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OBIDOVA GULNOZANING

"PARABOLIK TIPLI TENGLAMALARNI MAPLE PAKETI YORDAMIDA YECHISH"

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BITIRUV MALAKAVIY ISH

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MUNDARIJA

Kirish

1. Masalaning qo’yilishi: Hususiy hosilali differensial tenglamalarni har xil sinfdagi masalalarni yechish uchun, aynan formula bo’yicha hisoblash, uning umumiy yechimini topish amaliy jihatdan juda muhim masala hisoblanadi.

Bitiruv malakaviy ishida Maple paketi orqali *sterjenda issiqlik o’tkazuvchanlik tenglamasini va yarim to’g’ri chiziqda issiqlik o’tkazuvchanlik tenglamalarini Fur’ye usuli ya’ni o’zgaruvchilarni ajratish usuli yordamida yechishni* analitik, sonli usularini qo’llash hamda natijalarni grafik tarzda ifodalash vositalarini qo’llash borasida amalga oshiriladigan vazifalarni namoyish qilishdan iborat.

2. Mavzuning dolzarbligi: Turmush hayotimizda muhim ahamiyatga ega bo’lgan issiqlikning to’g’ri chiziq, tekislik va fazoda tarqalish jarayoni, shuningdek, diffuziya hodisasi parabolik tipli tenglamalar orqali o’rganiladi. Bu tenglamalar uchun ham to’lqin tenglamasi kabi chegaraviy va Koshi masalalari tenglama yechimini bir qiymatli ajratib olishga imkon yaratadi va ular belgilangan rejimga asosan tanlab olinadi.

Biz bu malakaviy bitiruv ishida chekli uzunlikdagi sterjenda qo’yilgan aralash masalalarning limitik holi sifatida aniqlangan chegaralanmagan uzunlikdagi sterjenda issiqlik tarqalish tenglamasiga qo’yilgan Koshi masalasining yechimi xuddi giperbolik tenglamalar uchun chegaraviy masalalarni yechishda qo’llanilgan o’zgaruvchilarni almashtirishyoki Fur’e usuli yordamida topilib, yechim Puasson integrali deb ataluvchi integral shaklda tasvirlanishini o’rgandik.

Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib, sterjenda issiqlik o’tkazuvchanlik tenglamasini va yarim to’g’ri chiziqda issiqlik o’tkazuvchanlik tenglamalarni Fur’ye usuli ya’ni o’zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali parabolik tipdagi tenglamalarni yechish jarayoni qoidaga mos ta’lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo’llanilishi ketma-ket tarzda keltirilgan, ya’ni parabolik tipdagi tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o'lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki o'lchovli grafigi tasvirlangan.

3. **Ishning maqsadi va vazifalari:** Bitiruv malakaviy ishining maqsadi *sterjenda issiqlik o'tkazuvchanlik tenglamasini va yarim to'g'ri chiziqda issiqlik o'tkazuvchanlik tenglamalarini Fur'ye usuli ya'ni o'zgaruvchilarni ajratish usuli yordamida yechishning* nazariy asoslarini muhim jixatlarini aniqlash, tanlangan parabolik tipdagi tenglamalarni yechishning maple tizimidagi vositalarini aniqlash, yechimning ikki o'lchovli animasiyali grafigi tasvirlash, bundan tashqari ba'zi misollar uchun bir qancha vaqt momentlarini ikki o'lchovli grafigi tasvirlash va amaliy jixatdan qo`llash uslublarini ko`rsatishdan iborat.

4. **Ilmiy tadqiqot usullari:** Bitiruv malakaviy ishining maqsad va vazifalarini bajarish maqsadida "Matematik fizika tenglamalari", "Differensial tenglamalar", "Kompyuter algebrasi tizimlari" fanlarining tadqiqot usullaridan foydalanildi.

5. **Ishning ilmiy ahamiyati:** Bu ish ilmiy tadqiqotlarni bajarish uchun Maple paketidan foydalanishga oid fundamental va amaliy ko'rsatmalar berilgan. Ularning barchasi matematika mutaxassislariga, shuningdek turli darajadagi ta'lim tizimiga ham tegishli.

6. **Ishning amaliy ahamiyati:** Bitiruv malakaviy ishdagi ma'lumotlar matematik xarakterdagi masalalarni aniq Maple tizimida ifodalashda keng doiradagi mutaxassislarga, magistrlar, talabalarga foydalanish uchun qo'shimcha uslubiy qo'llanma sifatida xizmat qilishi mumkin. Ma'lumotlarni ifodalashda yetarlicha qiziqarli misollar keltirilgan.

7. **Ishning tuzilishi:** Bitiruv ishi kirish, ikkita bob, xulosa va adabiyotlar ro'yxatidan iborat.

Birinchi bob to'rtta paragrafdan iborat. Birinchi paragrafda parabolik tipli tenglamalarga keltiriladigan fizik jarayonlar: issiqlik tarqalish va diffuziya tenglamalari qaralgan.

Ikkinchi paragrafda Issiqlik tarqalish tenglamasi yechimi uchun maksimal qiymat prinspi. Chegaraviy va Koshi masalasi yechimining yagonaligi o’rganilgan. Uchinchi paragrafda parabolik tipli tenglamalarga qo’yilgan chegaraviy masalalarni yechishning Fur’e usuli qaralgan. To’rtinchi paragrafda Maple paketi orqali sterjenda issiqlik o’tkazuvchanlik tenglamasini Fur’e usuli (o’zgaruvchilarni ajratish usuli) yordamida yechish o’rganilgan.

Ikkinchi bob beshta paragrafdan iborat. Birinchi paragrafda umumiy birinchi tur chegaraviy masala va uni yechishni soda holga keltirish usuli o’rganilgan. Ikkinchi paragrafda issiqlik o’tkazuvchanlik tenglamasi uchun birinchi chegaraviy masala qarab chiqilgan. Uchinchi paragrafda issiqlik o’tkazuvchanlik tenglamasi uchun 1-chegaraviy masalani yechimining yagonaligi va turg’unligi o’rganilgan. To’rtinchi paragrafda yarim to’g’ri chiziqda issiqlik o’tkazuvchanlik tenglamasi uchun birinchi va ikkinchi chegaraviy masala qarab o’tilgan. Beshinchi paragrafda Maple paketi orqali yarim chegaralangan sohada issiqlik o’tkazuvchanlik tenglamasini Fur’e usuli (o’zgaruvchilarni ajratish usuli) yordamida yechish o’rganilgan.

8. Olingan natijalarining qisqacha mazmuni: Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib, sterjenda issiqlik o’tkazuvchanlik tenglamasini va yarim to’g’ri chiziqda issiqlik o’tkazuvchanlik tenglamalarni Fur’ye usuli ya’ni o’zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali parabolik tipdagi tenglamalarni yechish jarayoni qoidaga mos ta’lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo’llanilishi ketma-ket tarzda keltirilgan, ya’ni parabolik tipdagi tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o’lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba’zi misollar uchun bir qancha vaqt momentlarini ikki o’lchovli grafigi tasvirlangan.

1-Bob. Sterjenda issiqlik o'tkazuvchanlik tenglamasini Fur'ye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish

1.1-§. Parabolik tipli tenglamalarga keltiriladigan fizik jarayonlar: issiqlik tarqalish va diffuziya tenglamalari

I. Asosiy masalalarning qo'yilishi

Fizikaning issiqlik harakati va gazlarning diffuziyasi bilan bog'liq masalalarini o'rganish odatda ikkinchi tartibli xususiy hosilali giperbolik tenglamalar orqali o'rganiladi. Bunday tenglamalarni kanonik shaklga keltirish bilan ularning eng sodda misoli sifatida

$$u_t = u_{xx}$$

issiqlikning Ox o'qi bo'ylab erkin tarqalish tenglamasi hisoblanadi. Bu shakldagi tenglamalar chekli uzulikdagi sterjen, yaxlit uzun metal o'tkazgich bo'ylab tarqalayotgan issiqlik miqdorini uning x nuqtasiga mos kesimning t vaqtdagi $u(x,t)$ temperaturasi orqali o'rganishda hosil bo'ladi.

II. Chekli sterjenda issiqlikning tarqalishi

Biz mavzuning bu qismida tashqi muhitdan issiqlikdan himoyalangan l uzunlikdagi yatarlicha ingichka sterjenni qaraymiz. Aniqlik uchun sterjenni Ox o'qi bo'ylab bir uchini $x=0$ nuqtaga, uning ikkinchi uchini esa $x=l$ nuqtaga joylashtiramiz. Sterjen x nuqtasiga mos kesimning t vaqtdagi temperaturasini $u(x,t)$ bilan belgilaymiz.

Dastlab eng sodda masalalarni qaraymiz. Faraz qilaylik, sterjen uchlarida doimiy T_1 va T_2 temperaturalar ushlab turilgan bo'lsa, u holda bir jinsli sterjenda biz issiqlikning uning nuqtalari bo'ylab chiziqli uzatilishiga ega bo'lamiz:

$$u(x) = T_1 + \frac{T_2 - T_1}{l} x \qquad (1.1.1)$$

Ma’lumki, bunda issiqlik yuqori temperaturali uchdan past temperaturali uchga tomon oqadi. Bunda biz agar issiqlik oqimi Ox o’qining musbat yo’nalishida bo’lsa, uni musbat, aks holda manfiy deb qaraymiz.

Bu holda sterjenning S kesim yuzidan birlik vaqt mobaynida oqib o’tgan issiqlik miqdori

$$Q = -k\frac{T_2 - T_1}{l}S = -kS\frac{\partial u}{\partial x} \qquad (1.1.2)$$

Formula bilan hisoblanadi. Bunda $k-$ sterjenning issiqlik o’tkazuvchanlik koeffitsiyenti bo’lib, u sterjen materialiga bog’liq.

Endi sterjenda issiqlik tarqalishining umumiy holini qaraymiz, ya’ni bu holda $u(x,t)$ funksiya qanday parametrlar orqali aniqlanishi va qanday qonuniyatga bo’ysinishi haqida to’xtalamiz.

1) Sterjenning x nuqtasiga mos ko’ndalang kesim yuzidan (t_1,t_2) vaqt mobaynida oqib o’tgan issiqlik miqdori

$$Q_1 = -S\int_{t_1}^{t_2} k(x)\frac{\partial u(x,t)}{\partial x}dt \qquad (1.1.3)$$

formula bilan ifodalanadi. k(x)- sterjen x nuqtasiga mos kesimning issiqlik $o’tkazuvchanlik\ koeffitsiyenti$.

2) Elementar fizikadan ma’lumki, bir jinsli issiqlik o’tkazuvchi jism temperaturasini Δu ga oshirish uchun unga

$$Q_2 = cm\Delta u = c\rho v\Delta u$$

miqdordagi issiqlik miqdorini berish kerak. Bunda $c-$ jismning solishtirma issiqlik sig’imi, m –jism massasi, ρ –jism zichligi, v- jism hajmi bo’lib, jism bir jinsli bo’lganligi uchun bu parametrlar doimiy, ya’ni jism nuqtalariga va vaqtga bog’liq emas.

Agarda sterjen bir jinsli bo’lmasa, bu qiymatlar sterjen nuqtalariga bog’liq bo’lib, unga berilgan issiqlik miqdori quyidagicha ko’rinish oladi:

$$Q_2 = S\int_{x_1}^{x_2} c(x)\rho(x)\Delta u(x,t)dx \qquad (1.1.3)$$

3) Sterjen ichki nuqtalarida issiqlik hosil bo’lishi mumkin. Bu issiqlik miqdori t vaqtda x nuqtadagi issiqlik manbalarining $\mathrm{F(x,t)}$ zichligi bilan tavsiflanadi. Ushbu issiqlik manbalarining sterjen (x_1, x_2) qismiga $(\mathrm{t_1}, \mathrm{t_2})$ vaqt mobaynida bergan jami issiqlik miqdori

$$Q_2 = S\int_{t_1}^{t_2}\int_{x_1}^{x_2} F(x,t)dxdt \qquad (1.1.5)$$

formula bilan beriladi.

Ushbu topilgan uchta issiqlik miqdorlari orqali sterjen (x_1, x_2) qismi uchun $(\mathrm{t_1}, \mathrm{t_2})$ vaqt oralig’ida issiqlik balansi tenglamasi tuzib, sterjenda issiqlikning tarqalish tenglamasini hosil qilishimiz mumkin bo’ladi. Buning uchun energiyaning saqlanish qonuni va (1.1.3), (1.1.3) va (1.1.5) formulalardan foydalansak, quyidagi tenglamani hosil qilamiz:

$$\int_{t_1}^{t_2}\left[k(x_2)\frac{\partial u(x_2,t)}{\partial x} - k(x_1)\frac{\partial u(x_1,t)}{\partial x}\right]dt + \int_{t_1}^{t_2}\int_{x_1}^{x_2} F(x,t)dxdt = \qquad (1.1.6)$$

(1.1.6) sterjenda issiqlik tarqalishining integral ko’rinishdagi tenglamasidir. Undagi integrallarga o’rta qiymat haqidagi teoremani qo’llab,
issiqlik tarqalishining differensial formadagi

$$\frac{\partial}{\partial x}\left(k(x)\frac{\partial u(x,t)}{\partial x}\right) + F(x,t) = c(x)\rho(x)\frac{\partial u}{\partial t} \qquad (1.1.7)$$

Agar sterjen bir jinsli bo’lsa (1.1.7) tenglamada $\mathrm{k, c}$ va ρ lar doimiy bo’lib, (1.1.7) tenglama quyidagi ko’rinishni oladi:

$$u_t(x,t) = a^2 u_{xx}(x,t) + f(x,t), \qquad (1.1.8)$$

bunda

$$a^2 = \frac{k}{c\rho}, \quad f(x,t) = \frac{F(x,t)}{c\rho}.$$

Agarda sterjenda tashqi issiqlik manbalari bo’lmasa, $\mathrm{F(x,t) = 0}$ bo’lib, issiqlik tarqalish tenglamasi quyidagi sodda ko’rinishga keladi:

$$u_t(x,t) = a^2 u_{xx}(x,t). \qquad (1.1.9)$$

III. Gaz diffuziyasi tenglamasi

Agar muhit turli gazlar bilan notekis to’ldirilgan bo’lsa, u holda yuqori konsentratsiyali nuqtalardan past konsentratsiyali nuqtalarga tomon gaz diffuziyasi kuzatiladi. Ushbu hodisa notekis aralashgan suyuqlik aralashmalarida ham uchraydi. Ushbu harakatni biz gaz tarqalayotgan trubka x nuqtasining t ondagi $u(x,t)$ gaz yoki suyuqlik konsentratsiyasi orqali tavsiflaymiz. Biz soddalik uchun trubkada gaz yoki suyuqlik manbalari yoq va uning ichki devorlarida diffuziya sodir bo'lmaydi deb faraz qilamiz.

Nernst qonuniga asosan, trubka x nuqtasidan dt vaqt intervalida oqib o'tgan gaz massasi

$$dQ = -D(x,t)\frac{\partial u(x,t)}{\partial x}S(x)dt = W(x,t)S(x)dt$$

formula bilan beriladi. Bunda D - diffuziya koeffisienti, S - trubka ko'ndalang kesim yuzi, $W(x,t)$ - gaz vaqt birligida birlik yuzadan oqib o'tgan gaz massasi bo'lib, *diffuziya oqimi zichligi* deyiladi.

Konsentratsiyaning ta'rifidan V hajmdagi gaz miqdori

$$Q = uV$$

ga teng bo'ladi. Bundan gaz konsentratsiyasi Δu ga o'zgarganda trubkaning (x_1, x_2) qismida gaz massasining o'zgarishi uchun

$$\Delta Q = \int_{x_1}^{x_2} c(x)\Delta u(x,t)S(x)dx$$

ifodani hosil qilamiz. Trubkaning har bir nuqtasi ko'ndalang kesimi bir xil bo'lsin, ya'ni $S(x) = S = const$ deb qaraymiz.

Trubka (x_1, x_2) qismi uchun (t_1, t_2) vaqt intervalida gaz massasi balansi tenglamasi quyidagi ko'rinishda bo'ladi:

$$S\int_{t_1}^{t_2}\left[D(x_2,t)\frac{\partial u(x_2,t)}{\partial x} - D(x_1,t)\frac{\partial u(x_1,t)}{\partial x}\right]dt = S\int_{x_1}^{x_2} c(x)\left[u(x,t_2) - u(x,t_1)\right]dx.$$

Ushbu integrallarga ham o'rta qiymat haqidagi teoremani qo'llab, gaz yoki suyuqlik diffuziya uchun differensial shakldagi tenglamaga ega bo'lamiz:

$$\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial u(x,t)}{\partial x}\right) = c(x)\frac{\partial u(x,t)}{\partial t}. \qquad (1.1.10)$$

Ko'rinib turibdiki, (1.1.10) diffuziya tenglamasi ham xuddi sterjenda issiqlik tarqalish tenglamasi (1.1.9) ga o'xshash ko'rinishga ega. Ulardagi asosiy farq noma'lum funksiya shu fizik jarayonni xarakterlovchi turli kattaliklarni ifodalaydi.

Agar shu bo'lim boshida yo'qligi talab qilingan trubkada manbalar bo'lishi yoki uning devorlari ham diffuziya jarayoniga ishtirok etishi mumkinligi hisobga olinsa, diffuziyaning issiqlik tarqalish tenglamasining umumiyroq ko'rinishidagi (1.1.7) yoki (1.1.8) ga o'xshash differensial tenglamalarni hosil qilgan bo'lar edik.

Xuddi shu kabi issiqlikning fazoda tarqalish masalasi ham parabolik tipdagi tenglamalarga keltiriladi. Bu jarayon issiqlik tarqalayotgan muhitning (x, y, z) nuqtasining t vaqtdagi temmperaturasi $u(x, y, z, t)$ orqali tavsiflanadi. Bu holda ham Fur'e qonunidan va issiqlik balansi tenglamasidan foydalanib issiqlikning fazoda tarqalish jarayonini to'rt o'zgaruvchili $u(x, y, z, t)$ funksiyaga nisbatan ikkinchi tartibli xususiy hosilali

$$c\rho u_t = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right) + F(x, y, z, t).$$

Bunda $k = k(x, y, z)$- issiqlik o'tkazuvchanlik koeffitsiyenti. Agar muhit bir jinsli bo'lsa $c = const,\ \rho = const$ va $k = const$ bo'lib, yuqoridagi tenglama

$$u_t = a^2(u_{xx} + u_{yy} + u_{zz}) + f(x, y, z, t)$$

ko'rinishga keladi. Bu yerda

$$a^2 = \frac{k}{c\rho}, \quad f = \frac{F}{c\rho}.$$

IV. Chegaraviy masalalarning qo'yilishi

Avvalgi mavzularda ta'kidlanganidek, issiqlik tarqalish va diffuziya tenglamalarini ifodalovchi matematik modellar ikkinchi tartibli xususiy hosilali tenglamalardan iborat bo'lib, bu tenglamalar cheksiz ko'p yechimga ega. Bu tenglamalar qaralayotgan jarayonni bir qiymatli aniqlashi uchun unga shu charayonni tavsiflovchi qo'shimcha shartlar ilova qilinishi lozim.

Issiqlik tarqalish tenglamalarida t bo'yicha birinchi tartibli xususiy hosila ishtirok etayotganligi uchun *boshlang'ich shart sifatida* jarayonning boshida sterjen nuqtalarida o'rnatilgan temperaturani ifodalovchi shart, ya'ni $u(x,t)$ funksiyaning tajriba boshlangan t_0 ondagi qiymati berilishidan iborat bo'ladi:

$$u(x,t_0)=\varphi(x). \qquad (1.1.11)$$

Bunda $\varphi(x),\ 0\le x\le \ell$ - berilgan uzluksiz funkisya, ℓ - sterjen uzunligi. Odatda tajriba boshlangan t_0 vaqtni sanoq boshi deb olinadi, ya'ni $t_0=0$.

Faraz qilaylik, sterjen 0x o'qi boylab gorizontal joylashgan bo'lib, uning bir uchi $x=0$ nuqtada, ikkinchi uchi esa $x=l$ nuqtada bo'lsin. Uning uchlaridagi temperatura rejimiga asoslanib *chegaraviy shartlar* turli ko'rinishlarda qo'yilishi mumkin. Xuddi to'lqin tenglamasiga qo'yilgani kabi issiqlik tarqalish va diffuziya tenglamalariga ham asosan uch tipdagi chegaraviy shartlar qo'yiladi:

`1) Sterjenning $x=0$ uchida vaqt davomida $u(0,t)=\mu_1(t)$ harorat, $x=\ell$ uchida esa $u(\ell,t)=\mu_2(t)$ harorat belgilangan bo'lsin. Bunda $\mu_1(t)$ va $\mu_2(t)$ lar biror $[0,T]$ vaqt oralig'ida aniqlangan berilgan funksiyalar, T- jarayon kuzatiladigan vaqt uzunligi. Sterjen uchlarida berilgan

$$\left.\begin{array}{l} u(0,t)=\mu_1(t) \\ u(\ell,t)=\mu_2(t) \end{array}\right\}$$

ko'rinishdagi chegaraviy shartga birinchi tipdagi chegaraviy shart deb yuritamiz.

2) Sterjen uchlari kesim yuzidan oqib o'tuvchi issiqlik oqimi belgilangan bo'lsin.

Masalan uning $x=0$ cheti kesimidan vaqt davomida o'tuvchi $Q_1(0,t)$ belgilangan rejimga bo'singan bo'lsa

$$Q_1(0,t)=-k\frac{\partial u(0,t)}{\partial x}$$

tenglik bajariladi. Bundan sterjenning $x=0$ uchida $u_x(0,t)=\nu_1(t)=-\dfrac{Q_1(0,t)}{k}$ shart bajarilishi lozimligiga kelamiz.

Xuddi shu kabi sterjen $x=\ell$ cheti kesimidan vaqt davomida o'tuvchi $Q_2(l,t)$ belgilangan rejimga bo'singan bo'lsa

$$Q_2(\ell,t)=-k\frac{\partial u(\ell,t)}{\partial x}$$

tenglik bajariladi. Bundan sterjenning $x=\ell$ uchida $u_x(\ell,t)=\nu_2(t)=-\frac{Q_2(\ell,t)}{k}$ shart bajarilishi lozimligiga kelamiz. Shunday qilib sterjen uchlarida issiqlik oqimi o'zgarishi belgilangan rejimga bo'sinishi talab qilinganda, issiqlik tarqalish tenglamasiga qo'shimcha

$$\left.\begin{aligned} u_x(0,t)&=\nu_1(t)\\ u_x(\ell,t)&=\nu_2(t)\end{aligned}\right\}$$

chegaraviy shartlarning bajarilishi lozim ekanligiga kelamiz. Bu ko'rinishdagi chegaraviy shartlarga 2-tipdagi chegaraviy shartlar deb yuritamiz.

3) Faraz qilaylik sterjen temperaturasi vaqt davomida aniq va $\theta(t)$ qonuniyat boyicha o'zgaruvchi tashqi muhit bilan issiqlik almashinuvi belgilangan rejimga bo'sinsin. Bu holda sterjen $x=0$ va $x=l$ uchlari uchun qo'yiladigan qo'shimcha shartlar

$$\left.\begin{aligned} u_x(0,t)&=h_1\{u(0,t)-\theta(t)\}\\ u_x(\ell,t)&=h_2\{u(\ell,t)-\theta(t)\}\end{aligned}\right\}$$

ko'rinishda ifodalanib, ularga odatda 3-tipdagi chegaraviy shartlar deb yuritiladi.

Bulardan tashqari sterjenning ikkala uchida ikki tipdagi chegaraviy shart qo'yilishi ham mumkin. Bu tipdagi chegaraviy shartlarga aralash tipdagi chegaraviy shartlar deb yuritamiz.

1-Ta'rif. *(7) issiqlik tarqalish masalasining (11) boshlang'ich shart va 1-tipdagi (mos ravishda 2-tipli, 3-tipli yoki aralash tipli) chegaraviy sharni qanoatlantiruvchi yechimini topish masalasiga 1-tur (mos ravishda 2-tur, 3-tur yoki aralash) chegaraviy masala deyiladi.*

Chegaraviy masalaning regulyar yechimi deganda issiqlik tenglamasining boshlang'ich va belgilangan chegaraviy shartlarni qanoatlantiruvchi hamda ikki marta uzluksiz differensiallanuvchi yechimiga aytiladi.

Ba'zan ta'riflangan chegaraviy masalalrdan tashqari uzunligi chegaralanmagan yoki juda ham uzun sterjenda issiqlikning tarqalish masalasini ham o'rganishga to'g'ri keladi. Bu holda sterjen bir uchi -∞ likda va ikkinchi uchini esa +∞ deb qarab, (1.1.7) tenglamaning faqat (1.1.11) boshlang'ich shartlarni qanoatlantiruvchi yechimini topish masalasiga duch kelamiz. Bu masala odatda Koshi masalasi deyiladi.

2-Ta'rif. *(7) issiqlik tarqalish masalasining* $-\infty < x < +\infty, t \geq 0$ *sohada aniqlangan va*

$$u(x,0) = \varphi(x), \ -\infty < x < +\infty$$

sartni qanoatlantiruvchi yechimini topish masalasiga issiqlik tarqalish tenglamasi uchun qo'yilgan Koshi masalasi deyiladi. Bunda $\varphi(x), -\infty < x < +\infty$ *berilgan funksiya.*

Xuddi shu kabi bir uchi chegaralanmagan sterjen uchun boshlang'ich shart va bitta chegaraviy shartni qanoatlantiruvchi yechimini topish haqidagi chegaraviy masalalar ham uchraydi.

1.2-§. Issiqlik tarqalish tenglamasi yechimi uchun maksimal qiymat prinsipi. Chegaraviy va Koshi masalasi yechimining yagonaligi

Ushbu va keyingi mavzularda biz alohida ta'kidlanmasa, o'zgarmas koeffitsiyentli

$$w_t = a^2 w_{xx} + \beta w_x + \gamma w \qquad (1.2.1)$$

issiqlik tarqalish masalasini qaraymiz. Ushbu tenglamada

$$w(x,t) = e^{\mu x + \lambda t} u(x,t)$$

almashtirish bajarib

$$u_t = \lambda e^{\mu x+\lambda t} u + e^{\mu x+\lambda t} u_t, \quad w_x = \mu e^{\mu x+\lambda t} u + e^{\mu x+\lambda t} u_x, w_{xx} = \mu^2 e^{\mu x+\lambda t} u + 2\mu e^{\mu x+\lambda t} u_x + e^{\mu x+\lambda t} u_{xx}$$

bo'lib, ularni yuqoridagi tenglamaga qo'sak , unga teng kuchli bo'lgan

$$u_t = a^2 u_{xx} + (2\mu a^2 + \beta) u_x + (\mu^2 a^2 + \beta\mu - \lambda + \gamma) u$$

tenglamaga kelamiz. Agar bu tenglamada

$$\mu = -\frac{\beta}{2a^2},\ \lambda = \gamma - \frac{\beta^2}{4a^2}$$

deb olsak so'ngi tenglama

$$u_t = a^2 u_{xx} \tag{1.2.2}$$

sodda ko'rinishga keladi. Demak (1.2.1) va (1.2.2) differensial tenglamalar bir vaqtda yechimga ega yoki ega bo'lmaydi. Shuning uchun (1.2.2) ko'rinishdagi tenglama yechimini tadqiq qilish yetarlidir. Quyidagi teoremada (1.2.2) issiqlik tarqalish tenglama yechimining ekstremal qiymatlari haqida so'z yuritiladi.

1-Teorema (Maksimal qiymat prinsipi). *Agar $u(x,t)$ funksiya yopiq $0 \le t \le T, 0 \le x \le \ell$ sohada aniqlangan va uzluksiz differensiallanuvchi bo'lib, $0 < t \le T, 0 < x < \ell$ sohada (1.2.2) tengllani qanoatlantirsa, u holda $u(x,t)$ funksiya o'zining eng katta va eng kichik qiymatiga yo boshlang'ich $t = 0$ vaqtda yoki sohaning chegaraviy nuqtalari $x = 0$ yoki $x = \ell$ nuqtalarda erishadi.*

Endi biz maksimal qiymat prinsipidan kelib chiqadigan natijalarga to'xtalamiz. Ushbu natijalardan eng muhimi chegaraviy masala yechimining yagonaligini isbotlashga tatbiqi hisoblanadi. Quyida biz yagonalik teoremasini keltiramiz.

2-Teorema. (1-chegaraviy masala yechimining yagonaligi)

$$u_t = a^2 u_{xx} + f(x,t),\ 0 < x < \ell, t > 0 \tag{1.2.3}$$

issiqlik tarqalish tenglamasi

$$u(x,0) = \varphi(x) \tag{1.2.4}$$

boshlang'ich shart va

$$u(0,t) = \mu_1(t),\ u(\ell,t) = \mu_2(t) \tag{1.2.5}$$

chegaraviy shartlarni qanoatlantiruvchi va $0 \le x \le \ell, 0 \le t \le T$ sohada aniqlangan, ikkinchi tartibgacha uzluksiz differensizllanuvchi yechimi yagonadir.

Endi maksimal qiymat prinsipidan to'g'ridan to'g'ri kelin chiqadigan natijalarni keltiramiz:

1-Natija. *Agar (1.2.2) tenglamaning ikkita $u_1(x,t)$ va $u_2(x,t)$ yechimlari uchun*

$$u_2(x,0) \geq u_1(x,0),$$

$$u_2(0,t) \geq u_1(0,t),\ u_2(\ell,t) \geq u_1(\ell,t)$$

tengsizliklar o'rinli bo'lsa u holda barcha $0 \leq x \leq \ell,\ 0 \leq t \leq T$ *lar uchun*

$$u_2(x,t) \geq u_1(x,t)$$

tengsizlik o'rinli bo'ladi.

2-Natija. *Agar (1.2.2) tenglamaning uchta* $u_1(x,t), u_2(x,t)$ *va* $u_3(x,t)$ *yechimlari uchun*

$$u_1(x,0) \leq u_2(x,0) \leq u_3(x,0),$$

$$u_1(0,t) \leq u_2(0,t) \leq u_3(0,t),\ u_1(\ell,t) \leq u_2(\ell,t) \leq u_3(\ell,t)$$

tengsizliklar o'rinli bo'lsa u holda barcha $0 \leq x \leq \ell,\ 0 \leq t \leq T$ *lar uchun*

$$u_1(x,t) \leq u_2(x,t) \leq u_3(x,t)$$

tengsizlik o'rinli bo'ladi.

3-Natija. *Agar ixtiyoriy* $\varepsilon > 0$ *soni va (1.2.2) tenglamaning ikkita* $u_1(x,t)$ *va* $u_2(x,t)$ *yechimlari uchun*

$$|u_2(x,0) - u_1(x,0)| \leq \varepsilon,$$

$$|u_2(0,t) - u_1(0,t)| \leq \varepsilon,\ |u_2(\ell,t) - u_1(\ell,t)| \leq \varepsilon$$

tengsizliklar o'rinli bo'lsa u holda barcha $0 \leq x \leq \ell,\ 0 \leq t \leq T$ *lar uchun*

$$|u_2(x,t) - u_1(x,t)| \leq \varepsilon$$

tengsizlik o'rinli bo'ladi.

3-Natija issiqlik tarqalish tenglamasiga qo'yilgan 1-tur chegaraviy masala yechimi unga qo'yilgan chegaraviy va boshlang'ich shartlarga uzluksiz bog'liqligini, ya'ni chegaraviy masala yechimining turg'unligini ifodalaydi.

Fizikada issiqlikning to'g'ri chiziq bo'ylab cheksizlikka yoki yetarlicha katta oraliqqa tarqalish masalasi muhim ahamiyatga ega hisoblanadi. Issiqlikning to'g'ri chiziq bo'ylab cheksizlikka tarqalish tenglamasiga qoyiladigan masala yechimi yagona bo'lishi uchun unga boshlang'ich shartdan tashqari yana qo'shimcha shartlar talab qilinadi. Bu masalada sterjen uchlari cheksizlikda deb faraz qilinganligi uchun chegaraviy shartlar qatnashmaydi. Odatda bu masala Koshi masalasi deb yuritiladi. Ushbu masalada yechimning qaralayotgan sohada chegaralanganligi muhim ahamiyatga ega.

1-Ta'rif. Agar shunday $M > 0$ son topilib, ixtiyoriy $-\infty < x < +\infty$ va $t \geq 0$ uchun

$$|u(x,t)| < M$$

tengsizlik bajarilsa, u holda $u(x,t)$ funksiyaga shu sohada chegaralangan deyiladi.

2-Ta'rif. $-\infty < x < +\infty, t > 0$ sohada

$$u_t = a^2 u_{xx} \qquad (1.2.6)$$

issiqlik tarqalish tenglamasining

$$u(x,0) = \varphi(x), \ -\infty < x < +\infty \qquad (1.2.7)$$

boshlang'ich shartni qanoatlantiruvchi uzluksiz va chegaralangan yechimini topish masalasiga issiqlik tarqalish tenglamasiga qo'yilgan Koshi masalasi deyiladi.

Endi ushbu masala yechimining yagonalik teoremasini keltiramiz.

3-Teorema. $-\infty < x < +\infty, t > 0$ sohada (1.2.6) tenglamaning (1.2.7) boshlangich shartni qanoatlantiruvchi uzluksiz va chegaralangan yechimi yagonadir.

Bu mavzuning asosiy mohiyati xuddi chegaraviy masalaning limitik holi sifatida uchlari cheksizlikda bo'lgan chegaralanmagan sterjenda issiqlikning tarqalish masalasi, ya'ni Koshi masalasi qo'yilishi va uni yechish usuli bilan tanishishdan iborat. Avvalgi mavzularda biz issiqlik tarqalish tenglamasiga qo'yilgan uch turdagi chegaraviy masalalar hamda Koshi masalasining qo'yilishi, ular yechimining yagonaligi masalasini hal etilishi bilan tanishgan edik.

Ayytilgandek biz uchlari cheksizlikda bo'lgan chegaralanmagan sterjenda issiqlikning tarqalish masalasining uzluksiz chegaralangan yechimini topish masalasi bilan tanishamiz. Ushbu fizik masala matematik model sifatida quyidagicha yoziladi:

Ta'rif.

$$u_t = a^2 u_{xx}, \ -\infty < x < +\infty, t > 0 \qquad (1.2.8)$$

issiqlik tarqalish tenglamasining

$$u(x,0)=\varphi(x),\ -\infty<x<+\infty \qquad \textit{(1.2.9)}$$

boshlang'ich shartlarni qanoatlantiruvchi uzluksiz va chegaralangan yechimini topish masalasiga chegaralanmagan sterjenda issiqlik tarqalish tenglamasi uchun Koshi masalasi deyiladi.

Bu masala issiqlik tarqalayotgan sterjenning uzunligi ahamiyatga ega bo'lmagan va asosiy e'tibot sterjen ichki nuqtalaridagi temperaturaning yetarlicha kichik vaqt intervalidagi holatiga qaratilgan fizik masalalarda hosil bo'ladi. Endi bu masalaning yechimini topish bilan shug'ullanamiz.

(1.2.8)-(1.2.9) Koshi masalasining nolmas chegaralangan yechimini o'zgaruvchilarni ajratish usulida izlaymiz, ya'ni yechimni

$$u(x,t)=X(x)T(t) \qquad (1.2.10)$$

ko'rinishda izlaymiz. (1.2.10) ifodani (1.2.8) ga qo'yib xuddi Fur'e usulidagi kabi quyidagi tenglamalarni olamiz:

$$\frac{X''}{X}=\frac{T'}{a^2T}=-\lambda^2.$$

Bu tenglama esa o'z navbatida ikkita

$$T'+a^2\lambda^2T=0, \qquad (1.2.11)$$

$$X''+\lambda^2X=0 \qquad (1.2.12)$$

oddiy differensial tenglamalarga ajraladi. Bu oddiy differensial tenglamalarni yechish bilan biz tanishmiz:

$$T_\lambda(t)=C_1(\lambda)e^{-(\lambda a)^2t},\ X_\lambda(x)=C_2(\lambda)e^{i\lambda x},$$

bunda $C_1(\lambda)$ va $C_2(\lambda)$ ixtiyoriy chekli doimiylar. Bularga va (1.2.10) ga asosan (1.2.8) tenglamaning chegaralangan yechimi uchun

$$u_\lambda(x,t)=C(\lambda)e^{-(\lambda a)^2t+i\lambda x}$$

ifodani hosil qilamiz. (1.2.8) chiziqli tenglama bo'lganligi uchun bu yechimlarning λ bo'yicha yig'indisi ham yana yechim bo'ladi:

$$u(x,t)=\int_{-\infty}^{+\infty}C(\lambda)e^{-(\lambda a)^2t+i\lambda x}d\lambda. \qquad (1.2.13)$$

Yechimning bu ko’rinishidagi noma’lum $C(\lambda)$ koeffitsiyentlarni (1.2.9) boshlang’ich shartdan foydalanib topamiz

$$u(x,0)=\varphi(x)=\int_{-\infty}^{+\infty}C(\lambda)e^{i\lambda x}d\lambda. \qquad (1.2.14)$$

(1.2.14) da teskari Fur’e almashtirishlarini qo’llash bilan noma’lum koeffitsiyentlarni topamiz:

$$C(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\varphi(s)e^{-i\lambda s}ds. \qquad (1.2.15)$$

$C(\lambda)$ koeffitsiyentlarning (1.2.15) ifodasini (1.2.13) ga qo’yish bilan qo’yilgan (1.2.8)-(1.2.9) Koshi masalasi yechimini hosil qilamiz

$$u(x,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{+\infty}\varphi(s)e^{-i\lambda s}ds\right)e^{-(\lambda a)^2t+i\lambda x}d\lambda=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{+\infty}e^{-(\lambda a)^2t+i\lambda(x-s)}d\lambda\right)\varphi(s)ds.$$

Oxirgi tenglikdagi ichki integralni hisoblaymiz:

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-(\lambda a)^2t+i\lambda(x-s)}d\lambda=\frac{1}{2\sqrt{\pi a^2t}}e^{-\frac{(x-s)^2}{4a^2t}}. \qquad (1.2.16)$$

Natijada qaralayotgan Koshi masalasi yechimi uchun quyidagi integral tasvirni olamiz

$$u(x,t)=\frac{1}{2\sqrt{\pi a^2t}}\int_{-\infty}^{+\infty}e^{-\frac{(x-s)^2}{4a^2t}}\varphi(s)ds. \qquad (1.2.17)$$

Ba’zan (1.2.8)-(1.2.9) Koshi masalsining topilgan (1.2.17) ko’rinishdagi yechimini

$$u(x,t)=\int_{-\infty}^{+\infty}G(x,s,t)\varphi(s)ds \qquad (1.2.18)$$

yoziladi. Bunda

$$G(x,s,t)=\frac{1}{2\sqrt{\pi a^2t}}e^{-\frac{(x-s)^2}{4a^2t}}$$

funksiya odatda issiqlik tarqalish tenglamasining fundamental yechimi deb aytiladi. Bu funksiya quyidagicha fizik ma'no kasb etadi: Agar boshlang'ich $t = t_0$ vaqtda sterjen s nuqtasida $Q = c\rho$ issiqlik miqdori ajralgan bo'lsa, u holda

$$G(x,s,t-t_0) = \frac{1}{c\rho 2\sqrt{\pi a^2(t-t_0)}} e^{-\frac{(x-s)^2}{4a^2(t-t_0)}}$$

funksiya sterjen x nuqtasining t ondagi temperaturasini ifodalaydi. Bundan tashqari $G(x,s,t-t_0)$ funksiya o'zining (x,t) o'zgaruvchilar bo'yicha $u_t = a^2 u_{xx}$ issiqlik tarqalish tenglamasining yechimi bo'ladi. Haqiqatan ham integralni hadlab differensiallash haqidagi teoremani qo'llab quyidagi xususiy hosilalarni topamiz:

$$G_x(x,s,t-t_0) = -\frac{1}{2\sqrt{\pi}} \frac{x-s}{2\left[a^2(t-t_0)\right]^{\frac{3}{2}}} e^{-\frac{(x-s)^2}{4a^2(t-t_0)}},$$

$$G_{xx}(x,s,t-t_0) = \frac{1}{2\sqrt{\pi}} \left\{ -\frac{1}{2\left[a^2(t-t_0)\right]^{\frac{3}{2}}} + \frac{(x-s)^2}{4\left[a^2(t-t_0)\right]^{\frac{5}{2}}} \right\} e^{-\frac{(x-s)^2}{4a^2(t-t_0)}},$$

$$G_t(x,s,t-t_0) = \frac{a^2}{2\sqrt{\pi}} \left\{ -\frac{1}{2\left[a^2(t-t_0)\right]^{\frac{3}{2}}} + \frac{(x-s)^2}{4\left[a^2(t-t_0)\right]^{\frac{5}{2}}} \right\} e^{-\frac{(x-s)^2}{4a^2(t-t_0)}}.$$

Bularni o'rniga qo'yib haqiqatan ham $G_t = a^2 G_{xx}$ ekanligiga ishonch hosil qilamiz. Hisoblashlarga ishonch hosil qilish uchun yuqoridagi xusuiy hosilalarni va tenglama yechimi ekanligini mustaqil tekshirib chiqing.

Koshi masalasidagi boshlang'ich shartdagi berilgan φ funksiya uzluksiz va chegaralangan funksiyadir. Koshi masalasining integral tasviri, ya'ni (1.2.17) formula odatda chegaralanmagan sohada Koshi masalasi yechimi uchun Puasson formulasi hamda undagi integralga esa Puasson integrali deg yuritiladi.

1.3-§. Parabolik tipli tenglamalarga qo'yilgan chegaraviy masalalarni yechishning Fur'e usuli

Bu mavzuning asosiy mohiyati xuddi biz to'lqin tenglamasiga qo'yilgan chegaraviy yoki aralash masalani yechishda qo'llanilgan Fur'e usulining issiqlik tarqalish tenglamasiga tadbiq etishdan iboratdir. Avvalgi mavzularda biz issiqlik tarqalish tenglamasiga qo'yilgan uch turdagi chegaraviy masalalarning qo'yilishi, ular yechimining yagonaligi masalasini hal etilishi bilan tanishgan edik.

Ayytilgan usul bilan biz dastlab issiqlikning bir jinsli tenglamasiga qoyilgan bir jinsli 1-tur chegaraviy masala misolida tanishamiz, ya'ni quyydagi masalani yechamiz:

$$u_t = a^2 u_{xx},\ 0 < x < \ell, t > 0 \qquad (1.3.1)$$

issiqlik tarqalish tenglamasining

$$u(x,0) = \varphi(x),\ 0 \le x \le \ell \qquad (1.3.2)$$

boshlang'ich hamda

$$u(0,t) = 0,\ \ u(\ell,t) = 0,\ \ t \ge 0 \qquad (1.3.3)$$

chegaraviy shartni qanoatlantiruvchi va $0 \le x \le \ell,\ t \ge 0$ sohada aniqlangan ikkinchi tartibgacha uzluksiz yechimini topish talab qilinadi. Bunda $\varphi(x)$ berilgan uzluksiz differensizllanuvchi funksiya bo'lib, $\varphi(0) = 0$ shartni qanoatlantiradi.

Ushbu masalaning izlangan ko'rinishdagi nolmas yechimi mavjud deb faraz qilib

$$u(x,t) = X(x)T(t) \ne 0 \qquad (1.3.4)$$

ko'rinishda izlaymiz. Yechimning kerakli xususiy hosilalarni topib (1.3.1) tenglamaga qo'ysak, u quyidagi tenglamaga teng kuchli bo'ladi:

$$X(x)T'(t) = a^2 X''(x)T(t),\ 0 < x < \ell, t > 0.$$

Xuddi to'lqin tenglamasidagi kabi, bu tenglamaninig ikkala qismini nolmas $a^2 X(x)T(t)$ ifodaga bo'lib quyidagi tenglamalarga kelamiz:

$$\frac{X''(x)}{X(x)} = \frac{T'(t)}{a^2 T(t)} = -\lambda\ .$$

Bu tenglama biz avval tanishganimiz kabi ikkita oddiy differensial tenglamalarga ajraladi:

$$\left.\begin{array}{r} X''(x) + \lambda X(x) = 0 \\ T'(t) + \lambda a^2 T(t) = 0 \end{array}\right\} \qquad (1.3.5)$$

(1.3.3) chegaraviy shartlar quyidagi ko'rinishni oladi:

$$\left.\begin{matrix} u(x,0)=0 \\ u(x,\ell)=0 \end{matrix}\right\} \quad \Rightarrow \quad \left.\begin{matrix} X(0)T(t)=0 \\ X(\ell)T(t)=0 \end{matrix}\right\} \quad \Rightarrow \quad \left.\begin{matrix} X(0)=0 \\ X(\ell)=0 \end{matrix}\right\},$$

chunki $T(t)=0$ bo'lsa (1.3.4) ga asosan $u(x,t)$ yechimning aynan nolga tengligiga kelamiz. Bu esa shartga ko'ra mumkin emas.

Shunday qilib biz $X(x),\ 0\le x\le \ell$ funksiya uchun quyidagi qo'shimcha masalaga keldik:

$$X''(x)+\lambda X(x)=0 \tag{1.3.6}$$

tenglamaning

$$X(0)=0,\ X(\ell)=0 \tag{1.3.7}$$

shartlarni qanoatlantiruvchi nolmas yechimini topish lozim. Odatda bu masala issiqlik tarqalish tenglamiga qo'yilgan bir jinsli 1-tur chegaraviy masalaga mos Shturm-Liuvill masalasi deb yuritiladi. (1.3.6) tenglama nolmas yechimga ega bo'ladigan λ ning qiymatiga Shturm-Liuvill masalasi xos qiymati va unga mos nolmas $X(x)$ yechimga esa unga mos xos funksiya deyiladi.

Shturm-Liuvill masalasi yechimini $X(x)=Ce^{kx}$ ko'rinishda izlaymiz. U holda ikkinchi tartibli oddiy chiziqli (1.3.6) differensial tenglamaning xarakteristik tenglamasi deb ataluvchi

$$k^2+\lambda=0 \tag{1.3.8}$$

tenglamaga kelamiz. Ushbu chala kvadrat tenglamaning yechimi λ qiymatining ishorasiga (manfiy, nol yoki musbatligiga) bog'liq. Shuning uchun ham bu uchta holni alohida-alohida qarab chiqamiz.

1-hol. $\lambda<0$ manfiy bo'lsin. Bu holda (1.3.8) tenglama ikkita har xil haqiqiy $k_{1,2}=\pm\sqrt{-\lambda}$ ildizlarga ega bo'lib, (1.3.6) tenglamaning umumiy yechimi

$$X(x)=Ae^{\sqrt{-\lambda}\,x}+Be^{-\sqrt{-\lambda}}$$

ko'rinishda bo'ladi. Bundagi A va B koeffitsiyentlarni shunday tanlaymizki, (1.3.7) chegaraviy shartlar ham bajarilsin. Bu shartlar asosida quyidagi chiziqli tenglamalar sistemasiga kelamiz:

$$\left.\begin{matrix} X(0)=0 \\ X(\ell)=0 \end{matrix}\right\} \quad \Rightarrow \quad \left.\begin{matrix} A+B=0 \\ Ae^{\sqrt{-\lambda}\ell}+Be^{-\sqrt{-\lambda}\ell}=0 \end{matrix}\right\} \quad \Rightarrow \quad \left.\begin{matrix} A=-B \\ B(e^{\sqrt{-\lambda}\ell}-e^{-\sqrt{-\lambda}\ell})=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} A=0 \\ B=0 \end{matrix}\right\}.$$

Oxirgi tenglik $\lambda < 0$ bo'lganda ($\lambda \neq 0$ bo'lishi yetarli) $e^{\sqrt{-\lambda}\ell} \neq e^{-\sqrt{-\lambda}\ell}$ ga asoslanib yozilgan. Demak $\lambda < 0$ bo'lgan holda Shturm-Liuvill masalasi faqat nol yechimga ega bo'lib, xos qiymat va xos funksiyaga ega emas ekan. Endi ikkinchi holni qaraymiz.

2-hol. $\lambda = 0$ **bo'lsin.** Bu holda (1.3.6) tenglama $X''(x) = 0$ ko'rinishni oladi. Uning umumiy yechimi $X(x) = Ax + B$ ko'rinishda bo'ladi. Bunda ham A va B koeffitsiyentlarni shunday tanlaymizki, (1.3.7) chegaraviy shartlar ham bajarilsin:

$$\left.\begin{array}{l} X(0)=0 \\ X(\ell)=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} B=0 \\ A\ell=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} A=0 \\ B=0 \end{array}\right\}.$$

Demak $\lambda = 0$ bo'lgan holda ham Shturm-Liuvill masalasi faqat nol yechimga ega bo'lib, xos qiymat va xos funksiyaga ega bo'lmas ekan. Endi so'ngi holni qaraymiz.

3-hol. $\lambda > 0$ **musbat bo'lsin.** Bu holda (1.3.8) tenglama ikkita qo'shma kompleks $k_{1,2} = \pm i\sqrt{\lambda}$ ildizlarga ega bo'lib, (1.3.6) tenglamaning umumiy yechimi

$$X(x) = A\cos\sqrt{\lambda}\ x + B\sin\sqrt{\lambda}\ x$$

ko'rinishda bo'ladi. Bu holda ham umumiy yechimdagi ixtiyoriy A va B koeffitsiyentlarni shunday tanlaymizki, (1.3.7) chegaraviy shartlar bajarilsin. Bu shartlar asosida quyidagi chiziqli tenglamalar sistemasiga kelamiz:

$$\left.\begin{array}{l} X(0)=0 \\ X(\ell)=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} A=0 \\ B\sin\sqrt{\lambda}\ell=0 \end{array}\right\}.$$

Oxirgi sistemadan $X(x) \neq 0$ bo'lganligidan (*B=0* bo'lsa *X(x)=0* bo'lar edi)

$$\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2, n = 1,2,3,\cdots$$

ekanligini va ularha mos yechim o'zgarmas ko'paytuvchi aniqligida

$$X_n(x) = \sin\frac{n\pi}{\ell}x.$$

Demak $\lambda > 0$ bo'lgan holda Shturm-Liuvill masalasi musbat $\lambda_n = \left(\frac{n\pi}{\ell}\right)^2, n = 1,2,3,\cdots$ xos qiymatlarga va ularga mos $X_n(x) = \sin\frac{n\pi}{\ell}x$ xos funksiyalarga ega bo'lib, xos funksiyalar skalyar ko'paytmasi uchun n ≠ m ∈ N bo'lganda

$$\left(X_n, X_m\right) = \int_0^{\ell} \sin\frac{n\pi}{\ell}x \sin\frac{m\pi}{\ell}x dx = \frac{1}{2}\left(\frac{1}{n-m}\sin(n-m)\pi - \frac{1}{n+m}\sin(n+m)\pi\right) = 0$$

tenglikni, ya'ni ortogonallik shartini qanoatlantiradi.

Demak (1.3.1) tenglama faqatgina $\lambda_n = \left(\frac{n\pi}{\ell}\right)^2, n = 1,2,3,\cdots$ bo'lgandagina nolmas yechimga ega bo'lar ekan. $\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2, n = 1,2,3,\cdots$ bo'lganda (1.3.5) sistemaning ikkinchi tenglamasi yechimi quyidagicha tasvirlanadi:

$$T_n(t) = C_n e^{-a^2\lambda_n t}. \tag{1.3.9}$$

U holda (1.3.4) ga asosan

$$u_n(x,t) = T_n(t)X_n(x) = C_n e^{-\left(\frac{n\pi}{\ell}a\right)^2 t} \sin\frac{n\pi}{\ell}x.$$

(1.3.1) ning chiziqli differensial tenglama ekanligidan va hozircha yaqinlashishi noma'lum bolgan

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi}{\ell}a\right)^2 t} \sin\frac{n\pi}{\ell}x \tag{1.3.10}$$

qator har biri uning yechimidan iboratligidan yig'indi ham (1.3.1) tenglamaning (1.3.3) chegaraviy shartlarni qanoatlantiruvchi yechimi bo'ladi.
Bundan esa bu koeffitsiyentlar uchun

$$C_n = \varphi_n = \frac{2}{\ell}\int_0^{\ell} \varphi(x)\sin\frac{n\pi}{\ell}x dx \tag{1.3.11}$$

formula o'rinli ekanligini olamiz.

Shunday qilib biz koeffitsiyentlari (1.3.11) formulalar bilan aniqlanuvchi (1.3.10) qatorning yaqinlashuvchi bo'lib, uning yig'indisi $u(x,t)$ ning (ya'ni qatorning) x bo'yicha ikki marta va t bo'yicha bir marta differensiallanuvchanligini ko'rsatsak, qo'yilgan (1.3.1)-(1.3.3) masalaning yechimini topgan bo'lamiz.

Shu maqsadda biz

$$\sum_{n=1}^{\infty}\frac{\partial u_n}{\partial t} \text{ va } \sum_{n=1}^{\infty}\frac{\partial^2 u_n}{\partial x^2}$$

qatorlarning tekis yaqinlashuvchiligini ko'rsatamiz. Buning uchun funksional qatorlar tekis yaqinlashuvchi bo'lishligi haqidagi Veyershtrass teoremasini tadbiq etamiz. Faraz qilaylik $t_0 > 0$ ixtiyoriy son va $t \geq t_0$ bo'lsin.

$$\left|\frac{\partial u_n}{\partial t}\right| = \left|-C_n\left(\frac{n\pi}{\ell}a\right)^2 e^{-\left(\frac{n\pi}{\ell}a\right)^2 t}\sin\frac{n\pi}{\ell}x\right| \leq |C_n|\left(\frac{n\pi}{\ell}a\right)^2 e^{-\left(\frac{n\pi}{\ell}a\right)^2 t}.$$

Shartga ko'ra $\varphi(x)$ funksiya yopiq $0 \leq x \leq \ell$ sohada uzluksiz bo'lganligi uchun chegaralangan bo'ladi, ya'ni shunday M > 0 son topilib barcha $0 \leq x \leq \ell$ lar uchun $|\varphi(x)| < M$ tengsizlik bajariladi. U holda

$$|C_n| = \frac{2}{\ell}\left|\int_0^{\ell}\varphi(x)\sin\frac{n\pi}{\ell}x dx\right| \leq \frac{2}{\ell}\int_0^{\ell}|\varphi(x)|dx < 2M$$

tengsizlik o'rinli bo'ladi. Bundan foydalansak quyidagi munosabatga ega bo'lamiz:

$$\left|\frac{\partial u_n}{\partial t}\right| < 2M\left(\frac{n\pi}{\ell}a\right)^2 e^{-\left(\frac{n\pi}{\ell}a\right)^2 t_0}.$$

Umumiy hadi

$$a_n = 2M\left(\frac{n\pi}{\ell}a\right)^2 e^{-\left(\frac{n\pi}{\ell}a\right)^2 t_0}$$

bo'lgan

$$\sum_{n=1}^{\infty} a_n$$

musbat hadli qatorning yaqinlashuvchi ekanligini ma'lum alomatlar, masalan Dalamber (yoki Koshi) alomatidan foydalanib

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 0$$

ekanligini ko'rsatish mumkin. Demak Veyershtrass teoremasiga ko'ra (1.3.10) qatorni t bo'yicha $t \geq t_0 > 0$ sohada istalgan marta differensiallash mumkin. $t_0 > 0$

ning ixtiyoriyligidan bu tasdiq istalgan $t>0$ uchun o'rinli bo'lib, (1.3.10) qator yig'indisi (1.3.1) tenglamani va (1.3.2)-(1.3.3) shartlarni qanoatlantiradi.

Endi (1.3.10) qatorni x bo'yicha $0 \le x \le \ell$ sohada istalgan marta diffrensiallash mumkinligini ko'rsatamiz. Xuddi yuqoridagi kabi mulohazalar yuritib, quyidagi baholashga ega bo'lamiz:

$$\left|\frac{\partial^2 u_n}{\partial x^2}\right| < 2M\left(\frac{n\pi}{\ell}\right)^2 e^{-\left(\frac{n\pi}{\ell}a\right)^2 t_0}$$

Bundan esa (1.3.10) qator yaqinlashuvchi va uning yig'indisi (1.3.1) tenglamaning (1.3.2)-(1.3.3) shartlarni qanoatlantiruvchi yechimi ekanligini olamiz.

Bir jinsli bo'lmagan to'lqin tenglamasiga qo'yilgan chegaraviy masala uchun Fur'e usulining qo'llanilishi

Endi biz yuqorida qollagan usulni issiqlik tarqakishning bir jinsli bo'lmagan, ya'ni sterjenda issiqlik manbalari ta'siri kuzatilgan hilga tadbiq etamiz. Bu holda biz

$$u_t = a^2 u_{xx} + f(x,t),\ 0 < x < \ell, t > 0 \qquad (1.3.12)$$

issiqlik tarqalish tenglamasining

$$u(x,0) = 0,\ 0 \le x \le \ell \qquad (1.3.13)$$

bir jinsli boshlang'ich shartni hamda

$$u(0,t) = 0,\ \ u(\ell,t) = 0,\ \ t \ge 0 \qquad (1.3.14)$$

chegaraviy shartlarni qanoatlantiruvchi va $0 \le x \le \ell, t \ge 0$ sohada aniqlangan ikkinchi tartibgacha uzluksiz aynan nolga teng bo'lmagan yechimini topishdan iboratdir.

Ushbu masala yechimini oldingi masaladagi Shturm-Liuvill masalasi

$$X_n(x) = \sin\frac{n\pi}{\ell}x$$

xos funksiyalari bo'yicha Fur'e qatori ko'rinishida izlaymiz, ya'ni

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t)\sin\frac{n\pi}{\ell}x. \qquad (1.3.15)$$

Xuddi shu kabi (1.3.12) tenglama o'ng tomonidagi $f(x,t)$ funksiyani ham t ni hozircha parametr sifatida qarab, Fur'e qatoriga yoyamiz:

$$f(x,t)=\sum_{n=1}^{\infty} f_n(t)\sin\frac{n\pi}{\ell}x\,. \qquad (1.3.16)$$

Bunda

$$f_n(t)=\frac{2}{\ell}\int_0^{\ell} f(x,t)\sin\frac{n\pi}{\ell}xdx\,.$$

Yechimning izlangan (1.3.15) ifodasini va $f(x,t)$ funksiyaning (1.3.16) ifodasini (1.3.12) tenglamaga qo'yib

$$\sum_{n=1}^{\infty}\left\{\left(\frac{n\pi}{\ell}a\right)^2 u_n(t)+u_n^{'}(t)-f_n(t)\right\}\sin\frac{n\pi}{\ell}x=0=\sum_{n=1}^{\infty}0\cdot\sin\frac{n\pi}{\ell}x\,.$$

Ikki teng funksiyalar Fur'e koeffitsiyentlari teng bo'lganligi uchun oxirgi tenglamadan

$$\left(\frac{n\pi}{\ell}a\right)^2 u_n(t)+u_n^{'}(t)-f_n(t)=0 \qquad (1.3.17)$$

oddiy differensial tenglamaga kelamiz. Endi (1.3.13) boshlang'ich shartlarni qaraymiz.

$$u(x,0)=\sum_{n=1}^{\infty}u_n(0)\sin\frac{n\pi}{\ell}x=0\,.$$

Demak (1.3.13) boshlang'ch shart $u_n(t)$ uchun qo'yilgan

$$u_n(0)=0 \qquad (1.3.18)$$

shartga o'tar ekan. (1.3.17) birinchi tartibli oddiu chiziqli differensial tenglamaning (1.3.18) shartni qanoatlantiruvchi yechimi bizga oddiy differensial tenglamalar kursidan ma'lum bo'lgan formulada yechiladi:

$$u_n(t)=\int_0^{\ell} f_n(\tau)e^{-\left(\frac{n\pi}{\ell}a\right)(t-\tau)}d\tau\,.$$

$u_n(t)$ ning bu ifodasini (1.3.15) ga qo'yib, qo'yilgan (1.3.12)-(1.3.14) masalaning yechimiga ega bo'lamiz:

$$u(x,t)=\sum_{n=1}^{\infty}\left\{\int_0^{\ell} f_n(\tau)e^{-\left(\frac{n\pi}{\ell}a\right)(t-\tau)}d\tau\right\}\sin\frac{n\pi}{\ell}x\,.$$

Ushbu yechimni $f_n(t)$ ning ifodasini o'rniga qo'yish va qator tekis yaqinlashuvchanligiga asosan uni hadlab integrallash mumkin ligidan foydalanib

$$u(x,t)=\int_0^t\int_0^\ell G(x,\xi,t-\xi)\,f(\xi,\tau)\,d\xi d\tau$$

ko'rinishda yozish mumkin. Bunda

$$G(x,y,z)=\frac{2}{\ell}\sum_{n=1}^{\infty}e^{-\left(\frac{n\pi}{\ell}a\right)z}\sin\frac{n\pi}{\ell}x\sin\frac{n\pi}{\ell}\xi.$$

Odatda ushbu $G(x,y,z)$ funksiyani nuqtaviy issiqlik manba funksiyasi deb aytiladi.

1.4-§. Maple paketi orqali sterjenda issiqlik o'tkazuvchanlik tenglamasini Fur'ye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish

Sterjenda issiqlik o'tkazuvchanlik tenglamasini qaraymiz. Buning uchun quyidagi bir jinsli tenglama

$$\frac{\partial}{\partial t}\mathrm{u}(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x)\right)$$

boshlang'ich shartlar bilan yechish

$$\mathrm{u}(0,x)=\mathrm{f}(x).$$

```
> restart;
```

Bir jinsli tenglama va uning yechimini o'zgaruvchilarni ajratish usuli orqali izlaymiz:

```
> PDE:=diff(u(t,x),t)=a^2*diff(u(t,x),x,x);
struc:=pdsolve(PDE,HINT=T(t)*X(x));
```

$$PDE:=\frac{\partial}{\partial t}\mathrm{u}(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x)\right)$$

$$struc:=(\mathrm{u}(t,x)=\mathrm{T}(t)\,\mathrm{X}(x))\ \&where\left[\left\{\frac{d}{dt}\mathrm{T}(t)=\_c_1\,\mathrm{T}(t),\ \frac{d^2}{dx^2}\mathrm{X}(x)=\frac{\_c_1\,\mathrm{X}(x)}{a^2}\right\}\right]$$

```
> dsolve(diff(T(t),t)=_c[1]*T(t));
dsolve(diff(X(x),`$`(x,2))=_c[1]*X(x)/a^2);
```

$$\mathrm{T}(t)=\_C1\,\mathbf{e}^{(\_c_1\,t)}$$

$$\mathrm{X}(x)=\_C1\,\mathbf{e}^{\left(\frac{\sqrt{\_c_1}\,x}{a}\right)}+\_C2\,\mathbf{e}^{\left(-\frac{\sqrt{\_c_1}\,x}{a}\right)}$$

Quyidagicha almashtirish olamiz:

$\_c_1 \; = = - \; \lambda^2$

> dsolve(diff(T(t),t)=-lambda^2*T(t)*a^2);

dsolve(diff(X(x),`$`(x,2))=-lambda^2*X(x));

$$\mathrm{T}(t) = \_C1\, \mathbf{e}^{(-\lambda^2 a^2 t)}$$

$$\mathrm{X}(x) = \_C1 \sin(\lambda\, x) + \_C2 \cos(\lambda\, x)$$

Natijada umumiy yechim quyidagicha bo'ladi:

> u(t,x):=(C1*sin(lambda*x)+C2*cos(lambda*x))*exp(-lambda^2*a^2*t);

$$\mathrm{u}(t, x) := (C1 \sin(\lambda\, x) + C2 \cos(\lambda\, x))\, \mathbf{e}^{(-\lambda^2 a^2 t)}$$

$\mathrm{u}(t, x)$ - funksiya ixtiyoriy λ uchun tenglamani yechimi bo'ladi (bu yerda λ - uzluksiz - ∞ dan + ∞ da qiymatlarini o'zgartiruvchi parametr), xar bir λ uchun $\mathrm{C1}(\lambda)$ va $\mathrm{C2}(\lambda)$ mos koeffesiyentlar bo'ladi.

Shuning uchun quyidagiga ega bo'lamiz:

>

u[lambda](t,x):=(C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t);

$$u_\lambda(t, x) := (\mathrm{C1}(\lambda) \sin(\lambda\, x) + \mathrm{C2}(\lambda) \cos(\lambda\, x))\, \mathbf{e}^{(-\lambda^2 a^2 t)}$$

Natijada chiziqli bir jinsli tenglamani λ parametrga bog'lik yechimini quyidagicha tasvirlaymiz:

> u(t,x):=int(u[lambda](t,x), lambda=-infinity..infinity);

$$\mathrm{u}(t, x) := \int_{-\infty}^{\infty} (\mathrm{C1}(\lambda) \sin(\lambda\, x) + \mathrm{C2}(\lambda) \cos(\lambda\, x))\, \mathbf{e}^{(-\lambda^2 a^2 t)}\, d\lambda$$

$\mathrm{C1}(\lambda)$ va $\mathrm{C2}(\lambda)$ koeffisiyentlarni topish uchun boshlang'ich shartlardan foydalanamiz:

> u_0(t,x):=eval(subs(t=0, u(t,x)))=f(x);

$$\mathrm{u\_0}(t, x) := \int_{-\infty}^{\infty} \mathrm{C1}(\lambda) \sin(\lambda\, x) + \mathrm{C2}(\lambda) \cos(\lambda\, x)\, d\lambda = \mathrm{f}(x)$$

Bu ifoda $\mathrm{f}(x)$ funksiyani Fur'e integraliga yoyish bilan mos tushadi:

> f(x)=(1/(2*Pi))*int(int(f(xi)*cos(lambda*(xi-x)),xi=-infinity..infinity),lambda=-infinity..infinity);

$$f(x) = \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda(\xi - x))\,d\xi\,d\lambda\right)$$

Demak, $C1(\lambda)$ va $C2(\lambda)$ koeffisiyentlar qiyudagicha ifodalanadi:

> C1(lambda):=(1/(2*Pi))*int(f(xi)*sin(lambda*xi),xi=-infinity..infinity);
C2(lambda):=(1/(2*Pi))*int(f(xi)*cos(lambda*xi),xi=-infinity..infinity);

$$C1(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\,\xi)\,d\xi\right)$$

$$C2(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\,\xi)\,d\xi\right)$$

> u(t,x):=int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity);
u(t,x):=combine(int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity));

$$u(t,x) := \int_{-\infty}^{\infty}\left(\frac{1}{2}\left(\frac{\sin(\lambda\,x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\,\xi)\,d\xi\right) + \frac{1}{2}\left(\frac{\cos(\lambda\,x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\,\xi)\,d\xi\right)\right)\mathbf{e}^{(-\lambda^2 a^2 t)}\,d\lambda$$

$$u(t,x) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2}\,\frac{\mathbf{e}^{(-\lambda^2 a^2 t)}\,f(\xi)\cos(-\lambda\,x + \lambda\,\xi)}{\pi}\,d\xi\,d\lambda$$

Olingan ifoda almashtirish mumkin.

Buning uchun quyidagi integralni qaraymiz:

> int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi), lambda = -infinity .. infinity);

$$\int_{-\infty}^{\infty} \mathbf{e}^{(-\lambda^2 a^2 t)}\cos(-\lambda\,x + \lambda\,\xi)\,d\lambda$$

Integral ostidagi funksiyada o'zgaruvchilarni almashtirish olamiz:

> simplify(subs({xi=-v*a*t^(1/2)+x,lambda=w/(a*sqrt(t))},exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi)));

$$e^{(-w^2)}\cos(w\,v)$$

> Int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi),lambda = -infinity .. infinity)=(1/(a*sqrt(t)))*int(exp(-w^2)*cos(w*v),w = -infinity .. infinity);

$$\int_{-\infty}^{\infty} e^{(-\lambda^2 a^2 t)}\cos(-\lambda\,x+\lambda\,\xi)\,d\lambda = \frac{\sqrt{\pi}\,e^{\left(-\frac{v^2}{4}\right)}}{a\sqrt{t}}$$

> Int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi),lambda=-infinity..infinity)= subs(v=(x-xi)/a/t^(1/2),1/a/t^(1/2)*Pi^(1/2)*exp(-1/4*v^2));

$$\int_{-\infty}^{\infty} e^{(-\lambda^2 a^2 t)}\cos(-\lambda\,x+\lambda\,\xi)\,d\lambda = \frac{\sqrt{\pi}\,e^{\left(-\frac{(x-\xi)^2}{4a^2 t}\right)}}{a\sqrt{t}}$$

Natijada yechimni ko'rinishi quyidagicha:

> u(t,x):=(1/(2*a*sqrt(Pi*t)))*int(f(xi)*exp(-1/4*(x-xi)^2/a^2/t),xi = -infinity .. infinity);

$$u(t,x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\,t}}\int_{-\infty}^{\infty} f(\xi)\,e^{\left(-\frac{(x-\xi)^2}{4a^2 t}\right)}\,d\xi\right)$$

Tenglamani yechishga misollar

1- misol

> restart;

Bir jinsli tenglamani

$$\frac{\partial}{\partial t}u(t,x) = a^2\left(\frac{\partial^2}{\partial x^2}u(t,x)\right)$$

boshlang'ich shartlar bilan yeching

$$u(0,x) = f(x)\,,$$

bu yerda $f(x)$ funksiya quyidagicha berilgan:

> a:=1;L:=1;alpha:=1;l:=L/3;

f(xi):=u0*exp(-gamma^2*xi^2);

$$a := 1$$

$$L := 1$$

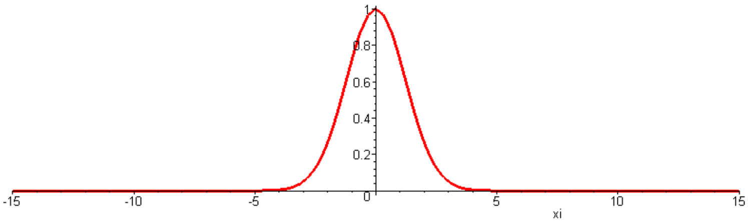
$$\alpha := 1$$

$$l := \frac{1}{3}$$

$$f(\xi) := u0\, e^{(-\gamma^2 \xi^2)}$$

```
> u0:=1;gamma:=0.5;
plot(f(xi), xi=-15..15, color=red,thickness=3);
```

$$u0 := 1$$



Yechish uchun 1.1. formuladan foydalanamiz:

```
> u(t,x):=1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*exp(-1/4*(x-xi)^2/a^2/t),xi = -infinity ..
infinity);
```

$$u(t,x) := \frac{1}{2} \frac{\begin{cases} \dfrac{2\, e^{\left(\frac{x^2 \gamma^2}{-4\gamma^2 t - 1}\right)} u0 \sqrt{\pi}}{\sqrt{\dfrac{4\gamma^2 t + 1}{t}}} & \operatorname{csgn}\left(\gamma^2 + \dfrac{1}{4t}\right) = 1 \\ \infty & \text{otherwise} \end{cases}}{\sqrt{\pi t}}$$

```
> u0:=1;beta:=0.5; a:=1;
```

$$u0 := 1$$

$$\beta := 0.5$$

$$a := 1$$

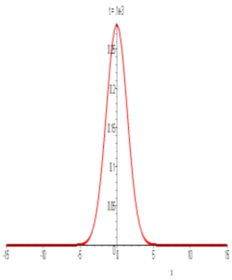
Tenglamani yechimi:

```
> with(plots):
u(t,x):=(1/(2*a*sqrt(Pi*t)))*exp(1/(-4*beta^2*a^2*t-
1)*x^2*beta^2)*u0/((4*beta^2*a^2*t+1)/a^2/t)^(1/2);
```

$$u(t,x) := \frac{1}{2} \frac{e^{\left(\frac{0.25 x^2}{-1.00 t - 1}\right)}}{\sqrt{\pi t} \sqrt{\dfrac{1.00 t + 1}{t}}}$$

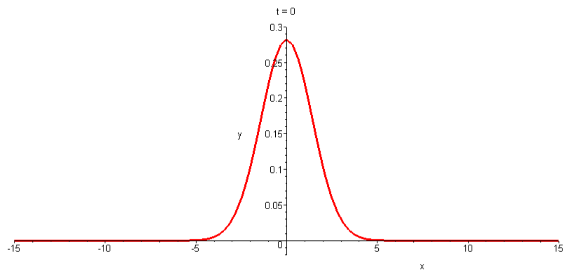
Olingan yechimni ikki o'lchamli animirlangan grafik ko'rinishida tasvirlaymiz:

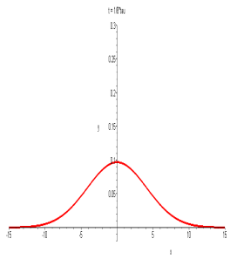
```
> animate(plot,[u(t,x),x=-15..15], t=0.0001..60, frames=30,thickness=3);
```

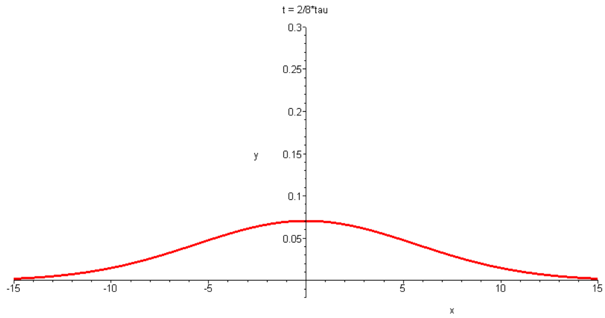


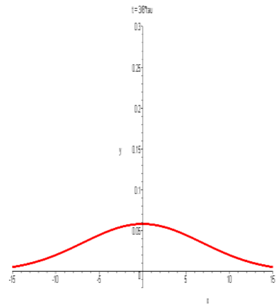
Olingan yechimni vaqtni bir nechta momentlarida ikki o'lchamli grafik ko'rinishida tasvirlaymiz:

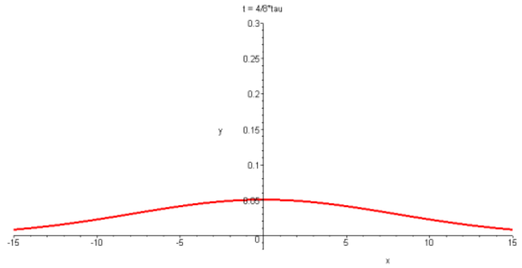
```
> tau:=60:
u_1(x):=subs(t=tau*0.000001,u(t,x)):
u_2(x):=subs(t=tau*(1/8),u(t,x)):
u_3(x):=subs(t=tau*(2/8),u(t,x)):
u_4(x):=subs(t=tau*(3/8),u(t,x)):
u_5(x):=subs(t=tau*(4/8),u(t,x)):
u_6(x):=subs(t=tau*(5/8),u(t,x)):
u_7(x):=subs(t=tau*(6/8),u(t,x)):
u_8(x):=subs(t=tau*(7/8),u(t,x)):
plot(u_1(x),x=-15..15,y=-0.02..0.3,title="t = 0", color=red,thickness=3);
plot(u_2(x),x=-15..15,y=-0.02..0.3,title="t = 1/8*tau",color=red,thickness=3);
plot(u_3(x),x=-15..15,y=-0.02..0.3,title="t = 2/8*tau",color=red,thickness=3);
plot(u_4(x),x=-15..15,y=-0.02..0.3,title="t = 3/8*tau",color=red,thickness=3);
plot(u_5(x),x=-15..15,y=-0.02..0.3,title="t = 4/8*tau",color=red,thickness=3);
plot(u_6(x),x=-15..15,y=-0.02..0.3,title="t = 5/8*tau",color=red,thickness=3);
plot(u_7(x),x=-15..15,y=-0.02..0.3,title="t = 6/8*tau",color=red,thickness=3);
plot(u_8(x),x=-15..15,y=-0.02..0.3,title="t = 7/8*tau",color=red,thickness=3);
```

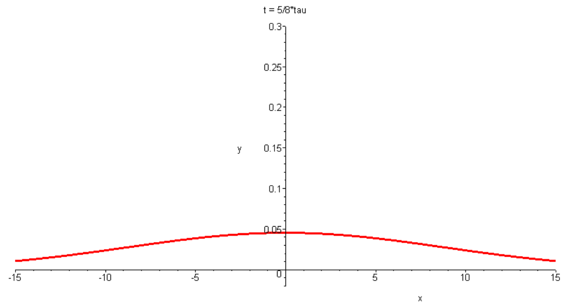


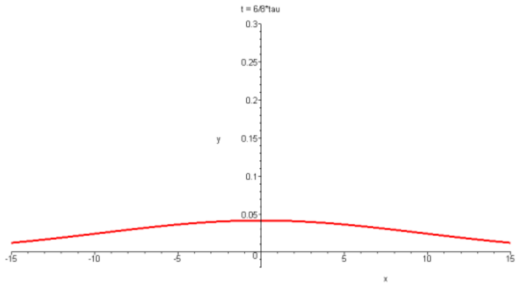


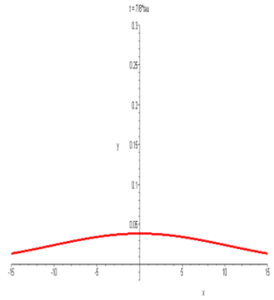












2 - Misol

```
> restart;
```

Bir jinsli tenglamani

$$\frac{\partial}{\partial t}\mathrm{u}(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x)\right)$$

boshlang'ich shartlar bilan yeching

$$\mathrm{u}(0,x)=\mathrm{f}(x)\ ,$$

bu yerda $\mathrm{f}(x)$ funksiya quyidagicha berilgan:

```
> a:=1;L:=2;alpha:=1;
f(x):=x->piecewise(x<-L,0, x<0,alpha*(1+x/L),x<L,alpha*(1-x/L), x>L,0);
```

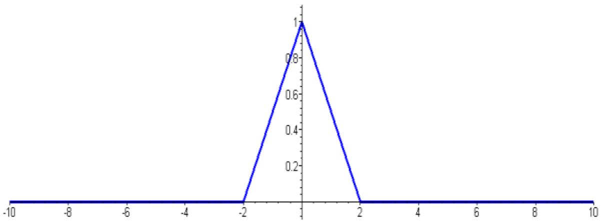
$$a := 1$$

$$L := 2$$

$$\alpha := 1$$

$$\mathrm{f}(x) := x \rightarrow \mathrm{piecewise}\left(x < -L,\ 0,\ x < 0,\ \alpha\left(1+\frac{x}{L}\right),\ x < L,\ \alpha\left(1-\frac{x}{L}\right),\ L < x,\ 0\right)$$

```
> plot(f(x),-10..10,-0.1..1.1, numpoints=400,color=blue,thickness=3);
```



> restart;

f(xi):=xi->piecewise(xi<-L,0, xi<0,alpha*(1+xi/L),xi<L,alpha*(1-xi/L), x>L,0);

$$f(\xi) := \xi \to \text{piecewise}\left(\xi < -L, 0, \xi < 0, \alpha\left(1+\frac{\xi}{L}\right), \xi < L, \alpha\left(1-\frac{\xi}{L}\right), L < x, 0\right)$$

Yechish uchun 1.1. formuladan foydalanamiz:

> u(t,x):=simplify(1/2*1/a/(Pi*t)^(1/2)*int(alpha*(1+xi/L)*exp(-1/4*(x-xi)^2/a^2/t),xi = -L .. 0)+1/2*1/a/(Pi*t)^(1/2)*int(alpha*(1-xi/L)*exp(-1/4*(x-xi)^2/a^2/t),xi = 0 .. L));

$$u(t,x) := \frac{1}{2}\alpha\left(\sqrt{\pi}\sqrt{t}\,\mathrm{erf}\left(\frac{L+x}{2a\sqrt{t}}\right)L + 2at\,\mathbf{e}^{\left(-\frac{(L+x)^2}{4a^2t}\right)} + x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{L+x}{2a\sqrt{t}}\right) - 4at\,\mathbf{e}^{\left(-\frac{x^2}{4a^2t}\right)} - 2x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{x}{2a\sqrt{t}}\right) + \sqrt{\pi}\sqrt{t}\,\mathrm{erf}\left(\frac{L-x}{2a\sqrt{t}}\right)L + 2at\,\mathbf{e}^{\left(-\frac{(L-x)^2}{4a^2t}\right)} - x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{L-x}{2a\sqrt{t}}\right)\right) \Big/ \left(\sqrt{\pi}\sqrt{t}\,L\right)$$

> a:=1;L:=2;alpha:=1;

$$a := 1$$

$$L := 2$$

$$\alpha := 1$$

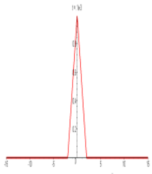
Olingan yechimni ikki o'lchamli animirlangan grafik ko'rinishida tasvirlaymiz:

> with(plots):

u(t,x):=-1/2*alpha*(-Pi^(1/2)*t^(1/2)*erf(1/2*(L+x)/a/t^(1/2))*L-2*a*t*exp(-1/4*(L+x)^2/a^2/t)-t^(1/2)*x*Pi^(1/2)*erf(1/2*(L+x)/a/t^(1/2))+4*a*t*exp(-1/4/a^2/t*x^2)+2*t^(1/2)*x*Pi^(1/2)*erf(1/2/a/t^(1/2)*x)-Pi^(1/2)*t^(1/2)*erf(1/2*(L-x)/a/t^(1/2))*L-2*a*t*exp(-1/4*(L-x)^2/a^2/t)+t^(1/2)*x*Pi^(1/2)*erf(1/2*(L-x)/a/t^(1/2)))/Pi^(1/2)/t^(1/2)/L;

animate(plot,[u(t,x),x=-15..15], t=0.0001..15, frames=30,thickness=3);

$$u(t,x) := -\frac{1}{4}\left(-2\sqrt{\pi}\sqrt{t}\,\mathrm{erf}\left(\frac{2+x}{2\sqrt{t}}\right) - 2\,t\,\mathbf{e}^{\left(-\frac{(2+x)^2}{4\,t}\right)} - x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2+x}{2\sqrt{t}}\right) + 4\,t\,\mathbf{e}^{\left(-\frac{x^2}{4\,t}\right)} + 2\,x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{x}{2\sqrt{t}}\right) - 2\sqrt{\pi}\sqrt{t}\,\mathrm{erf}\left(\frac{2-x}{2\sqrt{t}}\right) - 2\,t\,\mathbf{e}^{\left(-\frac{(2-x)^2}{4\,t}\right)} + x\sqrt{t}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2-x}{2\sqrt{t}}\right)\right) \Big/ \left(\sqrt{\pi}\sqrt{t}\right)$$



3 - Misol

> restart;

Bir jinsli tenglamani

$$\frac{\partial}{\partial t}u(t,x) = a^2\left(\frac{\partial^2}{\partial x^2}u(t,x)\right)$$

boshlang'ich shartlar bilan yeching

$$u(0,x) = f(x)\,,$$

bu yerda $f(x)$ funksiya quyidagicha berilgan:

```
> a:=1;L:=2;alpha:=1;
f(x):=x->piecewise(x<-L,0, x<L,alpha, x>L,0);
```

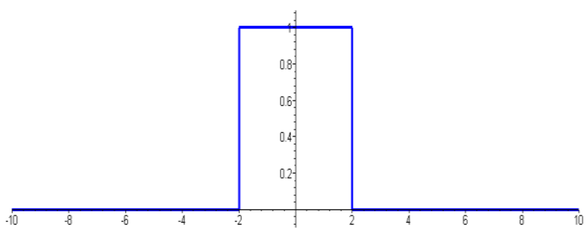
$$a := 1$$

$$L := 2$$

$$\alpha := 1$$

$$f(x) := x \to \mathrm{piecewise}(x < -L, 0, x < L, \alpha, L < x, 0)$$

```
> plot(f(x),-10..10,-0.1..1.1, numpoints=400,color=blue,thickness=3);
```




```
> restart;
> f(xi):=xi->piecewise(xi<-L,0, xi<L,alpha, xi>L,0);
```

$$f(\xi) := \xi \to \mathrm{piecewise}(\xi < -L, 0, \xi < L, \alpha, L < \xi, 0)$$

Yechish uchun 1.1. formuladan foydalanamiz:

> u(t,x):=simplify(1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*exp(-1/4*(x-xi)^2/a^2/t),xi = -L .. L));

$$u(t,x) := \frac{1}{2} c \left(\operatorname{erf}\left(\frac{L+x}{2\,a\,\sqrt{t}} \right) + \operatorname{erf}\left(\frac{L-x}{2\,a\,\sqrt{t}} \right) \right)$$

> a:=1;L:=2;alpha:=1;

$$a := 1$$

$$L := 2$$

$$\alpha := 1$$

Tenglamani yechimi:

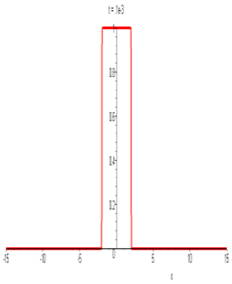
> with(plots):

u(t,x) := 1/2*(erf(1/2*(L+x)/a/t^(1/2))+erf(1/2*(L-x)/a/t^(1/2)));

$$u(t,x) := \frac{1}{2} \operatorname{erf}\left(\frac{2+x}{2\sqrt{t}} \right) + \frac{1}{2} \operatorname{erf}\left(\frac{2-x}{2\sqrt{t}} \right)$$

Olingan yechimni ikki o'lchamli animirlangan grafik ko'rinishida tasvirlaymiz:

> animate(plot,[u(t,x),x=-15..15], t=0.0001..15, frames=30,thickness=3);



2-Bob. **Yarim to'g'ri chiziqda issiqlik o'tkazuvchanlik tenglamasini Fur'ye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish**

2.1-§. Umumiy 1-tur chegaraviy masala va uni yechishni sodda holga keltirish usuli

Endi issiqlik tarqalishining bir jinsli bo'lmagan tenglamasiga qo'yilgan bir jinsli bo'lmagan boshlang'ich va 1-tur chegaraviy shartni qanoatlantiruvchi yechimni topish masalasini, ya'ni

$$u_t = a^2 u_{xx} + f(x,t),\ 0 < x < \ell, t > 0 \qquad (2.1.1)$$

issiqlik tarqalish tenglamasining

$$u(x,0) = \varphi(x),\ 0 \le x \le \ell \qquad (2.1.2)$$

boshlang’ich shartni hamda

$$u(0,t)=\mu_1(t),\ \ u(\ell,t)=\mu_2(t),\ \ t\ge 0 \qquad (2.1.3)$$

chegaraviy shartlarni qanoatlantiruvchi va $0\le x\le \ell,\, t\ge 0$ sohada aniqlangan ikkinchi tartibgacha uzluksiz aynan nolga teng bo’lmagan yechimini topish masalasini qaraymiz. Bunda $\varphi(x),\, 0\le x\le \ell$ va $\mu_i(t),\ \ t\ge 0, i=1,2$ lar berilgan funksiyalar bo’lib, o’z argumentlarining uzluksiz differensiallanuvchi funksiyalaridir.

Ushbu masalani yordamchi kiritish bilan avval o’rganilgan soddaroq chegaraviy masalalarni yechishga keltirish mumkin. Haqiqatan ham (2.1.1) tenglamaning yechimini

$$u(x,t)=v(x,t)+w(x,t) \qquad (2.1.4)$$

ko’rinishda izlasak va undan kerakli xususiy hosilalarni olib (2.1.1), (2.1.2) va (2.1.4) ga qo’ysak va unda $w(x,t)$ yordamchi funksiyani

$$\mu_1(t)=w(0,t),\ \ \mu_2(t)=w(\ell,t)$$

shartlarni qanoatlantirsin deb, masalan

$$w(x,t)=\mu_1(t)+\frac{x}{\ell}[\mu_2(t)-\mu_1(t)] \qquad (2.1.5)$$

kabi tanlasak (bunday funksiyalar yagona emas), $v(x,t)$ funksiya uchun (2.1.1)-(2.1.3) masalaga o’xshash bo’lgan

$$v_t=a^2v_{xx}+f_1(x,t),\ 0<x<\ell, t>0$$

$$v(x,0)=\varphi_1(x),\ 0\le x\le \ell$$

$$v(0,t)=v(\ell,t)=0,\ \ t\ge 0$$

masalani yechish masalasiga kelamiz. Bunda

$$f_1(x,t)=f(x,t)+a^2w_{xx}(x,t)-w_t(x,t)\,,\ \varphi_1(x)=\varphi(x)-w(x,0)$$

aniq ko’rinishga ega bo’lgan berilgan uzluksiz differensiallanuvchi funksiyalardir. Bu masalani (2.1.1)-(2.1.3) masalani yechish usulidagi kabi yechib, uni va (2.1.5) ni (2.1.3) ga qo’yib, (2.1.1) issiqlik tarqalish tenglamasining (2.1.2) va (2.1.3) shartlarni qanoatlantiruvchi yechimini olamiz.

To’g’ri to’rtburchak membranasi tebranishlari

Membrana deb bizlar juda yupqa plenkani tushunamiz, u torga o’xshab faqat cho’zilishga ishlaydi, bukilishga ishlamaydi. Agarda membrana tekis tortilishi kuchi ta’sirda bo’lsa, muvozanat holatida *(x, y)* tekislikda joylashgan bo’lsa, (bizlar faqat *Oz* o’qiga parallel siljishlarni o’rganamiz) shunda *(x, y)* membrana nuqtasini siljishi *x, y, t* o’zgaruvchilarni u funksiyasi bo’ladi va bu funksiya tor tenglamasiga o’xshab analogik quyidagi differensial tenglamani qanoatlantiradi

$$u_{tt} = a^2(u_{xx} + u_{yy}) + f(x, y, t), \qquad (2.1.6)$$

Bu yerda

$$a = \sqrt{\frac{T_0}{p}},$$

P- membrana sirtining zichligi, ρ- f tashqi kuch yoki yuklama.

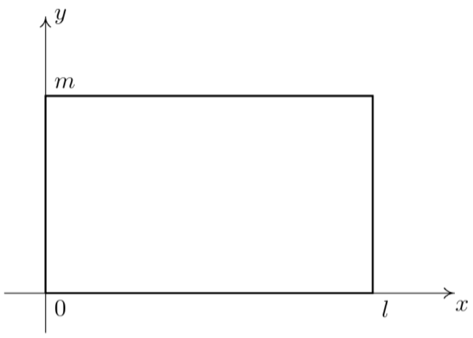
(2.1.6) differensial tenglamadan tashqari bizlar chegaraviy shartlarni inobatga olish kerak. $u$funksiya S kontorda ya’ni membrana chegarasida qonoatlantiradigan shartlar

$$u = 0. \qquad (2.1.7)$$

Holatni qaraymiz. Bundan tashqari boshlang’ich shartlar berish kerak ya’ni boshlang’ich momentlarni barcha nuqtalarini siljishi

$$u|_{t=0} = \varphi 1(x, y), \; u_t|_{t=0} = \varphi 2(x, y) \qquad (2.1.8)$$

To’rtburchakli membrana ozod tebranishlari ko’zatamiz



Kontur *(x, y)* tekislikda

$$x = 0, x = l, y = 0, y = m$$

ko’rinishdagi to’g’riturtburchakni xosil qiladi. Tashqi kuch yo’q deb hisoblaymiz ya’ni $f=0$

$$u_{tt}=a^2(u_{xx}+u_{yy}), \tag{2.1.9}$$

(2.1.9) tenglamani (2.1.7), (2.1.8) shartlarni qanoatlantiruvchi yechimini topamiz.

Furye usulidan foydalanib (2.1.9) tenglamani yechimini qo’yidagi shakldа izlaymiz

$$(\alpha\cos\omega t+\beta\sin\omega t)U(x,y) \tag{2.1.10}$$

Bu esa bizlarga quyidagini beradi.

$$-\omega^2(\alpha\cos\omega t+\beta\sin\omega t)U(x,y)=a^2(U_{xx}+U_{yy})(\alpha\cos\omega t+\beta\sin\omega t),$$

$\frac{\omega^2}{a^2}=k^2$, belgilash keritamiz.

U funksiya uchun qo’yidagi tenglamani olamiz

$$U_{xx}+U_{yy}+k^2U=0,$$

Bu tenglamani xususiy yechimini quyidagi ko’rinishda axtaramiz

$$U(x,y)=X(x)Y(y),$$

Bu yerda

$$X''(x)Y(y)+X(x)Y''(y)+k^2X(x)Y(y)=0,$$

$$\frac{X''(x)}{X(x)}=-\frac{Y''(y)+k^2Y(y)}{Y(y)}=-\lambda^2,\lambda=const.$$

Shunday qilib quyidagi sistemani hosil qilamiz

$$\begin{cases}X''(x)+\lambda^2X(x)=0\\ Y''(y)+\mu^2Y(y)=0;\end{cases}\qquad \mu^2=k^2-\lambda^2. \tag{2.1.11}$$

(2.1.11) tenglamalar bizlarga $X(x)$ va $Y(y)$ funksiyalarni quyidagi umumiy ko’rinishlari beradi

$$X(x)=C_1\sin\lambda x+C_2\cos\lambda x,$$

$$Y(y)=C_3\sin\mu x+C_4\cos\mu x.$$

$u=0,$ shartdan bizlar $U(x,y)=0$ xosil qilamiz. Bu shart esa qo'yidagi shartlarga ajralib ketadi.

$\mathrm{X}(0)=0, X(l)=0, \mathrm{Y}(0)=0, Y(m)=0$

Bu yerda aniq ko'rinib turibdiki $C_2=C_4=0,$ va agarda bizlar $C_1 \neq C_3$ ligidan va doimiy ko'paytuvchilarni olib tashqariga chiqarsak

$$X(x)=\sin\lambda l,$$
$$Y(y)=\sin\mu m,$$

Bu yerda

$$\sin\lambda l=0, \quad \sin\mu m=0. \tag{2.1.12}$$

(2.1.12) tenglamadan

$\lambda=\frac{\pi n_1}{l}, \quad \mu=\frac{\pi n_2}{m}, \quad n_1, n_2 \in N$ larni topomiz so'ngra esa k doimiy qiymatlarini topish mumkin,

$$\mathrm{k}^2\mathrm{n}_1\mathrm{n}_2=\lambda^2\mathrm{n}_1+\mu^2\mathrm{n}_2=\pi^2\left(\frac{n_1^2}{l^2}+\frac{n_2^2}{m^2}\right)$$

va ω chastotani topilgan

$$\omega^2{}_{n_1n_2}=a^2k^2{}_{n_1n_2}=a^2\pi^2\left(\frac{n^2}{l^2}+\frac{n_2^2}{m^2}\right)$$

(2.1.12) tenglamaga qo'yib qo'yidagini hosil qilamiz

$$\left(\alpha_{n_1n_2}\cos\omega_{n_1n_2}t+\beta_{n_1n_2}\sin\omega_{n_1n_2}t\right)\sin\frac{\pi n_1x}{l}\sin\frac{\pi n_2y}{m}.$$

α, β doimiy boshlang'ich shartlarda aniqlanadi

$$u=\sum_{n_1,n_2=1}^{\infty}\left(\alpha_{n_1n_2}\cos\omega_{nqn_2}t+\beta_{n_1n_2}t\right)\sin\frac{\pi n_1x}{l}\sin\frac{\pi n_2y}{m},$$

$$u_t=\sum_{n_1,n_2=1}^{\infty}\omega_{n_1n_2}\left(\beta_{n_1n_2}\cos\omega_{n_1n_2}t-\alpha_{n_1n_2}\sin\omega_{n_1n_2}t\right)\sin\frac{\pi n_1x}{l}\sin\frac{\pi n_2y}{m},$$

Yo'qoridagi formulada $t=0$ deb olsak va (2.1.10) formulaga asoslanib qo'yidagilarni hosil qilamiz.

$$u_t\big|_{t=0}=\varphi 1(x,y)=\sum_{n_1,n_2=1}^{\infty}\alpha_{n_1n_2}\sin\frac{\pi n_1x}{l}\sin\frac{\pi n_2y}{m},$$

$$u_t\big|_{t=0} = \varphi 2(x,y) = \sum_{n_1,n_2=1}^{\infty} \beta_{n_1 n_2}\omega_{n1n2} \sin\frac{\pi n_1 x}{l}\sin\frac{\pi n_2 y}{m},$$

Bu formulada φ_1 φ_2 funksiyalarni ikkinchi tur Furye qatori yoyilmasi bo'ladi. Va α β kaeffesiyentlar esa qo'yidagi formula bo'yicha aniqlanadi

$$\alpha_{n_1 n_2} = \frac{4}{lm}\int_0^l \int_0^m \varphi_{1(\xi,\eta)} \sin\frac{\pi n_1 \xi}{l}\sin\frac{\pi n_2 \eta}{m}\, d\xi dn,$$

$$\beta_{n_1 n_2} = \frac{4}{\omega_{n_1 n_2} lm}\int_0^l \int_0^m \varphi_{2(\xi,\eta)} \sin\frac{\pi n_1 \xi}{l}\sin\frac{\pi n_2 \eta}{m}\, d\xi dn,$$

Bularni hammasi qo'yilgan masalani yechimini beradi. Bizlar qaragan membrana tor bilan nimasi bilan farq qiladi. Torda har qanday chastotada o'zini tebranishlariga mos bo'ladigan vabirnechta qismga ajraladigan va va tugun yordamida bir necha qismlarga ajraladi toning o'zini formasi mos keladi.

Membrana uchun bitta chastotaga membrana bir necha formasi har xil tugunlar bilan formasiga mos kelishi mumkin. Boshqacha aytganda membrana tebranish chastotasi nolga Doiradi.

Doira membranasini tebranishlari. Bessel tenglamasi

Doira membranasini erkin tebranishlarini tekshirish markazi koordinata boshida bo'lgan radiusi l Doira kontorida membranasini siljitmasdan hisoblaymiz. To'g'ri burchakli koordinatalarga (x,y) mos (r,θ) qutb koordinatalarini kiritamiz.

$$u\big|_{r=l} = 0.$$

$$u_{tt} = a^2(u_{xx} + u_{yy})$$

Tenglamani xususiy yechimini

$$(\alpha\cos\omega t + \beta\sin\omega t)U(r,\theta).$$

ko'rinishda izlaymiz.

$U(r,\theta)$ funksiya uchun qo'yidagi differensial tenglamani olamiz.

$$U_{xx} + U_{yy} + k^2 U = 0 \qquad (2.1.13)$$

Faqat yangi (r,θ) o'zgaruvchiga almashtirish kerak, buni uchun Laplas operatorini ifodalash yetarli.

$$\Delta U = U_{xx} + U_{yy}. \tag{2.1.14}$$

Uch o'zgaruvchili Laplas operatori quyidagicha bo'ladi

$$\Delta U = U_{xx} + U_{yy} + U_{zz}$$

Silindrik koordinatalarda ifodalaymiz

$$\begin{cases} x = \rho\cos\varphi, \\ y = \rho\sin\varphi, \\ z = z; \end{cases}$$

Shu ko'rinishda

$$\Delta U = \frac{1}{\rho}\left(\rho U_{\rho\rho} + \frac{1}{\rho}U_{\varphi\varphi} + \rho U_{zz}\right)$$

U ni z dan bog'liqmas hisoblab (2.1.14) ni qutb koordinatalar orqali ifodalaymiz. Bundan keyin radius vektor uzunligini r harfi bilan belgilaymiz. Qutbiy burchakni θ harfi bilan belgilaymiz.

$$U_{xx} + U_{yy} = U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta}$$

Xuddi shunday (2.1.13) ni quyidagi ko'rinishda yozamiz.

$$U_{rr} + \frac{1}{r}U_r + \frac{1}{r^2}U_{\theta\theta} + k^2U = 0$$

Uning xususiy yechimini quyidagi ko'rinishda topamiz.

$$U(r,\theta) = T(\theta)R(r),$$

Bulardan quyidagilar keladi.

$$T(\theta)\left[R''(r) + \frac{1}{r}R'(r) + k^2R(r)\right] + \frac{1}{r^2}T''(\theta)R(r) = 0,$$

$$\frac{T''(\theta)}{T(\theta)} = -\frac{r^2R''(r) + rR'(r) + k^2r^2R(r)}{R(r)} = -\lambda^2, \lambda = const.$$

Ikkita tenglamani olamiz.

$$T''(\theta)+\lambda^2 T(\theta)=0 \qquad (2.1.15)$$

$$R''(r)+\frac{1}{r}R'(r)+\left(k^2-\frac{\lambda^2}{r^2}\right)R(r)=0 \qquad (2.1.16)$$

(2.1.15) tenglamani umumiy yechimini ko'rinishi quyidagicha bo'ladi.

$$T(\theta)=C\cos\lambda\theta+D\sin\lambda\theta.$$

U funksiya bir qiymatli davriy va davri 2π ga teng bo'lsin, u holda $T(\theta)$ funksiya xuddi yuqoridagidek xossalarga ega, λ butun son, λ faqat musbat qiymatlarni qabul qiladi, $\lambda=0,1,2,...,n,...$ ga mos $T(\theta)$ va $R(r)$ funksiyalarni quyidagicha ifodalaymiz.

$$T_0(\theta),T_1(\theta),T_2(\theta),...,T_n(\theta),...,R_0(0),R_1(r),R_2(r),...,R_n(r),...$$

Shu yo'l bilan cheksiz to'plamda (2.1.1) tenglamaniyechimini qo'yidagi ko'rinishini olamiz.

$$(\alpha\cos\omega t+\beta\sin\omega t)(C\cos\theta+D\sin n\theta)R_n(r),\ \omega=ak.$$

$R_n(r)$ funksiya (20) tenglamani qanoatlantiradi, agar λ ni n ga almashtirsak

$$R''_n(r)+\frac{1}{r}R'_n(r)+\left(k^2-\frac{n^2}{r^2}\right)R_n(r)=0$$

Integralning umumiy tenglamasi quyidagicha bo'ladi.

$$R_n(r)=C_1J_n(kr)+C_2K_n(kr),$$

Bu yerda $J_n(x)$ - birinchi tur Bessel funksiyasi va $K_n(x)$ - ikkinchi tur Bessel funksiyasi.

2.2-§ Issiqlik o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masala.

1. Birinchi chegaraviy masala yechimining mavjudligi. O'zgaruvchilarni ajratish usuli.

Birinchi chegaraviy masalaga kengroq to'xtalib o'tamiz:

$$[2.2.1]\begin{cases} u_t = a^2 u_{xx} + f(x,t), 0 < x < l, 0 < t \le T \\ u(0,t) = \mu_1(t), \qquad 0 \le t \le T \\ u(l,t) = \mu_2(t), \qquad 0 \le t \le T \\ u(x,0) = \phi(x), \qquad 0 \le x \le l \end{cases}$$

Yechimning mavjud va yagonaligini qarab o'tamiz, shu bilan birga turhunligini va **Grinn funksiyasini** qo'llashini qaraymiz. Birinchi chegaraviy masalaning yechima nima. Aniqki, birjinsli issiqlik o'tkazuvchanlik tenglamasi holatida $\tilde{u}(x,t)$ uzilishga ega bo'lgan funksiyalar tuplami qanoatlantiradi:

$$\tilde{u}(x,t) = const, \ (x,t) \in Q_T = \{(x,t) : (0;1) \times (0;T)\};$$
$$\tilde{u}(0,t) = \mu_1(t); \ 0 \le t \le T;$$
$$\tilde{u}(l,t) = \mu_2(t); \ 0 \le t \le T;$$
$$\tilde{u}(x,0) = \phi(x); \ 0 \le x \le l.$$

Shuning uchun funksiya dan uzluksizlikni talab qilamiz, bu talab bilan keyinchalik biz barcha funksiyani o'rganishdagi noqulayliklar bartaraf etamiz.

Ta'rif. u(x,t) funksiya **[2.2.1] issiqlik o'tkazuvchanlik tenglamasi uchun 1-chegaraviy masalasining yechimi** deyiladi, agar u quyidagi 3 shartni qanoatlantirsa:

$$1. \ u \in C[\overline{Q_T}];$$
$$2. \ u_t, u_{xx} \in C[Q_T];$$
$$3. \ u(x,t) \qquad [2.2.2]$$

Bir jinsli issiqlik o'tkazuvchanlik tenglamasi nolinchi chegaraviy shartlar bilan berilgan birinchi chegaraviy masala uchun yechimni topamiz:

$$[2.2.3] \quad \begin{cases} (1) \ u_t = a^2 u_{xx}, 0 < x < l, 0 < t \le T; \\ (2) \ u(0,t) = 0, \ 0 \le t \le T; \\ (3) \ u(l,t) = 0, \ 0 \le t \le T; \\ (4) \ u(x,0) = \phi(x), 0 \le x \le l. \end{cases}$$

Yechimni quyidagi yo'l bilan aniqlaymiz, avvalo berilgan tenglamani almashtirish yordamida biror *u(x,t)* funksiyani tuzatamiz, keyin esa, boshlang'ich shartlarga qo'yilgan ma'lum bir cheklanishlarda biz tuzgan funksiya 1-chi chegaraviy masalaning yechimi bo'lishini isbotlaymiz.

Yangi funksiyani aniqlaymiz:

$$v(x,t) = X(x)T(t.)$$

Funksiyamizni issiqlik o'tkazuvchanlik tenglamasiga qo'yib quyidagini hosil qilamiz: $X(x)T'(t) = a^2 X''(x)T(t).$

Tenglikning ikki tomonini ham $a^2 X(x)T(t)$ ga bo'lamiz:

$$\frac{T'(t)}{a^2 T(t)} = \frac{X''(x)}{X(x)}$$

O'ng va chap tomondagi funksiyalar har xil o'zgaruvchilarga bog'lik bo'lganligi tufayli, aniqki ularning har ikkalasi ham biror konstantaga teng bo'ladi, biz uni $-\lambda$ bilan belgilaymiz:

$$\frac{T'(t)}{a^2 T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$

Bundan 2 ta tenglamaga ega bo'lamz:

$$X''(x) + \lambda X(x) = 0; \qquad \textbf{(2.2.4)}$$

$v(x,t)$ funksiyamiz uchun chegaraviy shartlarni yozib olamiz:

$$\begin{cases} v(0,t) = 0; \\ \qquad\qquad\qquad t \in [0;T], \\ v(l,t) = 0. \end{cases}$$

Quyidagini hosil qilamiz:

$$\begin{cases} X(0) = 0 \\ X(l) = 0. \end{cases}$$

(2.2.4) ni xosil bo'lgan sistema bilan birlashtirsak, **Shturm-Liuvill masalasini** hosil qilamiz:

$$\begin{cases} X''(x) + \lambda X(x) = 0; \\ X(0) = 0; \\ X(l) = 0. \end{cases}$$

Barcha λ larni topish talab qilinadi.

Differensial tenglama kursidan malumki,

$$\begin{cases} \lambda_n = \left(\frac{\pi n}{l}\right)^2, n \in N \\ X_n(x) = c_n^1 \sin\left(\frac{\pi n}{l} x\right), n \in N \end{cases}$$

λ_n ni (2.2.4) ga qo'yib, quyidagi ko'rinishdagi tenglikni hosil qilamiz:

$$T_n'(t) + a^2\lambda_n T_n(t) = 0.$$

Yechim

$$T_n = c_n^2 \exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\} \text{ bo'ladi.}$$

$X_n(x)$ *va* $T_n(t)$ ni birlashtirib quyidagini hosil qilamiz:

$$v_n(x,t) = X_n(x)T_n(t) = c_n \sin\left(\frac{\pi n}{l}x\right)\exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\}$$

Qayd etib o'tamizki, xamma shunday funksiyalar (2.2.1) issiqlik o'tkazuvchanlik tenglamasining yechimi va (2.2.2), (2.2.3) chegaraviy shartlarni qanoatlantiradi. u(x,t) funksiyani qatorning yig'indisi sifatida aniqlaymiz:

$$u(x,t) = \sum_{n=1}^{\infty} v_n(x,t)$$

Takidlab o'tamizki bu chegaraviy shartlarni qanoatlantiradi. Konstantalarni shunday tanlaymizki, boshlang'ich shartlar bajarilsin:

$$\phi(x) = u(x,0) = \sum_{n=1}^{\infty} v_n(x,0) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{\pi n}{l}x\right)$$

Tenglikni $\sin\left(\frac{\pi m}{l}x\right)$ ga ko'paytiramiz (m-butun). $x \to s$ almashtirish olamiz va s bo'yicha integrallaymiz:

$$\int_0^l \phi(s)\sin\left(\frac{\pi m}{l}s\right)ds = \sum_{n=1}^{\infty} c_n \int_0^l \sin\left(\frac{\pi m}{l}s\right)\sin\left(\frac{\pi n}{l}s\right)ds.$$

$$\int_0^l \sin\left(\frac{\pi n}{l}x\right)\sin\left(\frac{\pi m}{l}x\right) = \begin{cases} 0, n \neq m; \\ \frac{l}{2}, n - m. \end{cases} \Rightarrow \int_0^l \phi(s)\sin\left(\frac{\pi m}{l}s\right)ds = \frac{l}{2}c_m \Rightarrow \qquad c_m = \frac{2}{l}\int_0^l \phi(s)\sin\left(\frac{\pi m}{l}s\right)ds$$

Natijada *u(x,t)* uchun quyidagi formulani hosil qilamiz:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{l}\left(\int_0^l \phi(s)\sin\left(\frac{\pi n}{l}s\right)ds\right)\sin\left(\frac{\pi n}{l}x\right)\exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\}. \qquad \textbf{(2.2.5)}$$

1. Fazoda issiqlik o'tkazuvchanlik tenglamasini chiqarilishi

Uch o'lchovli fazoda biror issiqlik o'tkazuvchi va koordinatalari (x, y, z) bo'lgan ixtiyoriy M nuqtaning temperaturasi t vakt momentida $u(x, y, z, t)$ funksiya ko'rinishida beriluvchi jismni qaraymiz. Ma'lumki, issiqlik potoki vektori uchun $\vec{W}$ quyidagi Fur'ye qonuni deb ataluvchi formula o'rinlidir.

$$\vec{W} = -k\ grad\, u$$

Bu yerda $k(x, y, z)$ - issiqlik o'tkazuvchanlik koeffitsiyenti.

Agar jism E^3 fazoda berilgan bo'lsa Ω soxaning chegarasi Σ bo'ladi. Shunda jismning issiqlik miqdori t vaqt momentida quyidagi formula bilan hisoblanadi:

$$Q_2 - Q_1 = \iiint\limits_{\Omega} c(M)\rho(M)u(M, t_2)d\tau_M - \iiint\limits_{\Omega} c(M)\rho(M)u(M, t_1)d\tau_M =$$
$$= (t_2 - t_1)\iiint\limits_{\Omega} c(M)\rho(M)u_t(M, t_3)d\tau_M$$

$[t_1; t_2]$ $\left(Q(t_1) = Q_1, Q(t_2) = Q_2\right)$ vaqt oralig'ini qaraymiz. Shunda

$$Q_2 - Q_1 = \iiint\limits_{\Omega} c(M)\rho(M)u(M, t_2)d\tau_M - \iiint\limits_{\Omega} c(m)\rho(M)u(m, t_1)d\tau_M$$

bo'ladi. Issiqlik miqdorining o'zgarishi tashqaridan issiqlik oqib kelish natijasida va ba'zi ichki manbaning (stoklarning) harakati tufayli ro'y beradi:

$$Q_2 - Q_1 = \int\limits_{t_1}^{t_2}\left[-\iint\limits_{\Sigma}(\vec{W}, \vec{n})dv\right]dt + \int\limits_{t_1}^{t_2}\left[\iiint\limits_{\Omega} F(M, t)d\tau\right]dt$$

Birinchi integral uchun Ostogradskiy-Gauss formulasini qo'llaymiz va o'rta qiymat haqidagi formulani esa ikkinchi integral uchun qo'llaymiz:

$$Q_2 - Q_1 = -\int\limits_{t_1}^{t_2}\left[\iiint\limits_{\Omega}(div\vec{W})d\tau\right]dt + (t_1 - t_2)\iiint\limits_{\Omega} F(M, t_4)d\tau$$

Bu yerda $t_4 \in [t_1; t_2]$ ga qarashli.

Lagranj formulasidan quyidagi silliq (buni faraz qilamiz) u funksiya uchun foydalanamiz:

$$u(M, t_2) - u(M, t_1) = u_t(M, t_3)(t_2 - t_1), \quad t_3 \in [t_1; t_2]$$

Bundan quyidagini hosil qilamiz:

$$Q_2 - Q_1 = \iiint\limits_{\Omega} c(M)\rho(M)u(M, t_2)d\tau_M - \iiint\limits_{\Omega} c(M)\rho(M)u(M, t_1)d\tau_M =$$
$$= (t_2 - t_1)\iiint\limits_{\Omega} c(M)\rho(M)u_t(M, t_3)d\tau_M$$

Demak,

$$\left(t_2-t_1\right)\iiint_{\Omega} c(M)\rho(M)u_t(M,t_3)d\tau_M=-\int_{t_1}^{t_2}\left[\iiint_{\Omega}(div\vec{W})drM\right]dt+(t_1-t_2)\iiint_{\Omega}F(M,t_4)dr\,.$$

Endi hamma integral uchun umumlashtirilgan o'rat qiymat formulani qo'llaymiz:

$$c(M_1)\rho(M_1)u_t(M_1,t_3)V_\Omega(t_2-t_1)=-div\vec{W}\Big|_{\substack{t=t_5\\ M=M_2}}V_\Omega(t_2-t_1)+F(M_3,t_4)V_\Omega(t_2-t_1)\,,$$

Bunda $t_5\in[t_1;t_2]; M_1,M_2\in\Omega,\ V_\Omega-\Omega$ ning hajmi bo'ladi. $V_\Omega(t_2t_1)$ ga qisqartirib, Ω dan olingan biror bir M_1,M_2 nuqtalar uchun quyidagini hosil qilamiz:

$$c(M_1)\rho(M_1)u_t(M_1,t_3)V_\Omega(t_2-t_1)=-div\vec{W}\Big|_{\substack{t=t_3\\ M=M_2}}+F(M_3,t_4)\,.$$

Endi biror M_0 nuqtagacha Ω ni qissak, $[t_1,t_2]$ kesma ham t_0 nuqtagacha qisiladi. Bundan ko'rinadiki M_1,M_2 nuqtalar M_0 ga o'tadi, t_3,t_4,t_5 lar esa t_0 ga. Bundan limitga o'tganda quyidagi hosil bo'ladi:

$$c(M_0)\rho(M_0)u_t(M_0,t_0)=-div\vec{W}\Big|_{\substack{t=t_0\\ M=M_0}}+F(M_0,t_0)$$

$\vec{W}$ uchun Fur'ye qonunini qo'llab quyidagini hosil qilamiz:

$$div\vec{W}=div(-kgradu)=-\frac{\partial}{\partial x}k\frac{\partial u}{\partial x}-\frac{\partial}{\partial y}k\frac{\partial u}{\partial y}-\frac{\partial}{\partial z}k\frac{\partial u}{\partial z}\Rightarrow$$

$$\Rightarrow c(M_0)\rho(M_0)u_t(M_0,t_0)=\frac{\partial}{\partial x}k\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}k\frac{\partial u}{\partial y}+\frac{\partial}{\partial z}k\frac{\partial u}{\partial z}+F(M_0,t_0)$$

M_0,t_0 nuqtalarni ixtiyoriy olganimiz sababli, hosil qilingan formulani butun $[t_1.t_2]$ va Ω ni soha uchun yoyish mumkin:

$$c(x,y,z)\rho(x,y,z)u_t(x,y,z,t)=\frac{\partial}{\partial x}(k(x,y,z)u_x(x,y,z,t))+\frac{\partial}{\partial y}(k(x,y,z)u_y(x,y,z,t))+$$
$$+\frac{\partial}{\partial z}(k(x,y,z)u_z(x,y,z,t))+F(x,y,z,t)$$

Bu ifoda **fazoda issiqlik o'tkazuvchanlik tenglamasi** deb nomlanadi.

c,ρ,k larni konstanta da deb olib, quyidagi tenglik hosil qilamiz:

$$u_t=a^2(u_{xx}+u_{yy}+u_{zz})+f(x,y,z,t),\qquad a^2=\frac{k}{c\rho},\qquad f=\frac{F}{c\rho}\qquad \textbf{(2.2.6)}$$

Agar u, f faqat x va t o'zgaruvchilari bilan bog'liq bo'lsa, u holda bu tenglik quyidagicha yoziladi:

$$u_t = a^2 u_{xx} + f(x,t) \qquad \textbf{(2.2.7)}$$

Fizik interpretasiyada bir jinsli yupqa sterjinda issiqlik o'tkazuvchanlik (yoyilish) tenglamasidir. (2.2.7) tenglamani biz keyinchalik **issiqlik o'tkazuvchi tenglamasi** deb yuritamiz.

Analogik fikrlashni boshqa bir fizik prosesslar uchun ham o'tkazishimiz mumkin, masalan diffuziya uchun. Agar $u(x.y.z.t)$- fazoda gazning konsentrasiyasi bo'lsa, u holda **diffuziya tenglamasi** quyidagicha bo'ladi:

$$cu_t = div(Dgradu) + F(x,y,z,t)$$

$$D - diffuziya \quad koeffitsiyenti$$
$$F - biror\ bir\ \ funksiya$$

2. <u>Bir fazoviy o'zgaruvchi bilan berilgan issiqlik o'tkazuvchanlik tenglamasi. Asosiy masalalarning qo'yilishi</u>

Quyidagi tenglamani qarab chiqamiz:

$$u_t = a^2 u_{xx} + f(x,t), \quad 0 < x < l, \quad 0 < t \le T$$

Agar bizga sterjinning boshlang'ich vaqt momentidagi temperaturasi malum bo'lsa, u holda biz <u>boshlang'ich shartga</u> ega bo'lamiz:

$$u(x,o) = \phi(x), \ 0 \le x \le l$$

Agar chetlarida temperaturani o'zgarishini bilsak, u holda ayrim <u>chegaraviy shartlar</u> **hosil qilamiz:**

$$x = l, 0 \le t \le T \qquad \begin{cases} (1)\ u(l,t) = \mu_2(t) - birinchi\ \ chegaraviy\ shart \\ (2)\ u_x(l,t) = \nu_2(t) - ikkinchi\ chegaraviy\ shart \\ (3)\ u_x(l.t) = -\lambda_2[u(l,t) - \theta_2(t)] - uchinchi\ chegaraviy\ shart \end{cases}$$

$$x = 0, 0 \le t \le T \qquad \begin{cases} (4)\ u(0,t) = \mu_1(t) - birinchi\ chegaraviy\ shart \\ (5)\ u_x(0,t) = \nu_1(t) - ikkinchi\ chegaraviy\ shart \\ (6)\ u_x(0,t) = -\lambda_1[u(0,t) - \theta_1(t)] - uchinchi\ chegaraviy\ shart \end{cases}$$

Bu shartlardan bir nechtasini tanlab har xil tipli masalalarni hosil qilamiz:

<u>Birinchi chegaraviy masala</u>

$$\begin{cases} u_t = a^2 u_{xx} + f(x,t), 0 < x < l, 0 < t \le T \\ u(0,t) = \mu_1(t), \qquad 0 \le t \le T \\ u(l,t) = \mu_2(t), \qquad 0 \le t \le T \\ u(x,o) = \phi(x), \qquad 0 \le x \le l \end{cases}$$

<u>Ikkinchi chegaraviy masala</u>

$$\begin{cases} u_t = a^2 u_{xx} + f(x,t), \ 0 < x < l, 0 < t \le T \\ u_x(0,t) = \nu_1(t), \qquad 0 \le t \le T \\ u_x(l,t) = \nu_2(t), \qquad 0 \le t \le T \\ u(x,0) = \phi(x), \qquad 0 \le x \le l \end{cases}$$

<u>Yarim to’g’ri chiziqdagi masala</u>

$$\begin{cases} u_t = a^2 u_{xx} + f(x,t), x > 0, 0 < t \le T \\ u(0,t) = \mu(t), \qquad 0 \le t \le T \\ u(x,0) = \phi(x), \qquad x \ge 0 \end{cases}$$

<u>Koshi masalasi</u>

$$\begin{cases} u_t = a^2 u_{xx} + f(x,t), -\infty < x < +\infty, 0 < t \le T \\ u(x,0) = \phi(x), \qquad -\infty < x < +\infty \end{cases}$$

2.3-§ Issiqlik o’tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalani yechimining yagonaligi va turg’unligi

Teorema 2.3 (Birinchi chegaraviy masalani yechimining yagonaligi).

Bizga $u_1(x,t), u_2(x,t)$ funksiyalar

$$C\left[\overline{Q_T}\right], \quad \frac{\partial^2 u}{\partial x^2}, \frac{\partial u_i}{\partial t} \in C\left[Q_T\right], \quad i = 1, 2$$

sinfdan olingan bo’lib, bu funksiyalarning ikkalasi ham [2.3.1] chegaraviy masalaning yechimi bo’lsa, shunda quyidagi tenglik o’rinli: $u_1(x,t) \equiv u_2(x,t)$

Isboti: Yangi $v(x,t) = u_1(x,t) - u_2(x,t)$ funksiya kiritamiz. Shunda $v \in C\left[\overline{Q_T}\right]$ $v_t, v_{xx} \in C[Q_T]$ bo’lishi aniq.

Bu funksiyamiz quyidagi masalaning yechimi bo’ladi

$$\begin{cases} v_t = a^2 u_{xx}, x \in (0,l), t \in (0,T] \\ v(0,t) = 0, t \in [0,T] \\ v(l,t) = 0, t \in [o,T] \\ v(x,0) = 0, x \in [0,l] \end{cases}$$

$v(x,t)$ funksiya uchun max prinsipining barcha shartlari bajarilishi aniq. Demak max prinsipini qo'llaganimizda $\begin{cases} \max\limits_{Q_T} v(x,t) = \max\limits_{\Gamma} v(x,t) = 0 \\ \min\limits_{\overline{Q_T}} v(x,t) = \min\limits_{\Gamma} v(x,t) = 0 \end{cases}$

$\Rightarrow v(x,t) \equiv 0 \Rightarrow u_1(x,t) \equiv u_2(x,t)$ teorema isbotlandi.

Lemma 1. Bizlarga $u_1(x,t)$ va $u_2(x,t)$ funksiyalar berilgan va quyidagi shartlar bajarilsin:

$$u_i \in C\left[\overline{Q_T}\right], \quad \frac{\partial^2 u_i}{\partial x^2}, \frac{\partial u_i}{\partial t} \in C\left[\overline{Q}_T\right], \quad \text{i=1,2}$$

va

$$\begin{cases} \dfrac{\partial u_i}{\partial t} \geq a^2 \dfrac{\partial^2 u_i}{\partial x^2}, x \in (0,l), t \in (0,T], i = 1,2 \\ u_1(0,t) \geq u_2(0,t), t \in [0,T] \\ u_1(l,t) \geq u_2(0,t), t \in [o,T] \\ u_1(x,0) \geq u_2(x,0), x \in [0,l] \end{cases}$$

o'rinli bo'lsa, shunda $\overline{Q_T}$ sohada $u_1(x,t) \geq u_2(x,t)$.

Teorema 2.4 (Birinchi chegaraviy masalani yechimining turg'unligi). Bizga $u_1(x,t), u_2(x,t)$ funksiyalar berilgan va quyidagi shartlar:

$$u_i \in C\left[\overline{Q_T}\right], \quad \frac{\partial^2 u_i}{\partial x^2}, \frac{\partial u_i}{\partial t} \in C\left[\overline{Q}_T\right], \quad \text{i=1,2}$$

$$\begin{cases} \dfrac{\partial u_i}{\partial t} = a^2 \dfrac{\partial^2 u_i}{\partial x^2}, x \in (0,l), t \in (0,T], i = 1,2 \\ u_i(0,t) = \mu_1^i(t), t \in [0,T], i = 1,2 \\ u_i(l,t) = \mu_2^i(t), t \in [o,T], i = 1,2 \\ u_i(x,0) = \phi_i(x), x \in [0,l], i = 1,2 \end{cases}$$

o'rinli bo'lsa, shunda

$$\max_{\overline{Q_T}} \left|u_1(x,t) - u_2(x,t)\right| = max\left\{ \max_{t\in[0,T]}\left|\mu_1^1(t) - \mu_1^2(t)\right|, \max_{t\in[0,T]}\left|\mu_2^1(t) - \mu_2^2(t)\right|, \max_{x\in[0,l]}\left|\phi_1(x) - \phi_2(x)\right| \right\}$$

tenglik o'rinli.

Umumiy chegaraviy masala yechimining yagonaligi

$$[2.3.2]\begin{cases} u_t = a^2u_{xx} + f(x,t); & 0 < t \le T,\ \ 0 < x < l; \\ \alpha_1 u(0,t) - a_2 u_x(0,t) = p(t); & 0 \le t \le T; \\ \beta_1 u(l,t) + \beta_2 u_x(l,t) = q(t); & 0 \le t \le T; \\ u(x,0) = \varphi(x); & 0 \le x \le l; \end{cases}$$

Bu yerda $\alpha_1 + \alpha_2 > 0;\ \ \beta_1 + \beta_2 > 0.$ -manfiy bo'lmagan o'zgarmaslar. Bu o'zgarmaslar uchun quyidagi shart bajarilishi kerak.

$$\alpha_1 + \alpha_2 > 0; \qquad \beta_1 + \beta_2 > 0;$$

Bu chegaraviy masala uchun quyidagi teorema o'rinli.

Teorema 2.5 (yagonalik). Faraz qilaylik, Q_T sohada $u_1, u_2(x,t)$ funksiyalar aniqlangan bo'lsin. Bu funksiyalar quyidagi shartlarni qanoatlantiradi:

$$u_i, \frac{\partial u_i}{\partial x} \in C[\overline{Q}_T], \frac{\partial^2 u_i}{\partial x^2}, \frac{\partial^2 u_i}{\partial t} \in C[Q_T], \qquad \text{i } = 1,2,$$

va bir xil [2.3.2] chegaraviy masalaning yechimlari bo'lsin.

Shunda $\overline{Q_T}$ sohada $u_1(x,t) = u_2(x,t)$

1. Koshi masalaning yechimining mavjudligi

Bir jinsli Koshi masalasini qaraymiz:

$$[2.3.3]\begin{cases} (1) \quad u_t = a^2u_{xx}, & -\infty < x < +\infty, \quad 0 < t < T; \\ (2) \quad u(x,0) = \varphi(x), & -\infty < x < +\infty. \end{cases}$$

[2.3.3] 1-chegaraviy masalani yechimini topayotganimizdek bu yerda ham oldin malum bir almashtirishlarni o'tkazamiz. So'ngra esa hosil bo'lgan funksiya yechim ekanligini ko'rsatamiz.

$$v(x,t) = X(x)T(t).$$

$v(x,t)$ funksiyadan issiqlik o'tkazuvchanlik tenglamasini qanoatlantirishini talab qilamiz:

$$T'(t)X(x) = a^2X''(x)T(t).$$

Ikkala tomonini $a^2X(x)T(t)$ ga bo'lamiz, shunda hosil bo'lgan tengliklar

quyidagicha: $\dfrac{X''(x)}{X(x)} = \dfrac{T'(t)}{a^2T(t)} = -\lambda^2;$

Bu yerda $\lambda = const > 0$ ikkita tenglama xosil bo'ladi:

$$X''(x) + \lambda^2 X(x) = 0; \qquad (2.3.4)$$

$$T'(t) + a^2\lambda^2 T(t) = 0. \qquad (2.3.5)$$

$X(x) = e^{i\lambda x}$ funksiya (2.3.4), tenglamaning yechimi bo'ladi. Xuddi shunday qilib $T(t) = e^{-a^2\lambda^2 t}$ funksiyamiz (2.3.5) tenglamaning yechimi bo'ladi. Demak,

$v(x,t) = e^{i\lambda x - a^2\lambda^2 t}$ birinchi tenglamaning yechimi bo'ladi.

$u_\lambda = A(\lambda)e^{i\lambda x - a^2\lambda^2 t}$ funksiya ham yechim bo'ladi (A (λ)-qandaydir funksiya)

Endi yakuniy funksiya quyidagicha aniqlanadi

$$u(x,t) = \int_{-\infty}^{+\infty} A(\lambda)e^{i\lambda x - a^2\lambda^2 t} d\lambda$$

boshlang'ich shartlani qanoatlantirishini talab qilamiz

$$u(x,0) = \varphi(x) = \int_{-\infty}^{+\infty} A(\lambda)e^{i\lambda x} d\lambda$$

Endi, Fur'ye almashtirishtirishlar nazariyasinidan kelib chiqgan holda $A(\lambda)$ quydagicha topamiz

$$A(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\lambda s}\varphi(s)ds\ .$$

Shunday qilib bizlar $u(x,t)$:funksiya uchun quydagi ko'rinishini xosil qilamiz

$$u(x,t) = \int_{-\infty}^{+\infty}\frac{1}{2\pi}\left[\int_{-\infty}^{+\infty} e^{-i\lambda s}\varphi(s)ds\right]e^{i\lambda x - a^2 i\lambda^2 t}d\lambda = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\frac{1}{2\pi}e^{i\lambda(x-s) - a^2\lambda^2 t}d\lambda\right]\varphi(s)ds.$$

$u(x,t)$: uchun yechim shunday ko'rinishga ega:

$$u(x,t) = \int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2 t}}\exp\left\{-\frac{(x-s)^2}{4a^2 t}\right\}\varphi(s)ds. \qquad (2.3.6)$$

$$G(x,s,t) = \frac{1}{\sqrt{4\pi a^2 t}}\exp\left\{-\frac{(x-s)^2}{4a^2 t}\right\},$$

belgilash kiritasak:

$$u(x,t) = \int_{-\infty}^{+\infty} G(x,s,t)\varphi(s)ds.$$

$G(x,s,t)$ funksiyamiz issiqlik o'tkazuvchanlik tenglamasini s-fiksirlangan bo'lganda qanoatlantirishini ko'rsatamiz:

$$G_x(x,s,t) = \frac{1}{\sqrt{4\pi a^2 t}}\exp\left\{-\frac{(x-s)^2}{4a^2 t}\right\}\left(-\frac{2(x-s)}{4a^2 t}\right);$$

$$G_t(x,s,t) = \frac{1}{2\sqrt{4\pi a^2 t^{\frac{3}{2}}}}\exp\left\{-\frac{(x-s)^2}{4a^2t}\right\} + \frac{1}{\sqrt{4\pi a^2 t}}\exp\left\{-\frac{(x-s)}{4a^2t}\right\}\frac{(x-s)^2}{4a^2t^2}$$

$$G_{xx}(x,s,t) = \frac{1}{\sqrt{4\pi^2 t}}\exp\left\{-\frac{(x-s)^2}{4a^2t}\right\}\frac{(x-s)^2}{4a^x t^2} + \frac{1}{\sqrt{4\pi a^2 t}}\exp\left\{-\frac{(x-s)^2}{4a^2t}\right\}\left(-\frac{2}{4a^2t}\right)$$

$G(x,s,t) = a^2 G_{xx}(x,s,t)$ ekanligini tekshirish oson.

Endi bizlar xosil bo'lgan funksiyamizni qandaydir boshlangich shartlarda mavjud ekanligini ko'rishimiz kerak.

Teorem 2.6 (Koshi masalasi yechimining mavjudlik teoremasi). [2.3.3] Koshi masalaning boshlang'ich shartlarini $\varphi(x)$ yordamidi aniqlangan bo'lsin va

$$\varphi(x) \in C(R), |\varphi(x)| \le M, \forall x \in R.$$

Shunda (2.3.6) formula bilan aniqlangan $u(x,t)$ funksiya $x \in R, t > 0$ bo'lganda uzluksiz bo'ladi, u_t, u_{xx} uzluksiz xosilalarga ega, agarda $x \in R, t > 0$ bo'lsa, va issiqliq o'tkazuvchanlik tenglamani qanoatlantiradi. $x \in R, t > 0$ va $\forall x_0 \in R \lim\limits_{\substack{t\to 0+\\ x\to x_0}} u(x,t) = \varphi(x_0)$ lar uchun

Izox: Teoremaning oxirgi sharti quyidagi ma'noga ega.

$$u(x,t) = \begin{cases} \int\limits_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi a^2 t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\varphi(s)ds, t > 0; \\ \varphi(x), t = 0. \end{cases}$$

$$(x,t) : x \in R, t \ge 0$$

da uzluksiz ekanligini oxirgi shart bildiradi.

Natija 1: Agarda teoremaning barcha shartlari $(\varphi(x) \in C(R), |\varphi(x)| \le M)$ bajarilsa, demak biz $u(x,t)$ funksiyamiz chegaralangan ekanligini xulosa qilishimiz mumkin.

$$|u(x,t)| = \int\limits_{-\infty}^{+\infty} |G(x,s,t)||\varphi(s)|ds \le M \int\limits_{-\infty}^{+\infty} G(x,s,t)ds = M.$$

Natija 2: Xuddi shunday qilib $(R \times R^+)$ fazoda $u(x,t)$ funksiyamiz cheksiz uzluksiz ekanligini xosil qilishimiz mumkin.

$$\frac{\partial^p u}{\partial x^k \partial t^m}(x,t) = \int\limits_{-\infty}^{+\infty} \frac{\partial^p G}{\partial x^k \partial t^m}(x,s,t)\varphi(s)ds, \quad (k+m=p)$$

bu integral esa tekis yaqinlashuvchi bulib, buni teorema isbotidagi tasdiklar orkali ko'rsatish mumkin.

Natija 3: Koshi masalasidagi shartlarni kabul qilib, biz issiqlik tarkalishining "cheksiz" tezligiga ega bo'lamiz.

Faraz qilaylik $\varphi(x) = u(x,o)$ uzluksiz funksiyamiz $[a;b]$ oralikdan boshqa barcha joyda nolga teng bo'lsin. U holda quyidagiga ega bo'lamiz.

$$u(x,t) = \int_a^b G(x,s,t)\varphi(s)ds > 0 \quad \forall t > 0, \forall x \in R$$

Teorema 2.7 (Koshi masalasi yechimining yagonaligi). Koshi masalasi berilgan bo'lsin. Faraz qilaylik $(R \times R^{+})$ fazoda bizlarga 2 ta uzluksiz $u_1, u_2(x,t)$ funksiyalar berilgan bo'lsin va ular **[2.3.3]** masalaning yechimlari bulib, quyidagi shartlarni qanoatlantirsin.

$$\begin{aligned} &|u_i(x,t)| \le M, \forall (x,t) \in R \times \overline{R}^{+}; \\ &\frac{\partial u_i}{\partial t}, \frac{\partial^2 u_i}{\partial t^2} \in C(R \times R^{+}) \end{aligned} \qquad i = 1,2$$

shunda

$$u_1(x,t) = u_{21}(x,t) \forall (x,t) \in (R \times \overline{R}^{+})$$

2.4-§. Yarim to'g'ri chiziqda issiqlik o'tkazuvchanlik tenglamasi uchun birinchi va ikkinchi chegaraviy masala

1. Yarim to'g'ri chiziqda qo'ydagi birinchi chegaraviy masala

Yarim to'g'ri chiziqda qo'ydagi birinchi chegaraviy masalani ko'rib chiqamiz:

$$[2.4.1] \begin{cases} u_{tt} = a^2 u_{xx}, & x > 0, t > 0; \\ u(0,t) = 0, & t \ge 0; \\ u(x,0) = \phi(x), & x \ge 0. \end{cases}$$

bu yerda $\phi(x) = 0$.

Butun Haqiqiy o'qda boshlang'ich shartni beruvchi $\phi(x)$ *funksiyani toq qilib davom ettirib yechimni topamiz:*

$$\Phi(x) = \begin{cases} \phi(x), & x \ge 0; \\ -\phi(-x), & x < 0. \end{cases}$$

Mos ravishda qo'ydagi Koshi masalasini ko'rib chiqamiz:

$$[2.4.2]\begin{cases} U_{tt}=a^2U_{xx}, & -\infty<x<+\infty, t>0;\\ U(0,t)=0, & t\ge 0;\\ U(x,0)=\Phi(x), & -\infty<x<+\infty. \end{cases}$$

Uning yechimi bizga malum:

$$U(x,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\Phi(s)ds.$$

Aytaylik $(x,t)\in(\overline{R}^+\times\overline{R}^+)$ *da* $u(x,t)=U(x,t)$. *Bu funksiya [2.4.1] ning yechimi ekanligini ko'rsatamiz . Koshi masalasining qo'yilishiga ko'ra ,*

$$\begin{cases} u_t=a^2u_{xx}, & x>0, t>0;\\ u(x,0)=\phi(x), & x\ge 0. \end{cases}$$

ekanligi malum. Chegaraviy shartni bajarilishini tekshiramiz:

$$u(0,t)=U(0,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{s^2}{4a^2t}\}\Phi(s)ds.$$

Integiral ostida juft va toq funksiyalarning ko'paytmasi turibdi , shuning uchun u nolga teng . Chegaravi shart bajarilayapdi. endi yechim uchun to'liq formulani olamiz:

$$u(x,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\Phi(s)ds=$$

$$=\int_{-\infty}^{0}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}(-\phi(-s))ds+\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\phi(s)ds=$$

$$=-\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x+s)^2}{4a^2t}\}\phi(s)ds+\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\phi(s)ds=$$

$$=\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\left[\exp\{-\frac{(x-s)^2}{4a^2t}\}-\exp\{-\frac{(x+s)^2}{4a^2t}\}\right]\phi(s)ds.$$

Demak,

$$u(x,t)=\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\left[\exp\{-\frac{(x-s)^2}{4a^2t}\}-\exp\{-\frac{(x+s)^2}{4a^2t}\}\right]\phi(s)ds. \qquad (2.4.3)$$

bu yarim to'g'ri chiziqda birinchi chegaraviy masalaning yechimi bo'ladi.

2. Yarim to'g'ri chiziqda ikkinchi chegaraviy masala

Yarim to'gri chiziqda ikkinchi chegaraviy masala qo'ydagi ko'rinishga ega:

$$[2.4.4]\begin{cases} u_{tt}=a^2u_{xx}, & x>0, t>0; \\ u_x(0,t)=0, & t\ge 0; \\ u(x,0)=\phi(x), & x\ge 0. \end{cases}$$

Yana yechimni topish uchun boshlang'ich shartni beruvchi funksiyani endi juft qilib davom ettiramiz:

$$\Phi(x)=\begin{cases} \phi(x), & x\ge 0; \\ \phi(-x), & x<0. \end{cases}$$

Boshlang'ich shartni o'zgartirib , quyidagi koshi masalasini olamiz:

$$\begin{cases} U_t=a^2U_{xx}, & -\infty<x<+\infty, t>0; \\ U(0,t)=0, & t\ge 0; \\ U(x,0)=\Phi(x), & -\infty<x<+\infty. \end{cases}$$

Xuddi shunday uning yechimi

$$U(x,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\Phi(s)ds$$ *funksiya bo'ladi.*

Aytaylik $(x,t)\in(\overline{R}^+\times\overline{R}^+)$ *da* $u(x,t)=U(x,t)$ *bo'lsin.*

Yana

$$\begin{cases} u_t=a^2u_{xx}, & x>0, t>0; \\ u(x,0)=\phi(x), & x\ge 0. \end{cases}$$

ekanligi aniq.

Chegaraviy masalaning bajarilishini tekshiramiz:

$$u_x(x,t)=U_x(x,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\left(-\frac{(x-s)}{2a^2t}\right)\exp\{-\frac{(x-s)^2}{4a^2t}\}\Phi(s)ds\Rightarrow$$

$$u_x(0,t)=U_x(0,t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\left(\frac{s}{2a^2t}\right)\exp\{-\frac{s^2}{4a^2t}\}\Phi(s)ds\ \forall t\ge 0.$$

Hosil bo'lgan integral ostida 2 ta juft va bitta toq funksiyaning ko'paytmas turibdi, demak u nolga Doiradi. Chegaraviy shart bajarilmoqda. [2.4.4] ning yechimi uchun qo'ydagi formilani xosil qilamiz:

$$u(x,t)=\int_{0}^{+\infty}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x+s)^2}{4a^2t}\}\phi(s)ds+\int_{-\infty}^{0}\frac{1}{\sqrt{4\pi a^2t}}\exp\{-\frac{(x-s)^2}{4a^2t}\}\phi(-s)ds=$$

$$= \int_0^{+\infty} \frac{1}{\sqrt{4\pi a^2 t}} \left[\exp\{-\frac{(x-s)^2}{4a^2 t}\} + \exp\{-\frac{(x+s)^2}{4a^2 t}\} \right] \phi(s) ds.$$

Bu yarim to'g'ri chiziqda 2-chegaraviy masalaning yechimidir.

2.5-§. Maple paketi orqali yarim chegaralangan sohada issiqlik o'tkazuvchanlik tenglamasini Fur'ye usuli (o'zgaruvchilarni ajratish usuli) yordamida yechish

Yarim chegaralangan sterjenda issiqlik tarqalish jarayonini qaraymiz. Buning uchun

$$\frac{\partial}{\partial t} u(t,x) = a^2 \left(\frac{\partial^2}{\partial x^2} u(t,x) \right)$$

bir jinsli tenglamani

$$u(0,x) = f(x)$$

boshlang'ich shart bilan va quyidagi chegaraviy shartlar bilan yeching:

1. Sterjenning uchi $x = 0$ mahkamlangan:

$$\frac{\partial}{\partial x} u(t,0) = 0 \text{ .}$$

yoki

2. O'zgarmas temperaturada sterjenning uchi $x = 0$ nuqtada mahkamlangan:

$$u(t,0) = T0$$

```
> restart;
```

Bir jinsli tenglama va uning yechimini o'zgaruvchilarni ajratish usuli bilan yechamiz:

```
> PDE:=diff(u(t,x),t)=a^2*diff(u(t,x),x,x);
struc:=pdsolve(PDE,HINT=T(t)*X(x));
```

$$PDE := \frac{\partial}{\partial t} u(t,x) = a^2 \left(\frac{\partial^2}{\partial x^2} u(t,x) \right)$$

$$struc := (u(t,x) = T(t)\, X(x)) \ \&where \left[\{ \frac{d}{dt} T(t) = \_c_1 \, T(t), \frac{d^2}{dx^2} X(x) = \frac{\_c_1 \, X(x)}{a^2} \} \right]$$

```
> dsolve(diff(T(t),t)=_c[1]*T(t));
dsolve(diff(X(x),`$`(x,2))=_c[1]*X(x)/a^2);
```

$$T(t) = \_C1\, e^{(\_c_1\, t)}$$

$$X(x) = \_C1\, e^{\left(\frac{\sqrt{\_c_1}\, x}{a}\right)} + \_C2\, e^{\left(-\frac{\sqrt{\_c_1}\, x}{a}\right)}$$

O'zgaruvchilarni ajratuvchi o'zgarmas almashtirish olamiz:

$\_c_1 \; == -\; \lambda^2$

```
> dsolve(diff(T(t),t)=-lambda^2*T(t)*a^2);
dsolve(diff(X(x),`$`(x,2))=-lambda^2*X(x));
```

$$T(t) = \_C1\, e^{(-\lambda^2 a^2 t)}$$

$$X(x) = \_C1 \sin(\lambda x) + \_C2 \cos(\lambda x)$$

Natijada umumiy yechim quyidagi ko'rinishni oladi:

```
> u(t,x):=(C1*sin(lambda*x)+C2*cos(lambda*x))*exp(-lambda^2*a^2*t);
```

$$u(t, x) := (C1 \sin(\lambda x) + C2 \cos(\lambda x))\, e^{(-\lambda^2 a^2 t)}$$

$u(t, x)$ funksiya - ixtiyoriy λ uchun tenglamani yechimi bo'lib (bu yerda λ - qiymati - ∞ dan + ∞ gacha intervalda bo'lgan ixtiyoriy o'zgaruvchi parametr), bu yerda har bir λ uchun $C1(\lambda)$ va $C2(\lambda)$ koeffisiyentlar mos tushadi.

Shuning uchun quyidagiga ega bo'lamiz:

```
>
u[lambda](t,x):=(C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-
lambda^2*a^2*t);
```

$$u_\lambda(t, x) := (C1(\lambda) \sin(\lambda x) + C2(\lambda) \cos(\lambda x))\, e^{(-\lambda^2 a^2 t)}$$

Natijada chiziqli bir jinsli tenglamani yechimini λ parametrga bog'liq bo'lgan yechimning superpozisiyasi ko'rinishida tasvirlash mumkin:

```
> u(t,x):=int(u[lambda](t,x), lambda=-infinity..infinity);
```

$$u(t, x) := \int_{-\infty}^{\infty} (C1(\lambda) \sin(\lambda x) + C2(\lambda) \cos(\lambda x))\, e^{(-\lambda^2 a^2 t)}\, d\lambda$$

$C1(\lambda)$ va $C2(\lambda)$ koeffisiyentlarni aniqlashda boshlang'ich shartlardan foydalanamiz:

```
> u_0(t,x):=eval(subs(t=0, u(t,x)))=f(x);
```

$$u\_0(t,x) := \int_{-\infty}^{\infty} C1(\lambda)\sin(\lambda x) + C2(\lambda)\cos(\lambda x)\, d\lambda = f(x)$$

Bu ifoda $f(x)$ funksiyani Fur'ye integraliga yoyish bilan ustma ust tushadi:

> f(x)=(1/(2*Pi))*int(int(f(xi)*cos(lambda*(xi-x)),xi=-infinity..infinity),lambda=-infinity..infinity);

$$f(x) = \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda(\xi - x))\, d\xi\, d\lambda\right)$$

Demak, $C1(\lambda)$ va $C2(\lambda)$ koeffisiyentlar quyidagicha ifodalanadi:

> C1(lambda):=(1/(2*Pi))*int(f(xi)*sin(lambda*xi),xi=-infinity..infinity);
C2(lambda):=(1/(2*Pi))*int(f(xi)*cos(lambda*xi),xi=-infinity..infinity);

$$C1(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\xi)\, d\xi\right)$$

$$C2(\lambda) := \frac{1}{2}\left(\frac{1}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\xi)\, d\xi\right)$$

> u(t,x):=int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity);
u(t,x):=combine(int((C1(lambda)*sin(lambda*x)+C2(lambda)*cos(lambda*x))*exp(-lambda^2*a^2*t),lambda = -infinity .. infinity));

$$u(t,x) := \int_{-\infty}^{\infty} \left(\frac{1}{2}\left(\frac{\sin(\lambda x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\sin(\lambda\xi)\, d\xi\right) + \frac{1}{2}\left(\frac{\cos(\lambda x)}{\pi}\int_{-\infty}^{\infty} f(\xi)\cos(\lambda\xi)\, d\xi\right)\right) e^{(-\lambda^2 a^2 t)}\, d\lambda$$

$$u(t,x) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2}\,\frac{e^{(-\lambda^2 a^2 t)} f(\xi)\cos(-\lambda x + \lambda\xi)}{\pi}\, d\xi\, d\lambda$$

Olingan ifodani almashtirish mumkin.

Buning uchun quyidagi integralni qaraymiz:

> int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi), lambda = -infinity .. infinity);

$$\int_{-\infty}^{\infty} \mathbf{e}^{(-\lambda^2 a^2 t)} \cos(-\lambda x + \lambda \xi)\, d\lambda$$

O\zgaruvchilarni almashtirish olamiz va integral ostidagi ifodani almashtiramiz:

```
> simplify(subs({xi=-v*a*t^(1/2)+x,lambda=w/(a*sqrt(t))},exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi)));
```

$$\mathbf{e}^{(-w^2)} \cos(w\, v)$$

```
> Int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi),lambda = -infinity .. infinity)=(1/(a*sqrt(t)))*int(exp(-w^2)*cos(w*v),w = -infinity .. infinity);
```

$$\int_{-\infty}^{\infty} \mathbf{e}^{(-\lambda^2 a^2 t)} \cos(-\lambda x + \lambda \xi)\, d\lambda = \frac{\sqrt{\pi}\, \mathbf{e}^{\left(-\frac{v^2}{4}\right)}}{a\sqrt{t}}$$

```
> Int(exp(-lambda^2*a^2*t)*cos(-lambda*x+lambda*xi),lambda=-infinity..infinity)= subs(v=(x-xi)/a/t^(1/2),1/a/t^(1/2)*Pi^(1/2)*exp(-1/4*v^2));
```

$$\int_{-\infty}^{\infty} \mathbf{e}^{(-\lambda^2 a^2 t)} \cos(-\lambda x + \lambda \xi)\, d\lambda = \frac{\sqrt{\pi}\, \mathbf{e}^{\left(-\frac{(x-\xi)^2}{4 a^2 t}\right)}}{a\sqrt{t}}$$

Natijada yechim quyidagi ko'rinishga ega bo'ladi:

```
> u(t,x):=(1/(2*a*sqrt(Pi*t)))*int(f(xi)*exp(-1/4*(x-xi)^2/a^2/t),xi = -infinity .. infinity);
```

$$\mathrm{u}(t, x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}} \int_{-\infty}^{\infty} \mathrm{f}(\xi)\, \mathbf{e}^{\left(-\frac{(x-\xi)^2}{4 a^2 t}\right)} d\xi\right)$$

Sterjen yarim chegaralangan holda $x=0$ nuqtada chegarviy shart zaruriydir.

1. sterjenning uchi $x=0$ nuqtada mahkamlangan:

$$\frac{\partial}{\partial x}\mathrm{u}(t, 0) = 0 .$$

Bu holda $\mathrm{f}(x)$ ni manfiy yarim o'qda juft davom ettiramiz:

$$\mathrm{f}(x) = \mathrm{f}(-x) .$$

```
> u_x(t,x):=diff(u(t,x),x);
```

$$u\_x(t,x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} -\frac{1}{2}\frac{f(\xi)(-\xi+x)\,\mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}}{a^2 t}\,d\xi\right)$$

O'zgaruvchini almashtirish olamiz:

> u0_x:=subs(x=0,u_x(t,x));

$$u0\_x := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} \frac{1}{2}\frac{f(\xi)\,\xi\,\mathbf{e}^{\left(-\frac{\xi^2}{4a^2t}\right)}}{a^2 t}\,d\xi\right)$$

Bu yerda integral ostidagi ifoda toq; shuningdek integral nolga teng va $x=0$ nuqtadagi chegaraviy shart bajariladi.

Bunda yechimni quyidagi ko'rinishda yozish mumkin:

> u(t,x):=1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*(exp(-1/4*(-xi+x)^2/a^2/t)+exp(-1/4*(xi+x)^2/a^2/t)),xi=-infinity..infinity);

$$u(t,x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{\infty} f(\xi)\left(\mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4a^2t}\right)}+\mathbf{e}^{\left(-\frac{(\xi+x)^2}{4a^2t}\right)}\right)d\xi\right)$$

2. O'zgarmas temperaturada sterjenning uchi $x=0$ nuqtada mahkamlangan ($0<x$): $u(t,0)=T0$.

Bu masalani yechish uchun chegaraviy shartni bir jinsliga almashtiramiz:

> U(t,x)=u(t,x)-T0;

F(x)=f(x)-T0;

$$U(t,x)=u(t,x)-T0$$

$$F(x)=f(x)-T0$$

Bunda $F(x)$ ni manfiy o'q bo'yicha toq davom ettiramiz:

$$F(x)=\ -F(-x).$$

Bu holda masalani yechimi:

> U(t,x):=1/2*1/a/(Pi*t)^(1/2)*int(F(xi)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. infinity);

$$U(t,x) := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}}\int_{-\infty}^{\infty} F(\xi)\, \mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4\,a^2\,t}\right)}\, d\xi\right)$$

Natijada quyidagiga ega bo'lamiz:

> F(xi):=f(xi)-T0;

u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)+T0)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. infinity);

$$F(\xi) := f(\xi) - T0$$

$$u(t,x) := T0 + \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}}\int_{-\infty}^{\infty} (f(\xi)+T0)\, \mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4\,a^2\,t}\right)}\, d\xi\right)$$

$F(\xi) := f(\xi) + T0$ funksiyaning toq ekanligini hisobga olib, quyidagiga ega bo'lamiz:

> u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)-T0)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. 0)+1/2*1/a/(Pi*t)^(1/2)*int((f(xi)-T0)*exp(-1/4*(xi+x)^2/a^2/t),xi = 0 .. infinity);

$$u(t,x) := T0 + \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}}\int_{-\infty}^{0} (f(\xi)-T0)\, \mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4\,a^2\,t}\right)}\, d\xi\right) + \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}}\int_{0}^{\infty} (f(\xi)-T0)\, \mathbf{e}^{\left(-\frac{(\xi+x)^2}{4\,a^2\,t}\right)}\, d\xi\right)$$

yoki

> u(t,x):=T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi))*(exp(-1/4*(-xi+x)^2/a^2/t)-exp(-1/4*(xi+x)^2/a^2/t)),xi=0..infinity)-T0*Integr;

$$u(t,x) := T0 + \frac{1}{2}\left(\frac{1}{a\sqrt{\pi\, t}}\int_{0}^{\infty} f(\xi)\left(\mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4\,a^2\,t}\right)} - \mathbf{e}^{\left(-\frac{(\xi+x)^2}{4\,a^2\,t}\right)}\right) d\xi\right) - T0\ Integr$$

bu yerda:

> Integr:=1/2/a/(Pi*t)^(1/2)*Int((T0)*exp(-1/4*(-xi+x)^2/a^2/t),xi = -infinity .. 0)-1/2/a/(Pi*t)^(1/2)*Int((T0)*exp(-1/4*(xi+x)^2/a^2/t),xi = 0 .. infinity);

$$Integr := \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{-\infty}^{0} T0\, \mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4a^2 t}\right)} d\xi\right) - \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{0}^{\infty} T0\, \mathbf{e}^{\left(-\frac{(\xi+x)^2}{4a^2 t}\right)} d\xi\right)$$

Bu integrallarni hisoblash uchun almashtirish olamiz:

$\xi = x - 2\,a\,t^{\left(\frac{1}{2}\right)} y\,,\ \ \xi = -x + 2\,a\,t^{\left(\frac{1}{2}\right)} z\,.$

> subs(xi=x-2*a*t^(1/2)*y,exp(-1/4*(-xi+x)^2/a^2/t));

subs(xi=-x+2*a*t^(1/2)*y,exp(-1/4*(xi+x)^2/a^2/t));

$$\mathbf{e}^{(-y^2)}$$

$$\mathbf{e}^{(-y^2)}$$

> I1:=simplify((1/a/(Pi*t)^(1/2))*int(exp(-y^2),y = -infinity .. x/(2*a*t^(1/2)))*2*a*t^(1/2))/2;

I2:=simplify((1/a/(Pi*t)^(1/2))*int(exp(-y^2),y = x/(2*a*t^(1/2)) ..infinity)*2*a*t^(1/2))/2;

$$I1 := \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{x}{2\,a\sqrt{t}}\right)$$

$$I2 := -\frac{1}{2}\,\mathrm{erf}\left(\frac{x}{2\,a\sqrt{t}}\right) + \frac{1}{2}$$

> Integr:=simplify(I1-I2);

$$Integr := \mathrm{erf}\left(\frac{x}{2\,a\sqrt{t}}\right)$$

Shuningdek, quyidagiga ega bo'lamiz:

> u(t,x):=collect(T0+1/2*1/a/(Pi*t)^(1/2)*int((f(xi))*(exp(-1/4*(-xi+x)^2/a^2/t)-exp(-1/4*(xi+x)^2/a^2/t)),xi=0..infinity)-T0*Integr,T0);

$$\mathrm{u}(t,x) := \left(-\mathrm{erf}\left(\frac{x}{2\,a\sqrt{t}}\right) + 1\right) T0 + \frac{1}{2}\left(\frac{1}{a\sqrt{\pi t}}\int_{0}^{\infty} \mathrm{f}(\xi)\left(\mathbf{e}^{\left(-\frac{(-\xi+x)^2}{4a^2 t}\right)} - \mathbf{e}^{\left(-\frac{(\xi+x)^2}{4a^2 t}\right)}\right) d\xi\right)$$

Tenglamani yechishga misollar

1 - misol

> restart;

Bir jinsli tenglamani

$$\frac{\partial}{\partial t} u(t, x) = a^2 \left(\frac{\partial^2}{\partial x^2} u(t, x) \right)$$

quyidagi chegaraviy shart (sterjen uchi $x = 0$ nuqtada mahkamlangan):

$$\frac{\partial}{\partial x} u(t, 0) = 0 .$$

va boshlang'ich shart bilan yeching

$$u(0, x) = f(x) ,$$

bu yerda $f(x)$ funksiya quyidagi ko'rinishda berilgan:

```
> a:=1;l:=4;L:=6;alpha:=1;
f(x):=x->piecewise(x<l,0, x<L,alpha, x>L,0);
```

$$a := 1$$

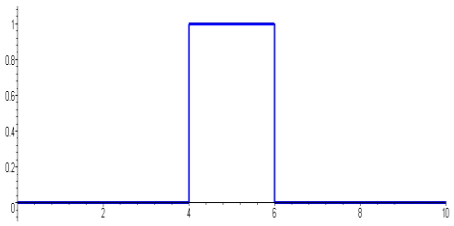
$$l := 4$$

$$L := 6$$

$$\alpha := 1$$

$$f(x) := x \rightarrow \text{piecewise}(x < l, 0, x < L, \alpha, L < x, 0)$$

```
> plot(f(x),0..10,-0.1..1.1, numpoints=400,color=blue,thickness=3);
```




```
> restart;
f(xi):=xi->piecewise(xi<l,0, xi<L,alpha, xi>L,0);
```

$$f(\xi) := \xi \rightarrow \text{piecewise}(\xi < l, 0, \xi < L, \alpha, L < \xi, 0)$$

Yechish uchun yuqorida olingan formuladan foydalanamiz:

```
>  u(t,x):=simplify(1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*(exp(-1/4*(-xi+x)^2/a^2/t)+exp(-1/4*(xi+x)^2/a^2/t)),xi = l .. L));
```

$$u(t, x) := -\frac{1}{2} c \left(\text{erf}\left(\frac{l - x}{2 a \sqrt{t}} \right) + \text{erf}\left(\frac{l + x}{2 a \sqrt{t}} \right) - \text{erf}\left(\frac{L - x}{2 a \sqrt{t}} \right) - \text{erf}\left(\frac{L + x}{2 a \sqrt{t}} \right) \right)$$

```
> a:=1;l:=4;L:=6;alpha:=1;
```

$$a := 1$$

$$l := 4$$

$$L := 6$$

$$\alpha := 1$$

Tenglamaning yechimi:

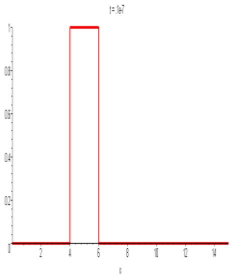
```
> with(plots):
u(t,x):=-1/2*(erf(1/2*(l-x)/a/t^(1/2))+erf(1/2*(l+x)/a/t^(1/2))+erf(1/2*(-
L+x)/a/t^(1/2))-erf(1/2*(L+x)/a/t^(1/2)));
```

Warning, the name changecoords has been redefined

$$u(t, x) := -\frac{1}{2}\operatorname{erf}\left(\frac{4-x}{2\sqrt{t}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{4+x}{2\sqrt{t}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{-6+x}{2\sqrt{t}}\right) + \frac{1}{2}\operatorname{erf}\left(\frac{6+x}{2\sqrt{t}}\right)$$

Olingan yechimni ikki o'lchovli animasiyali grafik ko'rinishida tasvirlaymiz:

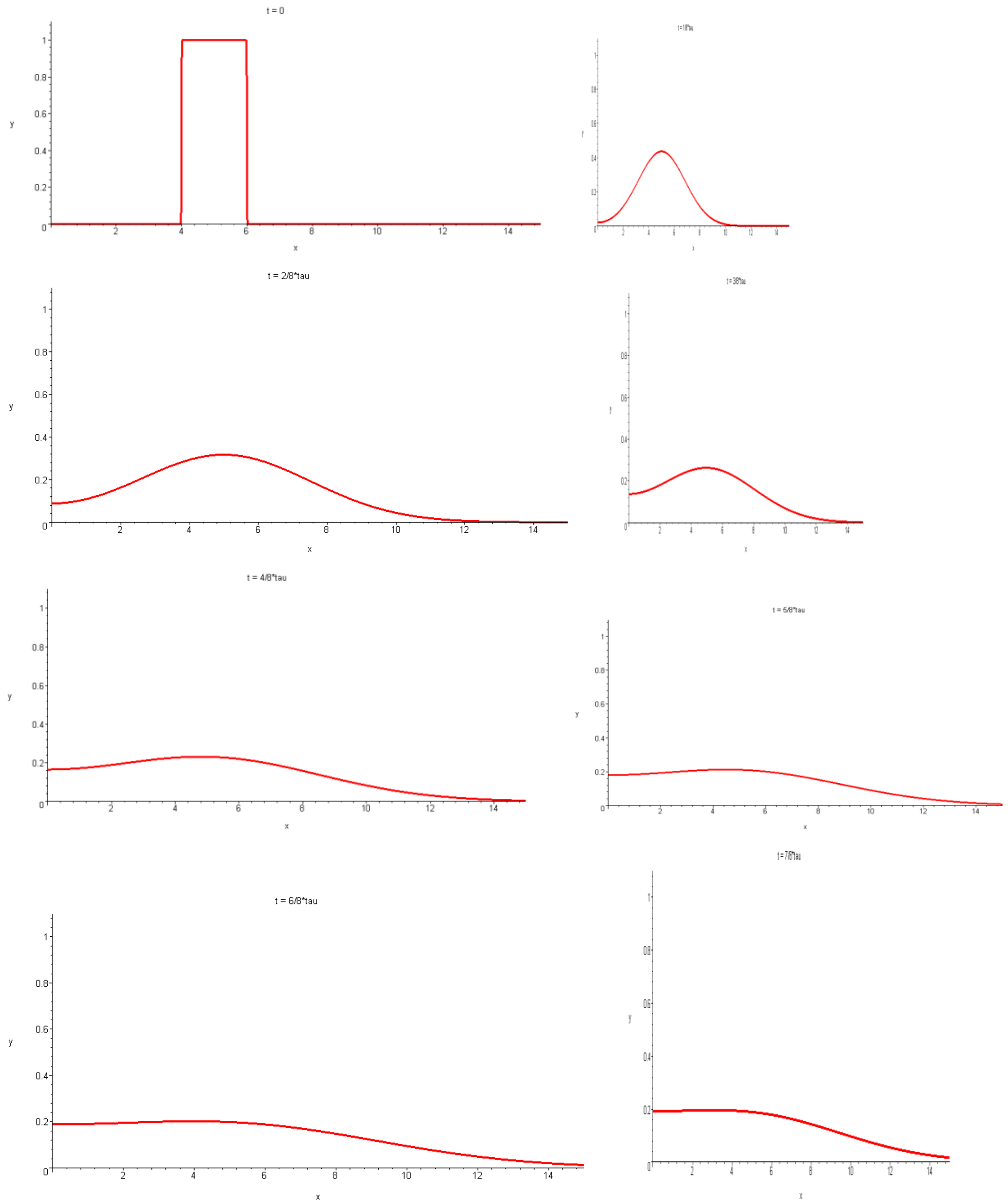
```
> animate(plot,[u(t,x),x=0..15], t=0.00000001..12, frames=60,thickness=3);
```



Olingan yechimni bir nechta vaqt momentlarida ikki o'lchovli grafik ko'rinishida tasvirlaymiz:

```
> tau:=12:
u_1(x):=subs(t=tau*0.000001,u(t,x)):
u_2(x):=subs(t=tau*(1/8),u(t,x)):
u_3(x):=subs(t=tau*(2/8),u(t,x)):
u_4(x):=subs(t=tau*(3/8),u(t,x)):
u_5(x):=subs(t=tau*(4/8),u(t,x)):
u_6(x):=subs(t=tau*(5/8),u(t,x)):
u_7(x):=subs(t=tau*(6/8),u(t,x)):
u_8(x):=subs(t=tau*(7/8),u(t,x)):
plot(u_1(x),x=0..15,y=-0.02..1.1,title="t = 0", color=red,thickness=3);
plot(u_2(x),x=0..15,y=-0.02..1.1,title="t = 1/8*tau",color=red,thickness=3);
```

```
plot(u_3(x),x=0..15,y=-0.02..1.1,title="t = 2/8*tau",color=red,thickness=3);
plot(u_4(x),x=0..15,y=-0.02..1.1,title="t = 3/8*tau",color=red,thickness=3);
plot(u_5(x),x=0..15,y=-0.02..1.1,title="t = 4/8*tau",color=red,thickness=3);
plot(u_6(x),x=0..15,y=-0.02..1.1,title="t = 5/8*tau",color=red,thickness=3);
plot(u_7(x),x=0..15,y=-0.02..1.1,title="t = 6/8*tau",color=red,thickness=3);
plot(u_8(x),x=0..15,y=-0.02..1.1,title="t = 7/8*tau",color=red,thickness=3);
```



2 - misol

```
> restart;
```

Bir jinsli tenglamani

$$\frac{\partial}{\partial t}\mathrm{u}(t,x)=a^2\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(t,x)\right)$$

quyidagi chegaraviy shart (o'zgarmas temperaturada sterjenning uchi $x=0$ nuqtga mahkamlangan ($0<x$):

$$\mathrm{u}(t,0)=T0$$

va boshlang'ich shart bilan yeching

$$\mathrm{u}(0,x)=\mathrm{f}(x)\ ,$$

bu yerda $\mathrm{f}(x)$ funksiya quyidagi ko'rinishda berilgan:

```
> T0:=0; a:=1;l:=4;L:=6;alpha:=1;
f(x):=x->piecewise(x<l,T0, x<L,alpha+T0, x>L,0);
```

$$T0:=0$$

$$a:=1$$

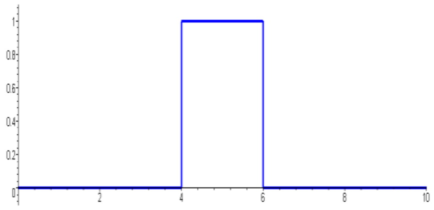
$$l:=4$$

$$L:=6$$

$$\alpha:=1$$

$$\mathrm{f}(x):=x\rightarrow \mathrm{piecewise}(x<l, T0, x<L, \alpha+T0, L<x, 0)$$

```
> plot(f(x),0..10,-0.1..1.1, numpoints=400,color=blue,thickness=3);
```




```
> restart;
f(xi):=xi->piecewise(xi<l,T0, xi<L,alpha+T0, xi>L,0);
```

$$\mathrm{f}(\xi):=\xi\rightarrow \mathrm{piecewise}(\xi<l, T0, \xi<L, \alpha+T0, L<\xi, 0)$$

Yechish uchun yuqoridagi formuladan foydalanamiz:

```
> u(t,x):=simplify((-erf(1/2*x/a/t^(1/2))+1)*T0+1/2*1/a/(Pi*t)^(1/2)*int(f(xi)*(exp(-1/4*(-xi+x)^2/a^2/t)-exp(-1/4*(xi+x)^2/a^2/t)),xi = l .. L));
```

$$\mathrm{u}(t,x):=-T0\,\mathrm{erf}\left(\frac{x}{2\,a\,\sqrt{t}}\right)+T0-\frac{1}{2}\,c\,\mathrm{erf}\left(\frac{l-x}{2\,a\,\sqrt{t}}\right)+\frac{1}{2}\,c\,\mathrm{erf}\left(\frac{l+x}{2\,a\,\sqrt{t}}\right)+\frac{1}{2}\,c\,\mathrm{erf}\left(\frac{L-x}{2\,a\,\sqrt{t}}\right)-\frac{1}{2}\,c\,\mathrm{erf}\left(\frac{L+x}{2\,a\,\sqrt{t}}\right)$$

> T0:=0; a:=1;l:=4;L:=6;alpha:=1;

$$T0 := 0$$

$$a := 1$$

$$l := 4$$

$$L := 6$$

$$\alpha := 1$$

Tenglamani yechimi:

> with(plots):

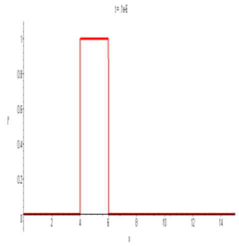
u(t,x):=-T0*erf(1/2*x/a/t^(1/2))+T0+1/2*erf(1/2*(-l+x)/a/t^(1/2))+1/2*erf(1/2*(l+x)/a/t^(1/2))+1/2*erf(1/2*(L-x)/a/t^(1/2))-1/2*erf(1/2*(L+x)/a/t^(1/2));

Warning, the name changecoords has been redefined

$$u(t, x) := \frac{1}{2}\operatorname{erf}\left(\frac{-4+x}{2\sqrt{t}}\right) + \frac{1}{2}\operatorname{erf}\left(\frac{4+x}{2\sqrt{t}}\right) + \frac{1}{2}\operatorname{erf}\left(\frac{6-x}{2\sqrt{t}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{6+x}{2\sqrt{t}}\right)$$

Представим полученные решения в виде двумерных анимированных графиков:

> animate(plot,[u(t,x),x=0..15, y=-0.1..1.1], t=0.0000001..12, frames=60,thickness=3);



Olingan yechimni bir nechta vaqt momentlarida ikki o'lchovli grafik ko'rinishida tasvirlaymiz:

> tau:=12:

u_1(x):=subs(t=tau*0.000001,u(t,x)):

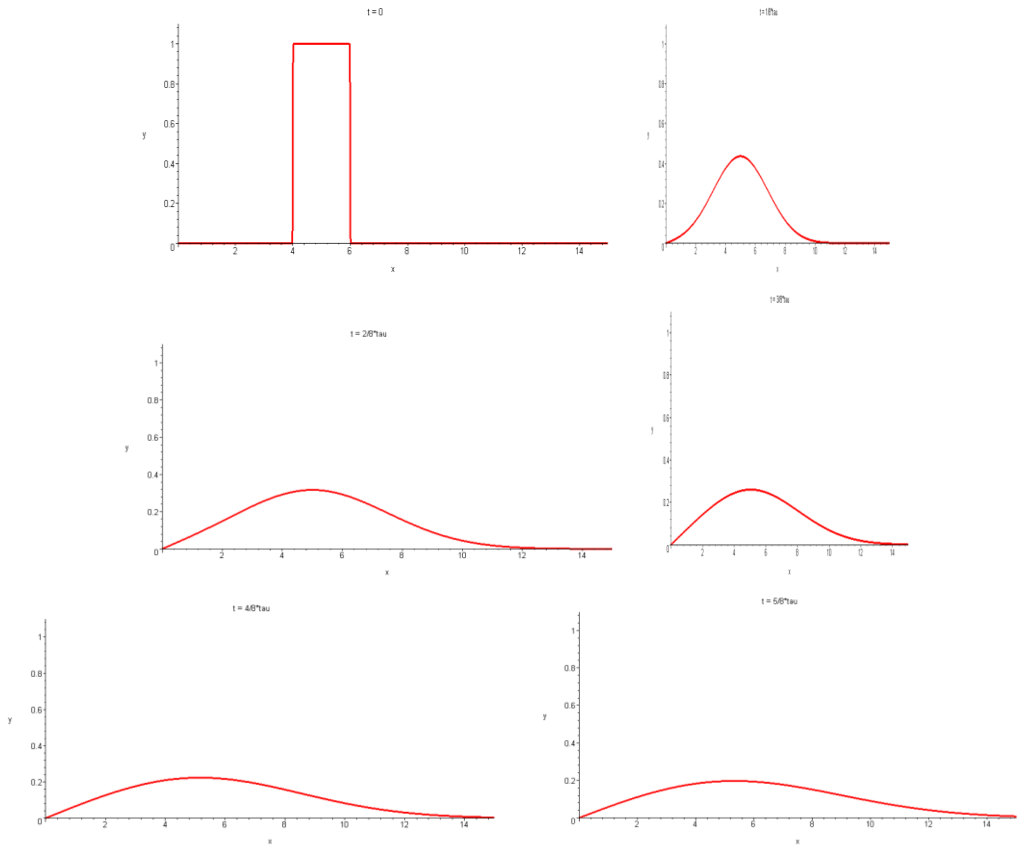
u_2(x):=subs(t=tau*(1/8),u(t,x)):

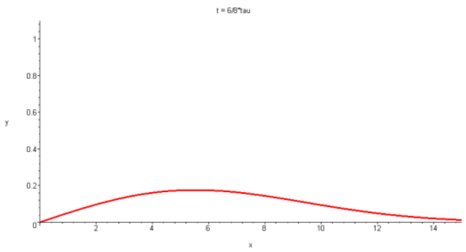
u_3(x):=subs(t=tau*(2/8),u(t,x)):

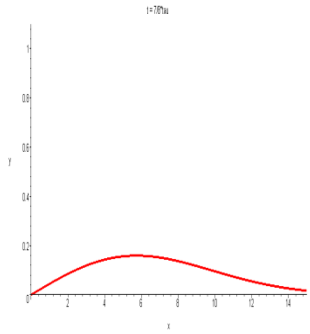
u_4(x):=subs(t=tau*(3/8),u(t,x)):

u_5(x):=subs(t=tau*(4/8),u(t,x)):

```
u_6(x):=subs(t=tau*(5/8),u(t,x)):
u_7(x):=subs(t=tau*(6/8),u(t,x)):
u_8(x):=subs(t=tau*(7/8),u(t,x)):
plot(u_1(x),x=0..15,y=-0.02..1.1,title="t = 0", color=red,thickness=3);
plot(u_2(x),x=0..15,y=-0.02..1.1,title="t = 1/8*tau",color=red,thickness=3);
plot(u_3(x),x=0..15,y=-0.02..1.1,title="t = 2/8*tau",color=red,thickness=3);
plot(u_4(x),x=0..15,y=-0.02..1.1,title="t = 3/8*tau",color=red,thickness=3);
plot(u_5(x),x=0..15,y=-0.02..1.1,title="t = 4/8*tau",color=red,thickness=3);
plot(u_6(x),x=0..15,y=-0.02..1.1,title="t = 5/8*tau",color=red,thickness=3);
plot(u_7(x),x=0..15,y=-0.02..1.1,title="t = 6/8*tau",color=red,thickness=3);
plot(u_8(x),x=0..15,y=-0.02..1.1,title="t = 7/8*tau",color=red,thickness=3);
```







Xulosa

Turmush hayotimizda muhim ahamiyatga ega bo’lgan isssiqlikning to’g’ri chiziq, tekislik va fazoda tarqalish jarayoni, shuningdek diffuziya hodisasi parabolik tipli tenglamalar orqali o’rganiladi. Bu tenglamalar uchun ham to’lqin tenglamasi kabi chegaraviy va Koshi masalari tenglama yechimini bir qiymatli ajratib olishga imkon yaratadi va ular belgilangan rejimga asosan tanlab olinadi.

Biz bu malakaviy bitiruv ishida chekli uzunlikdagi sterjenda qo’yilgan aralash masalalarning limitik holi sifatida aniqlagan chegaralanmagan uzunlikdagi sterjenda issiqlik tarqalish tenglamasiga qo’yilgan Koshi masalasining yechimi xuddi giperbolik tenglamalar uchun chegaraviy masalalrni yechishda qo’llanilgan o’zgaruvchilarni almashtirish yoki Fur’e usuli yordamida topilib, yechim Puasson integrali deb ataluvchi integral shaklda tasvirlanishini o’rgandik.

Bitiruv malakaviy ishida, Maple matematik paketidan foydalanib, sterjenda issiqlik o’tkazuvchanlik tenglamasini va yarim to’g’ri chiziqda issiqlik o’tkazuvchanlik tenglamalarni Fur’ye usuli ya’ni o’zgaruvchilarni ajratish usuli yordamida yechish keltirilgan. Maple paketi orqali parabolik tipdagi tenglamalarni yechish jarayoni qoidaga mos ta’lim berish uchun qiziqarli misollar yordamida tasvirlangan. Maple paketini har bir turdagi masalani yechishga qo’llanilishi ketma-ket tarzda keltirilgan, ya’ni parabolik tipdagi tenglamalarni yechishda misollarga quyidagicha tavsif berilgan: hisoblash formulasi, analitik va sonli yechimi, shuningdek, yechimning ikki o’lchovli animasiyali grafigi tasvirlangan, bundan tashqari ba’zi misollar uchun bir qancha vaqt momentlarini ikki o’lchovli grafigi tasvirlangan.