y_0\ \ y_1\ \ y_2 \ldots y_n \end{cases} \qquad (3.1)$$

Bu jadvalni asosida berigan funktsiyani polinomi yoki ko'phadini

$$P_n(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n \qquad (3.2)$$

topish uchun quyidagicha shart qo'yamiz: jadvalning har bir x_k, k=0,1,2,...,n nuktada

$$P_n(x_k)\approx f(x_k)=y_k \qquad (3.3)$$

munosabat urinla bo'lsin. Bunday masala interpolyatsiyalash deyiladi. Topilgan ko'phadini interpolyatsiya ko'phadi deyiladi. Topilgan interpolyatsiya ko'phadi asosida biror $[x_k, x_{k+1}]$ oralikka tegishli x ning taqribiy qiymatini topish masalasini xam yechamiz.

Ikkinchi tartibli

$$P_2(x)=a_0x^2+a_1x+a_2 \qquad (3.4)$$

bu ko'phadining koeffitsentlarini

$$P_2(x_i)=y_i\,,\ i=0,1,2 \qquad (3.5)$$

shart saosida topish masalasini kuyamiz. Xakikatan xam $x=x_0$, $x= x_1$, $x=x_2$ larda (12.5) shart asosida kuyidagi sistemani tuzamiz:

$$a_0x_0^2+a_1x_0+a_2=y_0$$
$$a_0x_1^2+a_1x_1+a_2=y_1$$
$$a_0x_2^2+a_1x_2+a_2=y_2$$

Bu sistemadagi nomalumlar- koeffitsentlarini

$$D=\begin{vmatrix} X_0^2 & x_0 & 1 \\ x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \end{vmatrix}=(x_1-x_0)(x_2-x_0)(x_3-x_0)\neq 0$$

bo'lganda topish mumkin. Lyokin yuqori tartibli ko'phadilarni topishda tuziladigan sistemalarni yechish kiyinlashadi. SHuning uchun Lagranj usulidan foydalanamiz.

3.1.2. Lagranj interpolyatsiyalash formulasi va uning axamiyati.

Yuqoridagi jadval asosida topiladigan ko'phadini quyidagicha tanlaymiz:

$$\begin{aligned} P_n(x)=&a_0(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)+\\ &+a_1(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n)+ \qquad (3.6)\\ &+\ldots\ldots\ldots\ldots\ldots\ldots+\\ &+a_n(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_{n-1}) \end{aligned}$$

bunda n=2 uchun

$$P_2(x)=a_0(x-x_1)(x-x_2)+a_1(x-x_0)(x-x_2)+a_2(x-x_1)(x-x_2) \qquad (3.7)$$

Bu a_0, a_1, a_2 koeffitsentlarini topish uchun

$$P_2(x_0)=y_0,\ P_2(x_1)=y_1,\ P_2(x_2)=y_2$$

SHartga asosan kuyidagi sistemani topamiz:

$$a_0(x-x_1)(x-x_2)=y_0$$
$$a_1(x-x_0)(x-x_2)=y_1$$
$$a_2(x-x_1)(x-x_2)=y_2$$

bundan:

$$a_0=y_0/(x-x_1)(x-x_2),$$
$$a_1=y_1/(x-x_0)(x-x_2),$$
$$a_2=y_2/(x-x_1)(x-x_2)$$

Endi izlanaetgan interpolyatsiya ko'phadini yozamiz:

$$P_2(x)=y_0\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}+y_1\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+y_2\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

SHuningdek n=3 bo'lganda:

$$P_3(x)=y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}+$$
$$+y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

Demak Lagranj interpolyatsiya ko'phadini umumiy holda quyidagicha yozamiz:

$$P_n(x)=\sum_{j=0}^{n} y_j \prod_{i\neq j} \frac{(x-x_i)}{(x_j-x_i)} \quad , \qquad (3.8)$$

3.1-Masala. Quyidagi, y=ln x funktsiya asosida tuzilgan

X	2	3	4	5
Y	0.6931	1.0986	1.3863	1.6094

Jadvaldan foydalanib Lagranj interpolyatsiya ko'phadini toping va bu ko'phadilar yerdamida ln 3.5 ni hisoblang.

Echish.

$$L_3(x)=\frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(1-5)}0.6981+\frac{(x-2)(x-4)(x-5)}{(3-2)(3-4)(3-5)}1.0986+$$

$$+\frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)}1.3865+\frac{(x-2)(x-3)(x-4)}{(5-2)(5-3)(5-4)}1.6094=$$

$$=0.0089\,x^3-0.1387x^2+0.9305\,x-0.6841$$

Hosil bo'lgan ko'phadga asosan

$$\ln 3{,}5\approx L(3{,}5)=0.0089*(3.5)^3-0.1387(3{,}5)^2+0.9805(3.5)-0.684=$$

$$=0.31-1.701+3.2567-0.6841=1.25145$$

bo'ladi.

3.1 - Maple dasturi

Lagranj interpolyatsiya ko'phadini va x=3.5 dagi L(x) ning qiymati aniqlash.

Jadvalga asosan ko'phadni topish

```
> with(CurveFitting):
> PolynomialInterpolation([2,3,4,5],
[0.6931,1.0986,1.3865,1.6094], x, form=Lagrange );
```

K 0.1161833333 (x K 3) (x K 4) (x K 5)
C 0.5493000000 (x K 2) (x K 4) (x K 5)
K 0.6931500000 (x K 2) (x K 3) (x K 5)
C 0.2682333333 (x K 2) (x K 3) (x K 4)

```
> with(CurveFitting):
> PolynomialInterpolation([[2,0.6931], [3,1.0986],[4,1.3865],
[5,1.6094]],x);
```

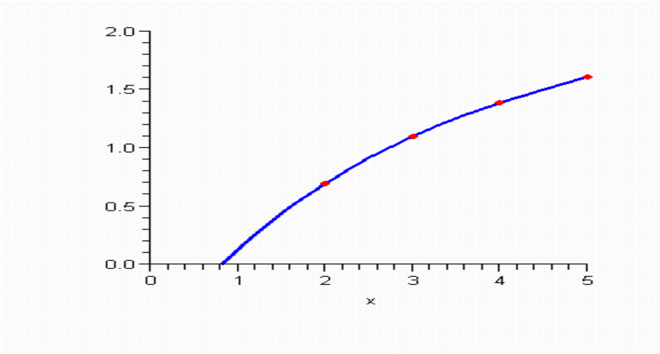
$0.008200000000x^3$ K $0.1307000000x^2$ K 0.644100000
C $0.8992000000x$

```
> PolynomialInterpolation([2,3,4,5],[0.6971,1.0986,1.3863,
1.6094], 3.5, form=Lagrange );
```

1.253600000

Jadvalga asosan topilgan ko'phadni grafigini qurish

```
> with(stats):with(plots):
> plot([p,[[2,0.6931],[3,1.0986],[4,1.3863],[5,1.6094]]],
x=0..5,0..2, style =[line,point],color = [blue,red],
thickness=2);
```



3.1-rasm

```
5'------------   3.1- Byeysik tili dasturi ----------------------
10 DIM Y(20),X(20)
20 READ N
30 FOR I=1 TO N :READ X(I) : NEXT I
40 FOR I=1 TO N :READ Y(I) : NEXT I
50 GOSUB 210 : PRINT ": X";
60 FOR I=1 TO N :PRINT " :";USING"##.###";X(I);
70 NEXT I : PRINT " :":GOSUB 210 :PRINT ": Y";
80 FOR I=1 TO N :PRINT " :";USING"##.###";Y(I);
90 NEXT I : PRINT " :":GOSUB 210
100 READ N1
110 FOR K=1 TO N1
120 READ X1
130 S=0
140 FOR I=1 TO N :P=1 : FOR J=1 TO N
150 IF I=J THEN 170
160 P=P*(X1-X(J))/(X(I)-X(J))
170 NEXT J
172 S=S+P*Y(I)
174 NEXT I
180 PRINT :PRINT TAB(19);"Y(";USING"###.###";X1;
190 PRINT ")=";USING"###.#####";S: NEXT K
200 END
210 PRINT "---";: FOR  I=1 TO N :PRINT "--------";
220 NEXT I : PRINT "--" : RETURN
230 DATA 2,3,4,2.5,6
240 DATA 1.583,1.436,1.372,1.238,1.084
250 DATA 2,3.5,4.1
260 END
```

```
---------------------------------------------------
: X : 2.000 : 3.000 : 4.000 : 5.000 : 6.000 :
---------------------------------------------------
: Y : 1.583 : 1.436 : 1.372 : 1.238 : 1.084 :
---------------------------------------------------
              Y(  3.500)= 1.4079
              Y(  4.100)= 1.3625
```

3.2.Kichik kvadratlar usuli.
Tajriba natijalarinig chiziqli va parabolik bog'laninshini aniqlash.

Maqsad: Kichik kvadratlar usulida tajriba natijalarida topilgan qiymatlar orasidagi chiziqli va parabolik bog'laninshini aniqlash.

Reja:

3.2.1. Kichik kvadratlar usuli
3.2.2. To'g'ri chiziqi bog'laninsh tenlamasini aniqlash.
3.2.3. Ikkinch darajali bog'laninsh tenglamasini topish.
3.2.4. Chiziqsiz bog'laninsh tenglamasini topish.

3.2.**1. Kichik kvadratlar usuli**

Aytaylik tajriba natijalari quyidagi jadval asosida berilgan bo'lsin.

x	x_1,	x_2	x_3	...	x_n
y	y_1,	y_2	y_3	...	y_n

Bu ikki o'zgaruvchi orasidagi bog'lanish formulasinianalinik usulda aniqlash masalasini yechamiz. Bu masalani kichik kvadratlar usuli bilan yechamiz. Buning uchun bog'laninshni ifodalovchi funktsiyalar turini tanlaymiz. Masalan:

1) chiziqli bog'laninsh: $y=ax+b$
2) parabolik bog'laninsh: $y=ax^2+bx+c$

Bu bog'lanishlarni aniqlashda ularning koeffitsentlarini aniqlash asosiy masala hisoblanadi. Umumiylik uchun izlanayotgan funktsiyani

$$y=F(x,a,b,c)$$

ko'rinishda oxtaramiz. Bu bog'lanish koeffitsentlarini aniqlash uchun berilgan jadval asocida

$$F(x_i,a,b,c)=y_i\ ,\ i=1,2,...,n$$

shartni yozamiz. Izlanayotgan funktsiya va y_i lar orasidagi farq minimum yoki kichik bo'lish shartini topamiz. Buning uchun quyidagi funktsianalni tuzamiz:

$$F(a,b,c)=\sum[y_i - F(x_i,a,b,c)]^2, \quad i=1,2,...,n$$

Bu F(a,b,c) funktsianing minimumini topish uchun quyidagi zaruriy sharttan foydalanamiz.

$$F'_a(a,b,c)=0,\ F'_b(a,b,c)=0.\ F'_c(a,b,c)=0$$

Ya'ni

$$\sum[y_i - F(x_i,a,b,c)]\ F'_a(a,b,c)=0$$
$$\sum[y_i - F(x_i,a,b,c)]\ F'_b(a,b,c)=0$$
$$\sum[y_i - F(x_i,a,b,c)]\ F'_c(a,b,c)=0$$

Ushbu sistemeni yechish bilan a,b,c larni topamiz va jadvalni ifodalovchi bog'lanish funktsiasini topamiz.

3.2.**2. To'g'ri chiziqi bog'laninsh tenlamasini aniqlash.**

CHiziqli bog'lanish $F(x_i,a,b)=a\,x_i +b$, uchun $F'_a= x_i$, $F'_b=1$

$$\sum[y_i - ax_i - b)]^2\ xi=0$$
$$\sum[y_i - ax_i - b)]^2\ 1=0$$

$$\left.\begin{matrix}\left(\sum x_i^{\ 2}\right)a + \left(\sum x_i\right)b = \sum x_i \cdot y_i \\ \left(\sum x_i\right)a + nb = \sum y_i\end{matrix}\right\}$$

Bu sistemani *a.b* larga nisbatan yechamiz:

$$a = \frac{\sum x_i \cdot y_i - \sum x_i \cdot \sum y_i}{\sum x_i^2 - \left(\sum x_i\right)^2}, \quad b = \frac{\sum y_i \cdot \sum x_i^2 - \sum x_i \cdot \sum x_i y_i}{\sum x_i^2 + \left(\sum x_i\right)^2}$$

3.2.**3. Ikkinch darajali bog'laninsh tenglamasini topish.**

Parabolik bog'laninsh: $F(x_i,a,b,c)=ax_i^2+bx_i+c$ uchun

$$F'_a=x_i^2,\ F'_b=x_i,\ F'_s=1$$
$$\sum[y_i - ax_i^2 - bx_i - c)]\ x_i^2=0$$
$$\sum[y_i - ax_i^2 - bx_i - c)]\ x_i=0$$
$$\sum[y_i - ax_i^2 - bx_i - c)]\ 1=0$$

Bu sistemani quyidagicha yozamiz

$$\begin{cases}\left(\sum x_i^{\ 4}\right)a + \left(\sum x_i^{\ 3}\right)b + \left(\sum x_i^{\ 2}\right)c = \sum y_i x_i^{\ 2} \\ \left(\sum x_i^{\ 3}\right)a + \left(\sum x_i^{\ 2}\right)b + \left(\sum x_i\right)c = \sum y_i x_i \\ \left(\sum x_i^{\ 2}\right)a + \left(\sum x_i\right)b + nc = \sum y_i\end{cases}$$

va uni biror usul bilan yechib *a,b,c* larni topamiz.

3.2- Masala. Tajriba natijasida topilgan quyidagicha o'lchov natijalarining bog'lanishini aniqlang.

3.1-jadval

X_i	0.5	1.0	1.5	2.0	2.5	3.0
y_i	6.0	5.0	3.7	2.6	1.6	0.6

3.1- masalada berilgan jadval asosida yuqoridagi kichik kvadratlar usuli bilan bog'lanishlarni aniqlovchi va hisoblovchi dasturlari asosida topilgan natijalarni keltiramiz:

3.2-Maple dasturi

y=a+bx **chiziqli bog'lanishni aniqlash(3.1-jadval)**

1. Bog'lanishni aniqlash.

```
> with(Statistics):
> X := Vector([0.5, 1, 1.5, 2, 2.5, 3], datatye=float):
  Y := Vector([6,5, 3.7, 2.6, 1.6, 0.6], datatye=float):
> Fit(a+b*t, X, Y, t);
```

7.08000000000000274 K 2.18857142857142950 *t*

2. Bog'lanishni grafigini qurish.

```
> with(stats):with(plots):
```

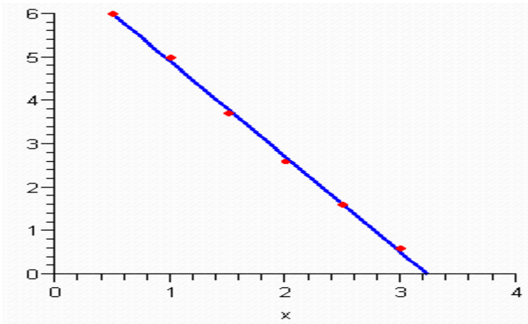
```
> r2:=rhs(fit[leastsquare[[x,y], y=a*x+b, {a,b}]]([[0.5, 1.0,
1.5, 2.0,2.5,3.0],[6.0, 5.0, 3.7, 2.6, 1.6, 0.6]]));
```

$$r2 := 7.080000000 \quad K \quad 2.188571429 \quad x$$

```
> plot([r2,[[0.5,6],[1,5],[1.5,3.7],[2,2.6],[2.5,1.6],[3,0.6]]],
x=0..4,0..6,style =[line,point],color = [blue,red],thickness=2);
```

3. **Bog'lanishni grafigini muloqat oynasida qurish.**

```
> with(CurveFitting):
Interactive([[0.5,6],[1,5],[1.5,3.7],[2,2.6],[2.5,1.6],[3,0.6]],
t);
```



3.2-rasm

```
4 '--------------- 3.2- BYEYSIK TILI Dasturi ----------------
5 ' Kichik kvadratlar usulida
6 'To'g'ri  chiziqli bog'lanish aniklash
10 REM MNK chiziqli grafik bilan
20 REM SAVE"mhklin2.bas",a
30 DIM X(20),Y(20)
40 CLS:SCREEN 9
50 INPUT "Xiva Yi lar soni N=";N
60 SX=0:SY=0:SXX=0:SXY=0
70 FOR I=1 TO N:READ X(I),Y(I):NEXT I
80 FOR I=1 TO N
90 SX=SX+X(I):SY=SY+Y(I)
100 SXX=SXX+X(I)^2:SXY=SXY+X(I)*Y(I)
110 NEXT I:
130 B0=( N*SXY -SX*SY)/((N*SXX)- (SX)^2)
140 B1=(SXX*SY- SX*SXY)/(N*(SXX)- (SX)^2)
150 PRINT "Sx="SX;"Sy=";SY;"Sxx=";SXX;"Syx="SYX
160 PRINT "B0="B0; "  B1="B1
170 PRINT "y=("B0")x+("B1")"
180 INPUT "Xini kriting=";X:Y1=B0*X+B1
190 PRINT "y("X")="Y1
200 INPUT "Grafik.kerakmi Xa=1/yo'q=0";T
210 IF T=1 THEN 220 ELSE END
220 LINE(0,137)-(600,137), 5
230 LINE(250,0)-(250,600), 5
240 FOR I=1 TO 10
250 PSET(X(I)*40+250,-Y(I)/5*40*.64+136),3:NEXT I
260 FOR X=X(1) TO X(10) STEP .1
270 PSET(X*40+250,-(B0*X+B1)/5*40*.64+136),4:NEXT X
280 DATA 0.5,6,1,5,1.5,3.7,2,2.6,2.5,1.6,3,0.6
290 END
```

RUN
(x,y) lar sonini kriting N=? 6
(x,y) juftlik larni kriting:
b1= -0.873
b0= 11.773
y(X)= 11.77259 *EXP(-.8733742 *x)

argumentni kiriting x=? 1
y(1)= 4.915532

3.3-Maple dasturi

$y=ax^2+bx+c$ parabolik bog‘lanishni aniqlash(3.1-jadval)

1. Bog‘lanishni aniqlash.

```
> with(Statistics):
> X := Vector([0.5, 1, 1.5, 2, 2.5, 3], datatye=float):
Y := Vector([6,5, 3.7, 2.6, 1.6, 0.6], datatye=float):
Fit(a+b*t+c*t^2, X, Y, t);
```

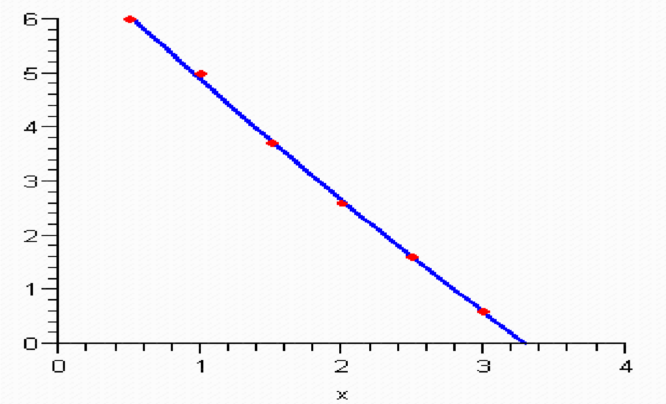
$$7.28000000000000380 \; K \; 2.48857142857143244 t$$
$$C \; 0.0857142857142865894 t^2$$

2. Bog‘lanishni grafigini qurish.

```
> with(stats):with(plots):
> r3:=rhs(fit[leastsquare[[x,y], y=a*x^2+b*x+c]]
([[0.5, 1.0, 1.5, 2.0,2.5,3.0], [6.0, 5.0, 3.7, 2.6, 1.6,
0.6]]));
```

$$r3 := 0.08571428571 x^2 \; K \; 2.488571429 x \; C \; 7.28000000($$

```
> plot([r3,[[0.5, 6],[1,5],[1.5, 3.7],[2, 2.6],[2.5, 1.6],
[3,0.6]]],x=0..4,0..6,style =[line,point],
color=[blue,red],thickness=2);
```



3.3-rasm

Gauss usulini qo‘llaymiz va uning hisoblash dasturini beramiz:

```
4 ‘--------------- 6.2- DASTUR ----------------
5 ‘ KICHIK KVADRATLAR USULIDA
6 ‘KO‘PHAD  KO‘RINISHIDAGI  BOG‘LANISH ANIKLASH
10 REM SAVE"MNK-POL.BAS",A
12 CLS
20 INPUT "POLINOM DARAJASINI KRITING"; M
30 M = M + 1
32 DIM A(20, 20), B(40), C(20), X(20), Y(20)
40 INPUT "(X,Y) LAR SONI N="; N
60 FOR I = 1 TO N: READ X, Y
70 'PRINT "I="; I: INPUT "X,Y LARNI KRITING="; X, Y
80 F = 1: FOR J = 1 TO 2 * M - 1: IF J > M THEN 100
90 B(J) = B(J) + Y: Y = Y * X
100 C(J) = C(J) + F: F = F * X: NEXT J, I
110 FOR I = 1 TO M: K = I
120 FOR J = 1 TO M
130 A(I, J) = C(K): K = K + 1: NEXT J, I
```

```
140 FOR I = 1 TO M - 1: FOR J = I + 1 TO M
150 A(J, I) = -A(J, I) / A(I, I)
160 FOR K = I + 1 TO M
170 A(J, K) = A(J, K) + A(J, I) * A(I, K)
180 NEXT K: B(J) = B(J) + A(J, I) * B(I)
190 NEXT J, I
200 X(M) = B(M) / A(M, M)
210 FOR I = M - 1 TO 1 STEP -1
220 H = B(I)
230 FOR J = I + 1 TO M: H = H - X(J) * A(I, J): NEXT J
240 X(I) = H / A(I, I): NEXT I
250 PRINT "POLINOM KOEFISENTLAPI"
260 FOR I = 0 TO M - 1
270 PRINT "A("; I; ")="; USING "###.###"; X(I + 1): NEXT I
280 INPUT "XNI KIRIT="; Z: S = 0
290 FOR I = M TO 2 STEP -1
300 S = (S + X(I)) * Z: NEXT I
310 PRINT "Y(X)="; S + X(1): GOTO 280
312 DATA 0.5,6,1,5,1.5,3.37,2,2.6,2.5,1.6,3,0.6
320 END
polinom darajasini kriting N=? 2
(x,y) lar soni h=? 6
polinom KOEFISENTlARI:
B( 0 )=  7.280
B( 1 )= -2.489
B( 2 )=  0.086
Xni kirit=? 1
Y(x)= 4.877143
Xni kirit=? 2
Y(x)= 2.645713
Ok
```

3.2.4. **Chiziqsiz bog'laninsh tenglamasini topish.**

Tajriba natijasida topilgan x va y o'zgaruvchilar orasida bog'lanish quyidagi jadval ko'rinishida berilgan bo'lsin.

Yuqoridagi kichik kvdratlar usuli asosida

3.2-jadval

x	1	2	3	4	5	6	7	8
y	12.2	6.8	5.2	4.6	3.9	3.7	3.5	3.2

jadval uchun quyidagi bog'lanishlarning parametrlarini aniqlovchi formulalar asosida xisoblash dasturlarini beramiz.

1. $\acute{o} = b_0 + \frac{b_1}{x}$ giperbolik bog'lanishni b_0 , b_1 parametrlarini aniqlovchi quyidagi formulalarni yozamiz:

$$\begin{cases} b_0 N + b_1 \sum_{i=1}^{N} \frac{1}{x_i} = \sum_{i=1}^{N} y_i \\ b_0 \sum_{i=1}^{N} \frac{1}{x_I} + b_1 \sum_{i=1}^{N} \frac{1}{x_I^2} = \sum_{i=1}^{N} \frac{y_i}{x_i} \end{cases}$$

Xisoblash dasturini beramiz:

3.4-Maple dasturi

$\acute{o} = a + \frac{b}{x}$ **giperbolik bog'lanishni aniqlash(3.2-jadval).**

1) *Bog'lanishni aniqlash.*

> with(Statistics):

> X := Vector([1, 2, 3, 4, 5,6,7,8], datatye=float):

Y := Vector([12.2,6.8,5.2,4.6,3.9, 3.7, 3.5, 3.2], datatye=float):
> Fit(a+b/t, X, Y, t);

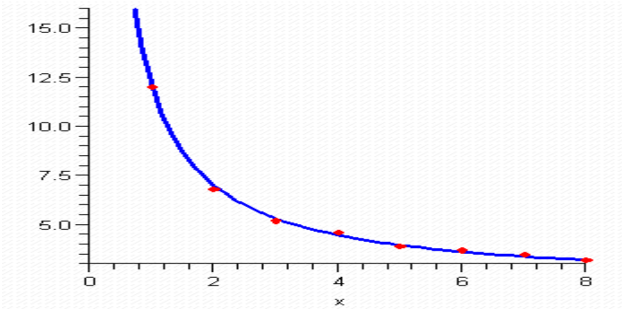
$$1.93576189703930290 \quad C \quad \frac{10.1601752307910243}{t}$$

2) Bog'lanishni grafigini qurish.

```
> with(plots):
> r4:=rhs(fit[leastsquare[[x,y], y=a+b/x]]( [[1, 2, 3,
4,5,6,7,8],[12.2,6.8, 5.2, 4.6,3.9,3.7, 3.5, 3.2]]));
```

$$r4 := 1.935761897 \quad C \quad \frac{10.16017523}{x}$$

```
> plot([r4,[[1,12],[2,6.8],[3, 5.2],[4, 4.6],[5,
3.9],[6,3.7],[7,3.5],[8,3.2]]],x=0..8,3..16,style
=[line,point],color = [blue,red],thickness=2);
```



3.4-rasm

```
4 '--------------- 3.4 BYEYSIK TILI Dasturi ----------------
5 ' Kichik kvadratlar usulida
6 'Giperbolik  ko'rinishidagi  bog'lanish aniklash
8REM SAVE"GI'BOG.bas",a
10 REM Giperbolik bog'lanish
20 INPUT "(x,y) lar soni N=";N
30  S1=0:S2=0:S3=0:S4=0
40 PRINT"x(i),y(i) larni kiriting:"
42 FOR I=1 TO N:'RINT"i=";I;
50 'INPUT "xi=";X
52 'INPUT "yi=";Y
54 READ X,Y :'RINT USING "###.##";X;Y
60 S1=S1+1/X:S2=S2+1/(X^2)
70 S3=S3+Y:S4=S4+Y/X
80 NEXT I
90 E=N*S2-S1^2:F=S3*S2-S4*S1
100 K=N*S4-S1*S3:B0=F/E:B1=K/E
110 PRINT"y(x)=";USING "###.###";B0;
112 PRINT"+";USING "###.###";B1;:'RINT"/x"
120 INPUT "arguvtnt x=";X
130 PRINT"y(x)=";USING "###.###";B0+B1/X
140 GOTO 120
142 DATA 1,12.2,2,6.8,3,5.2,4,4.6,5,3.9,6,3.7,7,3.5,8,3.2
150 END
Ok
RUN
(x,y) lar soni N=? 8
x(i),y(i) larni kiriting:
i= 1   1.00 12.20
i= 2   2.00  6.80
i= 3   3.00  5.20
i= 4   4.00  4.60
i= 5   5.00  3.90
```

i= 6   6.00 3.70
i= 7   7.00 3.50
i= 8   8.00 3.20
y(x)= 1.936+ 10.160/x
arguvtnt x=? 2
y(x)= 7.016
arguvtnt x=? 1
y(x)= 12.096
arguvtnt x=? 4
y(x)= 4.476

Yuqorida topilgan bog‘lanishlarni aniqlash qoidalari asosida quyidagi jadvalni tuzamiz.

3.3-jadval

T/r	Bog‘lanish tenglamasi	Kichik kvadratlar usulida bog‘lanish koeffitsentlarini aniqlovchi tenglamalar sistemasi
1	$y=a+bx$	$an+b\Sigma x=\Sigma y$ $a\Sigma x+b\Sigma x^2=\Sigma(xy)$
2	$lgy=a+bx$	$an+b\Sigma x=\Sigma lgy$ $a\Sigma x+b\Sigma x^2=\Sigma(xlgy)$
3	$y=a+blgx$	$an+b\Sigma lgx=\Sigma y$ $a\Sigma lgx+b\Sigma(lgx)^2=\Sigma(ylgx)$
4	$lgy=a+blgx$	$an+b\Sigma lgx=\Sigma y$ $a\Sigma lgx+b\Sigma(lgx)^2=\Sigma(lgxlgy)$
5	$y=ab^x$ *yoki* $lgy=lga+blgx$	$an+b\Sigma lgx=\Sigma lgy$ $lga\Sigma lgx+lgb\Sigma x^2=\Sigma(lgxlgy)$
6	$y=a+bx+cx^2$	$an+b\Sigma x+c\Sigma x^2=\Sigma y$ $a\Sigma x+b\Sigma x^2+c\Sigma x^3=\Sigma(xy)$ $a\Sigma x^2+b\Sigma x^3+c\Sigma x^4=\Sigma(x^2y)$
7	$y=a+bx+cx^2+dx^3$	$an+b\Sigma x+c\Sigma x^2+d\Sigma x^3=\Sigma y$ $a\Sigma x+b\Sigma x^2+c\Sigma x^3+d\Sigma x^4=\Sigma(xy)$ $a\Sigma x^2+b\Sigma x^3+c\Sigma x^4+d\Sigma x^5=\Sigma(x^2y)$ $a\Sigma x^3+b\Sigma x^4+c\Sigma x^5+d\Sigma x^6=\Sigma(x^3y)$
8	$y=a+bx+c\sqrt{x}$	$an+b\Sigma x+c\Sigma\sqrt{x}=\Sigma y$ $a\Sigma x+b\Sigma x^2+c\Sigma\sqrt{x}^3=\Sigma(xy)$ $a\Sigma\sqrt{x}+b\Sigma\sqrt{x}^3+c\Sigma x=\Sigma(\sqrt{xy})$
9	$ó=ab^xc^{x^2}$ *yoki* $lgy=lga+xlgx+x^2lgc$	$nlga+lgb\Sigma x+lgc\Sigma x^2=\Sigma lgy$ $lga\Sigma x+lgb\Sigma x^2+lgc\Sigma x^3=\Sigma(xlgy)$ $lga\Sigma x^2+lgb\Sigma x^3+lgc\Sigma x^4=\Sigma(x^{2lg}y)$

3-laboratoriya ishi bo’yicha mustaqil ishlash uchun topshiriqlar

Quyidagi jadval uchun:

1)Lagranj interpolyatsiya ko’hadini toping;

2)kichik kvadratlar usulida to’g’ri chiziqi va ikkinch darajali bog’lanishini aniqlang.

Variant 1

X	0,43	0,48	0,55	0,62	0,70	0,75
Y	1,63597	1,73234	1,87686	2,03345	2,22846	2,35973

Variant 2

X	0,02	0,08	0,12	0,17	0,23	0,30

Y	1,02316	1,09590	1,14725	1,21483	1,30120	1,40976

Variant 3

X	0,35	0,41	0,47	0,51	0,56	0,64
Y	2,739	2,300	1,968	1,787	1,595	1,345

Variant 4

X	0,41	0,46	0,52	0,60	0,65	0,72
Y	2,574	2,325	2,093	1,862	1,749	1,620

Variant 5

X	0,68	0,73	0,80	0,88	0,93	0,99
Y	0,808	0,894	1,029	1,209	1,340	1,523

Variant 6

X	0,11	0,15	0,21	0,29	0,35	0,40
Y	9,054	6,616	4,691	3,351	2,739	2,365

Variant 7

X	1,375	1,380	1,385	1,390	1,395	1,400
Y	5,041	5,177	5,320	5,470	5,629	5,797

Variant 8

X	0,115	0,120	0,125	0,130	0,135	0,140
Y	8,657	8,293	7,958	7,648	7,362	7,096

Variant 9

X	0,150	0,155	0,160	0,165	0,170	0,175
Y	6,616	6,399	6,196	6,005	5,825	5,655

Variant 10

X	0,180	0,185	0,190	0,195	0,200	0,205
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 11

X	0,210	0,215	0,220	0,225	0,230	0,235
Y	4,831	4,722	4,618	4,519	4,424	4,333

Variant 12

X	1,415	1,420	1,425	0,430	0,435	0,440
Y	0,888	0,889	0,890	0,891	0,892	0,893

Variant 13

X	0,33	0,38	0,45	0,52	0,60	0,65
Y	1,63597	1,73234	1,87686	2,03345	2,22846	2,35973

Variant 14

X	0,03	0,09	0,13	0,18	0,24	0,31
Y	1,02316	1,09590	1,14725	1,21483	1,30120	1,40976

Variant 15

X	0,25	0,31	0,37	0,41	0,46	0,54
Y	2,739	2,300	1,968	1,787	1,595	1,345

Variant 16

X	0,21	0,26	0,32	0,40	0,45	0,52
Y	2,574	2,325	2,093	1,862	1,749	1,620

Variant 17

X	0,38	0,43	0,50	0,58	0,63	0,69
Y	0,808	0,894	1,029	1,209	1,340	1,523

Variant 18

X	0,31	0,35	0,41	0,49	0,55	0,60
Y	9,054	6,616	4,691	3,351	2,739	2,365

Variant 19

X	1,175	1,180	1,185	1,190	1,195	1,200
Y	5,041	5,177	5,320	5,470	5,629	5,797

Variant 20

X	0,215	0,220	0,225	0,230	0,235	0,240
Y	8,657	8,293	7,958	7,648	7,362	7,096

Variant 21

X	0,250	0,255	0,260	0,265	0,270	0,275
Y	6,616	6,399	6,196	6,005	5,825	5,655

Variant 22

X	0,280	0,285	0,290	0,295	0,300	0,305
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 23

X	0,310	0,315	0,320	0,325	0,330	0,335
Y	4,831	4,722	4,618	4,519	4,424	4,333

Variant 24

X	1,315	1,320	1,325	0,330	0,335	0,340
Y	0,888	0,889	0,890	0,891	0,892	0,893

Variant 25

X	0,315	0,320	0,325	0,330	0,335	0,340
Y	8,657	8,293	7,958	7,648	7,362	7,096

Variant 26

X	0,450	0,455	0,460	0,465	0,470	0,475
Y	6,616	6,399	6,196	6,005	5,825	5,655

Variant 27

X	0,580	0,585	0,590	0,595	0,600	0,605
Y	5,615	5,466	5,326	5,193	5,066	4,946

Variant 28

X	0,410	0,415	0,420	0,425	0,430	0,435
Y	4,831	4,722	4,618	4,519	4,424	4,333

Variant 29

X	0,315	0,320	0,325	0,330	0,335	0,340
Y	0,888	0,889	0,890	0,891	0,892	0,893

Variant 30

X	2,315	2,320	2,325	2,330	2,335	2,340
Y	0,888	0,889	0,890	0,891	0,892	0,893

4-LABORATORIYA ISHI

Aniq integralni taqribiy hisoblash

Maqsad: Aniq integralni taqribiy hisoblash usullarini o'rganish

Reja:

4.**1. To'g'ri to'rtburchaklar formulasi.**

4.2. Trapetsiyalar formulasi.

4.3. Simpson yoki parabola formulasi.

Integrallanuvchi f(x) funktsiyaning boshlang'ichini mahlum funktsiyalar orqali ifodalash mumkin bo'lmaganda, f(x) funktsiya jadval yoki grafik usulda berilganda integrallni taqribiy hisoblashga to'gri keladi.

Aytaylik [*a*,*b*] oraliqda f(x) funktsiya grafigi yordamida x=*a*, x=*b* xamda y=0(Ox) chiziqlar bilan chegaralangan yuzani hisoblash kerak bo'lsin.

Quyida bunday yuzalarni taqribiy hisoblash usullarini ko'ramiz.

4.1. To'g'ri to'rtburchaklar formulasi

Berilgan [*a;b*] oraliqda qadami h=(b-a)/n bo'linish nuqtalari.

$$x_0=a,\ x_i=x_{i-1}+h,\ i=1, 2, 3, 4, 5, \ldots n$$

bo'lganda f(x) funktsiya bo'yicha olingan aniq integralni taqribiy hisoblash formulasi quyidagicha bo'ladi.

$$\int_{в}^{a} f(x)dx = h\sum_{i=1}^{n} f(x_1) = h(y_1 + y_2 + \ldots . y_n)$$

4.2. Trapetsiyalar formulasi

Berilgan [a;v] oraliqda qadami $h = \frac{b-a}{n}$ va bo'linish nuqtalari.

$$x_0=0,\ x_k=x_{k-1}+ h,\ k=1, 2, \ldots n$$

bo'lganda aniq integralni hisoblash formulasi quyidagicha bo'ladi.

$$\int_{a}^{в} f(x)dx = h(\frac{f(a)+f(b)}{2} + \sum_{r=1}^{n-1} f(x_k))$$

4.3. Simpson yoki parabola formulasi

Berilgan [*a*,*b*] oraliqda qadami $h = \frac{b-a}{n}$ va bo'linish nuqtalari.

$x_0=0,\ x_k=x_{k-1}+h,\ \ k=1, 2, \ldots .n$

bo'lganda aniq integralni taqribiy hisoblash formulasi quyidagicha bo'ladi.

$$\int_{a}^{b} f(x)dx = \frac{h}{3}[f(a)+f(b)+4\sum_{k=1}^{n} f(x_{2x-1})+2\sum_{k=1}^{n-1} f(x_{2k})]$$

4.1-masala.

Quyidagi $\int_{2}^{3,5} \frac{dx}{\sqrt{5+4x-x^2}}$

aniq integralni:

1) To'gri to'rtburchak formulasi bilan.
2) Trapetsiya formulasi bilan.
3) Simpson formulasi bilan.

Bo'linishlar soni 10 bo'lganda Ye=0,001 aniqlikda hisoblang.

Echish.

Integral ostidagi funktsiya qiymatlarini bo'linish qadami

$h=(b-a)/n=(3,5-2)/10=0,15$

bo'lganda, bo'linish nuqtalari $x_i=a+ i\ h,\ i=1,10$

bo'lsa, nuqtalarni [2;3,5] oraliqda aniqlab, bu nuqtalarda integral ostidagi funktsiya qiymatlarini topamiz.

$x_0=2,00$ y_0 $y_0 = f(2) = \frac{1}{\sqrt{5+42-3^2}} = 0,3333$

$x_1=2,15$ $y_1 = f(2,5) = \frac{1}{\sqrt{5+4(2,5)-(2,5)^2}} = 0,3388$

$x_2=2,30$ $y_2=f(2,30)=0,3350$ $x_3=2,45$ $y_3=f(2,45)=0,3371$

$x_4=2,60$ $y_4=f(2,60)=0,3402$ $x_5=2,75$ $y_5=f(2,75)=0,3443$

$x_6=2,90$ $y_6=f(2,90)=0,3494$ $x_7=3,05$ $y_7=f(3,05)=0,3558$

$x_8=3,20$ $y_8=f(3,20)=0,3637$ $x_9=3,35$ $y_9=f(3,35)=0,3733$

$x_{10}=3,50$ $y_{10}=f(3,50)=0,3849$

Yukoridagi x va u qiymatlarga asosan to'g'ri **to'rtburchak** formulasiga asosan.

$\int_{2}^{3,5} \frac{dx}{\sqrt{5+4x-x^2}}$ =0,15(0,3333+0,3388+0,3350+0,3371+0,3402+0,3443+0,3494+ +0,3858+0,3637+0,3733+0,3849)=0,5755

Trapetsiya formulasiga asosan.

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}} = 0{,}15\left(\frac{0{,}3333+0{,}3849}{2}+0{,}3388+0{,}3350+0{,}3371+0{,}3402+\right.$$

$$\left.+0{,}3443+0{,}3494+0{,}3858+0{,}3637+0{,}3733\right)=0{,}15*3{,}49178=0{,}52376$$

Simpon formulasiga asosan:

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}} \approx \frac{0{,}15}{3}[0{,}3333\text{-}0{,}3849+4(0{,}3338+0{,}3371+0{,}3443+0{,}3558+$$

$$+0{,}3733)+2(0{,}3350+0{,}3402+0{,}3494+0{,}3637)]=0{,}05(1{,}0515+4*1{,}7443+$$

$$+2*1{,}3883)=0{,}05*10{,}8013=0{,}54265.$$

4.1-Maple dasturi

Berilgan integralni :

$$\int_2^{3,5}\frac{dx}{\sqrt{5+4x-x^2}}$$

1. Boshlang'ich funktsiyasini topish:

```
> Int( 1/sqrt(5+4*x-x^2), x)=int( 1/sqrt(5+4*x-x^2), x );
```

$$\int \frac{1}{\sqrt{5+4x-x^2}}\,dx = \arcsin\left(-\frac{2}{3}+\frac{1}{3}x\right)$$

2. 10 xona aniqlikda taqribiy hisoblash.

```
> I1 := 1/sqrt(5+4*x-x^2):
Int(I1, x = 2..3.5) = evalf(Int(e1, x=2..3.5, digits=10,
method=_Dex));
```

$$\int_2^{3.5}\frac{1}{\sqrt{5+4x-x^2}}\,dx = 0.5235987756$$

```
> evalf(Int( 1/sqrt(5+4*x-x^2), x=2..3.5 ));
```

0.5235987756

```
>  evalf[25](Int( 1/sqrt(5+4*x-x^2), x=2..3.5 ));
```

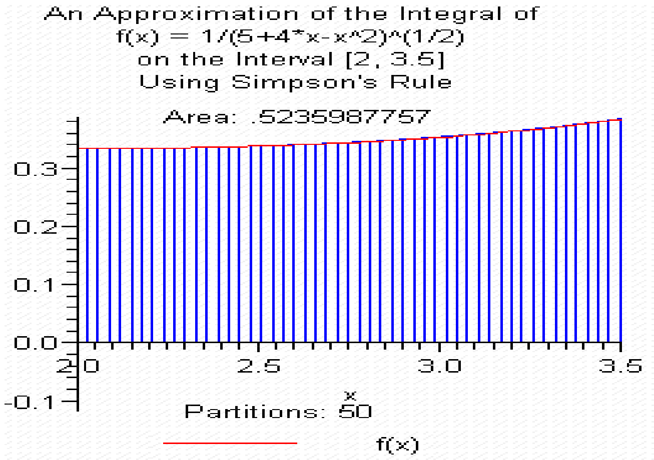
0.5235987755982988730771072

3. Trapetsiya va Simpson usulida hisoblash yuza grafigini qurish.

```
>  with(Student[Calculus1]):
AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = trapezoid);
AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = trapezoid,
output =plot);
AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = trapezoid,
output = plot, artition = 50);
AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = trapezoid,
output = animation);
```

0.5237590264

```
> with(Student[Calculus1]):
> AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = simpson,
output = plot);
> AroximateInt(1/sqrt(5+4*x -x^2), 2..3.5,method = simpson,
output = plot, artition = 50);
```



4.1-rasm

1.Aniq integralni to‘g’ri to‘rtburchak usulida xisoblash dasturlari:

```
8 ‘------4.1- BYEYSIK TILI  DASTUR--------
10 REM TURTBURCHAK USULI.
20 DEF FNF (X)=1/SQR (5+4*X-X*X)
22 PRINT: PRINT
22 PRINT “Anik integralni to‘gri to‘rtburchak usulida”
24 PRINT “                    taqribiy hisoblash.”
28 REM Aniq integral chegaralari a,b va aniqlikni kiritish
30 READ A, B, EPS
50 N=10
60 H=(B-A)/N
70 S=0
80 FOR I=1 TO N
90 X=A+H*I
100 S=S+FNF (X)
110 NEXT I
120 S=S*H
122 REM Aniq integral qiymatini baholash
130 IF N><10 THEN 150
140 N=N+10:Z=S:GOTO 60
150 IF ABS (S-Z)>EPS THEN 140
160 N=N-10:H=(B-A)/N
170 FOR I=0 TO N
180 X=A+H*I
190 PRINT “X (“;USING “###.####”;I
200 PRINT “)=“;USING “###.####”;X;
210 PRINT “ F(“;USING “###.####”;I;
220 PRINT “)=”;USING “###.####”;FNF(X)
230 NEXT I
240 PRINT
250 PRINT “Integralning kiymati=”;USING “##.####”;S
252 REM Aniq integral chegaralari a,b va aniqlik qiymatlari
260 DATA 2, 3.5, 0.01
270 END
```

Aniq integralni to‘gri to‘rtburchak usulida
taqribiy hisoblash.

X(0)=2,000	F(0)=0,3333
X(1)= 2,075	F(1)= 0,3334
X(2)= 2,150	F(2)= 0,3338
X(3)= 2,225	F(3)= 0,3343
X(4)= 2,300	F(4)= 0,3350
X(5)= 2,375	F(5)= 0,3360
X(6)= 2,450	F(6)= 0,3371
X(7)= 2,525	F(7)= 0,3386

X(1)= 2,600	F(1)= 0,3402
X(9)= 2,675	F(9)= 0,3421
X(10)= 2,750	F(10)=0,3443

integralning kiymati=0.3249

2. Aniq integralni trapetsiya usulida hisoblash dasturlari:

```
1 '-------------  4.2 - DASTUR --------------------
2 REM SAVE"a:inttr'1",a
5 DEF FNF(X)=1/sqr(5+4*X- X^2)
6 DIM X(80)
8 PRINT"Berilganlarni kiriting:"
10 INPUT "Integral chegaralari a,b=";A,B
12 INPUT "Bolinishlar soni N=";N
14 INPUT "Aniqlik qiymati E=";E
20 H=(B-A)/N
25 X(0)=A : S=(FNF(A)+FNF(B))/2
30 FOR I=1 TO N-1
40 X(I)=X(I-1)+H
80 S=S+FNF(X(I))
90 NEXT I
92 S=S*H
94 REM Aniq integral qiymatini baholash
95 IF ABS(S-S1)<E THEN 120
100 N=2*N:S1=S:GOTO 20
120 PRINT "S=";S
130 END
```

```
Berilganlarni kiriting:
Integral chegaralari a,b=? 2,3.5
Bolinishlar soni N=? 10
Aniqlik qiymati E=? 0.001
S= .523639
```

3. Aniq integralni Simpson usulida hisoblash dasturlari:

```
2 '------------  4.3 - DASTUR -------------------

3 PRINT "Aniq integralni Simpon usulida yechish"
5 REM SAVE"SIM51",A
30 READ  A,B,N,E
40 DEF FNF(X)=1/SQR(5+4*X-X^2)
70 X=A : H=(B-A)/N : C=1
90 FOR I=1 TO N-1
110 X=X+H : S=S+(C+3)*FNF(X)
112 C=-C
120 NEXT I
122 S=S*H/3
180 IF ABS(S-S1)<E THEN 210
190 N=2*N : S1=S : GOTO 70
210 PRINT "Integralni qiymati S=";S
220 DATA 2,3.5,10,0.01
230 END
        RUN
        Aniq integralni Sim'on usulida yechish
        Integralni qiymati S= .5210937
```

4-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi integrallarni:

1) to'g'ri to'rtburchak; 2)trapetsiya; 3) Simpson usulida hisoblang.

1. $\int_1^{3,5} \frac{\ln x}{x\sqrt{1+\ln x}}dx$ **2 .** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(tg^2 x + ctg^2 x)dx$

3. $\int_1^4 \frac{1}{x}\ln^2 xdx$ **4.** $\int_2^3 \frac{1}{x\lg x}dx$

5. $\int_0^{\ln 2}\sqrt{e^x - 1}dx$ **6.** $\int_0^1 xe^x \sin xdx$

7. $\int_0^2 \frac{1}{\sqrt{9+x^3}}dx$ **8.** $\int_1^{2.5}\frac{1}{x^2}\sin\frac{1}{x}dx$

9. $\int_0^{\sqrt{3}} xarctgxdx$ **10.** $\int_0^3 \arcsin\sqrt{\frac{x}{1+x}}dx$

11. $\int_1^3 x^x(1+\ln x)dx$ **12.** $\int_0^1 \frac{dx}{\sqrt{1+3x+2x^2}}$

13. $\int_1^2 \frac{1}{x}\sqrt{x^2+0.16}dx$ **14.** $\int_0^1 \frac{xarctgx}{\sqrt{1+x^2}}dx$

15. $\int_0^2 \frac{e^{3x}+1}{e^x+1}dx$ **16.** $\int_0^{1.99} x^2\sqrt{4-x^2}dx$

17. $\int_0^{\pi} e^x \cos^{-2} xdx$ **18.** $\int_1^e (x\ln x)^2 dx$

19. $\int_{-1}^{2} \arccos\sqrt{\frac{x}{1+x}}dx$ **20.** $\int_{0}^{1} \frac{(x^2+4)dx}{(x^2+1)\sqrt{x^4+1}}$

21. $\int_{1}^{1.5} \sin x \ln(tgx)dx$ **22.** $\int_{0}^{1.5} \frac{e^x(1+\sin x)}{1+\cos x}dx$

23. $\int_{0}^{3/4} (x+1)/\sqrt{x^2+1}dx$ **24.** $\int_{0}^{1} \frac{dx}{(3\sin x+2\cos x)^2}$

25. $\int_{1}^{2} (\frac{\ln x}{x})^3 dx$ **26.** $\int_{1}^{2} \frac{x^3}{\sqrt{x}+3}dx$

27. $\int_{1}^{2} \frac{x}{x^4+3x^2+2}dx$ **28.** $\int_{0}^{\pi/2} \sqrt{1-\frac{1}{4}\sin 2x}dx$

29. $\int_{0}^{\pi/2} \sqrt{2+\cos x}dx$ **30.** $\int_{\ln 2}^{\ln 3} \frac{e^{2x}}{e^x-e^{-x}}dx$

5-LABORATORIYA ISHI

5.1.Birinchi tartibli oddiy differentsial tenglama uchun Koshi masalasini taqribiy yechish.
5.2. Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini Eyler usulida taqribiy yechish

Maqsad: Birinchi tartibli oddiy differentsial tenglama uchun Koshi masalasini taqribiy yechish usullarini o'rganish.

Reja:

5.1. Eyler usuli
5.2. Birinchi tartibli differentsial tneglamalar sistemasi Eyler usulida yechish

5.1. Eyler usuli

Aytaylik bizga birinchi tartibli

$$\frac{dy}{dx} = f(x;y) \quad \text{yoki} \quad y_0 = f(x;y) \qquad (5.1)$$

differentsial tenglama berilgan bo'lib, [x;v] kesmada

$$x=x_0,\ y=y_0 \qquad (5.2)$$

boshlang'ich shartni kanoatlantiruvchi yechimni taqribiy hisoblash masalasi quyilgan bo'lsin. Bu masala Koshi masalasi deyiladi. Bu masalani taqribiy yechishning bir necha usullarni ko'ramiz.

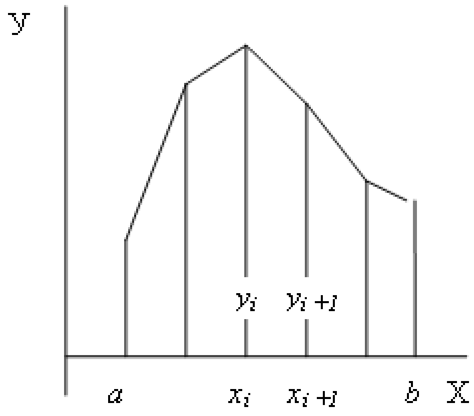
Berilgan $[x_0;b]$ kesmani n ta teng bo'lakka bo'lib bo'linish nuqtalari orasidagi qadam

$$h=(b-x_0)/n \qquad (5.3)$$

bo'lganda, bu nuqtalar koordinatalari

$$x_i=x_{i-1}+h, \quad i=1, 2, \ldots n \qquad (5.4)$$

bo'ladi. x_0 va y_0 lardan foydalanib tenglama yechimining qiymatlarini taqriban quyidagicha hisoblaymiz.



5.1-rasm

$$y_1=y_0+hf(x_0;y_0)$$
$$y_2=y_1+hf(x_1;y_1)$$
$$y_3=y_2+hf(x_2;y_2) \qquad (5.5)$$

$$y_n=y_{n-1}+hf(x_{n-1};y_{n-1})$$

natijada izlanaetgan yechimni kanoatlantiruvchi

$$(x_0;y_0), (x_1;y_1), (x_2;y_2), \ldots\ldots, (x_n;y_n)$$

nuqtalarni aniqlaymiz. Bu nuqtalarni tutashtiruvchi sinik chiziq Eyler chiziqi(5.1-rasm) deb ataladi va u tenglama yechimining taqribiy grafigini ifodalydi.

5.1-masala. Quyidagi.

$$y^1 = x + \cos\left(\frac{y}{\sqrt{5}}\right)$$

birinchi tartibli defferitsial tenglamaning

$$x_0=1,3 \qquad y_0=2,6$$

boshlang'ich shartni kanoatlantiruvchi [1,8;2,8] oraliqda yechimini h=0,1 qadami bilan, e=0,001 aniqlikda hisoblang.

Eyler usuli

Echish:

1. Berilgan differentsial tenglamani Eyler usulida yechamiz, [1,8;2,8] oraliqda

$$n=\frac{b-a}{h}=\frac{2.8-1.8}{0.1}=10$$

bo'lganidan oraliqni n=10 ta bo'lakka ajratamiz. Bo'linish nuqtalarini:

$$x_i=x_{i-1}+h \qquad i=1,2,\ldots,10$$

formulaga asosan topamiz.

$$x_1=x_0+h=1.8+0.1=1.9$$
$$x_2=x_1+h=1.9+0.1=2.0$$

shuningdek

$x_3=2.1$, $x_4=2.2$, $x_5=2.3$, $x_6=2.4$, $x_7=2.5$, $x_8=2.6$, $x_9=2.7$, $x_{10}=2.8$

Berilgan tenglamaning ung tomonidagi

$$F(x;y)=x+\cos(y/\sqrt{5})$$

funktsiyaga asosan, Eyler qoidasi bilan quyidagi

$$y_{i+1}=y_i+h\,f(x_i;y_i), \qquad i=1,2,\ldots,10$$

formulaga asosan berilgan differitsial tenglama yechimining qiymatlarini quyidagicha topamiz.

$$y_1=y_0+hf(x_0;y_0)=y_0+h(x_0+\cos(y_0/\sqrt{5}))=$$

$=2.6+0.1(1.8+\cos(2.6/\sqrt{5}))=2.6+0.1(1.8+0.3968)=2.81968$

$y_2=y_1+h\ f(x_1;y_1)=y_1+h(x_1+\cos(y_1/\sqrt{5}))=$

$=2.819+0.1(1.9+\cos(2.819/\sqrt{5}))=2.819+0.1(1.9+0.3968)=3.03948$

SHuningdek, quyidagilarni topamiz:

$y_3=3.261,\ y_4=3.4831,\ y_5=3.7045,\ y_6=3.926$

$y_7=4.1478,\ y_8=4.3701,\ y_9=4.5931,\ y_{10}=4.8173$

5.1-Maple dasturi

5.1-masalani Eyler usulida yechish.

```
> dsol1 := diff(y(x),x) = cos(y(x)/sqrt(5)) + x;
```

$$dsol1 := \frac{d}{dx}\, y(x) = \cos\left(\frac{1}{5}\, y(x)\sqrt{5}\right) + x$$

```
> init1 := y(1.8)=2.6;
```

$$init1 := y(1.8) = 2.6$$

```
> Digits := 20:
ans2:=dsolve({dsol1,init1},numeric,method=classical[heunform],
output=array([1.9,2.0,2.1,2.2]), stepsize=0.001);
```

$$ans2 := \begin{bmatrix} [x, y(x)] \\ [1.9 \quad 2.8201058808] \\ [2.0 \quad 3.0408294458] \\ [2.1 \quad 3.2619000478] \end{bmatrix}$$

5.1- Byeysik tili dasturi

```
10 DEF FNE (X,Y)=X+COS(Y/SQR (5))
20  PRINT:  PRINT
30 PRINT “Birinchi tartibli differentsial tenglama ”
40 PRINT “                    Y1=F(X,Y) uchun”
50 PRINT “Koshi masalasini Eyler usulida”
60 PRINT “       taqribiy yechimini topish”
70 REM “boshlang’ich qiymat ,qadam”
72 REM “berilgan kesma yuqori chegarasi:”
80 READ X, Y, H, B
82 REM “boshlang’ich qiymat ,qadam”
84 REM “berilgan kesma yuqori chegarasi qiymatlari:”
88 DATA 1.8, 2.6, 0.1, 2.8
90 N=(B-X)/H
100 FOR I=1 TO N
110 Y=Y+H*FNE(X,Y)
120 X=X+H
130 PRINT “X (“;USING “###.###”;1:
140 PRINT “)=”;USING “###.###”;X;
150 PRINT “ F(“;USING “###.###”;I;
160 PRINT “)=”;USING “###.###”;Y
170 NEXT I
180  END
RUN
```

Birinchi tartibli differentsial tenglama

Y^1=F(X,Y) uchun

Koshi masalasini Eyler usulida

taqribiy yechimini topish.

X(1)= 1.900	Y(1)= 2.8197
X(2)= 2.000	Y(2)= 3.0402
X(3)= 2.100	Y(3)= 3.2611
X(4)= 2.200	Y(4)= 3.4823
X(5)= 2.300	Y(5)= 3.7037
X(6)= 2.400	Y(6)= 3.9251
X(7)= 2.500	Y(7)= 4.1468
X(1)= 2.600	Y(1)= 4.3688
X(9)= 2.700	Y(9)= 4.5914
X(10)= 2.800	Y(10)= 4.8150

5-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

Quyidagi birinchi tartibli differietsial tenglamalar uchun Koshi masalasinining taqribiy yechimini h=0.1 qadam bilan Eyler usulida toping.

1)	$y'=x/(x+y)$	$y(0)=1$	[0,1]
2)	$y'-2y=3e^x$	$y(0,3)=1,415$	[0,1;0,5]
3)	$y'=x+y^2$	$y(0)=0$	[0;0,3]
4)	$y'=y^2-x^2$	$y(1)=1$	[1;2]
5)	$y'=x^2+y^2$	$y(0)=0.27$	[0;1]
6)	$y'+xy(1-y^2)=0$	$y(0)=0.5$	[0;1]
7)	$y'=x^2-xy+y^2$	$y(0)=0.1$	[0;1]
8)	$y'=(2y-x)/y$	$y(1)=2$	[1;2]
9)	$y'=x^2+xy+y^2+1$	$y(0)=0$	[0;1]
10)	$y'+y=x^3$	$y(1)=-1$	[1;2]
11)	$y'=xy+e^y$	$y(0)=0$	[0;0.1]
12)	$y'=2xy+x^2$	$y(0)=0$	[0;0.5]
13)	$y'=x+\sin\frac{y}{3}$	$y(0)=1$	[0;1]
14)	$y'=e^x-y^2$	$y(0)=0$	[0;0.4]
15)	$y'=2x+\cos y$	$y(0)=0$	[0;0.1]
16)	$y'=x^3+y^2$	$y(0)=0.5$	[0;0.5]
17)	$y'=xy^3-y$	$y(0)=1$	[0;1]
18)	$y'=y^2e^x-2y$	$y(0)=1$	[0;1]
19)	$y'=\frac{1}{y^2-x}$	$y(1)=0$	[1;2]
20)	$y'=\frac{x^2+1}{e^x}$	$y(1)=1$	[1;2]
21)	$y'=e^x\cos y/x$	$y(1)=1$	[1;2]
22)	$y'=e^x\sin y/x$	$y(1)=1$	[1;2]
23)	$y'\cos x-y\sin x=2x$	$y(0)=0$	[0;1]
24)	$y'=y\,tgx-\frac{1}{\cos^3 x}$	$y(0)=0$	[0;1]
25)	$y'+y\cos x=\cos x$	$y(0)=0$	[0;1]

26)	$y'=\frac{y}{x}+tg\frac{x}{y}$	y(0)=0	[0;1]
27)	$y'=(1+\frac{y-1}{2x})^2$	y(1)=1	[1;2]
28)	$xy'-\frac{y}{x+1}-x=0$	y(1)=1/2	[1;2]
29)	$y'=\frac{y}{x}(1+lny-lnx)$	y(1)=e	[1;2]
30)	$y^3xdx=(x^2y+2)dy$	y(0.348)=2	[0;1]

5.2. Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini Eyler usulida taqribiy yechish

Maqsad: Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini takribiy yechishda Eyler usuli va uni ikkinchi tartibli differentsial tneglamalarni yechishda qo'llash.

Reja:

1. Birinchi tartibli differentsial tneglamalar sistemasi uchun Koshi masalasini takribiy yechish.
2. Ikkinchi tartibli differentsial tneglamalarni yechish

Quyidagi

$$y'=f_1(x,y,z),\quad z'=f_2(x,y,z,)\qquad (5.6)$$

Birinchi tartibli diffirintsial tenglamalar sistemasini [a, v] oraliqdagi boshlang'ich

$$y(x_0)=y_0\quad,\quad z(x_0)=z_0\qquad (5.6)$$

SHartlarni kanoatlantiruvchi yechimni kiymatini topish uchun Eyler usulini qo'llaymiz.

Eyler usuli.

Berilgan differentsial tenglamalar sistemasining oraliqdagi yechimini topish uchun

Xi=X0+ih . i=0,1,2,…,n

larni topib, har bir tenglama uchun Eyler usulini ko'llaymiz.

Yi+1=Yi+hf1(Xi; Yi; Zi) (5.7)

Zi+1=Yi+hf2(Xi; Yi; Zi)

Natijada differentsial tenglamalar sistemasi yechimining taqribiy qiymatini topamiz.

Y(Xi)=Yi Z(Xi)=Zi i=1,2,3,4,5,…,n

Quyidagi 5.1-masalani yordamida berilgan ikkinchi tartibli differentsial tenglamani yechimini birinchi tartibli differentsial tenglamalar sistemasiga keltirib uchimni topish qulayligini ko'rsatamiz.

5.2-masala. Quyidagi

$$y''+y'/x+y=0$$

Differentsial tenglamani

y(1)=0.77, y′(1)=-0.44

boshlang'ich shartlarni qanoatlantiruvchi, qadami *h=01* **bo'lgan, [1;1.5] oralikdagi yechimi Eyler usuli bilan topilsin.**

Yechish.

Berilgan differentsial tenglamada

$$y'=z. \; y''=z'$$

almashtirish qilib, quyidagi birinchi tartibli differentsial tenglamalar sistemasiga kelamiz.

$$\begin{cases} y' = z \\ z' = -z/x - y \end{cases} \qquad (5.6)$$

boshlangich shartlari esa

$$Y(1)=077 \qquad Z(1)=-044$$

kabi yoziladi

Bu holda (5.6) differentsial tenglamalar sistemasiga asosan.

$$\begin{cases} f_1(x.y.z) = z = 0.x + 0.y + z \\ f_2(x.y.z) = -z/x - y \end{cases}$$

Endi hosil bulgan(*) differentsial tenglamalar sistemani Eyler va Runge- Kutta usuli bilan yechimini topamiz.

Eyler usulida yechishda (16.3) formulaga asosan

$$X_i=X_0+ih$$
$$Y_{i+1}=Y_i+hf1(X_i,Y_i,Z_i)$$
$$Z_{i+1}=Z_i+hf2(X_i,Y_i,Z_i)$$
$$I=0,1,2,3,...$$

$I=0 \;\; X_0=1.0, Y_0=0.77, Z_0=-0.44$

$$Y_1=Y_0+hf1(X_0,Y_0,Z_0)=0.77+0.1(Z_0)=0.726$$
$$Z_1=Z_0+hf_2(X_0,Y_0,Z_0)=-0.44+0.1(-Z_0/X_0-Y_0)=-0.473$$

$I=1 \; X_1=1.1, \; Y_1=0.726, \; Z_1=-0.473$

$$Y_2=Y_1+hf_1(X_1,Y_1,Z_1)=0.726+0.1(-0.473)=0.679$$
$$Z_2=Z_1+hf_2(X_1,Y_1,Z_1)=-0.473+0.1(-0.473/1.1-0.726)=-0.503$$

Bu koidani $I=2,3,4,5$ lar uchun ketma-ket takrorlab tenglamalar sistemasi yechimining

$$Y_3=0.6284, \; Y_4=0.5756, \; Y_5=0.5205$$

qiymatini xisoblab topamiz.

Differentsial tenglamalar sistemasiga qo'yilgan Koshi masalasi yechimini Eyler usuli bilan topish(5.2-masala).

5.2-Maple dasturi

```
> dsys1:={diff(y(x),x$1)=z(x), diff(z(x),x$1)=-z(x)/x-y(x),
         y(1) = 0.77, z(1) =-0.44};
```

$$dsys1 := \left\{ y(1) = 0.77, z(1) = -.44, \frac{d}{dx}\, y(x) = z(x), \frac{d}{dx}\, z(x) = -\frac{z(x)}{x} - y(x) \right\}$$

```
> dsol1:=dsolve(dsys1, numeric,output=listprocedure,range=1..2):
    dsol1y:=subs(dsol1,y(x)):dsol1z:=subs(dsol1,z(x)):
> dsol1y(1), dsol1z(1);
```

0.77, −.44

```
> dsol1y(1.1), dsol1z(1.1);
```

0.7244058886424936760 4−.471314281199129617

```
> dsol1y(1.2), dsol1z(1.2);
```

0.6758539649217229423 4−.499122324226440343

```
> dsol1y(1.3), dsol1z(1.3);
```

0.6247043854650458793 5−.523241777133150330

```
> dsol1y(1.4), dsol1z(1.4);
```

0.5713337128502281623 8−.543517968966493242

```
> dsol1y(1.5), dsol1z(1.5);
```

0.5161330236349786192 3−.559826882961881883

5.2-BYEYSIK TILI DASTURI

```
10 DEF FNF(X,Y,Z)=Z+0*X+0*Y
12 DEF FNA(X,Y,Z)=-Z/X-Y
20 REM "boshlang'ich qiymat,qadam, berilgan kesma yuqori chegarasi:"
22 READ X,Y,Z,H,B
24 REM "boshlang'ich qiymat ,qadam, berilgan kesma yuqori chegarasi qiymatlari:"
26 DATA 1, 0, 77, -0.44, 0.1, 1.5
40 PRINT: PRINT
50 PRINT TAB(5): "Birinchi tartibli differentsial tenglamalar sistemasi"
60 PRINT TAB(90): " Y'=F(X,Y,Z),  Z'=F1(X,Y,Z) UCHUN"
70 PRINT TAB(95): "Koshi masalasini Eyler usulida "
80 PRINT TAB(20): "taqribiy yechimini topish"
90 PRINT
100 N=(B-X)/H
110 FOR I=1 TO N
120 Y1=X+H*FNE(X,Y,Z)
130 Z1=Z+H*FNA(X,Y,Z)
140 Y=Y1: Z=Z1: X=X+H
150 PRINT "   X(";USING "##";I;
160 PRINT ")=";USING "##. ###";X;
170 PRINT "   Y(";USING "##";I;
180 PRINT ")=";USING "###. ####";Y;
190 PRINT "   Z(";USING "##";I;
210 PRINT ")=";USING "###. ####";Z;
220 NEXT I
240 END
```

Birinchi tartibli differentsial tenglamalar sistemasi
Y'=F(X, Y, Z) , Z'=F1(X, Y , Z) uchun

```
Koshi masalasini Eyler usulida
taqribiy yechimini topish
```

X(1)=1 .100	Y(1)=0 . 7260	Z(1)=-0. 4730
X(2)=1 .200	Y(2)=0 . 6787	Z(2)=-0. 5026
X(3)=1 .300	Y(3)=0 . 6284	Z(2)=-0. 5286
X(4)=1 .400	Y(4)=0 . 5756	Z(3)=-0. 5508

X(5)=1 .500 Y(5)=0 . 5205 Z(5)=-0. 5690

5-laboratoriya ishi bo'yicha mustaqil ishlash uchun topshiriqlar

1. Quyidagi birinchi tartibli differentsial tenglamalar sistemasi uchun Koshi masalasini Eyler usulida taqribiy yechimini toping.

1. $\begin{cases} y' = \cos(y+2z)+3, \\ z' = 2/(x+3x^2)+y+x, \end{cases}$ y(0)=1, z(0)=0.05

2. $\begin{cases} x' = \sin(2x^2)+t+y \\ y' = t+x-3y^2+1 \end{cases}$ x(0)=1 y(0)=0.5

3. $\begin{cases} x' = \sqrt{(t^2+2x^2)+y} \\ y' = \cos(3y+x), \end{cases}$ x0)=0.5 y(0)=1

4. $\begin{cases} x' = \ln(6t+\sqrt{2t^2+y^2}) \\ y' = (2t^2+x^2) \end{cases}$ x(0)=1, y(0)=0.5

5. $\begin{cases} x' = e^{-(x^2+y^2)}+0.15t \\ y' = 6x^2+y \end{cases}$ x(0)=0.5 y(0)=1

6. $\begin{cases} y' = z/x+\sqrt{(x+y)} \\ x' = 2z^2/(x(y-1))+z/x \end{cases}$ x(1)=1/3 y(1)=0

7. $\begin{cases} y' = (z-y)x \\ z' = (z+y)x, \end{cases}$ y(0)=1 z(0)=1

8. $\begin{cases} y' = \cos(y+2z)+2 \\ z' = 2/(x+2y^2)+x+1 \end{cases}$ y(0)=1 z(0)=1

9. $\begin{cases} y' = e^{-(y^2+z^2)}+2x \\ z' = 2y^2+z, \end{cases}$ y(0)=0.5 z(0)1

10. $\begin{cases} y' = y + 2z - \sin z^2 \\ z' = -y - 3z + x(e^{(x^2/2)} - 1) \end{cases}$ y(0)=0 z(0)=0

11. $\begin{cases} y' = -z + xy \\ z' = z^2/y \end{cases}$ y(1)=1 z(1)= -0.5

12. $\begin{cases} y' = (z-1)/z \\ z' = 1/(y-x), \end{cases}$ y(0)=-1 z(0)=1

13. $\begin{cases} y' = 2xy/(x^2 - y^2 - z^2) \\ `z' = 2xz/(x^2 - y^2 - z^2), \end{cases}$ y(1)=2 z(1)=1

14. $\begin{cases} y' = z/(z-y)^2 \\ z' = y/(z-y)^2 \end{cases}$ y(0)=1 z(0)=2

15. $\begin{cases} y' = -y/x + xz \\ z' = -2y/x^3 + z/x \end{cases}$ y(1)=1 z(1)=2

16. $\begin{cases} dx/dt = x - 2y \\ dy/dt = x - y, \end{cases}$ x(0)=1 z(0)=1

17. $\begin{cases} dy/dx = z - y \\ dz/dx = -y - z \end{cases}$ y(0)=2.23 z(0)=1.05

18. $\begin{cases} dy/dx = 1 - 1/z \\ dz/dx = 1/(y-x) \end{cases}$ y(0)=2.12 z(0)=1,13

19. $\begin{cases} dy/dx = x/yz \\ dz/dx = x/y^2 \end{cases}$ y(0)=1 z(0)=2

20. $\begin{cases} dy/dx = -z \\ dz/dx + 4y = 0, \end{cases}$ y(0)=1.2 z(0)=-2

21. $\begin{cases} y' = z/x \\ z' = 2z^2/(x(y-1) + z/x) \end{cases}$ u(1)=0 z(1)=1/3

22. $\begin{cases} y' = (z-y)x \\ z' = (z+y)x \end{cases}$ y(0)=1 z(0)=1

23. $\begin{cases} y' = \cos(y + 2z) + 2 \\ z' = 2/(x + 2y^2) + x + 1 \end{cases}$ y(0)=1 z(0)=005

24. $\begin{cases} y' = e^{-(y^2+z^2)} + 2x \\ z' = 2y^2 + z \end{cases}$ y(0)=05 z(0)=1

25. $$\begin{cases} y'=(z-y)y \\ z'=(z+y)z \end{cases}$$ $y(0)=1.05$ $z(0)=2$

2.Quydagi ikkinchi tartibli differentsial tenglamalar uchun Koshi masalasining yechimini topishda, ikkinchi tartibli differentsial tenglamani birinchi tartibli differentsial tenglamalar sistemasiga keltirib Eyler usulida taqribiy yechimini toping.

1. $y''=1/cosx-y;$	$y(0)=1;$	$yo(0)=0,$	[0;0.5],	$h=0.1$
2. $(1+x{\wedge}2)y''+(yo)^2+1=0,$	$y(0)=1,$	$yo(0)=1,$	[0; 0.5],	$h=0.05$
3. $y''+2yo+2y=2e^{-x}cosx,$	$y(0)=1,$	$yo(0)=0,$	[0; 0.5],	$h=0.05$
4. $y''+4y=e^{3x}(13x-7),$	$y(0)=0,$	$yo(0)=-1,$	[0.1],	$h=01$
5. $y''+4yo+4y=0,$	$y(0)=1,$	$yo(0)=-1,$	[0,1],	$h=01$
6. $y''-y=sinx+cos3x,$	$y(0)=1.8,$	$yo(0)=-05,$	[0;2],	$h=02$
7. $y''-3yo=e^{5x},$	$y(0)=2.2,$	$yo(0)=0.8,$	[0;02],	$h=002$
8. $y''+y=cosx,$	$y(0)=08,$	$yo(0)=2,$	[0;1],	$h=08$
9. $y''-yo-6y=2e^{4x},$	$y(0)=1.433,$	$yo(0)=-0.367,$	[0;1],	$h=01$
10. $y-2yo+y=5xe^{x},$	$y(0)=1,$	$yo(0)=2,$	[0;1],	$h=0.1$
11. $y''+yo-6y=3x^{2-x}-x,$	$y(0)=-0.9,$	$yo(0)=3.2$	[0;1],	$h=0.1$
12. $8yoo+2yo-3y=x+5,$	$y(0)=1/9,$	$yo(0)=-7/12,$	[0;1],	$h=0.1$
13. $y''-4yo+5y=3x,$	$y(0)=1.48,$	$yo(0)=3.6,$	[0;0.5],	$h=005$
14. $y''-5yo+6y=e^{x},$	$y(0)=0,$	$yo(0)=0,$	[0;0.2],	$h=0.02$
15. $y''-3yo+2y=x^2+3x,$	$y(0)=5.1,$	$yo(0)=4.2,$	[0;1],	$h=01$
16. $y''+(1/x)yo-(1/x)y=8x,$	$y(1)=4,$	$yo(1)=4,$	[1, 1.5],	$h=005$
17. $x^2y''+xyo=0,$	$y(1)=5,$	$yo(1)=-1,$	[1; 1.5],	$h=005$
18. $y''-2yo+y=xe^{x},$	$y(0)=1,$	$yo(0)=2,$	[0;05],	$h=005$
19. $y''-3yo+2y=2sinx,$	$y(0)=2,$	$yo(0)=3.2,$	[0;1],	$h=0.1$
20. $x{\wedge}2y''+2.5yox-y=0,$	$y(0)=2,$	$yo(1)=3.5,$	[0;1],	$h=0.1$
21. $4xy''+2yo+y=0,$	$y(1)=1.3817,$	$yo(1)=-0.1505,$	[1;2],	$h=0.1$
22. $x^2y''-4xyo+6y=2,$	$y(1)=1.43,$	$yo(1)=2.3,$	[1,2],	$h=0.1$
23. $y''-y=e^{2x}(x-1),$	$y(0)=11/9,$	$yo(0)=-11/9,$	[0;1],	$h=01$
24. $y''-3yo-2y=cos2x,$	$y(0)=1.95,$	$y(0)=2.7,$	[0;05],	$h=005$
25. $y''-05yo-05y=3e^{x/2},$	$y(0)=-4,$	$yo(0)=-2.5,$	[0;1],	$h=01$
26. $y''+4yo=sinx+sin2x,$	$y(0)=1,$	$yo(0)=-23/12,$	[0;1],	$h=01$
27. $y''+y=x^2-x+2,$	$y(0)=1,$	$yo(0)=0,$	[0;1],	$h=0.1$
28. $x^2y''-2y=0,$	$y(1)=5/6,$	$yo(1)=2/3,$	[1;2],	$h=01$
29. $y''+4yo+4y=2x-3,$	$y(0)=-1/4,$	$yo(0)=-1/2,$	[0;05],	$h=005$
30. $y''+y=x{\wedge}^{2}-x+2,$	$y(0)=1,$	$yo(0)=0,$	[0;1],	$h=01$

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Mundarija