Xurramov Sh.R.

OLIY MATEMATIKA

MISOL VA MASALALAR
NAZORAT TOPSHIRIQLARI

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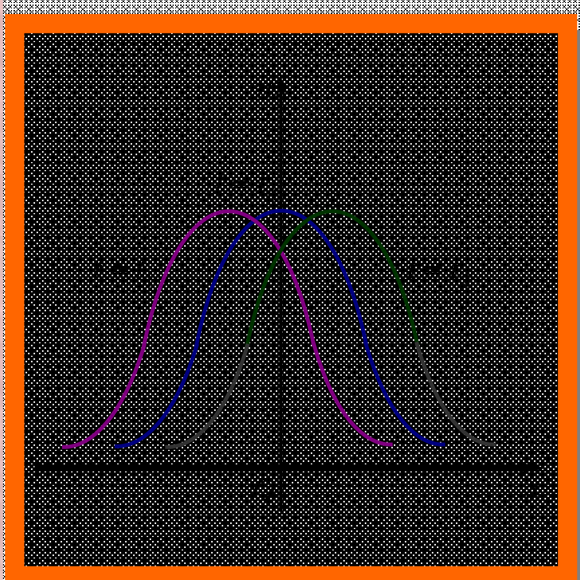

Ehtimollar nazariyasi va matematik statistika

Kompleks o'zgaruvchli funksiyalar nazariyasi

Operatsion hisob

Matemtik fizika tenglamalari

**O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI**

SH. R. XURRAMOV

OLIY MATEMATIKA

MASALALAR TO‘PLAMI
NAZORAT TOPSHIRIQLARI

III QISM

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi oliy ta’lim muassasalari uchun o‘quv qo‘llanma sifatida tavsiya etgan

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Ushbu o‘quv qo‘llanma oily ta’lim muassasalarining texnika va texnologiya yo‘nalishlari bakalavrlari uchun «Oliy matematika» fani dasturi asosida yozilgan bo‘lib, fanning ehtimollar nazariyasi va matematik statistika, kompleks o’zgaruvchili funksiyalar nazariyasi, operatsion hisob va matematik fizika tenglamalari, kabi maxsus bo‘limlariga oid materiallarni o‘z ichiga oladi.

Qo‘llanmada zarur nazariy tushunchalar, qoidalar, teoremalar va formulalar keltirilgan va ularning mohiyati misol va masalalar yechimlarida tushuntirilgan, mustahkamlash uchun mashqlar, nazorat ishi va laboratoriya ishlari uchun topshiriqlar berilgan.

Taqrizchilar:

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SO‘Z BOSHI

Qo‘llanma oliy ta’lim muassasalari texnika va texnologiya bakalavr ta’lim yo‘nalishlari Davlat ta’lim standartlariga mos keladi va fanning o‘quv dasturlariga to‘la javob beradigan tarzda bayon qilingan.

Ushbu o‘quv qo‘llanma bakalavr ta’lim yo‘nalishlarining talabalari uchun mo‘ljallangan bo‘lib, fanning ehtimollar nazariyasi va matematik statistika, kompleks o’zgaruvchili funksiyalar nazariyasi, operatsion hisob va matematik fizika tenglamalari, kabi maxsus bo‘limlari bo‘yicha materiallarni o‘z ichiga oladi.

Qo‘llanmaning har bir bo‘limi zarur nazariy tushunchalar, ta’riflar, teoremalar va formulalar bilan boshlangan, ularning mohiyati misol va masalalarning yechimlarida tushuntirilgan, shu bo‘limga oid amaliy mashg‘ulot darslarida va mustaqil uy ishlarida bajarishga mo‘ljallangan ko‘p sondagi mustahkamlash uchun mashqlar javoblari bilan berilgan.

Har bir bo‘limning oxirida nazorat ishi va laboratoriya ishlari uchun topshiriiqlar variantlari keltirilgan.

Qo‘llanmani yozishda oily texnika o‘quv yurtlarining bakalavrlari uchun oily matematika fanining amaldagi dasturida tavsiya qilingan adabiyotlardan hamda o‘zbek tilida chop etilgan zamonaviy darslik va o‘quv qo‘llanmalardan keng foydalanilgan.

Qo‘llanma haqida bildirilgan fikr va mulohazalar mamnuniyat bilan qabul qilinadi.

Muallif

O‘quv qo‘llanmada *quyidagi belgilashlardan* foydalanilgan:

– muhim ta’riflar;

– «alohida e’tibor bering»;

, – misol yoki masala yechimining boshlanishi va oxiri;

Shuningdek, muhim teorema va formulalar to‘g‘ri to‘rtburchak ichiga olingan.

I bob
EHTIMOLLAR NAZARIYASI VA MATEMATIK STATISTIKA

1.1. EHTIMOLLARNI BEVOSITA HISOBLASH

Hodisalar algebrasi. Kombinatorika elementlari. Ehtimolning ta'riflari

1.1.1. *Hodisa* deb sinashlar natijasida, ya'ni tayin shartlar majmuasi bajarilganda ro'y berishi mumkin bo'lgan har qanday faktga aytiladi.

Hodisalar lotin alfavitining bosh harflari bilan belgilanadi. Masalan, tangani tashlash - sinash, $A-$gerbli tomon tushishi, $B-$raqamli tomon tushishi hodisalar.

Sinash natijasida albatta ro'y beradigan hodisaga *muqarrar hodisa* deyiladi va U bilan belgilanadi.

Sinash natijasida mutlaqo ro'y bermaydigan hodisaga *mumkin bo'lmagan hodisa* deyiladi va V bilan belgilanadi.

Sinash natijasida ro'y berishi ham ro'y bermasligi ham mumkin bo'lgan hodisa *tasodifiy hodisa* deb ataladi.

Agar sinash natijasida bir nechta hodisalardan hech birini boshqalariga nisbatan ro'y berishi mumkinroq deyishga asos bo'lmasa, bunday hodisalarga *teng imkoniyatli hodisalar* deyiladi.

Agar sinash natijasida bir nechta hodisalardan bittasi va faqat bittasining ro'y berishi muqarrar hodisa bo'lsa, bunday hodisalar *yagona mumkin bo'lgan hodisalar* deb ataladi.

Sinash natijasida ro'y berisi mumkin bo'lgan har bir hodisaga *elementar hodisa* deyiladi. Barcha elementar hodisalar to'plami *hodisalar maydoni* deyiladi va Ω orqali belgilanadi.

Hodislar maydonida bitta sinash bilan bog'liq bo'lgan A va B hodisalar ustida qo'shish, ayirish va ko'paytirish amallari aniqlangan.

◎ A va B *hodisalarning yig'indisi* (yoki *birikmasi*) deb ulardan hech bo'lmaganda bittasi ro'y berishidan iborat bo'lgan hodisaga aytiladi va $A+B$ (yoki $A\cup B$) bilan belgilanadi.

◎ A va B *hodisalarning ko'paytmasi* (yoki *kesishmasi*) deb ularning birgalikda ro'y berishidan iborat bo'lgan hodisaga aytiladi va $A\cdot B$ (yoki $A\cap B$) bilan belgilanadi.

A va B *hodisalarning ayirmasi* deb, A hodisa ro‘y berganda B hodisaning ro‘y bermasligidan iborat bo‘lgan hodisaga aytiladi va $A\backslash B$ bilan belgilanadi.

A hodisaga *qarama-qarshi hodisa* (yoki A hodisaning *inkori*) deb, A hodisaning ro‘y bermasligidan iborat bo‘lgan $\overline{A}$ hodisaga aytiladi. Qarama- qarshi hodisalar uchun $A\cdot\overline{A}=V$ va $A+\overline{A}=\Omega$ bo‘ladi.

Agar A hodisa ro‘y berganida B hodisa albatta ro’y bersa, bunda *A hodisa B hodisani ergashtiradi* (yoki A hodisa B hodisaga *kiradi*) deyiladi va $A\subset B$ deb yoziladi. Agar $A\subset B$ va $B\subset A$ bo‘lsa, u holda A va B ekvivalent hodisalar deyiladi va $A=B$ kabi belgilanadi.

Hodisalar ustida amallar quyidagi xossalarga ega:

1^o. $A+B=B+A$, $A\cdot B=B\cdot A$;

2^o. $(A+B)+C=A+(B+C)$, $(A\cdot B)\cdot C=A\cdot(B\cdot C)$;

3^o. $A\cdot(B+C)=A\cdot B+A\cdot C$, $A+B\cdot C=(A+B)\cdot(A+C)$;

4^o. $A+\overline{A}=\Omega$, $A\cdot\overline{A}=V$;

5^o. $A\cdot\Omega=A$, $A+\Omega=\Omega$;

6^o. $A+A=A$, $A\cdot A=A$;

7^o. $A\backslash B=A\cdot\overline{B}$;

8^o. $\overline{V}=\Omega$, $\overline{\Omega}=V$, $\overline{\overline{A}}=A$;

9^o. $\overline{A\cdot B}=\overline{A}+\overline{B}$; $\overline{A+B}=\overline{A}\cdot\overline{B}$.

Bir vaqtda ro‘y bermaydigan A va B hodisalarga *birgalikda bo‘magan hodisalar* deyiladi. Birgalikda bo‘lmagan hodisalar uchun $A\cdot B=V$ bo‘ladi.

Ikkitasi birgalikda bo‘lmagan $A_1,A_2,...,A_n$ hodisalarga *juf - jufti bilan birgalikda bo‘lmagan hodisalar* deyiladi.

Agar $A_1,A_2,...,A_n$ hodisalar birgalikda bo‘lmagan va yagona mumkin bo‘lgan hodisalar bo‘lsa, u holda *bu hodisalar to‘la guruh tashkil etadi* deyiladi. To‘la guruh tashkil etuvchi hodisalar uchun $A_i\cdot A_j=V\ (i\neq j)$

va $A_1+A_2+...+A_n=U$ bo‘ladi.

Ω hodisalar maydonining S hodisalar sinfi *hodisalar algebrasi* deyiladi, agar:

1. $V\in S,\ \Omega\in S$ bo‘lsa;
2. $A\in S$ dan $\overline{A}\in S$ kelib chiqsa;
3. $A\in S,\ B\in S$ dan $A+B\in S,\ A\cdot B\in S$ kelib chiqsa.

1-misol. Oyin kubigi ikki marta tashlanadi. Sinash natijasi - birinchi va ikkinchi tashlashda tushgan ochkolarga mos sonlar juftligi. Sinashning jami elementar hodisalari to‘plami Ω ni va quyidagi hodisalarning elementar hodisalari to'plamlarini toping: $A-$har ikkala holda 3 ga karrali ochkolar tushishi; $B-6$ ochko bir marta ham tushmasligi; $C-$har ikkala holda 4 dan katta ochkolar tushushi; $D-$ har ikkala holda bir xil ochkolar tushushi.

Oyin kubigi tashlanganda oltita elementar natija - 1, 2, 3, 4, 5, 6 ochko tushishi hodisalari mavjud. Shu sababli Ω to'plam va ko'rsatilgan hodisalarga mos to'plamlar quyidagicha bo'ladi:

$$\Omega=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),...,(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\},$$

$$A=\{(3,3),(6,3),(3,6),(6,6)\},$$

$$B=\{(1,1),(1,2)(1,3),(1,4),(1,5),...,(5,1),(5,2),(5,3),(5,4),(5,5)\},$$

$$C=\{(5,5),(5,6),(6,5),(6,6)\},\ D=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}.$$

2-misol. Ixtiyoriy A,B hodisalar uchun A va $\overline{A+B}$ hodisalarning birgalikda bo'lishi yoki birgalikda bo'lmasligini tekshiring.

$A\cdot\overline{A+B}=(9^o$ xossaga ko'ra)$=A\cdot(\overline{A}\cdot\overline{B})=(2^o$ xossaga ko'ra)=

$=(\overline{A}\cdot A)\cdot B=(4^o$ xossaga ko'ra)$=V\cdot B=V.$

Demak, A va $\overline{A+B}$ hodisalar birgalikda emas.

1.1.2. Bir qancha kombinatorika masalalari ikkita qoida asosida yechiladi.

Qo'shish qoidasi. Agar A_1 element n_1 usul bilan, A_2 element boshqa bir n_2 usul bilan, A_3 element birinchi ikki usuldan farqli bo'lgan n_3 usul bilan va shu kabi A_k element birinchi $(k-1)$ usuldan farqli bo'lgan n_k usul bilan tanlangan bo'lsa, u holda ko'rsatilgan elementlardan istalgan bittasi $n_1+n_2+...+n_k$ usul bilan tanlanishi mumkin.

Ko'paytirish qoidasi. Agar A_1 element n_1 usul bilan tanlangan bo'lsa, har bir shunday tanlashdan keyin A_2 element n_2 usul bilan tanlangan bo'lsa va shu kabi har bir $(k-1)$ marta tanlashdan keyin A_k element n_k usul bilan tanlangan bo'lsa, u holda barcha elementlar $A_1,A_2,...,A_k$ tartibda $n_1\cdot n_2\cdot...\cdot n_k$ usul bilan tanlanishi mumkin.

n ta elementdan m ta $(0<m\le n)$ elementni tanlashning ikki sxemasi mavjud: takrorlashlarsiz (qaytarib qo'yishlarsiz) va takrorlash (qaytarib qo'yish) bilan.

Takrorlashlarsiz tanlash sxemasi

n ta elementdan k tadan o'rinlashtirish deb yoki elementlarining tartibi yoki ularning tarkibi bilan farq qiluvchi k ta elementdan tashkil topgan birikmaga aytiladi.

n ta elementdan k tadan o‘rinlashtirishlar soni

$$A_n^k = n(n-1)....(n-(k-1)) = \frac{n!}{(n-k)!}$$

tenglik bilan aniqlanadi.

n *ta elementli o‘rin almashtirish* deb faqat tartibi bilan farq qiluvchi n ta elementdan tashkil topgan birikmaga aytiladi.

n ta elementli o‘rin almashtirishlar soni quyidagi formula bilan topiladi:

$$P_n = A_n^n = n!$$

n *ta elementdan* k *tadan guruhlash* deb hech bo‘lmaganda bitta elementi bilan farq qiluvchi k ta elementdan tashkil topgan birikmaga aytiladi.

n ta elementdan k tadan guruhlashlar soni

$$C_n^k = \frac{A_n^k}{P_k} = \frac{n!}{(n-k)! \cdot k!}$$

kabi topiladi.

Takrorlash bilan tanlash sxemasi

n *ta elementdan* k *tadan takrorlash bilan o‘rinlashtirish* deb yoki elementlarining tartibi yoki ularning takrorlanish soni bilan farq qiluvchi k ta elementdan tashkil topgan birikmaga aytiladi.

n ta elementdan k tadan takrorlash bilan o‘rinlashtirishlar soni

$$\widetilde{A}_n^k = n^k$$

tenglik bilan topiladi.

n ta elementli birikmada k ta har xil element bo‘lib, bunda birinchi element n_1 marta, ikkinchi element n_2 marta, va shu kabi k – element n_k marta takrorlangan hamda $n_1 + n_2 + ... + n_k = n$ bo’lsin. Bu birikmaning n ta elementl o‘rin almashtirishiga n *ta elementli takrorlash bilan o‘rin almashtirish* deyiladi.

n ta elementli takrorlash bilan o‘rin almashtirishlar soni

$$P_n(n_1, n_2, ...n_k) = \frac{n!}{n_1! \cdot n_2! \cdot ... \cdot n_k!}$$

tenglik bilan aniqlanadi.

Agar n ta elementdan k tadan guruhlashda elementlar qayta tartibga olish bajarilmasdan qaytarilsa, u holda n *ta elementdan* k *tadan takrorlash bilan guruhlash* deyiladi.

n ta elementdan k tadan takrorlash bilan guruhlashlar soni

$$\widetilde{C}_n^k = C_{n+m-1}^m$$

kabi aniqlanadi.

3-misol. 0,3,4,5,6,8 raqamlaridan nechta uch xonali son tuzish mumkin?

Uch xonali sonni tuzish jarayoni uchta harakatdan iborat: birinchi harakat- birinchi raqamni tanlash; ikkinchi harakat- ikkinchi raqamni tanlash; uchinchi harakat- uchinchi raqamni tanlash. Uch xonali sonning birinchi raqami 0 boʻlmasligi kerak. Shu sababli birinchi harakatni besh usul bilan bajarish mumkin. Ikkinchi raqam berilgan sonlardan istalgani boʻlishi mumkin va ikkinchi harakatni olti usul bilan bajarish mumkin. Oxirgi raqam juft bo'lishi kerak. Shu sababli uchinchi harakatni toʻrt usul bilan (0,4,6,8 raqamlarini tanlash orqali) bajarish mumkin. U holda koʻpaytirish qoidasiga koʻra tanlash usullari soni $5 \cdot 6 \cdot 4 = 120$ ga teng.

Shunday qilib, masalaning shartini qanoatlantiruvchi 120 ta son mavjud.

4-misol. Bankning yangi prezidenti oʻnta direktorlar orasidan yangi ikkita vitse-prezidentini tanlashi lozim. Prezident ixtiyorida nechta tanlash usuli mavjud, agar: 1) vitse-prezidentlardan birining (birinchisining) lavozimi boshqasining lavozimidan yuqori boʻlsa; 2) vitse-prezidentlar teng lavozimga ega boʻlsa.

1) 10 ta talabgordan ikkita har xil lavozimga 2 ta nomzodni tanlash usullari soni 10 ta elementdan 2 tadan oʻrinlashtirishlar soniga teng boʻladi, ya'ni

$$A_{10}^2 = 10 \cdot 9 = 90.$$

2) 10 ta talabgordan ikkita bir xil lavozimga 2 ta nomzodni tanlash usullari soni 10 ta elementdan 2 tadan guruhlashlar soniga teng boʻladi, ya'ni

$$C_{10}^2 = \frac{10!}{2! \cdot 8!} = \frac{10 \cdot 9}{1 \cdot 2} = 45.$$

1.1.3. Hodisa obʻektiv roʻy berish imkoniyati darajasining sonli koʻrsatgichiga *hodisaning ehtimoli* deyiladi.

Ehtimolning matematik ta'rifi. Har bir A hodisaga bu hodisaning ehtimoli deb ataluvchi biror $P(A)$ oʻlchov mos qo'yiladi va bu oʻlchov quyidagi aksiomalarni qanoatlantiradi:

1^o. Har qanday hodisa uchun $0 \le P(A) \le 1$ boʻladi;

2^o. Muqarrar hodisaning ehtimoli $P(U) = 1$ boʻladi;

3^o. Roʻy bermaydigan hodisaning ehtimoli $P(V) = 0$ boʻladi;

4^o. Agar $A \cdot B = V$ boʻlsa, u holda $P(A+B) = P(A) + P(B)$, ya'ni birgalikda boʻlmagan A va B hodisalar yigʻindisining ehtimoli shu hodisalar ehtimollarining yigʻindisiga teng boʻladi.

Agar sinashning natijalari hodisalarning to‘la guruhini tashkil etsa va teng imkoniyatli bo‘lsa, ya’ni ular yagona mumkin bo‘lgan, birgalikda bo‘lmagan va teng imkoniyatli hodisalar bo‘lsa, u holda bu natijalarga *elementar natijalar* deyiladi. Bunda sinash “klassik” deb yuritiladi. Sinashning o‘rganilayotgan hodisaning ro‘y berishiga olib keladigan elementar natijalariga sinashning hodisa ro‘y berishiga *qulaylik tug‘diruvchi natijalari* deyiladi.

E h t i m o l n i n g k l a s s ik t a’ r i f i ga binoan A *hodisaning ehtimoli* deb, sinashning A hodisa ro‘y berishiga qulaylik tug‘diruvchi natijalari soni m ning sinashning barcha elementar natijalari soni n ga nisbatiga aytiladi va $P(A)$ bilan belgilanadi:

$$P(A)=\frac{m}{n}.$$

Bunda A hodisa ro‘y berishiga qulaylik tug‘diruvchi natijalar “moyil hodisalar” deb yuritiladi.

A *hodisaning nisbiy chastotasi* deb hodisa ro‘y bergan sinashlar soni m^* ning aslida o‘tkazilgan jami sinashlar soni n^* ga nisbatiga aytiladi va $P^*(A)$ bilan belgilanadi:

$$P^*(A)=\frac{m^*}{n^*}.$$

E h t i m o l n i n g s t a t i s t i k t a’ r i f i ga binoan **s**inash shartlari o‘zgarmaganda A hodisaning nisbiy chastotasi tebranadigan songa A *hodisaning ehtimoli* deyiladi.

E h t i m o l n i n g g o m e t r i k t a’ r i f i ga binoan A *hodisaning ehtimoli* deb A hodisaga moyil soha o‘lchamining butun soha o‘lchamiga nisbatiga aytiladi, ya’ni

$$P(A)=\frac{mesg}{mesG}. \qquad (1.3)$$

Ehtimolning klassik, statistik va geometrik ta’riflari matematik ta’rifning barcha aksiomalariga bo‘ysinadi.

5-misol. Oyin kubigi bir marta tashlanganda toq ochko tushishi ehtimolini toping.

⊜ Oyin kubigi tashlanganda oltita elementar natija - 1, 2, 3, 4, 5, 6 ochko tushishi hodisalari mavjud. Barcha $n=6$ ta elementar natijalar teng imkoniyatli va to‘la guruh tashkil qiladi. $A-$toq ochko tushishi hodisasi bo‘lsin. A hodisa ro‘y berishiga $m=3$ ta natija - 1, 3 va 5 ochkolar tushishi hodisalari moyil bo‘ladi.

U holda ehtimolning klassik ta’rifiga ko‘ra

$$P(A)=\frac{3}{6}=\frac{1}{2}.$$

6-misol. Qutida 3 ta oq, 5 ta yashil va 2 ta ko‘k sharlar bor. Qutidan tavakkaliga olingan sharning rangli bo‘lishi ehtimolini toping.

$A-$olingan sharning rangli bo‘lishi hodisasi bo‘lsin. Sinash 10 ta teng imkoniyatli elementar hatijalardan iborat bo‘lib, ulardan 7 tasi olingan shar rangli (yashil, ko‘k) bo‘lishiga, ya’ni A hodisaga moyil bo‘ladi.

Demak,

$$P(A)=\frac{7}{10}=0{,}7.$$

7-misol. Oltita bIr xil varaqqa alohida A, B, E, I, F, L harflari yozilgan. Bola varaqlarni tavakkaliga oladi va chapdan o‘ngga qarab ketma-ket yoyadi: 1) 3 ta varaq olinganda “*AQL*”, “*FIL*” so‘zlarining chiqishi ehtimollarini; 2) 6 ta varaq olinganda “*ALIFBE*” so‘zining chiqishi ehtimolini toping.

1) $A-3$ ta varaq olinganda “*AQL*” so‘zi chiqishi hodisasi bo‘lsin. *Q* harfi varaqlarda yo‘q. Shu sababli $m=0$ va $P(A)=0$.

$B-3$ ta varaq olinganda “*FIL*” so‘zi chiqishi hodisasi bo‘lsin. So‘zdagi harflar varaqlarda bir martadan uchraydi. Shu sababli $m=1$. Oltita harfdan uchtadan birikmalar soni 6 ta elementdan 3 tadan o‘rinlashtirishlar soniga teng:

$$A_6^3=6\cdot 5\cdot 4=120.$$

Demak,

$$P(B)=\frac{1}{120}.$$

2) $C-6$ ta varaq olinganda “*ALIFBE*” so‘zi chiqishi hodisasi bo‘lsin. So‘zdagi harflar varaqlarda bir martadan uchraydi. Shu sababli $m=1$. Oltita harfdan oltitadan birikmalar soni 6 ta elementlidan o‘rin almashtirishlar soniga teng:

$$P_6=6!=720.$$

Demak,

$$P(C)=\frac{1}{720}.$$

8-misol. Qirqma alfavitning 10 ta harfidan “*MATEMATIKA*” so‘zi tuzilgan. Bu harflar sochilib ketgan va qaytadan ixtiyoriy tartibda yig‘ilgan. Quyidagi so‘zlar chiqishi ehtimollarini toping: 1) “*MATEMATIKA*”,
2) “*KATET*”.

1) A – “*MATEMATIKA*” so‘zi chiqishi hodisasi bo‘lsin. Sinashning mumkin bo‘lgan teng imkoniyatli elementar hatijalari 10 ta elementdan o‘rin almashtirish sonidan iborat, ya’ni $n = P_{10} = 10!$. A hodisaga moyil hodisalar soni $m = 2!\cdot 3!\cdot 2!$ ga teng, chunki matematika so‘zida “*M*” 2 marta, “*A*” 3 marta, “*T*” 2 marta takrorlanadi.

Demak,

$$P(A) = \frac{m}{n} = \frac{2!\cdot 3!\cdot 2!}{10!} = \frac{1}{151200}.$$

Bu masalani boshqacha yechish mumkin: 10 ta harfning takrorlash bilan orin almashtirishlar soni

$$P_{10}(3,2,2,1) = \frac{10!}{3!\cdot 2!\cdot 2!\cdot 1!} = 151200.$$

Bundan

$$P(A) = \frac{1}{P_{10}(3,2,2,1)} = \frac{1}{151200}.$$

2) B – “*KATET*” so‘zi chiqishi hodisasi bo‘lsin. Sinashning mumkin bo‘lgan teng imkoniyatli elementar hatijalari 10 ta elementdan 5 tadan o‘rinlashtirishdan iborat:

$$n = A_{10}^{5} = \frac{10!}{5!}.$$

B hodisaga moyil hodisalar soni $m = 2!$, chunki katet so‘zida “*T*” 2 marta takrorlanadi.

Shunday qilib,

$$P(B) = \frac{m}{n} = \frac{2!\cdot 5!}{10!} = \frac{1}{15120}.$$

9-misol. Qutida 10 ta detal bo‘lib, ulardan 7 tasi standart. Tavakkaliga: 1) 4 ta detal olinganda, ularning hammasi standart bo‘lishi ehtimolini toping; 2) 5 ta detal olinganda, ularning 3 tasi standart bo‘lishi ehtimolini toping.

1) Sinashning mumkin bo‘lgan elementar natijalari soni 10 detaldan 4 ta detalni olish usullari soniga, ya’ni C_{10}^{4} ga teng. A hodisaga moyil natijalar soni 7 detaldan 4 ta detalni olish usullari soni C_{7}^{4} ga teng.

Demak,

$$P(A)=\frac{C_7^4}{C_{10}^4}=\frac{\frac{7!}{4!\cdot 3!}}{\frac{10!}{4!\cdot 6!}}=\frac{7!\cdot 6!}{10!\cdot 3!}=\frac{4\cdot 5\cdot 6}{8\cdot 9\cdot 10}=\frac{1}{6}.$$

2) Sinashning mumkin bo‘lgan elementar natijalari C_{10}^5 ga teng. Ulardan $C_7^3\cdot C_3^2$ tasi tanlangan detallar ichida 3 tasi standart bo‘lishi hodisasi B ga moyil.

Shu sababli

$$P(B)=\frac{C_7^3\cdot C_3^2}{C_{10}^5}=\frac{\frac{7!}{3!\cdot 4!}\cdot\frac{3!}{2!\cdot 1!}}{\frac{10!}{5!\cdot 5!}}=\frac{7!\cdot 3!\cdot 5!\cdot 5!}{10!\cdot 3!\cdot 4!\cdot 2!}=\frac{5}{12}.$$

10-misol. Yetti qavatli uyning liftiga birinchi qavatda 3 kishi kirdi. Ularning har biri ikkidan ettigacha bo‘lgan istalgan qavatda liftdan chiqishi mumkin. Quyidagi hodisalarning ro‘y berishi ehtimollarini toping: $A-$ularning barchasi 5-qavatda liftdan chiqisi; $B-$ularning barchasi bitta qavatda liftdan chiqisi; $C-$ulardan har biri turli qavatda liftdan chiqisi.

Yo‘lovchilarning har biri ikkidan ettinchi qavatgacha 6 usul bilan liftdan chiqishi mumkin. Bunda 6 ta elementdan 3 tadan takrorlash bilan o‘rinlashtirishlar soni, ya’ni sinashning mumkin bo‘lgan elementar natijalari soni $\widetilde{A}_6^3=6^3=216$ ga teng bo‘ladi. Ulardan A hodisaga $m_1=1$ ta natija moyil, B hodisaga $m_2=6$ ta natija (barcha yo‘lovchi yoki 2-qavatda, yoki 3-qavat-da,…, yoki 7-qavatda liftdan chiqadi) moyil, C hodisaga $m_3=C_6^3=20$ ta natija (yo‘lovchilar 6 ta qavatdan 3ta qavatda liftdan chiqadi) moyil.

Bundan

$$P(A)=\frac{1}{216};\qquad P(B)=\frac{6}{216}=\frac{1}{36};\qquad P(C)=\frac{20}{216}=\frac{5}{54}.$$

11-misol. Ikkita o‘yin kubigi baravar tashlanganda quyidagi hodisalarning ro‘y berishi ehtimollarini toping: $A-$ tushgan ochkolar yig‘indisi 6 ga teng bo‘lishi; $B-$ tushgan ochkolar ko‘paytmasi 6 ga teng bo‘lishi; $C-$tushgan ochkolar yig‘indisi ularning ko‘paytmasidan katta bo‘lishi.

Har bir o‘yin kubigi tashlanganda oltita elementar natija ro‘y berishi mumkin. Bunda 6 ta elementdan 2 tadan takrorlash bilan o‘rinlashtirishlar soni, ya’ni sinashning mumkin bo‘lgan elementar natijalari soni $\widetilde{A}_6^2=6^2=36$

ga teng bo‘ladi. Bu elementar natijalardan A hodisaga $m_1 = 5$ tasi moyil: kubiklarda (1,5), (2,4), (3,3), (4,2), (5,1) ochkolar tushishi, B hodisaga $m_2 = 4$ tasi moyil: kubiklarda (1,6), (2,3), (3,2), (6,1) ochkolar tushishi, C hodisaga $m_3 = 11$ tasi moyil: kubiklarda (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (3,1), (4,1), (5,1), (6,1) ochkolar tushish.

Demak,

$$P(A) = \frac{5}{36}; \quad P(B) = \frac{4}{36} = \frac{1}{9}; \quad P(C) = \frac{11}{36}.$$

12-misol. Do‘konda bor bo‘lgan uch turdagi 5, 7 va 13 ta sovutgichdan 21 tasi sotilgan. Har bir turdagi sovutgichlarning sotilishi ehtimollari bir xil bo‘lsa, do‘konda sotilmasdan qolgan sovutgichlarning ehtimollarini toping: 1) bir turdagi; 2) har xil turdagi.

1) $A-$bir turdagi sovutgichlar sotilmasdan qolgan bo‘lishi hodisasi bo‘lsin. 25 ta sovutgichdan 4 ta sovutgich $n = C_{25}^4$ usul bilan sotilmasdan qolishi mumkin. Birinchi turdagi 4 ta sovutgichni olishlar soni $m_1 = C_5^4$ ga, ikkinchi turdagi - $m_2 = C_7^4$ ga va uchinchi turdagi - $m_3 = C_{13}^4$ ga teng. U holda A hodisaning ro‘y berishiga birikmalarni qo‘shish qoidasiga ko‘ra $m = m_1 + m_2 + m_3 = C_5^4 + C_7^4 + C_{13}^4$ ta hodisa moyil bo‘ladi.

Demak,

$$P(A) = \frac{m}{n} = \frac{C_5^4 + C_7^4 + C_{13}^4}{C_{25}^4} = \frac{5 + 35 + 715}{12650} = 0{,}06.$$

2) $B-$har xil turdagi sovutgichlar sotilmasdan qolgan bo‘lishi hodisasi bo‘lsin. B hodisa uchta variantdan birida ro‘y berishi mumkin. Birinchi variantda birinchi, ikkinchi va uchunchi turdagi sovutgichlardan mos ravishda 1, 1, 2 tasi, ikkinchi variantda - 1, 2, 1 tasi va uchinchi variantda esa - 2, 1, 1 tasi sotilmasdan qolsa.

U holda birikmalarni ko‘paytirish qoidasiga ko‘ra birinchi, ikkinchi va uchinchi variantga mos ravishda $m_1 = C_5^1 C_7^1 C_{13}^2$ ta, $m_2 = C_5^1 C_7^2 C_{13}^1$ ta va $m_3 = C_5^2 C_7^1 C_{13}^1$ ta hodisa moyil bo‘ladi.

Bundan

$$P(B) = \frac{m}{n} = \frac{m_1 + m_2 + m_3}{n} = \frac{C_5^1 C_7^1 C_{13}^2 + C_5^1 C_7^2 C_{13}^1 + C_5^2 C_7^1 C_{13}^1}{C_{25}^4} =$$

$$= \frac{5 \cdot 7 \cdot 78 + 5 \cdot 21 \cdot 13 + 10 \cdot 7 \cdot 13}{12650} = 0{,}396.$$

13-misol. Yuk mashinasiga ortish vaqtida 400 ta tarvuzdan 25 tasi yorilganligi aniqlandi. Tarvuzlar yorilishi hodisasining nisbiy chastotasini toping.

$A-$tarvuzlarning yorilishi hodisasi bo'lsin. Masalaning shartiga ko'ra $n^*=400,\ m^*=25.$

U holda

$$P^*(A)=\frac{m^*}{n^*}=\frac{25}{400}=0,063.$$

14-misol. Radiusi R ga teng doiraga nuqta tavakkaliga tashlangan. Nuqtaning doira bilan doiraga ichki chizilgan muntazam uchburchak orasiga tushishi ehtimolini toping.

s uchburchakning yuzasi, S doiraning yuzasi, $A-$nuqtaning doira bilan uchburchak orasiga tushishi hodisasi bo'lsin. Radiusi R ga teng doiraning yuzasi $S=\pi R^2$ ga, unga uchki chizilgan uchburchakning yuzasi $s=\frac{3\sqrt{3}R^2}{4}$ ga teng bo'ladi.

U holda ehtimolning geometrik ta'rifiga ko'ra

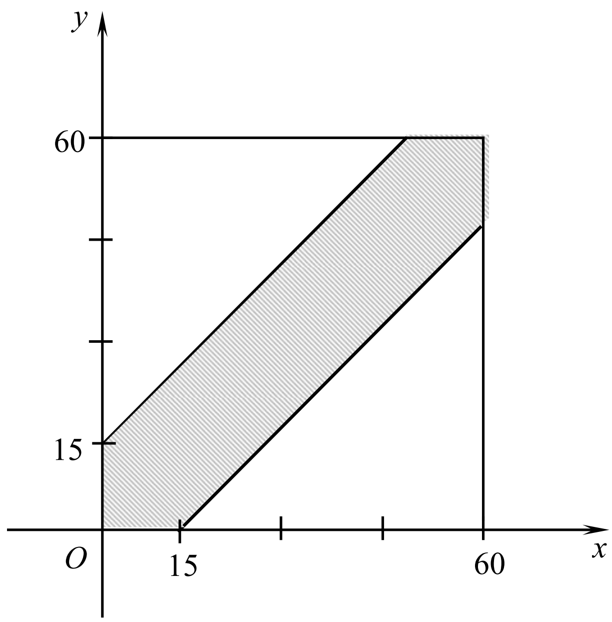
$$P=(A)=\frac{S-s}{S}=\frac{4\pi R^2-3\sqrt{3}R^2}{4\pi R^2}=\frac{4\pi-3\sqrt{3}}{4\pi}.$$

15-misol (Uchrashuv haqidagi masala). Ikki talaba ertalab coat 9 bilan 10 orasida tayin joyda uchrashishga kelishib olishdi. Oldin kelgan talaba ikkinchi talabani 15 minut davomida kutib, agar u kelmasa, qaytib ketadi. Agar har bir talaba kelish vaqtini kelishuv soatida tavakkaliga tanlasa, ularning uchrashishi ehtimolini toping.

$x-$birinchi talabaning kelish vaqti, $y-$ikkinchi talabaning kelish vaqti bo'lsin. Agar masshtab birligi qilib minut olinsa, x va y ning qabul qilishi mumkin bo'lgan qiymatlari

$$0\le x\le 60,\ 0\le y\le 60$$

bo'ladi. Bu qiymatlar Oxy koordinatalar tekisligida tomoni 60 ga teng bo'lgan kvadratni aniqlaydi (1-shakl).



1-shakl.

Bu kvadratning nuqtalari uchrashuvchilar vaqtini ifodalaydi:

$$\Omega = \{(x,y): 0 \le x \le 60,\ 0 \le y \le 60\}.$$

Bunda barcha elementar natijalar teng imkoniyatli, chunki talabalar tavakkaliga keladi.

$A-$talabalar uchrashishi hodisasi bo‘lsin. A hodisa talabalarning kelish vaqtlari orasidagi ayirmaning moduli 15 dan katta bo‘lmaganda ro‘y beradi, ya’ni

$$A = \{(x,y): |x-y| \le 15\}.$$

$|x-y| \le 15$ tengsizlik 1-shaklda bo‘yalgan sohani aniqlaydi. Bu sohada yotuvchi nuqtalar A hodisaga moyil bo‘ladi.

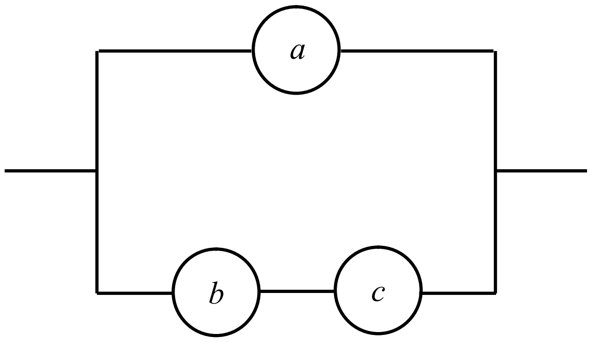
U holda ehtimolning geometrik ta’rifiga ko‘ra izlanayotgan ehtimol

$$P(A) = \frac{60^2 - 2\cdot\frac{1}{2}\cdot 45\cdot 45}{60^2} = \frac{7}{16}.$$

Mashqlar

1.1.1. Tanga uch marta tashlanadi. Sinash natijasi - tanganing yuqori tomonida gerb (g) yoki raqam (r) chiqichi. Sinashning jami elementar natijalari to‘plami Ω ni va quyidagi hodisalarning elementar natijalari to‘plamlarini toping: $A-$gerb rosa bir marta chiqishi; $B-$raqam bir marta ham chiqmasligi; $C-$gerblar raqamlardan ko‘p chiqishi; $D-$ gerb ketma-ket ikki martadan ko‘p chiqishi.

1.1.2. Elektr zanjiri 2-shaklda keltirilgan sxemada tuzilgan. $A-a$ elementning ishdan chiqishi, $B-b$ elementning ishdan chiqiushi, $C-c$ elementning ishdan chiqishi hodisalari bo‘lsa, D va $\overline{D}$ hodisalarning ifodasini yozing, bu yerda $D-$zanjirning uzilishi hodisasi.



2-shakl.

1.1.3. Tasodifiy sonlar to‘plamidan tavakkaliga bitta son olingan. Agar $A-$olingan son 0 bilan tugaydi, $B-$olingan son 5 ga bo‘linadi hodisalari bo‘lsa, $B\setminus A$ va $A\cap B$ qanday hodisalar bo‘ladi?

1.1.4. Ixtiyoriy A, B hodisalar uchun quyidagilarni isbotlang:
1) $(A+B)\cdot(A+\overline{B})=A;$ 2) $(A+B)\cdot(\overline{A}+B)\cdot(A+\overline{B})=A\cdot B.$

1.1.5. Guruhdagi yigirmata talabadan anjumanga yuborish uchun uchtasini tanlash kerak. Tanlashning nechta usuli mavjud?

1.1.6. Erkin Anvar bilan xafalashib qolgani uchun u bilan bitta avtobusda ketishni istamaydi. Talabalar turar joyidan institutgacha coat 7 bilan 8 oralig‘ida beshta avtobus jo‘naydi. Oxirgi avtobusga eta olmagan talaba mashg‘ulotga kech qoladi. Erkin bilan Anvar har xil avtobuslarda nechta usul bilan mashg‘ulotga kechikmasdan borishlari mumkin?

1.1.7. “DADA” so‘zida harflarning o‘rnini almashtirish orqali nechta har xil so‘z tuzish mumkin?

1.1.8. Oltita bir xil varaqqa 3,3,4,5,5,5 sonlari yozilgan. Bu varaqlar ketma-ket joylashtirilsa, nechta olti xonali son tuzish mumkin?

1.1.9. Do‘konga 30ta televizor keltirildi. Ulardan 5 tasi bilinmas nuqsonga ega. Tekshirish uchun tavakkaliga tanlangan televizorning nuqsonsiz bo‘lishi ehtimolini toping.

1.1.10. “*TARVUZ*” so‘zidan tavakkaliga bitta harf tanlangan. Bu harfning: 1) “*M*” harfi bo‘lishi ehtimolini toping; 2) unli harf bo‘lishi ehtimolini toping.

1.1.11. Qutida 3 ta qizil, 7 ta ko‘k va 5 ta oq shar bor. Qutidan tavakkaliga olingan sharning: 1) oq rangda bo‘lishi ehtimolini toping; 2) qizil rangda bo‘lishi ehtimolini toping; 3) yashil rangda bo‘lishi ehtimolini toping; 4) rangli bo‘lishi ehtimolini toping.

1.1.12. Domino toshlari to‘plamidan (28 ta tosh) tavakkaliga olingan toshda: 1) 5 ochko bo‘lishi ehtimolini toping; 2) 4 ochko yoki 6 ochko bo‘lishi ehtimolini toping; 3) chiqqan ochkolar yig‘indisi 8 ga teng bo‘lishi ehtimolini toping.

1.1.13. Birinchi qutida 1 dan 5 gacha raqamlangan sharlar, ikkinchi qutida 6 dan 10 gacha raqamlangan sharlar bor. Har bir qutidan tavakkaliga bittadan shar olingan. Olingan sharlarda raqamlar yig‘indisi: 1) 7 dan kichik bo‘lmasligi ehtimolini toping; 2) 11 ga teng bo‘lishi ehtimolini toping; 3) 11 dan kichik bo‘lmasligi ehtimolini toping.

1.1.14. Abonent telefon raqamini terayotib nomerning oxirgi uchta raqamini eslay olmadi va bu raqamlar har xil ekanini bilgan holda ularni tavakkaliga terdi. Raqamning toʻgʻri terilishi ehtimolini toping.

1.1.15. Ikkita oʻyin kubigi tashlanganda: 1) tushgan raqamlar ko'paytmasi 12 ga teng boʻlishi ehtimolini toping; 2) tushgan raqamlar yigʻindisi 10 ga teng boʻlishi ehtimolini toping; 3) 2 raqam tushishi ehtimolini toping.

1.1.16. BIr xil varaqqa alohida yozilgan 10 ta variantdan 8 tasi tavakkaliga tanlangan va bir qatorda oʻtirgan 8 ta talabaga tarqatilgan. Quyidagi hodisalarning roʻy berishi ehtimollarini toping: 1) birinchi va ikkinchi variantlar tarqatilmay qolishi; 2) birinchi va ikkinchi variantlar yonma-yon oʻtirgan talabalarga tushishi; 3) variantlar ketma-ket tartibda tarqatilishi.

1.1.17. Toʻrtta bIr xil varaqqa alohida A, L, L, O harflari yozilgan. Bola varaqlarni tavakkaliga oladi va chapdan oʻngga qarab ketma-ket yoyadi:
1) 3 ta varaq olinganda "*LOL*", "*OLA*" soʻzlarining chiqishi ehtimolini;
2) 4 ta varaq olinganda "*LOLA*" soʻzining chiqishi ehtimolini toping.

1.1.18. Qirqma alfavitning 10 ta harfidan "*STATISTIKA*" soʻzi tuzilgan. Bu harflar sochilib ketgan va qaytadan ixtiyoriy tartibda yigʻilgan. Quyidagi soʻzlar chiqishi ehtimollarini toping: 1) "*STATISTIKA*", 2)"*KATTA*", 3) "*ISTAK*".

1.1.19. Toʻqqiz qavatli uyning liftiga birinchi qavatda 4 kishi kirdi. Ularning har biri ikkidan toʻqqizgacha boʻlgan istalgan qavatda liftdan chiqishi mumkin. Quyidagi hodisalarning roʻy berishi ehtimollarini toping: 1) barcha yoʻlovchi 6-qavatda liftdan chiqisi; 2) barcha yoʻlovchi bitta qavatda liftdan chiqisi; 3) har bir yoʻlovchi turli qavatda liftdan chiqisi.

1.1.20. Guruhdagi 30 talabadan 5 tasi a'lochi. Tavakkaliga tanlangan uchta talabaning: 1) barchasi a'lochi boʻlishi ehtimolini toping; 2) birortasi ham a'lochi boʻlmasligi ehtimolini toping.

1.1.21. "36 dan 6 ta sportloto" oʻyinida sportning sonlar bilan belgilangan 36 turidan 6 turi tavakkaliga tanlanadi. Bunda: 1) 6 ta son toʻgʻri topilishi hodisasining ehtimolini toping; 2) 4 ta son toʻgʻri topilishi hodisasining ehtimolini toping.

1.1.22. Qutida 6 ta oq va 4 ta qora shar bor. Tavakkaliga: 1) 3 ta shar olinganda ularning hammasi oq bo‘lishi ehtimolini toping; 2) 5 ta shar olinganda ulardan 2 tasi qora bo‘lishi ehtimolini toping; 3) 2 ta shar olinganda ularning turli rangda bo‘lishi ehtimolini toping.

1.1.23. Savatdagi 10 ta tennis to‘pidan 4 tasi yangi. Ulardan 3 tasi tavakkaliga tanlandi. Tanlangan to‘plardan 2tasi yangi bo‘lishi ehtimolini toping.

1.1.25. Do‘konda 30 ta televizor bo‘lib, ulardan 20 tasi import. Barcha televizorlarning sotilishi ehtimoli bir xil bo‘lsa, 5 ta sotilgan televizordan 3 tasi import bo‘lishi ehtimolini toping.

1.1.25. Qutida 5 ta ko‘k, 4 ta qizil va 3 ta yashil rangli qalamlar bor. Tavakkaliga 3 ta qalam olingan. 1) barcha qalam bir xil rangli bo‘lishi ehtimolini toping; 2) qalamlar har xil rangli bo‘lishi ehtimolini toping; 3) 2 ta qalam ko‘k rangli va bitta qalam yashil rangli bo‘lishi ehtimolini toping.

1.1.26. Futbol bo‘yicha musobaqada 18 ta jamoa qatnashadi va ulardan 5 tasi oliy liga a’zosi. Jamoalar tavakkaliga 9 tadan ikki guruhga bo‘linadi. Quyidagi hodisalarning ro‘y berishi ehtimollarini toping: 1) $A -$ oily liganing barcha jamoalari bitta guruhga tushishi; 2) $B -$ oily liganing 2 tasi jamoasi guruhlardan biriga va 3 tasi ikkinchisiga tushishi.

1.1.27. Qarta dastasi (52 ta) tavakkaliga 26 tadan ikki qutiga solinadi. Quyidagi hodisalarning ro‘y berishi ehtimollarini toping: 1) har bir qutida 2 tadan tuz bo‘lishi; 2) qutilardan birida birorta tuz bo‘lmasligi, ikkinchisida 4 tuz bo‘lishi; 3) qutilardan birida 1 ta, ikkinchisida 3 ta tuz bo‘lishi.

1.1.28. Mergan nishonga qarata 20 ta o‘q uzdi va 12 o‘qning nishonga tekkanligi aniqlandi. Merganning nishonga tekkizishi hodisasining nisbiy chastotasini toping.

1.1.29. Texnik nazorat bo‘limi 120 ta mahsulotdan 6 tasi sifatsiz ekanini aniqladi. Sifatsiz mahsulot chiqishi hodisasining nisbiy chastotasini toping.

1.1.30. Radiusi R ga teng doiraga nuqta tavakkaliga tashlangan. Uning shu doiraga ichki chizilgan muntazam ko‘pburchakka tushishi ehtimolini toping: 1) uchburchakka; 2) to‘rtburchakka; 3) oltiburchakka.

1.1.31. Radiusi R ga teng sharga nuqta tavakkaliga tashlangan. Uning shu sharga ichki chizilgan kubga tushishi ehtimolini toping.

1.2. EHTIMOLLARNI TOPISHNING ASOSIY FORMULALARI

Ehtimollarni qo‘shish teoremalari. Ehtimollarni ko‘paytirish teorimalari. To‘la ehtimol formulasi. Bayes formulasi

1.2.1. 1-teorema. Birgalikda bo‘lmagan A va B hodisalar yig‘indisining ehtimoli shu hodisalar ehtimollarining yig‘indisiga teng, ya’ni

$$P(A+B)=P(A)+P(B).$$

Juft-lufti bilan birgalikda bo‘lmagan $A_1, A_2, ..., A_n$ hodisalar uchun

$$P(A_1+A_2+...+A_n)=P(A_1)+P(A_2)+...+P(A_n).$$

2- teorema. Juft-jufti bilan birgalikda bo‘lmagan to‘la guruh tashkil etuvshi $A_1, A_2, ..., A_n$ hodisalar ehtimollarining yig‘indisi birga teng, ya’ni

$$P(A_1)+P(A_2)+...+P(A_n)=1$$

1-natija. Qarama - qarshi hodisalar ehtimollarining yig‘indisi birga teng, ya’ni

$$p+q=1,$$

bu yerda $p=P(A)$, $q=P(\overline{A})$.

1-misol. Otyin kubigi tashlanganda 3 ochko yoki 4 ochko tushishi hodisalarining ehtimolini toping.

$A-3$ ochko tushishi hodisasi, $B-4$ ochko tushishi hodisasi bo‘lsin. U holda $P(A)=\frac{1}{6}$, $P(B)=\frac{1}{6}$ bo‘ladi.

A va B hodisalar birgalikda bo‘lmagan hodisalar. Shu sababli

$$P(A+B)=P(A)+P(B)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}.$$

2-misol. $A, B, C, D-$to‘la guruh tashkil qiluvchi hodisalar va $P(A)=0{,}3$, $P(B)=0{,}4$, $P(C)=0{,}1$ bo‘lsa, D hodisaning ehtimolini toping.

A, B, C, D hodisalar to‘la guruh tashkil qilgani sababli

$$P(A)+P(B)+P(C)+P(D)=1.$$

Bundan

$$P(D)=1-\big(P(A)+P(B)+P(C)\big)=1-(0{,}3+0{,}4+0{,}1)=0{,}2.$$

3-misol. 8 ta oq va 4 ta rangli shar solingan qutidan tavakkaliga 5 ta shar olinadi. Olingan sharlar orasida hech bo‘lmaganda bitta rangli shar bo’lishi ehtimolini toping.

$A -$ olingan sharlar orasida hech bo‘lmaganda bitta rangli shar bo‘lishi hodisasi bo‘lsin. U holda $\overline{A} -$ olingan sharlar orasida rangli shar bo‘lmasligi hodisasi bo‘ladi.

$P(\overline{A})$ ni topamiz. 12 ta sharlar orasidan 5 ta sharni $n = C_{12}^5$ usul bilan olish mumkin. 8 ta oq shardan 5 ta sharni $m = C_8^5$ usul bilan olish mumkin.

U holda

$$P(\overline{A}) = \frac{C_8^5}{C_{12}^5} = \frac{\frac{8!}{5!\cdot 3!}}{\frac{12!}{5!\cdot 7!}} = \frac{8!\cdot 7!}{12!\cdot 3!} = \frac{4\cdot 5\cdot 6\cdot 7}{9\cdot 10\cdot 11\cdot 12} = \frac{7}{99}.$$

Bundan

$$P(A) = 1 - P(\overline{A}) = 1 - \frac{7}{99} = \frac{92}{99}.$$

1.2.2. Agar A hodisaning ro‘y berishi B hodisaning ro‘y berishi yoki ro‘y bermasligiga bog‘liq bo‘lmasa, A va B hodisalarga *bog‘liqmas hodisalar* deyiladi.

Agar A hodisaning ro‘y berishi B hodisaning ro‘y berishi yoki ro‘y bermasligiga bog‘liq bo‘lsa, A va B hodisalarga *bog‘liq hodisalar* deyiladi.

A hodisaning B hodisa ro‘y berdi degan shartda hisoblangan ehtimoliga A hodisaning B hodisa ro‘y berishi shartidagi *shartli ehtimoli* deyiladi va $P_B(A)$ (yoki $P(A/B)$) bilan belgilanadi.

3-teorema. A va B hodisalar ko‘paytmasining ehtimoli hodisalardan birining ehtimoli bilan ikkinchisining birinchi hodisa ro‘y berishi shartidagi shartli ehtimoli ko’paytmasiga teng, ya’ni

$$P(A\cdot B) = P(A)\cdot P_A(B) = P(B)\cdot P_B(A).$$

n ta $A_1, A_2, ..., A_n$ hodisalar uchun

$$P(A_1\cdot A_2\cdot A_3\cdot ...\cdot A_n) = P(A_1)\cdot P_{A_1}(A_2)\cdot P_{A_1A_2}(A_3)\cdot ...\cdot P_{A_1A_2...A_{n-1}}(A_n).$$

$A_1, A_2, ..., A_n$ hodisalardan istalgan bittasining ro‘y berishi qolganlarining har qanday ko‘paytmasi ro‘y berishi yoki ro‘y bermasligiga bog‘liq bo‘lmasa, bu hodisalarga *birgalikda bog‘liqmas hodisalar* deyiladi.

2-natija. Birgalikda bog‘liqmas $A_1, A_2, ..., A_n$ hodisalar ko‘paytmasining ehtimoli ularning ehtimollari ko‘paytmasiga teng, y’ani

$$P(A_1\cdot A_2\cdot ...\cdot A_n) = P(A_1)\cdot P(A_2)\cdot ...\cdot P(A_n).$$

Xususan, bir xil p ehtimolga ega $A_1, A_2, ..., A_n$ hodisalar uchun

$$P(A_1\cdot A_2\cdot ...\cdot A_n) = p^n.$$

4-teorema. Birgalikda bog‘liqmas $A_1, A_2, ..., A_n$ hodisalardan hech bo‘lmaganda bittasining ro‘y berishdan iborat bo‘lgan A hodisaning ehtimoli

$$P(A) = 1 - q_1 q_2 ... q_n$$

bo‘ladi.

Xususan, bir xil p ehtimolga ega $A_1, A_2, ..., A_n$ hodisalar uchun

$$P(A) = 1 - q^n.$$

4-misol. 3 ta oq va 5 ta qora shar solingan qutidan tavakkaliga ketma-ket 3 ta shar olinadi. Olingan sharlar qutiga qaytarilmaydi. Qutidan olingan har uchala sharning qora bo‘lishi ehtimolini toping.

A_i – olingan i – sharning qora bo‘lishi hodisasi bo‘lsin, bunda $i = 1,2,3$. U holda izlanayotgan ehtimol $P(A_1 \cdot A_2 \cdot A_3)$ ga teng bo‘ladi.

Ehtimollarni ko‘paytirish teoremasiga ko‘ra

$$P(A_1 \cdot A_2 \cdot A_3) = P(A_1) \cdot P_{A_1}(A_2) \cdot P_{A_1 A_2}(A_3).$$

Birinchi shar olinayotganda qutida 8 ta shar bo‘lib, ulardan 5 tasi qora shardan iborat. Demak, $P(A_1) = \frac{5}{8}.$

Ikkinchi shar olinayotganda A_1 hodisa ro‘y bergan bo‘ladi. Shu sababli qutida 3 ta oq va 4 ta qora shar qoladi. Shu sababli $P_{A_1}(A_2) = \frac{4}{7}$ bo‘ladi.

Shu kabi $P_{A_1 A_2}(A_3) = \frac{3}{6}.$ Shunday qilib,

$$P(A_1 \cdot A_2 \cdot A_3) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}.$$

5-misol. Zavod ishlab chiqargan mahsulotining 5% i yaroqsiz. Nazorat uchun zavod ishlab chiqargan mahsulotlar orasidan tavakkaliga 20 ta detal olingan. Olingan detallar orasida hech bo‘lmaganda bittasi yaroqsiz bo‘lishi ehtimolini toping.

A_i – olingan i – detalning yaroqsiz bo‘lishi hodisasi bo‘lsin, bu yerda $i = \overline{1,20}$. U holda $P(A_i) = 0,05$ ga teng bo‘ladi. Bunda olingan detallar orasida hech bo‘lmaganda bittasi yaroqsiz bo‘lishi hodisasi $A = A_1 + A_2 + A_3 + ... + A_{20}$ bo‘ladi. Yo‘lga qo‘yilgan texnologik jarayonda $A_1, A_2, A_3, ..., A_{20}$ hodisalarni birgalikda bog‘liqmas hodisalar desa bo‘ladi.

U holda izlanayotgan ehtimol

$$p(A) = 1 - \prod_{i=1}^{20} P\left(\overline{A_i}\right) = 1 - (1 - P(A_i))^{20} = 1 - 0,95^{20} \approx 0,64.$$

6-misol. Ikkita kimyoviy reaktorning bir soat davomida to‘xtovsiz ishlashi ehtimoli mos ravishda $0{,}75$ va $0{,}8$ ga teng. Quyidagi hodisalarning ehtimollarini toping:

$B-$ bir soat davomida har ikkala reaktorlarning ishdan chiqishi;

$C-$ bir soat davomida har ikkala teaktorning to‘xtovsiz ishlashi;

$D-$ uch soat davomida har ikkala teaktorning to‘xtovsiz ishlashi;

$E-$bir soat davomida hech bo‘lmaganda bitta reaktorning to‘xtovsiz ishlashi;

$F-$bir soat davomida faqat bitta reaktorning to‘xtovsiz ishlashi.

A_1-birinchi reaktorning bir soat davomida to‘xtovsiz ishlashi hodisasi, A_2-ikkinchi reaktorning bir soat davomida to‘xtovsiz ishlashi hodisasi bo‘lsin.

Masalaning shatrtiga ko‘ra: $P(A_1)=0{,}75,\ P(A_2)=0{,}8.$

Bundan

$$P(\overline{A}_1)=1-0{,}75=0{,}25,\quad P(\overline{A}_2)=1-0{,}8=0{,}2.$$

Bir soat davomida har ikki reaktorning ishdan chiqishi hodisasi $B=\overline{A}_1\overline{A}_2$ bo‘ladi. $\overline{A}_1$ va $\overline{A}_2$ hodisalar bog‘liqmas hodisalar bo‘lgani uchun hodisalarni ko‘paytirish teoremasiga ko‘ra

$$P(B)=P(\overline{A}_1\overline{A}_2)=P(\overline{A}_1)\cdot P(\overline{A}_2)=0{,}25\cdot 0{,}2=0{,}05.$$

Bir soat davomida har ikkala teaktorning to‘xtovsiz ishlashi hodisasi $C=A_1A_2$ bo‘ladi. Bundan

$$P(C)=P(A_1A_2)=P(A_1)\cdot P(A_2)=0{,}75\cdot 0{,}8=0{,}6.$$

Uch soat davomida har ikkala teaktorning to‘xtovsiz ishlashi hodisasi $D=CCC$ bo‘ladi. U holda

$$P(D)=P(CCC)=P(C)\cdot P(C)\cdot P(C)=0{,}6\cdot 0{,}6\cdot 0{,}6=0{,}216.$$

Bir soat davomida hech bo’lmaganda bitta reaktorning to‘xtovsiz ishlashi hodisasi (E hodisa) har ikkala reaktorning ishdan chiqishi hodisasiga (B hodisaga)qarama - qarshi hodisa bo‘ladi: $E=\overline{B}$.

Demak,

$$P(E)=1-P(B)=1-0{,}05=0{,}95.$$

Bir soat davomida faqat bitta reaktorning to’xtovsiz ishlashi hodisasi $F=A_1\overline{A}_2+\overline{A}_1A_2$ bo‘ladi.

U holda ehtimollarni qo‘shish va ko‘paytirish teoremalariga ko‘ra

$$P(F)=P(A_1\overline{A}_2+\overline{A}_1A_2)=P(A_1\overline{A}_2)+P(\overline{A}_1A_2)=$$
$$=P(A_1)\cdot P(\overline{A}_2)+P(\overline{A}_1)\cdot P(A_2)=0{,}75\cdot 0{,}2+0{,}8\cdot 0{,}25=0{,}35.$$

7-misol. Uchta merganning o‘qni nishonga tekkizish ehtimollari mos ravishda $P_1 = 0,4,\ P_2 = 0,5,\ P_3 = 0,9$ ga teng. Uchala mergan baravariga o‘q uzganda nishonning yakson bo‘lishi ehtimolini toping. Bunda nishon yakson bo‘lishi uchun unga bitta o‘q tegishi kifoya.

Masalaning shartiga ko‘ra: $q_1 = 1 - p_1 = 1 - 0,4 = 0,6$, $q_2 = 0,5$, $q_3 = 0,1$. Merganlarning nishonga tekkazishi hodisalari bog‘liqmas, chunki har bir mergan nishonga mustaqil o‘q uzadi. Shu sababli, bitta oqning nishonga tegishi, ya’ni nishonning yakson bo‘lishi hodisasining ehtimoli

$$P(A) = 1 - q_1 q_2 q_3 = 1 - 0,6 \cdot 0,4 \cdot 0,2 = 0,952$$

bo‘ladi.

5-teorema. Birgalikda bo‘lgan A va B hodisalar yig‘indisining ehtimoli shu hodisalar ehtimollari yig‘indisidan ularning birgalikda ro‘y berishi ehtimolini ayrilganiga teng, ya’ni

$$P(A + B) = P(A) + P(B) - P(A \cdot B).$$

Uchta hodisa uchun

$$P(A + B + C) =$$
$$= P(A) + P(B) + P(C) - P(A \cdot B) - P(A \cdot C) - P(B \cdot C) + P(A \cdot B \cdot C).$$

8-misol. Aditoriyadagi 100 ta talabadan 50 tasi ingliz tilini, 40 tasi fransuz tilini va 35 tasi nemis tilini biladi. Ingliz va fransuz tillarini 20 talaba, ingliz va nemis tillarini 8 talaba, fransuz va nemis tillarini 10 talaba biladi. Har uchala tilni 5 talaba biladi. Bitta talaba auditoriyadan tashqariga chiqdi. Quyidagi hodisalarning ehtimollarini toping:

$A -$ auditoriyadan chiqqan talabaning ingliz yoki fransuz tillarini bilishi;

$B -$ auditoriyadan chiqqan talabaning birorta tilni bilmasligi.

$C -$ auditoriyadan chiqqan talabaning ingliz tilini bilishi hodisasi, $D -$ auditoriyadan chiqqan talabaning fransuz tilini bilishi hodisasi, $E -$ auditoriyadan chiqqan talabaning nemis tilini bilishi hodisasi bo‘lsin.

Masalaning shatrtiga ko‘ra:

$$P(C) = 0,5,\ P(D) = 0,4,\ P(E) = 0,35,$$

$$P(CD) = 0,2,\ P(CE) = 0,08,\ P(DE) = 0,1,\ P(CDE) = 0,05.$$

Auditoriyadan chiqqan talabaning ingliz yoki fransuz tillarini bilishi hodisasi $A = C + D$ bo‘ladi. C va D hodisalar birgalikda bo‘lgan va bog‘liq hodisalar. Shu sababli

$$P(A) = P(C + D) = P(C) + P(D) - P(CD) = 0,5 + 0,4 - 0,2 = 0,7.$$

Auditoriyadan chiqqan talabaning birorta tilni bilmasligi hodisasi $B = \overline{C} \cdot \overline{D} \cdot \overline{E}$ bo‘ladi. Bu hodisaning ehtimolini topish uchun hodisalar ustida amallarning xossalari va birgalikda bo‘lgan uchta hodisani qo‘shish formulasidan foydalanamiz:

$$P(B)=P(\overline{C}\cdot\overline{D}\cdot\overline{E})=P(\overline{C+D+E})=1-P(C+D+E)=$$
$$=1-(P(C)+P(D)+P(E)-P(CD)-P(CE)-P(DE)+P(CDE))=$$
$$=1-(0{,}5+0{,}4+0{,}35-0{,}2-0{,}08-0{,}1+0{,}05)=0{,}08.$$

1.2.3. Birgalikda bo’lmagan $B_1,B_2,...,B_n$ hodisalar to‘la guruh tashkil etsin. A hodisa bu hodisalardan biri ro‘y berganda ro‘y bersin. Bunda $B_1,B_2,...,B_n$ larga *gipotezalar* deyiladi.

U holda

$$P(A)=P(B_1)P_{B_1}(A)+P(B_2)P_{B_2}(A)+...+P(B_n)P_{B_n}(A) \quad \text{yoki} \quad P(A)=\sum_{i=1}^{n}P(B_i)P_{B_i}(A)$$

bo‘ladi. Bu formulaga *to‘la ehtimol formulasi* deyiladi.

9-misol. Birinchi qutida 2 ta oq va 6 ta qora, ikkinchi qutida 4 ta oq va 2 ta qora shar bor. Birinchi qutidan tavakkaliga ikkita shar olinadi va ikkinchi qutiga solinadi. Shundan keyin ikkinchi qutidan olingan sharning oq bo‘lishi ehtimolini toping.

$A-$ikkinchi qutidan olingan shar oq bo‘lishi, B_1,B_2,B_3-birinchi qutidan ikkinchi qutiga solingan sharlar mos ravishda 2 ta oq, 2 ta turli rangda, 2 ta qora bo‘lishi hodisalari bo‘lsin.

U holda $P(B_1)=\frac{C_2^2}{C_8^2}=\frac{1}{28}, \quad P(B_2)=\frac{C_2^1C_6^1}{C_8^2}=\frac{12}{28}, \quad P(B_3)=\frac{C_6^2}{C_8^2}=\frac{15}{28};$

$$P_{B_1}(A)=\frac{6}{8}, \quad P_{B_2}(A)=\frac{5}{8}, \quad P_{B_3}(A)=\frac{4}{8}.$$

$B_1,B_2,B_3-$to‘la guruh tashkil etadi.

Demak, to‘la ehtimol formulasiga ko‘ra

$$P(A)=\frac{1}{28}\cdot\frac{6}{8}+\frac{12}{28}\cdot\frac{5}{8}+\frac{15}{28}\cdot\frac{4}{8}=\frac{9}{16}.$$

1.2.4. $B_1,B_2,...,B_n$ gipotezalar bo‘lib, ularning $P(B_1),P(B_2),...,P(B_n)$ ehtimollari berilgan bo‘lsin. Tajriba o‘tkazilib, uning natijasida A hodisa ro‘y bersin va $P_{B_1}(A),P_{B_2}(A),...,P_{B_n}(A)$ shartli ehtimollar ma’lum bo‘lsin.

U holda

$$P_A(B_i)=\frac{P(B_i)P_{B_i}(A)}{\sum_{i=1}^{n}P(B_i)P_{B_i}(A)}$$

bo‘ladi. Bu formulaga *Bayes formulasi* deyiladi.

$10-$misol. Savdo firmasiga ikkita korxonadan lampalar keltirilgan bo‘lib, ulardan 30% i birinchi korxonada ishlab chiqarilgan. Lampaning yaroqli bo‘lishi ehtimollari korxonalar uchun mos ravishda 0,8 va 0,6 ga teng. Tanlangan lampa tekshirilganda yaroqli chiqdi. Uning birinchi korxonada ishlab chiqarilganligi ehtimolini toping.

Ikkita gipotezani qaraymiz: B_1-lampa birinchi korxonada ishlab chiqarilgan, B_2-lampa ikkinchi korxonada ishlab chiqarilgan.

Masalaning shartiga ko‘ra:

$$P(B_1)=0{,}3,\ P(B_2)=0{,}7,\ P_{B_1}(A)=0{,}8,\quad P_{B_2}(A)=0{,}6.$$

Tajriba natijasida tekshirilgan lampa yaroqli chiqqan, ya’ni A hodisa ro‘y bergan. U holda Bayes formulasiga binoan lampaning birinchi korxonada ishlab chiqarilganlipi ehtimoli

$$P_A(B_1)=\frac{0{,}3\cdot 0{,}8}{0{,}3\cdot 0{,}8+0{,}7\cdot 0{,}6}\approx 0{,}364.$$

Mashqlar

1.2.1. 6 ta oq va 5 ta qora shar solingan qutidan tavakkaliga 2 ta shar olinadi. Olingan har ikkala sharning bir xil rangli bo‘lishi ehtimolini toping.

1.2.2. Fabrikada bir nechta tikuv mashinasida ish bajariladi. Smena davomida bitta tikuv mashinasini tuzatish talab etilishi ehtimoli 0,3 ga, ikkita tikuv mashinasini tuzatish talab etilishi ehtimoli 0,21 ga, ikkitadan ortiq tikuv mashinasini tuzatish talab etilishi ehtimoli 0,09 ga teng. Smena davomida tikuv mashinalarini tuzatish talab etilishi ehtimolini toping.

1.2.3. Qutida 5 ta standart va 2 ta nostandart detallar bor. Qutidan navbat bilan tavakkaliga bittadan detal olinadi. Ikkinchi olingan detalning standart bo‘lishi ehtimolini toping: 1) agar detal qutiga qaytarilsa; 2) agar detal qutiga qaytarilmasa.

1.2.4. O‘quv zalida ehtimollar nazariyasidan 8 ta darslik bo‘lib, ulardan 3 tasi lotin alifbosida yozilgan. Talaba tavakkaliga ketma- ket 2 ta darslik oldi. Olingan darsliklarning har ikkalasi lotin alifbosida yozilgan bo‘lishi ehtimolini toping.

1.2.5. Ikki to‘pdan bir yo‘la o‘q uzishda nishonga o‘q tegishi ehtimoli 0,95 ga teng. Agar ikkinchi to‘pdan bitta o‘q uzishda o‘qning nishonga tegishi ehtimoli 0,8 ga teng bo‘lsa, bu ehtimolni birinchi to‘p uchun toping.

1.2.6. Ikkita to‘quv dastgohining bir soat davomida to‘xtovsiz ishlashi ehtimoli mos ravishda 0,6 va 0,85 ga teng. Bir soat davomida faqat bitta to‘quv dastgohining to‘xtovsiz ishlashi ehtimolini toping.

1.2.7. 6 ta oq va 2 ta rangli shar solingan qutidan tavakkaliga 4 ta shar olinadi. Olingan sharlar ichida hech bo‘lmaganda bitta rangli shar bo‘lishi ehtimolini toping.

1.2.8. 100 ta lotereya biletidan 5 tasi yutuqli. Hech bo‘lmaganda bitta biletda yutuq bo‘lishi ehtimolini toping: 1) agar 2 ta bilet olingan bo‘lsa; 2) agar 4 ta bilet olingan bo‘lsa.

1.2.9. Ikkita qutidan birinchisida 4 ta oq va 8 ta qora shar va ikkinchisida 6 ta oq va 3 ta qora shar bor. Har bir qutidan tavakkaliga bittadan shar olinadi. Olingan sharlardan hech bo‘lmaganda bittasi oq bo‘lishi ehtimolini toping.

1.2.10. 52 qartali dastadan bir vaqtda 4 ta qarta olinadi. Qartalarning har xil turda bo‘lishi ehtimolini toping, agar: 1) qartalar dastaga qaytarilmasa; 2) qartalar qutiga qaytarilsa.

1.2.11. Talaba o‘quv dasturidagi 25 ta savoldan 20 tasini biladi. Talaba o‘qituvchi tomonidan berilgan uchta savolni bilishi ehtimolini toping.

1.2.12. Qutida 8 ta oq, 6 ta qiil va 4 ta yashil shar bor. Qutidan tavakkaliga ketma-ket 3 ta shar olinadi va qutiga qaytarilmaydi. Olingan sharlarning birinchisi oq, ikkinchisi qizil va uchinchisi yashil bo‘lishi ehtimolini toping.

1.2.13. Talabaning uchta test sinovidan o‘tishi ehtimollari mos ravishda 0,9, 0,8 va 0,9 ga teng. Talabaning: 1) faqat uchinchi sinovdan o‘tishi; 2) faqat bitta sinovdan o‘tishi; 3) har uchala sinovdan o‘tishi; 4) hech bo‘lmaganda ikkita sinovdan o‘tishi; 5) hech bolmaganda bitta sinovdan o‘tishi ehtimolini toping.

1.2.14. Uchta mergan nishonga qarata bittadan o‘q uzishdi. Merganlarning o‘qni nishonga tekkazishi ehtimollari mos ravishda 0,7, 0,8 va 0,6 ga teng bo‘lsa, quyidagi hodisalarning ehtimollarini toping: 1) faqat Ikkinchi merganning nishonga tekkazishi; 2) faqat bitta merganning nishonga tekkazishi; 3) har uchala merganning nishonga tekkazishi; 4) hech bo‘lmaganda ikkita merganning nishonga tekkazishi; 5) hech bo‘lmaganda bitta merganning nishonga tekkazishi.

1.2.15. Ko'prikka bitta bomba tushsa, u yakson bo'ladi. Ko'prikka tushishi ehtimollari 0,5, 0,6, 0,7, 0,8 bo'lgan 4 ta bomba tashlangan. Ko'prikning yakson bo'lishi ehtimolini toping.

1.2.16. Uchta bog'liqmas sinashda hodisaning hech bo'lmaganda bir marta ro'y berishi ehtimoli 0,9919 ga teng. Hodisaning ehtimoli barcha sinashlarda o'zgarmas bo'lsa, hodisaning bitta sinashda ro'y berishi ehtimolini toping.

1.2.17. Basketbolchining bir tashlashda koptokni savatga tushirish ehtimoli 0,6 ga teng. 0,784 dan kam bo'lmagan ehtimol bilan hech bo'lmaganda bir marta savatga tushirish uchun basketbolchi koptokni savatga kamida necha marta tashlashi kerak?

1.2.18. Qimmatli qog'ozlar bozorida har bir aksiya paketi aksiyadorga 0,5 ehtimol bilan foyda keltiradi. Hech bo'lmaganda bitta aksiya paketida 0,96875 ehtimol bilan foyda ko'rilishi uchun kamida nechta har xil firmalarning aksiyasini sotib olish kerak?

1.2.19. Ikkita o'yin kubigi tashlanmoqda. Hech bo'lmaganda bitta kubikda 5 ochko tushishi ehtimolini toping.

1.2.20. Ikki zambarakdan bir-biriga bog'liq bo'lmagan holda nishon o'qqa tutilmoqda. Birinchi zambarakdan otilgan snaryadning nishonga tegishi ehtimoli 0,7 ga, ikkinchi zambarakdan otilgan snaryadning nishonga tegishi ehtimoli 0,8 ga teng. Nishonga bitta snaryad tegsa yakson bo'ladi. Nishonning yakson bo'lishi ehtimolini toping.

1.2.21. Agar $P(A)=0,6,\ P(A+B)=0,8,\ P(A\cdot B)=0,5$ bo'lsa, $P(B),\ P_A(B)$ va $P_B(A)$ ni toping.

1.2.22. Agar $P(A)=0,5,\ P_A(B)=0,8,\ P_B(A)=0,6$ bo'lsa, $P(B),\ P(A\cdot B)$ va $P(A+B)$ ni toping.

1.2.23. O'q teshib o'tishi mumkin bo'lgan joylari ikkita dbigateli va uchuvchi kabinasi bo'lgan samolyotga qarata o'q uzilmoqda. Samolyotni urib tushurish uchun o'q har ikkala dvigatelga yoki uchuvchi kabinasiga tegishi kerak. O'qning birinchi dvigatelga tegishi ehtimoli 0,8 ga, ikkinchi dvigatelga tegishi ehtimoli 0,7 ga, kabinaga tegishi ehtimoli 0,6 ga teng. Samolyot qismlarining shikastlaninshi o'zaro bog'liq bo'lmasa, samolyotning urib tushirilishi ehtimolini toping.

1.2.24. Qutida 2 ta oq va 3 ta qora shar bor. Ikkita talaba qutidan navbati bilan bittadan shar oladi va qutiga qaytaradi. Birinchi bo'lib oq sharni olgan talaba yutuqqa ega bo‘ladi. Birinchi talabaning yutuqqa ega bo‘lishi ehtimolini toping.

1.2.25. Birinchi qutida 5 ta oq va 15 ta qora, ikkinchi qutida 3 ta oq va 9 ta qora shar bor. Birinchi qutidan tavakkaliga bitta shar olinadi va ikkinchi qutiga solinadi. Keyin ikkinchi qutidan bitta shar olinadi. Bu sharning qora bo‘lishi ehtimolini toping.

1.2.26. Yig‘uv sexiga birinchi sexdan 40%, ikkinchi sexdan 60% detal keltirilgan. Birinchi sexda 90%, ikkinchi sexda 95 % standart detallar tayyorlanadi. Tavakkaliga olingan detalning standart bo‘lishi ehtimolini toping.

1.2.27. Musabaqada 3 ta sport ustasi, 4 ta sport ustaligiga nomzod va 5 ta birinchi razryadli sportchi qatnashmoqda. Sport ustasining o‘qni nishonga tekkazishi ehtimoli 0,9 ga, sport ustaligiga nomzodningg o‘qni nishonga tekkazishi ehtimoli 0,85 ga va birinchi razryadli sportchining o‘qni nishonga tekkazishi ehtimoli 0,75 ga teng. Otilgan o‘q nishonga tegdi. Nishonga tekkazgan qatnashchining sport ustasi bo‘lishi ehtimolini toping.

1.2.28. Do‘konga uchta firmadan mahsulot keltirilgan: birinchi firmadan 20%, ikkinchi firmadan 46% va uchinchi firmadan 34%. Firmalar mahsulotlarining yaroqsiz bo‘lishi ehtimoli mos ravishda 0,03, 0,02, 0,01 ga teng. Do‘kondan tavakkaliga olingan mahsulot yaroqsiz chiqdi. Uning birinchi firma mahsuloti bo‘lishi ehtimolini toping.

1.2.29. Savdo firmasiga uchta ta’minotchi tomonidan 1:4:5 nisbatda televizorlar keltirildi. Birinchi, ikkinchi va uchinchi ta’minotchilardan keltirilgan televizorlarning mos ravishda 98%, 88% va 92%iga kafolat muddatida tuzatish talab qilinmaydi. Savdo firmasiga keltirilgan televizorlarga kafolat muddatida: 1) tuzatish talab qilinmasligi; 2) tuzatish talab qilinishi ehtimolini toping. 3) keltirilgan televizor kafolat muddatida tuzatildi. Uning qaysi ta’minotchidan keltirilganligi ehtimolliroq?

1.2.30. Do‘konga uchta korxonadan 5:8:7 nisbatda mahsulot keltirildi. Korxonalar mahsulotlarining yaroqli bo‘lishi ehtimoli mos ravishda 0,9, 0,85, 0,75 ga teng. Quyidagi ehtimollarni toping: 1) sotilgan mahsulotning yaroqsiz bo‘lishi; 2) sotilgan mahsulotning yaroqli bo‘lishi; 3) sotilgan mahsulot yaroqli chiqdi, uning uchinchi korxonada ishlab chiqarilganligi.

1.3. SINASHLARNING TAKRORLANISHI

Bernulli sxemasi. Bernulli formulasi. Muavr - Laplas teoremalari. Puasson teoremasi

1.3.1. Agar sinashlar natijalarining har qanday kombinasiyasi bog‘liqmas hodisalardan iborat bo‘lsa, bunday sinashlarga *bog‘liqmas sinashlar* deyiladi.

n ta bog‘liqmas sinashlar takrorlanayotgan, ya’ni ketma-ket o‘tkazilayotgan bo‘lsin. Agar sinashlarning har birida A hodisaning ro‘y berishi ehtimoli bir xil $P(A)=p$ ga teng bo‘lsa, bu sinashlarga A hodisaga nisbatan bog‘liqmas sinashlar deyiladi. Sinashlarning bunday ketma-ketligi *Bernulli sxemasi* deb ataladi.

1.3.2. Bernulli sxemasi uchun chekli sondagi n ta sinashlar ketma-ketligida A hodisaning rosa m marta ro‘y berishi ehtimoli

$$P_n(m)=\frac{n!}{m!\cdot(n-m)!}p^m q^{n-m}$$

formula bilan topiladi. Bu formulaga *Bernulli formulasi* deyiladi.

Agar Bernulli sxemasining n ta sinashlarida A hodisaning m_0 marta ro‘y berishi ehtimoli $P_n(m_0)$ istalgan m larda $P_n(m)$ ehtimollardan katta bo‘lsa, u holda m_0 songa *eng ehtimolli son* deyiladi.

Bu son

$$np-q\le m_0\le np+p$$

qo’sh teksizlikdan topiladi. Bunda: 1) $np-q$ kasr son bo‘lsa bitta m_0 son mavjud bo‘ladi; 2) $np-q$ butun son bo‘lsa ikkita m_0 va m_0+1 sonlari mavjud bo‘ladi; 3) np butun son bo‘lsa bitta $m_0=np$ son mavjud bo‘ladi.

1-misol. Har bir sinashda A hodisaning ro‘y berishi ehtimoli 0,7 ga teng. n ta sinashda $n=8$ va $n=9$ da A hodisa ro‘y berishining eng ehtimolli sonlarini va bu sonlarning ehtimolini toping.

⊖ Masalaning shartiga ko’ra $p=0{,}7,\ q=0{,}3.$

1) $n=8$ bo‘lsin. U holda

$$8\cdot 0{,}7-0{,}3\le m_0\le 8\cdot 0{,}7+0{,}7 \quad \text{yoki} \quad 5{,}3\le m_0\le 6{,}3.$$

Bundan

$$m_0=6, \qquad P_8(6)=C_8^6\cdot 0{,}7^6\cdot 0{,}3^2=0{,}296.$$

2) $n=9$ bo‘lsin. U holda

$$9\cdot 0{,}7-0{,}3\le m_0\le 9\cdot 0{,}7+0{,}7 \quad \text{yoki} \quad 6\le m_0\le 7.$$

Bundan $m_{01} = 6$, $m_{02} = 7$. Bu sonlarning ro‘y berishi ehtimollari bir xil bo‘ladi:

$$P_9(6) = P_9(7) = C_9^7 \cdot 0{,}7^7 \cdot 0{,}3^2 = 0{,}267.$$

2-misol. Bitta o‘q uzishda o‘qning nishonga tegishi ehtimoli 0,8 ga teng. Nishonga tekkizishlarning eng ehtimolli soni 15 ga teng bo‘lishi uchun nishonga nechta o‘q uzilishi kerak?

Masalaning shartiga ko’ra $m_0 = 15$, $p = 0{,}8$, $q = 0{,}2$.

Tengsizlikni tuzamiz: $n \cdot 0{,}8 - 0{,}2 \le 15 \le n \cdot 0{,}8 + 0{,}8$

Bundan

$$n \cdot 0{,}8 \le 15{,}2 \quad \text{yoki} \quad n \le 19,$$

$$n \cdot 0{,}8 \ge 14{,}2 \quad \text{yoki} \quad n \ge 17{,}75.$$

Demak, nishonga 18 ta yoki 19 ta o‘q uzilishi kerak.

Bernulli sxemasi uchun chekli sondagi n ta sinashlar ketma-ketligida A hodisaning kamida m_1 marta va ko‘pi bilan m_2 marta ro‘y berishi ehtimoli

$$P_n(m_1, m_2) = P_n(m_1) + P_n(m_1 + 1) + ... + P_n(m_2)$$

formula bilan topiladi.

$3-$misol. O‘simlik urug‘ining 70%i unib chiqadi. Oltita ekilgan urug‘dan: 1) rosa 4 tasi unib chiqishi ehtimolini; 2) 4 tadan kam bo‘lmagani unib chiqishi ehtimolini; 3) 4 tadan ko‘p bo‘lmagani unib chiqishi ehtimolini; 4) hech bo‘lmaganda bittasi unib chiqishi ehtimolini; 5) 2 tadan 4 tagacha bo‘lgani unib chiqishi ehtimolini toping.

O‘simlik urug‘ining unib chiqish ehtimoli $p = 0{,}7$ ga teng va unib chiqmasligi ehtimoli $q = 1 - p = 1 - 0{,}7 = 0{,}3$ ga teng. U holda:

1) $P_6(4) = C_6^4 \cdot 0{,}7^4 \cdot 0{,}3^2 = 0{,}324;$

2) $P_6(m \ge 4) = P_6(4) + P_6(5) + P_6(6) =$

$$= C_6^4 \cdot 0{,}7^4 \cdot 0{,}3^2 + C_6^5 \cdot 0{,}7^5 \cdot 0{,}3^1 + C_6^6 \cdot 0{,}7^6 \cdot 0{,}3^0 = 0{,}744;$$

3) $P_9(m \le 4) = P_6(0) + P_6(1) + P_6(2) + P_6(3) + P_6(4) =$

$$= C_6^0 \cdot 0{,}7^0 \cdot 0{,}3^6 + C_6^1 \cdot 0{,}7^1 \cdot 0{,}3^5 + C_6^2 \cdot 0{,}7^2 \cdot 0{,}3^4 + C_6^3 \cdot 0{,}7^3 \cdot 0{,}3^3 + C_6^4 \cdot 0{,}7^4 \cdot 0{,}3^2 = 0{,}5798;$$

4) $P_9(m \ge 1) = 1 - P_6(0) = 1 - C_6^0 \cdot (0{,}7)^0 \cdot (0{,}3)^6 = 0{,}999271;$

5) $P_9(2 \le m \le 4) = P_6(2) + P_6(3) + P_6(4) =$

$$= C_6^2 \cdot 0{,}7^2 \cdot 0{,}3^4 + C_6^3 \cdot 0{,}7^3 \cdot 0{,}3^3 + C_6^4 \cdot 0{,}7^4 \cdot 0{,}3^2 = 0{,}5687.$$

1.3.3. Bernulli sxemasi uchun n va m sonlarining yetarlicha katta qiymatlarida $P_n(m)$ va $P_n(m_1, m_2)$ ehtimollar Muavr-Laplas teoremalari asosida topiladi.

1-teorema (*Muavr-Laplasning lokal teoremasi*). Agar A hodisaning ro‘y berishi ehtimoli har bir sinashda o‘zgarmas va p $(0<p<1)$ ga teng bo‘lsa, u holda n sonining yetarlicha katta qiymatlarida

$$P_n(m) \approx \frac{1}{\sqrt{npq}}\varphi(x)$$

bo‘ladi, bu yerda $\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}, \quad x = \frac{m-np}{\sqrt{npq}}.$

$\varphi(x)$ funksiya jadvallashtirilgan bo‘lib, uning qiymatlari jadvali 1-ilovada berilgan.

$4-$misol. O‘g‘il bola tug‘ilishi ehtimoli 0,51 ga teng bo‘lsa, 80 ta chaqaloqdan 45 tasi o‘g‘il bola bo‘lishi ehtimolini toping.

⊖ Shartga ko’ra $n=80, \quad m=45, \quad p=0{,}51, \quad q=0{,}49$.

Muavr-Laplasning lokal teoremasi bilan topamiz:

$$P_{80}(45) \approx \frac{1}{\sqrt{npq}}\varphi(x) = \frac{1}{\sqrt{80\cdot 0{,}51\cdot 0{,}49}}\varphi(x) = \frac{1}{4{,}4712}\varphi(x),$$

bu yerda $x = \frac{m-np}{\sqrt{npq}} = \frac{45-80\cdot 0{,}51}{\sqrt{80\cdot 0{,}51\cdot 0{,}49}} = 0{,}9393.$

1-ilovadagi jadvaldan topamiz: $x=0{,}9393$ da $\varphi(x)=0{,}2567.$

Demak,

$$P_{80}(45) = \frac{1}{4{,}4712}\cdot 0{,}2567 = 0{,}0574.$$ ⊗

2-teorema (*Muavr-Laplasning integral teoremasi*). Agar A hodisaning ro‘y berishi ehtimoli har bir sinashda o‘zgarmas va p $(0<p<1)$ ga teng bo‘lsa, u holda n sonining yetarlicha katta qiymatlarida

$$P_n(m_1, m_2) \approx \Phi(x_2) - \Phi(x_1)$$

bo‘ladi, bu yerda $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_0^x e^{-\frac{t^2}{2}}dt, \qquad x_1 = \frac{m_1-np}{\sqrt{npq}}, \qquad x_2 = \frac{m_2-np}{\sqrt{npq}}.$

$\Phi(x)$ funksiya jadvallashtirilgan bo‘lib, uning qiymatlari jadvali 2-ilovada berilgan.

$5-$misol. Ishlov berishda o‘rtacha 10%i detal yaroqsiz chiqadi. Ishlov berilgan 400 ta detallar orasida yaroqli detal: 1) 340 tadan 380 tagacha bo‘lishi ehtimolini; 2) 380 tadan ko‘p bo‘lmasligi ehtimolini toping.

⊖ 1) Sartga ko‘ra $n=400, \quad m_1=340, \quad m_2=380, \; p=0{,}9, \quad q=0{,}1$.

Muavr-Laplasning integral teoremasi bilan topamiz:

$$P_{400}(340 \le m \le 380) \approx \Phi(x_2) - \Phi(x_1),$$

bu yerda

$$x_2 = \frac{m_2 - np}{\sqrt{npq}} = \frac{380 - 400 \cdot 0,9}{\sqrt{400 \cdot 0,9 \cdot 0,1}} = 3,33, \quad x_1 = \frac{m_1 - np}{\sqrt{npq}} = \frac{340 - 400 \cdot 0,9}{\sqrt{400 \cdot 0,9 \cdot 0,1}} = -3,33.$$

2-ilovadagi jadvaldan topamiz: $x_2 = 3,33$ da $\Phi(3,33) = 0,499565$.

U holda

$$P_{400}(340 \le m \le 380) \approx \Phi(3,33) - \Phi(-3,33) = 2\Phi(3,33) = 0,99913.$$

2) Shu kabi topamiz:

$$P_{400}(0 \le m \le 380) \approx \Phi(x_2) - \Phi(x_1) = \Phi(3,33) - \Phi\left(\frac{0 - 400 \cdot 0,9}{\sqrt{400 \cdot 0,9 \cdot 0,1}}\right) =$$

$$= \Phi(3,33) - \Phi(-60) = 0,499565 + 0,5 \approx 1.$$ ◆

1.3.4. Bernulli sxemasi uchun n sonining etarlicha katta va p sonining etarlicha kichik qiymatlarida $P_n(m)$ Puasson teoremasi asosida topiladi.

3-teorema (*Puasson teoremasi*). $n \to \infty$ da $\lambda = np$ ($\lambda -$o'zgarmas musbat son) bo'lsin. Agar A hodisaning ro'y berishi ehtimoli har bir sinashda o'zgarmas va p ga teng bo'lsa, u holda

$$P_n(m) \approx e^{-\lambda} \cdot \frac{\lambda^m}{m!}$$

bo'ladi.

Puassan formulasi odatda $n \ge 50$ va $np \le 10$ bo'lganida ishlatiladi.

$P_n(m)$ funksiyaning qiymatlari 3-ilovada keltirilgan.

$6-$misol. Zavoddan omborga 5000 ta sifatli buyumlar yuborildi. Har bir buyumning yo'lda shikastlanishi ehtimoli 0,0002 ga teng. Buyumlar orasidan yo'lda: 1) rosa 3 tasi shikastlanishi ehtimolini; 2) 3 tadan ko'p bo'lmagani shikastlanishi ehtimolini; 3) 3 tadan ko'p bo'lgani shikastlanishi ehtimolini toping.

◆ Shartga ko'ra: $p = 0,0002,\ n = 5000.$ Bundan $\lambda = 5000 \cdot 0,0002 = 1.$

U holda (3-ilovadagi jadvaldan foydalanildi):

1) $P_{5000}(3) \approx \frac{1}{3!}e^{-1} = \frac{1}{e \cdot 3!} = 0,0613.$

2) $P_{5000}(0 \le m \le 3) \approx \frac{1}{0!}e^{-1} + \frac{1}{1!}e^{-1} + \frac{1}{2!}e^{-1} + \frac{1}{3!}e^{-1} =$

$$= 0,3679 + 0,3679 + 0,1839 + 0,0613 = 0,981.$$

3) $P_{5000}(m > 3) = 1 - P_{5000}(0 \le m \le 3) = 1 - 0,981 = 0,019.$ ◆

Mashqlar

1.3.1. Oilada 5 ta farzand bor. Qiz bola va o‘g‘il bola tug‘ilishi ehtimollari teng bo‘lsa, oilada: 1) 3 ta qiz bola bo‘lishi ehtimolini; 2) o‘g‘il bola 3 tadan ko‘p bo‘lmasligi ehtimolini toping.

1.3.2. Bankka har beshinchi mijoz qo‘ygan omonatining foizini olgani kelishi ma’lum bo‘lsa, bankka kelgan 6 ta mijozdan: 1) faqat 2 tasi omonat foizini olishi ehtimolini; 2) hech bo‘lmaganda 1 tasi omonat foizini olishi ehtimolini toping.

1.3.3. Qurilish kompaniyasida o‘tkazilgan auditorlik tekshiruvida auditor tavakkaliga 6 ta hisob varaqasini tanlaydi. Hisob varaqasining 4%ida xatoliarga yo‘l qo‘yilgan bo‘lsa, auditorning: 1) 2 ta hisob varaqasida xato topishi; 2) hech bo‘lmaganda 1 ta hisob varaqasida xato topishi ehtimolini toping.

1.3.4. Ikkita teng kuchli raqib shaxmat o‘ynayotgan bo‘lsin. $P_4(2),\ P_6(3),\ P_8(4)$ yutib olish ehtimollarni o‘sish tartibida yozing.

1.3.5. Nazoratchi mahsulotlardan 24 ta namunasini tekshiradi. Har bir mahsulotning sotishga yaroqli deb topilishi ehtimoli 0,6 ga teng. Nazoratchi sotishga yaroqli deb topadigan namunalarning eng ehtimolli sonini toping.

1.3.6. Agar 49 ta erkli sinashda hodisa ro‘y berishining eng ehtimolli soni 30 ga teng bo‘lsa, har bir sinashda hodisa ro‘y berishi ehtimolini toping.

1.3.7. Tavakkaliga tanlangan detalning nostandart bo‘lishi ehtimoli 0,1 ga teng. Standart detallarning eng ehtimolli soni 50 ga teng bo‘lishi uchun necha detal olinishi kerak?

1.3.8. n ta sinashning har birida ijobiy natija olish ehtimoli 0,3 ga teng. Bu sinashlarda hodisa ro‘y berishining eng ehtimolli soni 30 ga teng bo‘lishi uchun nechta sinash o‘tkazilishi kerak?

1.3.9. Auksionda o‘rtacha 20% aksiya boshlang‘ich qiymatida sotiladi. 5 aksiyalar paketidan: 1) rosa 4 tasi; 2) 2 tadan 4 tagacha bo‘lgani; 3) 2 tadan kam bo‘lgani; 4) 2 tadan ko‘p bo‘lmagani; 5) hech bo‘lmaganda 2 tasi; 6) eng ehtimolli soni boshlang‘ich qiymatida sotilishi ehtimollarini toping.

1.3.10. Qutida 10 ta oq va 5 ta qora sharlar bor. Qutidan tavakkaliga ketma-ket 6 ta shar olinadi. Bunda olingan har bir shar keyingi shar olinishidan oldin qutiga qaytariladi va sharlar aralashtiriladi. Olingan sharlardan: 1) rosa 3 tasi; 2) 3 tadan 5 tagacha bo‘lgani; 3) 3 tadan kam bo‘lgani; 4) 3 tadan ko‘p bo‘lgani; 5) hech bo‘lmaganda 3 tasi; 6) eng ehtimolli soni oq chiqishi ehtimolini toping.

1.3.11. Ehtimollar nazariyasidan namunaviy hisob ishini 50% talaba muvaffaqiyatli bajaradi. Namunaviy hisob ishini 400 talabadan: 1) 180 talaba muvaffaqiyatli bajarishi; 2) 180 tadan kam bo‘lmagan talaba muvaffaqiyatli bajarishi ehtimolini toping.

1.3.12 Korxonada ishlab chiqarilgan mahsulotning yaroqsiz chiqishi ehtimoli 0,2 ga teng. 400 ta mahsulotdan: 1) 100 tasi yaroqsiz chqushi; 70 tadan 130 tagacha bo‘lgani yaroqsiz chiqishi ehtimolini toping.

1.3.13. Zavod tomonidan ishlab chiqarilgan telefon apparatlarining 50%i birinchi nav mahsulot bo‘lishi ma’lum. Zavod ishlab chiqargan 1000 telefon apparatidan: 1) 120 tasi birinchi nav bo‘lishi; 2) eng ehtimolli soni birinchi nav bo‘lishi; 3) 120 tadan kam bo‘lmagani birinchi nav bo‘lishi; 4) kamida 120 tasi va ko‘pi bilan 520 tasi birinchi nav bo‘lishi ehtimolini toping.

1.3.14. Merganning bitta o‘q uzishda nishonga tekkazishi ehtimoli 0,8 ga teng. 400 ta o‘q uzishda merganning: 1) 300 ta o‘qni nishonga tekkazishi; 2) kamida 300 ta va ko‘pi bilan 360 ta o‘qni nishonga tekkazishi; 3) kamida 280 ta va ko‘pi bilan 360 ta o‘qni nishonga tekkazishi; 4) eng ehtimolli sondagi o‘qni nishonga tekkazishi ehtimolini toping.

1.3.15. Fakultetda 1825 talaba bor. Fakultet talabalaridan 3 tasining tug‘ilgan kuni 21 mart bo‘lishining ehtimolini toping (bir yilda 365 kun bor bo‘lsin).

1.3.16. Har bir o‘q uzishda o‘qning nishonga tegishi ehtimoli 0,0001 ga teng. 5000 ta o‘q uzishda ikkitadan kam bo‘lmagan o‘qning nishonga tegishi ehtimolini toping.

1.3.17. Yo‘lovchining poyezdga kechikish ehtimoli 0,01 ga teng. 800 ta yo‘lovchidan poyezdga kechikadigan yo‘lovchilarning eng ehtimolli sonini aniqlang va shu sondagi kechikuvchilarning ehtimolini toping.

1.3.18. Kredit kartasining egasi kredit kartasini hafta davomida yo‘qotishi ehtimoli 0,001 ga teng. Bank 2000 ta mijozga karta bergah bo‘lsa, kelayotgan haftada: 1) hech bo‘lmaganda bitta kartaning yo‘qolishi; 2) rosa bitta kartaning yo‘qolishi ehtimolini toping.

1.3.19. Zavod bazaga 10000 ta standart buyum jo‘natdi. Yo‘lda buyumning o‘rtacha 0.02% i shikastlanadi. Yo‘lda 10000 buyumdan: 1) rosa 3 tasi shikastlanishi; 2) kamida 3 tasi shikastlanishi; 3) 9997 tasi shikastlanmasligi; 4) ko‘pi bilan 3 tasi shikastlanishi ehtimolini toping.

1.3.20. Do‘konga 1000 shisha idishda madanli shuv yuborildi. Tashish vaqtida idishning sinib qolishi ehtimoli 0,003 ga teng. Do‘konga: 1) rosa 2 ta singan idish keltirilishi; 2) 2 tadan kam singan idish keltirilishi; 3) 2 tadan ko‘p singan idish keltirilishi; 4) kamida bitta singan idish keltirilishi ehtimolini toping.

1.4. TASODIFIY MIQDORLAR

Diskret va uzluksiz tasodifiy miqdorlar. Taqsimot funksiyasi. Taqsimot zichligi. Tasodifiy miqdorlar ustida amallar. Tasodifiy miqdorning funksiyasi

1.4.1. Sinash natijasida mumkin bo‘lgan qiymatlaridan oldindan ma’lum bo‘lmagan birini qabul qiladigan miqdorga *tasodifiy miqdor* deyiladi.

Tasodifiy miqdorlar lotin alfavitining bosh harflari $X, Y, \ldots$ bilan, ularning mumkin bo‘lgan qiymatlari tegishli kichik harflar $x, y, \ldots$ bilan belgilanadi. X tasodifiy miqdorning x qiymatni qabul qilishi hodisasi $X = x$ deb, bu hodisaning ehtimoli $P(X = x)$ kabi belgilandi.

Tasodifiy miqdorlar diskret va uzluksiz tasodifiy miqdorlarga bo‘linadi.

Mumkin bo‘lgan qiymatlari chekli yoki sanoqli cheksiz ketma- ketlikdan iborat bo‘lgan miqdorga *diskret tasodifiy miqdor* deyiladi.

Taqsimot qonunining jadval usuldagi berilishida jadvalning (matritsaning) birinchi satriga o‘sish tartibida barcha mumkin bo‘lgan qiymatlar, ikkinchi satrida ularga mos ehtimollar qo‘yiladi. Bunday jadvalga *tasodifiy miqdorning taqsimot qatori* (*matritsasi*) deyiladi.

Taqsimot qonunining grafik usuldagi berilishida koordinatalar tekisligining abssissalar o‘qida tasodifiy miqdorning mumkin bo‘lgan qiymatlari, ordinatalar o‘qida ularga mos ehtimollar qo‘yiladi va $(x_1; p_1)$, $(x_2; p_2), \ldots, (x_n; p_n)$ nuqtalar kesmalar bilan tutashtiriladi. Hosil bo‘lgan shaklga *taqsimot ko’pburchagi* deyiladi.

Analitik usulda x_i va p_i orasidagi bog‘lanish $P(X = x_i)_i = \varphi(x_i)$ ko‘rinishdagi formula shaklida yoki taqsimot funksiyasi bilan beriladi.

1-misol. Bitta sinash o‘tkazilgan. Bunda A hodisaning ro‘y berishi ehtimoli $P(A)=p$ ga teng. A hodisaning ro‘y berishidan iborat X tasodifiy miqdorning taqsimot qatorini toping.

X miqdor ikkita $x_1=0,\ x_2=1$ qiymatlar qabul qiladi.

$$P(A)=p, \quad P(\overline{A})=1-p=q.$$

Bundan

$$p_1=P(X=0)=q,\ \ p_2=P(X=1)=p.$$

Demak,

$$X:\begin{pmatrix} 0 & 1 \\ q & p \end{pmatrix}.$$

2-misol. Qutida 10 shar bo‘lib, ulardan 8 tasi oq. Tavakkaliga 2 ta shar olingan. Olingan sharlar oq bo‘lishining taqsimot qonunini toping.

X – olingan sharlar oq bo’lishi soni bo’sin. Uning mumkin bo’lgan qiymatlari: $x_1=0,\ x_2=1,\ x_3=2$. Bu qiymatlarga mos ehtimollarni topamiz:

$$P(X=0)=\frac{C_8^0C_2^2}{C_{10}^2}=\frac{1\cdot 2}{10\cdot 9}=\frac{1}{45}, \qquad P(X=1)=\frac{C_8^1C_2^1}{C_{10}^2}=\frac{8\cdot 2}{45}=\frac{16}{45},$$

$$P(X=2)=\frac{C_8^2C_2^0}{C_{10}^2}=\frac{8\cdot 7}{1\cdot 2\cdot 45}=\frac{28}{45}.$$

Demak,

$$X:\begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{45} & \frac{16}{45} & \frac{28}{45} \end{pmatrix}.$$

Mumkin bo‘lgan qiymatlari chekli yoki cheksiz oraliqni butunlay to‘ldiradigan miqdorga *uzluksiz tasodifiy miqdor* deyiladi.

Uzluksiz tasodifiy miqdorning taqsimot qonuni taqsimot funksiyasi yoki taqsimot zichligi orqali beriladi.

1.4.2. X tasodifiy miqdorning x dan kichik qiymat qabul qilish ehtimoli $F(x)=P(X<x)$ ga X tasodifiy miqdorning *taqsimot funksiyasi* deyiladi.

Taqsimot funksiyasi quyidagi xossalarga ega.

1^o. $0\le F(x)\le 1$.

2^o. Taqsimot funksiyasi butun sonlar o‘qida kamaymaydigan funksiya.

3^o. $P(x_1\le X<x)=F(x_2)-F(x_1)$.

4^o. $F(-\infty)=0$, $F(+\infty)=1$.

1-natija. Tasodifiy miqdorning barcha mumkin bo‘lgan qiymatlari $(a;b)$ oraliqqa tegishli bo‘lsa, $x \le a$ da $F(x)=0$ va $x \ge b$ da $F(x)=1$ bo‘ladi.

2-natija. Uzluksiz tasodifiy miqdorning tayin qiymat qabul qilishi ehtimoli nolga teng.

3-natija. Uzluksiz tasodifiy miqdorning $(x_1;x_2)$ oraliqqa tushishi ehtimoli bu oraliqning ochiq yoki yopiq bo‘lishiga bog‘liq bo‘lmaydi, ya’ni

$$P(x_1 < X < x_2) = P(x_1 \le X < x_2) = P(x_1 < X \le x_2) = P(x_1 \le X \le x_2).$$

3-misol. Tasodifiy miqdorning taqsimot qatori berilgan.

$$X:\begin{pmatrix} -1 & 3 & 6 \\ 0{,}3 & 0{,}5 & 0{,}2 \end{pmatrix}.$$

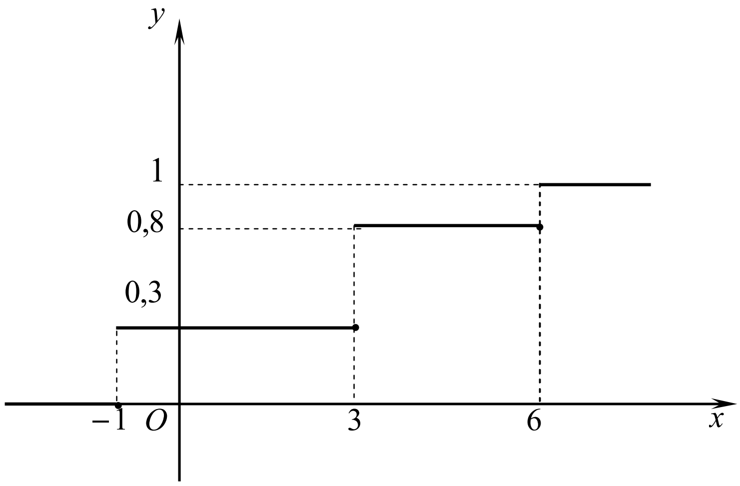
Taqsimot funksiyasini toping va uning grafigini chizing.

$x$ga har xil qiymatlar berib, ular uchun $F(x)=P\left(X<x\right)$ larni topamiz:

1. $x \le -1$ da $F(x)=0$;
2. $-1 < x \le 3$ da $F(x)=P(X<x)=P(X=-1)=0{,}3;$
3. $3 < x \le 6$ da $F(x)=P(X<x)=P(X=-1)+P(X=3)=0{,}3+0{,}5=0{,}8;$
4. $x>6$ da $F(x)=P(X<x)=(P(X=-1)+P(X=2))+P(X=5)=0{,}8+0{,}2=1.$

Demak (3-shakl),

$$F(x)=\begin{cases} 0, & agar\ x \le -1, \\ 0{,}3, & agar\ -1 < x \le 3, \\ 0{,}8, & agar\ 3 < x \le 6, \\ 1, & agar\ x > 6. \end{cases}$$



3-shakl.

4-misol. X tasodifiy miqdor taqsimot qonuni bilan berilgan:

$$F(x)=\begin{cases} 0, & agar\ x \le 3, \\ (x-3)^2, & agar\ 3 < x \le 4, \\ 1, & agar\ x > 4. \end{cases}$$

Tasodifiy miqdorning: 1) $X=3{,}5$ qiymat qabul qilishi ehtimolini; 2) (2;3,5) va (3,5;4,5) oraliqlarga tushishi ehtimollarini; 3) zichlik funksiyasini toping.

1) 1-natijaga ko‘ra $F(3.5)=0.$

2) 3^o xossaga ko‘ra:

$$P(2<X<3{,}5)=F(3{,}5)-F(2)=0{,}5^2-0=0{,}25,$$
$$P(3{,}5<X<4{,}5)=F(4{,}5)-F(3{,}5)=1-0{,}5^2=0{,}75.$$

3) Taqsimot zichliginning ta’rifiga ko‘ra

$$f(x)=F'(x)=\begin{cases}0, & agar\ x\le 3,\\ 2(x-3), & agar\ 3<x\le 4,\\ 0, & agar\ x>4.\end{cases}$$

1.4.3. X uzluksiz tasodifiy miqdor ehtimoli o‘rtacha zichligining $\Delta x\to 0$ dagi limitiga uzluksiz tasodifiy miqdor ehtimolining *taqsimot zichligi* deyiladi va $f(x)$ bilan belgilanadi:

$$f(x)=\lim_{\Delta x\to 0}\frac{F(x+\Delta x)-F(x)}{\Delta x}=F'(x).$$

Taqsimot zichligi quyidagi xossalarga ega.

1^o. $f(x)\ge 0$.

2^o. $F(x)=\int\limits_{-\infty}^{x} f(x)dx$.

3^o. $P(x_1\le X<x_2)=\int\limits_{x_1}^{x_2} f(x)dx$.

4^o. $\int\limits_{-\infty}^{+\infty} f(x)dx=1$.

4-natija. Tasodifiy miqdorning mumkin bo‘lgan qiymatlari $[a;b]$ esmaga tegishli bo‘lsa

$$\int\limits_a^b f(x)dx=1.$$

5-misol. Uzluksiz tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\begin{cases}0, & agar\ x<0,\\ c\sin x, & agar\ 0\le x<\pi,\\ 0, & agarx>\pi.\end{cases}$$

c va $F(x)$ ni toping.

4-natijaga ko‘ra $\int\limits_0^{\pi} c\sin x dx=1$ yoki

$$c\int\limits_0^{\pi}\sin x dx=c(-\cos x)\Big|_0^{\pi}=c(-\cos\pi+\cos 0)=2c=1.$$

Bundan $c = \frac{1}{2}$. U holda:

$$x < 0 \text{ da} \quad F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} 0dx = 0;$$

$$0 \le x \le \pi \text{ da} \quad F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)dx =$$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{x} \frac{1}{2}\sin x dx = \frac{1}{2}(-\cos x)\Big|_{0}^{\pi} = \frac{1}{2}(1-\cos x);$$

$$x > 0 \text{ da} \quad F(x)\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{\pi} \frac{1}{2}\sin x dx + \int_{\pi}^{\infty} 0dx = -\frac{1}{2}\cos x\Big|_{0}^{\pi} = 1.$$

Demak,

$$F(x) = \begin{cases} 0, \ agar \ x < 0, \\ \frac{1}{2}(1-\cos x), \ agar \ 0 \le x \le \pi, \\ 0, \ agar \ x > \pi. \end{cases}$$

1.4.4. Agar ikkita tasodifiy miqdorlardan birining taqsimot qonuni ikkinchisining qanday mumkin bo‘lgan qiymat qabul qilishigan qat’iy nazar o‘zgarmasa, bu miqdorlarga *bog‘liqmas tasodifiy miqdorlar* deyiladi.

X – tasodifiy miqdor x_i $(i = \overline{1,n})$ qiymatlarni va Y – tasodifiy miqdor y_j $(j = \overline{1,m})$ qiymatlarni qabul qilsin. Bunda X va Y tasodifiy miqdorlarning bog‘liqmas bo‘lishi $X = x_i$ va $Y = y_i$ tasodifiy hodisalar i va j larning istalgan qiymatlarida bog‘liqmas bo‘lishini anglatadi. Aks holda tasodifiy miqdorlar *bog‘liq* deyiladi.

Ikkita diskret tasodifiy miqdor berilgan bo’lsin:

$$X : \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix}, \qquad Y : \begin{pmatrix} y_1 & y_2 & \dots & y_n \\ p'_1 & p'_2 & \dots & p'_n \end{pmatrix}.$$

X tasodifiy miqdorning k *o‘zgarmas songa ko’paytmasi* deb $k \cdot x_i$ qiymatlarni p_i $(i = \overline{1,n})$ ehtimol bilan qabul qiladigan $k \cdot X$ tasodifiy miqdorga aytiladi.

X tasodifiy miqdorning m- *darajasi* deb x_i^m qiymatlarni p_i $(i = \overline{1,n})$ ehtimol bilan qabul qiladigan X^m tasodifiy miqdorga aytiladi.

X va Y tasodifiy miqdorlarning *algebraik yig‘indisi* deb $x_i \pm y_j$ qiymatlarni $p_{ij} = P(X = x_i, Y = y_j)$, $i = \overline{1,n}$, $j = \overline{1,m}$ ehtimol bilan qabul qiladigan $X \pm Y$ tasodifiy miqdorga aytiladi. Bu yerda $p_{ij} = P(X = x_i, Y = y_j)$ ifoda X

miqdor x_i qiymatni, Y miqdor y_j qiymatni qabul qilishi ehtimolini, ya'ni $X = x_i$ va $Y = y_j$ hodisalarning birgalikda ro'y berishi ehtimolini ifodalaydi.

Agar X va Y tasodifiy miqdorlar bog'liqmas bo'lsa, u holda ehtimollarni ko'paytirish teoremasiga asosan $p_{ij} = p_i \cdot p'_j$ bo'ladi, bu yerda $p_i = P(X = x_i), \quad p'_j = P(Y = y_j)$. Bunda bir xil qiymatli yig'indilar (ayirmalar) hosil bo'lsa, u holda bu qiymatlarning mos ehtimollari qo'shilgan holda birlashtiriladi va yangi jadvalda yoziladi.

Tasodifiy miqdorlarning ko'paytmasi ham shu kabi aniqlanadi. Bunda jadvalning yo'qori satrida yig'indilar o'rnida mos ko'paytmalar qo'yiladi.

$6-$misol. X tasodifiy miqdor berilgan:

$$X : \begin{pmatrix} -3 & 1 & 3 \\ 0,3 & 0,5 & 0,2 \end{pmatrix}.$$

Tasodifiy miqdorlarning taqsimot qonunlarini toping:

1) $Y = 2X$; 2) $Z = X^2$.

1) Y tasodifiy miqdorning qiymatlarini topamiz: $2 \cdot (-3) = -6$, $2 \cdot 1 = 2$, $2 \cdot 3 = 6$. Ular mos ravishda 0,3, 0,5, 0,2 ehtimollarga ega bo'ladi.

Demak,

$$Y : \begin{pmatrix} -6 & 2 & 6 \\ 0,3 & 0,5 & 0,2 \end{pmatrix}.$$

2) Z tasodifiy miqdorning qiymatlarini topamiz: $(-3)^2 = 9, \quad 1^2 = 1, \quad 3^2 = 9$. Ular mos ravishda 0,3, 0,5, 0,2 ehtimollarga ega bo'ladi. Bunda $Z = 9$ qiymat 0,3 ehtimolli (-3)ni kvadratga ko'tarishdan va 0,2 ehtimolli $(+3)$ni kvadratga ko'tarishdan hosil bo'ladi. U holda ehtimollarni qo'shish teormasiga ko'ra $P(Z = 9) = 0,3 + 0,2 = 0,5.$

Shunday qilib,

$$Z : \begin{pmatrix} 1 & 9 \\ 0,5 & 0,5 \end{pmatrix}.$$

7-misol. X va Y bog'liqmas tasodifiy miqdorlar berilgan:

$$X : \begin{pmatrix} -1 & 1 \\ 0,4 & 0,6 \end{pmatrix}, \quad Y : \begin{pmatrix} 1 & 2 & 3 \\ 0,3 & 0,5 & 0,2 \end{pmatrix}.$$

Tasodifiy miqdorlarning taqsimot qonunlarini toping:

1) $Z = X + Y$, 2) $U = X \cdot Y$.

⊖ 1) Quyidagi jadvalni tuzamiz:

$Z:$

z_{ij}	$-1+1$	$-1+2$	$-1+3$	$1+1$	$1+2$	$1+3$
p_{ij}	$0{,}4\cdot 0{,}5$	$0{,}4\cdot 0{,}2$	$0{,}4\cdot 0{,}3$	$0{,}6\cdot 0{,}5$	$0{,}6\cdot 0{,}2$	$0{,}6\cdot 0{,}3$

Bir xil qiymatli yig'indilar turgan ustunlarni birlashtirib, ushbu taqsimot qonunini hosil qilamiz:

$$Z:\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0{,}20 & 0{,}08 & 0{,}42 & 0{,}12 & 0{,}18 \end{pmatrix}.$$

2) $U = X\cdot Y$ kupaytmaning taqsimot qonunini shu kabi topamiz:

$$U:\begin{pmatrix} -1 & -2 & -3 & 1 & 2 & 3 \\ 0{,}20 & 0{,}08 & 0{,}12 & 0{,}30 & 0{,}12 & 0{,}18 \end{pmatrix}.$$ ⊖

X, $Y-$ uzluksiz bog'liqmas tasodifiy miqdorlar, $f_1(x)$, $f_2(y)-$mos taqsimot zichliklari bo'lsin. U holda $Z = X+Y$ tasodifiy miqdorning taqsimot zichligi quyidagi formulalardan biri bilan topiladi:

$$g(z)=\int_{-\infty}^{\infty} f_1(x)f_2(z-x)dx \quad \text{yoki} \quad g(z)=\int_{-\infty}^{\infty} f_2(y)f_1(z-y)dy\,.$$

8-misol. X va Y bog'liqmas tasodifiy miqdorlar taqsimot zichliklari bilan berilgan:

$$f_1(x)=e^{-x}\ (0\le x<\infty),\quad f_2(y)=\frac{1}{2}e^{-\frac{y}{2}}\ (0\le x<\infty).$$

$Z = X+Y$ tasodifiy miqdorning taqsimot zichligini toping.

⊖ Argumentlarning mumkin bo'lgan qiymatlari manfiy emas. Berilgan funksiyaning taqsimot zichligini yuqoridan keltirilgan formulalarning birinchisi bilan topamiz:

$$g(z)=\int_0^z e^{-x}\left(\frac{1}{2}e^{-\frac{z-x}{2}}\right)dx=\frac{1}{2}\int_0^z e^{-x}e^{-\frac{z-x}{2}}dx=$$

$$=\frac{1}{2}e^{-\frac{z}{2}}\int_0^z e^{-\frac{x}{2}}dx=\frac{1}{2}e^{-\frac{z}{2}}\cdot\left(-2e^{-\frac{x}{2}}\right)\Bigg|_0^z=e^{-\frac{z}{2}}\left(1-e^{-\frac{z}{2}}\right).$$

Demak,

$$z\in(0;\infty)\ \text{da}\ \ g(z)=e^{-\frac{z}{2}}\left(1-e^{-\frac{z}{2}}\right),\ z\notin(0;\infty)\ \text{da}\ \ g(z)=0.$$ ⊖

1.4.5. $X-$tasodifiy miqdor, $\varphi(x)-$aniqlanish sohasi X tasodifiy miqdorning mumkin bo‘lgan qiymatlari to‘plamidan iborat funksiya bo‘lsin.

X *tasodifiy miqdorning funksiyasi* deb har bir sinashda $y=\varphi(x)$ qiymatlar qabul qiladigan $Y=\varphi(X)$ funksiyaga aytiladi, bu yerda $x-$shu sinashda X tasodifiy miqdor qabul qiladigan qiymat.

X diskret tasodifiy miqdor berilgan bo‘lsin:

$$X:\begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & \dots & p_n \end{pmatrix}.$$

X tasodifiy miqdorning mumkin bo‘lgan qiymatlari sohasida $y=\varphi(x)$ funksiya aniqlangan va monoton bo‘lsin. U holda $Y=\varphi(X)$ mumkin bo‘lgan qiymatlari $\varphi(x_1), \varphi(x_2),...,\varphi(x_n)$ bo‘lgan yangi tasodifiy miqdor bo‘ladi Bunda Y tasodifiy miqdorning $y=\varphi(x_i)$ qiymatni qabul qilish ehtimoli X tasodifiy miqdorning x_i qiymatni qabul qilish ehtimoliga teng bo‘ladi, ya’ni $P(Y=y_i)=P(X=x_i)$.

Demak, $Y=\varphi(X)$ tasodifiy miqdor

$$Y:\begin{pmatrix} \varphi(x_1) & \varphi(x_2) & \dots & \varphi(x_n) \\ p_1 & p_2 & \dots & p_n \end{pmatrix}.$$

taqsimot qonuniga ega bo‘ladi.

$\varphi(x)$ funksiya X tasodifiy miqdorning mumkin bo‘lgan qiymatlari sohasida monoton bo‘lmasa, $Y=\varphi(X)$ miqdor X ning turli qiymatlarida bir xil qiymatlar qabul qilishi mumkin. U holda avval yo‘qorida keltirilgan ko‘rinishdagi jadval tuziladi, keyin X ning bir xil qiymatli ustunlari mos ehtimollari qo‘shilgan holda birlashtiriladi va yangi jadval tuziladi.

X uzluksiz tasodifiy miqdor bo‘lib, uning taqsimot zichligi $f(x)$ bo‘lsin.

Agar $y=\varphi(x)$ funksiya monoton, differensiallanuvchi bo‘lib, uning teskari funksiyasi $x=\psi(y)$ bo’lsa, u holda Y tasodifiy miqdorning taqsimot zichligi

$$g(y)=f(\psi(y))\cdot|\psi'y)|$$

tenglikdan topiladi.

Agar $y=\varphi(x)$ funksiya monoton bo‘lmasa, u holda X tasodifiy miqdorning mumkin bo‘lgan qiymatlari oralig‘i $\varphi(x)$ funksiya monoton bo‘ladigan oraliqlarga ajratiladi. Har bir monotonlik oralig‘i uchun $g_k(y)$ taqsimot zichligi aniqlanadi va ularning yig’indisi $g(y)=\sum_k g_k(y)$ topiladi.

9-misol. X tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\begin{cases}\dfrac{1}{\pi}, & agar\ \ x\in\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right),\\[2ex] 0, & agar\ \ x\notin\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right).\end{cases}$$

$Y=\sin X$ tasodifiy miqdorning taqsimot zichligini toping.

$y=\sin x$ funksiya $\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right)$ oraliqda monoton.

U holda $x=\psi(y)=\arcsin y$ teskari funksiya mavjud, bu yerda $y\in(-1;1)$.

Bundan $\psi'(y)=\dfrac{1}{\sqrt{1-y^2}}$. Taqsimot zichligini topamiz:

$$g(y)=\frac{1}{\pi}\left|\frac{1}{\sqrt{1-y^2}}\right|=\frac{1}{\pi\sqrt{1-y^2}}.$$

Demak,

$$g(y)=\begin{cases}\dfrac{1}{\pi\sqrt{1-y^2}}, & agar\ \ y\in(-1;1),\\[2ex] 0, & agar\ \ y\notin(-1;1).\end{cases}$$

Mashqlar

1.4.1. Tanga uch marta tashlanadi. Raqamli tomon tushishi sonining taqsimot qonunini toping.

1.4.2. Talabaning uchta fandan test nazoratini topshirishi ehtimollari 0,7, 0,8 va 0,6 ga teng. Talaba topshiradigan test nazoratlari sonining taqsimot qonunini toping.

1.4.3. Guldondagi 5 ta atirguldan 2 tasi oq. Bir vaqtda olingan 2 ta gulda oq gul bo‘lishi sonining taqsimot qonunini toping.

1.4.4. Qutidagi 7 ta sharning 4 tasi oq. Qutidan ketma-ket birinchi shar oq chiqqanicha shar olinadi. Sharlar olinishi sonining taqsimot qonuni toping.

1.4.5. 6 ta detal solingan qutida 4 ta standart detal bor. Tavakkaliga 3 ta detal olingan. Olingan detallar orasidagi standart detallar sonidan iborat tasodifiy miqdorning taqsimot funksiyasini toping.

1.4.6. Ikkita o‘yin kubigi tashlangan. Juft ochkolar chiqishi sonining taqsimot funksiyasini topng.

1.4.7. Merganning bitta o‘q uzishda nishonga tekkazishi ehtimoli 0,8 ga teng va bu ehtimol har bir o‘q uzilgandan keyin 0,1 ga kamayadi. Uchta o‘q uzilgan. Nishonga tegadigan o‘qlar sonidan iborat tasodifiy miqdorning taqsimot funksiyasini toping.

1.4.8. Talabaning kerakli kitobni kutubxonadan topishi ehtimoli 0,4 ga teng. Talaba 4 ta kutubxonaga borishi mumkin. Talabaning kutubxonaga borishi sonining taqsimot funksiyasini toping.

1.4.9. Tasodifiy miqdorning taqsimot qatori berilgan:

$$X:\begin{pmatrix} -2 & 1 & 2 & 3 \\ 0{,}1 & 0{,}3 & 0{,}4 & 0{,}2 \end{pmatrix}.$$

1)Taqsimot funksiyasini toping va uning grafigini chizing;
2) $P(X<2),\quad P(1\le X<3)$ ehtimollarni toping.

1.4.10. Tasodifiy miqdorning taqsimot qatori berilgan:

$$X:\begin{pmatrix} -1 & 1 & 2 & 3 \\ 0{,}3 & 0{,}38 & 0{,}12 & 0{,}2 \end{pmatrix}.$$

1)Taqsimot funksiyasini toping va uning grafigini chizing;
2) $P(X<-1),\ P(-1\le X<2)$ ehtimollarni toping.

1.4.11. X uzluksiz tasodifiy miqdor taqsimot qonuni bilan berilgan:

$$F(x)=\begin{cases} 0, \ agar\ x\le -2, \\ a+b\arcsin\dfrac{x}{2}, \ agar\ -2<x\le 2, \\ 1, \ agar\ x>2. \end{cases}$$

Toping: 1) a va b ni; 2) $P(X=1)$ ni, 3) $P(-3\le X<1)$ ni; 4) $f(x)$ ni.

1.4.12. X uzluksiz tasodifiy miqdor taqsimot qonuni bilan berilgan:

$$F(x)=\begin{cases} 0, \ agar\ x\le -\pi, \\ a(\cos x+b), \ agar\ -\pi<x\le 0, \\ 1, \ agar\ x>0. \end{cases}$$

Toping: 1) a va b ni; 2) $P\left(X=\dfrac{\pi}{100}\right)$ ni, 3) $P\left[-\dfrac{\pi}{3};\dfrac{\pi}{2}\right]$ ni; 4) $f(x)$ ni.

1.4.13. X tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\begin{cases} 0, & agar\ x\le 1,\\ c(x-1), & agar\ 1<x\le 2,\\ 0, & agar\ x>2. \end{cases}$$

Toping: 1) c ni; 2) $F(x)$ ni; 3) $P(1{,}4<X<1{,}9)$ ni.

1.4.14. X tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\frac{2c}{e^{x}+e^{-x}},\quad -\infty<x<+\infty.$$

Toping: 1) c ni; 2) $F(x)$ ni; 3) $P(0\le X\le \ln\sqrt{3})$ ni.

1.4.15. Kompyuter qurilmasi buzulmasdan ishlash vaqtidan iborat tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\frac{1}{T}e^{-\frac{x}{T}},\quad x\ge 0.$$

Toping: 1) $F(x)$ ni; 2) $P(T\le X\le 2T)$ ni.

1.4.16. Soliq to‘lovchi yillik daromadididan iborat tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\begin{cases} \dfrac{a}{x_0}\left(\dfrac{x_0}{x}\right)^{a+1}, & agar\ x\ge x_0,\\ 0, & agar\ x<x_0, \end{cases}\quad a>0.$$

Toping: 1) $F(x)$ ni; 2) $P(x_0\le X\le 2x_0)$ ni.

1.4.17. Ikkita avtomat-stanokda bir xil detal ishlab chiqariladi. Har bir stanokda smena davomida yaroqsiz detallar ishlab chiqarish sonining taqsimot qonuni berilgan:

$$X:\begin{pmatrix} 0 & 1 & 2\\ 0{,}1 & 0{,}6 & 0{,}3 \end{pmatrix},\qquad Y:\begin{pmatrix} 2 & 2\\ 0{,}5 & 0{,}5 \end{pmatrix}.$$

Smena davomida har ikkala stanokda yaroqsiz detallar ishlab chiqarish sonining taqsimot qonunini toping.

1.4.18. X diskret tasodifiy miqdor taqsimot qonuni bilan berilgan:

$$X:\begin{pmatrix} -2 & -1 & 0 & 1 & 2\\ 0{,}1 & 0{,}2 & 0{,}3 & 0{,}3 & 0{,}1 \end{pmatrix}.$$

1) $Y=X^2+1$; 2) $Z=X+|X|$ miqdorlarning taqsimot qonunlarini toping.

1.4.19. Ikkkita mergan nishonga 2 tadan o‘q uzdi. Ularning nishonga tekkazishi ehtimollari 0,6 va 0,7 ga teng. Jami nishonga tegishlar sonining taqsimot qonunini toping.

1.4.20. X va Y yzluksiz o‘zaro bog‘liqmas tasodifiy miqdorlar zichlik funksiyalari bilan berilgan: $f(x) = ae^{-ax}, \ x \geq 0, \quad f(y) = ae^{-ay}, y \geq 0$.
$Z = X + Y$ miqdorning zichlik funksiyasini toping.

1.4.21. X va Y yzluksiz o‘zaro bog‘liqmas tasodifiy miqdorlar zichlik funksiyalari bilan berilgan: $f_1(x) = \frac{1}{4}e^{-\frac{x}{4}}, \ x \geq 0, \quad f_2(y) = \frac{1}{7}e^{-\frac{y}{7}}, \ y \geq 0.$
$Z = X + Y$ tasodifiy miqdorning zichlik funksiyasini toping.

1.4.22. X yzluksiz tasodifiy miqdor zichlik funksiyasi $f(x)$ bo‘lsa, $Y = 5X$ tasodifiy miqdorning zichlik funksiyasini toping.

1.4.23. X yzluksiz tasodifiy miqdor zichlik funksiyasi bilan berilgan: $f(x) = e^{-x}, \ x \geq 0.$ $Y = e^{-X}$ miqdorning zichlik funksiyasini va taqsimot qonunini toping.

1.5. TASODIFIY MIQDORNING SONLI XARAKTERISTIKALARI

Tasodifiy miqdorning matematik kutilishi. Tasodifiy miqdorning dispersiyasi va o‘rtacha kvadratik chetlashishi. Boshlang‘ich va markaziy momentlar

1.5.1. X diskret tasodifiy miqdorning *matematik kutilishi* $M(X)$ deb, X miqdorning mumkin bo‘lgan qiymatlari bilan mos ehtimollar ko‘paytmalarining yig‘indisiga teng songa aytiladi, ya’ni

$$M(X) = x_1 p_1 + x_2 p_2 + ... + x_n p_n = \sum_{l=1}^{n} x_i p_i \quad \text{yoki} \quad M(X) = \sum_{l=1}^{\infty} x_i p_i .$$

Mumkin bo‘lgan qiymatlari $[a;b]$ kesmaga ($(-\infty;\infty)$ oraliqqa) tegishli bo‘lgan X uzluksiz tasodifiy miqdorning matematik kutilishi deb

$$M(X) = \int_a^b x f(x) dx \quad \left(M(X) = \int_{-\infty}^{\infty} x f(x) dx \right)$$

songa aytiladi, bu yerda $f(x)$ – taqsimot zichligi.

1-misol. Ovchining 4 ta o‘qi bor. Ovchi ilvasinga qarata bitta o‘q tekkazganigacha yoki o‘qlari tugaganigacha o‘q uzadi. Birinchi o‘q uzishda o‘qning ilvasinga tegishi ehtimoli 0,6 ga teng va bu ehtimol har bir uzilgan o‘qdan keyin 0,1 ga kamayadi. Ovchining sarflagan o‘qlari sonining matematik kutilishini toping

⊖ Avval tasodifiy miqdorning taqsimot qonunini topamiz.

$A_i\ (i=\overline{1,4})-$ i- o‘qning ilvasinga tegishi hodisasi bo‘lsin.

$X-$sarflangan o‘qlar soni 1,2,3, 4 ga teng bo‘lishi mumkin.

Bitta o‘q sarflanishining ehtimoli otilgan birinchi o‘qning ilvasinga tegishi hodisasining ehtimoliga teng bo‘ladi: $P(X=1)=P(A_1)=0{,}6;$

Ikkita o‘q sarflanishining ehtimoli otilgan birinchi o‘qning ilvasinga tegmasligi va ikkinchi o‘qning tegishi hodisalarining birgalikda ro‘y berishidan iborat hodisaning ehtimoliga teng bo‘ladi:

$$P(X=2)=P(\overline{A}_1\cdot A_2)=P(\overline{A}_1)\cdot P(A_2)=0{,}4\cdot 0{,}5=0{,}2;$$

Shu kabi, uchta o‘q sarflanishining ehtimoli:

$$P(X=3)=P(\overline{A}_1\cdot \overline{A}_2\cdot A_3)=P(\overline{A}_1)\cdot P(\overline{A}_2)\cdot P(A_3)=0{,}4\cdot 0{,}5\cdot 0{,}4=0{,}08;$$

To‘rtta o‘q sarflanishining ehtimoli otilgan birinchi uchta o‘qning ilvasinga tegmasligi va to‘rtinchi o‘qning tegishi hodisalarining birgalikda ro‘y berishidan hamda to‘rttala o‘qning ham ilvasinga tegmasligidan iborat bo‘g‘liqmas hodisaning ehtimoliga teng bo‘ladi:

$$P(X=4)=P(\overline{A}_1\cdot \overline{A}_2\cdot \overline{A}_3\cdot A_4)+P(\overline{A}_1\cdot \overline{A}_2\cdot \overline{A}_3\cdot \overline{A}_4)=$$
$$=P(\overline{A}_1)\cdot P(\overline{A}_2)\cdot P(\overline{A}_3)\cdot P(A_4)+P(\overline{A}_1)\cdot P(\overline{A}_2)\cdot P(\overline{A}_3)\cdot P(\overline{A}_4)=$$
$$=0{,}4\cdot 0{,}5\cdot 06\cdot 0{,}3+0{,}4\cdot 0{,}5\cdot 0{,}6\cdot 0{,}7=0{,}12.$$

Demak, X tasodifiy miqdorning taqsimot qonuni

$$X=\begin{cases} 1 & 2 & 3 & 4 \\ 0{,}6 & 0{,}2 & 0{,}08 & 0{,}12 \end{cases}.$$

Bundan

$$M(X)=1\cdot 0{,}6+2\cdot 0{,}2+3\cdot 0{,}08+4\cdot 0{,}12=1{,}72\,.$$ ⊖

2-misol. X uzluksiz tasodifiy miqdor

$$f(x)=\begin{cases} 0, & agar\ x<a, \\ 3x^2, & agar\ a\le x\le b, \\ 0, & agar\ x>b \end{cases}$$

zichlik funksiyasi bilan berilgan. $M(X)$ ni toping.

⊖ $$M(X)=\int_a^b xf(x)dx=\int_0^1 x3x^2dx=\frac{3x^4}{4}\bigg|_0^1=\frac{3}{4}.$$ ⊖

Matematik kutilish quyidagi xossalarga ega.

1^o. $M(C)=C$, $C-$o‘garmas miqdor.

2^o. $M(kX)=kM(X),\ k=const$.

4^o. $M(X\pm Y)=M(X)\pm M(Y)$.

5^o. $M(X\cdot Y)=M(X)\cdot M(Y)$, $X,Y-$bog‘liqmas tasodifiy miqdorlar.

1.5.2. X tasodifiy miqdorning *dispersiyasi* $D(X)$ deb, tasodifiy miqdorning o‘z matematik kutilishi $M(X)$dan chetlashishi kvadratining matematik kutilishiga aytiladi, ya’ni

$$D(X)=M(X-M(X))^2.$$

Bu formulani quyidagi ko‘rinishlarda ifodalash mumkin:

1) diskret tasodifiy miqdorlar uchun

$$D(X)=\sum_{l=1}^{n}(x_i-M(X))^2p_i,\qquad D(X)=\sum_{l=1}^{\infty}(x_i-M(X))^2p_i\ ;$$

2) uzluksiz tasodifiy miqdorlar uchun

$$D(X)=\int_a^b(x-M(X))^2f(x)dx,\quad D(X)=\int_{-\infty}^{\infty}(x-M(X))^2f(x)dx.$$

Matematik kutilishni topishda amaliy jihatdan qo‘llashga qulayroq bo‘lgan quyidagi formulalardan foydalaniladi:

1) diskret tasodifiy miqdorlar uchun

$$D(X)=\sum_{l=1}^{n}x_i^2p_i-M^2(X),\qquad D(X)=\sum_{l=1}^{\infty}x_i^2p_i-M^2(X);$$

2) uzluksiz tasodifiy miqdorlar uchun

$$D(X)=\int_a^b x^2f(x)dx-M^2(X),\qquad D(X)=\int_{-\infty}^{\infty}x^2f(x)dx-M^2(X).$$

Dispersiya quyidagi xossalarga ega.

1^o. $D(C)=0$, $C-$o‘zgarmas miqdor.

2^o. $D(kX)=k^2D(X)$, $k=const$.

3^o. $D(X\pm Y)=D(X)+D(Y)$, $X,Y-$bog‘liqmas tasodifiy miqdorlar.

X tasodifiy miqdorning *o‘rtacha kvadratik chetlashishi* $\sigma(X)$ deb uning dispersiyasidan olingan kvadrat ildizga aytiladi, ya’ni

$$\sigma(X)=\sqrt{D(X)}.$$

3-misol. Nazorat ishi 3 ta test savolidan iborat. Har bit test savolida 4 ta javob keltirilgan bo‘lib, ulardan bittasi to‘g‘ri. Oddiy topishda to‘g‘ri javoblar sonining matematik kutilishi va dispersiyasini toping.

$p-$har bir test savolida to‘g‘ri javob topilishining ehtimoli bo‘lsin.

Masalaning shartiga ko‘ra: $p=\frac{1}{4},\ q=\frac{3}{4}.$

$X-$ topilgan to‘g‘ri javoblar soni bo‘sin. Uning mumkin bo‘lgan qiymatlari: $x_1=0,\quad x_2=1,\quad x_3=2, x_4=3.$ Bu qiymatlarning ehtimollarini topamiz:

$$P(X=0)=C_3^0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3=\frac{27}{64},\qquad P(X=1)=C_3^1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2=\frac{27}{64},$$

$$P(X=2)=C_3^2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^1=\frac{9}{64},\qquad P(X=3)=C_3^3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0=\frac{27}{64}.$$

Demak,

$$X:\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{27}{64} & \frac{27}{64} & \frac{9}{64} & \frac{1}{64}\end{pmatrix}.$$

Bundan

$$M(X)=1\cdot\frac{27}{64}+2\cdot\frac{9}{64}+3\cdot\frac{1}{64}=\frac{3}{4};$$

$$D(X)=M(X^2)-(M(X))^2=1\cdot\frac{27}{64}+4\cdot\frac{9}{64}+9\cdot\frac{1}{64}-\left(\frac{3}{4}\right)^2=\frac{9}{8}-\frac{9}{16}=\frac{9}{16};$$

$$\sigma(X)=\sqrt{D(X)}=\sqrt{\frac{9}{16}}=\frac{3}{4}.$$ ⊂

4-misol. X uzluksizt tasodifiy miqdor taqsimot zichligi bilan berilgan:

$$f(x)=\begin{cases} 0,\ agar\ x\le-\frac{\pi}{2}, \\ \frac{1}{2}\cos x,\ agar\ -\frac{\pi}{2}<x\le\frac{\pi}{2}, \\ 0,\ agar\ x>\frac{\pi}{2}. \end{cases}$$

$M(X),\ D(X),\ \sigma(X)$ larni toping.

⊃ $$M(X)=\int_a^b xf(x)dx=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\cos xdx=\left|\begin{matrix} u=x,\ du=dx, \\ dv=\cos xdx,\ v=\sin x\end{matrix}\right|=$$

$$=\frac{1}{2}\left(x\sin x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin xdx\right)=\frac{1}{2}\cos x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=0;$$

$$D(X)=\int_a^b x^2f(x)dx-M^2(X)=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x^2\cos xdx=\left|\begin{matrix} u=x^2,\ du=2xdx, \\ dv=\cos xdx,\ v=\sin x\end{matrix}\right|=$$

$$= \frac{1}{2}\left(x^2 \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x \sin x dx \right) = \left| \begin{matrix} u = x, \ du = dx, \\ dv = \sin x dx, \ v = -\cos x \end{matrix} \right| = \frac{\pi^2}{4} + x\cos x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx =$$

$$= \frac{\pi^2}{4} - \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi^2}{4} - 2 = 0{,}465;$$

$$\sigma(X) = \sqrt{D(X)} = \sqrt{0{,}465} = 0{,}682.$$

5-misol. Ikkita to‘quv mashinasida bir xil mahsulot ishlab chiqariladi. Har bir mashina uchun bir smenada ishlab chiqariladigan nuqsonli mahsulotlar sonidan iborat tasodifiy miqdorning taqsimot qonunlari berilgan:

$$X: \begin{pmatrix} 0 & 1 & 2 \\ 0{,}2 & 0{,}4 & 0{,}4 \end{pmatrix}, \qquad Y: \begin{pmatrix} 1 & 2 & 3 \\ 0{,}3 & 0{,}5 & 0{,}2 \end{pmatrix}.$$

$Z = 6X - 5Y + 8$ tasodifiy miqdorning matematik kutilishi va dispersiyasini toping.

Avval X va Y tasodifiy miqdorlarning matematik kutilishi va dispersiyalarini topamiz.

X tasodifiy miqdor uchun:

$$M(X) = 1 \cdot 0{,}4 + 2 \cdot 0{,}4 = 1{,}2;$$

$$D(X) = 1 \cdot 0{,}4 + 4 \cdot 0{,}4 - (1{,}2)^2 = 2 - 1{,}44 = 0{,}56.$$

Y tasodifiy miqdor uchun:

$$M(Y) = 1 \cdot 0{,}3 + 2 \cdot 0{,}5 + 3 \cdot 0{,}2 = 1{,}9;$$

$$D(Y) = 1 \cdot 0{,}3 + 4 \cdot 0{,}5 + 9 \cdot 0{,}2 - (1{,}9)^2 = 4{,}1 - 3{,}61 = 0{,}49.$$

$Z = 6X - 5Y + 8$ tasodifiy miqdorning sonli xarakteristikalarini matematik kutilish va dispersiyaning xossalaridan foydalanib, topamiz:

$$M(Z) = M(6X - 5Y + 8) = M(6X) - M(5Y) + M(8) =$$
$$= 6M(X) - 5M(Y) + 8 = 6 \cdot 1{,}2 - 5 \cdot 1{,}9 + 8 = 7{,}2 - 9{,}5 + 8 = 5{,}7;$$
$$D(Z) = D(6X - 5Y + 8) = D(6X) - D(5Y) + D(8) =$$
$$= 36D(X) - 25D(Y) + 0 = 36 \cdot 0{,}56 - 25 \cdot 0{,}49 = 20{,}16 - 12{,}25 = 7{,}91.$$

1.5.3. X tasodifiy miqdorning *k-tartibli boshlang‘ich momenti* α_k deb X^k miqdorning matematik kutilishiga aytiladi, ya’ni

$$\alpha_k = M(X^k).$$

Xususan, $\alpha_1 = M(X)$, $\alpha_2 = M(X^2)$. Bundan $D(X) = \alpha_2 - \alpha_1^2$.

X tasodifiy miqdorning *k-tartibli markaziy momenti* β_k deb $(X - M(X))^k$ miqdorning matematik kutilishiga aytiladi, ya’ni

$$\beta_k = M((X - M(X)^k).$$

Xususan, $\beta_1 = 0$, $\beta_2 = D(X)$.

Mashqlar

1.5.1. Tasodifiy miqdorning taqsimot qatori berilgan:

$$X:\begin{pmatrix} -1 & 0 & 1 & 2 \\ 0{,}2 & 0{,}1 & 0{,}3 & 0{,}4 \end{pmatrix}.$$

Uning matematik kutilishini, dispersiyasini va o‘rtacha kvadratik chetlashishini toping.

1.5.2. Tasodifiy miqdorning taqsimot qatori berilgan:

$$X:\begin{pmatrix} -1 & 0 & 1 & 2 & 2{,}5 & 3 \\ 0{,}1 & 0{,}1 & 0{,}2 & 0{,}3 & 0{,}2 & 0{,}1 \end{pmatrix}.$$

Uning matematik kutilishini, dispersiyasini va o‘rtacha kvadratik chetlashishini toping.

1.5.3. Haydovchi manzilgacha yo‘lda 5 ta svetoforga duch keladi. Haydovchining har svetofordan to‘xtamasdan o‘tishi ehtimoli $\frac{1}{3}$ ga teng. Haydovchining birinchi to‘xtashigacha yoki manzilga yetishigacha o‘tadigan svetoforlar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

1.5.4. Qutida 6 ta oq va 4 ta qora shar bor. Qutidan tavakkaliga ketma-ket 5 ta shar olinadi. Bunda olingan har bir shar keyingi shar olinishidan oldin qutiga qaytariladi va sharlar aralashtiriladi. Olingan sharlar oq chiqishlari sonidan iborat X tasodifiy miqdorning taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

1.5.5. Ikkita merganning o‘q uzishda nishonga tekkazishlari sonidan iborat tasodifiy miqdorlarning taqsimot qonunlari berilgan:

$$X:\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0{,}20 & 0{,}25 & 0{,}20 & 0{,}20 & 0{,}25 \end{pmatrix}, \quad Y:\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}05 & 0{,}15 & 0{,}20 & 0{,}20 & 0{,}25 & 0{,}15 \end{pmatrix}.$$

$Z = 5X - 4Y + 7$ miqdorning matematik kutilishini va dispersiyasini toping.

1.5.6. Ikkita stanokda tayyorlangan nuqsonli detallar sonidan iborat tasodifiy miqdorlarning taqsimot qonunlari berilgan:

$$X:\begin{pmatrix} 0 & 2 & 4 & 6 \\ 0{,}1 & 0{,}4 & 0{,}2 & 0{,}3 \end{pmatrix}, \quad Y:\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0{,}2 & 0{,}3 & 0{,}2 & 0{,}2 & 0{,}1 \end{pmatrix}.$$

$Z = 5X + 2Y + 9$ miqdorning matematik kutilishini va dispersiyasini toping.

1.5.7. A hodisa ro'y berishlari sonining matematik kutilishi va dispersiyasini toping: 1) bitta sinashda; 2) n ta sinashda.

1.5.8. X tasodifiy miqdorning taqsimot qatori ikkita mumkin bo'lgan qiymatdan iborat. Tasodifiy miqdorning bu qiymatlardan birini qabul qilishi ehtimoli 0,8 ga teng. Agar $M(X)=3{,}2,\ D(X)=0{,}16$ bo'lsa tasodifiy miqdorning taqsimot funksiyasini toping.

1.5.9. X tasodifiy miqdorning taqsimot qatori ikkita mumkin bo'lgan qiymatdan iborat. Tasodifiy miqdorning bu qiymatlardan birini qabul qilishi ehtimoli 0,6 ga teng. Agar $M(X)=1{,}4,\ D(X)=0{,}24$ bo'lsa tasodifiy miqdorning taqsimot funksiyasini toping.

1.5.10. X diskret tasodifiy miqdor uchta mumkin bo'lgan $x_1,\ x_2$ va $x_3\ (x_1<x_2<x_3)$ qiymatga ega bo'lib, $P(X=x_i)=p_i\ (i=1,2,3)$ bo'lsin. Quyida berilganlar asosida X tasodifiy miqdorning taqsimot funksiyasini toping:

1) $x_3=3,\ p_1=0{,}2,\ p_2=0{,}4,\ M(X)=1{,}6,\ D(X)=1{,}44;$

2) $x_1=1,\ x_2=2,\ p_3=0{,}5,\ M(X)=2{,}5,\ D(X)=0{,}25;$

3) $x_1=0,\ x_2=2,\ p_2=0{,}3,\ M(X)=1{,}8,\ D(X)=2{,}76;$

4) $x_1=-1,\ p_1=0{,}3,\ x_3=1,\ p_3=0{,}3,\ M(X)=0.$

1.5.11. Taqsimot funksiyasi bilan berilgan X tasodifiy miqdorning matematik kutilishi, dispersiyasi va o'rtacha kvadratik chetlashishini toping:

1) $F(x)=\begin{cases} 0, & agar\ x\le 0, \\ cx^2, & agar\ 0<x\le 1, \\ 1, & agar\ x>1 \end{cases};$ 2) $F(x)=\begin{cases} 0, & agar\ x\le 2, \\ c(x^3-8), & agar\ 2<x\le 3, \\ 1, & agar\ x>3 \end{cases};$

3) $F(x)=\begin{cases} 0, & agar\ x\le a, \\ 0{,}25x^2, & agar\ a<x\le b, \\ 1, & agar\ x>b \end{cases};$ 4) $F(x)=\begin{cases} 0, & agar\ x\le 0, \\ cx^3, & agar\ 0<x\le 2, \\ 1, & agar\ x>2 \end{cases}.$

1.5.12. Taqsimot zichligi bilan berilgan X tasodifiy miqdorning matematik kutilishi va dispersiyasini toping:

1) $f(x)=2x-2,\ x\in(1;2];$ 2) $f(x)=\frac{1}{2}\sin x,\ x\in(0;\pi];$

3) $f(x)=3^x\ln 3,\ x\in(-\infty;0];$ 4) $f(x)=cxe^{-x},\ x\in[0;+\infty).$

1.6. TASODIFIY MIQDORNING TAQSIMOT QONUNLARI

Diskret tasodifiy miqdorning taqsimot qonunlari. Uzluksiz tasodifiy miqdorning taqsimot qonunlari

16.1. *1. Binominal taqsimot*

Mumkin bo‘lgan qiymatlari $k = \{0,1,...,n\}$ ning ehtimollari

$$P_n(k) = C_n^k p^n q^{n-k}$$

formula bilan aniqlanadigan diskrit tasodifiy miqdorning taqsimot qonuniga *binominal taqsimot* deyiladi.

Binominal taqsimot n ta bogliqmas sinashlarning har birida bir xil p ga teng ehtimol bilan ro‘y beradigan A hodisaning ro‘y berishlari soni $X = k$ ni ifodalaydi. Binominal taqsimot qonuni mahsulot sifatini statistik nazorat qilish nazariyasi va amaliyotida, ommaviy xizmatlar nazariyasida, otish nazariyasida, mustahkamlik nazariyasida qo‘llaniladi.

Binominal taqsimotning sonli xarakteristikalari

$$M(X) = np, \quad D(X) = npq, \quad \sigma(X) = \sqrt{npq}$$

tengliklar bilan topiladi.

1-misol. Guruhda 16 ta talaba bo‘lib, ulardan 12 tasi xorijiy tillini biladi. Guruhdan tavakkaliga 3 ta talaba tanlanadi. Tanlanmada xorijiy tilni biladigan talabalar sonining taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

A – talabaning xorijiy tilni bilishi hodisasi bo‘lsin.

U holda

$$p = P(A) = \frac{12}{16} = \frac{3}{4}, \quad q = P(\overline{A}) = 1 - \frac{3}{4} = \frac{1}{4}$$

bo‘ladi.

X – tanlanmada xorijiy tilni biladigan talabalar soni bo’lsin.

X ning mumkin bo’lgan qiymatlari: 0, 1, 2, 3.

Bu qiymatlarning ehtimollarini topamiz:

$$P(X=0) = C_3^0 p^0 q^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}; \qquad P(X=1) = C_3^1 p^1 q^2 = \frac{3!}{1!2!}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 = \frac{9}{64};$$

$$P(X=2) = C_3^2 p^2 q^1 = C_3^2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1 = \frac{27}{64}; \qquad P(X=3) = C_3^3 p^3 q^{03} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}.$$

Demak, X tasodifiy miqdorning taqsimot qonuni

$$X:\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{64} & \frac{9}{61} & \frac{27}{64} & \frac{27}{64} \end{pmatrix}.$$

Bernulli taqsimot qonunining sonli xarakteristikalarini topamiz:

$$M(X)=3\cdot\frac{3}{4}=\frac{9}{4},\qquad D(X)=3\cdot\frac{3}{4}\cdot\frac{1}{4}=\frac{9}{16}.$$

2-misol. Qurilma o‘zaro erkli ishlaydigan 15 ta elementdan iborat. Har bir elementning bitta sinashda ishdan chiqishi ehtimoli 0,3 ga teng. Sinash natijasida ishdan chiqqan elementlar sonining taqsimot qonuni turini aniqlang. Tasodifiy miqdorning sonli xarakteristikalarini toping.

$A-$ Har bir elementning bitta sinashda ishdan chiqishi hodisasi bo‘lsin. U holda $p=P(A)=0{,}3,\ \ q=P(\overline{A})=1-0{,}3=07$ bo‘ladi.

$X-$sinash natijasida ishdan chiqqan elementlar soni bo‘lsin. X tasodifiy miqdor binominal taqsimotga bo‘ysinadi.

U holda

$$M(X)=15\cdot 0{,}3=4{,}5,\quad D(X)=15\cdot 0{,}3\cdot 0{,}7=3{,}15,\quad \sigma(X)=\sqrt{D(X)}=1{,}77.$$

2. Puasson taqsimoti

Mumkin bo‘lgan qiymatlari $k=\{0,1,...,k,...\}$ning ehtimollari

$$P_n(k)=\frac{\lambda^k e^{-\lambda}}{k!},\ \ \lambda=np$$

formula bilan aniqlanadigan diskret tasodifiy miqdorning taqsimot qonuniga *Puasson taqsimoti* deyiladi.

Avtomatik liniyadagi to‘xtashlar soni, normal rejimdagi murakkab sistemadagi buzilishlar soni, t vaqt mobaynida telefon stansiyasiga keluvchi chaqiruvlar soni, katta partiyadagi nuqsonli detallar soni, radiaktiv manbadan tarqalayorgan α zarralar soni Puasson qonuni bilan taqsimlanadi.

Puasson taqsimotining sonli harakteristikalari

$$M(X)=\lambda,\ D(X)=\lambda,\ \sigma(X)=\sqrt{\lambda}.$$

formulalar bilan aniqlanadi.

3-misol. 800 ta urchuqning har birida $t$vaqt ichida ipning uzilishi ehtimoli 0,005 ga teng. Ko‘rsatilgan vaqt ichida beshtagacha ipning uzilishlari sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini va o‘rtacha kvadratik chetlashishini toping.

$X-t$vaqt ichida ipning uzilishlari sonidan iborat tasodifiy miqdor bo‘lsin. X parametrlari $n = 800$, $p = 0{,}005$ bo’lgan Puasson taqsimotga ega bo‘ladi. Bunda $\lambda = np = 800 \cdot 0{,}005 = 4$.

U holda 3-ilovadan foydalanib, topamiz:

$$P(X=0) = e^{-4} = 0{,}0183, \qquad P(X=1) = \frac{4}{1!} \cdot e^{-4} = 0{,}0733,$$

$$P(X=2) = \frac{4^2}{2!} \cdot e^{-4} = 0{,}1465, \qquad P(X=3) = \frac{4^3}{3!} \cdot e^{-4} = 0{,}1954,$$

$$P(X=4) = \frac{4^4}{4!} \cdot e^{-4} = 0{,}1954, \quad P(X=5) = \frac{4^5}{5!} \cdot e^{-4} = 0{,}1563.$$

Demak, X tasodifiy miqdorning taqsimot qonuni

$$X : \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}0183 & 0{,}0733 & 0{,}1465 & 0{,}1954 & 0{,}1954 & 0{,}1563 \end{pmatrix}.$$

Taqsimot qonunining sonli xarakteristikalarini topamiz:

$$M(X) = \lambda = 4, \qquad D(X) = \lambda = 4.$$

4-misol. Bug‘doy urug’lari orasida 0,2% begona urug‘ bor. Tavakkaliga 5000 ta urug‘ tanlansa, tanlanmada begona urug‘lar sonidan iborat tasodifiy miqdorning sonli xarakteristikalarini va 20 ta begona urug‘ chiqishi ehtimolini toping.

Sinashlar soni etarlicha katta $(n = 5000)$ va hodisaning ehtimoli etarlicha kichik $(p = 0{,}002)$ bo‘lganida begona urug‘lar sonidan iborat tasodifiy miqdor Puasson taqsimotiga ega bo‘ladi. Bunda $\lambda = np = 10$.

U holda

$$M(X) = D(X) = \lambda = 10.$$

Tanlangan 5000 ta urug‘ orasida 20 ta begona urug‘ chiqishi ehtimolini topamiz:

$$P_{5000}(20) = \frac{10^{20} e^{-10}}{20!} = 0{,}0019.$$

3. Geometrik taqsimot

Mumkin bo‘lgan qiymatlari $k = \{1, 2, ..., k, ...\}$ ning ehtimollari

$$P(k) = pq^{k-1}$$

formula bilan aniqlanadigan diskrit tasodifiy miqdorning taqsimot qonuniga *geometrik taqsimot* deyiladi.

Geometrik taqsimot k ta bogliqmas sinashlarning har birida bir xil va p ga teng ehtimol bilan ro‘y beradigan A hodisaning birinchi ijobiy natijagacha ro‘y berishlari soni $X = k$ ni ifodalaydi. Masalan, nishonga birinchi tekkazguncha o‘q otishlar soni, uskuna birinchi to‘xtaguncha sinashlar soni, tangani birinchi gerb chiqqanicha tashlashlar soni, mahsulotni birichi nuqsonlisi chiqguncha tekshirishlar soni va hokazo.

Geometrik taqsimotining sonli harakteristikalari:

$$M(X)=\frac{1}{p},\ D(X)=\frac{q}{p^2},\ \sigma(X)=\frac{\sqrt{q}}{p}$$

5-misol. Uskuna mustahkamligi sinovdan o‘tkazilmoqda. Sinashlar uskunaning ishdan chiqishiga qadar o‘tkaziladi. Har bir sinashda uskunaning ishdan chiqishi ehtimoli 0,1 ga teng. Muvaffaqiyatli o‘tkazilgan sinashlar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

$X-$muvaffaqiyatli o‘tkazilgan sinashlar sonidan iborat tasodifiy miqdor bo‘lsin. X parametrlari $p=0{,}1,\ q=1-p=0{,}9$ bo‘lgan geometrik taqsimotga ega.

Demak, X tasodifiy miqdorning taqsimot qonuni

$$X:\begin{pmatrix} 1 & 2 & 3 & \ldots & k & \ldots \\ 0{,}1 & 0{,}09 & 0{,}081 & \ldots & 0{,}1\cdot 0{,}9^{k-1} & \ldots \end{pmatrix}.$$

Taqsimot qonunining sonli xarakteristikalarini topamiz:

$$M(X)=\frac{1}{p}=\frac{1}{0{,}1}=10,\quad D(X)=\frac{q}{p^2}=\frac{0{,}9}{0{,}1^2}=90,\quad \sigma(X)=\sqrt{90}\approx 9{,}49.$$

4. Gipergeometrik taqsimot

Mumkin bo’lgan qiymatlari $k=\{1,2,\ldots,\min(n,K)\}$ ning ehtimollari

$$P(k)=\frac{C_K^k C_{N-K}^{n-k}}{C_N^n}$$

formula bilan aniqlanadigan diskrit tasodifiy miqdorning taqsimot qonuniga *gipergeometrik taqsimot* deyiladi, bu yerda $k\le N,\ n\le N,\ n,\ K,\ M-$natural sonlar.

K tasi ma‘lum xossaga ega bo‘lgan N ta elementlar to‘plamidan tavakkaliga joyiga qaytarilmasdan olingan n ta elementdagi berilgan xossaga ega bo‘lgan elementlar sonidan iborat $X=k$ tasadifiy miqdor gipergeometrik taqsimotga ega bo‘ladi. Gipergeometrik taqsimot sanoat mahsulotlarining qabuliga oid statistik nazorati amaliyotida, tanlanma tekshirishlarni tashkil qilish bilan bog‘liq masalalarda va boshqa sohalarda keng qo‘llaniladi.

$n,\ K,\ N$ parametrli gipergeometrik taqsimotning sonli xarakteristikalari

$$M(X)=n\frac{K}{N},\quad D(X)=n\frac{K}{N-1}\left(1-\frac{K}{N}\right)\left(1-\frac{n}{N}\right),\quad \sigma(X)=\sqrt{D(X)}$$

formulalar bilan topiladi.

6-misol. Firmaning sotuvga qo‘yilgan 20 ta kompyuteridan 7 tasida nosozlik mavjud. Tavakkaliga 5 ta kompyuter tanlangan. Tanlanmada nosoz kompyuterlar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishi va dispersiyasini va olingan kompyuterlar orasida nosozlari bo‘lmasligi ehtimolini toping.

X – nosoz kompyuterlar sonidan iborat tasodifiy miqdor bo‘lsin. X parametrlari $N=20,\ K=7,\ n=5$ bo‘lgan gipergeometrik taqsimotga ega.

U holda

$$P(X=0)=\frac{C_7^0C_{13}^5}{C_{20}^5}=0{,}083,\qquad P(X=1)=\frac{C_7^1C_{13}^4}{C_{20}^5}=0{,}323,$$

$$P(X=2)=\frac{C_7^2C_{13}^3}{C_{20}^5}=0{,}387,\qquad P(X=3)=\frac{C_7^3C_{13}^2}{C_{20}^5}=0{,}176,$$

$$P(X=4)=\frac{C_7^4C_{13}^1}{C_{20}^5}=0{,}03,\qquad P(X=5)=\frac{C_7^5C_{13}^0}{C_{20}^5}=0{,}001.$$

Demak, X tasodifiy miqdorning taqsimot qonuni

$$X:\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}083 & 0{,}323 & 0{,}387 & 0{,}176 & 0{,}030 & 0{,}001 \end{pmatrix}.$$

Taqsimot qonunining sonli xarakteristikalarini topamiz:

$$M(X)=5\cdot\frac{7}{20}=1{,}75,\qquad D(X)=5\cdot\frac{7}{19}\left(1-\frac{7}{20}\right)\left(1-\frac{5}{20}\right)=0{,}898.$$

Olingan kompyuterlar orasida nosozlari bo’lmasligi ehtimoli $P(X=0)=0{,}083$ ga teng.

1.6.2. *1. Tekis taqsimot*

Taqsimot zichligi

$$f(x)=\begin{cases}\dfrac{1}{b-a}, & agar\ a\le x\le b,\\[2mm] 0, & agar\ a<x\ \ yoki\ \ x>b.\end{cases}$$

ko‘rinishda berilgan uzluksiz tasodifiy miqdorning taqsimotiga $[a;b]$ kesmada *tekis taqsimot* deyiladi.

Tekis taqsimotning integral funksiyasi

$$F(x)=\begin{cases} 0, & agar\ x<a, \\ \dfrac{x-a}{b-a}, & agar\ a\le x\le b, \\ 1, & agar\ x>b \end{cases}$$

kabi aniqlandi.

$[a;b]$ kesmada tekis taqsimlangan tasodifiy miqdorning $[\alpha;\beta)\subset[a;b]$ oraliqqa tegishli qiymat qabul qilishi ehtimoli

$$P(\alpha\le X<\beta)=\frac{\beta-\alpha}{b-a}$$

tenglik bilan topiladi.

Tekis taqsimot qonuni yo‘lovchining ma‘lum oraliq bilan xarakatlanuvchi transportni kutish vaqtini aniqlashda, sonni butun qismga yaxlitlash xatoligini tahlil qilishda, kuzatishlarni statistik modellashtirishda va ommaviy xizmatning ko‘pchilik masalalarida qo‘llaniladi.

Tekis taqsimotning sonli xarakteristikalari

$$M(X)=\frac{b+a}{2},\quad D(X)=\frac{(b-a)^2}{12},\quad \sigma(X)=\frac{b-a}{2\sqrt{3}}$$

tengliklar bilan aniqlanadi.

7-misol. Bir soat ichida bekatga faqat bitta avtobus kelib toxtaydi. $t=0$ vaqtda bekatga kelgan yo‘lovchining avtobusni 10 minutdan ortiq kutmasligi ehtimolini toping.

$t=0$ vaqtda bekatga kelgan yo‘lovchining avtobusni kutish vaqti $[0;1]$ oraliqda tekis taqsimlangan X tasodifiy miqdor bo‘ladi.

Shu sababli

$$f(x)=\begin{cases} 0, & agar\ x\le 0, \\ 1, & agar\ 0<x\le 1, \\ 0, & agar\ x>0. \end{cases}$$

Bundan $b=1,\ a=0,\ \alpha=0,\ \beta=10\min=\frac{1}{6}c.$

U holda

$$P\left(0<X<\frac{1}{6}\right)=\frac{\beta-\alpha}{b-a}=\frac{\frac{1}{6}-0}{1-0}=\frac{1}{6}.$$

2. *Ko'rsatkichli taqsimot*

Taqsimot zichligi

$$f(x)=\begin{cases} ae^{-ax}, & agar\ x>0, \\ 0, & agar\ x<0 \end{cases}$$

ko'rinishda berilgan uzluksiz tasodifiy miqdorning taqsimotiga *ko'rsatkichli taqsimot* deyiladi, bu yerda $a>0$.

Ko'rsatkichli taqsimotning integral funksiyasi

$$F(x)=\begin{cases} 1-e^{-ax}, & agar\ x>0, \\ 0, & agar\ x<0 \end{cases}$$

formula bilan topiladi.

Ko'rsatkichli taqsimotga ega tasodifiy miqdorning $[\alpha;\beta)$ oraliqqa tegishli qiymat qabul qilishi ehtimoli

$$P(\alpha \le X < \beta)=e^{-\alpha a}-e^{-\beta a}$$

tenglik bilan aniqlanadi.

Ko'rsatkichli taqsimot qonuni ommaviy xizmatlar nazariyasida, fizikada va mustahkamlik nazariyasida muhim ahamiyatga ega.

Ko'rsatkichli taqsimotning sonli xarakteristikalari:

$$M(X)=\frac{1}{a}, \quad D(X)=\frac{1}{a^2}, \quad \sigma(X)=\frac{1}{a}$$

8-misol. X uzluksiz tasodifiy miqdorning taqsimot zichligi berilgan:

$$f(x)=\begin{cases} 0, & agar\ x<0, \\ 2e^{-2x}, & agar\ x\ge 0. \end{cases}$$

$M(X),\ D(X),\ \sigma(X)$ larni toping.

Taqsimot qonunining sonli xarakteristikalarinitopamiz:

$$M(X)=\frac{1}{a}=\frac{1}{2}=0{,}5,\ D(X)=\frac{1}{a^2}=\frac{1}{4}=0{,}25,\ \sigma(X)=\frac{1}{a}=0{,}5.$$

3. *Normal taqsimot*

Taqsimot zichligi

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-a)^2}{2\sigma^2}}$$

ko'rinishda berilgan uzluksiz tasodifiy miqdorning taqsimotiga *normal taqsimot* deyiladi, bu yerda $a\in R,\ \sigma>0$ parametrlar.

Normal taqsimotning integral funksiyasi

$$F(x)=\frac{1}{2}+\Phi\left(\frac{x-a}{\sigma}\right),$$

formula bilan topiladi, bu yerda $\Phi-$ Laplas funksiyasi.

Normal taqsimot ehtimoliy–statistika nazariyasi va amliyotida markaziy o‘rin egallaydi. Bu taqsimot yordamida bir nechta muhim taqsimotlar hosil qilingani uning yuqori darajada nazariy ahamiyatga ega ekanini ko‘rsatadi.

Normal taqsimotga ega tasodifiy miqdorning $[\alpha,\beta)$ oraliqqa tegishli qiymat qabul qilish ehtimoli

$$P(\alpha\le X<\beta)=\Phi\left(\frac{\beta-a}{\sigma}\right)-\Phi\left(\frac{\alpha-a}{\sigma}\right)$$

formula bilan, $[a-\delta,\ a+\delta)$ oraliqqa tegishli qiymat qabul qilish ehtimoli

$$P(|x-a|<\delta)=2\Phi\left(\frac{\delta}{\sigma}\right)$$

formula bilan aniqlanadi.

Normal taqsimotning sonli xarakteristikalari:

$$M(X)=a,\ D(X)=\sigma^2,\ \sigma(X)=\sigma$$

9-misol. Aksiyaning kundalik bahosi matematik kutilishi 15 ming so‘m va o‘rtacha kvadratik chetlashishi 0,2 ming so‘mga teng normal taqsimotga ega bo‘lsa, aksiyaning narxi: 1) 15,3 ming so‘mdan ortiq bo‘lmasligi; 2). 15,4 ming so‘mdan kam bo‘lmasligi; 3) 14,9 dan 15,3 ming so‘m oralig‘ida bo‘lishi ehtimollarini toping. Aksiya kundalik bahosining o‘zgarish chegarasini toping.

Masalaning shartiga ko‘ra tasodifiy miqdor normal taqsimlangan. Bunda $a=15,\ \sigma=0{,}2.$ Berilgan parametrlarda normal taqsimot uchun 2-ilovadan foydalanib, topamiz:

1) Aksiyaning narxi 15,3 ming so‘mdan ortiq bo‘lmasligi ehtimoli

$$P(0\le X\le 15{,}3)=\Phi\left(\frac{15{,}3-15}{0{,}2}\right)-\Phi\left(\frac{0-15}{0{,}2}\right)=$$

$$=\Phi(1{,}5)-\Phi(-7{,}5)=0{,}4332+0{,}5=0{,}9332.$$

2) Aksiyaning narxi ming so‘mdan kam bo‘lmasligi ehtimoli

$$P(X\ge 15{,}4)=1-P(0\le X\le 15{,}4)=1-\left(\Phi\left(\frac{15{,}4-15}{0{,}2}\right)-\Phi\left(\frac{0-15}{0{,}2}\right)\right)=$$

$$=1-\left(\Phi(2)-\Phi(-7{,}5)\right)=1-(0{,}4772+0{,}5)=0{,}0228.$$

3) Aksiyaning narxi 14,9 dan ming so‘m oralig‘ida bo‘lishi ehtimoli

$$P(14{,}9 \le X \le 15{,}3) = \Phi\left(\frac{15{,}3-15}{0{,}2}\right) - \Phi\left(\frac{14{,}9-15}{0{,}2}\right) =$$

$$= \Phi(1{,}5) - \Phi(-0{,}5) = 0{,}4332 + 0{,}1915 = 0{,}6247.$$

Uch sigma qoidasiga ko‘ra $|X-a| \le 3\sigma$ hodisa ehtimoli deyarli muqarrar hodisa bo‘ladi. Demak, aksiya kundalik bahosining o‘zgarish chegarasi

$$|X-15| \le 3 \cdot 0{,}2 \quad \text{yoki} \quad 14{,}4 \le X \le 15{,}6.$$

Mashqlar

1.6.1. Korxona tomonidan ishlab chiqarilgan mahsulotning 20%i qo‘shimcha sozlashni talab qiladi. Korxona mahsulotlaridan tavakkaliga 5 ta mahsulot tanlanadi. Tanlanmada sozlashni talab qiladigan mahsulotlar sonining taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

1.6.2. Statistik ma’lumotlarga ko‘ra qiz bola tug‘ilishi ehtimoli 0,51 ga teng. 4 ta farzandli oilada qiz bolalar sonining taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

1.6.3. Do‘konga ikkita firmadan 2:3 nisbatda soat keltirilgan va ulardan 20 tasi sotilgan. Birinchi firmada ishlab chiqarilgan soatlarning sotilganlari sonidan iborat tasodifiy miqdorning sonli xarakteristikalarini toping.

1.6.4. 20ta pul-buyum lotereyasi sotib olingan. Har bir biletga yutuq chiqishi ehtimoli 0,15 ga teng bo‘lsin. Sotib olingan biletlarga yutuq chiqishlari sonining sonli xarakteristikalarini toping.

1.6.5. Darslik 10000 nusxada chop etilgan. Har bir nusxaning noto‘g‘ri muqavalanishi ehtimoli 0,0001ga teng. Noto‘g‘ri muqavalangan darsliklar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishi va dispersiyasini toping.

1.6.6. Do‘konga jo‘natilgan 1000 ta idishdagi madanli suvning yo‘lda shikastlanishi ehtimoli 0,002 ga teng. Madanli suvning yo‘lda shikastlanishlari sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishi va dispersiyasini toping.

1.6.7. Bitta o‘q uzishda o‘qning nishonga tegishi ehtimoli 0,01 ga teng. 200 ta otilgan o‘qning nishonga tegishlari sonidan iborat tasodifiy miqdorning sonli xarakteristikalarini va kamida 5 ta va ko‘pi bilan 10 ta o‘qning nishonga tegishi ehtimolini toping.

1.6.8. Bankka keluvchi mijozlar soni Puasson taqsimotiga bo‘ysunadi va o‘rta hisobda bir daqiqada bankka 5 ta mijoz kiradi. 1) navbatdagi bir daqiqada bankka bitta mijoz kirishi; 2) navbatdagi bir daqiqada bankka kamida uchta mijoz kirishi ehtimolini toping.

1.6.9. Nishonning yakson etilishi ehtimoli 0,05 ga teng. Birinchi o‘q tekkunicha o‘q otilmoqda. Otilgan o‘qlar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini, dispersiyasini va nishoh yakson etilishi uchun 5 tadan kam bo‘lmagan otishlar talab etilishi ehtimolini toping.

1.6.10. Imtihonda talabaga qo‘shimcha savollar berilmoqda. Talabaning berilgan har qanday savolga javob berishi ehtimoli 0,85 ga teng. Talaba berilgan savolga javob bera olmagan zahoti imtihon to‘xtatiladi. Talabaga berilgan qo‘shimcha savollar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini, dispersiyasini toping.

1.6.11. Chapaqaylar aholining taxminan bir foizini tashkil qilsa, o’nta chapaqayni topish uchun nechta kishi ro‘yxatdan o’tkazilishi kerak?

1.6.12. Do‘konga keltirilgan q‘ol telefonlarining 75 foizi nuqsonga ega bo’lsa, xaridor nuqsonsiz telefon sotib olishi uchun o’rtacha nechta telefonni sinab ko‘rishi kerak?

1.6.13. Guruhda 15 talaba bo‘lib, ularning 9 nafari a’lochilar. Ro‘yxat bo‘yicha tavakkaliga 3 ta talaba tahlab olindi. A’lochi talabalar sonidan iborat tasodifiy miqdorning taqsimot qonunini, matematik kutilishini va dispersiyasini toping.

1.6.14. Sportlotto o‘yinda 45 tadan tavakkaliga tanlangan 6 ta sport turidan 3, 4, 5 , 6 ta sport turlarini topgan qatnashchilar pul yutuqlariga ega bo‘ladi, bunda topilgan sport turlari ortishi bilan yutuq miqdori ham ortib boradi. Tavakkaliga tanlangan 6 ta sport turidan topilgan sport turlari sonidan iborat X tasodifiy miqdorning taqsimot qonunini, matematik kutilishi va dispersiyasini toping. Pul yutuqlari olinishi ehtimolini aniqlang.

1.6.15. Metro poezdi har 3 minutda bekatga keladi. Yo‘lovchi bekatga istalgan vaqtda kelsa, uning poezdni yarim minutdan ko‘p kutmasligi ehtimolini toping. Yo‘lovchining poezdni kutish vaqtidan iborat tasodifiy miqdorning zichlik funksiyasini, matematik kutilishini va o‘rtacha kvadtatik chetlashishini aniqlang.

1.6.16. Chorrahaga yashil rangi har 2 minutda yonuvchi svetofor qo‘yilgan. Qizil rangga kelgan avtomobilning bu svetofor oldida turib qolishi $(0;2)$ oraliqda tekis taqsimlangan tasodifiy miqdor bo‘lsa, uning: 1) zichlik funksiyasini; 2) o‘rtacha kutish vaqtini; 3) o‘rtacha kvadratik og‘ishini toping.

1.6.17. Televizorning to‘xtovsiz ishlashi ehtimoli

$$f(x) = 0{,}02e^{-0{,}02x} \quad (x > 0)$$

qonunga ega bo‘lsa, quyidagilarni toping: 1) tasodifiy miqdorning matematik kutilishi va dispersiyasini; 2) televizorning 50 soat to‘xtovsiz ishlashi ehtimolini.

1.6.18. X tasodifiy miqdor

$$f(x) = 3e^{-3x} \ (x > 0)$$

differensial funksiya bilan berilgan. 1) $M(X)$, $D(X)$ ni hisoblang; 2) X ning $(0{,}13;0{,}7)$ oraliqqa tushishi ehtimolini toping.

1.6.19. Detalni o‘lchashda o‘rtacha kvadratik chetlashishi $\sigma = 10$ mm. bo‘lgan normal taqsimotga bo‘ysinuvchi tasodifiy xatolar uchraydi. Detalda uchta bog‘liqmas o‘lchash o‘tkazilgan bo‘lsa, hech b‘lmaganida bitta o‘lchashda xatolik modul bo‘yicha 2 mm. dan oshmasligi ehtimolini toping.

1.6.20. Valning diametri sistematik xatolarsiz o‘lchanadi. $X-$o‘lchamlarning nisbiy xatolari bo‘lsin. U o‘rtacha kvadratik chetlashishi $\sigma = 10$mm.ga teng normal taqsimotga bo‘ysunsa, o‘lchash absolut qiymati bo‘yicha 15 mm.dan ortiq bo‘lmaydigan xato bilan o‘tkazilishi ehtimolini toping.

1.6.21. Poyezd 100 ta vagondan tashkil topgan. Har bir vagonning massasi matematik kutilishi 65 tonna va o‘rtacha kvadratik chetlashishi $\sigma = 0{,}9$ tonnadan iborat. Lokomativ 6600 tonna massali sostavni tortishi mumkin, aks holda ikkinchi lokomativni qo‘shishga to‘g‘ri keladi. Ikkinchi lokomativ kerak bo‘lmasligi ehtimolini toping.

1.6.22. Avtomat sharchalar tayyorlaydi. $X-$sharcha diametri bo‘lsin. Bu diametrning loyihadagi o‘lchamdan chetlashishi absolut qiymat bo‘yicha 0,7 mm.dan kichik bo‘lsa, sharcha yaroqli hisoblanadi. X tasodifiy miqdor $\sigma = 0{,}4$ o‘rtacha kvadratik chetlashish bilan normal taqsimlangan bo‘lsa, tayyorlangan 100 ta sharchadan nechtasi yaroqli bo‘ladi?

1.7. EHTIMOLLAR NAZARIYASINING LIMIT TEOREMALARI

Chebishev teoremasi. Bernulli teoremasi. Markaziy limit teorema

1.7.1. Lemma (*Chebishev tengsizligi*). X **-**ixtiyoriy tasodifiy miqdor, $M(X)$, $D(X)-$mos ravishda uning matematik kutilishi va dispersiyasi, $\varepsilon-$istalgan musbat son bo‘lsin . U holda

$$P(|X-M(X)|<\varepsilon)\geq 1-\frac{D(X)}{\varepsilon^2}$$

tengsizlik bajariladi.

1-teorema (*Chebishev teoremasi*). $X_1, X_2..., X_n$ juft–jufti bilan bog‘liqmas bir xil taqsimotga ega tasodifiy miqdorlar ketma–ketligi, ya’ni $M(X_i)=a,\ D(X_i)=\sigma\ (i=\overline{1,n})$ bo‘lsin. U holda istalgan $\varepsilon>0$ son uchun

$$\lim_{n\to\infty}\left(\left|\frac{X_1+X_2+...+X_n}{n}-a\right|\leq\varepsilon\right)=1$$

bo‘ladi.

1-misol. Dengizning chuqurligi sistematik xatolarga ega bo‘lmagan uskuna bilan o‘lchanadi. O‘lchashlarning o‘rtacha kvadratik chetlashishi 15 m. dan oshmaydi. 0,9 dan kam bo‘lmagan ehtimol bilan o‘lchashlarning o‘rta arifmetigi (dengiz chuqurligi) a dan modul bo‘yicha 5 m. dan kam farq qiladi deb tasdiqlash uchun nechta bog‘liqmas o‘lchashlar o‘tkazilishi kerak?

Dengiz chuqurligining n ta bog‘liqmas o‘lchashlar natijalarini X_i bilan belgilaymiz. Masalaning shartiga ko‘ra: $\varepsilon=5,\ D(X)=\sigma^2=225.$

Chebishev tengsizligini qanoatlantiruvchi n ni topamiz:

$$P\left\{\left|\frac{1}{n}\sum_{i=1}^{n}X_i-a\right|<5\right\}\geq 1-\frac{225}{25n}\geq 0{,}9\,.$$

Bundan $0{,}1\geq\frac{9}{n}$ yoki $n\geq 90.$

Demak, 90 dan kam bo‘lmagan o‘lchashlar o‘tkazilishi kerak.

2-misol. Texnologik uskuna tayyorlanayotgan detal uzunligining o‘rtacha kvadratik chetlashishi bu ozunlikhning matematik kutilishishidan 0,05 sm. dan ko‘p bo‘lmasligini ta’minlaydi. 50 ta detal o‘lchangan. Bu o‘lchashlarning o‘rta arifmetigi haqiqiy matematik kutilishdan 0,02 dan ortiq bo‘lmagan chetlashishining ehtimolini toping.

Masalaning shartiga ko‘ra: $\varepsilon = 0{,}02,\ D(X) = 0{,}05^2,\ n = 50.$

U holda Chebishev tengsizligiga ko’ra

$$P\left\{\left|\frac{1}{50}\sum_{i=1}^{50} X_i - a\right| < 0{,}02\right\} \geq 1 - \frac{0{,}05^2}{50 \cdot 0{,}02^2} = 0{,}875.$$

1.7.2. 2- teorema (*Bernulli teoremasi*). $k-$ Bernulli sxemasidagi n ta sinashda A hodisaning ro‘y berishlari soni, $p - A$ hodisaning bitta sinashda ro‘y berishi ehtimoli bo‘lsin. U holda

$$\lim_{n\to\infty}\left(\left|\frac{k}{n} - p\right| \leq \varepsilon\right) = 1$$

bo‘ladi.

3-misol. Qo‘lyozmaning bitta betida xato bo‘lishi ehtimoli 0,2 ga teng. 400 betdan iborat qo‘lyozmada xato bo’lishining nisbiy chastotasi mos ehtimoldan modul bo‘yicha 0,05 dan kam farq qilishi ehtimolini toping.

Masalada berilishicha: $p = 0{,}2,\ q = 0{,}8,\ n = 400,\ \varepsilon = 0{,}05.$

Bernulli teoremasiga ko‘ra

$$P\left\{\left|\frac{1}{n} - 0{,}2\right| < 0{,}005\right\} \geq \left(1 - \frac{pq}{n\varepsilon^2}\right) = 1 - \frac{0{,}2 \cdot 0{,}8}{400 \cdot 0{,}05^2} = 0{,}84.$$

1.7.3. 3-teorema (*Markaziy limit teorema*). Agar $X_1, X_2, ..., X_n -$ bog‘liqmas tasodifiy miqdorlar bo‘lib, chekli $M(X_i) = a$ matematik kutilish va $D(X_i) = \sigma^2$ dispersiyaga ega bo‘lgan bir xil taqsimot qonuniga ega bo‘lsa, u holda n cheksiz ortganida

$$\frac{\sum_{i=1}^{n} X_i - na}{\sigma\sqrt{n}}$$

ning taqsimot qonuni matematik kutilishi 0 va dispersiyasi 1 bo‘lgan normal taqsimotga yaqinlashadi.

4-misol. Har biri $[0;4]$ kesmada tekis taqsimlangan 75 ta bog‘liqmas tasodifiy miqdorlar yig‘indisining zichligi uchun taqribiy ifodani toping va yig‘indi 120 dan kam 160 gacha bo‘lishi ehtimolini toping.

$X = \sum_{i=1}^{75} X_i$, bunda $X_i -$ $[0;4]$ kesmada tekis taqsimlangan tasodifiy miqdorlar bo‘lsin.

U holda $a_i = M(X_i) = \frac{4+0}{2} = 2,\ D(X_i) = \frac{(4-0)^2}{12} = \frac{4}{3}$ bo‘ladi.
Demak, markaziy limit teoremasining shartlari bajariladi.

Bundan

$$m_x = M\left(\sum_{i=1}^{75} X_i\right) = \sum_{i=1}^{75} M(X_i) = 75 \cdot 2 = 150, \quad \sigma_x^2 = D\left(\sum_{i=1}^{75} X_i\right) = 75 \cdot \frac{4}{3} = 100.$$

U holda

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}} = \frac{1}{10 \cdot \sqrt{2\pi}} e^{-\frac{(x-150)^2}{200}}.$$

Yig‘indi 120 dan 160 gacha bo‘lishi ehtimolini topamiz:

$$P(120 \le X \le 160) = \Phi\left(\frac{160-150}{10}\right) - \Phi\left(\frac{120-150}{10}\right) =$$
$$= \Phi(1) + \Phi(3) = 0,3413 + 0,4987 \approx 084.$$

Mashqlar

1.7.1. Chorvachilik fermasida suvning o‘rtacha sarfi kuniga 1000 *l*. Bu tasodifiy miqdorning o‘rtacha kvadratik chetlashishi 200 *l* dan oshmaydi. Ixtiyoriy tanlangan kunda fermadagi suv sarfi 2000*l* dan oshmasligi ehtimolini toping.

1.7.2. Har bir sinashda hodisaning ro‘y berishi ehtimoli 0,25 ga teng. Agar 800 ta sinash o‘tkazilishi kerak bo‘lsa, *A* hodisaning ro‘y berishi soni 150 dam 250 gacha bo‘lishi ehtimolini toping.

1.7.3. Avtomatdan standart detalning chiqishi ehtimoli 0,96 ga teng. Ghebeshev tengsizligidan foydalanib, 2000 detal orasida nostandart detallar soni 60 tadan 100 tagacha bo‘lishini baholang.

1.7.4. Depozitga qo‘yilgan aksiyalarning talab qilinishi ehtimoli 0,08 ga teng. 1000 ta mijozdan kamida 70 tasi va ko‘pi bilan 90 tasi aksiyalarini talab qilishi ehtimolini toping.

1.7.5. 900 ta sinashning har birida hodisaning ro‘y berishi ehtimoli 0,7 ga teng. Bernulli teoremasidan foydalanib, hodisaning ro‘y berishlari soni 600 tadan 660 tagacha bo‘lishi ehtimolini toping.

1.7.6. O‘g‘il va qiz bola tug‘ilishi ehtimollari bir xil bo’lsa, 1000 ta tug‘ilgan bola orasida qiz bolalar soni 465 bilan 535 orasida bo‘lishi ehtimolini Bernulli teoremasi orgali toping.

1.7.7. X tasodifiy miqdor $X:\begin{pmatrix} 0,3 & 0,6 \\ 0,2 & 0,8 \end{pmatrix}$ taqsimot qonuniga ega. $|X - M(X)| < 0,2$ bo‘lishi ehtimolini toping.

1.7.8. X tasodifiy miqdor $X:\begin{pmatrix} 3 & 5 \\ 0,6 & 0,4 \end{pmatrix}$ taqsimot qonuniga ega. $|X - M(X)| < 1,3$ bo‘lishi ehtimolini toping.

1.7.9. X_i bog‘liqmas miqdorlar [0;1] kesmada tekis taqsimlangan. $Y = \sum_{i=1}^{100} X_i$ tasodifiy miqdorning taqsimot qonunini va $P(55 < Y < 70)$ ni toping.

1.7.10. [0;0,25] kesmada tasodifiy ravishda 162 ta son olingan. Ularning yig‘indisi 22 bilan 26 orasida bo‘lishi ehtimolini toping.

1.8. TANLANMANING XARAKTERISTIKALARI

Tanlanmamaning statistik taqsimoti. Statistik taqsimotning grafik tasvirlari. Statistik taqsimotning sonli xarakteristikalari

1.8.1. Tekshirilayotgan biror (sifatiy yoki miqdoriy) alomat bo‘yicha kuzatilayotgan barcha obyektlar to‘plamiga *bosh to‘plam* deyiladi. Bosh to‘plamdagi obyektlar soni *bosh to‘plamning hajmi* deb ataladi.

Bosh to‘plamdan tasodifiy ravishda tanlab olingan obyektlar to‘plamiga *tanlanma* yoki *tanlanma to‘plam* deyiladi. Tanlanmadagi obyektlar soni *tanlanmaning hajmi* deb ataladi.

Tanlanmaning har bir elementi *varianta* deb ataladi. Tartiblangan tanlanmaga *variasiya qatori* deyiladi.

Tanlanmada x_1 varianta n_1 marta, $x_2 - n_2$ marta va $x_k - n_k$ marta kuzatilgan bo‘lsin. Bunda n_i kattalikka x_i variantaning *chastotasi*, $w_i = \frac{n_i}{n}$ kattalikka esa uning *nisbiy chastotasi* deyiladi, bu yerda $n = \sum_{i-1}^{k} n_i$ – tanlanmaning hajmi. Nisbiy chastotalar uchun $\sum_{i=1}^{n} w_i = 1$ bo‘ladi.

Variantalar va ularga mos chastotalardan yoki nisbiy chastotalardan tashkil topgan

x_i	x_1	x_2	$\cdots$	x_k
n_i	n_1	n_2	$\cdots$	n_k

yoki

x_i	x_1	x_2	$\cdots$	x_k
w_i	w_1	w_2	$\cdots$	w_k

jadvalga tanlanmaning *statistik taqsimoti* yoki *statistik qator* deyiladi.

Variantalarning x sondan kichik bo'lgan qiymatlari nisbiy chastotasi

$$F^*(x) = \frac{n_x}{n}$$

tanlanmaning *emperik* (*statistik*) *taqsimot funksiyasi* deb ataladi, bu yerda $n_x - x$ qiymatdan kichik bo'lgan variantlari soni.

Nazariy taqsimot funksiyani $F(x)$ tanlanmaning emperik taqsimot funksiyasi $F^*(x)$ bilan baholanadi.

1.8.2. *Chastotalar poligoni* deb $(x_1;n_1)$, $(x_2;n_2),...,(x_k;n_k)$ nuqtalarni tutashtiruvchi siniq chiziqqa aytiladi.

Nisbiy chastotalar poligoni deb $(x_1;w_1)$, $(x_2;w_2),...,(x_k;w_k)$ nuqtalarni tutashtiruvchi siniq chiziqqa aytiladi.

X uzluksiz belgi bo'lsa yoki kuzatishlar soni katta bo'lganida X belgining kuzatilayotgan qiymatlari tushadigan oraliq h uzunlikdagi intervallarga bo'linadi, har bir interval uchun shu intervallarga tushgan variantalarning chastotalari (nisbiy chastotalari) aniqlanadi va intervalli statistik qator tuziladi. Bunda statistik qatorning birinchi satrida $[x_0;x_1),[x_1;x_2),...,[x_{k-1};x_k)$ intervallar yoziladi. Bu intervallar bir xil $h = x_1 - x_0 = x_2 - x_1 = ... = x_k - x_{k-1}$ uzunlikda tanlanad. Intervallarning uzunligi $h = \dfrac{x_{\max} - x_{\min}}{1 + 3{,}322\lg n}$ tenglik bilan aniqlanadi, bu yerda $x_{\max} - x_{\min}$ –belgining eng katta va eng kichik qiymatlari orasidagi ayirma; $m = 1 + 3{,}322\lg n$ – intervallar soni. Bunda birinchi intervalning boshi sifatida $x_0 = x_{\min} - \dfrac{h}{2}$ olinadi. Statistik qatorning ikkinchi satriga har bir intervalga tushgan chastotalat (nisbiy chastotalar) qo'yiladi.

Chastotalar gistogrammasi (*nisbiy chastotalar gistogrammasi*) deb asoslari h uzunlikdagi intervallardan va balandliklari $\dfrac{n_i}{n}$ $\left(\dfrac{w_i}{n}\right)$ sonlardan iborat bo'lgan to'g'ri to'rtburchaklardan tuzilgan pog'anasimon figuraga aytiladi.

Nisbiy chastotalar gistogrammasini shunday silliq chiziq $f^*(x)$ bilan tutashtirish mumkinki, $f^*(x)$ chiziq bilan chegaralangan egri chiziqli trapesiya yuzasi gistogramma yuzasiga teng bo‘ladi.

$f^*(x)$ chiziqqa *nisbiy chastotalar taqsimotining emperik funksiyasi* deyiladi.

Nazariy taqsimot zichligi $f(x)$ nisbiy chastotalar taqsimotining emperik funksiyasi $f^*(x)$ bilan baholanadi.

1-misol. 10 ta talabadan iborat guruhda oliy matematikadan o‘tkazilgan oraliq nazoratda quyidagi ballar to‘plangan: 2,5,4,0,2,5,0,4,2,1. Tanlanmaning chastotalar va nisbiy chastotalar statistik taqsimotlarini toping, emperik taqsimot funksiyasini tuzing va grafigini chizing, nisbiy chastotalar poligonini chizing.

Tanlanmaning 0,0,1,2,2,2,4,4,5,5 variasiya qatori bo‘yicha variantalarni, chastotalarni va nisbiy chastotalarni topamiz:

$$x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 4,\ x_5 = 5;$$

$$n_1 = 2,\ n_2 = 1,\ n_3 = 3,\ n_4 = 2,\ n_5 = 2;$$

$$w_1 = 0{,}2,\ w_2 = 0{,}1,\ w_3 = 0{,}3,\ w_4 = 0{,}2,\ w_5 = 0{,}2,$$

bu yerda

$$\sum_{i=1}^{5} n_i = 2+1+3+2+2 = 10, \quad \sum_{i=1}^{5} w_i = 0{,}2+0{,}1+0{,}3+0{,}2+0{,}2 = 1.$$

Chastota va nisbiy chastotalarning statistik qatorlarini tuzamiz:

x_i	0	1	2	4	5
n_i	2	1	3	2	2

x_i	0	1	2	4	5
w_i	0,2	0,1	0,3	0,2	0,2

Bu qatorlar asosida emperik taqsimot funksiyasini tuzamiz:

1. $x_1 = 0$ eng kichik varianta. Demak, $x \le 0$ lar uchun $F^*(x) = 0$;

2. $x < 1$ tengsizlikni qanoatlantiruvchi variantalar uchun $x_1 = 0$ nisbiy chastota $w_1 = 0{,}2$ varianta bilan kuzatilgan. Demak, $0 < x \le 1$ lar uchun $F^*(x) = 0{,}2;$

3. $1 < x \le 2$ larda $F^*(x) = 0{,}2 + 0{,}1 = 0{,}3$;

4. $2 < x \le 4$ larda $F^*(x) = 0{,}3 + 0{,}3 = 0{,}6$;

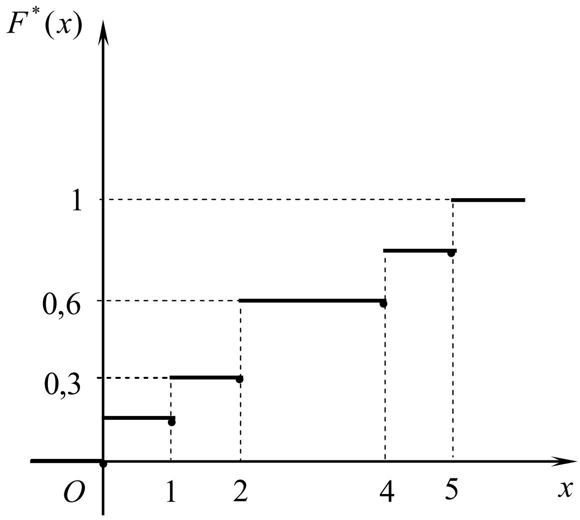
5. $4 < x \le 5$ larda $F^*(x) = 0{,}6 + 0{,}2 = 0{,}8;$

6. $x = 5$ eng katta varianta bo’lgani uchun $x > 5$ larda $F^*(x) = 1$.

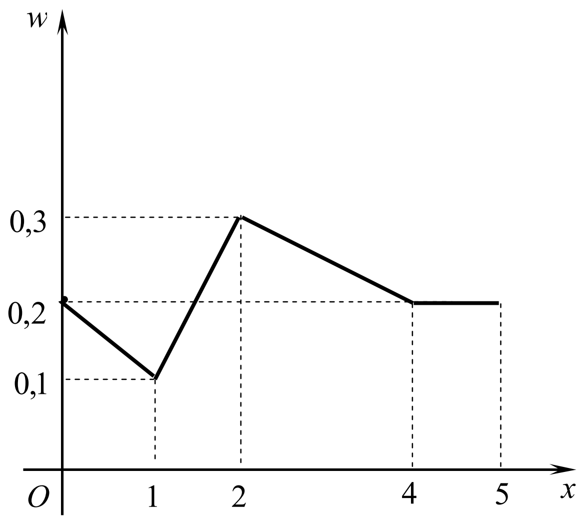
Demak, (4-shakl):

$$F^*(x) = \begin{cases} 0, & x \le 0, \\ 0{,}2, & 0 < x \le 1, \\ 0{,}3, & 1 < x \le 2, \\ 0{,}6, & 2 < x \le 4, \\ 0{,}8 & 4 < x \le 5, \\ 1, & x > 1. \end{cases}$$

Koordinatalar tekisligida koordinatalari $(x_i; w_i)$ bo‘lgan nuqtalarni belgilaymiz, ularni kesmalar bilan tutashtiramiz va nisbiy chastotalar poligonini hosil qilamiz (5-shakl).



4-shakl.



5-shakl.

2 – misol. Tavakkaliga tanlangan 20 ta talabaning bo‘yi (sm. aniqligida) o‘lchangan va quyidagi natijalar olingan:

171, 160, 163, 162, 156, 159, 176, 172, 164, 158,
162, 166, 162, 167, 171, 157, 167, 158, 169, 174.

Intervalli statistik qatorni tuzing va nisbiy chastotalar gistogrammasini chizing.

Olingan natijalarni o‘sish tartibida joylashtiramiz:

156, 157, 158, 158, 159, 160, 162, 162, 162, 163,
164, 166, 167, 167, 169, 171, 171, 172, 174, 176.

Bunda $x_{\min} = 156$, $x_{\max} = 176$ va Sterdjess formulasiga ko‘ra

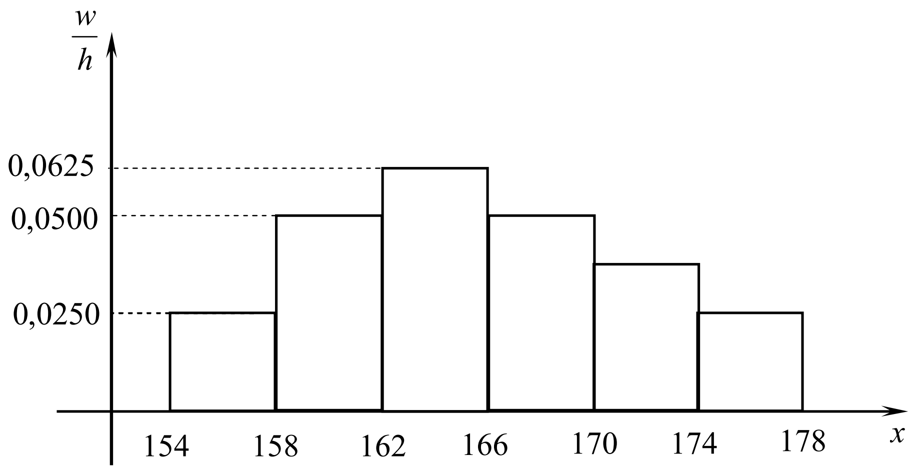
$$h = \frac{176 - 156}{1 + 3{,}322 \lg 20} \approx 4.$$

U holda $x_0 = 156 - \frac{4}{2} = 154.$ Berilganlarni $m = 1 + 3{,}322 \ln 20 \approx 6$ ta intervalga ajratamiz: $[154;158),[158;162),[162;166),[166;170),[170,174),[174;178).$

Har bir intervalga tushuvchi talabalar sonini (chastotalarni), nisbiy chastotalarni aniqlaymiz va intervalli statistik qatorni tuzamiz:

x_i	[154;158)	[158;162)	[162;166)	[166;170)	[170;174)	[174;178)
n_i	2	4	5	4	3	2
w_i	0,1	0,2	0,25	0,2	0,15	0,1
$\frac{w_i}{h}$	0,025	0,05	0,0625	0,05	0,0375	0,025

Bundan 6-shakldagi gistogrammani hosil qilamiz.



6-shakl.

1.8.3. Hajmi n ga teng tanlanmaning statistik taqsimoti berilgan bo'lsin:

x_i	x_1	x_2	...	x_k
n_i	n_1	n_2	...	n_k

Tanlanma o'rta qiymat (*variasiya qatorining o'rta arifmetigi*) $\overline{X}$ deb tanlanma barcha qiymatlarining o'rta arifmetigiga aytiladi, ya'ni

$$\overline{X} = \frac{1}{n}\sum_{l=1}^{k} X_i = \frac{1}{n}\sum_{l=1}^{k} x_i n_i .$$

Tanlanma dispersiya (*variasiya qatorining dispersiyasi*) S^2 deb tanlanma qiymatlarining tanlanma o'rta qiymatdan chetlashishi kvadratining o‘rta arifmetigiga aytiladi, ya’ni

$$\overline{D} = \frac{1}{n}\sum_{l=1}^{k}(X_i - \overline{X})^2 = \frac{1}{n}\sum_{l=1}^{k}(x_i - \overline{X})^2 n_i .$$

Tanlanma dispersiya uchun

$$\overline{D} = \frac{1}{n}\sum_{l=1}^{k} x_i^2 \cdot n_i - (\overline{X})^2 = \overline{X^2} - (\overline{X})^2$$

tenglik o‘rinli bo‘ladi.

Tanlanma o‘rtacha kvadratik chetlashish $\overline{\sigma} = \sqrt{\overline{D}}$ formula bilan topiladi.

X uzluksiz belgi uchun tanlanma sonli xarakteristikalar yuqorida keltirilgan formulalar kabi aniqlanadi. Bunda $x_1, x_2, ..., x_k$ qiymatlar sifatida $[x_0; x_1), [x_1; x_2), ..., [x_{k-1}; x_k)$ intervallarning $\frac{x_0 + x_1}{2}, \frac{x_1 + x_2}{2}, ..., \frac{x_{k-1} + x_k}{2}$ o‘rtalari olinadi.

Ayrim tanlanmalar uchun tanlama o‘rta qiymatni va dispersiyasni hisoblashni osonlashtiruvchi formulalardan foydalanish mumkin. Bunda berilgan x_i $(i = \overline{1,k})$ variantalar o‘rniga shartli $u_i = \frac{x_i - c}{h}$ variantalar olinadi. Bu yerda $c, h -$hisoblashni osonlashtiradigan qilib tanlanuvchi o‘zgarmaslar.

Bu holda avval shartli variantalarda tanlanma o‘rta qiymat va dispersiya

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{k} u_i n_i, \qquad \overline{D}_u = \frac{1}{n}\sum_{i=1}^{k} u_i^2 n_i - \left(\frac{1}{n}\sum_{i=1}^{k} u_i n_i\right)^2$$

tengliklar bilan topiladi.

Keyin $\overline{X} = \overline{U} \cdot h + c$, $\overline{D} = \overline{D}_u \cdot h^2$ tengliklar orqali berilgan variantalarga qaytiladi.

3-misol. Do‘konda kompyuterlarni 10 kunlik sotishdan quyidagi natijalar olingan: 1,2,4,0,2,5,0,4,2,5. Tanlanmaning sonli xarakteristikalarni toping.

⊜ Tanlanmaning 0,0,1,2,2,2,4,4,5,5 variasiya qatori bo‘yicha variantalar va chastotalarni topamiz:

$$x_1 = 0,\ x_2 = 1,\ x_3 = 2,\ x_4 = 4,\ x_5 = 5;$$

$$n_1 = 2,\ n_2 = 1,\ n_3 = 3,\ n_4 = 2,\ n_5 = 2;$$

Sonli xarakteristikalarn hisoblaymiz:

$$\overline{X} = \frac{1}{10}(0 \cdot 2 + 1 \cdot 1 + 2 \cdot 3 + 4 \cdot 2 + 5 \cdot 2) = 2{,}5;$$

$$\overline{D} = \frac{1}{10}((0 - 2{,}5)^2 \cdot 2 + (1 - 2{,}5)^2 \cdot 1 + (2 - 2{,}5)^2 \cdot 3 +$$

$$+ (4 - 2{,}5)^2 \cdot 2 + (5 - 2{,}5)^2 \cdot 2) = 3{,}25; \qquad \overline{\sigma} = \sqrt{3{,}25} \approx 1{,}8.$$ ⊝

4-misol. Hajmi 10 ga teng tanlanmaning statistik taqsimoti berilgan:

x_i	178	184	186	188
n_i	2	4	3	1

Tanlanma o'rta qiymat va tanlanma dispersiyani toping.

$u_i = x_i - 183$ shartli variantaga o'tamiz va

u_i	-5	1	3	5
n_i	2	4	3	1

tanlanmani hosil qilamiz. U holda

$$\overline{U} = \frac{1}{10}(-5\cdot 2 + 1\cdot 4 + 3\cdot 3 + 5\cdot 1) = 0{,}8;$$

$$\overline{D}_u = \frac{1}{10}(25\cdot 2 + 1\cdot 4 + 9\cdot 3 + 25\cdot 1) - (0{,}8)^2 = 10{,}6 - 0{,}64 = 9{,}96.$$

Bundan

$$\overline{X} = 0{,}8\cdot 1 + 183 = 183{,}8. \quad \overline{D} = 9{,}96\cdot 1 = 9{,}96.$$

Mashqlar

1.8.1. Do'konda sovutgichlarni va televizorlarni o'n kunlik sotishdan quyidagi natijalar olingan:

1) 2,3,2,1,0,2,4,3,0,1 (sovutgich); 2) 1,1,2,2,,3,5,2,5,1,0 (televizor).

Tanlanmalarning : 1) nisbiy chastotalar statistik taqsimotini toping; 2) emperik taqsimot funksiyasini tuzing va grafigini chizing; 3) nisbiy chastotalar poligonini chizing.

1.8.2. Yuk tashish bilan shug'ullanadigan korxonanik haftalik tashilgan yuklar hajmi (tonnada) kuzatilganda quyidagi natijalar olingan:

1) 1-haftada:

157,160,170,183,159,153,182,186,171,155,178,179,175,165,154, 156,166,179,155,158,173,171,167,175,167,173,163,164,169,172;

2) 2-haftada:

183,166,179,155,169,172,178,157,179,163,171,164,160,175,165, 155,159,158,154,170,165,186,167,173,153,182,171,173,167,175.

Har bir hafta uchun tanlanmaning intervalli statistik qatorini tuzing va nisbiy chastotalar gistogrammasini chizing.

1.8.3.10 ta abiturientdan iborat guruhda matematikadan test nazorati o‘tkazilgan. Bunda har bir abiturient 5 ballgacha to‘plashi mumkin bo‘ladi. Nazoratda quyidagi natijalar olingan:

1)1-guruh uchun: 4,4,5,3,3,1,5,5,2,5;   2) 2-guruh uchun: 3,4,5,0,1,2,3,4,5,4.
Har bir guruh uchun tanlanmaning sonli xarakteristikalarini toping.

1.8.4. Ulgurji savdoni tashkil qilishda erkaklar poyafzalining o‘rtacha o‘lchamini bilish maqsadida tajriba o‘tkazilgan. Bunda do‘kondan ma’lum vaqtda xaridorlar tomonidan sotib olingan erkaklar poyafzalining o‘lchami kuzatilgan va natijada quyidagi tanlanma olingan:

39,43,42,40,44,39,42,41,41,40,42,41,42,45, 43,44,40,43,41,42,
41,43,38,41,42,40,43,40,44,41,43,41,39,45,43,46,42,43,42,40,
43,42,41,43,39,44,40,43,41,42,41,43,42,45,44,42,41,42,40,44.

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

1.8.5. Elektr zanjirdagi kuchlanish tasodifiy xarakterga ega bo‘lgan kuchlanishning ulanishiga bog‘liq. Zanjirdagi kuchlanishning tebranishini o’rganish uchun ma’lum vaqt oralig‘ida voltning o‘ndar bir bo‘lagi aniqligida 30 ta o‘lchash o‘tkazilgan va quyidagi variasiya qatori olingan:

215,0, 215,5, 215,9, 216,4, 216,8, 217,3, 217,5, 218,1, 218,6, 218,9,
219,2, 219,4, 219,7, 219,8, 220,0, 220,2, 220,3, 220,5,220,7, 220,9,
221,3, 221,6, 221,9, 222,3, 222,6, 222,9, 223,4, 224,0, 224,5, 225,0.

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

1.8.6. Hajmi 20 ga teng tanlanmaning statistik taqsimoti berilgan:

x_i	2560	2600	2620	2650	2700
n_i	2	3	10	4	1

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

1.8.7. Hajmi 100 ga teng tanlanmaning statistik taqsimoti berilgan:

x_i	156	160	164	168	172	176	180
n_i	10	14	26	28	12	8	2

Tanlanma o‘rta qiymat va tanlanma dispersiyani toping.

1.9. TAQSIMOT NOMA'LUM PARAMETRLARINING STATISTIK BAHOLARI

Parametrlarni baholash. Nuqtaviy baholar. Intervalli baholar

1.9.1. Bosh to'plam X belgisining θ parametrni o'z ichiga olgan $\varphi(x,\theta)$ taqsimot funksiyasi yoki taqsimot zichligi berilgan bo'lsin. Bunda θ parametr, masalan, Puasson taqsimotining λ parametri bo'lishi mumkin.

Nazariy taqsimot θ parametrining $\tilde{\theta}_n$ *statistik bahosi* (*bahosi*) deb berilgan tanlashga bog'liq bo'lgan uning taqribiy qiymatiga aytiladi.

Agar $M(\tilde{\theta})=\theta$ bo'lsa $\tilde{\theta}$ bahoga θ parametr uchun *siljimagan baho* deyiladi. Bunda $M(\tilde{\theta})=\theta$ shart $\tilde{\theta}$ baho sistematik xatolikka ega bo'lmasdan, faqat tasodifiy xatoliklarga ega bo'lishini bildiradi.

Agar $\lim\limits_{n\to\infty} M(\tilde{\theta})=\theta$ bo'lsa, $\tilde{\theta}$ bahoga θ parametr uchun *asimptotik siljimagan baho* deyiladi.

Agar $M(\tilde{\theta})\neq\theta$ bo'lsa, $\tilde{\theta}$ bahoga θ parametr uchun *siljigan baho* deyiladi.

Agar $\lim\limits_{n\to\infty} P(|\tilde{\theta}-\theta|<\varepsilon)=1$ bo'lsa $\tilde{\theta}$ bahoga θ parametr uchun *asosli baho* deyiladi. Asosli baho uchun tanlanma hajmi oshgan sayin yetarlicha katta ehtimollik bilan $\tilde{\theta}\approx\theta$ bo'ladi.

1-teorema. $\tilde{\theta}$ siljimagan baho uchun $\lim\limits_{n\to\infty} D(\tilde{\theta}_n)=0$ bo'lsa, u holda $\tilde{\theta}$ asosli baho bo'ladi.

θ parametrning $\tilde{\theta}$ siljimagan bahosi θ parametrning barcha mumkin bo'lgan baholari orasida eng kichik dispersiyaga ega bo'lsa $\tilde{\theta}$ bahoga θ parametr uchun *samarali baho* deyiladi. Samarali $\tilde{\theta}$ bahoning qiymatlari θ parametrga boshqa baholarga nisbatan yaqinroq joylashgan deyish mumkin. Bahoning samaraligi $e\tilde{\theta}=\dfrac{D(\tilde{\theta}_n^e)}{D(\tilde{\theta}_n)}$ kattalik bilan baholanadi, bu yerda $\tilde{\theta}_n^e$ – sama rali baho. $e\tilde{\theta}$ birga qancha yaqin bo'lsa, baho shuncha samarali bo'ladi.

Agar $\lim\limits_{n\to\infty} e\tilde{\theta}=1$ bo'lsa, $\tilde{\theta}$ bahoga θ parametr uchun *asimptotik samarali baho* deyiladi.

Bir vaqtda siljimaganlik, asoslilik va samaralilik xossalariga ega bo'lgan bahoga *bir qiymatli baho* deyiladi.

1.9.2. Bosh to‘plam noma’lum parametrining taqribiy qiymati sifatida olinadigan statistikaga uning *nuqtaviy bahosi* deyiladi.

2- teorema. $X_1, X_2, \ldots, X_n$ bosh to‘plamdan olingan tanlanma va $M(X_i) = M(X) = a$, $D(X_i) = D(X)$ $(i = \overline{1,n})$ bo‘lsin. U holda $\overline{X} = \frac{1}{n}\sum_{i=1}^{k} X_i$ tanlanma o‘rta qiymat $M(X)$ matematik kutilish uchun siljimagan va asosli baho bo‘ladi.

3- teorema. $X_1, X_2, \ldots, X_n$ bosh to‘plamdan olingan tanlanma va $M(X_i) = M(X) = a$, $D(X_i) = D(X)$ $(i = \overline{1,n})$ bo‘lsin. U holda $S^2 = \frac{n}{n-1} \cdot \overline{D}$ tuzatilgan tanlanma dispersiya $D(X)$ dispersiya uchun siljimagan va asosli baho bo‘ladi.

4- teorema. n ta bo‘g‘liqmas sinashlarda A hodisa ro‘y berishining $\frac{m}{n}$ nisbiy chastotasi har bir sinashda A hodisa ro‘y berishi ehtimoli $p = P(A)$ uchun siljimagan, asosli va samarali baho bo’ladi.

5- teorema. Tanlanmaning taqsimot funksiyasi $F^*(x)$ tasodifiy miqdorning taqsimot funksiyasi $F(x)$ uchun siljimagan va asosli baho bo‘ladi.

Bosh to‘plam a matematik kutilishga va σ^2 dispersiyaga ega bo‘lsa, $\overline{X}$ tanlanma o‘rta qiymat parametrlari a va $\frac{\sigma^2}{n}$ bo‘lgan normal taqsimotga ega bo‘ladi. Bunda

$$P(\alpha < \overline{X} < \beta) = \Phi\left(\sqrt{n}\,\frac{\beta - a}{\sigma}\right) - \Phi\left(\sqrt{n}\,\frac{\alpha - a}{\sigma}\right), \quad P\left(\left|\overline{X} - a\right| < \varepsilon\right) = 2\Phi\left(\frac{\varepsilon\sqrt{n}}{\sigma}\right)$$

bo‘ladi.

1-misol. Hajmi 50 ga teng bo’lgan tanlanmaning statistik taqsimoti berilgan:

x_i	3	5	8	11
n_i	14	10	12	14

Bosh o‘rta qiymatning siljimagan bahosini toping.

Bosh o‘rta qiymatning siljimagan bahosi tanlanma o‘rta qiymat bo‘ladi. Uni topamiz:

$$\overline{X} = \frac{1}{50}(3 \cdot 14 + 5 \cdot 10 + 8 \cdot 12 + 11 \cdot 14) = 6{,}84.$$

2-misol. Hajmi 51 ga teng bo‘lgan tanlanma bo‘yicha dispersiyaning siljigan bahosi topilgan: $\overline{D}=7$. Bosh to‘plam dispersiyasining siljimagan bahosini toping.

⊖ Bosh to‘plam dispersiyasining siljimagan bahosi tuzatilgan dispersiya bo‘ladi:

$$S^2=\frac{n}{n-1}\overline{D}=\frac{51}{50}\cdot 7=7{,}14.$$

3-misol. Tanga n marta tashlanadi. Har bir tashlashda gerb tomon tushishi ehtimoli p ga teng. Sinash oxirida tanga gerb tomoni bilan m marta tushdi. $\tilde{\theta}=\frac{m}{n}$ baho $\theta=p$ parametr uchun siljimagan baho bo‘lishini ko‘rsating.

⊖ Sinash natijalari soni m Bernulli taqsimotiga bo‘ysunadi. Shu sababli $M(m)=np$ bo‘ladi. Bundan

$$M(\tilde{\theta})=M\left(\frac{m}{n}\right)=\frac{1}{n}M(m)=\frac{1}{n}\cdot np=p=\theta,$$

ya’ni $M(\tilde{\theta})=\theta.$

Demak, $\tilde{\theta}=\frac{m}{n}$ baho $\theta=p$ parametr uchun siljimagan baho bo‘ladi.

4-misol. O‘rik sharbati 200 *ml*. hajmli idishlarga quyiladi. Quyuvchi avtomat shunday sozlanganki, uning to‘ldirish xatoligi $\sigma \pm 10\,ml.$ ga teng. Iqishlar karton qutilarga 25 donadan qadoqlanadi. Xaridor qadoqlangan qutining o‘rtacha og‘irligi ko‘rsatilgan miqdordan kam bo‘lmasligini talab qiladi. Xaridor ishlab chiqarilgan mahsulotni qabul qilishi uchun ishlab chiqaruvchi avtomatni 205 *ml*. quyadigan qilib sozlab qo‘ydi. Tasodifan tanlangan qadoqlangan qutining og‘irlik tekshiruvidan o‘tmasligi ehtimolini toping.

⊖ Idishning o‘rtacha to‘ldirilishi 205 *ml*., o‘rtacha kvadratik og‘ishi 10 *ml*. Tasodifiy tanlanma sharbat bilan to‘ldirilgan 25 ta idishlardan iborat. $n=25$ hajmli mumkin bo‘lgan barcha tanlanmalar uchun o‘rtacha og‘irlikning taqsimoti normal taqsimotga bo‘ysinadi. Bunda idishning o‘rtacha to‘ldirilishi 205 *ml*.ga, o‘rtacha kvadratik og‘ishi $\frac{\sigma}{\sqrt{n}}=\frac{10}{\sqrt{25}}=2$ *ml*.ga teng bo‘ladi.

Qdoqlangan qutidagi idishlarning o‘rtacha to‘ldirilishi 200 *ml*. dan kam bo‘lsa, quti sifat nazoratidan o‘tmaydi (xaridor talabiga javob bermaydi).

Demak, izlanayotgan ehtimol

$$P(\overline{X} < 200) = P(0 < \overline{X} < 200) = \Phi\left(\frac{200-205}{2}\right) - \Phi\left(\frac{-205}{2}\right) =$$
$$= \Phi(-2{,}5) - \Phi(-102{,}5) = -0{,}4938 - (-0{,}5) = 0{,}0062.$$

Nuqtaviy baholarni aniqlashning *momentlar usuli*da taqsimot no'malum parametrining baholarini aniqlash uchun taqsimotning nazariy momentlari tanlanma asosida topilgan mos momentlarga tenglashtiriladi. Bunda:

- aqsimot bitta θ parametrga bog'liq bo‘lsa, uning bahosini topish uchun $M(X) = \overline{X}$ tenglama yechiladi;
- taqsimot ikkita θ_1 va θ_2 parametrga bog'liq bo‘lsa, ularning bahosi $\begin{cases} M(X) = \overline{X}, \\ D(X) = \overline{D} \end{cases}$ sistemadan topiladi va hokazo.

Bahoni aniqlashning *maksimal ishonchlilik* usuli $x_1, x_2, \ldots, x_n$ tanlanma asosida tuzilgan ishonchlilik funksiyasi deb ataluvchi

$$L(x_1, x_2, ..., x_n, \theta) = f(x_1, \theta) \cdot f(x_2, \theta) \cdot ... \cdot f(x_n, \theta) \quad \text{yoki} \quad L(x, \theta) = \prod_{i=1}^{n} f(x_i, \theta)$$

funksiyaga asoslanadi, bu yerda $f(x, \theta) - X$ uzluksiz tasodifiy miqdorning taqsimot zichligi. X diskret tasodifiy miqdor uchun $L(x, \theta) = \prod_{i=1}^{n} P(x_i, \theta)$ bo‘ladi, bu yerda $P(x_i, \theta) = P(X = x_i, \theta)$.

Maksimal ishonchlilik usulida θ parametrning nuqtaviy bahosi sifatida $L(x, \theta)$ ishonchlilik funksiyasi maksimumga erishadigan $\tilde{\theta}$ qiymat olinadi.

$L(x, \theta)$ va $\ln L(x, \theta)$ funksiyalarning bir xil qiymatlarda maksimumga erishishini hisobga olib, maksimal ishonchlilik usulida θ parametrning bahosini aniqlash uchun $\frac{d(\ln L(x, \theta))}{d\theta} = 0$ tenglama yechimlari topiladi. Keyin bu yechimlar orasidan maksimum bo‘lgan $\tilde{\theta}$ ajratiladi.

Bahoni aniqlashning *eng kichik kvadratlar usuli*da baho tanlanma qiymatlarining aniqlanayotgan bahodan chetlashishi kvadratilarining yig‘indisini minimallashtirish orqali topiladi, ya'ni $\tilde{\theta}$ baho $F(\theta) = \sum_{i=1}^{n} (X_i - \theta)^2$ funksiyani minimumlashtiruvchi qiymat bo‘ladi.

5-misol. Normal taqsimlangan X tasodifiy miqdor a va σ^2 parametrlarining bahosini momentlar usuli bilan toping.

$x_1, x_2, \ldots, x_n$ tanlanma asosida $a = M(X) = \theta_1$ va $\sigma^2 = D(X) = \theta_2$

noma'lum parametrlarning nuqtaviy baholarini topamiz. Bunda momentlar usuliga ko'ra:

$$\begin{cases} M(X) = \overline{X}, \\ D(X) = \overline{D} \end{cases} \text{ yoki } \begin{cases} a = \overline{X}, \\ \sigma^2 = \overline{D} \end{cases}$$

Demak, normal taqsimot parametrlarining baholari: $\widetilde{\theta}_1 = \overline{X}$ va $\widetilde{\theta}_2 = \overline{D}$. ◉

6-misol. Ko'rsatkichli taqsimotga ega X tasodifiy miqdor a parametrining bahosini maksimal ishonchlilik usuli bilan toping.

◉ Ko'rsatkichli taqsimotga ega X tasodifiy miqdor

$$f(x,a) = ae^{-ax}, \; x > 0$$

zichlik funksiyaga ega bo'ladi.

Uning ishonchlilik funksiyasini tuzamiz:

$$L(x,\theta) = \prod_{i=1}^{n} \theta \cdot e^{-\theta \cdot X_i} = \theta^n e^{-\theta \cdot \sum_{i=1}^{n} X_i}.$$

Bu funksiyani logarifmlaymiz:

$$\ln(L(x,\theta)) = \ln \theta^n e^{-\theta \cdot \sum_{i=1}^{n} X_i} = n \ln \theta - \theta \sum_{i=1}^{n} X_i.$$

Ishonchlilik tenglamasini tuzamiz:

$$\frac{d \ln(L(x,\theta))}{d\theta} = \left. \left(\frac{n}{\theta} - \sum_{i=1}^{n} X_i \right) \right|_{\theta = \widetilde{\theta}} = 0.$$

Bundan

$$\widetilde{\theta} = \frac{n}{\sum_{i=1}^{n} X_i} = \frac{n}{\overline{X}}.$$

$$\frac{d^2 \ln(L(x,\theta))}{d\theta^2} = \left. \left(-\frac{n}{\theta^2} \right) \right|_{\theta = \widetilde{\theta}} = -\frac{n}{\widetilde{\theta}^2} < 0$$ bo'lgani uchun $\widetilde{\theta} = \frac{n}{\overline{X}}$ baho $\theta = a$ parametrning bahosi bo'ladi. ◉

7-misol. Puasson taqsimotiga ega X tasodifiy miqdor λ parametrining bahosini eng kichik kvadratlar usuli bilan toping.

◉ $F(\theta) = \sum_{i=1}^{n} (X_i - \theta)^2$ funksiyani minimumga tekshiramiz:

$$F'(\theta) = \left. \left(\sum_{i=1}^{n} (X_i - \theta)^2 \right)' \right|_{\theta = \widetilde{\theta}} = \sum_{i=1}^{n} 2(X_i - \widetilde{\theta}) \cdot (-1) = 0.$$

Bundan

$$\sum_{i=1}^{n} X_i - n\widetilde{\theta} = 0 \quad \text{yoki} \quad \widetilde{\theta} = \frac{1}{n} \sum_{i=1}^{n} X_i = \overline{X}.$$

$$F''(\theta) = \left(-2\sum_{i=1}^{n}\theta\right)'\Bigg|_{\theta=\tilde{\theta}} = -2\sum_{i=1}^{n}(-1) = 2n > 0$$ bo‘lgani uchun $\tilde{\theta} = \overline{X}$ baho $\theta = \lambda$ parametrning bahosi bo‘ladi.

1.9.3. $\theta -$no’malum parametr, $\theta_1, \theta_2 - x_1, x_2, \ldots, x_n$ tanlanma elementlarining ikkita funksiyasi bo‘lsin, bu yerda $\theta_1 < \theta_2$.

Agar $P(\theta_1 < \theta < \theta_2) = \gamma$ tenglik bajarilsa $(\theta_1; \theta_2)$ intervalga θ parametrning *ishonchli intervali* deyiladi. Bunda θ_1 va θ_2 qiymatlarga ishonchli intervalning quyi va yuqori chegarasi, γ ga *ishonchli ehtimol* deyiladi.

Agar ishonchli interval $\tilde{\theta}$ nuqtaviy bahoga nisbatan simmetrik tanlansa, ya’ni bu interval $(\tilde{\theta} - \varepsilon; \tilde{\theta} + \varepsilon)$ bo‘lsa, $P(|\tilde{\theta} - \theta| < \varepsilon) = \gamma$ bo‘ladi. Bunda $\varepsilon > 0$ son bahoning aniqlik ko‘rsatkichi bo‘ladi.

Ishonchli interval uchun γ ishonchli ehtimol oldindan beriladi. Uning qiymati yechilayotgan masalaning mohiyatiga bog‘liq bo‘ladi. Odatda γ sifatida birga yaqin bo‘lgan qiymatlar, masalan, 0,95, 0,99, 0,999 olinadi.

Ishonchli iIntervalning chegaralarini ifodalovchi θ_1 va θ_2 sonlar bilan aniqlanadigan baho *intervalli baho* deyiladi.

Matematik kutilish uchun intervalli baholar

X belgili a va σ^2 parametrli normal taqsimlangan tanlanma berilgan bo‘lsin. Bunda $\gamma -$berilgan.

*σ parametr ma’lum bo‘lgan hol*da $\left(\overline{X} - \frac{t\sigma}{\sqrt{n}}; \overline{X} + \frac{t\sigma}{\sqrt{n}}\right)$ interval $a = M(X)$ no’malum parametr uchun γ ishonchlilik bilan ishonchli interval bo‘ladi. Bunda t kattalik $\Phi(t) = \frac{\gamma}{2}$ tenglikdan 2- ilova asosida topiladi; $\varepsilon = \frac{t\sigma}{\sqrt{n}}$ kattalik bahoning aniqligini belgilaydi.

*σ parametr noma’lum bo‘lgan hol*da $\left(\overline{X} - \frac{t_\gamma S}{\sqrt{n}}; X + \frac{t_\gamma S}{\sqrt{n}}\right)$ interval $a = M(X)$ no’malum parametr uchun γ ishonchlilik bilan ishonchli interval bo‘ladi. Bunda t_γ kattalik Styudent kreteriyasi qiymatlari jadvali (4-ilova) asosida topiladi; $\varepsilon = \frac{t_\gamma S}{\sqrt{n}}$ kattalik bahoning aniqligini belgilaydi.

8-misol. X belgili tanlanma $\sigma = 20$ parametr bilan normal taqsimlangan. X tasodifiy miqdor ustida 5 ta kuzatish o’tkazilgan va $x_1 = -25$, $x_2 = 34$, $x_3 = -20$, $x_4 = 10$, $x_5 = 21$ natijaga erishilgan. $\gamma = 0,95$ ishonchlilik bilan a parametr uchun ishonchli intervalni toping.

$\overline{X}$ ni topamiz:

$$\overline{X} = \frac{1}{5}(-25 + 34 - 20 + 10 + 21) = 4.$$

$\Phi(t) = \dfrac{\gamma}{2} = \dfrac{0,95}{2} = 0,475$ uchun 2-ilovadagi jadvaldan topamiz: $t = 1,96$.

U holda $\varepsilon = \dfrac{t\sigma}{\sqrt{n}} = \dfrac{1,96 \cdot 20}{\sqrt{5}} = 17,5.$

a parametr uchun ishonchli intervalni aniqlaymiz:

$$(4 - 17,5; 4 + 17,5) \quad \text{yoki} \quad (-13,5; 21,5).$$

9-misol. Jamg‘arma bozori ayrim aksiyalarining daromadliligi o‘rganilmoqda. 15 kunda tasodifiy tanlanma o’rtacha kvadratik og’ishi $S = 3,5\%$, o’rtacha (yillik) daromadlilik $\overline{X} = 10,37\%$ ga teng ekanini kuzatildi. Aksiyalarning daromadliligi normal taqsimot qonuniga bo‘ysinadi. O’rganilayotgan aksiyalar uchun 95% li ishonchli intervalni toping.

Bosh to‘plam o‘rtacha kvadratik chetlashishi σ noma’lum.

Shu sababli $n = 15$, $\gamma = 0,95$ uchun 4-ilovadagi jadvaldan topamiz:

$t_\gamma = t(n;\gamma) = t(15;0,95) = 2,15.$

U holda

$$\overline{X} \pm \frac{t_\gamma S}{\sqrt{N}} = 10,37 \pm \frac{2,15 \cdot 3,5}{\sqrt{15}} = 10,37 \pm 1,94.$$

Bundan $(8,43;12,31)$.

Demak, o‘rganilayotgan aksiyalarning haqiqiy daromadliligi $0,95$ ishonchlilik bilan $(8,43;12,31)$ intervalda yotadi.

O‘rtacha kvadratik chetlashish uchun intervalli baholar

X belgili a va σ^2 parametrli normal taqsimlangan tanlanma berilgan bo‘lsin. Bunda $a, \sigma -$ no’malum, $\gamma -$ berilgan.

σ no’malum parametrning γ ehtimolli ishonchli intervalini topish uchun avval tanlanmaning qiymatlari bo’yicha erkinlik darajasi $n-1$ bo‘lgan χ^2 taqsimotli $\chi^2 = \dfrac{(n-1)S^2}{\sigma^2}$ asodifiy miqdorning $(X_1; X_2)$ intervalga tushishi ehtimoli $P\left(S\sqrt{\frac{n-1}{X_1}}<\sigma<S\sqrt{\frac{n-1}{X_2}}\right)=\gamma$ va $\sqrt{\frac{n-1}{X_1}}$, $\sqrt{\frac{n-1}{X_2}}$ qiymatlarning jadvallari (5-ilova) asosida $q=q(\gamma,n)$ topiladi.

Keyin σ no'malum parametr uchun γ ishonchlilik bilan ishonchli interval

$$S(1-q)<\sigma<S(1+q), \text{ agar } q<1 \text{ bo'lsa,}$$
$$0<\sigma<S(1+q), \text{ agar } q\geq 1 \text{ bo'lsa}$$

tengsizliklardan aniqlanadi.

10-misol. Biror kattalik bitta asbob yordamida sistematik xatolarsiz 10 marta oʻlchangan boʻlib, bunda oʻlchashlardagi tasodifiy xatolarning oʻrta kvadratik chetlashishi $0{,}6$ ga teng chiqqan. Asbob aniqligini $0{,}95$ ishonchlilik bilan toping.

5- ilovadagi jadvaldan va $\gamma=0{,}95$ ga mos q ni topamiz:

$$q=q(10;0{,}95)=0{,}65.$$

U holda

$$0{,}6(1-0{,}65)<\sigma<0{,}6(1+0{,}65)$$

yoki

$$0{,}21<\sigma<0{,}99.$$

Mashqlar

1.9.1. Hajmi 70 ga teng boʻlgan tanlanmaning statistik taqsimoti berilgan:

1)

x_i	1	4	9	20
n_i	16	20	22	12

;

2)

x_i	-2	6	10	18
n_i	22	15	23	10

Bosh oʻrtacha qiymatning siljimagan bahosini toping.

1.9.2. Hajmi 60 ga teng boʻlgan tanlanma boʻyicha dispersiyaning siljigan bahosi topilgan: 1) $\overline{D}=8{,}26$; 2) $\overline{D}=7{,}67$.
Bosh toʻplam dispersiyasining siljimagan bahosini toping.

1.9.3. Bosh toʻplamdan olingan oʻrta qiymat λ parametrli Puasson taqsimoti uchun siljimagan baho boʻlishini koʻrsating.

1.9.4. Bosh to‘plamdan olingan o‘rta qiymat λ parametrli Puasson taqsimoti uchun asosli baho bo‘lishini ko‘rsating.

1.9.5. Bosh to‘plamning o‘rtachasi $X = 1{,}03$ ga va o‘rtacha kvadratik chetlashishi $\sigma = 400$ ga teng. Bosh to‘plamdan hajmi 100 ga teng bo‘lgan tanlanma olingan. Tanlanmaning o‘rtachasi $\overline{X}$ uchun kutilayotgan qiymatni va o‘rtacha kvadratik chetlashishni toping.

1.9.6. Bosh to‘plamning o‘rtachasi $X = 22{,}5$ ga va o‘rtacha kvadratik chetlashishi $\sigma = 16$ ga teng. Bosh to‘plamdan hajmi 200 ga teng bo‘lgan tanlanma olingan. Tanlanmaning o‘rtachasi $\overline{X}$ uchun kutilayotgan qiymatni va o‘rtacha kvadratik chetlashishni toping.

1.9.7. Do‘konga kirgan xaridorning do‘konda bo‘lishi vaqti o‘rta hisobda 14 daqiqaga, uning o‘rtacha kvadratik chetlashishi 4 daqiqaga teng. Tavakkaliga tanlangan 6 ta xaridorning kamida 12 daqiqa do‘konda bo‘lishi ehtimolini toping.

1.9.8. Do‘konda bir kunda o‘rtacha 1000 ta kitob sotiladi. Bir kunlik o‘rtacha savdo hajmining o‘rtacha kvadratik chetlashishi 100 ga teng bo‘lsa, to‘rt kunlik savdoning o‘rtacha 900 va 1100 dona kitob orasida bo‘lishi ehtimolini toping.

1.9.9. Bernulli sxemasidagi noma’lum natija ehtimolini momentlar usuli bilan toping.

1.9.10. Tekis taqsimotga ega X tasodifiy miqdor a va b parametrlarining bahosini momentlar usuli bilan toping.

1.9.11. Puasson taqsimotiga ega X tasodifiy miqdor λ parametrining bahosini maksimal ishonchlilik usuli bilan toping.

1.9.12. Tanga 10 marta tashlanganda 6 marta gerb tomon tushdi. Gerb tomon tushishi ehtimolini maksimal ishonchlilik usuli bilan baholang.

1.9.13. Bosh to‘plamning normal taqsimlangan X belgisining no’malum matematik kutilishi a ni 0,99 ishonchlilik bilan baholash uchun ishonchli intervalni toping. Bosh o‘rtacha kvadratik chetlashish σ, tanlanmaning o‘rta qiymati $\overline{X}$ va tanlanmaning hajmi n berilgan:

1) $\sigma = 5,\ \overline{X} = 16{,}3,\ n = 25;$ 2) $\sigma = 4,\ \overline{X} = 12{,}4,\ n = 16;$

3) $\sigma = 3,\ \overline{X} = 10{,}1,\ n = 9;$ 4) $\sigma = 6,\ \overline{X} = 14{,}5,\ n = 36.$

1.9.14. Audit tekshiruvchi tavakkaliga 50 ta to‘lov hisoblarini tahlil qilib, ularning o‘rtacha miqdori 1100 so‘mga va o‘rtacha kvadratik hetlashishi 287 so‘mga tengligini aniqladi. O‘rtacha to‘lov hisoblari uchun 90% li ishonchli intervalni toping.

1.9.15. Normal taqsimlangan bosh to‘plamning o‘rtacha kvadratik chetlashish $\sigma = 3$ ga teng. Bosh to‘plamning tanlanma o‘rta qiymat bo‘yicha matematik kutilishi bahosining aniqligi $\varepsilon = 0{,}2$ bo’lsa, tanlanmaning minimal hajmini 0,95 ishonchlilik bilan aniqlang.

1.9.16. Normal taqsimlangan bosh to‘plamning o‘rtacha kvadratik chetlashish $\sigma = 5$ ga teng. Bosh to‘plamning tanlanma o‘rta qiymat bo‘yicha matematik kutilishi bahosining aniqligi $\varepsilon = 0{,}4$ katta bo‘lmasa, tanlanmaning minimal hajmini 0,9 ishonchlilik bilan aniqlang.

1.9.17. Bosh to‘plamdan $n = 16$ hajmli tanlanma olingan:

1)

x_i	-2	2	4	6
n_i	5	4	4	3

2)

x_i	-1	1	3	5
n_i	6	2	2	6

Bosh to‘plamning normal taqsimlangan X belgisining no’malum matematik kutilishini o‘rtacha qiymati 0,95 ishonchlilik bilan ishonchli interval yordamida baholang.

1.9.18. Biror fizik kattalikni bir xil aniqlikda 16 marta o‘lchash ma’lumotlari bo‘yicha o‘lchash natijalarining o‘rtacha arifmetik qiymati $\overline{X} = 42{,}8$ va tuzatilgan o‘rtacha kvadratik chetlashshi $s = 8$ topilgan. O‘lchanayotgan kattalikning haqiqiy qiymatini 0,999 ishonchlilik bilan baholang.

1.9.19. Bir xil aniqlikdagi 15 ta o‘lchash bo‘yicha o‘rtacha kvadratik chetlashish aniqlangan: 1) $S = 0{,}12$; 2) $S = 0{,}16$; 3) $S = 0{,}24$; 4) $S = 0{,}19$. O‘lchash aniqligini 0,99 ishonchlilik bilan toping.

1.9.20. Biror fizik kattalik bitta asbob bilan (sistematik xatolarsiz) 8 marta o‘lchangan. Bunda o‘lchash tasodifiy xatolarning o‘rtacha kvadratik chetlashshi $s = 0{,}25$ bo‘lib chiqdi. Asbob aniqligini 0,99 ishonchlilik bilan aniqlang.

1.10. STATISTIK GIPOTEZALARNI TEKSHIRISH

Statistik gipotezani tekshirish sxemasi.
Normal taqsimot o‘rta qiymati uchun statistik gipotezani tekshirish.
Normal taqsimot dispersiyasi uchun statistik gipotezani tekshirish.
Bosh to‘plam haqidagi statistik gipotezani tekshirish

1.10.1. Noma’lum taqsimot qonunining ko‘rinishi yoki parametri haqidagi har qanday taxminga *statistik gipoteza* deyiladi.

Tekshirilayotgan gipoteza *nolinchi gipoteza* deb ataladi va H_0 bilan belgilanadi. H_0ga mantiqan zid bo‘lgan gipotezaga *raqobatli* gipoteza deyiladi va H_1 bilan belgilanadi.

H_0 gipoteza biror qoida bilan qabul qilinishi yoki rad etilishi mumkin. Bu qoidaga H_0 gipotezani tekshirishning *statistik mezoni* deyiladi.

Gipotezalarni tekshirishda avval $X_1, X_2, \dots, X_n$ tanlanma natijalari asosida mezonning statistikasi deb ataluvchi $K = K(X_1; X_2; ...; X_n)$ funksiya tanlanadi.

K statistik mezon tanlangach, uning mumkin bo’lgan qiymatlari to‘plami S ikkita kesishmaydigan S_1 va S_2 $(S_1 \cup S_2 = S,\ S_1 \cap S_2 = \varnothing)$ qism to‘plamlarga ajratiladi: S_1 – *gipotezani qabul qilish sohasi*; S_2 – *kritik soha*. Bunda: $K \in S_1$ bo‘lsa, H_0 gipoteza qabul qilinadi; $K \in S_2$ bo‘lsa, H_0 gipoteza rad etiladi.

S_1 va S_2 to‘plamlarni ajratuvchi nuqtalarga *kritik nuqtalar* deyiladi va K_{kr} bilan belgilanadi. Bunda $K > K_{kr} > 0$ tengsizlik bilan aniqlanuvchi $(K_{kr}; +\infty)$ oraliqqa o‘ng tomonlama kritik soha, $K < K_{kr} < 0$ tengsizlik bilan aniqlanuvchi $(-\infty; K_{kr})$ oraliqqa chap tomonlama kritik soha, $K < K_{1kr}$ va $K > K_{2kr}$ $(K_{2kr} > K_{1kr})$ tengsizliklar bilan aniqlanuvchi $(-\infty; K_{1kr}) \cup (K_{2kr}; +\infty)$ oraliqqa ikki tomonlama kritik soha deyiladi.

H_0 gipotezani tekshirishda noto‘g‘ri yechim qabul qilinishi mumkin, ya’ni ikki xil xatolikka yo‘l qolyilashi mumkin:

- 1-tur xatolik, bunda to‘g‘ri bo‘lgan H_0 gipoteza noto‘g‘ri deb rad etiladi;
- 2-tur xatolik, bunda noto‘g‘ri bo’lgan H_0 gipoteza to‘g‘ri deb qabul qilinadi.

1-tur xatolikka yo‘l qo‘yish ehtimoli α ga *statistik mezonning qiymatlilik darajasi* deyiladi.

2-tur xatolikka yo‘l qo‘yish ehtimoli β bilan belgilanafdi. 2-tur xatolikka yo‘l qo‘ymaslik ehtimoli $1-\beta$ ga *statistik mezonning quvvati* deyiladi.

1- tur xatolik ehtimoli α qaralayotgan masalaning mohiyatiga kelib chiqqan holda tadqiqotchi tomonidan belgilanadi. Berilgan α uchun β eng kichik bo‘lgan S_2 soha $P_{H_0}(K \in S_2) = \alpha$ tenglamadan topiladi.

Kritik nuqtalar K mezon uchun berilgan α qiymatlilik darajasiga qarab $P_{H_0}(K > K_{kr}) = \alpha$, $P_{H_0}(K < K_{kr}) = \alpha$, $P_{H_0}(K < K_{1kr}, K > K_{2kr}) = \alpha$ tenglamalarning biridan topiladi. Bu tenglamalarning ildizlari ko‘p ishlatiladigan mezonlar uchun odatda maxsus jadvallardan topiladi.

Statistik gipoteza quyidagi sxema asosida tekshiriladi:

1^o. X tasodifiy miqdor ustida n ta bog‘liqmas kuzatishlar o‘tkaziladi va $X_1, X_2, \ldots, X_n$ tanlanma hosil qilinadi;

2^o. Nolinchi H_0 va muqobil H_1 statistik gipotezalar kiritiladi;

3^o. $K = K(X_1, X_2, ..., X_n)$ statistik mezon tanlanadi va uning H_0 gipotezadagi $P_{H_0}(K)$ taqsimoti topiladi;

4^o. Qiymatlilik darajasi α belgilanadi;

5^o. $P_{H_0}(K \in S_2) = \alpha$ tenglama yordamida S_2 kritik soha topiladi;

6^o. $K = K(X_1, X_2, ..., X_n)$ statistik mezonda $X_1, X_2, ..., X_n$ tasodifiy miqdorlar o‘rniga tanlanmaning $x_1, x_2, ..., x_n$ qiymatlarini qo‘yib, statistik mezonning $K_t = K(x_1, x_2, ..., x_n)$ tuzatilgan qiymati hisoblanadi. Bunda: $K_t \in S_2$ bolsa H_0 gipoteza rad etilad; $K_t \in S_1$ bolsa H_0 gipoteza qabul qilinadi.

Gipotezalarni tekshirishda statistik mezon sifatida odatda normal taqsimot, χ^2 taqsimot, $t-$taqsimot va $F-$taqsimot tanlanadi.

Normal taqsimot bosh to‘plam dispersiyasi ma’lum bo‘lganda taqsimot o‘rta qiymati uchun statistik gipotezani tekshirishda, tanlanmaning ulushi uchun qo‘yiladigan gipotezalarni tekshirishda qo‘llaniladi.

χ^2 taqsimot o‘zgaruvchilar orasidagi bog‘lanishni tekshirishda, kuzatilayotgan taqsimotning biror standart taqsimotga muvofiqligi haqidagi gipotezani tekkshrishda ishlatiladi.

$t-$taqsimot bosh to‘plam dispersiyasi noma’lum bo‘lganda taqsimot o‘rta qiymati uchun statistik gipotezani tekshirishda tanlanadi.

$F-$taqsimot bosh to‘plamning dispersiyalarini solishtirish gipotezalarida qo‘llaniladi.

1.10.2. σ *parametr ma'lum bo'lgan hol*

1^o. X – tasodifiy miqdor $N(a,\sigma^2)$ normal taqsimotga ega bo'lisin.

2^o. Gipotezalarni kiritamiz: $H_0: \ a = a_0$, ya'ni bosh to'plam o'rta qiymati a_0 ga teng; $H_1: \ a > a_0$ (yoki $a < a_0$, yoki $a \neq a_0$).

3^o. Statistik mezon sifatida

$$Z = \sqrt{n}\frac{\overline{X} - a_0}{\sigma}$$

ni olamiz, bu yerda $\overline{X}$ – tanlanma o'rta qiymat.

H_0 gipoteza o'rinli bo'lganda $M(\overline{X}) = a_0$, $D(\overline{X}) = \frac{\sigma^2}{n}$ bo'ladi va Z statistik mezon $N(0,1)$ normal taqsimotga bo'ysinadi.

4^o. α qiymatlilik darajasini belgilaymiz.

5^o. α qiymatlilik darajasi bo'yicha S_2 kritik sohani topamiz. Bu soha H_1 gipotezaga bog'liq holda topiladi.

1) $H_1: \ a > a_0$ bo'lganda o'ng tomonlama $S_2 = (Z_{kr}; +\infty)$ kritik soha olinadi.

Berilgan α ga ko'ra

$$\alpha = P_{H_0}(Z \in S_2) = P_{H_0}(Z > Z_{kr}) = 1 - P_{H_0}(Z \leq Z_{kr}) = 1 - \Phi(Z_{kr}),$$

bu yerda $\Phi(x) = \frac{1}{2} + \Phi_0(x)$ bo'lib, $N(0,1)$ normal taqsimot funksiyasini ifodalaydi. Bunda kritik nuqta

$$\Phi_0(Z_{kr}) = \frac{1 - 2\alpha}{2}$$

tenglama asosida Laplas funksiyasining jadvalidan topiladi.

6^0. Kuzatuv natijalari bo'yicha Z_k ni hisoblanadi.

Bunda $Z_k > Z_{kr}$ bo'lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

2) $H_1: \ a < a_0$ bo'lganda chap tomonlama $S_2 = (-\infty; -Z_{kr})$ kritik soha olinadi.

Bunda

$$\alpha = P_{H_0}(Z \in S_2) = P_{H_0}(Z < -Z_{kr}) = \Phi(-Z_{kr}) = \frac{1}{2} + \Phi_0(-Z_{kr}) = \frac{1}{2} - \Phi_0(Z_{kr})$$

bo'lgani uchun kritik nuqta

$$\Phi_0(Z_{kr}) = \frac{1 - 2\alpha}{2}$$

Tenglama asosida Laplas funksiyasining jadvalidan topiladi.

Bunda $Z_k < -Z_{kr}$ bo'lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

3) $H_1: \ a \neq a_0$ bo'lganda ikki tomonlama $S_2 = (-\infty; -Z_{kr}) \cup (Z_{kr}; +\infty)$ kritik

soha olinadi. Bunda Z_{kr} kritik nuqta

$$\Phi_0(Z_{kr}) = \frac{1-\alpha}{2}$$

tenglamadan Laplas funksiyasining jadvali asosida topiladi.

Bunda $|Z_k| > Z_{kr}$ bo‘lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

$1-$misol. $\sigma^2 = 4,\ n = 36,\ \overline{X} = 6{,}4$. $H_0:\ a = 6$ gipotezani $H_1:\ a > 6$ gipotezada $0{,}05$ qiymatlilik darajasida tekshiring.

⊜ $\alpha = 0{,}05$ dan $\Phi_0(Z_{kr}) = \dfrac{1-2\alpha}{2} = \dfrac{1-0{,}1}{2} = 0{,}45.$

Bundan 2-ilovadagi jadvalga ko‘ra $Z_{kr} = 1{,}65$ va $S_2 = (1{,}65;+\infty)$.

Statistik mezonning kuzatilgan qiymatini hisoblaymiz:

$$Z_k = \sqrt{n}\frac{\overline{X} - a_0}{\sigma} = \sqrt{36}\frac{6{,}4-6}{2} = 1{,}2.$$

$Z_k = 1{,}2 < 1{,}65 = Z_{kr}$. Demak, H_0 gipoteza $\alpha = 0{,}05$ qiymatlilik darajasi bilan qabul qilinadi. ⊝

$2-$misol. $\sigma^2 = 9,\ n = 400,\ \overline{X} = 4{,}8$. $H_0:\ a = 5$ gipotezani $H_1:\ a < 5$ gipotezada $0{,}01$ qiymatlilik darajasida tekshiring.

⊜ $\alpha = 0{,}01$ uchun $\Phi_0(Z_{kr}) = \dfrac{1-2\alpha}{2} = \dfrac{1-0{,}2}{2} = 0{,}49.$

Bundan 2-ilovadagi jadvalga ko‘ra $Z_{kr} = 2{,}33$ va $S_2 = (-\infty;-2{,}33)$.

Statistik mezonning kuzatilgan qiymatini hisoblaymiz:

$$Z_k = \sqrt{n}\frac{\overline{X} - a_0}{\sigma} = \sqrt{400}\frac{4{,}8-5}{3} = -1{,}33.$$

$Z_k = -1{,}33 > -2{,}33 = Z_{kr}$. Demak, H_0 gipoteza $\alpha = 0{,}01$ qiymatlilik darajasi bilan qabul qilinadi. ⊝

$3-$misol. $\sigma^2 = 9,\ n = 81,\ \overline{X} = 0{,}8$. $H_0:\ a = 0$ gipotezani $H_1:\ a \neq 0$ gipotezada $0{,}1$ qiymatlilik darajasida tekshiring.

⊜ $\alpha = 0{,}1$ uchun $\Phi_0(Z_{kr}) = \dfrac{1-\alpha}{2} = \dfrac{1-0{,}1}{2} = 0{,}45.$

Bundan 2-ilovadagi jadvalga ko‘ra $Z_{kr} = 1{,}65$ va $S_2 = (-\infty;-1{,}65) \cup (1{,}65;+\infty)$.

Statistik mezonning kuzatilgan qiymatini hisoblaymiz:

$$Z_k = \sqrt{n}\frac{\overline{X} - a_0}{\sigma} = \sqrt{81}\frac{0{,}8-0}{3} = 2{,}4.$$

$Z_k = 2{,}4 > 1{,}65 = Z_{kr}$. Demak, H_0 gipoteza $\alpha = 0{,}01$ qiymatlilik darajasi bilan rad etiladi. ⊝

σ parametr noma'lum bo'lgan hol

1^o. X – tasodifiy miqdor $N(a,\sigma^2)$ normal taqsimotga ega bo'lsin.

2^o. Gipotezalarni kiritamiz: $H_0: a = a_0, H_1:\ a > a_0$ (yoki $a < a_0$, yoki $a \neq a_0$).

3^o. Statistik mezon sifatida

$$T = \sqrt{n-1}\frac{\overline{X} - a_0}{S}$$

funksiyani olamiz, bu yerda $\overline{X}$ – tanlanma o'rta qiymati, S^2 – tuzatilgan tanlanma dispersiya.

Bunda H_0 gipoteza o'rinli bo'lganda T statistik mezon parametri $k = n-1$ bo'lgan Styudent taqsimotiga bo'ysinadi.

4^o. α qiymatlilik darajasini belgilaymiz.

5^o. α qiymatlilik darajasi bo'yicha S_2 kritik sohani topamiz. Bu soha H_1 gipotezaga bog'liq holda topiladi.

1) $H_1:\ a > a_0$ bo'lganda o'ng tomonlama $S_2 = (T_{kr};+\infty)$ kritik soha olinadi.

Kritik nuqta T_{kr} berilgan α qiymatlilik darajasi va $k = n-1$ parametr bo'yicha Styudent taqsimotining kritik nuqtalari jadvalidan topiladi.

6^0. Kuzatuv natijalari bo'yicha T_k hisoblanadi.

Bunda $T_k > T_{kr}$ bo'lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

2) $H_1:\ a < a_0$ bo'lganda chap tomonlama $S_2 = (-\infty;-T_{kr})$ kritik soha olamiz. Bunda T_{kr} yuqoridagi kabi topiladi.

Bunda $T_k < -T_{kr}$ bo'lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

3) $H_1:\ a \neq a_0$ bo'lganda ikki tomonlama $S_2 = (-\infty;-T_{kr}) \cup (T_{kr};+\infty)$ kritik soha olinadi. Bunda kritik nuqta T_{kr} berilgan $\frac{\alpha}{2}$ qiymatlilik darajasi va $k = n-1$ parametr bo'yicha Styudent taqsimotining kritik nuqtalari jadvalidan topiladi.

Bunda $|T_k| > T_{kr}$ bo'lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

4 – misol. Elektr chiroqlari ishlab chiqaruvchi firmaning ma'lum turdagi chiroqlar uchun normalangan xizmat muddati 1500 soat qilib belgilangan ekan. Yangi ishlab chiqarilgan chiroqlar partiyasini tekshirish uchun $n = 10$ dona chiroq tanlanadi. Bu tanlanma uchun o'rtacha xizmat muddati $\overline{X} = 1410$ soatni va o'rtacha tuzatilgan kvatratik chetlashishi $S = 90$ soatni tashkil qilgan. Olingan ma'lumotlar ishlab chiqarilayotgan chiroqlarning xizmat muddati normalangan xizmat muddatidan farqlanadi degan xulosa chiqarishga asos bo'ladimi ($\alpha = 0{,}1$ da)?

Gipotezalarni kiritamiz: $H_0: a = 1500$, ya’ni tanlanma o‘rtacha 1500 soatga teng bo‘lgan bosh to‘plamdan olingan; $H_1: a \neq 1500$, ya’ni tanlanma o‘rtacha 1500 soatga teng bo‘lgan bosh to‘plamdan olinmagan.

Styudent taqsimotining kritik nuqtalari jadvalidan (6-ilova)

$$T_{kr} = t(\alpha; n-1) = t(0{,}1;9) = 1{,}83.$$

Statistik mezonning kuzatilgan qiymatini hisoblaymiz:

$$T_k = \sqrt{n-1}\frac{\overline{X} - a_0}{S} = \sqrt{10-1}\frac{1410-1500}{90} = -3.$$

$T_k = -3 < -1{,}83 = T_{kr}$ va H_0 gipoteza $\alpha = 0{,}1$ qiymatlilik darajasi bilan qabul qilinadi.

Demak, chiroqlarning o‘rtacha xizmat muddati o‘zgargan va u normalangan xizmat muddatini qanoatlantiradi degan xulosa chiqazish mumkin.

1.10.3. 1^o. X – tasodifiy miqdor $N(a,\sigma^2)$ normal taqsimotga ega bo‘lisin.

2^o. Gipotezalarni kiritamiz: $H_0: \sigma^2 = \sigma_0^2$, ya’ni bosh to‘plam dispersiyasi σ_0^2 ga teng; $H_1: \sigma^2 > \sigma_0^2$ (yoki $\sigma^2 < \sigma_0^2$, yoki $\sigma^2 \neq \sigma_0^2$).

3^o. Statistik mezon sifatida

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

olinadi, bu yerda S^2 – tanlanmaning tuzatilgan dispersiyasi.

4^o. α qiymatlilik darajasini belgilanadi.

5^o. α qiymatlilik darajasi bo‘yicha S_2 kritik sohani topamiz. Bu soha H_1 gipotezaga bog‘liq holda topiladi.

1). $H_1: \sigma^2 > \sigma_0^2$ bo‘lsin. Bunda o‘ng tomonlama $S_2 = (\chi_{kr}^2; +\infty)$ kritik soha tuziladi. S_2 sohaning kritik nuqtasi

$$\alpha = P_{H_0}(\chi^2 \in S_2) = P_{H_0}(\chi^2 > \chi_{kr}^2)$$

tenglama asosida berilgan $k = n-1$ va α bo‘yicha χ^2 taqsimotning kritik nuqtalari jadvalidan topiladi.

6^0. Kuzatuv natijalari bo‘yicha χ_k^2 hisoblanadi.

Bunda $\chi_k^2 > \chi_{kr}^2$ bo‘lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

2) $H_1: \sigma^2 < \sigma_0^2$ bo‘lsin. Bunda chap tomonlama $S_2 = [0; \chi_{kr}^2)$ kritik soha tuziladi. S_2 sohaning kritik nuqtasi

$\alpha = P_{H_0}(\chi \in S_2) = P_{H_0}(\chi^2 < \chi_{kr}^2) = 1 - P_{H_0}(\chi^2 > \chi_{kr}^2)$ yoki $P_{H_0}(\chi^2 > \chi_{kr}^2) = 1 - \alpha$

tenglama asosida berilgan $k = n-1$ va $\alpha - 1$ bo‘icha χ^2 taqsimotning kritik nuqtalari jadvalidan topiladi.

Bunda $\chi_k^2 > \chi_{kr}^2$ bo‘lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

3) $H_1:\ \sigma^2 \neq \sigma_0^2$ bo‘lsin. Bunda chap tomonlama $S_2' = [0; \chi_{1kr}^2)$ va o‘ng tomonlama $S_2'' = (\chi_{kr}^2; +\infty)$ sohalardan tashkil topgan $S_2 = S_2' + S_2''$ ikki tomonlama kritik soha tuziladi.

Berilgan α ga ko‘ra chap χ_{1kr}^2 va o‘ng χ_{2kr}^2 kritik nuqtalar

$\frac{\alpha}{2} = P_{H_0}(\chi^2 \in S_2') = P_{H_0}(\chi^2 \in S_2'')$ tenglamadan topiladi.

U holda

$$P_{H_0}(\chi^2 \in S_2) = P_{H_0}(\chi^2 \in S_2') + +P_{H_0}(\chi^2 \in S_2'') = \frac{\alpha}{2} + \frac{\alpha}{2} = \alpha$$

munosabat o‘rinli bo‘ladi.

$\frac{\alpha}{2} = P_{H_0}(\chi^2 \in S_2'') = P_{H_0}(\chi^2 > \chi_{2kr}^2)$ tenglama asosida berilgan $k = n-1$ va $\frac{\alpha}{2}$ bo‘yicha jadvaldan χ_{2kr}^2 kritik nuqta topiladi.

$\frac{\alpha}{2} = P_{H_0}(\chi^2 \in S_2') = P_{H_0}(\chi^2 < \chi_{1kr}^2) = 1 - P_{H_0}(\chi^2 > \chi_{1kr}^2)$ yoki $P_{H_0}(\chi^2 > \chi_{1kr}^2) = 1 - \frac{\alpha}{2}$ tenglik asosida berilgan $k = n-1$ va $1 - \frac{\alpha}{2}$ bo‘yicha jadvaldan χ_{1kr}^2 kritik nuqtani topamiz.

Bunda $\chi_{1kr}^2 < \chi_k^2 < \chi_{2kr}^2$ bo‘lsa H_0 gipoteza qabul qilinadi, aks holda rad etiladi.

$5-$misol. $n = 21,\ S^2 = 14{,}3,\ \alpha = 0{,}02,\ H_0:\ \sigma_0^2 = 6{,}7,\ H_1:\ \sigma^2 \neq 6{,}7$ bo‘lsa, H_0 gipotezani tekshiring.

7-ilovadagi jadvaldan $k = n-1 = 21-1 = 20$ va $\frac{\alpha}{2} = 0{,}01$ parametrlar bo‘yicha o‘ng kritik nuqta χ_{2kr}^2=37,6 va $k = n-1 = 21-1 = 20$ va $1 - \frac{\alpha}{2} = 0{,}99$ parametrlar bo‘yicha chap kritik nuqta χ_{1kr}^2=8,26 ekanini topamiz.

Statistik mezonning kuzatilgan qiymatini hisoblaymiz:

$$\chi_k^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{20 \cdot 14{,}3}{6{,}7} = 42{,}7.$$

$\chi_k^2 = 42{,}7 > 37{,}6 = \chi_{2kr}^2$ bo‘lgani uchun H_0 gipoteza $\alpha = 0{,}02$ qiymatlilik darajasi bilan rad etiladi.

1.10.4. Bosh to‘plam haqidagi gipotezalarni tekshirish muvofiqlik kriteriyalariga (mezonlariga) asoslanadi.

Muvofiqlik kriteriyasi deb taqsimot funksiyasining umumiy ko‘rinishi haqidagi gipotezani qabul qilish yoki rad etishga imkon beradigan kriteriyaga aytiladi.

Matematik statistikada Pirson (χ^2), Kolmogorov , Fisher, Smirnov va boshqa muvofiqlik kriteriyalari qo‘llaniladi.

Pirson kriteriyasi

1^o. X ustida o‘tkazilgan n ta bog‘liqmas kuzatishlar natijasida olingan tanlanma asosida

x_i	x_1	x_2	...	x_m
n_i	n_1	n_2	...	n_m

taqsimot qonuni olinadi, bu yerda n_i– emperik kuzatishlar chastotasi.

n_i chastotalarga mos n_i^* nazariy chastotalar taqsimotining turiga bog‘liq ravishda topiladi.

1) Taqsimot diskret bo‘lganda bu taqsimotdagi kuzatilgan x_i $(i=\overline{1,m})$ variantalarning $p_i = P(X = x_i)$ ehtimollari hisoblanadi va $n_i^* = np_i$ nisbiy chastotalar topiladi, bu yerda $n = \sum_{i=1}^{m} n_i$ – tanlanma hajmi.

2) Taqsimot uzluksiz bo‘lganda barcha variantalar yotgan $[a;b]$ ($a = \min x_i$, $b = \max x_i$, $i = \overline{1,m}$) kesmani bir xil uzunlikdagi m ta $(x_j;x_{j+1})$ qismiy oraliqlarga ajratiladi. Taqsimot qonuni (yoki tanlanma) asosida bu qismiy oraliqlarga tushgan x_i variantalar soni, ya’ni n_j $(j=\overline{1,m})$ emperik chastotalar aniqlanadi. Taxmin qilinayotgan taqsimot qonuni yordamida $P_j = P_{H_0}(x_j < X < x_{j+1})$ ehtimollar hisoblanadi. va $n_j^* = np_j$ nisbiy chastotalar topiladi, bu yerda $n = \sum_{i=1}^{m} n_j$ – tanlanmaning hajmi.

2^o. Gipotezalarni kiritamiz: H_0: taqsimot qonuni J qonundan iborat bo‘lsin.

3^o. Statistik mezon sifatida

$$\chi^2 = \sum_{j=1}^{m} \frac{(n_j - n_{j1}^*)^2}{n_j^*}$$

ni olinadi. Bunda H_0 gipoteza o‘rinli va $np_i > 5$ bo‘lsa, u holda $\sum_{i=1}^{m} \xi_i^2$ tasodifiy miqdor k erkinlik darajali χ^2 taqsimotga bo‘ysinadi.

k erkinlik daraja J taqsimotga bog‘liq holda $k = m - r - 1$ tenglikdan topiladi, bu yerda $r - J$ taqsimotning parametrlari soni. Masalan, J Puasson taqsimoti bo‘lsa $r = 1$, normal taqsimot bo‘lsa $r = 2$ bo‘ladi.

4^o. α qiymatlilik darajasini belgilaymiz.

5^o. α qiymatlilik darajasi bo‘yicha o‘ng tomonlama $S_2 = (\chi^2_{kr};+\infty)$ kritik sohani olamiz.

Bunda: $k \le 30$ bo‘lganda χ^*_{kr} kritik nuqta

$$\alpha = P_{H_0}(\chi^2 \in S_2) = P_{H_0}(\chi^2 > \chi^2_{kr})$$

tenglama bo‘yicha erkinlik darajasai $k = m - r - 1$ bo‘lgan χ^2 taqsimot jadvalidan topiladi; $n > 30$ bo‘lganda kritik nuqta normal taqsimotdan foydalanib topiladi.

6^0. Kuzatuv natijalari bo‘yicha χ^2_k ni hisoblaymiz.

Bunda $\chi^2_k > \chi^2_{kr}$ bo’lsa H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

Izohlar.

1. Pirson kriteriyasida tanlanma hajmi yetarlicha katta $(n \ge 50)$ bo‘lishi kerak;

2. Har bir $(x_j; x_{j+1})$ qismiy oraliq kamida 5–8 ta variantani o‘z ichiga olishi kerak;

3. Hisoblashni soddalashtirish uchun statistik mezonni $\chi^2 = \sum_{j=1}^{m} \frac{n_j^2}{n_j^*} - n$

kabi olish mumkin.

$6-$misol. Bosh to‘plam normal taqsimotga bo‘ysinishi haqidagi H_0 gipotezani tekshirish uchun tanlanma asosida emperik n_j va nazariy n_j^* chastotlar topilgan:

n_j	8	16	35	72	60	53	36
n_j^*	5	12	39	81	65	49	29

$\alpha = 0{,}05$ qiymatlilik darajasida H_0 gipotezani tekshiring.

Chastotalarning hajmlarini hisoblaymiz:

$$\sum_{j=1}^{7} n_j = 8 + 16 + 35 + 72 + 60 + 53 + 36 = 280;$$

$$\sum_{j=1}^{7} n_j^* = 5 + 12 + 39 + 81 + 65 + 49 + 29 = 280.$$

χ^2 kriteriyaning kuzatilgan χ_k^2 qiymatlarini jadval tarzida keltiramiz:

j	n_j	n_j^*	$n_j - n_j^*$	$(n_j - n_j^*)^2$	$\frac{(n_j - n_j^*)^2}{n_j^*}$
1	8	5	3	9	1,80
2	16	12	4	16	1,33
3	35	39	–4	16	0,41
4	72	81	–9	81	1,00
5	60	65	–5	25	0,38
6	53	49	4	16	0,33
7	36	29	7	49	1,69
$\sum$	280	280			6,94

Jadvalga muvofiq: $\chi_k^2 = 6{,}94,\ m = 7,\ r = 2,$ chunki taqsimot normal.

U holda $k = 7 - 2 - 1 = 4$. $k = 4,\ \alpha = 0{,}05$ parametrlarda 7-ilovadagi jadvalidan topamiz: $\chi_{kr}^2 = 9{,}5$. $\chi_k^2 = 6{,}94 < 9{,}5 = \chi_{rk}^2$.

Demak, H_0 gipoteza qabul qilinadi.

Kolmogorov kriteriyasi

1^o. X belgili bosh to‘plam va hajmi n ga teng bo‘lgan $X_1, X_2, ..., X_n$ tanlanma va $F_n^*(x)$ emperik taqsimot funksiyasi berilgan bo‘lsin.

2^o. Gipotezalarni kiritamiz: H_0: taqsimot qonuni $F(x)$ bo‘lsin.

3^o. Statistik mezon sifatida

$$D_n = \max\left|F(x) - F_n^*(x)\right|$$

olinadi.

Bunda istalgan uzluksiz $F(x)$ funksiya uchun

$$\lim_{n\to\infty} P\left(D_n < \frac{\lambda}{\sqrt{n}}\right) = K(\lambda)$$

bo‘ladi, bu yerda $K(\lambda) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2\lambda^2}$.

4^o. α qiymatlilik darajasini belgilaymiz.

5^o. $K(\lambda_\alpha) = 1 - \alpha$ tenglama ildizlari uchun ($n \geq 20$ da) tuzilgan

α	0,1	0,05	0,02	0,01	0,001
λ_α	1.224	1,358	1,520	1,627	1,950

jadvalidan α qiymatlilik darajasiga mos λ_α topiladi.

6^0. Kuzatuv natijalari bo‘yicha D_n ni hisoblaymiz.

Agar bunda $D_n > \frac{\lambda_\alpha}{\sqrt{n}}$ bo‘lsa, H_0 gipoteza rad etiladi, aks holda qabul qilinadi.

7 $-$misol. Tanga 4040 marta tashlanadi (Byuffon). $n_1 = 2048$ marta gerb tomon tushish va $n_2 = 1992$ marta raqam tomoni tushish kuzatildi. Bu natijalar tanganing simmetrikligi haqidagi H_0 gipotezaga mos kelishini Kolmogorov kriteriyasi bilan tekshiring $(\alpha = 0{,}05)$.

X tasodifiy miqdor ikkita qiymat qabul qiladi: $x_1 = -1$(raqam), $x_2 = 1$(gerb).

Bunda $H_0:\ P(X=-1) = P(X=1) = \frac{1}{2}.$

$F(x)$ va $F_n^*(x)$ larni tuzamiz:

x_i	-1	1
p_i	0,5	0,5

dan
$$F(x) = \begin{cases} 0, & agar\ x-1, \\ 0{,}5, & agar\ -1 < x \le 1, \\ 1, & agar\ x > 1; \end{cases}$$

x_i	-1	1
n_i	1992	2048
p_i^*	0,493	0,507

dan
$$F(x) = \begin{cases} 0, & agar\ x-1, \\ 0{,}493, & agar\ -1 < x \le 1, \\ 1, & agar\ x > 1. \end{cases}$$

Bundan
$$D_n = \max\left|F(x) - F_n^*(x)\right| = |\,0{,}5 - 0{,}493\,| = 0{,}007.$$

Kolmogorov taqsimoti jadvaliga ko‘ra
$$K(\lambda_\alpha) = 1 - 0{,}05 = 0{,}05 \text{ da } \lambda_\alpha = 1{,}358.$$

U holda
$$D_0 = \frac{1{,}358}{\sqrt{4040}} \approx 0{,}021.$$

Bunda
$$D_n = 0{,}007 < 1{,}358 = D_\lambda.$$

Demak, H_0 gipoteza qabul qilinadi.

Mashqlar

1.10.1. $\sigma^2 = 4,\ n = 36,\ \overline{X} = 6{,}2$ da H_0 gipotezani H_1 gipotezada α qiymatlilik darajasi bilan tekshiring: 1) $H_0 : a = 6,\ H_1 : a > 6,\ \alpha = 0{,}1;$ 2) $H_0 : a = 5,\ H_1 : a \neq 5,\ \alpha = 0{,}05;$ 3) $H_0 : a = 7,\ H_1 : a < 7,\ \alpha = 0{,}01.$

1.10.2. Atirgul ko'chatlarining bo'yi o'rtachasi $a = 43$ *sm.* va dispersiyasi $\sigma^2 = 9$ ga teng bo'lgan normal taqsimotga ega. 15 dona ko'chat o'tqazilishi kerak bo'lgan maydonga o'g'itlar normadan ikki barobar ko'p solingan. Bunda ko'chatlarning o'rtacha bo'yi 46 *sm.* ga etgan. Normadan ortiqcha solingan o'g'itlar foyda bermadi degan xulosa chiqarishga asos bormi?

1.10.3. $S = 1{,}2,\ n = 16,\ \overline{X} = 12{,}4$ da H_0 gipotezani H_1 gipotezada α qiymatlilik darajasi bilan tekshiring: 1) $H_0 : a = 11{,}8,\ H_1 : a \neq 11{,}8,\ \alpha = 0{,}02;$ 2) $H_0 : a = 12,\ H_1 : a > 12,\ \alpha = 0{,}05;$ 3) $H_0 : a = 13,\ H_1 : a < 13,\ \alpha = 0{,}1.$

1.10.4. Kofe 100 *gr.* li idishlarga avtomat uskunada qadoqlanadi. Qadoqlanayotgan idishning og'irligi aniq og'irlikdan farq qilsa uskuna sozlanadi. Ma'lum vaqtda qadoqlanayotgan idishlar ajratib olinib, ularning o'rtacha og'irligi tekshiriladi va og'irlikdan chtlasishi hisoblanadi. 30 dona qadoqlangan idishlar og'irliklari tahlili natijasida ularning o'rtacha og'irligi $\overline{X} = 102{,}4$ va tuzatilgan o'rtacha kvadratik chetlashishi $S = 18{,}54$ ekani aniqlangan. Uskunani cozlash zaruriyati bormi? (Qiymatlilik darajasi $\alpha = 0{,}05$).

1.10.5. $S^2 = 16{,}2,\ n = 21$ da H_0 gipotezani H_1 gipotezada α qiymatlilik darajasi bilan tekshiring:

1) $H_0 : \sigma_0^2 = 15,\ H_1 : \sigma^2 > 15,\ \alpha = 0{,}01;$

2) $H_0 : \sigma_0^2 = 17,\ H_1 : \sigma^2 < 17,\ \alpha = 0{,}05;$

3) $H_0 : \sigma_0^2 = 16,\ H_1 : \sigma^2 \neq 16,\ \alpha = 0{,}02.$

1.10.6. $S^2 = 0{,}24,\ n = 17$ da H_0 gipotezani H_1 gipotezada α qiymatlilik darajasi bilan tekshiring:

1) $H_0 : \sigma_0^2 = 0{,}18,\ H_1 : \sigma^2 > 0{,}18,\ \alpha = 0{,}05;$

2) $H_0 : \sigma_0^2 = 0{,}20,\ H_1 : \sigma^2 < 0{,}20,\ \alpha = 0{,}01;$

3) $H_0 : \sigma_0^2 = 0{,}16,\ H_1 : \sigma^2 \neq 0{,}16,\ \alpha = 0{,}02.$

1.10.7. Bosh to'plamdan olingan tanlanma asosida emperik n_j va nazariy n_j^* chastotalar topilgan:

1)

n_j	6	10	32	64	60	42	36
n_j^*	5	8	34	70	62	43	28

2)

n_j	4	8	30	60	66	40	32
n_j^*	4	5	28	63	62	42	36

$\alpha = 0{,}05$ qiymatlilik darajasida bosh to'plamning normal taqsimotga bo'ysinishi haqidagi H_0 gipotezani Pirson kriteriyasi bilan tekshiring.

1.10.8. Bosh to'plamdan olingan tanlanma asosida emperik n_j va nazariy n_j^* chastotalar topilgan:

1)

x_j	-2	-1	0	1	2	3	4
n_j	23	27	30	38	46	29	7
n_j^*	25	20	23	42	40	34	16

2)

x_j	-5	-3	-1	0	1	3	5
n_j	28	16	33	41	47	25	10
n_j^*	18	30	40	22	36	31	23

$\alpha = 0{,}01$ qiymatlilik darajasida bosh to'plamning normal taqsimotga bo'ysinishi haqidagi H_0 gipotezani Pirson kriteriyasi bilan tekshiring.

1.11. KORRELYATSION ANALIZ

Korrelyatsion bog'lanish. Chiziqli korrelyasiya. Chiziqli bo'lmagan korrelyasiya. Korrelyatsiya bog'lanishi zichligini baholash

1.11.1. X tasodifiy miqdorning mumkin bo'lgan har bir qiymatiga Y tasodifiy miqdorning mumkin bo'lgan bitta qiymati mos qo'yilgan bog'lanishga *funksional bog'lanish* deyiladi.

X tasodifiy miqdorning mumkin bo'lgan har bir qiymatiga Y tasodifiy miqdorning biror taqsimoti mos qo'yilgan bog'lanishga *statistik bog'lanish* deyiladi.

Statistik bog'langan X va Y tasodifiy miqdorlar uchun Y ning belgilangan X larda hisoblangan matematik kutilishiga (o'rta qiymatiga) *shartli o'rta qiymat* deyiladi va $\bar{y}_x$ bilan belgilanadi.

$\bar{y}_x$ va x lar orasidagi funksional bog‘lanishga X va Y tasodifiy miqdorlar orasidagi *korrelyasion bo‘g‘lanish* deyiladi.

Korrelyasiya bog‘lanishining tenglamasi $\bar{y}_x = \varphi(x)$ ga Y tasodifiy miqdorning X tasodifiy miqdor bo‘yicha *regressiya tenglamasi* deyiladi. Bunda $\varphi(x)$ funksiya *regressiya funksiyasi*, $\varphi(x)$ funksiyaning grafigi *regressiya chizig‘i* deb ataladi.

X va Y tasodifiy miqdorlar orasidagi korrelyatsiya bog‘lanishi odatda *korrelyatsiya jadvali* yordamida beriladi:

$X \backslash Y$	y_1	y_2	...	y_m	$\sum_{j=1}^{m} n_{ij}$
x_1	n_{11}	n_{12}	...	n_{1m}	$\sum_{j=1}^{m} n_{1j}$
x_2	n_{21}	n_{22}	...	n_{2m}	$\sum_{j=1}^{m} n_{2j}$
...	...	...	...	...	...
x_k	n_{k1}	n_{k2}	...	n_{km}	$\sum_{j=1}^{m} n_{kj}$
$\sum_{i=1}^{k} n_{ij}$	$\sum_{i=1}^{k} n_{i1}$	$\sum_{i=1}^{k} n_{i2}$	...	$\sum_{i=1}^{k} n_{im}$	n

Bunda $x_1, x_2, ..., x_k; y_1, y_2, ..., y_m - X$ va Y tasodifiy miqdorlarning kuzatilgan qiymatlari (diskret tasodifiy miqdorlar uchun) yoki intervallarining o‘rtalari (uzluksiz tasodifiy miqdorlar uchun); $n_{ij} -$ (x_i, y_i) juftlik uchraydigsan chastota; $n = \sum_{i=1}^{k} \sum_{j=1}^{m} n_{ij}$. Korrelyasiya jadvalini funksional bog‘lanish bilan almashtirish uchun jadvaldagi har bir x_i qiymatning $\bar{y}_i = \frac{\sum_{j=1}^{m} y_j n_{ij}}{n_i}$ shartli qiymatlari hisoblanadi, bu yerda $n = \sum_{j=1}^{m} n_{ij}$.

X va Y orasidagi korrellatsiya bog‘lanishining koordinata tekisligi nuqtalari bilan berilgan geometrik tasviriga *korrelyasiya maydoni* deyiladi.

Korrelyatsiya maydonininig $(x_i; \bar{y}_i)$ nuqtalarni tutashtiruvchi siniq chizig‘iga Y ning X bo‘yicha *emperik regressiya chizig‘i* deyiladi.

Agar emperik regressiya chizig‘i unga yaqin bo‘lgan biror silliq chiziq bilan almashtirilsa, bu chiziqqa *nazariy regressiya chizig‘i* deyiladi.

1.11.2. Chiziqli korrelyatsiyada nazariy regressiya tenglamasi

$$y_x = b_0 + b_1 x$$

ko‘rinishda izlanadi. Bunda b_0 va b_1 noma’lum parametrlarni bir nechta usul bilan topish mumkin.

Parametrlarni topishning *eng kichik kvadratlar usuli*da b_0 va b_1 noma‘lum parametrlar

$$S = \sum_{i=1}^{k} (y_{x_i} - \overline{y}_i)^2 n_i = \sum_{i=1}^{k} (b_0 + b_1 x_i - \overline{y}_i)^2 n_i$$

funksiyaning minununga erishishi shartidan keltirib chiqariladigan

$$\begin{cases} b_0 + b_1 \overline{x} = \overline{y}, \\ b_0 \overline{x} + b_1 \overline{x^2} = \overline{xy}, \end{cases}$$

sistemadan topiladi va Y ning X bo‘yicha regressiya tenglamasi

$$y_x - \overline{y} = \rho_{yx}(x - \overline{x})$$

ko‘rinishga keltiriladi. Bunda

$$\rho_{yx} = b_1 = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{\sigma}_x^2} = \frac{\mu}{\overline{\sigma}_x^2},$$

kattalik Y ning X bo‘yicha *tanlanma regressiya koeffitsiyenti* (yoki *regressiya koeffitsiyenti*) deb ataladi, bu yerda

$$\overline{x} = \frac{\sum_{i=1}^{k} x_i n_i}{n}, \quad \overline{y} = \frac{\sum_{j=1}^{m} y_j n_j}{n}, \quad \overline{x^2} = \frac{\sum_{i=1}^{k} x_i^2 n_i}{n}, \quad \overline{xy} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{m} x_i y_j n_{ij}}{n}.$$

$\mu = \overline{xy} - \overline{x} \cdot \overline{y}$ kattalikka tanlanma *korrelyatsiya momenti* yoki *tanlanma kovariatsiya* deyiladi.

X ning Y bo‘yicha regressiya tenglamasi

$$x_y - \overline{x} = \rho_{xy}(y - \overline{y}),$$

ko‘rinishga keltiriladi, bu yerda

$$\rho_{xy} == \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{y^2} - \overline{y}^2} = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{\sigma}_y^2} = \frac{\mu}{\overline{\sigma}_y^2}; \quad \overline{\sigma}_y^2 = \overline{y^2} - \overline{y}^2.$$

1.11.3. Chiziqli bo‘lmagan emperik regressiya chizig‘i parabolaga yaqin bo‘lsa, u holda uning nazariy regressiya tenglamasi

$$y_x = b_0 + b_1 x + b_2 x^2$$

ko‘rinishda izlanadi. Bunda b_0, b_1 va b_2 noma‘lum parametrlar

$$S = \sum_{i=1}^{k} (y_{x_i} - \overline{y}_i)^2 n_i = \sum_{i=1}^{k} (b_0 + b_1 x_i + b_2 x_i^2 - \overline{y}_i)^2 n_i$$

funksiyaning minununga erishishi shartidan keltirib chiqariladigan

$$\begin{cases} b_0\sum_{i=1}^{k} n_i + b_1\sum_{i=1}^{k} x_i n_i + b_2\sum_{i=1}^{k} x_i^2 n_i = \sum_{i=1}^{k} \overline{y}_i n_i, \\ b_0\sum_{i=1}^{k} x_i n_i + b_1\sum_{i=1}^{k} x_i^2 n_i + b_2\sum_{i=1}^{k} x_i^3 n_i = \sum_{i=1}^{k} x_i \overline{y}_i n_i, \\ b_0\sum_{i=1}^{k} x_i^2 n_i + b_1\sum_{i=1}^{k} x_i^3 n_i + b_2\sum_{i=1}^{k} x_i^4 n_i = \sum_{i=1}^{k} x_i^2 \overline{y}_i n_i \end{cases}$$

sistemadan topiladi.

Chiziqli bo‘lmagan

$$y_x = \frac{b}{x}, \quad y_x = ba^x, \quad y_x = be^x, \quad y_x = bx^n$$

ko‘rinishdagi korrelyatsiyalar tenglikning har ikkala tomonini logarifmlash va belgilashlar kiritish orqali chiziqli korrelyatsiyaga keltiriladi.

Ixtiyoriy chiziqli bo‘lmagan $\varphi(x)$ funksiyaning $y = a\varphi(x) + b$ ko‘rinishdagi chiziqli bo‘lmagan korrelyatsiyalarda $z = \varphi(x)$ belgilash kiritish orqali chiziqli korrelyatsiya hosil qilinadi.

1.11.4. Chiziqli korrelyatsiyada X va Y tasodifiy miqdorlar bog‘lanishi zichligining bahosi sifatida o‘rtacha kvadratik chetlashshlarning o‘zgarishiga asoslangan

$$r = \rho_{yx}\frac{\overline{\sigma}_x}{\overline{\sigma}_y} \qquad \text{yoki} \qquad r = \frac{\overline{xy} - \overline{x}\cdot\overline{y}}{\overline{\sigma}_x\overline{\sigma}_y}$$

kattalik olinadi. Bu kattalik *tanlanma koppelyatsiya koeffitsiyenti* (yoki *koppelyatsiya koeffitsiyenti*) deb ataladi. Korrelyatsiya koeffitsiyenti X bir $\overline{\sigma}_x$ ga ortganda Y ning ortacha nechta $\overline{\sigma}_y$ ga o‘zgarishini ko‘rsatadi.

Tanlanma korrelyatsiya koeffitsiyenti quyidagi xossalarga ega:

1^o. $-1 \le r \le 1$;

2^o. $|r|$ qancha katta bo‘lsa korrelyatsiya bog‘lanishi shuncha zich bo‘ladi;

3^o. $|r| = 1$da korrelyatsion bog‘lanish funksional bog‘lanishga aylanadi;

4^o. $r = 0$ bo‘lsa, X va Y tasodifiy miqdorlar orasida chiziqli bog‘lanish mavjud bo‘lmaydi.

Chishiqli bo‘lmagan korrelyatsiyada X va Y tasodifiy miqdorlar orasidagi korrelyatsiya bog‘lanishi zichligi *tanlanma korrelyatsion nisbat* deb ataluvchi ushbu

$$\eta_{yx} = +\sqrt{1 - \frac{\overline{\sigma}_x^2}{\overline{\sigma}_y^2}}$$

kattalik bilan baholanadi. Bunda $\eta_{yx} \to 1$ da X va Y tasodifiy miqdorlar orasidagi bog‘lanish zichlashib boradi va $\eta_{yx}=1$ da funksional bog‘lanishga aylanadi.

1-misol. Bir tipdagi 50 ta korxonaning sytkalik mahsulot ishlab chiqarishi Y (t) va asosiy ishlab chiqarish fondlari X (mln.so‘m) berilgan.

X \ Y	9	13	17	21	25	$\sum_{j=1}^{m} n_{ij}$
22,5	2	1	-	-	-	3
27,5	3	6	4	-	-	13
32,5	-	3	11	7	-	21
37,5	-	1	2	6	2	11
42,5	-	-	-	1	1	2
$\sum_{i=1}^{k} n_{ij}$	5	11	17	14	3	50

1) Y ning X bo‘yicha regressiya tenglamasini tuzing; 2) Asosiy ishlab chiqarish fondlari X va korxonalarning sutkalik ishlab chiqarishi Y orasidagi bog‘lanish zichligini baholang.

⊖ 1) Korrelyatsion jadvalga muvofiq har bir x_i uchun shartli o‘rta qiymatlarni topamiz:

$$\bar{y}_1=\frac{1}{3}(9\cdot 2+13\cdot 1)=10{,}3; \qquad \bar{y}_2=\frac{1}{13}(9\cdot 3+13\cdot 6+17\cdot 4)=13{,}3;$$

$$\bar{y}_3=\frac{1}{21}(13\cdot 3+17\cdot 11+21\cdot 7)=17{,}8;$$

$$\bar{y}_4=\frac{1}{11}(13\cdot 1+17\cdot 2+21\cdot 6+25\cdot 2)=20{,}3;$$

$$\bar{y}_5=\frac{1}{2}(21\cdot 1+25\cdot 1)=23.$$

Kerakli yig‘indilarni hisoblaymiz:

$$\sum_{i=1}^{5} x_i n_i=22{,}5\cdot 3+27{,}5\cdot 13+32{,}5\cdot 21+37{,}5\cdot 11+42{,}5\cdot 2=1605;$$

$$\sum_{i=1}^{5} x_i^2 n_i=22{,}5^2\cdot 3+27{,}5^2\cdot 13+32{,}5^2\cdot 21+37{,}5^2\cdot 11+42{,}5^2\cdot 2=52612{,}5;$$

$$\sum_{j=1}^{5} y_j n_j=9\cdot 5+13\cdot 11+17\cdot 17+21\cdot 14+25\cdot 3=846;$$

$$\sum_{i=1}^{5}\sum_{j=1}^{5} x_i y_j n_{ij}=22{,}5\cdot 9\cdot 2+22{,}5\cdot 1\cdot 13+...+42{,}5\cdot 1\cdot 21+42{,}5\cdot 1\cdot 25=27875.$$

Regressiya tenglamasining tanlanma xarakteristikalari va parametrlarini aniqlaymiz:

$$\overline{x}=\frac{1605}{50}=32{,}1; \qquad \overline{y}=\frac{846}{50}=16{,}92; \qquad \overline{\sigma}_x^2=\frac{52612{,}5}{50}-32{,}1^2=21{,}84;$$

$$\mu=\frac{27895}{50}-32{,}1\cdot 16{,}92=14{,}768; \qquad \rho_{yx}=\frac{14{,}768}{21{,}84}=0{,}6762.$$

Demak, regressiya tenglamasi:

$$y_x-16{,}92=0{,}6762(x-32{,}1)$$

yoki

$$y_x=0{,}6762x-4{,}79.$$

2) $\overline{\sigma}_y$ ni topish uchun avval

$$\sum_{j=1}^{5} y_j^2 n_j=9^2\cdot 5+13^2\cdot 11+17^2\cdot 17+21^2\cdot 14+25^2\cdot 3=15226$$

yig‘indini topamiz.

U holda

$$\overline{\sigma}_y^2=\frac{15226}{50}-16{,}92^2=18{,}2336.$$

Korrelyatsiya koeffitsiyentini topamiz:

$$r=0{,}6762\sqrt{\frac{21{,}84}{18{,}2336}}=0{,}6762\cdot 1{,}0944=0{,}74.$$

Demak, X va Y lar orasidagi bog‘lanish to‘g‘ri chiziqli va etarlicha zich.

Mashqlar

1.11.1. X va Y tasodifiy miqdorlar orasidagi bog‘lanish o‘rganilganida bu miqdorlarning mos qiymatlarini o‘lchash natijalari jadvali olingan:

1) x_i: 0,3 0,4 0,5 0,5 0,6 0,7 0,8 0,9 0,9 1,0 1,1 1,4

y_i: 0,2 0,8 1,2 1,1 1,8 2,5 3,1 3,4 3,8 4,1 4,4 5,9

Y ning X bo‘yicha $y_x=b_0+b_1x$ regressiya tenglamasini tuzing va ular orasidagi bog‘lanish zichligini baholang.

2) x_i: 1,3 1,5 1,5 1,8 1,9 2,0 2,1 2,4 2,4 2,5

y_i: 0,2 0,8 1,2 1,1 1,8 3,2 2,5 3,1 3,4 3,8

Y ning X bo‘yicha $y_x=b_0+b_1x+b_2x^2$ regressiya tenglamasini tuzing va ular orasidagi bog‘lanish zichligini baholang.

1-NAZORAT ISHI

1-2. Hodisalarning ehtimollarini topishga oid masalalarni yeching.

3. Berilfan X diskret tasodifiy miqdorning taqsimot qonunini va taqsimot funksiyasini toping. $F(X)$, $M(X)$, $D(X)$, $P(a < X \leq b)$ larni hisoblang.

1-variant

1. Qutida 7 ta qizil va 13 ta ko‘k qalam bor. Tavakkaliga olingan 3 ta qalamning: 1) barchasi bir xil rangli chiqishi; 2) kamida 2 tasi qizil rangli chiqishi ehtimolini toping.

2. Savdo firmasiga kompyuterlar uchta ta’minotchidan 8:5:7 nisbatda keltirilgan. Ta’minotchilar kompyuterlarining mos ravishda 85, 90 va 75 %i uchun kafolat muddatida ta’mir talab qilinmaydi. 1) Savdo firmasiga keltirilgan kompyuter uchun kafolat muddatida ta’mir talab qilinmasligi ehtimolini toping. 2) Sotilgan kompyuter kafolat muddatida ta’mirlandi. Uning birinchi ta’minotchidan keltirilgan bo‘lishi ehtimolini toping.

3. Savdo firmasidagi 10 ta kompyuterdan 4 tasi “Samsung” rusumli. Firmada 3 ta kompyuter sotilgam. $X-$ sotilgan kompyuterlarning “Samsung” rusumlilari sonidan iborat tasodifiy miqdor, $a=1,\ b=3.$

2-variant

1. Ko‘prik yakson bo‘lishi uchun unga bitta bomba tushishi etarli. Agar ko‘prikka tushishi ehtimollari 0,3, 0,4, 0,7, 0,8 ga teng bo’lgan 4 ta bomba tashlangan bo‘lsa, ko’prikning yakson bo‘lishi ehtimolini toping.

2.Uchta qutining har birida 6 ta qora va 4 ta oq shar bor. Birinchi qutidan tavakkaliga bitta shar olinadi va ikkinchisiga solinadi, keyin ikkinchi qutidan tavakkaliga bitta shar olinadi va uchinchi qutiga solinadi. Uchinchi qutidan tavakkaliga olingan sharning oq bo‘lishi ehtimolini toping.

3. Ikkita samolyot nishonga tekkuncha galma-galdan bomba tashlaydi. Birinchi samolyotning nishonni aniq mo‘ljalga olishi ehtimoli 0,7 ga, ikkinchisiniki 0,8 ga teng. Samolyotlarning har birida 3 tadan bomba bor. $X-$tashlangan bombalar soni, $a=3,\ b=5.$

3-variant

1. Ishchi 4 ta stanokka xizmat ko'rsatadi. Bir soat davomida stanoklarda ishchining sozlash uchun aralashuvi talab qilinmasligi ehtimollari mos ravishda 0,2, 0,25, 0,6 va 0,4 ga teng. Bir soat davomida birorta ham stanokda ishchining aralashuvi talab qilinmasligi ehtimolini toping.

2. Uchta ovchi ayiqqa qarata bir yo'la o'q uzushdi. Ayiq bitta o'q bilan o'ldirildi. Birinchi, ikkinchi va uchinchi ovchilarning nishonga tekkasishi ehtimollari mos ravishda 0,6, 0,5, 0,4 ga teng. Ayiq uchinchi mergan tomonidan o'ldirilgan bo'lishi ehtimolini toping.

3. Nazorat 3 ta test savolidan iborat. Hr bir testda 4 ta javob berilgan bo'lib, ulardan 1 tasi to'g'ri. $X-$topilgan to'g'ri. javoblar soni, $a=1,\ b=2.$

4-variant

1. Elektr zanjirida K_1 element ishdan chiqsa yoki K_2 va K_3 elementlar birgalikda ishdan chiqsa uzilish ro'y beradi. Elementlarning bir-biriga bog'liq bo'lmagan holda ishdan chiqishi ehtimollari mos ravishda 0,1, 0,2 va 0,3 ga teng bo'lsa, elektr zanjirida yzilish ro'y berishi ehtimolini toping.

2. Do'konga to'rtta zavodda tayyorlangan bir xil elektr yoritgichlari qabul qilib olindi: birinchisidan 350 dona, ikkinchisidan 625 dona, ucchinchisidan 245 dona va to'rtinchisidan 850 dona. Yoritgichlar 1500 soatdan ortiq vaqt yonishi ehtimollari zavodlar uchun mos ravishda 0,25, 0,30, 0,40 va 0,75 ga teng. Do'kon tokchalariga yoritgichlar aralashtirib terib chiqiladi. Sotilgan yoritgichning 1500 soatdan ko'p vaqt yonishi ehtimolini toping.

4. Oilada o'gil bola tug'ilishi ehtimoli 0,5 ga teng. $X-$ 3 farzandli oiladagi qiz bolalar soni, $a=1,\ b=2.$

5-variant

1. Do'konga mahsulot uchta firmadan 5:8:7 nisbatda keltirilgan. Firmalar mahsulotlarining mos ravishda 90, 85 va 75 %i standart. Do'kondan tavakkaliga olingan mahsulotning nostandart bo'lishi ehtimolini toping.

2. 15 ta sinov biletining har birida ikkitadan savol bo'lib, ular takrorlanmaydi. Talaba 25 ta savolga tayyorlangan. Talaba sinovdan o'tishi uchun tushgan biletdagi 2 ta savolga yoki biletdagi 1 ta savolga va 1 ta qo'shimcha savolga javob berishi yetarli bo'lsa, talabaning sinovdan o'tishi ehtimolini toping.

3. Merganning bitta otishda o‘qni nishonga tekkazishi ehtimoli 0,8 ga teng. U har bir tekkazgan o‘qi uchun 5 ochko oladi, xatosi uchun ochko olmaydi. $X-3$ ta o‘q otilganda merganning to‘plagan ochkolari soni, $a=0,\ b=2.$

6-variant

1. Qirqma alfavitning 7 ta harfidan "*ALLAQANDAY*" so‘zi tuzilgan. Bu harflar sochilib ketgan va qaytadan ixtiyoriy tartibda yig‘ilgan. Quyidagi so‘zlar chiqishi ehtimollarini toping: 1) "*ALLAQANDAY*", 2) "*QALAY*".

2. Omborga 1000 ta avtomobil shinasi keltirildi. Ularning 260 tasi birinchi korxonada, 400 tasi ikkinchi korxonada va 340 tasi uchinchi korxonada tayyorlangan. Shinaning nostandart bo‘lib chiqishi ehtimoli korxonalar uchun mos ravishda 0,08, 0,025 va 0,04 ga teng. Tavakkaliga olingan shina nostandart bo‘lib chiqdi. Bu shinaning ikkinchi korxonada tayyorlanganligi ehtimolini toping.

3. Omborda 12 % nostandart detal bor. Tavakkaliga 5 ta detal olinadi. $X-$ olingan detallardan nostandart detallar soni, $a=1,\ b=2.$

7-variant

1. 12 ta qiz va 18 ta o‘g‘il talabasi bor guruhdan 1-anjumanga 2 ta vakil jo‘natildi. So‘ngra yana 2 ta vakil 2-anjumanga jo‘natildi. Jo‘natilgan vakillarning barchasi qiz bola bo‘lishi ehtimolini toping.

2. Do‘konga uchta firmada ishlab chiqarilgan sovutgichlar keltirildi: ularning 30 % i birinchi, 50 % i ikkinchi va qolganlari uchinchi firmada ishlab chiqarilgan. Firmalar ishlab chiqargan sovutgichlarining mos ravishda 5, 4 va 3 %i nuqsonga ega. Sotilgan sovutchisninh nuqsonga ega bo‘lmasligi ehtimolini toping.

3. Avtomobil 4 ta svetoforga duch keladi.Svetoforlarning har biri 0,5 ehtimol bilan yo‘lni ochadi yoki harakatni taqiqlaydi. $X-$avtomobilning birinchi to‘xtashigacha o‘tgan svetoforlari soni, $a=2,\ b=4.$

8-variant

1. Qurilma bir-biriga bog‘liqsiz ishlaydigan beshta elementdan iborat. Ulardan bittasi buzilsa, qurilma ishlashdan to‘xtaydi. Bunda har bir element qurilma ishlay boshlaguncha ketma-ket almashtiriladi. 1) ikkita elementni almashtirishga to‘g‘ri kelishi ehtimolini toping. 2) to‘rtta elementni almashtirishga to‘g‘ri kelishi ehtimolini toping.

2. Talabalarning saralash sport musabaqasida I bosqichdan 6 ta, II bosqichdan 4 ta va III bosqichdan 5 ta talaba qatnashmoqda. Talabalarning institut terma jamoasiga qabul qilinishi ehtimollari bosqichlar uchun mos ravishda 0,9, 0,7, 0,8 ga teng. 1) Tavakkaliga tanlangan talabaning institut terma jamoasiga qabul qilinishi ehtimolini toping. 2) talaba terma jamoaga qabul qilingan bo‘lsa, uning II bosqichdan bo‘lishi ehtimolini toping.

3. Bankning mijozlari olgan kreditlarini 0,1 ga teng ehtimol bilan muddatida qaytaradi. $X-$ 5 ta mizozning olgan kreditini muddatida qaytarishlari soni, $a=2,\ b=3.$

9-variant

1. Institutning 60% talabasi sport bilan shug‘ullanadii, 40% talabasi turli to‘garaklarga qatnashadi, 20% talabasi ham sport bilan shug‘ullanadi ham to‘garaklarga qatnashadi. Muxbir tasodifiy ravishda bitta talabani suhbatga chorladi. Bu talaba: $A-$faqat sport bilan shug‘ullanadi hodisasining ehtimolini toping; $B-$keltirilgan faoliyatlardan faqat bittasi bilan shug‘ullanadi hodisasining ehtimolini toping.

2. Sinovga kelgan 10 talabaning 3 tasi a’lo, 4 tasi yaxshi, 2 tasi o‘rtacha va 1tasi yomon tayyorgarlikka ega. Sinov biletlarida 20 ta savol bor. A’lo tayyorgarlikka ega talaba barcha 20 ta savolga, yaxshi tayyorgarlikka ega talaba 16 ta savolga, o‘rtacha tayyorgarlikka ega talaba 10 ta savolga, yomon tayyorgarlikka ega talaba 5 ta savolga javob berishi mumkin. Tavakkaliga chaqirilgan talaba berilgan 3 ta istalgan savolga javob berdi. Bu talabaning : 1) yaxshi tayyorgarlikka ega bo‘lishi ehtimolini; 2) yomon tayyorgarlikka ega bo‘lishi ehtimolini toping.

3. Qutidagi 10 ta detaldan 4 tasi yaroqsiz. $X-$3 ta olingan detaldan yaroqsiz detallar soni, $a=1,\ b=2.$

10-variant

1. Kerakli detalning birinchi, ikkinchi va uchinchi qutida bo‘lishi ehtimollari mos ravishda 0,7; 0,8; 0,9 ga teng. Detalning: faqat uchinchi qutida bo‘lishi; 2) faqat ikkita qutida bo‘lishi; 3) har uchala qutida bo‘lishi ehtimollarini toping.

2. Bir turdagi sharlar solingan uchta qutidan birinchisida 6 ta oq va 6 ta qora shar, ikkinchisida 8 ta oq va 4 ta qora shar, uchinchisida 6 ta oq shar bor. Qutilardan bittasi tavakkaliga tanlanadi va qutidan 1 ta shar olinadi. Bu shar oq bo‘lsa, uning ikkinchi qutidan olingan bo‘lishi ehtimolini toping.

3. Firma buxgalteri hisobat xujjatlarida 5 % xatoga yo‘l qo‘yadi. $X-$ 3 ta tanlangan xujjatlardagi xatolar soni, $a=2,\ b=3$.

11-variant

1. To‘rtta ovchi nishonga qarata ushbu tartibda o‘q uzishga kelishib olishdi: navbatdagi ovchi undan oldingi ovchi nishonga tekkiza olmagan taqdirdagina o‘q uzadi. Har bir ovchining nishonga tekkazishi ehtimoli 0,8 ga teng bo‘lsa, 2 ta o‘q uzilishi kerak bo‘lishining ehtimolini toping.

2. O‘zbekiston havo yo‘llari yo‘nalishlarining 60 %i mahalliy, 30 %i MDH davlatlari va 10 %i xalqaro yo‘nalishlarda. Mahalliy yo‘nalishdagi yo‘lovchilarning 40 %i, MDH yo‘nalishdagi yo‘lovchilarning 60 %i, xalqaro yo‘nalishdagi yo‘lovchilarning 80 %i tadbirkorlik ishlari bilan yo‘lga chiqadi. Aeroportga kelgan yo‘lovchilardan bittasi tavakkaliga tanlandi. Bu yo‘lovchining xalqora yo‘nalishdagi tadbirkor bo‘lishi ehtimolini toping.

3. O‘yin kubigi 3 marta tashlanadi. $X-$ kubikda 6 ochko tushishlari soni, $a=0,\ b=2$.

12-variant

1. Ikkita avtomat detallar tayyorlaydi. Birinchi avtomatning nostandart detal tayyorlash ehtimoli 0,07 ga, ikkinchisiniki esa 0,09 ga teng. Ikkinchi avtomatning ishlab chiqarish unumdorligi birinchi avtomatning unumdorligidan uch marta yuqori. Tavakkaliga olingan detalninig standart bo‘lishi ehtimolini toping.

2. Kamondan otuvchi 15 ta sportchining 3 tasi sport ustasi, 6 tasi sport ustaligiga nomzod va 6 tasi birinchi razryadli. Sportchilarning o‘qni nishonga tekkazishi ehtimollari mos ravishda 0,9, 0,8, 0,7 ga teng. Tavakkaliga tanlangan sportchi otgan o‘q nishonga tegdi. Bu kamondan otuvchining sport ustasi bo‘lishi ehtimolini toping.

3. Ovchi o‘ljaga qarata o‘q tekkunicha o‘q uzadi, ammo 4 tadan ko‘p bo‘lmagan o‘q uzishga ulguradi. Bitta o‘q uzishda ovchining o‘ljaga tekkazishi ehtimoli 0,7 ga teng. $X-$ uzilgan o‘qlar soni, $a=1,\ b=3$.

13-variant

1. Talaba 25 ta savoldan 20 tasini biladi. Talaba biletdagi 4 ta savoldan kamida 3 tasiga javob bersa sinovdan o‘tgan hisoblanadi. Biletdagi birinchi savolga nazar tashlagan talaba uni bilishini aniqladi. Talabaning: 1) sinovni topshirishi ehtimolini; 2) sinovni topshira olmasligi ehtimolini toping.

2. 52 qartali dastadan bir yo‘la n $(n<52)$ ta qarta olindi. Ulardan biri ochib ko‘rilganda, u tuz bo‘lib chqdi. Keyin bu qarta olingan boshqa qartalar bilan birga aralashtirildi. So‘gra bu qartalardan ikkinchi marta bitta qarta olindi. Bu qartaning tuz bo‘lishi ehtimolini toping.

3. Korxona ishlab chiqargan 25 ta mahsulotning 6 tasi sifatsiz. $X-$ 3 ta tanlangan mahsulotlarning sifatsizlari soni, $a=1,\ b=3.$

14-variant

1. Ikkita o‘yinchi o‘yin kubigini navbatma-navbat tashlaydi. Kimning kubigida olti ochko tushsa, u yutgan hisoblanadi. 1) o‘yin kubigini birinchi bo‘lib tashlagan o‘yinchining yutishi ehtimolini toping; 2) o‘yin kubigini ikkinchi bo‘lib tashlagan o’yinchining yutishi ehtimolini toping.

2. Birinchi qutida 6 ta qora va 5 ta oq shar, ikkinchi qutida 8 ta qora va 4 ta oq shar bor. Birinchi qutidan tavakkaliga 3 ta shar olinadi va ikkinchisiga solinadi, keyin ikkinchi qutidan tavakkaliga 4 ta shar olinadi. Ikkinchi qutidan olingan barcha sharlarning oq bo‘lishi ehtimolini toping.

3. Firma mahsulotining 10 % i sifatsiz. Firma mahsulotlaridan tavakkaliga 6 tasi olingan. $X-3$ mahsulotdan sifatsizlari soni, $a=1,\ b=2.$

15-variant

1. Ikkita qutining birida 5 ta oq, 11 ta qizil va 8 ta yashil sharlar bor, ikkinchisida mos ravishda 10, 8 va 6 ta sharlar bor. Har ikkala qutidan tavakkaliga bittadan shar olinadi. Olingan har ikkala sharning bir xil rangli bo‘lishi ehtimolini toping.

2. To‘rtta mergan o‘zaro bog‘liq bo‘lmagan holda bitta nishonga bittadan o‘q uzdi. Merganlarning nishonga tekkazish ehtimollari mos ravishda 0,4, 0,6, 0,7 va 0,8 ga teng. Sinash tugagandan keyin nishondan uchta o‘qning izi topildi. To‘rtinchi merganning o‘qi xato ketganligi ehtimolini toping.

3. Ikkita o‘yin kubigi 2 marta tashlanadi. $X-$har ikkala kubiklarda juft ochkolar tushishlari soni, $a=1,\ b=2.$

16-variant

1. Benzin quyish shaxobchasi joylashgan shosse bo‘ylab o‘tayotgan yuk mashinalari sonining engil mashinalar soniga nisbati 7:2 kabi. Benzin olish uchun yuk masinasining 10%i, engil mashinaning 20%i shahobchaga kiradi. Benzin olish uchun kirib kelgan mashina yuk mashinasi bo‘lishi ehtimolini toping.

2. Birinchi jamoada 6 ta o‘g‘il va 4 ta qiz sportchi, ikkinchi jamoada 4 ta o‘g‘il va 6 ta qiz sportchi bor. Terma jamoaga birinchi jamoadan 6 ta va ikkinchi jamoadan 4 ta sportchi olindi. Terma jamoadan tavakkaliga bir o‘yinchi tanlanadi. Bu o‘yinchining qiz sportchi bo‘lishi ehtimolini toping.

3. Kompyuterning virus bilan kasallanishi ehtimoli 0,2 ga teng. $X-$ 4 ta tanlangan kompyuterning virus bilan kasallanishlari soni, $a=2,\ b=4.$

17-variant

1. Yig‘uvchiga kerakli detal birinchi, ikkinchi, uchinchi va to‘rtinchi qutida bo‘lishi ehtimollari mos ravishda 0,6, 0,6, 0,7 va 0,8 ga teng. Kerakli detalning: 1) ko‘pi bilan uchta qutida bo‘lishi; 2) kamida ikkita qutida bo‘lishi bo’lishi ehtimolini toping.

2. Plastmassa idishlar uchta avtomat mos ravishda idishlarning 40, 35 va 25 % ini ishlab chiqaradi. Birinchi avtomat ishlab chiqargan idishlarning 0,13 qisni, ikkinchi avtomat ishlab chiqargan idishlarning 0,025 qismi va uchinchi avtomat ishlab chiqargan idishlarning 0,025 qismi nostandart. Tanlangan standart idish uchinchi avtomatda tayyorlanganligi ehtimolini toping.

3. Darslik 100000 nusxada chop etilgan. Uning noto‘g‘ri muqovalangan bo‘lishi ehtimoli 0,0001 ga teng. $X-$noto‘g‘ri muqovalangan darsliklar soni, $a=100,\ b=1000.$

18-variant

1. Uchta mergan bir vaqtda nishonga o‘q uzmoqda. Merganlarning o‘qni nishonga tekkazishi ehtimollari mos ravishda 0,5, 0,6 va 0,8 ga teng. Nishonning 2 tadan kam bo‘lmagan o‘q bilan yakson bo’lishi ehtimolini toping.

2. Yo‘nalishning ikki bekati orasida avtobus ichida 3 ta yo‘lovchi ketmoqda. Yo‘lovchilardan har birining navbatdagi bekatda tushishi ehtimollari bog‘liqmas va 0,1 ga teng. Navbatdagi bekatda avtobusni 3 ta yo‘lovchi kutmoqda. Ularning avtobusga chiqishi ehtimollari bog‘liqmas va 0,3 ga teng. Avtobus navbatdagi bekatdan jo‘nagandan keyin, avtobus ichidagi yo‘lovchilar soni o‘zgarmasligi ehtimolini toping.

3. Qurilma 3 ta elementdan tashkil topgan. Bitta sinashda har qaysi elementning ishdan chiqishi ehtimoli 0,4 ga teng. $X-$ishdan chiqqan elementlar soni, $a=1,\ b=2.$

19-variant

1. Bir xil detal ishlab chiqariladigan uchta stanokning mehnat unumdorligi 1:3:6 ga teng. Saralanmagan detallar partiyasidan tavakkaliga 2 ta detal olindi. Olingan detallarning: 1) 1 tasi uchinchi stanokda ishlab chiqarilgan bo‘lishi; 2) har ikkala detal bitta stanokda ishlab chiqarilgan bo‘lishi ehtimolini toping.

2. Qutida 2 ta oq va 4 ta oq shar bor. Qutidan tavakkaliga 2 ta shar olinadi va rangiga qaralmasdan chetga qo‘yiladi. Keyin qutidan yana bitta shar olinadi. Bu sharning oq bo‘lishi ehtimolini toping.

3. Ovchining bitta o‘q uzishda o‘qni ayiqqa tekkazishi ehtimoli 0,7 ga teng va har bir otishdan keyin 0,1 ga kamayadi. $X-$ 4 ta otishda ayiqqa tekkan o‘qlar soni, $a=1,\ b=3.$

20-variant

1. Hakamlar hay’ati uchta hakamdan iborat. Birinchi va ikkinchi hakam bir-biriga bog‘liq bo‘lmagan holda 0,8 ehtimol bilan to‘g‘ri qaror qabul qiladi. Uchinchi hakam qaror qabul qilishi uchun tanga tashlaydi. Hakamlar hay’ati eng ko‘p ovozlar ntijasiga ko‘ra oxirgi qarorni qabul qiladi. Hakamlar hay’atining to‘g‘ri qaror qabul qilishi ehtimolini toping.

2. Uchta mergan nishonga qarata baravariga o‘q uzadi. Nisonning bitta o‘q tekkanda yakson bo‘lishi ehtimoli 0,1 ga, ikkita o‘q tekkanda yakson bo‘lishi ehtimoli 0,3 ga, uchta o‘q tekkanda yakson bo‘lishi ehtimoli 0,6 ga teng. Uchta o‘q uzildi va nishon yakson bo‘ldi. Nechta o‘qning nishonga tegishi ehtimolliroq bo‘ladi?

3. 5 ta atirguldan ikkitasi oq. 2 ta atirgul bir vaqtda uziladi. $X-$uzilgan oq atirgullar soni, $a=0,\ b=1.$

21-variant

1. Uchta quroldan otilgan oqning nishonga tegishi ehtimollari mos ravishda 0,8, 0,7, 0,9 ga teng. Hamma quroldan baravariga o‘q uzilganda ikkita o‘qning nishonga tegishi ehtimolini toping.

2. Kitob javoninda 20 ta kitob bo‘lib, ulardan 4 tasi uy egasi tomonidan o‘qib chiqilgan. Uy egasi tavakkaliga bitta kitob oladi va uni o‘qib chiqib, javonga qayta qo‘yadi. Keyin uy egasi javondan navbatdagi kitobni oladi. Bu kitobning o‘qilmagan bo‘lishi ehtimolini toping.

3. Ichida 5 ta oq va 7 ta qora shar bo‘lgan idishdan 4 ta shar olinadi. $X-$ olingan oq sharlar soni, $a=3,\ b=4.$

22-variant

1. Samolyotga qarata o‘q uzilmoqda. Samolyotni urib tushirish uchun o‘qlarni ikkita dvigatelga yoki uchuvchi kabinasiga tekkazish yetarli. O‘qning birinchi dvigatelga tegishi ehtimoli p_1 ga, ikkinchi dvigatelga tegishi ehtimoli p_2 ga va kabinaga tegishi ehtimoli p_3 ga teng bo‘lsa, samolyotning urib tushirilishi ehtimolini toping.

2. Ikkita qutining birida 12 ta qora va 10 ta oq shar, ikkinchisida 12 ta oq va 10 ta qora shar bor. Birinchi qutidan tavakkaliga ikkita shar olinadi va ikkinchisiga solinadi, keyin ikkinchi qutidan tavakkaliga bitta shar olinadi. Ikkinchi qutidan olingan shar qora bo‘lsa, birinchi qutidan olingan har ikkala sharning oq bo‘lishi ehtimolini toping.

3. Do‘kondagi 10 ta televizordan 4 tasi “Soni” rusumli. $X-$ sotilgan 3 ta televizorlarning “Soni” rusumlilari soni, $a=2,\ b=4$.

23-variant

1. Guruh 8 ta a’lochi, 7 ta yaxshi o‘qiydigan va 5 ta kuchsiz shug‘ullanuvchi talabalardan iborat. Sinovda a’lochi faqat a’lo baho olishi, yaxshi o’qiydigan talaba a’lo yoki yaxshi baho olishi, kuchsiz shug‘ullanuvchi talaba yaxshi, qoniqarli yoki qoniqarsiz baho olishi mumkin. Sinovga tasodifiy ravishda bitta talaba chaqiriladi. Bu talabaning sinovda yaxshi yoki a’lo baho olishi ehtimolini toping.

2. Zavod bir turdagi mahsulotlar ishlab chiqaradi: har bir mahsulot p_i ehtimol bilan nosozlikka ega. Tayyor mahsulotlar k ta nazoratchi tomonidan tekshiriladi. $i-$ $(i=1,2,...,k)$ nazoratchi mahsulotning nosozligini p_i ehtimol bilan aniqlaydi. Nosozlik aniqlansa mahsulot zavodga qaytariladi. 1) Mahsulotning zavodga qaytarilishi ehtimolini toping; 2) mahsulotning 2- nazoratchi tomonodan qaytarilishi ehtimolini toping; 3) mahsulotning barcha nazoratchilar tomonodan qaytarilishi ehtimolini toping.

6. Oilada qiz bola tug‘ilishi ehtimoli 0,5 ga teng. $X-$ 4 farzandli oiladagi o‘g‘il bolalar soni, $a=2,\ b=3$.

24-variant

1. Nishon k ta nuqtadan o‘qqa tutilmoqda. Har bir nuqtadan bir- biriga bog‘liq bo‘lmagan holda $p_i\ (i=\overline{1,k})$ ehtimol bilan o‘q uziladi. 1) hech bo‘lmaganda bitta oqning nishonga tegishi ehtimolini toping; 2) barcha o‘qlarning nishonga tegishi ehtimolini toping.

2. Mijozning bankdan olgan kreditini qaytarmaslik ehtimoli iqtisodiy o‘shish davrida 0,04 ga, iqtisodiy tanglik davrida 0,13 ga teng. Agar iqtisodiy o‘sish davri boshlanishi ehtimoli 0,65 ga teng bo‘lsa, tasodifiy tanlangan mijozning kreditini qaytarmasligi ehtimolini toping.

3. Qulfga 4 ta kalitdan bittasi tushadi. X –qulfni ochishga qilingan urinishlar soni, $a=0,\ b=3$.

25-variant

1. Qutida 3 ta oq va 7 ta qizil shar bor. Qutidan tavakkaliga ketma-ket qutiga qaytarilmasdan 2 ta shar olinadi. A – olingan birinchi sharning oq chiqishi, B – olingan ikkinchi sharning oq chiqishi, C – hech bo‘lmaganda bitta sharning oq chiqishi hodisalari bo‘lsa, $P_A(B),\ P_B(A),\ P_C(A)$ ehtimollarni toping.

2. Zavod ishlab chiqargan detalning 95%i standartlik talabiga javob beradi. Ishlab chiqarilgan detallar quyidagicha nazoratdan o‘tkazilad: agar detal standart bo‘lsa, u 0,98 ehtimol bilan yaroqli deb topiladi; agar detal nostandart bo‘lsa, u 0,06 ehtimol bilan yaroqli deb topiladi. 1) Tavakkaliga olingan detalning yaroqli deb topilishi ehtimolini toping; 2) bir marta nazoratdan o‘tgan detalning yaroqli deb topilishi ehtimolini toping.

3. Merganning bitta o‘q uzishda o‘qni nishonga tekkazishi ehtimoli 0,8 ga teng. U har bir tekkazgan o‘qi uchun 5 ochko oladi va xatosi uchun ochko olmaydi. X – 3 ta o‘q otilgnda merganning to‘plagan ochkolari soni, $a=0,\ b=2$.

26-variant

1. Avtobus yo‘nalishiga uchta avtobus bilan xizmat ko‘rsatiladi. Smena davomida yo‘nalishdagi avtobuslarda nosozliklar ro‘y berishi ehtimollari mos ravishda 0,2, 0,1, 0,08 ga teng. Smena davomida: 1) faqat bitta avtobusda nosozlik ro‘y birishi ehtimolini; 2) faqat ikkita avtobusda nosozlik ro‘y berishi ehtimolini toping.

2. Baliqchining ov qilish uchun uchta belgilangan joyi bo‘lib, u birinchi joyda 0,3 ehtimol bilan, ikkinchi joyda 0,2 ehtimol bilan va uchinchi joyda 0,6 ehtimol bilan baliq ovlashi mumkin. Baliqchi uch marta qarmoq tashlaganda bitta baliq ilindi. Bu baliqning birinchi joyda ilinganligi ehtimolini toping.

3. Ikkita mergan navbati bilan nishonga o‘q uzadi. Bitta o‘q uzishda xato ketish ehtimoli birinchi mergan uchun 0,2 ga, ikkinchisi uchun 0,4 ga teng. 4 ta o‘q uzilgan. X – nishonga tekkunicha otilgan o‘qlar soni , $a=1,\ b=3$.

27-variant

1. 52 ta qartali dastadan bir vaqtda 4 ta qarta olinadi. Olingan qartalarning har xil turda bo‘lishi ehtimolini toping: 1) qartalar qutiga qaytarilsa; 2) qartalar qutiga qaytarilmasa.

2. Bir turdagi mahsulotlarning ikkita partiyasi bor: birinchi partiyada 20 ta mahsulot bo‘lib, ulardan 6 tasi yaroqsiz; ikkinchi partiyada 15 ta mahsulot bo‘lib, ulardan 5 tasi yaroqsiz. Birinchi partiyadan 8 ta ikkinchi partiyadan 10 ta mahsulot olinadi. Olingan 18 ta mahsulot aralashtirilib, yangi partiya hosil qilinadi. Yangi partiyadan tavakkaliga olingan mahsulotning yaroqsiz chiqishi ehtimolini toping.

3. Qutidagi 8 ta detaldan 6 tasi yaroqli. $X-4$ ta olingan detalning yaroqlilari soni, $a=1,\ b=2.$

28-variant

1. Uchta basketbolchi savatga bittadan to‘p tashlaydi. Basketbolchilarning to‘pni savatga tushirishi ehtimollari mos ravishda 0,9, 0,8, 0,7 ga teng. 1) ikkita to‘pning savatga tushishi; 2) kamida ikkita to‘pning savatga tushishi; 3) hech bo‘lmaganda bitta to’pning savatga tushishi ehtimolini toping.

2. Yo‘lovchi chipta olishi uchun uchta kassaga mos ravishda 0,3, 0,2, 0,5 ehtimollar bilan murojat qilishi mumkin. Yo‘lovchi kelgan vaqtda kassalarda bo‘lgan chiptalarning mos ravishda 0,25, 0,4 va 0,35 qismlari sotib bo‘lingan. Yolovchi kassalardan biriga murojat qildi va chipta oldi. Bu chiptaning ikkinchi kasadan olingan bo‘lishi ehtimolini toping.

3. Ikkita tanga 3 martadan tashlanadi. $X-$gerbli tomon tushishlar soni, $a=1,\ b=2.$

29-variant

1. Nashriyot uchta aloqa bo‘limiga gazetalar jo‘natadi. Gazetalarning aloqa bo‘limlariga o‘z vaqtida etkazilishi ehtimollari mos ravishda 0,95, 0,9, 0,8 ga teng. Aloqa bo‘limlaridan: 1) faqat bittasi gazetalarni o‘z vaqtida olishi; 2) hech bo‘lmaganda bittasi gazetalarni o‘z vaqtida olishi ehtimolini toping.

2. 20 ta mergandan 6 tasining nishonga tekkazishi ehtimoli 0,7 ga, 7 tasiniki 0,8 ga, 5 tasiniki 0, 6 ga va 2 tasiniki 0, 5 ga teng. Tavakkaliga tanlangan mergan nishonga tekkaza olmadi. Uning qaysi guruhga tegishli bo‘lishi ehtimoli katta?

3. Biletda 3 ta misol bor. Misollarning to‘g‘ri yechilishi ehtimollari 0,9, 0,8 va 0,7 ga teng. $X-$ to‘g‘ri yechilgan misollar soni, $a=0,\ b=1.$

30-variant

1. Talabaning uchta yozma ishdan o'tishi ehtomollari mos ravishda 0,9, 0,8 va 0,9 ga teng. Talabaning: 1) 2 ta yozma ishdan o'tishi ehtimolini; 2) hech bo'lmaganda 2 ta yozma ishdan o'tishi ehtimolini toping.

2. Bir xil detal ishlab chiqariladigan uchta stanok 1:3:6 nisbatda ishlab chiqarish unumdorligiga ega. Stanoklarda ishlab chiqrilgan detallarning yaroqsiz bo'lishi ehtimollari mos ravishda 0,05, 0,15 va 0,05 ga teng. Ishlab chiqarilgan detallar orasidan tavakkaliga olingan detalning yaroqli bo'lishi ehtimolini toping.

3. Tanga 5 marta tashlanadi. $X-$ gerbli tomon tushishi soni, $a=3,\ b=4.$

LABORATORIYA ISHI

Chiziqli regressiya tenglamasini eng kichik kvadratlar usuli bilan toping. Hisoblashni soddalashtirish uchun $u_i=\dfrac{x_i-C_1}{h_1},\quad v_i=\dfrac{y_i-C_2}{h_2}$ almashtirishlardan foydalaning.

1.

Y \ X	5	10	15	20	25	30	n_y
30	2	6	-	-	-	-	8
40	-	5	3	-	-	-	8
50	-	-	7	40	2	-	49
60	-	-	4	9	6	-	19
70	-	-	-	4	7	5	16
n_x	2	11	14	53	15	5	$n=100$

2.

Y \ X	2	5	8	11	14	17	n_y
1	2	4	-	-	-	-	6
6	-	6	3	-	-	-	9
11	-	-	6	35	4	-	45
16	-	-	2	8	6	-	16
21	-	-	-	14	7	3	24
n_x	2	10	11	57	17	3	$n=100$

3.

Y \ X	5	10	15	20	25	30	n_y
45	2	4	-	-	-	-	6
55	-	3	5	-	-	-	8
65	-	-	5	35	5	-	45
75	-	-	2	8	17	-	27
85	-	-	-	4	7	3	14
n_x	2	7	12	47	29	3	$n=100$

4.

Y \ X	15	20	25	30	35	40	n_y
25	3	4	-	-	-	-	7
35	-	6	3	-	-	-	9
45	-	-	6	35	2	-	43
55	-	-	12	8	6	-	26
65	-	-	-	4	7	4	15
n_x	3	10	21	47	15	4	$n=100$

5.

Y \ X	5	10	15	20	25	30	n_y
210	3	5	-	-	-	-	8
220	-	4	4	-	-	-	8
230	-	-	7	35	8	-	50
240	-	-	2	10	8	-	20
250	-	-	-	5	6	3	14
n_x	3	9	13	50	22	3	$n=100$

6.

Y \ X	10	15	20	25	30	35	n_y
100	-	-	-	-	6	1	7
120	-	-	-	-	4	2	6
140	-	8	10	5	-	-	23
160	6	4	-	-	-	-	10
180	3	-	1	-	-	-	4
n_x	9	12	11	5	10	3	$n=50$

7.

$Y \backslash X$	45	50	55	60	65	70	n_y
30	-	-	-	-	8	4	12
35	-	1	6	20	30	12	69
40	1	2	10	40	35	5	93
45	-	1	10	8	2	-	21
50	-	2	2	1	-	-	5
n_x	1	6	28	69	75	21	$n=200$

8.

$Y \backslash X$	10	15	20	25	30	35	n_y
40	2	4	-	-	-	-	6
50	-	3	7	-	-	-	10
60	-	-	5	30	10	-	45
70	-	-	7	10	8	-	25
80	-	-	-	5	6	3	14
n_x	2	3	19	45	24	3	$n=100$

9.

$Y \backslash X$	15	20	25	30	35	40	n_y
15	4	1	-	-	-	-	5
25	-	6	4	-	-	-	10
35	-	-	2	50	2	-	54
45	-	-	1	9	7	-	17
55	-	-	-	4	3	7	14
n_x	4	7	7	63	12	7	$n=100$

10.

$Y \backslash X$	10	15	20	25	30	35	n_y
125	-	-	-	-	6	1	7
150	-	-	-	-	4	2	6
175	-	8	10	5	-	-	23
200	6	4	-	-	-	-	10
225	3	-	1	-	-	-	4
n_x	9	12	11	5	10	3	$n=50$

11.

Y \ X	45	50	55	60	65	70	n_y
30	-	1	-	-	-	-	1
35	1	2	5	12	-	-	20
40	-	3	2	8	-	-	13
45	-	-	1	-	7	-	8
50	-	-	-	-	4	4	8
n_x	1	6	8	20	11	4	$n=50$

12.

Y \ X	15	20	25	30	35	40	n_y
110	4	1	-	-	-	-	5
120	-	6	4	-	-	-	10
130	-	-	2	50	2	-	54
140	-	-	1	13	10	7	31
n_x	4	7	7	63	12	7	$n=100$

0

13.

Y \ X	15	20	25	30	35	40	n_y
15	4	1	-	-	-	-	5
25	-	6	4	-	-	-	10
35	-	-	2	50	2	-	54
45	-	-	1	13	10	7	31
n_x	4	7	7	63	12	7	$n=100$

0

14.

Y \ X	12	17	22	27	32	37	n_y
25	2	4	-	-	-	-	6
35	-	6	3	-	-	-	9
45	-	-	6	35	4	-	45
55	-	-	2	8	6	-	16
65	-	-	-	14	7	3	24
n_x	2	10	11	57	17	3	$n=100$

15.

Y \ X	4	6	8	10	12	14	n_y
3	7	21	10	-	-	-	38
8	-	5	15	10	-	-	30
13	-	-	11	10	4	-	25
18	-	-	-	4	1	2	7
n_x	7	26	36	24	5	2	$n=100$

16.

Y \ X	3	7	11	15	19	23	n_y
4	2	4	-	-	-	-	6
6	-	3	5	-	-	-	8
8	-	-	5	35	5	-	45
10	-	-	2	8	15	-	25
12	-	-	-	4	7	5	16
n_x	2	7	12	47	27	5	$n=100$

17.

Y \ X	25	30	35	40	45	50	n_y
30	-	-	-	-	8	4	12
35	-	1	6	20	30	12	69
40	1	2	10	40	35	5	93
45	-	1	10	8	2	-	21
50	-	2	2	1	-	-	5
n_x	1	6	28	69	75	21	$n=200$

18.

Y \ X	5	10	15	20	25	30	n_y
40	3	5	-	-	-	-	8
50	-	4	4	-	-	-	8
60	-	-	7	35	8	-	50
70	-	-	2	10	8	-	20
80	-	-	-	5	6	3	14
n_x	3	9	13	50	22	3	$n=100$

19.

Y \ X	10	15	20	25	30	35	n_y
50	-	-	-	-	6	1	7
70	-	-	-	-	4	2	6
90	-	8	10	5	-	-	23
110	6	4	-	-	-	-	10
130	3	-	1	-	-	-	4
n_x	9	12	11	5	10	3	$n=50$

20.

Y \ X	5	10	15	20	25	30	n_y
2	7	21	10	-	-	-	38
12	-	5	15	10	-	-	30
22	-	-	10	11	4	-	25
32	-	-	-	4	1	2	7
n_x	7	26	35	25	5	2	$n=100$

21.

Y \ X	10	15	20	25	30	35	n_y
5	2	4	-	-	-	-	6
15	-	3	7	-	-	-	10
25	-	-	5	30	10	-	45
35	-	-	7	10	8	-	25
45	-	-	-	5	6	3	14
n_x	2	3	19	45	24	3	$n=100$

22.

Y \ X	15	20	25	30	35	40	n_y
25	4	1	-	-	-	-	5
35	-	6	4	-	-	-	10
45	-	-	2	50	2	-	54
55	-	-	1	13	10	7	31
n_x	4	7	7	63	12	7	$n=100$

0

23.

Y \ X	20	25	30	35	40	45	n_y
25	2	4	-	-	-	-	6
35	-	6	3	-	-	-	9
45	-	-	6	35	4	-	45
55	-	-	2	8	6	-	16
65	-	-	-	14	7	3	24
n_x	2	10	11	57	17	3	$n=100$

24.

Y \ X	10	12	14	16	18	20	n_y
3	7	21	10	-	-	-	38
8	-	5	15	10	-	-	30
13	-	-	11	10	4	-	25
18	-	-	-	4	1	2	7
n_x	7	26	36	24	5	2	$n=100$

25.

Y \ X	3	7	11	15	19	23	n_y
12	2	4	-	-	-	-	6
14	-	3	5	-	-	-	8
16	-	-	5	35	5	-	45
18	-	-	2	8	15	-	25
20	-	-	-	4	7	5	16
n_x	2	7	12	47	27	5	$n=100$

26.

Y \ X	5	10	15	20	25	30	n_y
100	7	21	10	-	-	-	38
120	-	5	15	10	-	-	30
140	-	-	10	11	4	-	25
160	-	-	-	4	1	2	7
n_x	7	26	35	25	5	2	$n=100$

27.

Y \ X	5	10	15	20	25	30	n_y
25	2	6	-	-	-	-	8
35	-	5	3	-	-	-	8
45	-	-	7	40	2	-	49
55	-	-	4	9	6	-	19
65	-	-	-	4	7	5	16
n_x	2	11	14	53	15	5	$n=100$

28

Y \ X	5	8	11	14	17	20	n_y
1	2	4	-	-	-	-	6
6	-	6	3	-	-	-	9
11	-	-	6	35	4	-	45
16	-	-	2	8	6	-	16
21	-	-	-	14	7	3	24
n_x	2	10	11	57	17	3	$n=100$

29.

Y \ X	5	10	15	20	25	30	n_y
130	2	4	-	-	-	-	6
140	-	3	5	-	-	-	8
150	-	-	5	35	5	-	45
160	-	-	2	8	17	-	27
170	-	-	-	4	7	3	14
n_x	2	7	12	47	29	3	$n=100$

30.

Y \ 1	8	13	18	23	28	33	n_y
35	3	4	-	-	-	-	7
50	-	6	3	-	-	-	9
65	-	-	6	35	2	-	43
80	-	-	12	8	6	-	26
95	-	-	-	4	7	4	15
n_x	3	10	21	47	15	4	$n=100$

II bob
KOMPLEKS O‘ZGARUVCHILI FUNKSIYALAR NAZARIYASI

2.1. KOMPLEKS SONLAR

Kompleks son. Kompleks sonlar ustida amallar

2.1.1. z *kompleks son* deb ma’lum taryibda berilgan x va y haqiqiy sonlar juftiga aytiladi va $z=(x,y)$ yoki $z=x+iy$ deb yoziladi, bu yerda $x,\ y-$haqiqiy sonlar, $i\ (i^2=-1)-$ mavhum birlik.

$x\,,y$ larga mos ravishda kompleks sonning *haqiqiy* va *mavhum qismlari* deyiladi va $x=\operatorname{Re} z,\ y=\operatorname{Im} z$ kabi belgilanadi.

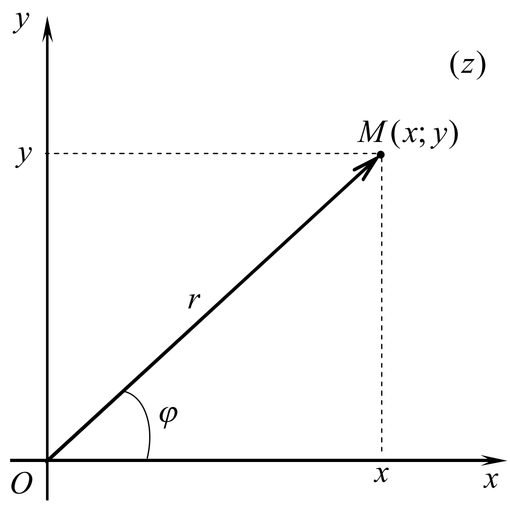
$z_1=(x,y_1)$ va $z_2=(x_2,y_2)$ kompleks sonlarida $x_1=x_2,\ y_1=y_2$ bo‘lsa, $z_1=z_2$ bo‘ladi.

$z=x+iy$ kompleks son nolga teng bo‘ladi, faqat $x=y=0$ bo‘lsa. Kompleks sonlar uchun “katta” va “kichik” tushunchalari kiritilmagan.

Mavhum qismlarining ishorasi bilan farq qiluvchi $z=x+iy$ va $\bar{z}=x-iy$ sonlarga *qo‘shma kompleks sonlar* deyiladi.

Haqiqiy va mavhum qismlarining ishorasi bilan farq qiluvchi $z_1=x+iy$ va $z_2=-x-iy$ sonlarga *qarama-qarshi kompleks sonlar* deyiladi.

Har bir $z=x+iy$ kompleks sonni geometrik jihatdan Oxy koordinatalar tekisligining $M(x;y)$ nuqtasi bilan ifodalash mumkin, bu yerda $x=\operatorname{Re} z,\ y=\operatorname{Im} z$. Shuningdek, koordinatalar tekisligining har bir $M(x;y)$ nuqtasini $z=x+iy$ kompleks sonning geometrik tasviri deb qarash mumkin (1-shakl).



1-shakl.

Oxy tekislikka *kompleks tekisligi* deyiladi va (z) kabi belgilanadi. Bunda, $z=x$ haqiqiy sonlar *haqiqiy o‘q* deb ataluvchi Ox o‘qning nuqtalari bilan aniqlanadi; $z=iy$ mavhum sonlar *mavhum o'q* deb ataluvchi Oy o'qning nuqtalari bilan aniqlanadi.

⇨ $z=x+iy$ kompleks sonni $M(x;y)$ nuqtaning radius vektori $\vec{r}=\overrightarrow{OM}$ bilan ifodalash mumkin (1-shakl). Bunda $\vec{r}$ vektorning uzunligiga *kompleks sonning moduli* deyiladi va $|z|$ yoki r bilan belgilanad; $\vec{r}$ vektorning Ox o'qning musbat yo'nalishi bilan hosil qilgan burchagiga *kompleks sonning argumenti* deyiladi va $Argz$ yoki φ bilan belgilanadi.

⇨ $z=0$ kompleks sonning argumenti aniqlanmagan. $z\neq 0$ kompleks sonning argumenti ko'p qiymatli bo'lib, $2\pi k$ $(k=0,-1,1,-2,2,...)$ qo'shiluvchigacha aniqlikda topiladi: $Argz=\arg z+2\pi k$, bu yerda $\arg z-(-\pi;\pi]$oraliqda yotuvchi *argumentning bosh qiymati*, ya'ni $-\pi<\arg z\leq\pi$ (ayrim hollarda argumentning bosh qiymati sifatida $[0;2\pi)$ oraliqqa tegishli qiymat olinadi).

Kompleks sonning algebraik shakli deb, $z=x+iy$ ifodaga, *kompleks sonning trigonometrik shakli* deb, $z=r(\cos\varphi+i\sin\varphi)$ ifodaga aytiladi. Bunda: $r=|z|$ modul $r=|z|=\sqrt{x^2+y^2}$ formula bilan bir qiymatli aniqlanadi; φ argument $\cos\varphi=\frac{x}{r},\ \sin\varphi=\frac{y}{r},\ t\varphi=\frac{y}{x}$ formulalardan topiladi.

Kompleks sonning algebraik shaklidan trigonometrik shakliga o'tishda kompleks son argumentining bosh qiymatini aniqlash etarli bo'ladi. Bu qiymat $-\pi<\arg z\leq\pi$ bo'lgani ucun $t\varphi=\frac{y}{x}$ tenglikdan topiladi:

$$\arg z=\begin{cases} arctg\frac{y}{x},\ I,IV\ choraklarning\ ichki\ nuqtalarida, \\ arctg\frac{y}{x}+\pi,\ II\ chorakning\ ichki\ nuqtalarida, \\ arctg\frac{y}{x}-\pi,\ III\ chorakning\ ichki\ nuqtalarida. \end{cases}$$

Eyler formulasi deb ataluvchi $\cos\varphi+i\sin\varphi=e^{i\varphi}$ ifoda yordamida $z=r(\cos\varphi+i\sin\varphi)$ tenglikdan $z=re^{i\varphi}$ ifoda keltirib chiqariladi. Bu ifodaga *kompleks sonning ko'rsatkichli* (*yoki eksponensial*) *shakli* deyiladi, bu yerda $r=|z|-$kompleks sonning moduli; $\varphi=\arg z+2\pi k$ $(k=0,-1,1,-2,2,...)-$kompleks sonning argumenti.

Eyler formulasiga ko'ra $e^{i\varphi}$ funksiya 2π davrli davriy funksiya bo'ladi. Shu sababli z kompleks sonni ko'rsatkichli shaklda yozish uchun kompleks son argumentining bosh qiymatini, ya'ni $\varphi=\arg z$ ni aniqlash etarli bo'ladi.

Teorema(algebraning asosiy teoremasi). $a_1, a_2, ..., a_n$ kompleks koef-fitsiyentli har qanday $P_n(z) = z^n + a_1 z^{n-1} + a_2 z^{n-2} + ... + a_n \quad (n \geq 1)$ ko'phad kompleks sonlar to'plamida hech bo'lmaganda bitta yechimga ega bo'ladi.

Xususan, $z^2 + pz + q = 0 \left(\frac{p^2}{4} - q < 0 \right)$ tenglama ikkita kompleks qo'shma $z_{1,2} = \alpha \pm i\beta$ yechimlarga ega bo'ladi, bu yerda $\alpha = -\frac{p}{2}, \ \beta = \sqrt{q - \frac{p^2}{4}}$.

1-misol. $z^2 - 6z + 13 = 0$ tenglamani yeching va $z^2 - 6z + 13$ kvadrat uchhadni ko'paytuvchilarga ajrating.

$$\alpha = -\frac{(-6)}{2} = 3, \ \beta = \sqrt{13 - \frac{(-6)^2}{4}} = 2.$$

U holda

$$z_1 = 3 + 2i, \quad z_2 = 3 - 2i;$$

$$z^2 - 6z + 13 = (z - z_1)(z - z_2) = ((z-3) - 2i)((z-3) + 2i).$$

2-misol. Berilgan kompleks sonlarni turli (algebraik, trigonometrik va ko'rsatkichli) shakllarda yozing:

1) $z_1 = 1 + i\sqrt{3};$ 2) $z_2 = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} \right);$

3) $z_3 = 5e^{iarctg\frac{3}{4}};$ 4) $z_4 = -\sin\frac{\pi}{8} + i\cos\frac{\pi}{8}.$

1) $z_1 = 1 + i\sqrt{3}$ sonning moduli va argumentini topamiz.

Bunda $x_1 = 1 > 0$ va $y_1 = \sqrt{3} > 0$.

U holda

$$r_1 = |z_1| = \sqrt{1^2 + (\sqrt{3})^2} = 2, \ \varphi_1 = \arg z_1 = arctg\frac{\sqrt{3}}{1} = \frac{\pi}{3}.$$

Bundan

$$z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \right).$$

$\cos\varphi + i\sin\varphi = e^{i\varphi}$ (Eyler formulasi) tenglikni qo'llab, topamiz:

$$z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \right) = 2e^{\frac{\pi}{3}i}.$$

Demak,

$$z_1 = 1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} \right) = 2e^{\frac{\pi}{3}i}.$$

2) Eyler formulasi bilan topamiz:

$$z_2 = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2e^{\frac{5\pi}{6}i}.$$

Kompleks sonning algebraik shaklini topamiz:

$$x_2 = r\cos\varphi = 2\cos\frac{5\pi}{6} = 2\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}, \quad y_2 = r\sin\varphi = 2\sin\frac{5\pi}{6} = 2\cdot\frac{1}{2} = 1,$$

$$z_2 = -\sqrt{3} + i.$$

Demak,

$$z_2 = -\sqrt{3} + i = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2e^{\frac{5\pi}{6}i}.$$

3) Eyler formulasiga ko‘ra

$$z_3 = 5e^{i\,arctg\frac{3}{4}} = 5\left(\cos\left(arctg\frac{3}{4}\right) + i\sin\left(arctg\frac{3}{4}\right)\right).$$

Bunda

$$\cos\left(arctg\frac{3}{4}\right) = \frac{1}{\sqrt{1 + tg^2\left(arctg\frac{3}{4}\right)}} = \frac{1}{\sqrt{1 + \left(\frac{3}{4}\right)^2}} = \frac{4}{5},$$

$$\sin\left(arctg\frac{3}{4}\right) = \sqrt{1 - \cos^2\left(arctg\frac{3}{4}\right)} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}.$$

U holda $z_3 = 5\left(\frac{4}{5} + i\frac{3}{5}\right) = 4 + 3i.$

Demak, $z_3 = 4 + 3i = 5\left(\cos\left(arctg\frac{3}{4}\right) + i\sin\left(arctg\frac{3}{4}\right)\right) = 5e^{i\,arctg\frac{3}{4}}.$

4) $z_4 = -\sin\frac{\pi}{8} + i\cos\frac{\pi}{8}$ kompleks son trigonometrik shakldа berilgan emas. Shu sababli shunday φ_4 burchakni topamizki, $\cos\varphi_4 = -\sin\frac{\pi}{8}$ va $\sin\varphi_4 = \cos\frac{\pi}{8}$ bo‘lsin. Bunday burchak $\varphi_4 = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$ bo‘ladi.

$\cos\frac{5\pi}{8} = -\frac{\sqrt{2-\sqrt{2}}}{2}$ va $\sin\frac{5\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$ ni hisobga olib, kompleks sonnning turli shakllarini yozamiz:

$$z_4 = -\frac{\sqrt{2-\sqrt{2}}}{2} + i\frac{\sqrt{2+\sqrt{2}}}{4} = \cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8} = e^{\frac{5\pi}{8}i}.$$

2.1.2. $z_1=(x,y_1)$ va $z_2=(x_2,y_2)$ *kompleks sonlarining yig'indisi* deb,

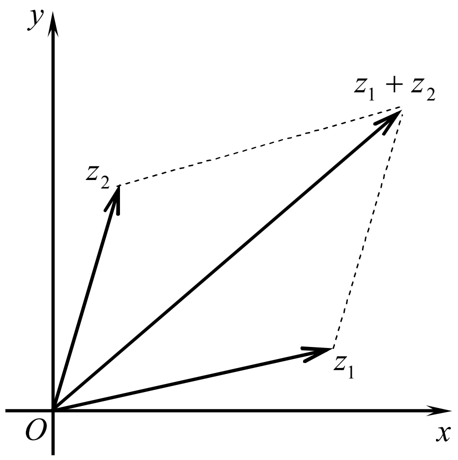
$$z_1+z_2=(x_1,y_1)+(x_2,y_2)=(x_1+x_2,y_1+y_2)$$

songa aytiladi. Demak,

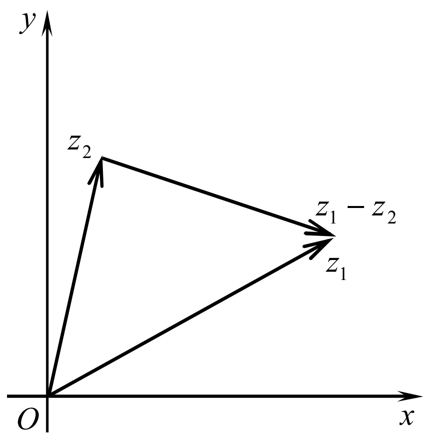
$$z_1+z_2=(x_1+x_2)+i(y_1+y_2)$$

Kompleks sonlar geometrik jihatdan vektorlar kabi qo'shiladi (2-shakl).

2-shakldan ko'rinadiki, $|z_1+z_2|\le|z_1|+|z_2|$. Bu tengsizlikka *uchburchak tengsizligi* deyiladi.



2-shakl.



3-shakl.

Kompleks sonlarni ayirish qo'shish amaliga teskari amal sifatida aniqlanadi.

z_1 va z_2 *kompleks sonlarning ayirmasi* deb, z_2 ga qo'shganda z_1 ni hosil qiluvchi z kompleks soniga aytiladi.

Agar $z_1=x_1+iy_1$ va $z_2=x_2+iy_2$ bo'lsa, bu ta'rifga ko'ra

$$z=z_1-z_2=(x_1-x_2)+i(y_1-y_2)$$

Kompleks sonlar geometrik jihatdan vektorlar kabi ayriladi (4-shakl).

4-shakldan bevosita ko'rinadiki, $|z_1-z_2|\ge|z_1|-|z_2|$.

Kompleks sonlar uchun

$$|z_1-z_2|=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=d$$

bo'ladi, ya'ni ikkita kompleks sonlar ayirmasining moduli tekislikda bu sonlarni ifodalovchi nuqtalar orasidagi masofaga teng.

Shu sababli, masalan $|z-2i|=1$ tenglik kompleks tekisligida $z_0=2i$ nuqtadan birga teng masofada yotuvchi nuqtalar to'plami z ni, ya'ni markazi $z_0=2i$ nuqtada joylashgan va radiusi birga teng aylanani aniqlaydi.

$z_1=(x,y_1)$ va $z_2=(x_2,y_2)$ *kompleks sonlarining ko'paytmasi* deb

$$z_1\cdot z_2=z=(x,y)=(x_1x_2-y_1y_2,x_2y_2+y_1x_2)$$

songa aytiladi. Demak,

$$z=z_1z_2=(x_1x_2-y_1y_2)+i(x_1y_2+y_1x_2).$$

Trigonometrik shaklda berilgan

$$z_1=r_1(\cos\varphi_1+i\sin\varphi_1) \text{ va } z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$$

kompleks sonlarni ko'paytmasi

$$z_1z_2=r_1r_2(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2))$$

kabi topiladi.

Demak, kompleks sonlar ko'paytirilganda ularning modullari ko'paytiriladi va argumentlari qo'shiladi.

Bu qoida istalgan sondagi ko'paytuvchilar uchun ham o'rinli bo'ladi. Xususan, agar n ta bir xil ko'paytuvchilar uchun

$$z^n=(r(\cos\varphi+i\sin\varphi))^n=r^n(\cos n\varphi+i\sin n\varphi).$$

Bu formulaga *Muavr formulasi* deyiladi.

Kompleks sonlarni bo'lish ko'paytirish amaliga teskari amal sifatida aniqlanadi.

z_1 va $z_2\neq 0$ *kompleks sonlarning bo'linmasi* deb, z_2 ga ko'paytirganda z_1 ni hosil qiluvchi z kompleks soniga aytiladi:

$$z=\frac{z_1}{z_2}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}.$$

Trigonometrik shaklda berilgan $z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$ kompleks sonini $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ kompleks soniga bo'linmasi

$$\frac{r_1(\cos\varphi_1+i\sin\varphi_1)}{r_2(\cos\varphi_2+i\sin\varphi_2)}=\frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2))$$

kabi aniqlanadi.

Shunday qilib, kompleks sonlar bo'linganda ularning mo'dullari bo'linadi va argumentlari ayriladi.

Kompleks sondan $n-$darajali ildiz chiqarish $n-$natural darajaga ko'tarish amaliga teskari amal sifatida aniqlanadi.

z *kompleks sonidan* $n-$*ildiz chiqarish* deb, $w^n=z$ tenglikni qanoatlantiruvchi w kompleks soniga aytiladi.

$z=r(\cos\varphi+i\sin\varphi)$ kompleks sonidan $w=\sqrt[n]{z}$ ildiz chiqarish quyidagi formula bilan topiladi:

$$w_k=\sqrt[n]{r(\cos\varphi+i\sin\varphi)}=\sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right),\ k=0,1,\ldots,n-1.$$

3-misol. $z_1 = 1+3i,\quad z_2 = -3+i$ bo‘lsa, $z_1+z_2,\ 2z_1-z_2,\ z_1\cdot z_2,\ \dfrac{z_1}{z_2}$ larni hisoblang.

$$z_1+z_2=(1+3i)+(-3+i)=-2+4i;\ 2z_1-z_2=(2+6i)-(-3+i)=5+5i;$$

$$z_1\cdot z_2=(1+3i)\cdot(-3+i)=(-3-3)+i(1-9)=-6-8i;$$

$$\frac{z_1}{z_2}=\frac{1+3i}{-3+i}=\frac{(1+3i)\cdot(-3-i)}{(-3+i)\cdot(-3-i)}=\frac{(-3+3)+i(-9-1)}{9+1}=-\frac{10i}{10}=-i.$$

4-misol. $z=\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$ bo‘lsa, z^6 ni hisoblang.

Avval kompleks sonni trigonometrik shaklga keltiramiz.

$x=\dfrac{\sqrt{3}}{2},\ y=-\dfrac{1}{2}$ bo‘lgani uchun $r=\sqrt{\left(\dfrac{\sqrt{3}}{2}\right)^2+\left(-\dfrac{1}{2}\right)^2}=1,\ \arg z=arctg\dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\dfrac{\pi}{6}.$

Bundan $z=\cos\left(-\dfrac{\pi}{6}\right)+i\sin\left(-\dfrac{\pi}{6}\right).$

Muavr formulasi bilan topamiz:

$$z^6=\cos\left(-\frac{\pi}{6}\cdot 6\right)+i\sin\left(-\frac{\pi}{6}\cdot 6\right)=\cos\pi-i\sin\pi=-1.$$

5-misol. $\sqrt[4]{-1}$ ning barcha ildizlarini toping.

Ildiz ostidagi ifodani trigonometrik shaklda yozamiz:

$$-1=\cos\pi+\sin\pi\ .$$

Bundan

$$\sqrt[4]{-1}=\cos\frac{\pi+2\pi k}{4}+i\sin\frac{\pi+2\pi k}{4},\ \ k=0,1,2,3.$$

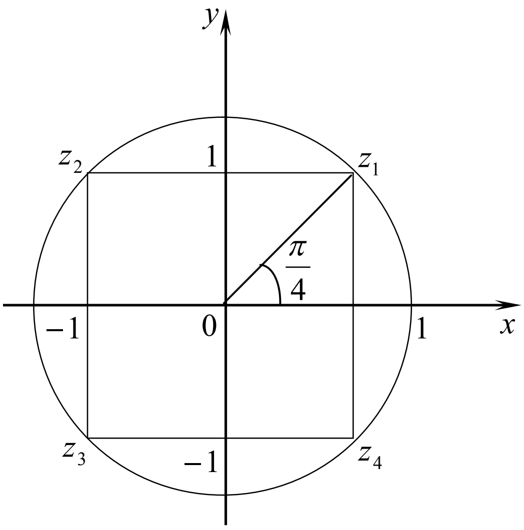
k ga qiymatlar berib, topamiz:

$$w_0=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}(1+i),$$

$$w_1=\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}(-1+i),$$

$$w_2=\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}=\frac{\sqrt{2}}{2}(-1-i),$$

$$w_3=\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}=\frac{\sqrt{2}}{2}(1-i).$$



4-shakl.

Bu ildizlar (z) kompleks tekisligida birlik aylanaga ichki chizilgan muntazam to‘rtburchakning (kvadratning) uchlarida yotadi (4-shakl).

Mashqlar

2.1.1. $z_1 = x - 3yi - y - \frac{2x}{i}$ va $z_2 = 2x + i^2 - 3xi - yi^3$ kompleks sonlar x, y ning qanday haqiqiy qiymatlarida qo‘shma bo‘ladi?

2.1.2. $z_1 = y + 2i^3 + 3 - 2xi$ va $z_2 = 3x + 8i + \frac{2y}{i} + 2i^2$ kompleks sonlar x, y ning qanday haqiqiy qiymatlarida teng bo‘ladi?

2.1.3. $z_1 = 3x - 2yi + 5i^3 - 1$ va $z_2 = 3y - i^3 + \frac{8x}{i} + 2i^2$ kompleks sonlar x, y ning qanday haqiqiy qiymatlarida qarama-qarshi bo‘ladi?

2.1.4. $z_1 = 5x + \frac{3y}{i^2} + 3yi + i^3$ va $z_2 = 3y(1+i) + \frac{5x}{i} - i^4$ kompleks sonlar x, y ning qanday haqiqiy qiymatlarida nolga teng bo‘ladi?

2.1.5. (z) tekislikda berilgan tenglamalarni yeching:

1) $z^2 + 6z + 25 = 0;$ 2) $2z^2 + iz + 1 = 0;$

3) $iz^2 - 2z + 3i = 0;$ 4) $z^2 - 6iz - 5 = 0.$

2.1.6. (z) tekislikda berilgan shartlar bilan qanday nuqtalar to‘plami aniqlanadi?

1) $\operatorname{Re} z = a;$ 2) $\operatorname{Im} z = b;$ 3) $r < |z| < R;$ 4) $\varphi < \arg z < \psi;$

5) $r < |z| < R,\ \varphi < \arg z < \psi,$ bu yerda $a, b, r, R, \varphi, \psi -$haqiqiy sonlar.

2.1.7. Berilgan kompleks sonlarni turli (algebraik, trigonometrik va ko‘rsatkichli) shakllarda yozing:

1) $z = -2 + 2\sqrt{3}i;$ 2) $z = \sqrt{3} - i;$

3) $z = \sqrt{3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right);$ 4) $z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right);$

5) $z = \sqrt{2}e^{-\frac{\pi}{4}i};$ 6) $z = \sqrt{13}e^{i\left(arctg\frac{2}{3} - \pi\right)};$

7) $z = 2\cos\frac{\pi}{3} - 2i\sin\frac{\pi}{3};$ 8) $z = -2\cos 45^o - 2i\sin 45^o.$

2.1.8. Berilgan kompleks sonlarning yig‘indisi, ayirmasi, ko‘paytmasi va bo‘linmasini toping:

1) $z_1 = -5 + 3i$ va $z_2 = 2 - 4i;$ 2) $z_1 = -3 - 4i$ va $z_2 = 2 + 3i.$

2.1.9. (z) tekislikda berilgan nuqtalar orasidagi masofani toping:

1) $1-3i$ va $4i$;
2) $1-5i$ va -4;
3) $1+3i$ va $3+2i$;
4) $8-3i$ va $2+5i$.

2.1.10. Hisoblang:

1) $\dfrac{2-3i}{1+2i}+(1-i)^2(1+i)$;
2) $\dfrac{1+3i}{-2+i}\cdot(-2i)+1$;
3) $(2+3i)^3-(2-3i)^3$;
4) $(-1+2i)^4-(1+2i)^4$.

2.1.11. Kompleks sonlarning haqiqiy va mavhum qismlarini toping:

1) $\dfrac{3\sqrt{3}-i^7}{2(\sqrt{3}+2i^3)}$;
2) $\dfrac{-1+i^5}{2+i}+\dfrac{8+19i^3}{40}$;
3) $\dfrac{3+i^5}{(1-i^3)(1+2i^7)}$;
4) $(i^5-5)\left(2i+\dfrac{3}{2-i^3}\right)$.

2.1.12. Berilgan kompleks sonlarning ko‘paytmasi va bo‘linmasini toping:

1) $z_1=4\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)$ va $z_2=2\left(\cos\dfrac{5\pi}{12}+i\sin\dfrac{5\pi}{12}\right)$;

2) $z_1=6\left(\cos\dfrac{5\pi}{6}+i\sin\dfrac{5\pi}{6}\right)$ va $z_2=\cos\left(-\dfrac{2\pi}{3}\right)+i\sin\left(-\dfrac{2\pi}{3}\right)$;

3) $z_1=8(\cos 135^o+i\sin 135^o)$ va $z_2=2(\cos 45^o+i\sin 45^o)$;

4) $z_1=8(\cos 90^o+i\sin 90^o)$ va $z_2=\cos 30^o+i\sin 30^o$.

2.1.13. Berilgan sonlarning darajasini toping:

1) $\left(\dfrac{\sqrt{2}}{2}\left(\cos\dfrac{7\pi}{36}+i\sin\dfrac{7\pi}{36}\right)\right)^{-9}$;
2) $(\sqrt{2}(\cos 20^0+i\sin 20^0))^{12}$;
3) $(2-2i)^9$;
4) $\left(\dfrac{1+i}{1-i}\right)^{10}$.

2.1.14. Berilgan sonlarning barcha ildizlarini toping:

1) $\sqrt{2-2\sqrt{3}i}$;
2) $\sqrt[3]{-i}$;
3) $\sqrt[4]{-8+8\sqrt{3}i}$;
4) $\sqrt[5]{1+i}$.

2.1.15. (z) tekislikda berilgan tenglamalarni yeching:

1) $z^2+i=0$;
2) $z^3-i=0$;
3) $z^3+8=0$;
4) $z^4+4=0$.
5) $z^2+4z+53=0$;
6) $z^4+26z^2+25=0$;
7) $z^3+2z^2+2z+1=0$;
8) $iz^4+1=0$;
9) $z^4+z^2=2$.

2.2. KOMPLEKS O‘ZGARUVCHINING FUNKSIYASI

Funksiya tushunchasi. Funksiyaning limiti va uzluksizligi. Kompleks o‘zgaruvchining asosiy elementar funksiyalari

2.2.1. Elementlari kompleks sonlardan iborat bo‘lgan D va E to‘plamlar berilgan va bunda D to‘plamning $z=x+iy$ soni (z) kompleks tekisligining nuqtasi, E to‘plamning $w=u+iv$ soni (w) kompleks tekisligining nuqtasi bo‘lsin.

Agar D to‘plamning har bir z soniga biror qonun yoki qoida bilan E to‘plamdagi yagona w soni mos qo‘yilgan bo‘lsa, D to‘plamda $w=f(z)$ *bir qiymatli funksiya* aniqlangan deyiladi. Bunda D to‘plamga $w=f(z)$ funksiyaning aniqlanish sohasi, E to‘plamning barcha $f(z)$ qiymatlari to‘plami E_1 ga $w=f(z)$ funksiyaning qiymatlar sohasi deyiladi.

Agar $E_1=E$, ya’ni E to‘plamning har bir nuqtasi $w=f(z)$ funksiyaning qiymatlar sohasi bo‘lsa, $w=f(z)$ funksiya D to‘plamni E to‘plamga *akslantiradi* deyiladi.

Agar har bir $z\in D$ songa bir nechta $w\in E$ sonlar mos qo‘yilsa, $w=f(z)$ funksiyaga *ko‘p qiymatli funksiya* deyiladi.

Bundan keyin D va E_1 to‘plamlari sohalar bo‘lgan $w=f(z)$ funksiyalarni qaraymiz. Kompleks tekisligining ochiqlilik va bog‘lamlilik xossalariga ega bo‘lgan nuqtalari to‘plamiga soha deyiladi: ochiqlilik xossasiga ko‘ra to‘plamning har bir nuqtasi biror atrofi bilan to‘plamga tegishli bo‘ladi; bog’lamlilik xossasiga ko‘ra to‘plamning istalgan ikki nuqtasini to‘plamda yotuvchi uzluksiz chiziq bilan tutashtirish mumkin bo‘ladi.

$w=f(z)$ funksiyani $u+iv=f(x+iy)$ yoki $f(x+iy)=u(x,y)+iv(x,y)$ ko‘rinishda yozish mumkin, bu yerda $(x,y)\in D;$ $u(x,y)=\operatorname{Re} f(z)-$funksiyaning haqiqiy qismi; $v(x,y)=\operatorname{Im} f(z)-$funksiyaning mavhum qismi.

Shunday qilib, kompleks o‘zgaruvchili funksiyaning berilishi ikkita haqiqiy o‘zgaruvchili funksiyaning berilishidan iborat bo‘ladi.

1-misol. Argumentning berilgan qiymatlarida $f(z)=z^3-3z^2+z$ funksiyaning qiymatlarini toping: 1) $z=i;$ 2) $z=1+i;$ 3) $z=3-i.$

Mavhum birlikning darajalari qiymatlarini hisobga olib, topamiz:

1) $f(i)=i^3-3i^2+i=-i+3+i=3;$

2) $f(1+i)=(1+i)^3-3(1+i)^2+(1+i)=1+3i+3i^2+i^3-3(1+2i+i^2)+1+i=$
$=1+3i-3-i-3(1+2i-1)+1+i=-1-3i;$

3) $f(3-i)=(3-i)^3-3(3-i)^2+(3-i)=$
$=27-27i+9i^2-i^3-3(9-6i+i^2)+3-i=$
$=27-27i-9+i-27+18i+3+3-i=-3-9i.$

2-misol. $f(z)=-z+i(\bar{z}+z^2)$ funksiyaning haqiqiy va mavhum qismlarini toping.

$$f(z)=u(x,y)+iv(x,y)=-(x+iy)+i(x-iy+(x+iy)^2)=$$
$$=-x-iy+i(x-iy+x^2+2xyi+i^2y^2)=-x-iy+i(x+x^2-y^2)-2xy+y=$$
$$=(y-x-2xy)+i(x-y+x^2-y^2)=(y-x-2xy)+i(x-y)\cdot(1+x+y).$$

Demak, $\operatorname{Re} f(z)=u(x,y)=y-x-2xy,\ \operatorname{Im} f(z)=v(x,y)=(x-y)\cdot(1+x+y).$

3-misol. Haqiqiy $\operatorname{Re} f(z)=u(x,y)=x^2-y^2$ va mavhum $\operatorname{Im} f(z)=2xy$ qismlariga ko‘ra $f(z)$ funksiyani toping.

$z=x+iy$ va $\bar{z}=x-iy$ dan topamiz: $x=\dfrac{z+\bar{z}}{2}$, $\quad y=\dfrac{z-\bar{z}}{2i}$.

U holda
$$f(z)=(x^2-y^2)+i(2xy)=\frac{z^2+2z\bar{z}+\bar{z}^2}{4}+\frac{z^2-2z\bar{z}+\bar{z}^2}{4}+2\frac{z+\bar{z}}{2}\cdot\frac{z-\bar{z}}{2}=$$
$$=\frac{z^2+\bar{z}^2}{2}+\frac{z^2-\bar{z}^2}{2}=z^2.$$

4-misol. $w=z-\sqrt{z}$ funksiyaning $z_0=i$ nuqtadagi barcha qiymatlarini toping.

$z_0=i=\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$ uchun topamiz:
$$\sqrt{z_0}=\cos\left(\frac{\pi}{4}+k\pi\right)+i\sin\left(\frac{\pi}{4}+k\pi\right).$$

U holda
$$w_k=i-\cos\left(\frac{\pi}{4}+k\pi\right)-i\sin\left(\frac{\pi}{4}+k\pi\right),\ k=0,1.$$

Bundan
$$w_0=i-\cos\left(\frac{\pi}{4}\right)-i\sin\left(\frac{\pi}{4}\right)=i-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}(-1+i\cdot(\sqrt{2}-1)),$$
$$w_1=i-\cos\left(\frac{\pi}{4}+\pi\right)-i\sin\left(\frac{\pi}{4}+\pi\right)=i+\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}(1+i\cdot(\sqrt{2}+1)).$$

Agar $w = f(z)$ funksiya bir qiymatli bo‘lsa, har bir $z_0 \in D \subset (z)$ nuqtaga biror $w_0 \in E \subset (w)$ nuqta mos keladi. Bunda w_0 nuqta z_0 nuqtaning (z) tekislikdagi *aksi*, z_0 nuqta esa w_0 nuqtaning (z) tekislikdagi *asli* deb ataladi.

Agar z o‘zgaruvchi z_0 nuqtadan o‘tuvchi l chiziqli ifodalasa va w o‘zgaruvchi w_0 nuqtadan o‘tuvchi L chiziqli ifodalasa, L chiziqqa l chiziqning (w) tekislikdagi aksi, l chiziqqa L chiziqning (z) tekislikdagi asli deyiladi.

5-misol. $z_0 = 1 + i$ nuqtaning $w = z^2 + 1$ akslantirishdagi shaklini toping.

$w_0 = f(z_0) = (z_0)^2 + i = (1+i)^2 + i = 1 + 2i - 1 + i = 3i.$

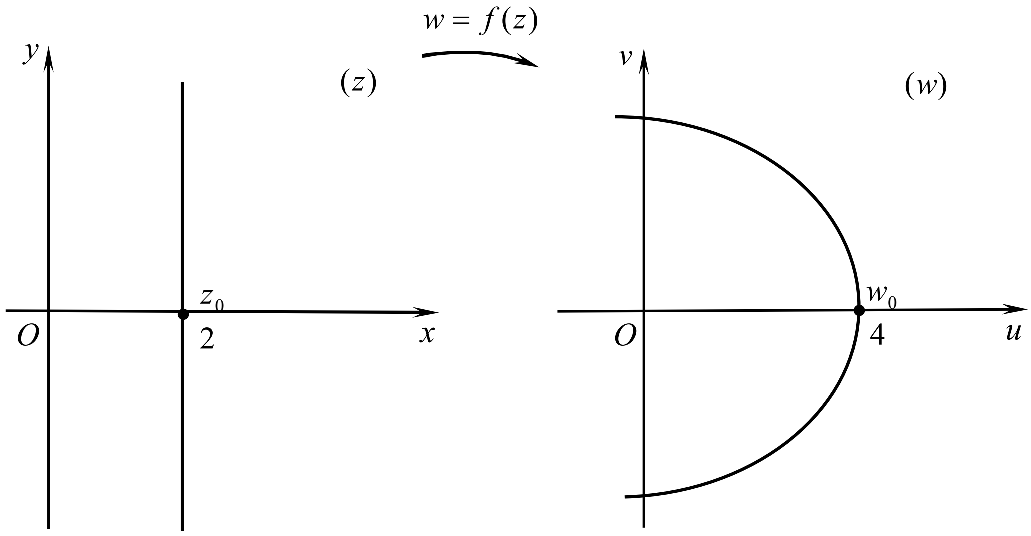
Demak, $w_0 = 3i.$

6-misol. $x = 2$ chiziqning $w = z^2$ akslantirishda (w) tekislikdagi aksini toping.

$w = z^2$ funksiyaning haqiqiy va mavhum qismlarini topamiz:

$$w = u(x,y) + iv(x,y) = (x+iy)^2 = (x^2 - y^2) + i \cdot (2xy).$$

Demak, $\begin{cases} u = x^2 - y^2, \\ v = 2xy. \end{cases}$



5-shakl.

$x = 2$ chiziqning nuqtalari $z = 2 + iy$ ko‘rinishga ega. $z_0 = x_0 + i \cdot 0$ da $w_0 = w(z_0) = 4$ bo‘ladi.

$x = 2$ da $\begin{cases} u = 4 - y^2, \\ v = 4y. \end{cases}$ Bundan $u = -\dfrac{1}{16}v^2 + 4$ kelib chiqadi.

Bu chizich (w) tekislikda parabolani ifodalaydi (5-shakl).

2.2.2. Bir qiymatli $w=f(z)$ funksiya z_0 nuqtaning biror atrofida aniqlangan bo‘lsin.

z_0 *nuqtaning* $\delta-$*atrofi* deb (z) kompleks tekisligining $|z-z_0|<\delta$ tengsizlikni qanoatlantiruvchi barcha z nuqtalari to‘plamiga aytiladi. Bu to‘plam markazi z_0 nuqtada bo‘lgan va radiusi δ ga teng ochiq (chegarasiz) doirada yotuvchi barcha z nuqtalardan tashkil topadi.

Agar $\forall\varepsilon>0$ son uchun z_0 nuqtaning shunday $\delta-$atrofi topilsaki, bu atrofning istalgan z nuqtasi (z_0 nuqta bundan istisno bo‘lishi mumkin) uchun $|f(z)-w_0|<\varepsilon$ tengsizlik bajarilsa, w_0 songa $w=f(z)$ funksiyaning z_0 nuqtadagi yoki $z\to z_0$ dagi limiti deyiladi va $\lim\limits_{z\to z_0} f(z)=w_0$ kabi yoziladi.

Agar $\lim\limits_{z\to z_0} f(z)=w_0$ limit mavjud bo‘lsa, u holda $\lim\limits_{(x,y)\to(x_0,y_0)} u(x,y)=u_0$ va $\lim\limits_{(x,y)\to(x_0,y_0)} v(x,y)=v_0$ limitlar ham mavjud bo‘ladi va aksincha.

Shunday qilib, kompleks o‘zgaruvchili funksiya limitining berilishi ikkita haqiqiy o‘zgaruvchili funksiya limitining berilishidan iborat bo‘ladi.

Bir (bir necha) haqiqiy o‘zgaruvchili funksiya uchun o‘rinli bo‘lgan arifmetik amallarning limiti haqidagi xossalar kompleks o‘zgaruvchili funksiyalar uchun ham o‘rinli bo‘ladi:

$$\lim_{z\to z_0}(f(z)\pm g(z))=\lim_{z\to z_0} f(z)\pm\lim_{z\to z_0} g(z);\quad \lim_{z\to z_0}(f(z)\cdot g(z))=\lim_{z\to z_0} f(z)\cdot\lim_{z\to z_0} g(z);$$

$$\lim_{z\to z_0}(c\cdot f(z))=c\cdot\lim_{z\to z_0} f(z),\ c=const;\qquad \lim_{z\to z_0}\frac{f(z)}{g(z)}=\frac{\lim\limits_{z\to z_0} f(z)}{\lim\limits_{z\to z_0} g(z)},\ \lim_{z\to z_0} g(z)\neq 0.$$

7-misol. $\lim\limits_{z\to 2i}\dfrac{z^2-iz+2}{z-2i}$ limitni toping.

$$\lim_{z\to 2i}\frac{z^2-iz+2}{z-2i}=\left(\frac{0}{0}\right)=\lim_{z\to 2i}\frac{z^2-2iz+iz-2i^2}{z-2i}=$$

$$=\lim_{z\to 2i}\frac{z(z-2i)+i\cdot(z-2i)}{z-2i}=\lim_{z\to 2i}(z+i)=3i.$$

Agar $\lim\limits_{z\to z_0} f(z)=f(z_0)$ bo‘lsa, $f(z)$ *funksiya* z_0 *nuqtada uzluksiz* deyiladi.

Shu kabi, agar $\lim\limits_{\Delta z\to 0}\Delta f(z)=0$ bo‘lsa, $f(z)$ *funksiya* z_0 *nuqtada uzluksiz* deyiladi.

Agar $w=f(z)$ funksiya D sohaning har bir nuqtasida uzluksiz bo‘lsa, u shu *sohada uzluksiz* deyiladi.

Kompleks o‘zgaruvchili funksiyaning moduli uchun haqiqiy o‘zgaruvchili funksiyaning limiti haqidagi barcha xossalar o‘rinli bo‘ladi.

8-misol. $w = |z|$ funksiyaning istalgan z da uzluksiz bo‘lishini ko‘rsating.

$w = |z|$ funksiya uchun $w_0 = |z_0|$ bo‘ladi.

(z) kompleks tekisligida $|z|$, $|z_0|$, $|z - z_0|$ qiymatlar uchburchakning tomonlarini ifodalaydi. Uchburchak tengsizligiga ko‘ra $||z| - |z_0|| \le |z - z_0|$. Bundan $|w - w_0| \le |z - z_0|$ kelib chiqadi.

$0 < \delta < \varepsilon$ bo‘lsin. U holda $|z - z_0| < \delta$ tengsizlikdan $|w - w_0| < \varepsilon$ tengsizlik keliib chiqadi. Bundan limit ta’rifiga ko‘ra, $\lim\limits_{z \to z_0} w = w_0$ yoki $\lim\limits_{z \to z_0} |z| = |z_0|$.

Demak, $w = |z|$ funksiya istalgan z da uzluksiz bo‘ladi.

2.2.3. ***Ko‘rsatkichli funksiya***: $w = e^z = e^x(\cos y + i \sin y)$.

Ko‘rsatkichli funksiyada $y = 0$ bo‘lsa $w = e^x$ bo‘ladi, ya’ni e^z funksiya $z = x$ haqiqiy qiymatlarda haqiqiy o’zgaruvchili ko‘rsatkichli funksiya bilan ustma-ust tushadi.

$w = e^z$ funksiya z o‘zgaruvchining istalgan qiymatida aniqlangan.

Ko‘rsatkichli funksiyaning xossalari:

1^0. $e^{z_1} \cdot e^{z_2} = e^{z_1 + z_2}$;

2^0. $\dfrac{e^{z_1}}{e^{z_2}} = e^{z_1 - z_2}$;

3^0. $(e^z)^n = e^{nz}$, $(n \in N)$;

4^0. $\lim\limits_{\text{Re}\, z \to -\infty} e^z = 0,\quad \lim\limits_{z \to +\infty} e^z = \infty$ (ma’noga ega emas);

5^0. $w = e^z$ funksiya asosiy davri $T_0 = 2\pi i$ bo‘lgan davriy funksiya.

Logarifmik funksiya: $Lnz = \ln r + i(\varphi + 2k\pi)$.

$e^w = z$ tenglikni qanoatlantiruvchi w soniga z kompleks sonning *logarifmi* deyiladi va $w = Lnz$ bilan belgilanadi.

e^w funksiya w ning har qanday qiymatida nolga teng bo‘lmagani sababli $z = x + iy \ne 0$ bo‘ladi, ya’ni nolning logarifmi mavjud bo‘lmaydi.

Logarifmik funksiya

$$Lnz = \ln r + i(\varphi + 2k\pi) \quad \text{yoki} \quad Lnz = \ln r + iArgz,$$

kabi aniqlanadi, bu yerda $r = |z|$, $\varphi = \arg z$, $Argz = \arg z + 2k\pi$.

$w = Lnz$ logarifmik funksiya ko‘p qiymatli funksiya. Bunda k ga tayin qiymat berish orqali bir qiymatli funksiya hosil qilinadi. $k = 0$ bo‘lgan holni, ya'ni $\ln r + i\varphi$ ni logarifmning bosh qiymati deyiladi va $\ln z$ bilan belgilanadi:

$$\ln z = \ln|z| + i\varphi.$$

Agar z haqiqiy son bo'lsa $\arg z = 0$ va $\ln z = \ln|z|$ bo'ladi, ya'ni haqiqiy musbat son logarifmining bosh qiymati bu sonning odatdagi natural logarifmi bilan ustma-ust tushadi.

Logarifmning xossalari kompleks sohada quyidagicha umumlashtiriladi:

1^0. $Ln(z_1 \cdot z_2) = Lnz_1 + Lnz_2$;

2^0. $Ln\left(\dfrac{z_1}{z_2}\right) = Lnz_1 - Lnz_2$;

3^0. $Lnz^n = nLnz$;

4^0. $Ln\sqrt[n]{z} = \dfrac{1}{n}\ln z$.

Darajali funksiya : $w = z^n$.

n natural son bo'lganda darajali funksiya

$$w = z^n = r^n(\cos n\varphi + i\sin n\varphi)$$

tenglik bilan aniqlanadi. Bu funksiya kompleks tekisligining barcha nuqtalarida aniqlangan va bir qiymatli.

$n = \dfrac{1}{m}$, $m \in N$ bo'lganda darajali funksiya

$$w = z^{\frac{1}{m}} = \sqrt[m]{z} = \sqrt[m]{|z|}\left(\cos\frac{\arg z + 2k\pi}{m} + i\sin\frac{\arg z + 2k\pi}{m}\right), \quad k = 0,1,...,m-1$$

tenglik bilan aniqlanadi. Bu funksiya kompleks tekisligining barcha nuqtalarida aniqlangan va ko'p qiymatli (m qiymatli). Bunda k ga tayin qiymat berish orqali bir qiymatli funksiya hosil qilinadi.

$n = \dfrac{l}{m}$, $l, m \in N$ bo'lganda darajali funksiya

$$w = z^{\frac{l}{m}} = \sqrt[m]{z^l} = \sqrt[m]{|z|^l}\left(\cos\frac{l(\arg z + 2k\pi)}{m} + i\sin\frac{l(\arg z + 2k\pi)}{m}\right)$$

tenglik bilan aniqlanadi. Bu funksiya kompleks tekisligining barcha nuqtalarida aniqlangan, ko'p qiymatli.

$\alpha = x + iy$ kompleks ko'rsatkichli darajali funksiya

$$w = z^\alpha = e^{\alpha Lnz}$$

tenglik bilan aniqlanadi. Bu funksiya kompleks tekisligining $z = 0$ nuqtadan boshqa barcha nuqtalarida aniqlangan va ko'p qiymatli.

Trigonometrik funksiyalar

$z = x + iy$ kompleks argumentning trigonometrik funksiyalari

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}, \quad tgz = \frac{\sin z}{\cos z}, \quad ctgz = \frac{\cos z}{\sin z}$$

tengliklar bilan aniqlanadi.

Haqiqiy z da bu tengliklardan haqiqiy o‘zgaruvchining trigonometrik funksiyalari kelib chiqadi.

Haqiqiy o‘zgaruvchining trigonometrik funksiyalari orasidagi munosabatlarni ifodalovchi barcha formulalar kompleks sohada ham o‘rinli bo‘ladi. Xususan,

$$\sin^2 z + \cos^2 z = 1, \qquad \sin 2z = 2\sin z \cos z,$$

$$\sin\left(z + \frac{\pi}{2}\right) = \cos z, \qquad \cos\left(z + \frac{\pi}{2}\right) = -\sin z,$$

$$\sin(z + \pi) = -\sin z, \qquad \cos(z + \pi) = -\cos z,$$

$$\sin\left(z + \frac{3\pi}{2}\right) = -\cos z, \qquad \cos\left(z + \frac{3\pi}{2}\right) = \sin z,$$

$$\sin(z + 2\pi) = \sin z, \qquad \cos(z + 2\pi) = \cos z,$$

$$\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2, \qquad \cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2 .$$

Kompleks o‘zgaruvchi trigonometrik funksiyalarining nollari haqiqiy o‘zgaruvchi trigonometrik funksiyalarining nollari kabi bo‘ladi:

$$z = k\pi,\ k \in Z \ \text{ da } \sin z = 0; \ z = \frac{\pi}{2} + k\pi,\ k \in Z \ \text{ da } \cos z = 0.$$

Kompleks tekislikda $\sin z$ va $\cos z$ funksiyalar chegaralanmagan, ya’ni

$$\lim_{\operatorname{Im} z \to \pm\infty} \sin z = \infty, \quad \lim_{\operatorname{Im} z \to \pm\infty} \cos z = \infty.$$

Giperbolik funksiyalar

$z = x + iy$ kompleks argumentning giberbolik funksiyalari deb

$$shz = \frac{e^{z} - e^{-z}}{2}, \quad chz = \frac{e^{z} + e^{-z}}{2}, \quad thz = \frac{shz}{chz}, \quad cthz = \frac{chz}{shz}$$

tengliklar bilan aniqlanadigan funksiyalarga aytiladi. Bunda shz va chz asosiy davri $T_0 = 2\pi i$ bo‘lgan davriy funksiyalar bo‘ladi, thz va $cthz$ asosiy davri $T_0 = \pi i$ bo‘lgan davriy funksiyalar bo‘ladi.

Haqiqiy z da bu tengliklardan haqiqiy o‘zgaruvchining giperbolik funksiyalari kelib chiqadi.

Kompleks sohada giperbolik va trigonometrik funksiyalar orasidagi quyidagi munosabatlar o‘rinli bo‘ladi:

$$shiz = i\sin z; \qquad \sin iz = ishz.;$$

$chiz = \cos z$; $\cos iz = chz$;

$thiz = itgz$; $tgiz = ithz$;

$cthiz = -ictgz$; $ctgiz = -icthz$.

Kompleks sohada giperbolik funksiyalarni bog‘lovchi quyidagi formulalar keltirib chiqarilgan:

$sh2z = 2shzchz.$; $ch2z = ch^2z + sh^2z$;

$ch^2z - sh^2z = 1$; $shz + chz = e^z$;

$sh(-z) = -shz,$ $ch(-z) = chz,$

$ch(z_1 + z_2) = chz_1chz_1 + shz_1shz_2$; $sh(z_1 + z_2) = shz_1chz_1 + shz_2chz_2$.

Kompleks $z = x + iy$ son uchun quyidagi munosabatlar o‘rinli bo‘ladi:

$\cos z = \cos x \cdot chy - i\sin x \cdot shy,$ $\sin z = \sin x \cdot chy + i\cos x \cdot shy,$

$chz = chx \cdot \cos y + ishx \cdot \sin y,$ $shz = shx \cdot \cos y + ichx \cdot \sin y.$

Teskari trigonometrik va teskari giperbolik funksiyalar

$\sin w = z$ tenglikni qanoatlantiruvchi w soniga z kompleks sonning *arksinusi* deyiladi va $w = Arc\sin z$ bilan belgilanadi. Bu funksiya $Arc\sin z = -iLn(iz + \sqrt{1 - z^2})$ kabi aniqlanadi.

Kompleks sohadagi boshqa teskari trigonometrik funksiyalar quyidagi formulalar bilan aniqlanadi:

$$Arc\cos z = -iLn(z + \sqrt{z^2 - 1}), \quad Arctgz = -\frac{i}{2}Ln\frac{i - z}{i + z} \ (z \neq \pm i),$$

$$Arcctgz = \frac{i}{2}Ln\frac{z - i}{z + i} \ (z \neq \pm i).$$

$shw = z$ tenglikni qanoatlantiruvchi w soniga z kompleks sonning *areasinusi* deyiladi va $w = Archz$ bilan belgilanadi. Bu funksiya $Arshz = Ln(z + \sqrt{z^2 + 1})$ kabi aniqlanadi.

Kompleks sohadagi boshqa teskari giperbolik funksiyalar quyidagi tengliklar bilan topiladi:

$$Archz = Ln(z + \sqrt{z^2 - 1}), \quad Arthz = \frac{1}{2}Ln\frac{1 + z}{1 - z}, \qquad Arcthz = \frac{1}{2}Ln\frac{z + 1}{z - 1}.$$

9-misol. Hisoblahg:

1) $Ln(1 - i)$; 2) i^{1+i}; 3) $\sin(2 - i)$; 4) $\cos(1 + i)$;

5) $ch(2 - 3i)$; 6) $Arc\cos 2$; 7) $Arctg(2i)$; 8) $Arcch(2i)$.

Hisoblarni kompleks o‘zgaruvchili elementar funksiyalarning formulalari va ular o‘rtasidagi munosabatlarni ifodalovchi tengliklardan foydalanib, bajaramiz.

1) $z=1-i$ sonning moduli va argumentini topamiz:

$$r=|z|=\sqrt{1^2+1^2}=\sqrt{2},\ \varphi=\arg z=arctg\left(\frac{-1}{1}\right)=-\frac{\pi}{4}.$$

Bundan

$$Ln(1-i)=\ln\sqrt{2}+i\cdot\left(-\frac{\pi}{4}+2k\pi\right)=\frac{1}{2}\ln 2+i\pi\left(-\frac{1}{4}+2k\right),\ k\in Z.$$

2) $i^{1+i}=e^{(1+i)Lni}=e^{(1+i)\cdot\left(\frac{\pi}{2}i+2k\pi i\right)}=e^{-\left(\frac{\pi}{2}+2k\pi\right)}\cdot e^{i\cdot\left(\frac{\pi}{2}+2k\pi\right)}=$

$=e^{Lni}\cdot e^{-\left(\frac{\pi}{2}+2k\pi\right)}=i\cdot e^{-\pi\cdot\left(\frac{1}{2}+2k\right)},\ k\in Z.$

3) $\sin(2-i)=\sin 2ch(-1)+i\cos 2sh(-1)=\sin 2ch1-i\cos 2sh1.$

4) $\cos(1+i)=\cos 1ch1-i\sin 1sh1.$

5) $ch(2-3i)=ch2\cos(-3)+ish2\sin(-3)=ch2\cos 3-ish2\sin 3.$

6) $Arc\cos 2=-iLn(2+\sqrt{2^2-1})=-iLn(2+\sqrt{3})=-\ln(2+\sqrt{3})+2k\pi,\ k\in Z.$

7) $Arctg(2i)=-\frac{i}{2}Ln\frac{i-2i}{i+2i}=-\frac{i}{2}Ln\left(-\frac{1}{3}\right)=$

$$=-\frac{i}{2}\left(\ln\frac{1}{3}+\pi i+2k\pi i\right)=\pi\left(\frac{1}{2}+2k\right)+i\frac{\ln 3}{2},\ k\in Z.$$

8) $Arcch(2i)=Ln(2i+\sqrt{(2i)^2-1})=Ln(2i+i\sqrt{5})=$

$$=Ln(i(2+\sqrt{5})=\ln(2+\sqrt{5})+i\pi\left(\frac{1}{2}+2k\right),\ k\in Z.$$

Mashqlar

2.2.1. $f(z)$ funksiyaning berilgan argumentdagi qiymatini toping:

1) $w=z^3-z,\ z=1+i;$ 2) $w=z^3-2z^2+5z,\ z=1-i;$

3) $w=z^7+z^5,\ z=\frac{1}{2}-i\frac{\sqrt{3}}{2};$ 4) $w=\frac{z}{|z|}+\frac{2}{z},\ z=2+2i.$

2.2.2. $f(z)$ funksiyaning z_1 va z_2 nuqtalardagi qiymatlarini toping:

1) $w=\frac{1}{\bar{z}},\ z_1=1+i,\ z_2=3-2i;$ 2) $w=\frac{1}{\bar{z}}-2i,\ z_1=1-i,\ z_2=\frac{i}{2}.$

2.2.3. $f(z)=u(x,y)+iv(x,y)$ (bu yerda $z=x+iy$) funksiyaning haqiqiy va mavhum qismlarini toping:

1) $f(z)=(\bar{z})^3+2i-1;$ 2) $f(z)=2i-z+iz^2;$

3) $f(z)=\mathrm{Re}(z^2)+i\,\mathrm{Im}(\bar{z}^2);$ 4) $f(z)=\mathrm{Re}(z^2+i)+i\,\mathrm{Im}(z^2-i).$

2.2.4. Haqiqiy $u(x,y)$ va mavhum $v(x,y)$ qismlariga ko‘ra $f(z)$ funksiyani toping:

1) $u = x,\ v = -y;$ 2) $u = x + y,\ v = x - y;$

3) $u = \dfrac{x}{x^2 + y^2},\ v = \dfrac{y}{x^2 + y^2};$ 4) $u = \dfrac{1}{x},\ v = \dfrac{1}{y};$

2.2.5. Haqiqiy o‘zgaruvchilarni kompleks shaklida yozing:

1) $x^2 + 2x + y^2 - y = 1;$ 2) $x + y + xy = 1.$

2.2.6. $w = f(z)$ funksiyaning berilgan nuqtadagi barcha qiymatlarini toping:

1) $w = \dfrac{\sqrt{z} + 1}{\sqrt{z} - 1},\ z_0 = i;$ 2) $w = \sqrt{i + \sqrt{z}},\ z_0 = -1.$

2.2.7. z_0 nuqtaning berilgan akslanishdagi shaklini toping:

1) $z_0 = 3 - 2i,\ w = \dfrac{z}{\bar{z}};$ 2) $z_0 = \dfrac{1 - i}{2},\ w = (z + i)^3.$

2.2.8. $y = 2$ chiziqning $w = z^2$ akslantirishda (w) tekislikdagi aksini toping.

2.2.9. $x = -1$ chiziqning $w = \bar{z}^2$ akslantirishda (w) tekislikdagi aksini toping.

2.2.10. Limitlarni toping:

1) $\lim\limits_{z \to i} \dfrac{z^2 + 3iz + 4}{z - i};$ 2) $\lim\limits_{z \to -3i} \dfrac{z^2 + 2iz + 3}{z + 3i};$

3) $\lim\limits_{z \to 0} \dfrac{1}{2i}\left(\dfrac{z}{\bar{z}} - \dfrac{\bar{z}}{z}\right);$ 4) $\lim\limits_{z \to \frac{\pi}{2}} \dfrac{e^{2iz} + 1}{e^{iz} - i}.$

2.2.11. $w = z^2$ funksiyaning istalgan z da uzluksiz bo‘lishini ko‘rsating.

2.2.12. $w = \bar{z}$ funksiyaning istalgan z da uzluksiz bo‘lishini ko‘rsating.

2.2.13. Hisoblang:

1) $e^{1-z};$ 2) $e^{\bar{z}};$ 3) $e^{(\bar{z}+i)^2};$ 4) $Ln(-5);$

5) $Ln(2\sqrt{3}i - 2);$ 6) $Lni;$ 7) $(-1)^i;$ 8) $(-3)^{-i};$

9) $(3 - 4i)^{1+i};$ 10) $\sin(1 + 2i);$ 11) $\sin i;$ 12) $ctg\pi i;$

13) $chi;$ 14) $sh(i - 2);$ 15) $th\pi i;$ 16) $Arc\sin 2;$

17) $Arctg3i;$ 18) $Arctg\sqrt{3};$ 19) $Arshi;$ 20) $Arch(-1).$

2.3. KOMPLEKS O‘ZGARUVCHI FUNKSIYASINI DIFFERENSIALLASH

Funksiyaning hosilasi. Differensiallash qoidalari. Analitik funksiyalar. Konform akslantirish

2.3.1. Kompleks tekislikdagi biror G sohada bir qiymatli $w=f(z)$ funksiya aniqlangan bo‘lib, $z\in G,\ z+\Delta z\in G,\ \Delta w=f(z+\Delta z)-f(z)$ bo‘lsin.

Agar Δz har qanday yo‘l (qonun) bilan nolga intilganda ham $\dfrac{\Delta w}{\Delta z}$ nisbat yagona limitga intilsa, bu limitga $f(z)$ funksiyaning z nuqtadagi hosilasi deyiladi va $f'(z)$, $w'(z)$, $\dfrac{df}{dz}$, $\dfrac{dw}{dz}$ larning biri bilan belgilanadi:

$$f'(z)=\lim_{\Delta z\to 0}\frac{\Delta w}{\Delta z}=\lim_{\Delta z\to 0}\frac{f(z+\Delta z)-f(z)}{\Delta z}.$$

z nuqtada hosilaga ega bo‘lgan $w=f(z)$ funksiyaga shu nuqtada *differensiallanuvchi* (yoki *monogen*) funksiya deyiladi.

Teorema. Biror G sohada aniqlangan $w=f(z)=u(x,y)+iv(x,y)$ funksiya shu sohaga tegishli z nuqtada differensiallanuvchi bo‘lishi uchun bu nuqtada $u(x,y)$, $v(x,y)$ funksiyalar differensiallanuvchi bo‘lishi va *Koshi-Riman* (yoki *Dalamber-Eyler*) shartlari deb ataluvchi

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$$

tengliklar bajarilishi zarur va yetarli.

Bu tengliklar Koshi-Riman shartlarining yagona shakli emas:

$z=re^{i\varphi}$ kompleks o‘zgaruvchi $f(z)=u(r,\varphi)+iv(r,\varphi)$ funksiyasining haqiqiy va mavhum qismlari

$$\frac{\partial u(r,\varphi)}{\partial r}=\frac{1}{r}\frac{\partial v(r,\varphi)}{\partial \varphi},\quad \frac{\partial v(r,\varphi)}{\partial r}=-\frac{1}{r}\frac{\partial u(r,\varphi)}{\partial \varphi}$$

tengliklar bilan bog‘lanadi, bu yerda $r,\varphi-(x;y)$ nuqtaning qutb koordinatalari;

$f(z)=R(x,y)e^{i\Phi(x,y)}$ funksiyaning moduli va argumenti

$$\frac{\partial R}{\partial x}=R\frac{\partial \Phi}{\partial y},\quad \frac{\partial R}{\partial y}=-R\frac{\partial \Phi}{\partial x},$$

tenliklar bilan bo‘glanadi.

Koshi-Riman shartlariga ko‘ra $w = f(z)$ funksiyaning z nuqtadagi hosilasini quyidagi formulalardan biri bilan topish mumkin:

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}, \quad f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}, \quad f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}, \quad f'(z) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y},$$

$$f'(z) = \frac{r}{z}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right), \quad f'(z) = \frac{1}{z}\left(\frac{\partial v}{\partial \varphi} - i\frac{\partial u}{\partial \varphi}\right)$$

2.3.2. Kompleks o‘zgaruvchi funksiyasi hosilasining ta’rifi haqiqiy o‘zgaruvchi funksiyasi hosilasining ta’rifiga o‘xshash bo‘lgani sababli kompleks o‘zgaruvchi funksiyasi uchun haqiqiy o‘zgaruvchi funksiyasining differensiallash qoidalariga o‘xshash qoidalar kelib chiqadi.

Jumladan:

1. $(f(z) \pm g(z))' = f'(z) \pm g'(z),\ f(z), g(z) -$ differensiallanuvchi funksiyalar;

2. $(f(z) \cdot g(z))' = f'(z) \cdot g(z) + f(z) \cdot g'(z);$

3. $\left(\dfrac{f(z)}{g(z)}\right)' = \dfrac{f'(z) \cdot g(z) - f(z) \cdot g'(z)}{g^2(z)},\ (g(z) \neq 0);$

4. $(f(\varphi(z)))' = f'_{\varphi}(\varphi) \cdot \varphi'_z(z)$, agar $\varphi(z)$ funksiya z nuqtada differensiallanuvchi va $f(w)$ funksiya $w = f(z)$ nuqtada differensiallanuvchi bo‘lsa;

5. $f'(z) = \dfrac{1}{(f^{-1}(w))'}$, agar biror z nuqtada $f(z)$ funksiya differensiallanuvchi, $f^{-1}(w)$ teskari funksiya mavjud va $(f^{-1}(w))' \neq 0$ bo‘lsa;

6. $w = e^z,\ w = \sin z,\ w = \cos z,\ w = shz,\ w = chz,\ w = z^n\ (n \in N)$ funksiyalar kompleks tekisligining har bir nuqtasida differensiallanuvchi, $w = tgz$ funksiya $z = \dfrac{\pi}{2} + k\pi\ (k \in Z)$ nuqtalardan boshqa nuqtalarda va $w = thz$ funksiya $z = \left(\dfrac{\pi}{2} + 2k\pi\right) \cdot i\ (k \in z)$ nuqtalardan boshqa nuqtalarda differensiallanuvchi;

7. $w = Lnz,\ w = a^z\ (a \neq 0,\ a - kompleks\ son),\ w = z^{\alpha}\ (kompleks\ son)$ funksiyalar uchun nolga teng bo‘lmagan istalgan z nuqtaning atrofida bir qiymatli barg topish mumkinki, bu funksiyalar z nuqtada differensiallanuvchi bo‘ladi;

8. 6-7 bandlarda keltirilgan funksiyalar uchun haqiqiy o‘zgaruvchi funksiyalarining hosilalar jadvaliga o‘xshash formulalar o‘rinli bo‘ladi.

Masalan, $(e^z)' = e^z,\ (tgz)' = \dfrac{1}{\cos^2 z}\left(z \neq \dfrac{\pi}{2} + k\pi\ (k \in Z)\right),\ (Lnz)' = \dfrac{1}{z}\ (z \neq 0).$

1-misol. $w = f(z)$ funksiya uchun $f'(z)$ hosila mavjud bo‘ladigan nuqtalarni ko‘rsating va bu nuqtalardagi hosilani toping:

1) $w = i\bar{z}$; 2) $w = 5xy - 9x + 18y + i(2x^2 - 2y^2)$; 3) $w = \cos z$; 4) $w = Lnz$.

1) $\bar{z} = x - iy$ ekanidan $w = i\bar{z} = i(x - iy) = y + ix$ bo‘ladi.
Bundan $u = y,\ v = x$ kelib chiqadi.

u va v funksiyalarning xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial y} = 1, \quad \frac{\partial v}{\partial x} = 1, \quad \frac{\partial v}{\partial y} = 0.$$

Bundan

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0, \quad \frac{\partial u}{\partial y} = 1 \neq -1 = -\frac{\partial v}{\partial x},$$

ya’ni Koshi-Riman tengliklarining ikkinchisi bajarilmaydi.

Demak, berilgan funksiya kompleks tekisligining hech bir nuqtasida hosilaga ega bo‘lmaydi.

2) $w = 5xy - 9x + 18y + i(2x^2 - 2y^2)$ da $u = 5xy - 9x + 18y$ va $v = 2x^2 - 2y^2$.

u va v funksiyalarning xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x} = 5y - 9, \quad \frac{\partial u}{\partial y} = 5x + 18, \quad \frac{\partial v}{\partial x} = 4x, \quad \frac{\partial v}{\partial y} = -4y.$$

Koshi-Riman shartlariga ko’ra funksiya hosilaga ega bo‘ladigan nuqtalarda

$$\begin{cases} 5y - 9 = -4y, \\ 5x + 18 = -4x \end{cases}$$

bo‘lishi kerak. Bundan $x = -2,\ y = 1$.

Demak, berilgan funksiya kompleks tekisligining $z_0 = -2 + i$ nuqtasida hosilaga ega bo‘ladi. Bu hosilani topamiz:

$$w'(z_0) = \left.\left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right)\right|_{\substack{x=-2\\ y=1}} = (5y - 9 + i4x)\big|_{\substack{x=-2\\ y=1}} = 5 \cdot 1 - 9 + i4 \cdot (-2) = -4 - 8i.$$

3) $w = \cos z = \cos x\,chy - i\sin x\,shy$ ekanidan $u = \cos x\,chy$ va $v = -\sin x\,shy$.
U holda

$$\frac{\partial u}{\partial x} = -\sin x\,chy, \quad \frac{\partial u}{\partial y} = \cos x\,shy, \quad \frac{\partial v}{\partial x} = -\cos x\,shy, \quad \frac{\partial v}{\partial y} = -\sin x\,chy.$$

Demak, berilgan funksiya uchun butun kompleks tekisligida Koshi-Riman shartlari bajariladi. Bu funksiyaning hosilasini topamiz:

$$w'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = -\sin x\,chy - i\cos x\,shy = -(\sin x\,chy + \cos x\,shy) = -\sin z.$$

4) $w = Lnz = \ln r + i(\varphi + 2k\pi)$ ekanidan $u = \ln r$ va $v = \varphi + 2k\pi.$

Koshi-Riman shartlarining qutb koordinatalardagi koʻrinishiga koʻra

$$\frac{\partial u}{\partial r} = \frac{1}{r}, \quad \frac{\partial u}{\partial \varphi} = 0, \quad \frac{\partial v}{\partial r} = 0, \quad \frac{\partial v}{\partial \varphi} = 1.$$

Bundan

$$\frac{\partial u}{\partial r} = \frac{1}{r} = \frac{1}{r}\frac{\partial v}{\partial \varphi}, \qquad \frac{\partial v}{\partial r} = 0 = -\frac{1}{r}\frac{\partial u}{\partial \varphi},$$

ya'ni Koshi-Riman shartlari $z = 0$ nuqtadan boshqa barcha nuqtalarda bajariladi. Bu funksiyaning hosilasini topamiz:

$$w'(z) = \frac{r}{z}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right) = \frac{r}{z}\left(\frac{1}{r} + i0\right) = \frac{1}{z}, \ z \neq 0.$$

2.3.3. G kompleks sohaning har bir nuqtasida differensiallanuvchi boʻlgan bir qiymatli $w = f(z)$ funksiyaga G sohada *analitik* (*golomorf*, *reguyar*) funksiya deyiladi.

z nuqtada va uning biror atrofida differensiallanuvchi boʻlgan bir qiymatli $w = f(z)$ funksiyaga z nuqtada *analitik* funksiya deyiladi.

Bir qiymatli $w = f(z)$ funksiya analitik boʻladigan nuqtalarga *toʻgʻri* (*regular*) nuqtalar deyiladi. Bir qiymatli $w = f(z)$ funksiya analitik boʻlmaydigan nuqtalarga esa *maxsus* nuqtalar deyiladi.

$w = f(z)$ analitik funksiyaning z nuqtadagi differensiali dw deb, Δw orttirmaning bosh qismiga aytiladi, ya'ni $dw = f'(z)\Delta z$ yoki $dw = f'(z)dz.$

Agar $w = f(z) = u(x, y) + iv(x, y)$ funksiya biror kompleks G sohada analitik boʻlsa, u holda bu funksiyaning haqiqiy va mavhum qismlari Laplas tenglamasi deb ataluvchi $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial^2 v} = 0$ tenglamanini qanoatlantiradi, ya'ni

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y} = 0, \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial^2 y} = 0$$

boʻladi.

Laplas tenglamasini qanoatlantiruvchi funksiyalarga *garmonik funksiyalar* deyiladi. Demak, analitik funksiyalarning haqiqiy va mavhum qismlari garmonik funksiyalar boʻladi.

2-misol. $w = f(z)$ funksiyaning analitik yoki analitik emasligini tekshiring: 1) $w = z^2$; 2) $w = z\operatorname{Re} z.$

1) $w = z^2 = (x + iy)^2 = x^2 - y^2 + i2xy$ ekanidan , $u = x^2 - y^2, v = 2xy.$

u va v funksiyalarning xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x}=2x,\quad \frac{\partial u}{\partial y}=-2y,\quad \frac{\partial v}{\partial x}=2y,\quad \frac{\partial v}{\partial y}=2x.$$

Koshi-Riman shartlarining bajarilishini tekshiramiz:

$$\frac{\partial u}{\partial x}=2x=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-2y=-\frac{\partial v}{\partial x}.$$

Demak, butun kompleks tekisligida Koshi-Riman shartlari bajariladi va berilgan funksiya bu tekislikning barcha nuqtalarida analitik bo‘ladi.

2) $w=z\operatorname{Re}z=(x+iy)x=x^2+ixy$ ekanidan

$$\frac{\partial u}{\partial x}=2x,\quad \frac{\partial u}{\partial y}=0,\quad \frac{\partial v}{\partial x}=y,\quad \frac{\partial v}{\partial y}=x$$

kelib chiqadi. Bu hosilalardan ko‘rinadiki, Koshi-Riman shartlari bajarilishi uchun $x=0,\ y=0$ bo‘lishi kerak. Demak, funksiya bitta $(0;0)$ nuqtada hosilaga ega. Shu sababli $w=z\operatorname{Re}z$ funksiya analitik emas.

2.3.4. (z) kompleks tekisligining biror G sohasida $w=f(z)$ analitik funksiya berilgan va G sohaning qo‘zg‘almas z_0 nuqtasida $f'(z_0)\neq 0$ bo‘lsin.

Bunda: 1) $|f'(z_0)|$ kattalik $w=f(z)$ akslanishning z_0 nuqtadagi cho‘zilish xarakterini aniqlaydi: $k=|f'(z_0)|$ kattalik $|f'(z_0)|>1$ bo‘lganda *cho’zilishni koeffitsiyenti*, $|f'(z_0)|<1$ bo’lganda esa *siqilish koeffitsiyenti* deyiladi (hosila modulining *geometrik ma’nosi*);

2) $\arg f'(z_0)$ kattalik l egri chiziqqa z_0 nuqtada o‘tkazilgan urunmani L egri chiziqqa w_0 nuqtada o‘tkazilgan urunmaga qadar burish burchagini (burilish burchagini)ifodalaydi (hosila argumentining *geometrik ma’nosi*).

z_0 nuqtada burchakning saqlanishi va cho‘zishning o‘zgarmasligi xossalariga ega bo‘lgan $w=f(z)$ akslantirishga *konform akslantirish* deyiladi.

Agar konform akslantirishda burchak hisobining yo‘nalishi ham o‘zgarmasa, bunday akslantirishga *I tur konform akslantirish* deyiladi.

Agar konform akslantirishda burchak hisobining yo‘nalishi qarama-qarshisiga o‘zgarsa, bunday akslantirishga *II tur konform akslantirish* deyiladi.

Shunday qilib, agar (z)kompleks tekisligining biror z_0 nuqtasida $w=f(z)$ funksiya analitik va $f'(z_0)\neq 0$ bo‘lsa, u holda bu nuqtada $w=f(z)$ akslantirish konform bo‘ladi.

3-misol. $w=z^2$ akslantirishning $z_0=\sqrt{2}(1+i)$ nuqtadagi cho‘zilish koeffitsiyentini va burilish burchagini toping.

⊖ $w'=2z$ ekanidan $w'\big|_{z=z_0}=2\sqrt{2}(1+i)$ bo‘ladi. U holda

$$k=|2\sqrt{2}(1+i)|=4,\quad \varphi=\frac{\pi}{4}.$$ ⊖

Mashqlar

2.3.1. $w=f(z)$ funksiya uchun $f'(z)$ hosila mavjud bo‘ladigan nuqtalarni ko‘rsating va bu nuqtalardagi hosilani toping:

1) $w=z\operatorname{Im}z;$ 2) $w=|z|^2;$
3) $w=\bar{z}\operatorname{Re}z^2;$ 4) $w=z^2-\operatorname{Re}(\bar{z}+2z);$
5) $w=\dfrac{1}{z-1};$ 6) $w=\dfrac{z}{|z|};$
7) $w=z^4;$ 8) $w=Ln(z^2);$
9) $w=\sin(iz);$ 10) $w=ish\bar{z}.$

2.3.2. $w=f(z)$ funksiyaning analitik yoki analitik emasligini tekshiring:

1) $w=(x^3-3xy^2)+i(3x^2y-y^3);$ 2) $w=(x^2+y^2)-i2xy;$
3) $w=e^{4z};$ 4) $w=\sin\dfrac{z}{4};$
5) $w=e^x\cos y+ie^x\sin y;$ 6) $w=\ln(z^2).$

2.2.3. Berilgan $v(x,y)$ mavhum qismiga ko‘ra $w=f(z)$ funksiyani toping:

1) $v(x,y)=2x^2-2y^2+x;$ 2) $v(x,y)=e^x\sin y.$

2.2.4. Berilgan $u(x,y)$ haqiqiy qismiga ko‘ra $w=f(z)$ funksiyani toping:

1) $u(x,y)=x^2-y^2-x;$ 2) $u(x,y)=chx\cos y.$

2.3.5. $w=f(z)$ akslantirishning z_0 nuqtadagi cho‘zilish koeffitsiyentini va burilish burchagini toping:

1) $w=z^3,\ z_0=1+i;$ 2) $w=z^2-z,\ z_0=1-i;$
3) $w=\sin z,\ z_0=0;$ 4) $w=ie^{2z},\ z_0=2\pi i.$

2.3.6. (w) kompleks tekisligining siqilish va cho‘zilish sohalarini aniqlang:

1) $w = z^2 + 2z;$ 2) $w = \frac{1}{z}.$

2.3.7. Berilgan akslanishda cho‘zilish koeffitsienti $k = 1$ bo‘ladigan nuqtalar to‘plamini aniqlang:

1) $w = z^2 - iz;$ 2) $w = -z^3.$

2.3.8. Berilgan akslanishda burilish burchagi $\varphi = 0$ bo‘ladigan nuqtalar to‘plamini aniqlang:

1) $w = z^2 + iz;$ 2) $w = -\frac{i}{z}.$

2.4. KOMPLEKS O‘ZGARUVCHI FUNKSIYASINI INTEGRALLASH

Kompleks o‘zgaruvchi bo‘yicha integrallash. Koshi teoremalari. Boshlang‘ich funksiya va aniqmas integral. Koshining integral formulasi

2.4.1. Kompleks tekislikdagi biror G sohada boshlang‘ich nuqtasi z_0 va oxirgi nuqtasi z_n bo‘lgan silliq yoki bo‘lakli silliq L egri chiziq berilgan bo‘lib, bu chiziqda kompleks o‘zgaruvchining $f(z)$ funksiyasi aniqlangan bo‘lsin.

L egri chiziqni $z_0, z_1, ..., z_n$ nuqtalar bilan n ta elementar $\gamma_0, \gamma_1, ..., \gamma_{n-1}$ yoylarga bo‘lamiz va quyidagi limitni qaraymiz:

$$\lim_{\lambda \to 0} \sum_{k=0}^{n-1} f(\xi_k) \Delta z_k, \tag{4.1}$$

bu yerda, $\xi_k \in \gamma_k$, $\Delta z_k = z_{k+1} - z_k$, $\lambda = \max\limits_{0 \le k \le n-1} \{l_k\}$, $l_k - \gamma_k$ yoy uzunligi, $k = \overline{0, n-1}.$

◎ Agar (4.1) chekli limit L egri chiziqni yoylarga bo‘lish usuliga va bu yoylarda ξ_k nuqtani tanlash usuliga bog‘liq bo‘lmagan holda mavjud bo‘lsa, bu limitga $f(z)$ funksiyadan *L egri chiuziq bo‘yicha olingan integral* deyiladi va $\int_L f(z)dz$ kabi belgilanadi:

$$\int_L f(z)dz = \lim_{\lambda \to 0} \sum_{k=0}^{n-1} f(\xi_k) \Delta z_k. \tag{4.2}$$

1-teorema (*funksiya integrallanuvchi bo'lishining etarli sharti*). Agar $f(z)$ funksiya L egri chiziqda uzluksiz bo'lsa, u holda u shu egri chiziq bo'yicha integrallanuvchi bo'ladi.

$\int_L f(z)dz$ integral haqiqiy o'zgaruvchili funksiyalarning ikkita II tur egri chiziqli integraliga keltiriladi:

$$\int_L f(z)dz = \int_L u(x,y)dx - v(x,y)dy + i\int_L v(x,y)dx + u(x,y)dy. \qquad (4.3)$$

Agar L egri chiziq $y = y(x)$ $(a \le x \le b)$ tenglama bilan berilgan bo'lsa, (4.3) integralda $y = y(x),\ dy = y'(x)dx$ o'rniga qo'yish bajariladi.

Agar $x = x(t),\ y = y(t)$ $(\alpha \le t \le \beta)$ tenglamalar L egri chiziqning parametrik tenglamalari bo'lsa, $z = z(t) = x(t) + iy(t)$ tenglamaga L egri chiziqning *kompleks parametrik tenglamasi* deyiladi.

Kompleks parametrik tenglamada (4.3) integral

$$\int_L f(z)dz = \int_\alpha^\beta f(z(t))z'(t)dt \qquad (4.4)$$

ko'rinishda yoziladi.

$f(z)$ funksiyadan L egri chiuziq bo'yicha olingan integral haqiqiy o'zgaruvchili funksiyaning aniq integrali va egri chiziqli integrali ega bo'ladigan xossalarga ega bo'ladi.

1-misol. Integrallarni berilgan egri chiziq bo'yicha hisoblang:

1) $\int_L (\bar{z} - \operatorname{Im} z)dz,\ L:\ z_1 = 0,\ z_2 = 2 + 4i$ nuqtalar orasidagi to'g'ri chiziq kesmasi;

2) $\int_L \frac{dz}{z-a},\ L:$ markazi $z_0 = a$ nuqtada bo'lgan R radiusli aylana, bunda aylanib o'tish soat strelkasi yo'nalishiga teskari;

3) $\int_L \operatorname{Im} z dz,\ L:\ y = 2x^2$ parabolaning $z_1 = 0,\ z_2 = 1 + 2i$ nuqtalar orasidagi yoyi;

4) $\int_L (y^2 + x + x^2 i)dz,\ L:\ z_1 = 2 - i,\ z_2 = 3 + 2i$ nuqtalar orasidagi to'g'ri chiziq kesmasi.

1) $z_1 = 0,\ z_2 = 2 + 4i$ nuqtalar orasidagi to'g'ri chiziq kesmasi $x = t, y = 2t$ $(0 \le t \le 2)$ parametrik tenglamalar bilan beriladi. Uning kompleks parametrik tenglamasi $z = t(1 + 2i)$ ko'rinishda bo'ladi. Bundan $dz = (1 + 2i)dt$.

U holda

$$\int_L (\bar{z} - \operatorname{Im} z)dz = \int_0^2 (t(1-2i) - 2t)(1+2i)dt =$$

$$= -(1+2i)^2\int_0^2 tdt = -(-3+4i)\frac{t^2}{2}\Bigg|_0^2 = 2(3-4i).$$

2) Markazi $z_0 = a$ nuqtada bo‘lgan R radiusli aylananing kompleks parametrik tenglamasi $z = a + R\cdot e^{it}$ $(0 \le t \le 2\pi)$ ko‘rinishda bo‘ladi. Bundan $z - a = R\cdot e^{it}$, $dz = Rie^{it}dt$. U holda

$$\int_L \frac{dz}{z-a} = \int_0^{2\pi}\frac{Rie^{it}dt}{R\cdot e^{it}} = i\int_0^{2\pi}dt = it\Big|_0^{2\pi} = 2\pi i.$$

3) $y = 2x^2$ parabolaning $z_1 = 0,\ z_2 = 1+2i$ nuqtalar orasidagi yoyida $0 \le x \le 1$, $dy = 4xdx$ bo‘ladi. Bunda $\operatorname{Im} zdz = y(dx + idy) = 2x^2(1+i4x)dx$ bo‘ladi.

U holda

$$\int_L \operatorname{Im} zdz = \int_0^1 (2x^2 + i8x^3)dx = \left(2\frac{x^3}{3} + i2x^4\right)\Bigg|_0^1 = \frac{2}{3} + 2i.$$

4) $z_1 = 2 - i,\ z_2 = 3 + 2i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi tenglamasini topamiz:

$$\frac{x-2}{3-2} = \frac{y+1}{2+1},\quad y = 3x - 7,\ 2 \le x \le 3.$$

U holda

$$\int_L (y^2 + x + x^2 i)dz = \int_2^3 ((3x-7)^2 + x + ix^2)(dx + i3dx) =$$

$$= \int_2^3 (9x^2 - 41x + 49 + ix^2)(1+3i)dx = \int_2^3 (6x^2 - 41x + 49 + i(28x^2 - 123x + 147))dx =$$

$$= \left(2x^3 - \frac{41}{2}x^2 + 49x\right)\Bigg|_2^3 + i\left(\frac{28}{3}x^3 - \frac{123}{2}x^2 + 147x\right)\Bigg|_2^3 = -\frac{31}{2} + i\frac{101}{6}.$$

2.4.2. 2-teorema (Koshining 1- teoremasi). Agar bir bog‘lamli yopiq G sohada $f(z)$ funksiya analitik bo‘lsa, u holda G sohada yotuvchi har qanday L yopiq kontur bo‘ylab $f(z)$ funksiyadan olingan integral nolga teng bo‘ladi, ya’ni

$$\oint_L f(z)dz = 0.$$

1-Natija. Agar bir bog‘lamli G sohada $f(z)$ funksiya analitik bo‘lsa, u holda $f(z)$ funksiyadan olingan integral integrallash yo‘liga bog‘liq bo‘lmasdan, balki bu yo‘lning boshlang‘ich z_0 va oxirgi z_n nuqtalariga bog‘liq bo‘ladi.

2-misol. $\int\limits_L \frac{dz}{z^2-1}$ integralni $|z-2i|=1$ aylana bo‘yicha hisoblang.

$\frac{1}{z^2-1}$ funksiya kompleks tekisligining $z_1=-1$ va $z_2=1$ nuqtalaridan boshqa barcha nuqtalarida analitik. Bu nuqtalar L aylanadan tashqarida yotadi. Shu sababli integral ostidagi funksiya L kontur bilan chegaralangan sohada analitik. U holda Koshining 1-teoremasiga ko‘ra $f(z)$ funksiyadan L kontur bo‘yicha olingan integral nolga teng, ya’ni

$$\int\limits_L \frac{dz}{z^2-1}=0.$$

3-teorema (Koshining 2-teoremasi). Agar ko‘p bog‘lamli yopiq G sohada $f(z)$ funksiya analitik bo‘lsa, u holda bu funksiyadan tashqi kontur bo‘yicha olingan integral ichki konturlar bo‘yicha olingan integrallar yig‘indisiga teng bo‘ladi:

$$\oint\limits_{L_0} f(z)dz=\oint\limits_{L_1} f(z)dz+\oint\limits_{L_2} f(z)dz+...+\oint\limits_{L_n} f(z)dz,$$

bunda barcha konturlar bo‘yicha yo‘nalish soat strelkasi yo‘nalishiga teskari olinadi.

2.4.3. G sohaning barcha nuqtalarida $F'(z=f(z)$ bo‘lsa, $F(z)$ funksiyaga G sohada $f(z)$ funksiyaning *boshlang‘ich funksiyasi* deyiladi.

$f(z)$ funksiyaning G sohadagi boshlang‘ich funksiyalari to‘plami $F(z)+C$ ga $f(z)$ funksiyaning *aniqmas integrali* deyiladi va $\int f(z)dz$ kabi belgilanadi.

Agar $F(z)$ funksiya $f(z)$ funksiyaning boshlang‘ich funksiyasi bo‘lsa,

$$\int\limits_L f(z)dz=\int\limits_{z_1}^{z_2} f(z)dz=F(z_2)-F(z_1) \qquad (4.5)$$

Nuyton-Leybnis formulasi o‘rinli bo‘ladi, bu yerda z_1, z_2-L egri chiziqning boshlang‘ich va oxirgi nuqtalari.

3-misol. Integrallarni hisoblang: 1) $\int\limits_i^{1-i} z^2dz$; 2) $\int\limits_0^i z\cos zdz.$

1) Integral ostidagi z^2 funksiya butun kompleks tekisligida analitik. Shu sababli uning $z_1=i$ nuqtadan $z_2=1-i$ nuqtagacha integrali integrallash yo‘liga bog‘liq bo‘lmaydi.

Bu integralni Nuyton-Leybnis formulasi bilan hisoblaymiz:

$$\int\limits_i^{1-i} z^2dz=\frac{1}{3}z^3\Big|_i^{1-i}=\frac{1}{3}(1-3i+3i^2-i^3-i^3)=\frac{1}{3}(1-3i-3+2i)=-\frac{2+i}{3}.$$

2) $z\cos z$ funksiya butun kompleks tekisligida analitik. Shu sababli bo‘laklab integrallash usulini va Nuyton-Leybnis formulasi qo‘llab, topamiz:

$$\int_0^i z\cos z dz = \left| \begin{array}{c} u = z,\ du = dz, \\ dv = \cos z dz,\ v = \sin z \end{array} \right| = z\sin z\Big|_0^i - \int_0^i \sin z dz =$$

$$= i\sin i + \cos z\Big|_0^i = i \cdot ish1 + \cos i - 1 = ch1 - sh1 - 1.$$

2.4.4. 5-teorema. $f(z)$ – bir bog‘lamli yopiq G sohada analitik, $L - G$ sohada yotuvchi D sohaning chegarasi bo‘lsin. U holda

$$f(z_0) = \frac{1}{2\pi i}\oint_L \frac{f(z)}{z - z_0}dz \qquad (4.6)$$

formula o‘rinli bo‘ladi. Bunda $z_0 - G$ sohaning ixtiyoriy ichki nuqtasi va L kontur bo‘yicha integrallar musbat, ya’ni soat strelkasi yo‘nalishiga teskari olinadi.

(4.6) tenglikning o‘ng tomonidagi *integral Koshi integrali deb ataladi.* (4.6) formulaga *Koshining* integral formulasi deyiladi.

Bu teorema ko‘p bog‘lamli sohalar uchun ham isbotlangan.

Bu teoremadan quyidagi natijalar kelib chiqadi.

2-Natija. z_0 nuqtada differensiallanuvchi har qanday $f(z)$ funksiya uchun barcha tartibli hosilalar mavjud bo‘ladi. Bunda n – tartibli hosila

$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_L \frac{f(z)}{(z - z_0)^{n+1}}dz \qquad (4.8)$$

formula bilan aniqlanadi.

3-Natija. z_0 nuqtadaning $f'(z_0)$ hosila mavjud bo‘lgan atrofida $f(z)$ funksiya yaqinlashuvchi

$$f(z) = f(z_0) + f'(z_0)(z - z_0) + \frac{f''(z_0)}{2!}(z - z_0)^2 + ... + \frac{f^{(n)}(z_0)}{n!}(z - z_0)^n + ... \qquad (4.9)$$

qator bilan berilishi mumkin.

4-misol. $\int_L \frac{ze^z}{(z^2+1)(z-1)}dz$ integralni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang, bunda:

1) $L: |z-1| = 1$ aylana; 2) $L: |z| = \frac{1}{2}$ aylana; 3) $L: |z - 1 + i| = \sqrt{2}$ aylana.

1) Integralni quyidagicha yozib olamiz:

$$\int_{|z-1|=1} \frac{ze^z}{(z^2+1)(z-1)}dz = \int_{|z-1|=1} \frac{\frac{ze^z}{z^2+1}}{z-1}dz.$$

Bunda $f(z)=\dfrac{ze^z}{z^2+1}$ funksiya $|z-1|<1$ sohada analitik. Va $z_0=1$ nuqta bu sohada yotadi. U holda (4.6) formulaga ko‘ra

$$\int\limits_{|z-1|=1}\frac{ze^z}{(z^2+1)(z-1)}dz=2\pi i\left.\frac{ze^z}{z^2+1}\right|_{z=1}=\pi ei.$$

2) $|z|<\dfrac{1}{2}$ sohada integral ostidagi $\dfrac{ze^z}{(z^2+1)(z-1)}$ funksiya analitik. U holda Koshining 1-teoremasiga ko‘ra

$$\int\limits_{|z|=\frac{1}{2}}\frac{ze^z}{(z^2+1)(z-1)}dz=0.$$

3) $L:|z-1+i|<\sqrt{2}$ sohada integral ostidagi funksiyaning maxraji nolga teng bo‘ladigan ikkita nuqta bor: $z_1=1,\ z_2=i.$ Shu sababli (4.6) formulani to‘g‘ridan-to‘g‘ri qo‘llab bo‘lmaydi. Integral ostidagi funksiyani sodda kasrlarga yoyamiz:

$$\frac{ze^z}{(z^2+1)(z-1)}=-\frac{(z-1)e^z}{2(z^2+1)}+\frac{e^z}{2(z-1)}.$$

Bunda har bir qo‘shiluvchi funksiyaning integrali uchun (4.6) formulani qo‘llab topamiz:

$$\int\limits_L\frac{ze^z}{(z^2+1)(z-1)}=-\frac{1}{2}\int\limits_L\frac{(z-1)e^z}{z^2+1}+\frac{1}{2}\int\limits_L\frac{e^z}{z-1}=-\frac{1}{2}\int\limits_L\frac{\frac{(z-1)e^z}{z+i}}{z-i}+\frac{1}{2}\int\limits_L\frac{e^z}{z-1}=$$

$$=-\pi i\left.\frac{(z-1)e^z}{z+i}\right|_{z=i}+\pi ie^z\Big|_{z=1}=\frac{\pi}{2}(1-i)e^i+\pi ei.$$

5-misol. $\displaystyle\int\limits_L\frac{chzdz}{(z+1)^2(z-1)}$ integralni $|z|=2$ aylana bo‘yicha hisoblang.

$(z+1)(z-1)$ ifoda $z_1=-1$ va $z_2=1$ nuqtalarda nolga teng. Bu nuqtalarni markaz deb olib, L_1 va L_2 aylanalarni shunday quramizki, bu aylanalar $|z|\le 2$ doirada to‘liq joylashsin. $L,\ L_1,\ L_2$ aylanalar bilan chegaralangan uch bog‘lamli sohaning barcha nuqtalarida $\dfrac{chz}{(z+1)^2(z-1)}$ funksiya analitik. U holda Koshining 2-teoremasiga ko‘ra

$$\int\limits_L\frac{chzdz}{(z+1)^2(z-1)}=\int\limits_{L_1}\frac{chzdz}{(z+1)^2(z-1)}+\int\limits_{L_2}\frac{chzdz}{(z+1)^2(z-1)}.$$

Bu integrallardan birinchisida integral ostidagi funksiyani

$$\frac{chz}{(z+1)^2(z-1)} = \frac{\dfrac{chz}{z-1}}{(z+1)^2}$$

ko‘rinishda yozib olamiz. $\dfrac{chz}{z-1}$ funksiya L_1 sohada analitik.

U holda (4.8) formulaga ko‘ra

$$\int_{L_1} \frac{chzdz}{(z+1)^2(z-1)} = \int_{L_1} \frac{\dfrac{chz}{z-1}}{(z+1)^2}dz = 2\pi i\left(\frac{chz}{z-1}\right)'\Bigg|_{z=-1} = \pi i\frac{2sh1-ch1}{2}.$$

Ikkinchi integralni shu kabi topamiz:

$$\int_{L_2} \frac{chzdz}{(z+1)^2(z-1)} = \int_{L_{21}} \frac{\dfrac{chz}{(z+1)^2}}{z-1}dz = 2\pi i\frac{chz}{(z+1)^2}\Bigg|_{z=1} = \pi i\frac{ch1}{2}.$$

U holda

$$\int_{L} \frac{chzdz}{(z+1)^2(z-1)} = \pi i\frac{2sh1-ch1}{2} + \pi i\frac{ch1}{2} = sh1\pi i.$$

6-misol. $\displaystyle\int_L \frac{z-2\sin z}{\left(z-\dfrac{\pi}{2}\right)^3}dz$ integralni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang, bunda: $L: |z-1|=2$ aylana.

$L: |z-1|=2$ aylana $z_0 = \dfrac{\pi}{2}$ nuqtani o‘z ichiga oladi.

U holda (4.8) gormulani qo‘llab, topamiz:

$$\int_L \frac{z-2\sin z}{\left(z-\dfrac{\pi}{2}\right)^3}dz = \frac{2\pi i}{2!}(z-2\sin z)''\Big|_{x=\frac{\pi}{2}} = \frac{2\pi i}{2}\cdot 2 = 2\pi i.$$

7-misol. $\displaystyle\int_0^{2\pi}\sin^4 xdx$ integralni hisoblang.

$z = e^{ix}$ o‘rniga qo‘yish bajaramiz.

Bundan $\sin x = \dfrac{1}{2i}\left(z-\dfrac{1}{z}\right)$, $\sin^6 x = \dfrac{1}{16}\dfrac{(z^2-1)^4}{z^4}$, $dx = \dfrac{dz}{iz}$.

U holda

$$\int_0^{2\pi}\sin^4 xdx = \frac{1}{16i}\int_{|z|=1}\frac{(z^2-1)^4}{z^5}dz = \frac{2\pi i}{16i\cdot 4!}((z^2-1)^4)^{(4)}\Big|_{z=0} = -\frac{3\pi}{4}.$$

Mashqlar

2.4.1. Integrallarni berilgan egri chiziq bo‘yicha hisoblang:

1) $\int_L (1+i-2\bar{z})dz,\ L:\ z_1=0,\ z_2=1+i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi;

2) $\int_L \mathrm{Re}(2\bar{z})dz,\ L:\ z_1=0,\ z_2=5-3i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi;

3) $\int_L z\,\mathrm{Im}\,z dz,\ L:\ z_1=1+i,\ z_2=2$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi;

4) $\int_L (z^2-3iz)dz,\ L:\ z_1=1,\ z_2=i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi;

5) $\int_L |z|\bar{z}dz,\ L:\ |z|=1$ yuqori yarim aylana, bunda aylanib o‘tish soat strelkasi yo‘nalishiga teskari;

6) $\int_L \frac{dz}{z-(3-2i)},\ L:\ |z-3+2i|=1$ yuqori yarim aylana, bunda aylanib o‘tish soat strelkasi yo‘nalishiga teskari;

7) $\int_L (z^2+z\bar{z})dz,\ L:\ y=x^2$ parabolaning $z_1=0,\ z_2=1+i$ nuqtalar orasidagi yoyi;

8) $\int_L \mathrm{Re}(z^2-z)dz,\ L:\ y=2x^2$ parabolaning $z_1=0,\ z_2=1+2i$ nuqtalar orasidagi yoyi;

9) $\int_L \sin z dz,\ L:\ z_1=0,\ z_2=i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi;

10) $\int_L e^{\bar{z}}dz,\ L:\ z_1=0,\ z_2=\pi+\pi i$ nuqtalar orasidagi to‘g‘ri chiziq kesmasi.

2.4.2. $\int_L \bar{z}dz$ integralni $z_1=-1,\ z_2=1$ nuqtagacha hisoblang:

1) L: to‘g‘ri chiziq kesmasi; 2) $L:\ |z|=1$ yuqori yarim aylana;

3) $L:\ y=1-x^2$ parabola yoyi.

2.4.3. $\int_L \mathrm{Im}\,z dz$ integralni $z_1=-i,\ z_2=i$ nuqtagacha hisoblang:

1) L:to‘g‘ri chiziq kesmasi; 2) $L:\ |z|=1$ o‘ng yarim aylana;

3) $L:\ x=1-y^2$ parabola yoyi.

2.4.4. $\int_L \frac{dz}{z+3}$ integralni $x=2\cos t,\ y=\sin t$ ellips bo‘yicha hisoblang.

2.4.5. $\int_L \frac{zdz}{(z-i)^2}$ integralni $|z|=\frac{1}{3}$ aylana bo‘yicha hisoblang.

2.4.6. Integrallarni hisoblang:

1) $\int\limits_{i-1}^{2+i}(3z^2+2z)dz;$ 2) $\int\limits_{i}^{2-i}(z^2-z+1)dz;$

3) $\int\limits_{\frac{\pi}{2}}^{i}\sin zdz;$ 4) $\int\limits_{0}^{i}(z+\cos z)dz;$

5) $\int\limits_{0}^{\pi-i\pi} chzdz;$ 6) $\int\limits_{1}^{i} ze^{z^2}dz;$

7) $\int\limits_{0}^{i}(\cos^2 z-2)dz;$ 8) $\int\limits_{0}^{2i} z\sin zdz.$

2.4.7. Integrallarni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang:

1) $\int\limits_L \frac{dz}{z^2+1}$, $L:|z-i|=1$ aylana; 2) $\int\limits_L \frac{\cos zdz}{z}$, $L:|z|=1$ aylana;

3) $\int\limits_L \frac{e^{z^2}dz}{z^2-4z}$, $L:|z-2|=3$ aylana; 4) $\int\limits_L \frac{\sin zdz}{z+i}$, $L:|z|=2$ aylana;

5) $\int\limits_L \frac{e^{2z}dz}{z-\pi i}$, $L:|z|=4$ aylana; 6) $\int\limits_L \frac{\sin\frac{\pi z}{2}dz}{z^2-1}$, $L:|z-1|=1$ aylana;

7) $\int\limits_L \frac{\sin z\sin(z-1)}{z^2-z}dz$, $L:|z|=2$ aylana; 8) $\int\limits_L \frac{\cos zdz}{z^2-\pi^2}$, $L:|z|=4$ aylana.

2.4.8. $\int\limits_L \frac{dz}{z^2+3z}$ integralni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang, bunda:

1) $L:|z-3|=1$ aylana; 2) $L:|z|=1$ aylana; 3) $L:|z+3|=1$ aylana.

2.4.9. $\int\limits_L \frac{e^{\pi z}dz}{z^2+iz}$ integralni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang, bunda:

1) $L:|z|=\frac{1}{2}$ aylana; 2) $L:|z-2|=1$ aylana; 3) $L:|z+i|=\frac{1}{2}$ aylana.

2.4.10. $\int\limits_L \frac{dz}{(z-1)^3(z+1)^3}$ integralni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang, bunda:

1) $L:|z-1|=1$ aylana; 2) $L:|z+1|=1$ aylana; 3) $L:|z|=R, R\neq 1$ aylana.

2.4.11. Integrallarni aylanib o‘tish soat strelkasiga teskari yo‘nalishda hisoblang:

1) $\int_L \frac{e^{2z}dz}{(z+3)^3}$, $L: |z+2|=2$ aylana; 2) $\int_L \frac{z\sin zdz}{(z-2)^3}$, $L: |z|=3$ aylana;

3) $\int_L \frac{\sin zdz}{z^2}$, $L: |z|=1$ aylana; 4) $\int_L \frac{dz}{(z+2)^3 z}$, $L: |z+2|=1$ aylana;

5) $\int_L \frac{\sin\frac{\pi z}{4}dz}{(z-3)(z-1)^2}$, $L: |z-i|=1$ aylana; 6) $\int_L \frac{\sin zdz}{(z+i)^3}$, $L: |z+i|=1$ aylana;

7) $\int_L \frac{1}{z^2}\cos\frac{\pi}{z+1}dz$, $L: |z|=\frac{1}{2}$ aylana; 8) $\int_L \frac{sh^2 zdz}{z^2}$, $L: |z|=1$ aylana.

2.5. KOMPLEKS HADLI QATORLAR

Sonli qatorlar. Darajali qatorlar. Teylor qatori. Analitik funksiyaning nollari. Loran qatori. Maxsus nuqtalarning klassifikasiyasi

2.5.1. ◎ Hadlari $c_1, c_2, ..., c_n, ...$ kompleks sonlardan iborat bo‘lgan

$$\sum_{n=1}^{\infty} c_n = c_1 + c_2 + c_3 + ... + c_n + ... \qquad (5.1)$$

qatorga kompleks sohada *sonli qator* deyiladi.

Hadlari $c_n = a_n + ib_n$ bo‘lgan (5.1) qatorni

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty}(a_n + ib_n) = \sum_{n=1}^{\infty} a_n + i\sum_{n=1}^{\infty} b_n$$

ko‘rinishda yozish mumkin, bu yerda a_n, b_n – haqiquy sonlar.

(5.1) qatorning birinchi n ta hadlarining yig‘indisi $S_n = \sum_{k=1}^{n} c_k = \sum_{k=1}^{n} a_k + i\sum_{k=1}^{n} b_k$ qatorning n*-qismiy yig‘indisi* deb ataladi.

◎ Agar qismiy yig‘indilar ketma-ketligi $\{S_n\}$ chekli limitga ega, ya’ni $\lim_{n\to\infty} S_n = S$ bo‘lsa, (5.1) qatorga *yaqinlashuvchi qator*, S ga qatorning *yig‘indisi* deyiladi.

◎ Agar $\{S_n\}$ ketma-ketlik chekli limitga ega bo‘lmasa, (5.1) qatorga *uzoqlashuvchi* qator deyiladi.

Kompleks hadli $\sum_{n=1}^{\infty} c_n$ qatorni yaqinlashishga tekshirish haqiqiy hadli ikkita $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlarni yaqinlashishga tekshirishga keltiriladi.

1-misol. $\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - i\frac{1}{3^n}\right)$ qatorning yaqinlashuvchi ekanini ko‘rsating va yig‘indisini toping.

Berilgan qatorning haqiqiy va mavhun qismlaridagi haqiqiy hadli qatorlarni yozib olamiz:

$$\sum_{n=o}^{\infty}\frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n}, \qquad -\sum_{n=o}^{\infty}\frac{1}{3^n} = -1 - \frac{1}{3} - \frac{1}{9} - \ldots - \frac{1}{3^n}.$$

Bu qatorlarning har ikkalasi cheksiz kamayuvchi geometrik progressiyalardan iborat. Shu sababli bu qatorlarning har ikkalasi yaqinlashuvchi va $S_1 = \frac{1}{1-\frac{1}{2}} = 2$ va $S_2 = \frac{-1}{1-\frac{1}{3}} = -\frac{3}{2}$ ga teng. U holda berilgan qator yaqinlashadi va uning yig‘indisi $S = 2 - i\frac{3}{2}$.

Kompleks hadli qatorlar nazariyasining asosiy ta’riflari, bir qancha teoremalari va ularning isbotlari haqiqiy hadli qatorlar nazariyasining mos ta’rif va teoremalariga o‘xshash bo‘ladi.

1-teorema (*qator yaqinlashishining zaruriy alomati*). Agar $\sum_{n=1}^{\infty} c_n$ qator yaqinlashuvchi bo‘lsa, u holda $\lim_{n\to\infty} c_n = 0$ bo‘ladi.

Agar (5.1) qator hadlarining absolut qiymatlaridan tashkil topgan

$$\sum_{n=1}^{\infty} |c_n| = |c_1| + |c_2| + |c_3| + \ldots + |c_n| + \ldots \qquad (5.2)$$

qator yaqinlashuvchi bo‘lsa, (5.1) qatorga *absolut yaqinlashuvchi qator* deyiladi.

Agar (5.1) qator yaqinlashuvchi va (5.2) qator uzoqlashuvchi bo‘lsa, (5.1) qator *shartli yaqinlashuvchi qator* deyiladi.

2-teorema. Agar (5.2) qator yaqinlashuvchi bo‘lsa, u holda (5.1) qator absolut yaqinlashadi.

Kompleks hadli qatorlarni yaqinlashishga tekshirishda haqiqiy hadli qatorlar uchun ma’lum bo‘lgan musbat hadli qatorlar yaqinlashishining yetarli alomatlari qo‘llaniladi.

2-misol. $\sum_{n=1}^{\infty}\left(\frac{2-i}{3}\right)^n$ qatorni yaqinlashishga tekshiring.

Berilgan qator hadlarining absolut qiymatlaridan iborat qatorni qaraymiz. Bu qator uchun $|c_n|=\left|\frac{2-i}{3}\right|^n$. Qatorni yaqinlashishga Koshining ildiz alomati bilan tekshiramiz:

$$\lim_{n\to\infty}\sqrt[n]{|c_n|}=\lim_{n\to\infty}\sqrt[n]{\left|\frac{2-i}{3}\right|^n}=\lim_{n\to\infty}\frac{|2-i|}{3}=\frac{\sqrt{5}}{3}<1.$$

Demak, $\sum_{n=1}^{\infty}\left|\frac{2-i}{3}\right|^n$ qator yaqinlashadi va berilgan qator absolut yaqinlashadi.

2.5.2. Hadlari $z=x+iy$ kompleks o‘zgaruvchining bir qiymatli $u_1(z),u_2(z),..u_n(z),...$ funksiyalaridan iborat bo‘lgan

$$\sum_{n=1}^{\infty}u_n(z)=u_1(z)+u_2(z)+...+u_n(z)+...$$

qatorga kompleks sohada *funksional qator* deyiladi.

Ushbu

$$\sum_{n=0}^{\infty}c_nz^n=c_0+c_1z+c_2z^2+...+c_nz^n+... \qquad (5.3)$$

ko‘rinishdagi funksional qatorga kompleks sohada *darajali qator* deyiladi, bu yerda c_n-kompleks sonlar (qatorning *koeffitsiyentlari*), $z=x+iy-$kompleks o‘zgaruvchi.

Shu bilan birga kompleks sohada

$$\sum_{n=0}^{\infty}c_n(z-z_0)^n \qquad (5.4)$$

$z-z_0$ ayirma darajalari bo‘yicha qator deb ataluvchi qator ham qaraladi, bu yerda z_0-kompleks son.

z argumentning (5.3) qator yaqinlashadigan nuqtalari to‘plamiga bu qatorning *yaqinlashish sohasi* deyiladi.

3-teorema (*Abel teoremasi*). Agar (5.3) darajali qator $z=z_0\neq 0$ nuqtada yaqinlashsa, u holda u $z$ning $|z|<|z_0|$ tengsizlikni qanoatlantiruvchi barcha nuqtalarida absolut yaqinlashadi.

1-natija. Agar (5.3) darajali qator $z=z_0$ nuqtada uzoqlashsa, u holda u z ning $|z|>|z_0|$ tengsizlikni qanoatlantiruvchi barcha nuqtalarida uzoqlashadi.

Demak, Abel teoremasiga ko‘ra shunday $R=|z_0|$son topiladiki, $z$ning $|z|<R$, tengsizlikni qanoatlantiruvchi barcha qiymatlarida (5.3) qator absolut yaqinlashadi. Bunda $|z|<R$ tengsizlik kompleks tekisligining markazi $z=0$ nuqtada yotuvchi radiusi R ga teng doiraning ichki nuqtalarini ifodalaydi.

R kattalikka (5.3) qatorning *yaqinlashish radiusi*, $|z|<R$ doiraga (5.3) qatorning *yaqinlashish doirasi* deyiladi. Bunda (5.3) qator: doira ichkarisida yaqinlashadi va doira tashqarisida uzoqlashadi; doiraning chegarasi bo‘lgan $|z|=R$ aylanada yaqinlashishi ham uzoqlashishi ham mumkiin.

(5.3) qatorning yaqinlashish radiusi $R=\lim\limits_{n\to\infty}\dfrac{1}{\sqrt[n]{|c_n|}}$ yoki $R=\lim\limits_{n\to\infty}\left|\dfrac{c_n}{c_{n+1}}\right|$ formula bilan topiladi.

(5.4) qatorning yaqinlashish doirasi $|z-z_0|<R$, ya’ni markazi $z=z_0$ nuqtada yotuvchi radiusi R ga teng doira hisoblanadi.

Darajali qator quyidagi xossalarga ega.

1°. Darajali qatorning yig‘indisi bu qatorning yaqinlashish doirasida analitik funksiya bo‘ladi.

2°. Darajali qatorni o‘zining yaqinlashish doirasida istalgan marta hadma-had differensiyallash (integrallash) mumkin. Darajali qatorni hadma-had differensiyallash (integrallash) natijasida hosil qilingan qatorning yaqinlashish doirasi ham berilgan qatorning yaqinlashish doirasi bilan bir hil bo‘ladi.

3-misol. $\sum\limits_{n=0}^{\infty}(2+2i)^n z^n$ qatorning yaqinlashish radiusini toping.

Berilgan qator uchun $|c_n|=|2+2i|^n=(\sqrt{8})^n$. U holda

$$R=\lim_{n\to\infty}\sqrt[n]{\frac{1}{|c_n|}}=\lim_{n\to\infty}\sqrt[n]{\frac{1}{(\sqrt{8})^n}}=\frac{1}{\sqrt{8}}=\frac{\sqrt{2}}{4}.$$

4-misol. $\sum\limits_{n=0}^{\infty}\dfrac{1}{n!}z^n$ qatorning yaqinlashish doirasini toping.

Berilgan qator uchun $c_n=\dfrac{1}{n!}$, $c_{n+1}=\dfrac{1}{(n+1)!}$. U holda

$$R=\lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|=\lim_{n\to\infty}\frac{(n+1)!}{n!}=\lim_{n\to\infty}n=\infty.$$

Demak, berilgan qator markazi $O(0;0)$ nuqtada bo‘lgan har qanday doirada yadinlashadi.

2.5.3. 4-teorema. $|z-z_0|<R$ doirada analitik bo‘lgan har qanday $f(z)$ funksiya bu doirada yagona tarzda

$$f(z)=\sum_{n=0}^{\infty}c_n(z-z_0)^n, \text{ bu yerda } c_n=\frac{f^{(n)}(z)}{n!}=\frac{1}{2\pi i}\oint_{l_r}\frac{f(\xi)}{(\xi-z_0)^{n+1}}d\xi \quad (n=1,2,3,...), \quad (5.5)$$

ko‘rinishda darajali qatorga yoyilishi mumkin, bu yerda, l_r – markazi z_0 bo‘lgan $|z-z_0|<R$ doirada yotuvchi ixtiyoriy aylana.

(5.5) qatorga $|z-z_0|<R$ doirada qaralayotgan $f(z)$ funksiyaning Teylor qatori deyiladi.

e^z, $\sin z$, $\cos z$, $\cos z$, $\ln(1+z)$, $(1+z)^\alpha$ funksiyalar $z_0=0$ nuqta atrofida Teylor qatoriga haqiqiy o‘zgaruvchili funksiyalardagi kabi yoyiladi:

1. $e^z=\sum_{n=0}^{\infty}\frac{z^n}{n!}=1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+...+\frac{z^n}{n!}+...\,,\quad R=\infty$;

2. $\sin z=\sum_{n=0}^{\infty}\frac{(-1)^n z^{2n+1}}{(2n+1)!}=z-\frac{z^3}{3!}+\frac{z^5}{5!}+...+\frac{(-1)^n z^{2n+1}}{(2n+1)!}+...\quad R=\infty$;

3. $\cos z=\sum_{n=0}^{\infty}\frac{(-1)^n z^{2n}}{(2n)!}=1-\frac{z^2}{2!}+\frac{z^4}{4!}+...+\frac{(-1)^n z^{2n}}{(2n)!}+...\,,\ R=\infty$;

4. $\ln(1+z)=\sum_{n=0}^{\infty}\frac{(-1)^n z^{n+1}}{n+1}=z-\frac{z^2}{2}+\frac{z^3}{3}+...+\frac{(-1)^{n-1}z^n}{n}+...\,,\ |R=1$;

5. $(1+z)^\alpha=1+\sum_{n=1}^{\infty}\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}z^n=$

$$=1+\frac{\alpha}{1!}z+\frac{\alpha(\alpha-1)}{2!}z^2+...+\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}z^n+...\,,\quad R=1,$$

xususan,

$$\frac{1}{1+z}=\sum_{n=0}^{\infty}(-1)^n z^n=1-z+z^2+...(-)^n z^n+...,\qquad \frac{1}{1-z}=\sum_{n=0}^{\infty}z^n=1+z+z^2+...+z^n+....$$

60.5-misol. $f(z)$ funksiyalarni z_0 nuqta atrofida Teylor qatoriga yoying:

1) $f(z)=sh3z,\ z_0=0;$ 2) $f(z)=\frac{1}{3-2z},\ z_0=3;$ 3) $f(z)=\ln z,\ z_0=1.$

1) e^{3z} va e^{-3z} funksiyalarning $z_0=0$ nuqta atrofidagi yoyilmalarini e^z funksiyaning $z_0=0$ nuqta atrofidagi yoyilmasida mos ravishda z ning o‘rniga $3z$ va $(-3z)$ ni qo‘yib topamiz:

$$e^{3z}=\sum_{n=0}^{\infty}\frac{(3z)^n}{n!}=1+3z+\frac{(3z)^2}{2!}+\frac{(3z)^3}{3!}+...+\frac{(3z)^n}{n!}+...,$$

$$e^{-3z}=\sum_{n=0}^{\infty}\frac{(-3z)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-1)^n z^n}{n!}=1-3z+\frac{(3z)^2}{2!}-\frac{(3z)^3}{3!}+...+\frac{(-1)^n(3z)^n}{n!}+....$$

U holda

$$sh3z=\frac{e^{3z}-e^{-3z}}{2}=3z+\frac{(3z)^3}{3!}+...+\frac{(3z)^{2n+1}}{(2n+1)!}+...=\sum_{n=0}^{\infty}\frac{3^{2n+1}}{(2n+1)!}z^{2n+1}.$$

Bu qator butun kompleks tekisligida yaqinlashadi.

2) Berilgan funksiyada almashtirishlar bajaramiz:

$$\frac{1}{3-2z}=\frac{1}{3-2(z-3+3)}=\frac{1}{-3-2(z-3)}=-\frac{1}{3}\cdot\frac{1}{1+\frac{2}{3}(z-3)}.$$

$\frac{1}{1+z}$ funksiyaning yoyilmasida z ning o‘rniga $\frac{2}{3}(z-3)$ni qo‘yib, topamiz:

$$\frac{1}{3-2z}=-\frac{1}{3}\cdot\left(1-\frac{2}{3}(z-3)+\frac{2^2}{3^2}(z-3)^2-\frac{2^3}{3^3}(z-3)^3\right)=$$

$$=-\frac{1}{3}+\frac{2}{3^2}(z-3)-\frac{2^2}{3^3}(z-3)^2+...+\frac{(-1)^{n+1}2^n}{3^{n+1}}(z-3)^n+...=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}2^n}{3^{n+1}}(z-3)^n.$$

Bu qator $\left|\frac{2}{3}(z-3)\right|<1$ yoki $|z-3|<\frac{3}{2}$ doirada yaqinlashadi.

3) $f(z)=\ln z$ funksiyaning hosilalarini topamiz:

$$f'(z)=\frac{1}{z},\ f''(z)=-\frac{1}{z^2},\ f'''(z)=\frac{2}{z^3},...,f^{(n)}(z)=(-1)^{n-1}\frac{(n-1)!}{z^n}.$$

U holda

$$c_n=\frac{1}{n!}(-1)^{n-1}\frac{(n-1)!}{z^n}\bigg|_{z=1}=\frac{(-1)^{n-1}}{n},\ n=1,2,...,\quad c_0=\ln 1=0.$$

Demak,

$$\ln z=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(z-1)^n.$$

Bu qatorning yaqinlashish radiusini topamiz:

$$R=\lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|=\lim_{n\to\infty}\frac{n+1}{n}=1.$$

Demak, bu qator $|z-1|<1$ doirada yaqinlashadi.

2.5.4. Agar $f(z_0)=0$ bo‘lsa, z_0 ga $f(z)$ *funksiyaning noli* deyiladi.

Agar $c_0=c_1=c_2=...=c_{m-1}=0$ va $c_m\neq 0$ bo‘lsa, $f(z)$funksiyaning z_0 nuqta atrofida darajali qatorga yoyilmasi

$$f(z)=c_m(z-z_0)^m+c_{m+1}(z-z_0)^{m+1}+...+c_n(z-z_0)^n+...$$

ko‘rinishda bo‘ladi. Bunda z_0 ga $f(z)$ *funksiyaning* m *karrali noli* deyiladi.

Agar $m=1$ bo‘lsa, z_0 ga $f(z)$ *funksiyaning oddiy noli* deyiladi.

Funksiyaning darajali qatorga yoyilmasini

$$f(z)=(z-z_0)^m\varphi(z), \text{ bu yerda } \varphi(z)=c_m+c_{m+1}(z-z_0)+...$$

ko‘rinishga keltirish mumkin. Bunda z_0 nuqta $\varphi(z)$ funksiya uchun nol bo‘lmaydi.

6-misol. $f(z)=z^2\sin z$ funksiyaning nollarini toping va ularning tartibimi aniqlang.

$f(z)=z^2\sin z=0$ tenglamaning yechimlari: $z_1=0,\ z_{2n}=n\pi\ (n\in Z)$. U holda

$$f'(z)=2z\sin z+z^2\cos z,\ f'(0)=0,\ f'(n\pi)=n^2\pi^2\cos n\pi\neq 0;$$

$$f''(z)=2\sin z+4z\cos z-z^2\sin z,\ f''(0)=0;$$

$$f'''(z)=6\cos z-6z\sin z-z^2\cos z,\ f'''(0)=6\neq 0.$$

Demak, $z_1=0$ nol 3-tartibli, $z_{2n}=n\pi$ nol oddiy (1-tartibli).

7-misol. $f(z)=\dfrac{z^7}{z-\sin z}$ funksiya uchun $z_0=0$ nolning tartibimi aniqlang.

$$f(z)=\frac{z^7}{z-\sin z}=\frac{z^7}{z-\left(z-\frac{z^3}{3!}+\frac{z^5}{5!}+...\right)}=\frac{z^7}{\frac{z^3}{3!}-\frac{z^5}{5!}+...}=\frac{z^4}{\frac{1}{3!}-\frac{z^2}{5!}+...}.$$

$\varphi(z)=\dfrac{1}{\frac{1}{3!}-\frac{z^2}{5!}+...}$ bo‘lsin. U holda $f(z)=z^4\varphi(z)$. Bunda $\varphi(z)$ funksiya $z_0=0$ nuqtada analitik va $\varphi(0)=6\neq 0$.

Demak, $z_0=0$ nol 4-tartibli.

2.5.5. 5-teorema. $r<|z-z_0|<R\ (0\le r<R\le\infty)$ halqada analitik bo‘lgan har qanday $f(z)$ funksiya bu halqada

$$f(z)=\sum_{n=-\infty}^{+\infty}c_n(z-z_0)^n, \text{ bu yerda } c_n=\frac{1}{2\pi i}\oint_L\frac{f(\xi)}{(\xi-z_0)^{n+1}}d\xi\ \ (n=0,\pm1,\pm2,...),\quad (5.6)$$

ko‘rinishda darajali qatorga yoyilishi mumkin, bu yerda, $L-$markazi z_0 bo‘lgan $r<|z-z_0|<R$ halqada yotuvchi ixtiyoriy aylana.

(5.6) qatorga $r<|z-z_0|<R$ halqada qaralayotgan $f(z)$ funksiyaning Loran qatori deyiladi.

Funksiyaning Loran qatoriga yoyilmasi ikkita $f_1(z)=\sum_{n=0}^{+\infty} c_n(z-z_0)^n$ va $f_2(z)=\sum_{n=1}^{+\infty}\frac{c_{-n}}{(z-z_0)^n}$ qismdan iborat. Bunda: $\sum_{n=0}^{+\infty} c_n(z-z_0)^n$ *qatorga Loran qatorining to‘g‘ri qismi* deyiladi, u $|z-z_0|<R$ doiraning ichkarisida $f_1(z)$ analitik funksiyaga yaqinlashadi; $\sum_{n=1}^{+\infty}\frac{c_{-n}}{(z-z_0)^n}$ *qatorga Loran qatorining bosh qismi* deyiladi, u $|z-z_0|>r$ doiraning tashqarisida $f_2(z)$ analitik funksiyaga yaqinlashadi.

$\sum_{n=-\infty}^{+\infty} c_n(z-z_0)^n$ qator $r<|z-z_0|<R$ halqaning ichkarisida $f(z)=f_1(z)+f_2(z)$ analitik funksiyaga yaqinlashadi.

8-misol. $f(z)$ funksiyalarni z_0 nuqta atrofida Loran qatoriga yoying:

1) $f(z)=\frac{1}{z^2+2z-3},\ z_0=1;$ 2) $f(z)=\frac{2z+1}{z^2+z-2},\ z_0=0;$

3) $f(z)=\cos\left(\frac{z}{z+1}\right),\ z_0=-1;$ 4) $f(z)=\frac{1}{(z-1)(z-3)},\ z_0=\infty.$

1) Berilgan funksiyani sodda kasrlar yig‘indisiga keltiramiz:

$$f(z)=\frac{1}{4(z-1)}-\frac{1}{4(z+3)}=f_1(z)-f_2(z).$$

$f_2(z)$ funksiya $|z-1|<4$ doirada analitik ($z=-3, z=\infty$ maxsus nuqtalar doiraga tushmaydi). Shu sababli uni $z_0=1$ nuqta atrofida Teylor qatoriga yoyish mumkin:

$$\frac{1}{4(z-3)}=\frac{1}{4(z-1)+4)}=\frac{1}{16}\cdot\frac{1}{1+\frac{1}{4}(z-1)}=\frac{1}{16}\sum_{n=0}^{\infty}(-1)^n\frac{(z-1)^n}{4^n}.$$

$f_1(z)$ funksiya $|z-1|>0$ sohada analitik. Bu funksiya $(z-1)$ ning darajalari bo‘yicha Loran qatori ko‘rinishida yozilgan.

Demak,

$$f(z)=f_1(z)-f_2(z)=\frac{1}{4(z-1)}-\sum_{n=0}^{\infty}(-1)^n\frac{(z-1)^n}{4^{n+2}}=\sum_{n=-1}^{\infty}\frac{(-1)^{n+1}(z-1)^n}{4^{n+2}}.$$

2) $z_1=-2$ va $z_2=1$ nuqtalarda $z^2+z-2=0$. U holda markazi $z_0=0$ nuqtada bo‘lgan uchta “halqa” mavjud bo‘ladiki, ularda $f(z)$ funksiya analitik bo‘ladi:

a) $|z|<1$ doira, b) $1<|z|<2$ halqa, c) $|z|>2$ soha -$|z|\le 2$ doiraning tashqi tomoni.

Bu “halqa”larning har birida $f(z)$ funksiyaning Loran qatorini topamiz.

a) Berilgan funksiyani sodda kasrlar yig‘indisiga keltiramiz:

$$f(z)=\frac{1}{z+2}+\frac{1}{z-1}=\frac{1}{2}\cdot\frac{1}{1+\frac{z}{2}}-\frac{1}{1-z}.$$

Bundan

$$f(z)=\frac{1}{2}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+...\right)-(1+z+z^2+z^3+...)=-\sum_{n=0}^{\infty}\frac{2^{n+1}-1}{2^{n+1}}z^n.$$

b) $f(z)$ funksiyani $1<|z|<2$ halqada qatorga yoyamiz.

$\frac{1}{2}\cdot\frac{1}{1+\frac{z}{2}}$ funksiyaning $\frac{1}{2}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+...\right)$ qatori bu halqada yaqinlashadi, chunki $|z|<2$.

$\frac{1}{1-z}$ funksiyaning $1+z+z^2+z^3+...$ qatori $|z|>1$ sohada uzoqlashadi.

Shu sababli $f(z)$ funksiyani quyidagicha yozib olamiz:

$$f(z)=\frac{1}{2}\cdot\frac{1}{1+\frac{z}{2}}+\frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}}.$$

U holda

$$f(z)=\frac{1}{2}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+...\right)+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+...=$$

$$=...+\frac{1}{z^3}+\frac{1}{z^2}+\frac{1}{z}+\frac{1}{2}-\frac{z}{4}+\frac{z^2}{8}+...=\sum_{n=1}^{\infty}\frac{1}{z^n}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{z^n}{2^n}.$$

c) $f(z)$ funksiyani $|z|>2$ uchun qatorga yoyamiz.

$\frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}}$. funksiyaning $+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+...=$ qatori $|z|>1$ sohadagi singari $|z|>2$ sohada yaqinlashadi.

$\frac{1}{2}\cdot\frac{1}{1+\frac{z}{2}}$ funksiyaning $\frac{1}{2}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+...\right)$qatori $|z|>2|$ da uzoqlashadi.

Shu sababli $f(z)$ funksiyani quyidagicha yozib olamiz:

$$f(z)=\frac{1}{z}\cdot\frac{1}{1+\frac{2}{z}}+\frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}}.$$

Bundan

$$f(z)=\frac{1}{z}\left(\left(1-\frac{2}{z}+\frac{4}{z^2}-\frac{z^3}{z^3}+...\right)+\left(1+\frac{1}{z}+\frac{1}{z^2}+...\right)\right)=\frac{2}{z}-\frac{1}{z^2}+\frac{5}{z^3}-\frac{7}{z^4}+....$$

Shunday qilib, bitta $f(z)$ funksiya uchun har xil halqalarda har xil ko‘rinishdagi Loran qatorlar hosil qilindi.

3) $\frac{z}{z+1}$ kasrni $1-\frac{1}{z+1}$ kabi yozib olamiz. U holda

$$\cos\left(\frac{z}{z+1}\right)=\cos\left(1-\frac{1}{z+1}\right)=\cos 1\cos\left(\frac{1}{z+1}\right)+\sin 1\sin\left(\frac{1}{z+1}\right).$$

$\sin z$ va $\cos z$ funksiyalarni Teylor qatoriga yoyilmasi formulalaridan foydalanib, topamiz:

$$\cos\left(\frac{z}{z+1}\right)=\cos 1\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\frac{1}{(z+1)^{2n}}+\sin 1\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{i}{(z+1)^{2n+1}}.$$

4) Cheksiz uzoqlashgan nuqtada $3<|z|<\infty$ bo‘ladi. U holda $\frac{1}{|z|}<\frac{3}{|z|}<1$ ni hisobga olib, topamiz:

$$f(z)=\frac{1}{(z-1)(z-3)}=-\frac{1}{2}\left(\frac{1}{z-1}-\frac{1}{z-3}\right)=-\frac{1}{2}\cdot\frac{1}{z\left(1-\frac{1}{z}\right)}+\frac{1}{2}\cdot\frac{1}{z\left(1-\frac{3}{z}\right)}=$$

$$=-\frac{1}{2z}\sum_{z=1}^{\infty}\frac{1}{z^n}+\frac{1}{2z}\sum_{n=1}^{\infty}\frac{3^n}{z^n}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{3^n-1}{z^{n+1}}.$$

2.5.6. $f(z)$ funksiya analitik bo‘lmaydigan nuqtalarga $f(z)$ *funksiyaning maxsus nuqtalari* deyiladi.

Agar $z=z_0$ maxsus nuqtaning biror atrofida $f(z)$ funksiyaning boshqa maxsus nuqta bo‘lmasa, z_0 nuqtaga $f(z)$ *funksiyaning ajralgan maxsus nuqtasi* deyiladi.

Agar z_0 nuqta $f(z)$ funksiyaning yakkalangan maxsus nuqtasi bo’lsa, u holda shunday $R>0$ son topiladiki, $0<|z-z_0|<R$ halqada $f(z)$ funksiya analitik bo‘ladi va (5.6) Loran qatoriga yoyiladi.

Maxsus nuqtalar $f(z)$ funksiyaning bu nuqtalar atrofidagi holatiga qarab uch turga bo‘linadi. Bu nuqtalarda $f(z)$ funksiyaning Loran qatoriga yoyilmasi turlicha bo‘ladi.

Agar $\lim\limits_{z\to z_0}f(z)$ chekli limit mavjud bo‘lsa, z_0 nuqtaga $f(z)$ funksiyaning *bartaraf qilinadigan maxsus nuqtasi* deyiladi. Bunday nuqtalarda Loran qatori bosh qismiga ega bo‘lmaydi.

Agar $\lim\limits_{z\to z_0}|f(z)|=\infty$ bo‘lsa, z_0 nuqtaga $f(z)$ funksiyaning *qutbi* deyiladi. Bunday nuqtalarda Loran qatorining bosh qismi chekli (nolga teng bo‘lmagan) sondagi hadlarga ega bo‘ladi. Agar z_0 nuqta $f(z)$ funksiyaning qutbi bo‘lib, Loran qatorining bosh qismi m ta hadga ega bo‘lsa, z_0 nuqtaga funksiyaning *m-tartibli qutbi* deyiladi. $m=1$ da z_0 nuqta *oddiy qutb* deyiladi.

Agar $\lim\limits_{z\to z_0} f(z)$ mavjud bo‘lmasa, z_0 nuqtaga $f(z)$ funksiyaning *muhim maxsus nuqtasi* deyiladi. Bunday nuqtalarda Loran qatorining bosh qismi cheksiz ko‘p hadlarga ega bo‘ladi.

Agar $f(z)$ funksiya cheksiz uzoqlashgan $z=\infty$ nuqtaning biror $R<|z|<\infty$ atrofida ($z=\infty$ nuqtadan boshqa nuqtalarda) analitik bo‘lsa, z nuqtaga $f(z)$ *funksiyaning ajralgan maxsus nuqtasi* deyiladi.

Cheksiz uzoqlashgan maxsus nuqta bartaraf qilinadigan maxsus nuqta, $m-$tartibli qutb yoki muhim maxsus nuqta bo‘lishi mumkin. Bu nuqtalardan birinchisida $z=\infty$ nuqtaning atrofida $f(z)$ funksiyaning Loran qatoriga yoyilmasi z ning musbat darajali hadlariga ega bo‘lmaydi, ikkinchisida chekli sondagi va uchinchisida cheksiz ko‘p musbat darajali hadlariga ega bo‘ladi.

Agar $w=0$ nuqta $\varphi(w)=f\left(\frac{1}{w}\right)$ funksiyaning bartaraf qilinadigan maxsus nuqtasi, qutbi ($m-$tartibli), muhim maxsus nuqtasi bo‘lsa, cheksiz uzoqlashgan $z=\infty$ nuqta $f(z)$ funksiyaning bartaraf qilinadigan maxsus nuqtasi, qutbi ($m-$tartibli), muhim maxsus nuqtasi bo‘ladi va aksincha.

Agar z_0 nuqta $f(z)$ funksiyaning $m-$tartibli noli bo‘lsa, u holda z_0 nuqta $\frac{1}{f(z)}$ funksiyaning $m-$tartibli qutbi bo‘ladi va aksincha.

9-misol. $f(z)$ funksiyalarning z_0 maxsus nuqtalari turini aniqlang:

$$1)\ f(z)=(1-z)tg\left(\frac{\pi z}{2}\right), z_0=1;\quad 2)\ f(z)=\frac{\sin z}{z^3+z^2-z-1}, z_0=-1;\quad 3)\ f(z)=e^{\frac{1}{z^2}}, z_0=0.$$

⊜ 1) Berilgan funksiyaning berilgan nuqtadagi limitini hisoblaymiz:

$$\lim_{z\to1}(1-z)tg\left(\frac{\pi z}{2}\right)=-\lim_{z\to1}(1-z)tg\left(\frac{\pi}{2}(1-z-1)\right)=\lim_{z\to1}(1-z)tg\left(\frac{\pi}{2}(1-z)-\frac{\pi}{2}\right)=$$

$$=\lim_{z\to1}(1-z)tg\left(\frac{\pi}{2}(1-z)\right)=\lim_{z\to1}\cos\left(\frac{\pi}{2}(1-z)\right)\cdot\frac{(1-z)\frac{\pi}{2}}{\sin\left(\frac{\pi}{2}(1-z)\right)\cdot\frac{\pi}{2}}=\frac{2}{\pi}.$$

Demak, z_0 – bartaraf qilinadigan maxsus nuqta.

2) Berilgan funksiya ustida almashtirishlar bajaramiz:

$$f(z)=\frac{\sin z}{z^3+z^2-z-1}=\frac{\sin z}{z^2(z+1)-(z+1)}=\frac{\sin z}{(z+1)^2(z-1)}=$$

$$=\frac{\frac{\sin z}{(z-1)}}{(z+1)^2}=\frac{1}{(z+1)^2}\varphi(z),\text{ bu yerda } \varphi(z)=\frac{\sin z}{(z-1)}.$$

$\varphi(z)$ funksiya $z_0=-1$ nuqtada analitik va $\varphi(-1)=\frac{\sin(-1)}{-2}\neq 0.$

Demak, z_0 – ikkinchi tartibli qutb.

3) $\lim\limits_{z\to 0} f(z)=\lim\limits_{z\to 0} e^{\frac{1}{z^2}}=e^{\frac{1}{0}}=e^{+\infty}=+\infty$.

Mavhum qismda $\lim\limits_{y\to 0} f(iy)=\lim\limits_{z\to 0} e^{\frac{1}{i^2y^2}}=e^{-\frac{1}{0}}=e^{-\infty}=0.$ Shunday qilib, $z_0=0$ nuqtada $f(z)=e^{\frac{1}{z^2}}$ funksiya limitga ega emas.

Demak, z_0 – muhim maxsus nuqta.

Mashqlar

2.5.1. Qatorlarni yaqinlashishga tekshiring:

1) $\sum\limits_{n=1}^{\infty}\frac{\cos\sqrt{n}+i\sin\sqrt{n}}{n^2}$;

2) $\sum\limits_{n=1}^{\infty}\frac{e^{i\frac{\pi}{n}}}{n}$;

3) $\sum\limits_{n=1}^{\infty}\frac{1}{(n+i)\sqrt{n}}$;

4) $\sum\limits_{n=1}^{\infty}\frac{\cos i\pi n}{2^n}$;

5) $\sum\limits_{n=1}^{\infty}\frac{ch i\frac{\pi}{n}}{n^{\ln n}}$;

6) $\sum\limits_{n=1}^{\infty}\frac{1}{n}+i\frac{1}{n\sqrt{n}}$.

2.5.2. Qatorlarni absolut yoki shartli yaqinlashishga tekshiring:

1) $\sum\limits_{n=1}^{\infty}\frac{i^n}{n}$;

2) $\sum\limits_{n=1}^{\infty}\frac{\sin in}{3^n}$;

3) $\sum\limits_{n=1}^{\infty}\frac{(\sin in)^n}{n sh^n n}$;

4) $\sum\limits_{n=1}^{\infty}\frac{(1+i)^n\sin in}{2^{\frac{n}{2}}\cos in}$.

2.5.3. Qatorlarning yaqinlashish radiusini toping:

1) $\sum_{n=1}^{\infty}\frac{z^n}{n^n}$; 2) $\sum_{n=1}^{\infty}\frac{n!}{3^n}z^n$;

3) $\sum_{n=1}^{\infty}\frac{n^n}{n!}z^n$; 4) $\sum_{n=1}^{\infty}n!e^{-n^2}z^n$;

5) $\sum_{n=1}^{\infty}\cos i^3 nz^n$; 6) $\sum_{n=1}^{\infty}e^{in}z^n$;

7) $\sum_{n=1}^{\infty}\sin\left(\frac{\pi}{n}i\right)z^n$; 8) $\sum_{n=1}^{\infty}\frac{1}{\sin^n(1+in)}z^n$.

2.5.4. Qatorlarning yaqinlashish doirasini toping:

1) $\sum_{n=1}^{\infty}\frac{(n+i)}{2^n}z^n$; 2) $\sum_{n=1}^{\infty}e^{2n\pi i}z^n$;

3) $\sum_{n=1}^{\infty}n^{\ln n}(z-2)^n$; 4) $\sum_{n=1}^{\infty}\cos in(z-1)^n$;

5) $\sum_{n=1}^{\infty}i^n z^n$; 6) $\sum_{n=1}^{\infty}\frac{1}{\ln^n in}z^n$;

7) $\sum_{n=1}^{\infty}\left(\frac{z}{1-i}\right)^n$; 8) $\sum_{n=1}^{\infty}\cos^n\frac{\pi i}{\sqrt{n}}$.

2.5.5. $f(z)$ funksiyalarni z_0 nuqta atrofida Teylor qatoriga yoying:

1) $f(z)=\frac{1}{2-z},\ z_0=0$; 2) $f(z)=\frac{1}{z^2+1},\ z_0=0$;

3) $f(z)=\frac{1}{5-3z},\ z_0=1$; 4) $f(z)=\frac{1}{(1-z)^2},\ z_0=0$;

5) $f(z)=\frac{z^2}{1-z^3},\ z_0=0$; 6) $f(z)=\frac{z-1}{2z^2+5z+2},\ z_0=-1$;

7) $f(z)=\frac{z}{z^2-2z-3},\ z_0=0$; 8) $f(z)=\frac{z-i}{z+i},\ z_0=i$;

9) $f(z)=\cos^4 z+\sin^4 z,\ z_0=0$; 10) $f(z)=sh^2\frac{z}{2},\ z_0=0$;

11) $f(z)=e^z,\ z_0=3$; 12) $f(z)=\cos z,\ z_0=-\frac{\pi}{4}$;

13) $f(z)=\ln(2-z),\ z_0=0$; 14) $f(z)=\ln(2-5z),\ z_0=-3$.

2.5.6. Funksiyaning nollarini toping va ularning tartibimi aniqlang:

1) $f(z)=1+\cos z$; 2) $f(z)=(z+i)^3(z-1)$;

3) $f(z)=1-e^z$; 4) $f(z)=(z+\pi i)shz$.

2.5.7. $f(z)$ funksiya uchun $z_0=0$ nolning tartibimi aniqlang:

1) $f(z)=\dfrac{z^3}{1-z-e^{-z}}$; 2) $f(z)=\dfrac{z^6}{z^2-\sin^2 z}$;

3) $f(z)=2(chz-1)-z^2$; 4) $f(z)=z^2(e^z-1)$.

2.5.8. $f(z)$ funksiyalarni z_0 nuqta atrofida Loran qatoriga yoying:

1) $f(z)=\dfrac{2z-3}{z^2-3z+2}$, $z_0=2$; 2) $f(z)=\dfrac{2}{z(3-z)}$, $z_0=0$;

3) $f(z)=\dfrac{1}{(z-1)(z-2)}$, $z_0=0$; 4) $f(z)=\dfrac{1}{z^2-z-6}$, $z_0=0$;

5) $f(z)=\dfrac{1-e^{-z}}{z^3}$, $z_0=0$; 6) $f(z)=z^3\cos\dfrac{1}{z}$, $z_0=0$;

7) $f(z)=\dfrac{\sin z}{z^2}$, $z_0=\infty$; 8) $f(z)=\dfrac{2}{z+i}$, $z_0=\infty$.

2.5.9. $f(z)$ funksiyalarni berilgan halqada Loran qatoriga yoying:

1) $f(z)=\dfrac{1}{z^2-3z+2}$, $0<|z-1|<1$; 2) $f(z)=\dfrac{1}{z-z^2}$, $0<|z|<1$;

3) $f(z)=\dfrac{1}{(z-1)(z-2)}$, $1<|z|<2$; 4) $f(z)=\dfrac{1}{z^2-5z+6}$, $2<|z|<3$.

2.5.10. $f(z)$ funksiyalarning maxsus nuqtalari turini aniqlang:

1) $f(z)=\dfrac{1+\cos z}{z-\pi}$; 2) $f(z)=z(e^{\frac{1}{z}}-1)$;

3) $f(z)=\dfrac{1}{z^3(1-\cos z)}$; 4) $f(z)=\dfrac{\sin z}{z^4}$;

5) $f(z)=\dfrac{1+z^2}{e^z}$; 6) $f(z)=(z-1)e^{\frac{1}{z-1}}$.

2.6. QOLDIQLAR NAZARIYASI

Qoldiqlar. Integrallarni qoldiqlar yordamida hisoblash

2.6.1. z_0- $f(z)$ analitik funksiyaning ajralgan maxsus nuqtasi, $L-$ markazi $f(z)$ funksiyaning analitiklik sohasida, ya’ni $0<|z-z_0|<R$ halqada yotuvchi z_0 nuqtada bo‘lgan L aylana bo‘lsin.

$f(z)$ analitik funksiyaning ajralgan maxsus z_0 nuqtadagi *qoldig'i* deb, L aylana bo'yicha musbat yo'nalishda olingan $\frac{1}{2\pi i}\oint_L f(z)dz$ integralning qiymatiga aytiladi va $\text{Res}\, f(z)$ kabi belgilanadi:

$$\text{Res}\, f(z) = \frac{1}{2\pi i}\oint_L f(z)dz. \qquad (6.1)$$

$f(z)$ funksiyaning ajralgan maxsus z_0 nuqtadagi qoldig'i bu funksiya Loran qatoriga yoyilmasining manfiy darajali birinchi hadi oldidagi koeffitsiyentiga teng bo'ladi:

$$\text{Re}\, sf(z_0) = c_{-1}.$$

$f(z)$ funksiyaning z_0 nuqtadagi qoldig'i maxsus nuqtaning turiga qarab, quyidagi formulalardan biri bilan hisoblanadi.

1. $z_0 -$ $f(z)$ funksiyaning *to'g'ri nuqtasi* yoki *bartaraf qilinadigan maxsus nuqtasi* bo'lsa

$$z_0 \neq \infty \text{ bo'lganda } \text{Res}\, f(z_0) = 0,$$

$$z_0 = \infty \text{ bo'lganda } \text{Res}\, f(z_0) = \lim_{z\to\infty} z\cdot(f(\infty) - f(z))$$

bo'ladi.

2. $z_0 -$ $f(z)$ funksiyaning *m-tartibli qutbi* bo'lsa

$$\text{Res}\, f(z_0) = \frac{1}{(m-1)!}\lim_{z\to z_0}\frac{d^{m-1}(f(z)(z-z_0)^m)}{dz^{m-1}} \qquad (6.2)$$

bo'ladi. Xususan, $n=1$, ya'ni oddiy qutb uchun

$$\text{Res}\, f(z_0) = \lim_{z\to z_0} f(z)(z-z_0). \qquad (6.3)$$

Agar $f(z)$funksiya z_0 nuqtada oddiy qutbga ega bo'lib, bu nuqtada analitik bo'lgan ikkita $\varphi(z)$ va $g(z)$ funksiyalarning nisbati, ya'ni $f(z) = \frac{\varphi(z)}{g(z)}$ ko'rinishda berilgan bo'lsa

$$\text{Res}\, f(z_0) = \frac{\varphi(z_0)}{g'(z_0)} \qquad (6.4)$$

bo'ladi.

Agar $f(z) = \frac{\varphi(z)}{(z-z_0)^m}$ bo'lib, bunda $\varphi(z)$ funksiya z_0 nuqtada analitik bo'lsa

$$\text{Res}\, f(z_0) = \frac{1}{(m-1)!}\varphi^{m-1}(z_0). \qquad (6.5)$$

bo'ladi.

3. $z_0 -$ $f(z)$ funksiyaning *muhim maxsus nuqtasi* bo‘lsa, u holda $f(z)$ funksiya $(z-z_0)$ning darajalari bo‘yicha Loran qatoridan yoyiladi va c_{-1} topiladi:

$$\operatorname{Re} sf(z_0)=c_{-1}, \quad \operatorname{Re} sf(\infty)=-c_{-1}.$$

Izoh. Agar $f(z)$ funksiya juft bo‘lsa, u holda

$$\operatorname{Res} f(z_0)=-\operatorname{Res} f(-z_0) \text{ va } \operatorname{Res} f(0)=\operatorname{Res} f(\infty)=0;$$

agar $f(z)$ funksiya toq bo‘lsa, u holda

$$\operatorname{Res} f(z_0)=\operatorname{Res} f(-z_0).$$

1-teorema. Agar $f(z)$ funksiya kengaytirilgan kompleks tekisligining chekli sondagi nuqtalaridan boshqa istalgan nuqtasida analitik bo‘lsa, u holda

$$2\pi i\sum_{n=1}^{k}\operatorname*{Res}_{z_n} f(z_n)+2\pi i\operatorname*{Res}_{\infty} f(z)=0 \tag{6.6}$$

bo‘ladi.

1- misol. $f(z)$ funksiyaning z_0 nuqtadagi qoldig‘ini toping:

1) $f(z)=\dfrac{\sin z^2}{z^3-3z^2},\ z_0=0;$ 2) $f(z)=\dfrac{ze^{iz}}{z^2+1},\ z_0=i;$

3) $f(z)=tgz,\ z_0=\dfrac{\pi}{2};$ 4) $f(z)=ctg^2 z,\ z_0=0;$

5) $f(z)=z^2\sin\dfrac{1}{z+1},\ z_0=-1;$ 6) $f(z)=z^5e^{\frac{1}{z^2}},\ z=\infty.$

1) $\lim\limits_{z\to 0} f(z)=\lim\limits_{z\to 0}\dfrac{\sin z^2}{z^2(z-3)}=\lim\limits_{z\to 0}\dfrac{\sin z^2}{z^2}\cdot\lim\limits_{z\to 0}\dfrac{1}{z-3}=-\dfrac{1}{3}.$

Demak, $z_0=0$ bartaraf qilinadigan maxsus nuqta. Shu sababli $\operatorname{Re} sf(0)=0.$

2) $\lim\limits_{z\to i} f(z)=\lim\limits_{z\to 0}\dfrac{ze^{iz}}{z^2+1}=\infty$. Demak, $z_0=i$ oddiy qutb.

U holda

$$\operatorname{Re} sf(i)=\lim_{z\to i}(z-i)f(z)=\lim_{z\to 0}\frac{(z-i)ze^{iz}}{(z-i)(z+i)}=\frac{1}{2e}.$$

3) $\lim\limits_{z\to\frac{\pi}{2}} tgz=\infty.$ Demak, $z_0=\dfrac{\pi}{2}$ oddiy qutb. $tgz=\dfrac{\sin z}{\cos z}$ deb yozish mumkin.

Bunda $\sin\dfrac{\pi}{2}=1\neq 0,\ \cos\dfrac{\pi}{2}=0,\ (\cos z)'\Big|_{z=\frac{\pi}{2}}=-\sin\dfrac{\pi}{2}=-1\neq 0$

U holda (6.4) formulaga ko‘ra

$$\operatorname{Res} f\left(\frac{\pi}{2}\right)=\left.\frac{\sin z}{(\cos z)'}\right|_{z=\frac{\pi}{2}}=-1.$$

4) $z_0=0$ 2-tartibli qutb, chunki $f(z)=ctg^2 z$ funksiyaning maxraji $z_0=0$ nuqtada 2-tartibli nolga ega. U holda (6.2) formulaga ko‘ra

$$\operatorname{Res} f(0)=\lim_{z\to 0}\frac{d}{dz}(z^2ctg^2 z)=\lim_{z\to 0}\left(2zctg^2 z-\frac{2z^2ctgz}{\sin^2 z}\right)=$$

$$=2\lim_{z\to 0} zctgz\left(\frac{\cos z\sin z-z}{\sin^2 z}\right)=2\lim_{z\to 0}\frac{z}{\sin z}\cos z\lim_{z\to 0}\frac{(\sin z\cos z-z)'}{(\sin^2 z)'}=$$

$$=2\lim_{z\to 0}\frac{-\sin^2+\cos^2 z-1)}{2\sin z\cos z}=-2\lim_{z\to 0}\frac{\sin z}{\cos z}=0.$$

5) $z_0=-1$ funksiyaning maxsus nuqtasi bo‘ladi. Bu nuqtaning turini aniqlash uchun $f(z)$ funksiyani $z_0=-1$ nuqta atrofida Loran qatoriga yoyamiz.

Bunda

$$z^2=(1+z-1)^2=(1+z)^2-2(1+z)+1,$$

$$\sin\frac{1}{1+z}=\frac{1}{1+z}-\frac{1}{3!(1+z)^3}+\frac{1}{5!(1+z)^5}-...$$

yoyilmalarni hisobga olsak

$$f(z)=\left(1-\frac{1}{3!}\right)\frac{1}{1+z}+\frac{2}{3!}\frac{1}{(1+z)^2}+\left(\frac{1}{5!}-\frac{1}{3!}\right)\frac{1}{(1+z)^3}+...+(2+(1+z))$$

qator kelib chiqadi. Bu qator cheksiz ko‘p manfiy darajali hadlarga ega. Demak, $z=-1$ muhim maxsus nuqta.

U holda

$$\operatorname{Res} f(-1)=c_{-1}=1-\frac{1}{3!}=\frac{5}{6}.$$

6) $f(z)=z^5e^{\frac{1}{z^2}}$ funksiyani $z=0$ nuqtada Loran qatoriga yoyamiz:

$$f(z)=z^5e^{\frac{1}{z^2}}=z^5+z^3+\frac{1}{2!}z+\frac{1}{3!z}+\frac{1}{4!z^3}+\frac{1}{5!z^5}+...$$

Bu qator $0<|z|<\infty$ halqada yaqinlashadi va cheksiz ko‘p manfiy darajali hadlarga ega. Demak, $z=0$ muhim maxsus nuqta. U holda (6.6) formulaga ko‘ra

$$\operatorname*{Res}_{\infty} f(z)=-\operatorname*{Res}_{0} f(z)=-\frac{1}{3!}=-\frac{1}{6}.$$

2.6.2. I. Agar $f(z)$ funksiya L kontur bilan chegaralangan $\overline{G}$ yopiq sohaning chekli sondagi $z_1, z_2, ..., z_k$ nuqtalaridan boshqa barcha nuqtalarida analitik bo‘lsa, u holda integralni hisoblashning

$$\oint_L f(z)dz = 2\pi i \sum_{n=1}^{k} \mathrm{Res} f(z_n) \tag{6.7}$$

formulasi o‘rinli bo‘ladi.

II. Qoldiqlar nazariyasini haqiqiy o‘zgaruvchining

$$\int_0^{2\pi} R(\cos x, \sin x)dx$$

ko‘rinishdagi integralini hisoblash uchun qo‘llash mumkin. Bu integral $z = e^{ix}$ o‘rniga qo‘yish orqali bu integral quyidagi integralga keltiriladi:

$$\oint_L R\left(\frac{z+z^{-1}}{2}; \frac{z-z^{-1}}{2}\right)\frac{dz}{iz}, \tag{6.8}$$

bu yerda, $L-$soat strelkasiga teskari yo‘nalishda aylanib o‘tiladigan $|z|=1$ aylana. Bu kompleks o‘zgaruvchining integrali qoldiqlar nazariyasi orqali hisoblanadi.

III. Ushbu

$$\int_{-\infty}^{+\infty} f(x)dx$$

xosmas integral $f(x)$ funksiyada x haqiqiy o‘zgaruvchi z kompleks o‘zgaruvchi bilan almashtirilganida hosil bo‘ladigan $f(z)$ funksiya uchun ayrim shartlar bajarilganida qoldiqlar nazariyasini qo‘llash orqali topilishi mumkin.

1) $f(z)$: a) chekli sondagi $z_1, z_2, ..., z_k$ maxsus nuqtalar hisobga olinmaganida yuqori yarim tekislikda analitik, $\mathrm{Im}(z_n) > 0\,(n = 1,2,...,k)$ va yarim tekislikning $z_1, z_2, ..., z_k$ nuqtalaridan boshqa nuqtalarida uzluksiz;

b) $\lim\limits_{z\to\infty} zf(z) = 0,\ \mathrm{Im}\, z \geq 0.$

U holda

$$\int_{-\infty}^{+\infty} f(x)dx = 2\pi i \sum_{n=1}^{k} \mathrm{Res}_{z=z_n} f(z) \tag{6.9}$$

bo‘ladi. Quyi yarim tekislik uchun bu tenglikning o‘ng tomoni minus ishora bilan olinadi.

2) $f(z)$: a) chekli sondagi $z_1, z_2, ..., z_k$ maxsus nuqtalar hisobga olinmaganida yuqori yarim tekislikda analitik, $\mathrm{Im}(z_n) > 0\,(n = 1,2,...,k)$ va yarim tekislikning $z_1, z_2, ..., z_k$ nuqtalaridan boshqa nuqtalarida uzluksiz;

b) $\lim\limits_{z\to\infty} zf(z)=0,\ \operatorname{Im} z\geq 0.$

U holda

$$\int_{-\infty}^{+\infty} e^{imx} f(x)dx = 2\pi i \sum_{n=1}^{k} \operatorname*{Res}_{z=z_n} (f(x)e^{imz}),\ m>0\,. \qquad (6.10)$$

Agar bundan tashqari $x\in R$ da $f(x)\in R$ bo‘lsa, u holda

$$\int_{-\infty}^{+\infty} f(x)\cos mx dx = -2\pi i \operatorname{Im}\left(\sum_{n=1}^{k} \operatorname*{Res}_{z=z_n} (f(z)e^{imz})\right),\ m>0, \qquad (6.11)$$

$$\int_{-\infty}^{+\infty} f(x)\sin mx dx = 2\pi i \operatorname{Re}\left(\sum_{n=1}^{k} \operatorname*{Res}_{z=z_n} (f(z)e^{imz})\right),\ m>0 \qquad (6.12)$$

bo‘ladi.

2- misol. Integrallarni hisoblang:

1) $\oint\limits_{|z|=\frac{1}{2}} \frac{\ln(z+2)}{z^2}dz;$ 2) $\oint\limits_{|z|=2} \frac{z^3}{z^4-1}dz;$

3) $\int\limits_{0}^{2\pi} \frac{dx}{1-4\cos x+4};$ 4) $\int\limits_{-\infty}^{+\infty} \frac{x^2 dx}{(1+x^2)^2}.$

1) $|z|\leq \frac{1}{2}$ doirada integral ostidagi funksiyaning faqat bitta $z=0$ maxsus nuqtasi yotadi. Bu nuqta integral ostidagi funksiya uchun 2-tartibli qutb. Shu sababli

$$\operatorname{Res} f(0) = \frac{1}{1!}\lim_{z\to 0}\frac{d}{dz}\left(\frac{\ln(z+2)}{z^2}\cdot z^2\right) = \lim_{z\to 0}\frac{1}{z+2} = \frac{1}{2}.$$

U holda (6.8) formulaga ko‘ra

$$\oint\limits_{|z|=\frac{1}{2}} \frac{\ln(z+2)}{z^2}dz = 2\pi i \operatorname{Res} f(0) = 2\pi i\cdot\frac{1}{2} = \pi i.$$

2) 1) $|z|\leq 2$ doirada integral ostidagi funksiyaning to‘rtta $z_1=1,\ z_2=-1,$ $z_3=i,\ z_4=-i$ maxsus nuqtalari yotadi. Bu doiradan tashqarida integral ostidagi funksiyaning faqat bitta cheksiz uzoqlashgan $z_5=\infty$ maxsus nuqtasi yotadi.

U holda 1-teoremaga ko‘ra $\sum\limits_{n=1}^{5} \operatorname*{Res}\limits_{z_n} \frac{z_n^3}{z_n^4-1}=0$

yoki

$$2\pi i \operatorname{Res} f(z_5) = -\,2\pi i \sum_{n=1}^{4} \operatorname*{Res}_{z_n} \frac{z_n^3}{z_n^4-1}.$$

$f(z)$ funksiyani Loran qatoriga yoyamiz:

$$\frac{z^3}{z^4-1}=z^3\cdot\left(-\frac{1}{1-z^4}\right)=z^3\cdot\left(-\frac{1}{1-(z^2)^2}\right)=$$

$$=z^3\cdot\left(1+\frac{1}{z^2}+\frac{1}{z^4}+\frac{1}{z^6}+...\right)=z^3+z+\frac{1}{z}+\frac{1}{z^3}+...$$

Bundan $\operatorname{Re}s\, f(z_5)=-1.$

Demak,

$$\oint\limits_{|z|=2}\frac{z^3}{z^4-1}dz=-2\pi i\operatorname{Re}sf(z_5)=-2\pi i\cdot(-1)=2\pi i.$$

3) Berilgan integralda (63.7) formula yordamida o‘zgaruvchini almashtiramiz:

$$\int\limits_0^{2\pi}\frac{dx}{1-4\cos x+4}=\oint\limits_{|z|=1}\frac{dz}{iz(1-2(z+z^{-1})+4)}=i\int\limits_{|z|=1}\frac{dz}{2z^2-5z+2}$$

Integral ostidagi finksiya ikkita oddiy qutbga ega: $z_1=\frac{1}{2},\ z_2=2.$

Ulardan z_1 nuqta birlik doirada yotadi.

Bunda $\operatorname{Re}s\, f(z_1)=\lim\limits_{z\to\frac{1}{2}}\frac{1}{2\left(z-\frac{1}{2}\right)(z-2)}\cdot\left(z-\frac{1}{2}\right)=-\frac{1}{3}.$

Demak,

$$\int\limits_0^{2\pi}\frac{dx}{1-4\cos x+4}=i\oint\limits_{|z|=1}\frac{dz}{2z^2-5z+2}=i\cdot 2\pi i\operatorname{Re}sf(z_1)=\frac{2\pi}{3}.$$

4) $\frac{z^2}{(1+z^2)^2}$ funksiya yuqori yarim tekislikning $z_1=i$ maxsus (ikkinchi tartibli qutb) nuqtasidan boshqa nuqtalarida analitik va uzluksiz, $\lim\limits_{z\to\infty}zf(z)=0,\ \operatorname{Im}z\geq 0.$

U holda

$$\int\limits_{-\infty}^{+\infty}\frac{x^2}{(1+x^2)^2}dx=2\pi i\operatorname{Re}sf(z_1).$$

Bunda $\operatorname{Re}sf(z_1)=\frac{1}{1!}\lim\limits_{z\to i}\frac{d}{dz}\left(\frac{z^2}{(z+i)^2(z-i)^2}\cdot(z-i)^2\right)=\lim\limits_{z\to i}\frac{2zi}{(z+i)^3}=\frac{1}{4i}.$

Demak,

$$\int\limits_{-\infty}^{+\infty}\frac{x^2dx}{(1+x^2)^2}=2\pi i\cdot\frac{1}{4i}=\frac{\pi}{2}.$$

Mashqlar

2.6.1. $f(z)$ funksiyaning z_0 nuqtadagi qoldig‘ini toping:

1) $f(z)=\dfrac{tgz}{z^2-6z},\ z_0=0;$ 2) $f(z)=\dfrac{e^{-\frac{1}{z^2}}}{z^3+1},\ z_0=0;$

3) $f(z)=\dfrac{\sin z^2}{z^3-\pi z^2},\ z_0=\pi;$ 4) $f(z)=\dfrac{z}{z^2-6z+8},\ z_0=4;$

5) $f(z)=\dfrac{e^z}{(z+1)^3(z-2)},\ z_0=2;$ 6) $f(z)=\dfrac{z+3}{z^3-z^2},\ z_0=1;$

7) $f(z)=\ln z\cdot\sin\dfrac{1}{z-1},\ z_0=1;$ 8) $f(z)=z^2\cos\dfrac{1}{z-1},\ z_0=1;$

8) $f(z)=\dfrac{\sin z}{(z-\pi)^2},\ z_0=\infty;$ 10) $f(z)=\dfrac{z+1}{z^2},\ z_0=\infty.$

2.6.2. $f(z)$ funksiyaning barcha maxsus nuqtalardagi va cheksiz uzoqlashgan nuqtadagi qoldig‘ini toping:

1) $f(z)=\dfrac{z^2}{(z-1)^3};$ 2) $f(z)=\dfrac{\sin\frac{1}{z}}{z-1};$

3) $f(z)=\dfrac{e^{\frac{1}{z}}}{1-z};$ 4) $f(z)=\dfrac{\cos z}{z^3-\frac{\pi}{2}z^2}.$

2.6.3. Integrallarni hisoblang:

1) $\displaystyle\oint_{|z|=3}\frac{dz}{z(z+2)(z+4)};$ 2) $\displaystyle\oint_{|z|=2}\frac{dz}{z^4+1};$

3) $\displaystyle\oint_{|z|=\frac{1}{2}}\frac{dz}{z^5-z^3};$ 4) $\displaystyle\oint_{|z|=\sqrt{2}}\frac{dz}{(z-1)^2(z^2+1)};$

5) $\displaystyle\int_0^{2\pi}\frac{dx}{3+\cos x};$ 6) $\displaystyle\int_0^{2\pi}\frac{\sin^2 xdx}{5+4\cos x};$

7) $\displaystyle\int_0^{2\pi}\frac{2dx}{2+\sqrt{3}\cos x};$ 7) $\displaystyle\int_0^{2\pi}\frac{dx}{2+\sin x};$

9) $\displaystyle\int_{-\infty}^{+\infty}\frac{dx}{(1+x^2)^3};$ 10) $\displaystyle\int_{-\infty}^{+\infty}\frac{x^2dx}{(1+4x^2)^2};$

11) $\displaystyle\int_{-\infty}^{+\infty}\frac{xe^{imx}dx}{1+x^2};$ 12) $\displaystyle\int_{-\infty}^{+\infty}\frac{\cos mxdx}{(1+x^2)^2};$

13) $\displaystyle\int_{-\infty}^{+\infty}\frac{x\sin xdx}{x^2+4x+8};$ 14) $\displaystyle\int_0^{+\infty}\frac{\sin^2 xdx}{4+x^2}.$

2-NAZORAT ISHI

1. z_1 va z_2 kompleks sonlari berilgan. a) ifodaning qiymatini toping; b) ildizning qiymatlarini hisoblang.

2. Funksiya differensiallanuvchi bo'ladigan nuqtalarni toping va funksiya hosilasining berilgan nuqtadagi qiymatini hisoblang.

3. Darajalai qatorning yaqinlashish doirasini toping.

1-variant

1. $z_1 = 4-4i,\ z_2 = 2-i\sqrt{2}:\ a)\ z_1^6 - 3iz_1z_2 - \dfrac{6z_1}{z_2};\ b)\ \sqrt[4]{z_1}.$

2. $w = (1-i\bar{z})^2,\ w'(1+i).$ **3.** $\sum\limits_{n=0}^{\infty}\dfrac{(z+i)^n}{n^3+n}.$

2-variant

1. $z_1 = -2i,\ z_2 = 4+i:\ a)\ 3z_1^6 - 2z_1z_2 + \dfrac{z_1}{z_2+1};\ b)\ \sqrt[3]{z_1}.$

2. $w = \dfrac{3}{1+z^2},\ w'(i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{(z+2-3i)^n}{(n+2)!}.$

3-variant

1. $z_1 = 1-\sqrt{3}i,\ z_2 = 3-2i:\ a)\ z_1^4 + 8z_1z_2 + \dfrac{iz_2}{z_1^2};\ b)\ \sqrt[3]{z_2}.$

2. $w = \dfrac{2}{1+z},\ w'(i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{(z-3-2i)^n}{1-3in}.$

4-variant

1. $z_1 = 4-4i,\ z_2 = 2-i\sqrt{2}:\ a)\ z_1^6 - 3iz_1z_2 - \dfrac{6z_1}{z_2};\ b)\ \sqrt[4]{z_1}.$

2. $w = \bar{z}\,\mathrm{Re}\,z,\ w'(0).$ **3.** $\sum\limits_{n=1}^{\infty} n^n z^n.$

5-variant

1. $z_1 = 2+2i,\ z_2 = 1+3i:\ a)\ z_1^6 - 3z_1z_2^2 + \dfrac{iz_2}{z_1};\ b)\ \sqrt[4]{z_1}.$

2. $w = \dfrac{(z+3i)^2}{\bar{z}},\ w'(1).$ **3.** $\sum\limits_{n=0}^{\infty}\dfrac{(z+i)^n}{1+in}.$

6-variant

1. $z_1 = 3-2i,\ z_2 = 5+3i:$ $a)$ $2z_1^6 + 3z_1z_2 - \dfrac{z_1}{z_2-1}$; $b)$ $\sqrt[3]{iz_1}$.

2. $w = 2z - (7z+i)^2,\ w'(-2-3i).$ **3.** $\sum_{n=0}^{\infty} \dfrac{(z+1-2i)^n}{3^n(n^2+1)}.$

7-variant

1. $z_1 = 2-3i,\ z_2 = 1-i\sqrt{2}:$ $a)$ $z_2^6 - 2iz_1z_2 - \dfrac{iz_1}{z_2}$; $b)$ $\sqrt[4]{iz_1}$.

2. $w = 8i(1+7z) - z^2,\ w'(1+i).$ **3.** $\sum_{n=1}^{\infty} (\sin in) z^n.$

8-variant

1. $z_1 = 4-2i,\ z_2 = 3-3i:$ $a)$ $2z_1^6 - 4z_1z_2 + \dfrac{z_1}{z_2-i}$; $b)$ $\sqrt[3]{iz_2}$.

2. $w = (3i+3-z)^2,\ w'(-1-i).$ **3.** $\sum_{n=0}^{\infty} n! z^n.$

9-variant

1. $z_1 = 2-3i,\ z_2 = \sqrt{3}+i:$ $a)$ $z_2^4 + 8iz_1z_2 - \dfrac{iz_1}{z_2^2}$; $b)$ $\sqrt[3]{iz_1}$.

2. $w = \dfrac{1}{i+z} + \dfrac{z}{2i},\ w'(-3i).$ **3.** $\sum_{n=1}^{\infty} (\cos in) z^n.$

10-variant

1. $z_1 = 2-3i,\ z_2 = 1+i\sqrt{2}:$ $a)$ $z_1^6 - 5iz_1z_2 - \dfrac{3z_1}{z_2}$; $b)$ $\sqrt[4]{iz_1}$.

2. $w = 2i(1+3z) - z^2,\ w'(i).$ **3.** $\sum_{n=1}^{\infty} \dfrac{(-1)^n (z-2i)^n}{(n+1)(n+2)}.$

11-variant

1. $z_1 = 4-3i,\ z_2 = 3-2i:$ $a)$ $z_2^6 - 3z_1z_2^2 + \dfrac{iz_1}{z_2}$; $b)$ $\sqrt[4]{iz_2}$.

2. $w = (iz)^2 - (i-\bar{z})^2,\ w'(-1-7i).$ **3.** $\sum_{n=1}^{\infty} \dfrac{(z+2i+1)^n}{n^2(1+i)^n}.$

12-variant

1. $z_1 = 3-\sqrt{3}i,\ z_2 = 2-4i:$ $a)$ $3z_2^6 - 4iz_1z_2 - \dfrac{z_1}{z_2+1}$; $b)$ $\sqrt[3]{iz_2}$.

2. $w = z\cdot |z|^2,\ w'(0).$ **3.** $\sum_{n=1}^{\infty} n^n (z-2)^{2n}$

13-variant

1. $z_1 = 2-4i,\ z_2 = 3-2i:\ a)\ z_1^6 + 3iz_1z_2 - \dfrac{z_1}{z_2 - i};\ b)\ \sqrt[3]{iz_2}.$

2. $w = z^2 + i\,|z|^2,\ w'(1+i).$ **3.** $\sum\limits_{n=0}^{\infty} \dfrac{2^n (z-i)^n}{(n+1)!}.$

14-variant

1. $z_1 = 3+2i,\ z_2 = 2-i\sqrt{2}:\ a)\ z_1^6 - 4iz_1z_2 + \dfrac{iz_2}{z_1};\ b)\ \sqrt[4]{z_1}.$

2. $w = (z+3i-1)^2,\ w'(1-i).$ **3.** $\sum\limits_{n=0}^{\infty} \dfrac{(z-3i)^{2n}}{n!}.$

15-variant

1. $z_1 = 1+3i,\ z_2 = \sqrt{2} - i:\ a)\ 2z_2^6 - 4iz_1z_2 + \dfrac{z_1}{z_2 + i};\ b)\ \sqrt[3]{z_1}.$

2. $w = \dfrac{2}{(z-1)^2},\ w'(-3i).$ **3.** $\sum\limits_{n=0}^{\infty} \dfrac{(z-i)^n}{n^2+1}.$

16-variant

1. $z_1 = 1-\sqrt{3}i,\ z_2 = 2-i:\ a)\ z_2^4 + 8iz_1z_2 + 3\dfrac{z_1}{z_2^2};\ b)\ \sqrt[3]{iz_2}.$

2. $w = (z+i)^3,\ w'(2i).$ **3.** $\sum\limits_{n=1}^{\infty} \dfrac{(z-2i)^n}{1+2in}.$

17-variant

1. $z_1 = 4+i,\ z_2 = 1-2i:\ a)\ z_2^6 - 3iz_1z_2 - \dfrac{iz_1}{z_2};\ b)\ \sqrt[4]{iz_2}.$

2. $w = (i-z)^3,\ w'(2i).$ **3.** $\sum\limits_{n=1}^{\infty} \dfrac{(z-i)^n}{in-1}.$

18-variant

1. $z_1 = 2+2i,\ z_2 = 1-i\sqrt{3}i:\ a)\ z_1^6 - 2iz_1z_2^2 + \dfrac{3z_1}{z_2};\ b)\ \sqrt[4]{iz_1}.$

2. $w = \dfrac{3i+1}{z-2},\ w'(1-i).$ **3.** $\sum\limits_{n=1}^{\infty} \dfrac{2^n z^n}{n!}.$

19-variant

1. $z_1 = 2-2i,\ z_2 = -2+2i:\ a)\ z_1^3 - 5iz_1z_2 - \dfrac{z_2}{z_1^2};\ b)\ \sqrt[3]{iz_1}.$

2. $w = \bar{z}\,\mathrm{Im}\,z - (3i-z)^2,\ w'(1).$

3. $\sum\limits_{n=1}^{\infty} \dfrac{z^n}{(n+2)!}.$

20-variant

1. $z_1 = 8+8i,\ z_2 = 2+3i:\ a)\ z_1^3 - 2iz_1z_2 + \dfrac{z_2}{2iz_1};\ b)\ \sqrt[4]{z_1}.$

2. $w = (i-z)^3 - 2z,\ w'(2+3i).$

3. $\sum\limits_{n=1}^{\infty} \dfrac{(1+i)^n (z-2)^n}{(n+1)(n+2)}.$

21-variant

1. $z_1 = -2i,\ z_2 = 3-4i:\ a)\ z_2^{10} - 4iz_1z_2^2 + \dfrac{8z_2}{z_1};\ b)\ \sqrt[3]{z_2}.$

2. $w = (3z-i)^2 + 2z,\ w'(2+3i).$

3. $\sum\limits_{n=0}^{\infty} \dfrac{(z+1+i)^n}{n7^n}.$

22-variant

1. $z_1 = \sqrt{2}(1+i),\ z_2 = \sqrt{2}-4i:\ a)\ z_2^2 + 5z_1z_2 + 3i\dfrac{z_1}{z_2^2};\ b)\ \sqrt[4]{z_1}.$

2. $w = \bar{z}\,\mathrm{Im}\,z,\ w'(i).$

3. $\sum\limits_{n=0}^{\infty} \dfrac{n!3^n z^n}{n^n}.$

23-variant

1. $z_1 = 1+i,\ z_2 = 2-i\sqrt{2}:\ a)\ iz_1^4 + 2z_1z_2 - \dfrac{iz_1}{i+z_2};\ b)\ \sqrt[4]{iz_1}.$

2. $w = (2i-z)^2 + \dfrac{1}{2i},\ w'(-1+i).$

3. $\sum\limits_{n=1}^{\infty} \dfrac{n(z-i-1)^n}{3^n}.$

24-variant

1. $z_1 = -1-i,\ z_2 = 3i:\ a)\ z_2^8 - 3z_1z_2^2 + \dfrac{3z_1}{iz_2};\ b)\ \sqrt[4]{iz_1}.$

2. $w = e^x(\cos y + i\sin y),\ w'(1).$

3. $\sum\limits_{n=1}^{\infty} \dfrac{(z-1)^n}{n^3}.$

25-variant

1. $z_1=-2i,\ z_2=4+i:\ a)\ 3z_1^6-4z_1z_2-\dfrac{z_2}{z_1+z_2};\ b)\ \sqrt[3]{z_1}.$

2. $w=(3-2z)^2+i,\ w'(1-i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{z^n}{(n+2)!}.$

26-variant

1. $z_1=8+8i,\ z_2=2+3i:\ a)\ z_1^3-2iz_1z_2+\dfrac{z_2}{2iz_1};\ b)\ \sqrt[4]{z_1}.$

2. $w=(1+iz)^2,\ w'(1-i).$ **3.** $\sum\limits_{n=0}^{\infty}\dfrac{(z-1)^{2n}}{4^n}.$

27-variant

1. $z_1=1+i,\ z_2=2-5i:\ a)\ z_1^{10}-3iz_1z_2^2+\dfrac{2z_2}{z_1+z_2};\ b)\ \sqrt[3]{z_2}.$

2. $w=(2+3z)^2-i,\ w'(1+i).$ **3.** $\sum\limits_{n=0}^{\infty}\dfrac{n(z+i)^n}{5^n}.$

28-variant

1. $z_1=3-4i,\ z_2=-2i:\ a)\ z_1^4-3z_1(z_2+2i)+(3i+1)\dfrac{z_2}{z_1};\ b)\ \sqrt[4]{z_2}.$

2. $w=(2i+z)^3-2i,\ w'(3-2i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{(z-2+3i)^n}{i^n(n+1)!}.$

29-variant

1. $z_1=1+i,\ z_2=2-i\sqrt{2}:\ a)\ iz_1^4+2z_1z_2-\dfrac{iz_1}{i+z_2};\ b)\ \sqrt[4]{iz_1}.$

2. $w=\dfrac{i-1}{2+z},\ w'(1+3i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{n^n(z-i)^n}{n+1}.$

30-variant

1. $z_1=3i,\ z_2=-1+i:\ a)\ 2z_2^8+z_1z_2^2+\dfrac{3iz_2}{6+4i+z_1};\ b)\ \sqrt[4]{z_1}.$

2. $w=\bar{z}\cdot|z|,\ w'(i).$ **3.** $\sum\limits_{n=1}^{\infty}\dfrac{(z+3+2i)^n}{n^2 5^n}.$

3-NAZORAT ISHI

1. Integralni Koshi formulasi bilan musbat yo‘nalishda hisoblang.
2. Funksiyani berilgan sohada Loran qatoriga yoying.
3. Integrallarni qoldiqlar orqali hisoblang.

1-variant

1. $\oint_L \frac{e^z}{z(1-z)^3}dz,\ L:|z|=2.$ **2.** $f(z)=\frac{z+2}{z^2-4z+3},\ 0<|z-1|<2.$

3. a) $\int_{-\infty}^{+\infty}\frac{(x-1)e^{ix}}{x^2-2x+2}dx;$ **b)** $\int_0^{2\pi}\frac{1}{2+\sin x}dx.$

2-variant

1. $\oint_L \frac{z^2-1}{z(z+2)^2(z+3)}dz,\ L:|z|<4.$ **2.** $f(z)=\frac{z^3}{(z-1)(z-2)},\ 0<|z-2|<1.$

3. a) $\int_0^{+\infty}\frac{x^4}{(1+2x^2)^4}dx;$ **b)** $\int_0^{2\pi}\frac{\sin^2 x}{5+4\cos x}dx.$

3-variant

1. $\oint_L \frac{z-3}{(z+i)(z-8)^2}dz,\ L:|z|\le 8.$ **2.** $f(z)=\frac{1}{(z^2-9)(z^2-4)},\ 2<|z|<3.$

3. a) $\int_{-\infty}^{+\infty}\frac{e^{ix}}{x^2+1}dx;$ **b)** $\int_0^{2\pi}\frac{1}{\sin^6 x+\cos^6 x}dx.$

4-variant

1. $\oint_L \frac{z^3}{(z-2)(z+i)^3}dz,\ L:|z+i|\le 1.$ **2.** $f(z)=\frac{1}{z^2+4},\ 0<|z+2i|<4.$

3. a) $\int_{-\infty}^{+\infty}\frac{(x+1)e^{-3ix}}{x^2-2x+5}dx;$ **b)** $\int_0^{2\pi}\frac{2+\cos x}{2-\sin x}dx.$

5-variant

1. $\oint_L \frac{z+1}{z(z-1)^2(z-3)}dz,\ L:|z|=2.$ **2.** $f(z)=\frac{z}{z^2+1},\ 0<|z-i|<2.$

3. a) $\int_{-\infty}^{+\infty}\frac{(x^3+5x)\sin x}{x^4+2x^2+2}dx;$ **b)** $\int_0^{2\pi}\frac{1}{2-\cos x}dx.$

6-variant

1. $\oint\limits_L \frac{3e^z-5}{(z^2+36)^2}dz,\ L:|z|\le 6.$

2. $f(z)=\frac{z}{(z^2+4)(z^2-1)},\ 1<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{(2x^3+13x)\sin x}{2x^4+6x^2+9}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{3+\sin x}dx.$

7-variant

1. $\oint\limits_L \frac{2e^z-3e^{-z}}{(z+i)^7}dz,\ L:|z|<\frac{3}{2}.$

2. $f(z)=\frac{7z-1}{z^2+z-2},\ 0<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{\cos x}{x^4+2x^2+2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{10+3\cos x}dx.$

8-variant

1. $\oint\limits_L \frac{z-6}{(z^2-1)(z-3)}dz,\ L:|z|<\frac{3}{2}.$

2. $f(z)=\frac{3-z}{z^2-2iz+8},\ 0<|z-4i|<6.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x^3\sin x}{x^4+4x^2+8}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{10+2\sin x}dx.$

9-variant

1. $\oint\limits_L \frac{e^{6z}-1}{(z-i)^6}dz,\ L:|z|<2.$

2. $f(z)=\frac{1}{z^2-3iz-2},\ 0<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x\cos x}{x^2-2x+10}dx;$

b) $\int\limits_0^{2\pi}\frac{\sin^2 x}{5+4\cos x}dx.$

10-variant

1. $\oint\limits_L \frac{z^3+1}{z^2(z+2)(z-3)}dz,\ L:|z|<4.$

2. $f(z)=\frac{z^3}{z^2-3z+2},\ 0<|z-1|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x\sin x}{(x^2+4)^2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{2\sin^2 x+3\cos^2 x}dx.$

11-variant

1. $\oint\limits_L \frac{2e^z-3e^{-z}}{(z-4)^5}dz,\ L:|z-4|<1.$

2. $f(z)=\frac{z}{(z^2-4)(z^2-1)^2},\ 1<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x^6}{(x^4+1)}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{(5+4\cos x)^2}dx.$

12-variant

1. $\oint\limits_L \frac{e^z+e^{-z}}{(z-1)^2(z-2)}dz,\ L:|z|\le 3.$

2. $f(z)=\frac{3-z}{z^2+2z-8},\ 0<|z-2|<6.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x^2}{(x^2+5ix-4)^2}dx;$

b) $\int\limits_0^{2\pi}\frac{\cos^2 2x}{5-4\cos x}dx.$

13-variant

1. $\oint_L \frac{e^z+e^{-z}}{(z^2+25)^2}dz,\ L:|z|\le 5.$

2. $f(z)=\frac{2}{z^2-1},\ 0<|z-3i|<\sqrt{10}.$

3. a) $\int_{-\infty}^{+\infty}\frac{e^{ix}}{x^2-2ix-1}dx;$

b) $\int_0^{2\pi}\frac{1}{17-8\cos x}dx.$

14-variant

1. $\oint_L \frac{1}{z(z^2-1)}dz,\ L:|z+1|\le \frac{1}{2}.$

2. $f(z)=\frac{4z}{z^2+4},\ 0<|z-2i|<4.$

3. a) $\int_{-\infty}^{+\infty}\frac{1}{x^2-2ix-1}dx;$

b) $\int_0^{2\pi}\frac{1}{(3+\cos x)^2}dx.$

15-variant

1. $\oint_L \frac{1}{(z^2+2z+1)(z-3)}dz,\ L:|z|\le 3.$

2. $f(z)=\frac{1}{z^2+iz+12},\ 0<|z-3i|<7.$

3. a) $\int_{-\infty}^{+\infty}\frac{(x-3)e^{ix}}{x^2-6x+9}dx;$

b) $\int_0^{2\pi}\frac{1}{5-3\cos x}dx.$

16-variant

1. $\oint_L \frac{1}{(z^2-5z+4)^2}dz,\ L:|z-1|=20.$

2. $f(z)=\frac{1}{z(z-1)},\ 0<|z-1|<1.$

3. a) $\int_{-\infty}^{+\infty}\frac{x\sin x}{x^2+2x+10}dx;$

b) $\int_0^{2\pi}\frac{1}{13-5\cos x}dx.$

17-variant

1. $\oint_L \frac{e^z}{(z-2)^2(z-3)}dz,\ L:|z-3|\le 1.$

2. $f(z)=\frac{z}{z^2-25},\ 0<|z-5|<10.$

3. a) $\int_0^{+\infty}\frac{x^6}{(x^4+16)^2}dx;$

b) $\int_0^{2\pi}\frac{\cos^2 x}{2-\sin^2 x}dx.$

18-variant

1. $\oint_L \frac{e^z-e^{-z}}{2z(1-z)^3}dz,\ L:|z-1|\le 1.$

2. $f(z)=\frac{1}{(z^2+1)(z^2+9)},\ 1<|z|<3.$

3. a) $\int_{-\infty}^{+\infty}\frac{1}{(x^2+1)^3}dx;$

b) $\int_0^{2\pi}\frac{\sin^2 x}{4+3\cos x}dx.$

19-variant

1. $\oint\limits_L \frac{e^{4z}}{(z+2)^2(z+3)}dz,\ L:|z|\leq 3.$

2. $f(z)=\frac{1}{(z^2-1)(z^2-9)},\ 1<|z|<3.$

3. a) $\int\limits_0^{+\infty}\frac{\cos 2x}{x^4+2x^2+2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{(8+\cos x)^2}dx.$

20-variant

1. $\oint\limits_L \frac{1}{z^2+4}dz,\ L:|z|<3.$

2. $f(z)=\frac{1}{(z^2-1)(z^2-4)},\ 1<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x}{(x^2+4x+13)^2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{6+\sin x}dx.$

21-variant

1. $\oint\limits_L \frac{z^2+3z+7}{(z-i)^2 z}dz,\ L:|z|\leq 2.$

2. $f(z)=\frac{1}{z^2+z},\ 0<|z|<1.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{(x-1)\cos 2x}{x^2-4x+5}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{\sin^2 x+5\cos^2 x}dx.$

22-variant

1. $\oint\limits_L \frac{e^{5z}-3}{z(z+1)^3}dz,\ L:|z|\leq 1.$

2. $f(z)=\frac{2z+3}{z^2+3z+2},\ 1<|z|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{x^4+1}{x^6+1}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{1-2a\cos x+a^2}dx.$

23-variant

1. $\oint\limits_L \frac{e^{z}+e^{-z}}{(z^2+9)^2}dz,\ L:|z-i|\leq 10.$

2. $f(z)=\frac{z^3}{z^2+1},\ 0<|z+i|<2.$

3. a) $\int\limits_0^{+\infty}\frac{x\cos x}{x^2-2x+10}dx;$

b) $\int\limits_0^{2\pi}\frac{\cos^2 x}{2-\sin^2 x}dx.$

24-variant

1. $\oint\limits_L \frac{z^2+5z-4}{z(1-z)^3}dz,\ L:|z|\leq 2.$

2. $f(z)=\frac{1}{z^2+iz+2},\ 1<|z-i|<3.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{1}{(x^2+1)^2(x^2+4)}dx;$

b) $\int\limits_0^{2\pi}\frac{\sin^2 x}{5+4\cos x}dx.$

25-variant

1. $\oint\limits_L \frac{z^3-2z+1}{(z-i)^2(z+2)^2}dz,\ L:|z|\le 2.$

2. $f(z)=\frac{1}{z^2-3iz-2},\ 0<|z-i|<1.$

3. a) $\int\limits_0^{+\infty}\frac{1}{(x^2+5ix-4)^2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{(3+2\cos x)^2}dx.$

26-variant

1. $\oint\limits_L \frac{z+5}{(z^2+1)(z+2)}dz,\ L:|z|\le 2.$

2. $f(z)=\frac{1}{1-z^2},\ 0<|z-1|<2.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{e^{ix}}{x^2+1}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{7+\cos x}dx.$

27-variant

1. $\oint\limits_L \frac{e^{4z}}{(z+i)^5}dz,\ L:|z|=3.$

2. $f(z)=\frac{1}{z^2+2iz+3},\ 0<|z-i|<4.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{(x+1)e^{-3ix}}{x^2-2x+5}dx;$

b) $\int\limits_0^{2\pi}\frac{\sin^2 x}{3+\cos x}dx.$

28-variant

1. $\oint\limits_L \frac{z^4-1}{z^2(z+3)(z-2)}dz,\ L:|z|<4.$

2. $f(z)=\frac{1}{(z+2)(z^2+1)},\ 1<|z|<2.$

3. a) $\int\limits_0^{+\infty}\frac{x\sin 2x}{x^2+4}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{2-2\cos x}dx.$

29-variant

1. $\oint\limits_L \frac{e^z+e^{-z}}{(z+1)^6}dz,\ L:|z+1|<1.$

2. $f(z)=\frac{3z-2}{z^2-iz+6},\ 0<|z+2i|<5.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{(x^3+5x)\sin x}{x^4+4x^2+8}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{(3+\cos x)^2}dx.$

30-variant

1. $\oint\limits_L \frac{e^z+e^{-z}}{(z-1)^2(z+1)}dz,\ L:|z|\le 1.$

2. $f(z)=\frac{4z-3}{z^2+3z+2},\ 0<|z+1|<1.$

3. a) $\int\limits_{-\infty}^{+\infty}\frac{1}{(x^2+4x+13)^2}dx;$

b) $\int\limits_0^{2\pi}\frac{1}{5-4\cos x}dx.$

III bob
OPERATSION HISOB

3.1. LAPLAS ALMASHTIRISHLARI

Originallar va tasvirlar. Laplas almashtirishining xossalari. Originallar va tasvirlar jadvali

3.1.1. Operatsion hisobning asosiy boshlang‘ich tushunchalari original – funksiya va tasvir – funksiya tushunchalari hisoblanadi.

$f(t)$ – haqiqiy t o‘zgaruvchining haqiqiy funksiyasi bo‘lsin.
Bunda t sifatida vaqt yoki koordinata tushuniladi.

Quyidagi shartlarni qanoatlantiruvchi $f(t)$ funksiyaga *original* deyiladi:

1. $t<0$ da $f(t)\equiv 0$;

2. $f(t)-t\geq 0$ da bo‘lakli uzluksiz, ya’ni $f(t)$ funksiya $t\geq 0$ da uzluksiz yoki $t\geq 0$ dagi ixtiyoriy chekli oraliqda chekli sondagi I tur uzilish nuqtalariga ega;

3. shunday $M>0$ va $s_0\geq 0$ sonlar topiladiki, barcha t lar uchun $|f(t)|\leq Me^{s_0 t}$ bo‘ladi, ya’ni t o‘sishi bilan $f(t)$ funksiya biror ko‘rsatkichli funksiyadan sekinroq o‘sib boradi. Bunda s_0 soniga $f(t)$ funksiyaning *o‘sish ko‘rsatkichi* deyiladi.

$f(t)$ *originalning tasviri* deb, $p=s+i\sigma$ kompleks o‘zgaruvchining

$$F(p)=\int_0^{\infty} f(t)e^{-pt}dt \qquad (1.1)$$

integrali bilan aniqlanadigan $F(p)$ funksiyaga aytiladi.

$f(t)$ originaldan $F(p)$ tasvirga o‘tishga *Laplas almashtirishi* deyiladi.

$f(t)$ original bilan $F(p)$ tasvir orsidagi moslik $F(p)\xrightarrow{\bullet} f(t)$ yoki $f(t)\xleftarrow{\bullet} F(p)$ deb yoziladi. Bunda “$\xrightarrow{\bullet}$” belgining yo‘nalishi hamma vaqt original tomonga bo‘ladi.

1- misol. Funksiyalarning tasvirini toping:

1) $\eta(t)=\begin{cases}1, & agar\ t\geq 0\ bo'lsa,\\ 0, & agar\ t<0\ bo'lsa;\end{cases}$ 2) $f(t)=e^{at},\ a\in Z.$

1) $\eta(t)$ – *Xevisaydning birlik funksiyasi* deb ataladi. Bu funksiya

yordamida originalning 2 – 3 shartlarini qanoatlantiruvchi istalgan funksiyani original-funksiyaga keltirish mumkin:

$$f(t)\eta(t)=\begin{cases} f(t), & agar\, t\geq 0 \;\; bo'lsa, \\ 0, & agar\, t<0 \;\; bo'lsa. \end{cases}$$

Bundan keyin bu original-funksiyani qisqacha $f(t)$ deb yozamiz.

$s=\operatorname{Re}p>0\;(s_0=0)$ da (1.1) formula bilan topamiz:

$$F(p)=\int_0^\infty 1\cdot e^{-pt}dt=\lim_{b\to\infty}\int_0^b e^{-pt}dt=\lim_{b\to\infty}\left(-\frac{1}{p}e^{-pt}\right)\Bigg|_0^b=\frac{1}{p}.$$

Demak,

$$\eta(t)\overset{\bullet}{\longleftarrow}\frac{1}{p}.$$

2) Berilgan funksiya original. Shu sababli (1.1) formula bilan topamiz:

$$F(p)=\int_0^\infty e^{at}e^{-pt}dt=\lim_{b\to\infty}\int_0^b e^{-(p-a)t}dt=-\lim_{b\to\infty}\left(\frac{1}{p-a}e^{-(p-a)t}\right)\Bigg|_0^b=$$

$$=\lim_{b\to\infty}\left(\frac{1}{p-a}-\frac{e^{-(p-a)b}}{p-a}\right)=\frac{1}{p-a},$$

bu yerda $\operatorname{Re}(p-a)>0$.

Demak,

$$e^{at}\overset{\bullet}{\longleftarrow}\frac{1}{p-a}\quad(\operatorname{Re}p>\operatorname{Re}a).$$

1-teorema (t*asvirning mavjudligi haqidagi teorema*). Har qanday chekli o‘sishga ega $f(t)$ original uchun $\operatorname{Re}p=s>s_0$ yarim tekislikda analitik bo‘lgan $F(p)$ tasvir mavjud (aniqlangan) bo‘ladi, bu yerda $s_0-f(t)$ funksiyaning o‘sish ko‘rsatkichi.

3.1.2. Operatsion hisobni qo‘llash asosan Laplas almashtirishining quyidagi xossasiga asoslanadi: agar original differensial tenglamani qanoatlantirsa, u holda bu originalga mos tasvir soddaroq bo‘lgan (hosilani o‘z ichiga olmaydigan) tenglamaga bo‘ysunadi. Shu sababli differensial tenglamalarni quyidagi sxema asosida yechish mumkin bo‘ladi:

1. Original qanoatlantiruvchi differensial tenglamadan Laplas almashtirishlari yordamida noma’lum originalga nisbatan differensial bo‘lmagan tenglamaga o‘tiladi;
2. Bu tenglamani yechish orqali noma’lum originalning tasviri topiladi;

3. Topilgan tasvir asosida izlanayotgan original hisoblanadi (bunga Laplas almashtirishiga qaytish formulalari orqali erishiladi).

Laplas almashtirishining quyidagi xossalari operatsion hisobning asosiy qoidalarini ifodalaydi.

Operatsion hisobning asosiy qoidalari

№	$F(p)\xrightarrow{\bullet} f(t)$	**Nomlanishi**
1	$\alpha\cdot F_1(p)+\beta\cdot F_2(p)\xrightarrow{\bullet}\alpha\cdot f_1(t)+\beta\cdot f_2(t)$	chiziqlilik
2	$\frac{1}{a}F\left(\frac{p}{a}\right)\xrightarrow{\bullet} f(at)$	o‘xshashlik
3	$e^{-p\tau}F(p)\xrightarrow{\bullet} f(t-\tau)$	kechikish
4	$F(p-a)\xrightarrow{\bullet} e^{at}f(t)$	siljish
5	$pF(p)-f(0)\xrightarrow{\bullet} f'(t)$	originalni differensiallash
6	$F'(p)\xrightarrow{\bullet} -tf(t)$	tasvirni differensiallash
7	$\frac{F(p)}{p}\xrightarrow{\bullet}\int_0^t f(\tau)d\tau$	originalni integrallash
8	$\int_p^\infty F(z)dz\xrightarrow{\bullet}\frac{f(t)}{t}$	tasvirni integrallash

2-misol. $\cos\omega t$ va $\sin\omega t$ funksiyaning tasvirini toping.

⊖ $f(t)$ funksiyaning tasvirini Laplas almashtirishining chiziqlilik xossasidan foydalanib topamiz:

$$\cos\omega t=\frac{1}{2}(e^{i\omega t}+e^{-i\omega t})\xleftarrow{\bullet}\frac{1}{2}\left(\frac{1}{p-i\omega}+\frac{1}{p+i\omega}\right)=\frac{p}{p^2+\omega^2},$$

$$\sin\omega t=\frac{1}{2i}(e^{i\omega t}-e^{-i\omega t})\xleftarrow{\bullet}\frac{1}{2i}\left(\frac{1}{p-i\omega}-\frac{1}{p+i\omega}\right)=\frac{\omega}{p^2+\omega^2}.$$ ⊘

3-misol. $f(t)=\cos t$ funksiyaning tasvirini toping.

⊖ 2-misoldan ma'lum bo'lgan $\cos\omega t \xleftarrow{\bullet} \dfrac{p}{p^2+\omega^2}$ moslikka Laplas almashtirishining o'xshashlik xossasini qo'llaymiz:

$$\cos t=\cos\left(\frac{1}{\omega}\omega t\right)\xleftarrow{\bullet}\frac{1}{\frac{1}{\omega}}\cdot\frac{\frac{\frac{p}{1}}{\omega}}{\frac{p^2}{\frac{1}{\omega^2}}+\omega^2}=\frac{p}{p^2+1}.$$ ⊖

4-misol. $f(t)=\begin{cases}0, & agar\ t<0, t\geq 6\ bo'lsa,\\ t, & agar\ 0\leq t\leq 3\ bo'lsa,\\ 6-t, & agar\ 3<t<6\ bo'lsa.\end{cases}$

funksiyaning tasvirini toping.

⊖ Original funksiyani Xevidaysning birlik funksiyasi yordamida bitta analitik funksiya ko'rinishida yozib olamiz:

$$f(t)=t\eta(t)-t\eta(t-3)+(6-t)\eta(t-3)-(6-t)\eta(t-6)$$

yoki

$$f(t)=t\eta(t)-(t-3+3)\eta(t-3)-(t-3-3)\eta(t-3)+(t-6)\eta(t-6).$$

Qavslarni ochamiz va o'xshash hadlarni ixchamlaymiz:

$$f(t)=t\eta(t)-2(t-3)\eta(t-3)+(t-6)\eta(t-6).$$

Bu originalga Laplas almashtirishining kechikish xossasini qo'llaymiz:

$$f(t)\xleftarrow{\bullet}=\frac{1}{p^2}-2\frac{1}{p^2}e^{-3p}+\frac{1}{p^2}e^{-6p}=\frac{(1-e^{-3p})^2}{p^2}=F(p).$$ ⊖

5-misol. $f(t)=e^{at}\cos\omega t$ funksiyaning tasvirini toping.

⊖ $f(t)$ funksiyaning tasvirini Laplas almashtirishining siljish xossasini qo'llab, topamiz:

$$e^{at}\cos\omega t\xleftarrow{\bullet}\frac{p-a}{(p-a)^2+\omega^2}.$$ ⊖

6-misol. Agar $f(0)=3,\ f'(0)=0,\ f''(0)=-2$ bo'lsa,

$f''(t)-2f''(t)-3f'(t)+2f(t)+2$ ifodaning tasvirini toping.

⊖ $f(t)\xleftarrow{\bullet}F(p)$ bo'lsin.

U holda originalni differensiallash formulalariga ko'ra

$$f'(t)\xleftarrow{\bullet}pF(p)-f(0)=pF(p)-3,$$

$$f''(t) \xleftarrow{\bullet} p^2F(p) - pf(0) - f'(0) = p^2F(p) - 3p,$$

$$f'''(t) \xleftarrow{\bullet} p^3F(p) - p^2f(0) - pf'(0) - f''(0) = p^3F(p) - 3p^2 + 2.$$

Shu bilan birga $2 = 2 \cdot \eta(t) = \dfrac{2}{p}$.

U holda

$$f''(t) - 2f''(t) - 3f'(t) + 2f(t) + 2 \xleftarrow{\bullet}$$

$$\xleftarrow{\bullet} p^3F(p) - 3p^2 + 2 - 2(p^2F(p) - 3p) - 3(pF(p) - 3) + 2F(p) + \frac{2}{p}.$$

7-misol. $f(t) = t\cos\omega t$ funksiyaning tasvirini toping.

62.2-misoldan ma'lumki, $\cos\omega t \xleftarrow{\bullet} \dfrac{p}{p^2 + \omega^2}$.

U holda tasvirni differensiallash xossasiga ko'ra

$$-t\cos\omega t \xleftarrow{\bullet} \left.\left(\frac{p}{p^2 + \omega^2}\right)'\right|_p$$

yoki

$$t\cos\omega t \xleftarrow{\bullet} \frac{p^2 - \omega^2}{(p^2 + \omega^2)^2}.$$

8-misol. $f(t) = \int_0^t e^{\tau} d\tau$ funksiyaning tasvirini toping.

62.1. 2)- misoldadan $a = 1$ da topamiz: $e^{\tau} \xleftarrow{\bullet} \dfrac{1}{p-1}$.

U holda originalni integrallash xossasiga ko'ra

$$\int_0^t e^{\tau} d\tau \xleftarrow{\bullet} \frac{\frac{1}{p-1}}{p} = \frac{1}{p(p-1)}.$$

9-misol. $f(t) = \dfrac{\sin t}{t}$ funksiyaning tasvirini toping.

2-misolda hosil qilingan moslikdan $\omega = 1$ da $\sin t \xleftarrow{\bullet} \dfrac{1}{p^2 + 1}$ kelib chiqadi. Bu moslikka tasvirni integrallash xossasini qo'llab, topamiz:

$$\frac{\sin t}{t} \xleftarrow{\bullet} \int_p^{\infty} \frac{1}{p^2 + 1} dp = arctgp\Big|_p^{\infty} = \frac{\pi}{2} - arctgp = arcctgp.$$

3.1.3. Amaliyotda ko‘p uchraydigan originallar va ularning tasvirlarini orasidagi mosliklarni o‘rnatuvchi jadvalni keltiramiz.

Originallar va tasvirlar jadvali

№	$f(t)$ **original**	$F(p)$ **tasvir**
1	1	$\frac{1}{p}$
2	e^{at}	$\frac{1}{p-a}$
3	$\sin \omega t$	$\frac{\omega}{p^2+\omega^2}$
4	$\cos \omega t$	$\frac{p}{p^2+\omega^2}$
5	sht	$\frac{\omega}{p^2-\omega^2}$
6	cht	$\frac{p}{p^2-\omega^2}$
7	$e^{at}\sin \omega t$	$\frac{\omega}{(p-a)^2+\omega^2}$
8	$e^{at}\cos \omega t$	$\frac{p-a}{(p-a)^2+\omega^2}$
9	$t^n (n - butun\ son)$	$\frac{n!}{p^{n+1}}$
10	$e^{at}t^n$	$\frac{n!}{(p-a)^{n+1}}$
11	$t\sin \omega t$	$\frac{2\omega p}{(p^2+\omega^2)^2}$
12	$t\cos \omega t$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$

10-misol. $f(t)=\sin 3t\cos 2t$ funksiyaning tasvirini originallar va tasvirlar jadvalidan foydalanib, toping.

$f(t)=\sin 3t\cos 2t=\frac{1}{2}(\sin 5t+\sin t)$ originalga originallar va tasvirlar jadvalining 3-formulasini va Laplas almashtirishining chiziqlilik xossasini qo‘llab, topamiz:

$$f(t)=\frac{1}{2}(\sin 5t+\sin t)\risingdotseq\frac{1}{2}\left(\frac{5}{p^2+5^2}+\frac{1}{p^2+1^2}\right)=\frac{3(p^2+5)}{(p^2+1)(p^2+25)}.$$

Mashqlar

3.1.1. Funksiyalarning tasvirini ta’rif asosida toping:

1) $f(t)=t^2;$
2) $f(t)=\sin 3t;$
3) $f(t)=e^{3t};$
4) $f(t)=sh2t.$

3.1.2. Funksiyalarning tasvirini Laplas almashtirishining xossalaridan foydalanib, toping:

1) $f(t)=\sin t-\cos t;$
2) $f(t)=3+2t;$
3) $f(t)=\sin^2 t;$
4) $f(t)=\cos^3 t;$
5) $f(t)=(t-1)^2;$
6) $f(t)=\cos^2(t-1);$
7) $f(t)=e^{-t}\cos 2t;$
8) $f(t)=e^{-t}t^3;$
9) $f(t)=\sin^3 t;$
10) $f(t)=t\cos\omega t;$
11) $f(t)=t^2e^t;$
12) $f(t)=t^2\cos t;$
13) $f(t)=\int_0^t \sin\tau d\tau;$
14) $f(t)=\int_0^t \cos^2\omega\tau d\tau;$
15) $f(t)=\frac{e^t-1}{t};$
16) $f(t)=\frac{1-\cos t}{t};$
17) $f(t)=tsh\omega t;$
18) $f(t)=te^{\omega t};$
19) $f(t)=\begin{cases}1, & agar\ 0\le t\le 1\ bo'lsa,\\ 2-t, & agar\ 1<t\le 2\ bo'lsa,\\ 0, & agar\ t<0,\ t>2\ bo'lsa;\end{cases}$
20) $f(t)=\begin{cases}t, & agar\ 0\le t\le 1\ bo'lsa,\\ 1, & agar\ 1<t\le 2\ bo'lsa,\\ 0, & agar\ t<0,\ t>2\ bo'lsa;\end{cases}$
21) $f(t)=\begin{cases}0, & agar\ t<a\ bo'lsa,\\ t-a, & agar\ a\le t\le b\ bo'lsa,\\ b-a, & agar\ t>b\ bo'lsa;\end{cases}$
22) $f(t)=\begin{cases}0, & agar\ t<0\ bo'lsa,\\ t, & agar\ 0\le t\le a\ bo'lsa,\\ a, & agar\ t>a\ bo'lsa.\end{cases}$

3.1.3. Funksiyalarning tasvirini originallar va tasvirlar jadvalidan foydalanib, toping:

1) $f(t) = 3e^{-t} + e^{t}\cos 3t;$ 2) $f(t) = \dfrac{1}{2^{t}} + 1;$

3) $f(t) = e^{t}\cos^{2} t;$ 4) $f(t) = \cos t \cos 3t;$

5) $f(t) = e^{-4t}\sin 3t \cos 2t;$ 6) $f(t) = 2\sin 2t + 3sh2t;$

7) $f(t) = shat\sin bt;$ 8) $f(t) = tchbt;$

9) $f(t) = t^{5} + 18\cos^{2} 5t;$ 10) $f(t) = e^{-3t}ch5t.$

3.2. TASVIR BO‘YICHA ORIGINALNI TOPISH

Riman-Mellin formulasi. Tasvirlarni ko‘paytirish teoremalari. Yoyish teoremalari

*Elementar usul*da tasvirdan originalga o‘tish bevosita tasvirlar jadvali yordamida amalga oshiriladi. Bunda $F(p)$ tasvirlar jadvalida mavjud bo‘lgan kasrlar yig‘indisiga keltiriladi va Laplas almashtirishining xossalarini qo‘llash orqali $f(t)$ topiladi.

1-misol. Berilgan tasvirlarning originallarini toping:

1) $F(p) = \dfrac{4}{p} + \dfrac{3}{p^{3}} + \dfrac{5}{p+3};$ 2) $F(p) = \dfrac{4p+1}{p^{2}+16};$

3) $F(p) = \dfrac{p}{p^{2} - 2p + 5};$ 4) $F(p) = \dfrac{e^{-p}}{(p-2)^{3}};$

5) $F(p) = \dfrac{p+1}{p(p-1)(p-2)(p-3)};$ 6) $F(p) = \dfrac{1}{(p+1)(p^{2}+4)}.$

1) Originallar va tasvirlar jadvalidan topamiz:

$$\frac{1}{p} \xrightarrow{\bullet} 1, \quad \frac{2!}{p^{3}} \xrightarrow{\bullet} t^{2}, \quad \frac{1}{p+3} \xrightarrow{\bullet} e^{-3t}.$$

$F(p)$ tasvirda mos almashtirishlar bajaramiz:

$$F(p) = 4 \cdot \frac{1}{p} + \frac{3}{2} \cdot \frac{2!}{p^{3}} + 5 \cdot \frac{1}{p+3}.$$

Bu tasvirga Laplas almashtirishining chiziqlilik xossasini qo‘llab, topamiz:

$$F(p) \xrightarrow{\bullet} 4 \cdot 1 + \frac{3}{2} \cdot t^{2} + 5 \cdot e^{-3t} = 4 + \frac{3t^{2}}{2} + 5e^{-3t}.$$

2) $F(p)$ tasvirni tasvirlar jadvalida bo‘lgan sodda kasrlar yig‘indisiga keltiramiz:

$$F(p)=4\cdot\frac{p}{p^2+16}+\frac{1}{4}\cdot\frac{4}{p^2+16}.$$

Bu tasvirda har bir yig‘indini mos original bilan almashtiramiz va Laplas almashtirishining chiziqlilik xossasini qo‘llab, topamiz:

$$F(p)\xrightarrow{\bullet}4\cdot\cos 4t+\frac{1}{4}\cdot\sin 4t=4\cos 4t+\frac{\sin 4t}{4}.$$

3) $F(p)$ tasvirni originalini tasvirlar jadvalining formulalari bilan topish mumkin bo‘lgan kasrlar yig‘indisiga keltiramiz:

$$F(p)=\frac{p}{p^2-2p+5}=\frac{(p-1)+1}{(p-1)^2+4}=\frac{p-1}{(p-1)^2+4}+\frac{1}{2}\cdot\frac{2}{(p-1)^2+4}$$

Bundan

$$F(p)\xrightarrow{\bullet}e^t\cos 2t+\frac{1}{2}e^t\sin 2t=\frac{e^t}{2}(2\cos 2t+\sin 2t).$$

4) Tasvirlar jadvalidan topamiz:

$$\frac{1}{(p-2)^3}=\frac{1}{2}\cdot\frac{2}{(p-2)^3}\xrightarrow{\bullet}\frac{1}{2}e^{2t}t^2=f(t).$$

Bu moslikka Laplas almashtirishining kechikish xossasini qo‘llaymiz:

$$\frac{e^{-p}}{(p-2)^3}\xrightarrow{\bullet}f(t-1)=e^{2(t-1)}(t-1)^2\eta(t-1).$$

5) $F(p)$ tasvirni sodda kasrlar yig‘indisiga keltiramiz:

$$F(p)=\frac{p+1}{p(p-1)(p-2)(p-3)}=\frac{A}{p}+\frac{B}{p-1}+\frac{C}{p-2}+\frac{D}{p-3}.$$

Bundan

$$A(p-1)(p-2)(p-3)+Bp(p-2)(p-3)+Cp(p-1)(p-3)+Dp(p-1)(p-2)=p+1.$$

Noma’lum koeffitsiyentlarni topamiz:

$$\begin{cases} p=0: & -6A=1,\\ p=1: & 2B=2,\\ p=2: & -2C=3,\\ p=3: & 6D=4, \end{cases}\qquad A=-\frac{1}{6},\ B=1,\ C=-\frac{3}{2},\ D=\frac{2}{3}.$$

Demak,

$$F(p)=-\frac{1}{6}\cdot\frac{1}{p}+\frac{1}{p-1}-\frac{3}{2}\cdot\frac{1}{p-2}+\frac{2}{3}\frac{1}{p-3}.$$

U holda tasvirlar jadvaliga ko‘ra

$$f(t) = -\frac{1}{6} + e^{-t} - \frac{3}{2}e^{-2t} + \frac{2}{3}e^{-3t}.$$

6) $F(p)$ tasvirni ikkita kasrning yig‘indisiga keltiramiz:

$$F(p) = \frac{1}{(p+1)(p^2+4)} = \frac{A}{p+1} + \frac{Bp+C}{p^2+4}.$$

Noma’lum koeffitsiyentlarni

$$A(p^2+4) + (Bp+C)(p+1) = 1$$

ayniyatdan topamiz. O‘zgaruvchining bir xil darajalari oldidagi koeffitsiyentlarni tenglab, topamiz:

$$A + B = 0,\ B + C = 0,\ 4A + C = 1.$$

Bundan $A = \frac{1}{5},\ B = -\frac{1}{5},\ C = \frac{1}{5}.$

Demak,

$$F(p) = \frac{1}{5}\left(\frac{1}{p+1} - \frac{p}{p^2+2^2} + \frac{1}{2}\cdot\frac{2}{p^2+2^2}\right).$$

U holda

$$f(t) = \frac{1}{5}\left(e^{-t} - \cos 2t + \frac{1}{2}\sin 2t\right).$$

Tasvirdan originalga o‘tish tasvirlar jadvali yordamida amalga oshmaydigan hollarda quyidagi usullardan biri qo‘llaniladi.

Birinchi (*umumiy*) *usul* $F(p)$ tasvirga mos $f(t)$ originalni *Riman-Mellin* formulasi yordamida topishdan iborat.

Ikkinchi usul avval $F(p)$ tasvirni ayrim originallarning tasvirlaridan iborat bo‘lgan kasrlar ko‘paytmasiga keltirishdan va keyin $f(t)$ originalni *tasvirlarni ko‘paytirish teoremalari* yordamida topishdan iborat.

Uchinchi usul $F(p)$ tasvirga mos $f(t)$ originalni *yoyish teoremalari* yordamida topishdan iborat.

3.2.1. 1-teorema. $f(t)-$ $t<0$ da $f(t)\equiv 0$, istalgan chekli oraliqda bo‘lakli-silliq, chekli o‘sishga ega funksiya va $F(p) \xrightarrow{\bullet} f(t)$ bo‘lsin.
U holda $f(t)$ differensiallanuvchi bo‘lgan har bir nuqtada

$$f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(p)e^{pt}dp \qquad (2.1)$$

bo‘ladi, bu yerda integral $\operatorname{Re} p = c > s_0$ to‘g‘ri chiziq bo‘ylab olinadi.

(2.1) formulaga *Riman-Mellin formulasi* deyiladi.

2-teorema (*originalning yagonaligi haqidagi teorema*). Agar $F(p)$ funksiya ikkita $f_1(t)$ va $f_2(t)$ originallarning tasviri bo‘lsa, u holda bu originallar o‘zlarining barcha uzliksizlik nuqtalarida ustma-ust tushadi.

(2.1) formuladan foydalanish uchun qoldiqlar nazariyasini qo‘llashga to‘g‘ri keladi. Ko‘p hollarda bu usul bilan originalni topish etarlicha murakkab hisoblash talab qiladi. Shu sababli odatda keltirilgan ikkinchi va uchinchi usullarning foydalanish ko‘rsatkichi yuqoriroq bo‘ladi.

3.2.2. Tasvirlarni ko‘paytirish teoremalari o‘rama tushunchasiga asoslanadi.

$f(t), g(t)-$ $(-\infty;+\infty)$ intervalda aniqlangan haqiqiy o‘zgaruvchi t ning bo‘lakli uzliksiz funksiyalari bo‘lib, $t<0$ da $f(t)=g(t)=0$ bo‘lsin. $f(t)$ va $g(t)$ *funksiyalarning o‘ramasi* deb,

$$\int_0^t f(\tau)g(t-\tau)d\tau \tag{2.2}$$

integralga aytiladi va $f*g(t)$ bilan belgilanadi.

3-teorema (***tasvirlarni ko‘paytirish teoremasi***). Agar $F_1(p)\xrightarrow{\bullet}f_1(t)$ va $F_2(p)\xrightarrow{\bullet}f_2(t)$ bo‘lsa, u holda $F_1(p)F_2(p)\xrightarrow{\bullet}f_1*f_2(t)$ bo‘ladi.

2-misol. $F(p)=\dfrac{1}{(p^2+\omega^2)^2}$ funksiyaning originalini toping.

$F(p)=\dfrac{1}{(p^2+\omega^2)^2}=\dfrac{1}{p^2+\omega^2}\cdot\dfrac{1}{p^2+\omega^2}$ va $\dfrac{1}{p^2+\omega^2}\xrightarrow{\bullet}\dfrac{1}{\omega}\cdot\sin\omega t$ ekanini inobatga olib, tasvirlarni ko‘paytirish teoremasini qo‘llaymiz:

$$F(p)\xrightarrow{\bullet}\int_0^t\frac{1}{\omega}\cdot\sin\omega t\cdot\frac{1}{\omega}\cdot\sin(t-\tau)d\tau=$$

$$=\frac{1}{2\omega^2}\int_0^t(\cos\omega(2\tau-t)-\cos\omega t)d\tau=$$

$$=\frac{1}{2\omega^2}\left(\frac{1}{2\omega}\sin(2\tau-t)\Big|_0^t-\cos\omega t\cdot\tau\Big|_0^t\right)=\frac{1}{2\omega^3}(\sin\omega t-\omega t\cdot\cos\omega t).$$

Demak,

$$\frac{1}{(p^2+\omega^2)^2}\xrightarrow{\bullet}\frac{1}{2\omega^3}(\sin\omega t-\omega t\cdot\cos\omega t).$$

4-teorema (*Dyuamel formulasi*). Agar $F_1(p)F_2(p)\xrightarrow{\bullet}f_1*f_2(t)$ va $f_1'(t)$ original bo‘lsa, u holda $pF_1(p)F_2(p)\xrightarrow{\bullet}\int_0^t f_1'(\tau)f_2(t-\tau)d\tau+f_1(0)f_2(t)$ bo‘ladi.

3-misol. $F(p)=\dfrac{2p^2}{(p^2+1)^2}$ funksiyaning originalini toping.

$$F(p)=\frac{2p^2}{(p^2+1)^2}=2p\cdot\frac{1}{p^2+1}\cdot\frac{p}{p^2+1} \text{ va } \frac{1}{p^2+1}\xrightarrow{\bullet}\sin t,\quad \frac{p}{p^2+1}\xrightarrow{\bullet}\cos t,$$

ekanini inobatga olib, Dyuamel formulasi bilan topamiz:

$$F(p)=2p\cdot\frac{1}{p^2+1}\cdot\frac{p}{p^2+1}\xrightarrow{\bullet}2\int_0^t\cos\tau\cos(t-\tau)d\tau+0=t\cos t+\sin t.$$

$$=\int_0^t\cos t d\tau+\int_0^t\cos(2\tau-t)d\tau=\cos t\cdot\tau\Big|_0^t+\frac{1}{2}\sin(2\tau-t)\Big|_0^t=t\cos t+\sin t.$$

3.2.3. 5-teorema (*birinchi yoyish teoremasi*). Agar $F(p)$ funksiya $p=\infty$ nuqtaning atrofida

$$F(p)=\sum_{n=0}^{\infty}\frac{c_n}{p^{n+1}}=\frac{c_0}{p}+\frac{c_1}{p^2}+\frac{c_2}{p^2}+\ldots$$

ko‘rinishda Loran qatori bilan berilgan bo‘lsa, u holda

$$f(t)=\sum_{n=0}^{\infty}c_n\cdot\frac{t^n}{n!}=c_0+c_1t+\ldots$$

$F(p)$ tasvirning originali bo‘ladi, ya’ni

$$F(p)=\sum_{n=0}^{\infty}\frac{c_n}{p^{n+1}}\xrightarrow{\bullet}\sum_{n=0}^{\infty}c_n\cdot\frac{t^n}{n!}=f(t).$$

4-misol. $F(p)=\dfrac{1}{p}\cos\dfrac{1}{p}$ funksiyaning originalini toping.

$F(p)$ funksiyani $\dfrac{1}{p}$ ning darajalari bo‘yicha qatorga yoyamiz:

$$F(p)=\frac{1}{p}\cos\frac{1}{p}=\frac{1}{p}\left(1-\frac{1}{2!p^2}+\frac{1}{4!p^4}-\frac{1}{6!p^6}+\ldots\right)=$$

$$=\frac{1}{p}-\frac{1}{2!p^3}+\frac{1}{4!p^5}-\frac{1}{6!p^7}+\ldots.$$

Birinchi yoyish teoremasini qo‘llab, topamiz:

$$f(t)=1-\frac{t^2}{2!2!}+\frac{t^4}{4!4!}-\frac{t^6}{6!6!}+\ldots.=\sum_{n=0}^{\infty}(-1)^n\frac{t^{2n}}{((2n)!)^2}.$$

6-teorema. Agar $F(p)=\dfrac{A(p)}{B(p)}$ to‘g‘ri ratsional kasr bo‘lib, uning $B(p)$

maxraji faqat oddiy $p_1, p_2, ..., p_n$ ildizlarga (nollarga) ega bo‘lsa, u holda

$$f(t) = \sum_{k=1}^{n} \frac{A(p_k)}{B'(p_k)} e^{p_k t} \tag{2.3}$$

funksiya $F(p)$ tasvirninig originali bo‘ladi.

5-misol. $F(p) = \dfrac{p^2 + 1}{p(p+1)(p-2)(p+3)}$ funksiyaning originalini toping.

Berilgan tasvir maxrajining barcha ildizlari haqiqiy va oddiy: $p_1 = 0$, $p_2 = -1$, $p_3 = 2$, $p_4 = -3$. Shu sababli (2.3) formulani qo‘llaymiz.

Bunda $A(p) = p^2 + 1$, $B(p) = p^4 + 2p^3 - 5p^2 - 6p$, $B'(p) = 4p^3 + 6p^2 - 10p - 6$.

U holda

$$\frac{A(p_1)}{B'(p_1)} = \frac{1}{-6} = -\frac{1}{6}, \quad \frac{A(p_2)}{B'(p_2)} = \frac{2}{6} = \frac{1}{3}, \quad \frac{A(p_3)}{B'(p_3)} = \frac{5}{30} = \frac{1}{6}, \quad \frac{A(p_4)}{B'(p_{14})} = \frac{10}{-30} = -\frac{1}{3}.$$

Bundan (2.3) formulaga ko‘ra

$$f(t) = -\frac{1}{6} + \frac{1}{3}e^{-t} + \frac{1}{6}e^{2t} - \frac{1}{3}e^{3t}.$$

Izoh. $f(t) = \sum_{k=1}^{n} \dfrac{A(p_k)}{B'(p_k)} e^{p_k t}$ formulada $c_k (k = 1,2,...,n)$ koeffitsiyentlar $F(p)$ funksiyaning oddiy qutblardagi qoldig‘i bo‘lishini ko‘rsatish mumkin, ya’ni

$$c_k = \frac{A(p_k)}{B'(p_k)} = \mathrm{Re}s F(p_k), \ k = 2,3,...n.$$

Agar $F(p) = \dfrac{A(p)}{B(p)}$ to‘g‘ri ratsional kasr bo‘lib, uning $B(p)$ maxraji $m_1, m_2, ..., m_n$ karrali $p_1, p_2, ..., p_n$ ildizlarga (nollarga) ega bo‘lsa, u holda

$$f(t) = \sum_{k=1}^{n} \frac{1}{(m_k - 1)!} \lim_{p \to p_k} \left(\frac{A(p)}{B(p)} e^{pt} (p - p_k)^{m_k} \right)^{(m_k - 1)} \tag{2.4}$$

bo‘ladi.

7-teorema (*ikkinchi yoyish teoremasi*). Agar $F(p) = \dfrac{A(p)}{B(p)}$ tasvir p ning kasr ratsional funksiyasi bo‘lib, $p_1, p_2, ..., p_n$ lar bu funksiyaning oddiy yoki karrali qutblari bo‘lsa, u holda $F(p)$ tasvirga mos original

$$F(p) = \frac{A(p)}{B(p)} \xrightarrow{\bullet} \sum_{k=1}^{n} \mathrm{Re}s(F(p_k) \cdot e^{p_k t}) = f(t) \tag{2.5}$$

formula bilan topiladi.

6-misol. $F(p)=\dfrac{1}{(p^2+4)^2}$ funksiyaning originalini toping.

Berilgan tasvir to‘g‘ri kasr funksiya. U ikkita ikki karrali $2i$ va $-2i$ qutbga ega. Shu sababli (2.4) formulani qo‘llaymiz:

$$f(t)=\frac{1}{(2-1)!}\lim_{p\to 2i}\frac{d}{dp}\left(\frac{(p-2i)^2e^{pt}}{(p-2i)^2(p+2i)^2}\right)+\frac{1}{(2-1)!}\lim_{p\to -2i}\frac{d}{dp}\left(\frac{(p+2i)^2e^{pt}}{(p-2i)^2(p+2i)^2}\right)=$$

$$=\lim_{p\to 2i}\frac{d}{dp}\left(\frac{e^{pt}}{(p+2i)^2}\right)+\lim_{p\to -2i}\frac{d}{dp}\left(\frac{e^{pt}}{(p-2i)^2}\right)=\lim_{p\to 2i}\frac{d}{dp}\left(\frac{-2e^{pt}}{(p+2i)^3}+\frac{te^t}{(p+2i)^2}\right)+$$

$$+\lim_{p\to -2i}\frac{d}{dp}\left(\frac{-2e^{pt}}{(p-2i)^3}+\frac{te^t}{(p-2i)^2}\right)=-\frac{2e^{2it}}{(4i)^3}+\frac{te^{2it}}{(4i)^2}-\frac{2e^{-2it}}{(-4i)^3}+\frac{te^{-2it}}{(-4i)^2}=$$

$$=\frac{1}{64}(-2i\cos 2t+2\sin 2t+4t(-\cos 2t-i\sin 2t))+$$

$$+\frac{1}{64}(2i\cos 2t+2\sin 2t+4t(-\cos 2t+i\sin 2t))=\frac{1}{8}(\sin 2t-t\cos 2t).$$

Mashqlar

3.2.1. Berilgan tasvirlarning originallarini toping:

1) $F(p)=\dfrac{2e^{-p}}{p^3}$;

2) $F(p)=\dfrac{e^{-3p}}{p+4}$

3) $F(p)=\dfrac{3p-2}{p^2-4}$;

4) $F(p)=\dfrac{p}{p^2+2p+2}$;

5) $F(p)=\dfrac{1}{p+2p^2+p^3}$;

6) $F(p)=\dfrac{1}{p^2(p^2+1)}$;

7) $F(p)=\dfrac{2p+3}{5p+4p^2+p^3}$;

8) $F(p)=\dfrac{1}{(p-1)(p^2-4)}$;

9) $F(p)=\dfrac{2p^3+p^2+2p+2}{p^5+2p^4+2p^3}$;

10) $F(p)=\dfrac{p}{p^3+1}$;

11) $F(p)=\dfrac{4e^{-3p}}{p^2+6p+10}$;

12) $F(p)=\dfrac{e^{-p}+e^{-2p}}{(p-2)^2}$;

13) $F(p)=\dfrac{2p^3+p^2+2p-1}{p^4-1}$;

14) $F(p)=\dfrac{p^3+p^2-1}{p^4-p^2}$;

15) $F(p)=\dfrac{p^3-2p-1}{p^3-3p^2+3p-1}$;

16) $F(p)=\dfrac{p}{(p^2-4)(p^2+1)}$.

3.2.2. Berilgan tasvirlarning originallarini tasvirlarni ko'paytirish teoremasi bilan toping:

1) $F(p)=\dfrac{p}{p^4-1}$; 2) $F(p)=\dfrac{1}{p^3(p-1)}$;

3) $F(p)=\dfrac{1}{(p^2+1)(p^2+9)}$; 4) $F(p)=\dfrac{p}{p^3(p^2+1)}$.

3.23. Berilgan tasvirlarning originallarini Dyuamel formulasi bilan toping:

1) $F(p)=\dfrac{p^3}{(p^2+1)^2}$; 2) $F(p)=\dfrac{p}{(p^2+1)(p^2+4)}$;

3) $F(p)=\dfrac{p}{(p-1)(p^2+1)}$; 4) $F(p)=\dfrac{p}{(p^2+1)(p^2+2p+2)}$.

3.2.4. Berilgan tasvirlarning originallarini birinchi yoyish teoremasi bilan toping:

1) $F(p)=\dfrac{1}{p}e^{-\frac{1}{p}}$; 2) $F(p)=\dfrac{1}{p(p^4+1)}$;

3) $F(p)=\sin\dfrac{1}{p}$; 4) $F(p)=\dfrac{p}{(p^2+1)}$;

5) $F(p)=\ln\left(1+\dfrac{1}{p}\right)$; 6) $F(p)=\dfrac{1}{p}e^{\frac{1}{p^2}}$.

3.2.5. Berilgan tasvirlarning originallarini ikkinchi yoyish teoremasi bilan toping:

1) $F(p)=\dfrac{1}{p^2-4p+3}$; 2) $F(p)=\dfrac{p^2+2}{p^3-p^2-6p}$;

3) $F(p)=\dfrac{1}{(p-1)(p^2-4)}$; 4) $F(p)=\dfrac{p}{p^4-5p^2+4}$;

5) $F(p)=\dfrac{1}{p(p-1)(p-2)(p-3)}$; 6) $F(p)=\dfrac{p+3}{p(p-1)(p-3)}$;

7) $F(p)=\dfrac{1}{p^3(p-1)}$; 8) $F(p)=\dfrac{1}{p^2(p-1)^2}$.

3.3. OPERATSION HISOBNING TATBIQLARI

Differensial tenglamalar va ularning sistemalarini yechish. Operatsion hisobning elektr zanjirlarini hisoblashlarga tatbiqi

3.3.1. Laplash almashtirishlari yordamida ayrim differensial tenglamalar (yoki ularning sistemalari) algebraik tenglamalarga keltiriladi. Bu algebraik tenglamalardan berilgan differensial tenglama (yoki sistema) yechimining tasviri topiladi va keyin topilgan tasvirga ko‘ra berilgan tenglamaning (yoki sistemaning) yechimi aniqlanadi.

O‘zgarmas koeffitsiyentli, chiziqli $n-$tartibli

$$y^{(n)}+a_1y^{(n-1)}+...+a_ny=f(t) \tag{3.1}$$

differensial tenglamaning

$$y(0)=y_0,\ y'(0)=y'_0,...,y^{(n-1)}(0)=y_0^{(n-1)} \tag{3.2}$$

boshlang‘ich shartlarni qanoatlantiruvchi $y(t)$ yechimini topish talab qilingan bo‘lsin.

$y(t)\overset{\bullet}{\longleftarrow}Y(p)=Y$ va $f(t)\overset{\bullet}{\longleftarrow}F(p)=F$ bo‘lsa, Laplas almashtirishining originalni differensiallash va chiziqlilik xossalariga ko‘ra

$$Y(p)=\frac{F(P)+R_{n-1}(p)}{Q_n(p)} \tag{3.3}$$

bo‘ladi. Bunda $Q_n(p),R_{n-1}(p)-$mos ravishda n va $n-1$ darajali ko‘phadlari.

(3.3) tenglamadan $Y(p)$ tasvirga mos $y(t)$ original aniqlanadi, ya’ni (3.1) tenglamaning (3.2) boshlang‘ich shartlarni qanoatlantiruvchi xususiy yechimi topiladi.

Xususan, agar barcha boshlang‘ich shartlar nolga teng, ya’ni

$$y(0)=y'(0)=...=y^{(n-1)}(0)=0$$

bo‘lsa, (3.3) tenglama

$$Y(p)=\frac{F(P)}{Q_n(p)} \tag{3.4}$$

ko‘rinishda bo‘ladi.

1-misol. Koshi masalasini yeching:

1) $y'+3y=e^t,\ y(0)=1;$

2) $y''+2y'+y=te^{-t},\ y(0)=1,y'(0)=2;$

3) $y''-2y'+2y=2e^t\cos t,\ y(0)=y'(0)=0;$

4) $y''' + 3y'' + 3y' + y = e^{-t}$, $y(0) = 1, y'(0) = y'' = 0.$

1) Misolning shartiga ko‘ra

$$Q_1(p) = p + 3, \quad R_0(p) = y(0) \cdot 1 = 1, \quad F(P) = \frac{1}{p-1}.$$

U holda (3.3) formuladan topamiz:

$$Y(p) = \frac{\frac{1}{p-1} + 1}{p+3} = \frac{p}{(p-1)(p+3)}.$$

$Y(p)$ tasvirni sodda kasrlar yig‘indisiga keltiramiz:

$$Y(p) = \frac{p}{(p-1)(p+3)} = \frac{A}{p-1} + \frac{B}{p+3}.$$

Bundan

$$A(p+3) + B(p-1) = p.$$

Noma’lum koeffitsiyentlarni topamiz:

$$\begin{cases} p = -3: \ -4B = -3, \\ p = 1: \ 4A = 1, \end{cases} \qquad A = \frac{1}{4}, \ B = \frac{3}{4}.$$

Demak,

$$Y(p) = \frac{1}{4} \cdot \frac{1}{p-1} + \frac{3}{4} \cdot \frac{1}{p+2}.$$

U holda tasvirlar jadvaliga ko‘ra

$$y(t) = \frac{1}{4} e^{t} + \frac{3}{4} e^{-3t}.$$

2) Misolning shartidan topamiz:

$$Q_2(p) = p^2 + a_1 p + a_2 = p^2 + 2p + 1 = (p+1)^2,$$

$$R_1(p) = y(0) \cdot (p + a_1) + y'_0 = p + 2 + 2 = p + 4, \quad F(P) = \frac{1}{(p+1)^2}.$$

U holda (3.3) formulaga ko‘ra

$$Y(p) = \frac{\frac{1}{(p+1)^2} + (p+4)}{(p+1)^2} = \frac{1}{(p+1)^4} + \frac{p+4}{(p+1)^2} =$$

$$= \frac{1}{(p+1)^4} + \frac{p+1+3}{(p+1)^2} = \frac{1}{p+1} + \frac{3}{(p+1)^2} + \frac{1}{(p+1)^4}.$$

Demak,

$$y(t) = e^{-t} + 3te^{-t} + \frac{1}{6} t^3 e^{-t}.$$

3) Misolning berilishiga ko‘ra barcha boshlang‘ish shartlar nolga teng va

$$Q_2(p) = p^2 + a_1 p + a_2 = p^2 - 2p + 2 = (p-1)^2 + 1, \qquad F(P) = 2\frac{p-1}{(p-1)^2+1}.$$

U holda (3.4) formulaga ko‘ra

$$Y(p) = \frac{2\dfrac{p-1}{(p-1)^2+1}}{(p-1)^2+1} = \frac{2(p-1)}{((p-1)^2+1)^2}.$$

Tasvirlar jadvaliga va Laplas almashtirishining siljish xossasiga ko‘ra

$$y(t) = te^t \sin t.$$

3) Bu misolda barcha boshlang‘ish shartlar nolga teng va

$$Q_3(p) = p^3 + a_1 p^2 + a_2 p + a_3 = p^3 + 3p^2 + 3p + 1 = (p+1)^3, \qquad F(P) = \frac{1}{p+1}.$$

U holda (3.4) formulaga ko‘ra $Y(p) = \dfrac{1}{(p+1)^4}.$

Bundan

$$y(t) = \frac{1}{6}t^3 e^{-t}.$$

Differensial tenglamalarni yechishda operatsion hisobning yoqorida keltirilgan usuli differensial tenglamaning o‘ng tomonidagi tasvirlar Laplas almashtirishining ta’rifi va xossalari yordamida oson topiladigan hollarda qo‘llaniladi. Differensial tenglamaning o‘ng tomonidagi tasvirlarni topish murakkab bo‘lgan hollarda Koshi masalasini Dyuamel integrali yordamida yechish mumkin bo‘ladi.

O‘zgarmas koeffitsiyentli, chiziqli $n-$tartibli

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = f(t) \tag{3.5}$$

differensial tenglamaning

$$y(0) = y'(0) = \ldots = y^{(n-1)}(0) = 0$$

boshlang‘ich shartlarni qanoatlantiruvchi $y(t)$ yechimini topish masalasi qo‘yilgan bo‘lsin.

Bu masala yordamchi masala, ya’ni chap tomoni (3.5) tenglamaning chap tomoni bilan bir xil bo‘lgan va o‘ng tomoni birga teng

$$z^{(n)} + a_1 z^{(n-1)} + \ldots + a_n z = 1 \tag{3.7}$$

differensial tenglamaning

$$z(0) = z'(0) = \ldots = z^{(n-1)}(0) = 0$$

boshlang‘ich shartlarni qanoatlantiruvchi $z(t)$ yechimini topish masalasi bilan birgalikda yechiladi va

$$Y(p)=\frac{F(P)}{Q_n(p)}, \quad Z(p)=\frac{1}{pQ_n(p)}, \tag{3.8}$$

hosil qilinadi, bu yerda $Q_n(p)=p^n+a_1p^{n-1}+...+a_{n-1}p+a_0$.

(3.8) formula yechimlaridan

$$Y(p)=pF(p)Z(p)$$

kelib chiqadi. Bu tenglikka Dyuamel formulasini qo‘llansa

$$pF(p)Z(p)=Y(p)\overset{\bullet}{\longrightarrow} y(t)=\int_0^t f(\tau)z'(t-\tau)d\tau \tag{3.9}$$

yoki

$$pF(p)Z(p)=Y(p)\overset{\bullet}{\longrightarrow} y(t)=\int_0^t z(\tau)f'(t-\tau)d\tau=f(0)z(t). \tag{3.10}$$

bo‘ladi.

(3.9) formula (yoki (3.10) formula) o‘zgarmas koeffitsiyentli chiziqli differensial tenglama uchun boshlang‘ich shartlari nollardan iborat bo‘lgan Koshi masalasi yechimini tenglamaning o‘ng tomonidagi $f(t)$ funksiyaning tasvirini topmasdan aniqlash imkonini beradi.

2-misol. $y''-y'=\dfrac{e^{2t}}{(1+e^t)^2}$ differensial tenglamaning $y(0)=y'(0)=0$ boshlang‘ich shartni qanoatlantiruvchi yechimini toping.

⬬ 1) Yordamchi Koshi masalasini qaraymiz:

$$z''-z'=1, \quad z(0)=z'(0)=0.$$

Bu masala uchun: $Q_1=p^2-p,\ 1\overset{\bullet}{\longleftarrow}\dfrac{1}{p}.$

U holda

$$Z(p)=\frac{1}{p(p^2-p)}=\frac{1}{p^2(p-1)}=\frac{A}{p}+\frac{B}{p^2}+\frac{C}{p-1}.$$

Bundan

$$Ap(p-1)+B(p-1)+Cp^2=1.$$

Noma’lum koeffitsiyentlarni topamiz:

$$\begin{cases} p=0: \ -B=1, \\ p=1: \quad C=1, \\ p^2: \ A+C=0, \end{cases} \qquad A=-1,\ B=1,\ C=1.$$

Demak,

$$Z(p) = -\frac{1}{p} + \frac{1}{p^2} + \frac{1}{p-1}.$$

U holda tasvirlar jadvaliga ko‘ra

$$z(t) = -1 - t + e^t.$$

(3.9) formulaga ko‘ra berilgan Koshi masalasining yechimini topamiz:

$$y(t) = \int_0^t \frac{e^{2\tau}}{(1+e^\tau)^2}(-1 - t + e^t)'\Big|_{t-\tau} d\tau = \int_0^t \frac{e^{2\tau}}{(1+e^\tau)^2}(e^{t-\tau} - 1)d\tau =$$

$$= e^t \int_0^t \frac{e^\tau}{(1+e^\tau)^2}d\tau - \int_0^t \frac{e^{2\tau}}{(1+e^\tau)^2}d\tau = \begin{pmatrix} 1 + e^\tau = u,\ du = e^\tau d\tau, \\ \tau = t\ da\ u = 1 + e^t,\ \tau = 0\ da\ u = 2 \end{pmatrix} =$$

$$= e^t \int_2^{1+e^t} \frac{du}{u^2} - \int_2^{1+e^t} \frac{u-1}{u^2}du = -e^t \frac{1}{u}\Bigg|_2^{1+e^t} - \left(\ln u + \frac{1}{u}\right)\Bigg|_2^{1+e^t} = \frac{1}{2}(e^t - 1) - \ln\left(\frac{1+e^t}{2}\right).$$

Agar boshlang‘ich shartlari nollardan iborat bo‘lmagan Koshi masalasi berilgan bo‘lsa, u holda bu masala avval sodda o‘rniga qo‘yishlar yordamida izlanayotgan funksiyani boshqa funksiyaga almashtirish orqali boshlang‘ich shartlari nollardan iborat bo‘lgan Koshi masalasiga keltiriladi va keyin yechiladi. Masalan, $y'' - y' = \dfrac{e^{2t}}{(1+e^t)^2} - 2,\ y(0) = 1,\ y'(0) = 2$ Koshi masalasi berilgan bo‘lsin. Bu masaladan

$$x(t) = y(t) - y(0) - ty'(0), \qquad x'(t) = y'(t) - y'(0), \qquad x''(t) = y''(t)$$

almashtirishlar orqali

$$x'' - x' = \frac{e^{2t}}{(1+e^t)^2},\ x(0) = x'(0) = 0$$

Koshi masalasi hosil qilinadi.

O‘zgarmas koeffitsiyentli chiziqli differensial tenglamalar sistemasini operatsion hisob usuli bilan yechish tartibi bitta differensial tenglamani bu usul bilan yechish tartibiga o‘xshash bo‘ladi. Bunda Laplas almashtirishi qo‘llanilgandan keyin izlanayotgan funksiyalarning tasvirlariga nisbatan chiziqli algebraik operator tenglamalar sistemasi hosil bo‘ladi. Avval operator tenglamalar sistemasini yechib, tasvirlar aniqlanadi va keyin tasvirlardan originallarga o‘tib, differensial tenglamalar sistemasining izlanayotgan yechimi topiladi.

3-misol. Chiziqli differensial tenglamalar sistemasini berilgan boshlang‘ich shartlarda yeching:

1) $\begin{cases} x'' - y' = 0, \\ x' - y'' = 2\cos t, \end{cases}$ $x(0) = y'(0) = 0,\ x'(0) = y(0) = 2;$

2) $\begin{cases} x' = y - z, \\ y' = x + y, \\ z' = x + z, \end{cases}$ $x(0) = 1,\ y(0) = 2,\ z(0) = 3;$

1) $x(t) \overset{\bullet}{\longleftarrow} X(p)$ va $y(t) \overset{\bullet}{\longleftarrow} Y(p)$ bo‘lsin.

U holda $x' \overset{\bullet}{\longleftarrow} pX - x(0) = pX, \quad x'' \overset{\bullet}{\longleftarrow} p^2 X - px(0) - x'(0) = p^2 X - 2,$
$y' \overset{\bullet}{\longleftarrow} pY - y(0) = pY - 2, \quad y'' \overset{\bullet}{\longleftarrow} p^2 Y - py(0) - y'(0) = p^2 Y - 2p,$

$\cos t \overset{\bullet}{\longleftarrow} \dfrac{p}{p^2+1}$ larni hisobga olib, berilgan sistemada $X(p)$ va $Y(p)$ ga nisbatan chiziqli operator tenglamalar sistemasiga o‘tamiz:

$$\begin{cases} p^2 X - 2 - pY + 2 = 0, \\ pX - p^2 Y + 2p = 2\dfrac{p}{p^2+1} \end{cases}$$

Bu algebraik tenglamalar sistemasini yechib, topamiz:

$$X(p) = \frac{2p^2}{p^4-1} = \frac{1}{p^2-1} + \frac{1}{p^2+1}, \quad Y(p) = \frac{2p^3}{p^4-1} = \frac{p}{p^2-1} + \frac{p}{p^2+1}.$$

Bundan tasvirlar jadvaliga ko‘ra

$$x(t) = sht + \sin t,\ y(t) = cht + \cos t.$$

2) $x(t) \overset{\bullet}{\longleftarrow} X(p)$, $y(t) \overset{\bullet}{\longleftarrow} Y(p)$ va $z(t) \overset{\bullet}{\longleftarrow} Z(p)$ bo‘lsin.

Bundan

$$x' \overset{\bullet}{\longleftarrow} pX - 1, \quad y' \overset{\bullet}{\longleftarrow} pY - 2, \quad z' \overset{\bullet}{\longleftarrow} pZ - 3.$$

Berilgan sistemani $X(p)$, $Y(p)$ va $Z(p)$ga nisbatan chiziqli operator tenglamalar sistemasiga keltiramiz:

$$\begin{cases} PX - Y - Z = 1, \\ X - (p-1)Y = -2, \\ X + (1-p)Z = -3. \end{cases}$$

Bundan

$$X(p) = \frac{p-2}{p(p-1)} = \frac{2}{p} - \frac{1}{p-1}, \quad Y(p) = \frac{2p^2 - p - 2}{p(p-1)^2} = -\frac{2}{p} + \frac{4}{p-1} - \frac{1}{(p-1)^2},$$

$$Z(p)=\frac{3p^2-2p-2}{p(p-1)^2}=-\frac{2}{p}+\frac{5}{p-1}-\frac{1}{(p-1)^2}.$$

Demak,

$$x(t)=2-e^t,\ y(t)=-2+4e^t-te^t,\ z(t)=-2+5e^t-te^t.$$

Opetatsion hisob yuqorida keltirilgan differensial tenglamalarni (yoki ularning sistemalarini) yechish usullari orqali bu tenglamalar (yoki sistemalar) bilan ifodalanuvchi texnika jarayonlariga, ya'ni texnika masalalarini yechishga tatbiq qilinadi.

3.3.2. Operatsion hisob usullari elektr zanjirlaridagi jarayonlarni hisoblashda keng qo'llaniladi. $i(t)$ va $u(t)$ mos ravishda elektr zanjiridagi tok va kuchlanish bo'lsin. Bunda operatsion hisob usulining qo'llanishi operator tok $I(p)\overset{\bullet}{\longrightarrow}i(t)$ va operator kuchlanish $Up)\overset{\bullet}{\longrightarrow}u(t)$ uchun Kirxgof qonunining bajarilishiga asoslanadi.

Om qonuniga ko'ra elektr zanjirining asosiy elementlari (R-qarshilik, U-induktivlik, C-sig'im) uchun quyidagi munosabatlarni yozish mumkin:

$$u_R(t)=Ri(t);\quad u_L(t)=L\frac{di(t)}{dt};\quad u_C(t)=\frac{1}{C}\int_0^t i(\tau)d\tau+u_C(0).$$

yoki

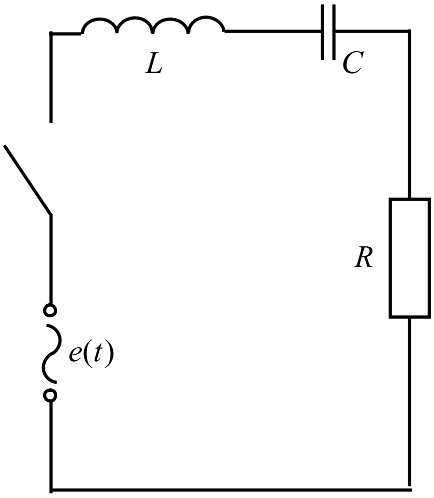
$$U_R(p)=RI(p);\quad U_L(t)=pLI(p)-Li(0);\quad u_C(t)=\frac{1}{pC}I(p)+\frac{1}{p}u_C(0).$$

Om qonunini zanjirning istalgan qismi uchun operator shaklida

$$U(p)=Z(p)I(p) \qquad (3.11)$$

kabi yozish mumkin, bu yerda $Z(p)-$zanjirning bu qismi uchun operator qarshilik.

Elektr zanjiridagi tebranishlar haqida masala. 1-shaklda berilgan zanjir uchun $i(t)$ tokni toping, bu yerda zanjir konturiga $e(t)=E$ o'zgarmas elektr yurituvchi kuch ulangan. Bunda boshlang'ich paytda konturdagi tok va kondensator zaryadi nolga teng. Zanjirdagi tokni toping.



1-shakl.

$e(t)=E\overset{\bullet}{\longleftarrow}\frac{1}{p}E$ ni hisobga olib, (3.11) tenglikdan topamiz:

$$Z(p)I(p)=\frac{E}{p}. \qquad (3.12)$$

Bunda 3-shaklda keltirilgan zanjir konturi uchun nolga teng boshlang‘ich shartlarda $Z(p)$ operator qarshilikni topamiz:

$$Z(p) = Z_L(p) + Z_C(p) + Z_R(P) = Lp + \frac{1}{Cp} + R$$

$Z(p)$ni (3.12) tenglikka qo‘yamiz:

$$I(p) = \frac{E}{pZ(p)} = \frac{E}{Lp^2 + Rp + \dfrac{1}{C}} = \frac{E}{L}\frac{1}{\left(p + \dfrac{L}{2R}\right)^2 + \left(\dfrac{1}{LC} - \dfrac{R^2}{4L^2}\right)}. \qquad (3.13)$$

$i(t)$ originalni topish uchun (3.13) tenglikning o‘ng tomonidagi kvadrat uchhad ildizlarining ko‘rinishiga bog‘liq bo‘lgan uchta holni ko‘rib chiqishga to‘g‘ri keladi.

1-hol. $\dfrac{1}{LC} - \dfrac{R^2}{4L^2} = \alpha^2 > 0$ bo‘lsin. Bu holda tasvirdan originalga o‘tib $i(t)$ tok uchun so‘nuvchi elektr tebranishlarini ifodalovchi

$$i(t) = \frac{E}{L\alpha} e^{-\frac{R}{2L}t} \sin \alpha t$$

yechimni hosil qilamiz.

2-hol. $\dfrac{R^2}{4L^2} - \dfrac{1}{LC} = \beta^2 > 0$ bo‘lsin. Bu holda

$$i(t) = \frac{E}{L\beta} e^{-\frac{R}{2L}t} sh\beta t$$

tok davriy bo‘lmaydi va zanjirda hech qanday tebranish sodir bo‘lmaydi.

3-hol. $\dfrac{1}{LC} - \dfrac{R^2}{4L^2} = 0$ bo‘lsin. Bu holda

$$i(t) = \frac{E}{L} t e^{-\frac{R}{2L}t}.$$

Demak, zanjirda hech qanday tebranishlar bo‘lmaydi.

Agar elektr sxemasiga ixtiyoriy ko‘rinishdagi $e(t)$ funksiya ta’sir qilayotgan bo‘lsa, u holda elektr zanjiridagi hisoblashlar uchun Dyuamel integralini qo‘llash yaxshi natija beradi. Bunda avval (3.11) formuladan sxemaga birlik $e(t) = \eta(t)$ funksiya ta’sir qilgandagi tok o‘zgarishi aniqlanadi:

$$I_1(p) = \frac{1}{pZ(p)}.$$

Keyin ixtiyoriy $e(t)$ funksiya ta’sir qilgandagi tok o‘zgarishi

$$I(p) = \frac{U(p)}{Z(p)} = pI_1(p)U(p)$$

fornula bilan aniqlanadi, bu yerda $U(p) \overset{\bullet}{\longrightarrow} e(t)$. Bu tenglikka Dyuamel formulasini qo'llansa

$$i(t) = e(0)i_1(t) + \int_0^t e'(\tau)i_1(t-\tau)d\tau = e(0)i_1(t) + \int_0^t e'(t-\tau)i_1(\tau)d\tau \qquad (3.14)$$

bo'ladi.

Mashqlar

3.3.1. Koshi masalasini yeching:

1) $y' + y = e^t, \ y(0) = 0;$

2) $y' + y = 2\cos t, \ y(0) = 0;$

3) $y'' + y' - 2y = 1, \ y(0) = 0, \ y'(0) = 2;$

4) $y'' - y' - 6y = 4, \ y(0) = 1, \ y'(0) = 0;$

5) $y'' + y' - 2y = e^{-t}, \ y(0) = 0, \ y'(0) = 1;$

6) $y'' - 2y' + y = e^t, \ y(0) = 0, \ y'(0) = 1;$

7) $y'' + y = \cos t, \ y(0) = -1, \ y'(0) = 1;$

8) $y'' + 2y' + y = \sin t, \ y(0) = 0, \ y'(0) = -1;$

9) $y'' - y = 2sht, \ y(0) = 0, \ y'(0) = 1;$

10) $y'' + y' = cht, \ y(0) = 0, \ y'(0) = 0;$

11) $y''' - y'' = e^t, \ y(0) = 1, \ y'(0) = 0, \ y''(0) = 0;$

12) $y''' - 3y'' + 3y' - y = 0, \ y(0) = 1, \ y'(0) = 0, \ y''(0) = 0.$

3.3.2. Diggerensial tenglamalarning umumiy yechimini toping:

1) $y'' + 2y' = te^{-2t};$ 2) $y'' + y' = e^{-t}\sin t.$

3.3.3. Koshi masalasini Dyuamel formulasidan foydalanib yeching:

1) $y'' + y' = \cos t, \ y(0) = y'(0) = 0;$ 2) $y''' - y' = \cos t, \ y(0) = y'(0) = y''(0) = 0;$

3) $y'' - y' = te^t, \ y(0) = y'(0) = 0;$ 4) $y'' - y' = \dfrac{e^{2t}}{2+e^t}, \ y(0) = y'(0) = 0.$

3.3.4. Chiziqli differensial tenglamalar sistemasini berilgan boshlang'ich shartlarda yeching:

1) $\begin{cases} x' - 3x - 4y = 0, \\ y' - 4x + 3y = 0, \end{cases} x(0) = 1, \ y(0) = 1;$ 2) $\begin{cases} x' - \ y + x = 0, \\ y' + 3y + x = 0, \end{cases} x(0) = 1, \ y(0) = 1;$

3) $\begin{cases} x' + y = 2e^t, \\ y' + x = 2e^t, \end{cases} x(0) = 1, \ y(0) = 1;$ 4) $\begin{cases} x' + x - y = e^t, \\ y' + y - x = e^t, \end{cases} x(0) = 1, \ y(0) = 1;$

5) $\begin{cases} x'' - y' = e^t, \\ x' + y'' - y = 0, \end{cases}$ $x(0) = y(0) = x'(0) = y'(0) = 0;$

6) $\begin{cases} 2x'' + x - y' = -3\sin t, \\ x + y' = -\sin t, \end{cases}$ $x(0) = 0,\ x'(0) = 1,\ y(0) = 0;$

7) $\begin{cases} x' = y + z, \\ y' = x + z, \\ z' = x + y, \end{cases}$ $x(0) = -1,\ y(0) = 1,\ z(0) = 0;$

8) $\begin{cases} x' + y = t, \\ y' + z = t^2 + 1, \\ z' + x = 2t + 1, \end{cases}$ $x(0) = 1,\ y(0) = 0,\ z(0) = 0.$

3.3.5. RL zanjir konturiga (R qarshilik va L induktivlik ketma-ket ulangan) $e(t) = E$ o‘zgarmas elektr yurituvchi kuch ulangan. Agar boshlang‘ich paytda konturdagi tok va kondensator zaryadi nolga teng bo‘lsa, zanjirdagi tokni toping.

3.3.6. RC zanjir konturiga (R qarshilik va C sig‘im ketma-ket ulangan) $e(t) = e^{\mu t}$ elektr yurituvchi kuch ulangan. Agar boshlang‘ich paytda konturdagi tok va kondensator zaryadi nolga teng bo‘lsa, zanjirdagi tokni toping.

4-NAZORAT ISHI

1. Berilgan funksiyaning tasvirini toping.
2. Funksiyaning berilgan tasviriga ko‘ra originalini toping.
3. Differensial tenglamaning berilgan boshlang‘ich shartlarni qanoatlantiruvchi yechimini operatsion hisob usulli bilan toping.
4. Differensial tenglamalar sistemasini operatsion hisob usulli bilan yeching.

1-variant

1. $f(t) = \dfrac{1 - ch4t}{t}.$ **2.** $F(p) = \dfrac{2p^2 + 5p + 5}{(p+2)(p^2 + 2p - 3)}.$

3. $y'' - 9y = \sin t - \cos t,\ y(0) = -3, y'(0) = 2.$ **4.** $\begin{cases} x' + x - y = e^t, \\ 2x' + y' + 2y = 0, \end{cases}$ $x(0) = 0, y(0) = 0.$

2-variant

1. $f(t)=4t^3-\dfrac{6}{7}e^{-2t}+12.$

2. $F(p)=\dfrac{2p^2+16p+9}{(p-2)(p^2+4p+4)}.$

3. $y''-2y'+5y=1-t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'+x+y=e^t, \\ y'-x+y=e^t,\ \ x(0)=1, y(0)=1. \end{cases}$

3-variant

1. $f(t)=5sh2t+8\cos 4t.$

2. $F(p)=\dfrac{3p^2-9p+16}{(p-2)(p^2-6p+13)}.$

3. $y''-y=te^t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'+x+2y=1, \\ y'+x-y=0,\ \ x(0)=1, y(0)=0. \end{cases}$

4-variant

1. $f(t)=2\sin 8t-5ch4t.$

2. $F(p)=\dfrac{-4p^2-3p-5}{p^3+2p^2+p}.$

2. $y''-9y=sht,\ \ y(0)=-1, y'(0)=3.$

4. $\begin{cases} x'-2x-8y=1, \\ y'-3x-4y=0,\ \ x(0)=2, y(0)=1. \end{cases}$

5-variant

1. $f(t)=\dfrac{\cos^2 2t}{t}.$

2. $F(p)=\dfrac{1-4p}{p^3-2p^2+p}.$

2. $y''-3y'+2y=-2\cos t,\ \ y(0)=1, y'(0)=0.$

4. $\begin{cases} x'-4x-2y=0, \\ y'-4x-6y=0,\ \ x(0)=1, y(0)=8. \end{cases}$

6-variant

1. $f(t)=3sh5t+7\cos\dfrac{t}{2}.$

2. $F(p)=\dfrac{p^2+15p+20}{(p-2)(p^2+2p+10)}.$

3. $y''-2y'-8y=2e^{2t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x-3y=3, \\ y'-x+y=1,\ \ x(0)=0, y(0)=1. \end{cases}$

7-variant

1. $f(t)=3t^5+8e^{-2t}+4.$

2. $F(p)=\dfrac{9p^2-21p-6}{(p+1)(p^2-5p+6)}.$

3. $y''+3y'+2y=e^{-t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-8x+3y=0, \\ y'-2x-y=0,\ \ x(0)=2, y(0)=9. \end{cases}$

8-variant

1. $f(t)=\dfrac{\sin 5t\sin 2t}{t}.$

2. $F(p)=\dfrac{3p+13}{(p-1)(p^2 2p+5)}.$

3. $y''+y'+y=7e^{2t},\ \ y(0)=1, y'(0)=4.$

4. $\begin{cases} x'-x-y=0, \\ y'-4x-y=1,\ \ x(0)=1, y(0)=0. \end{cases}$

9-variant

1. $f(t)=9\cos 3t-4sh5t.$

2. $F(p)=\dfrac{2p^2-10p+24}{(p-2)(p^2-2p-3)}.$

2. $y''-3y'+2y=e^t,\ \ y(0)=1, y'(0)=0.$

4. $\begin{cases} x'+2x-6y=1, \\ y'-2x=2,\ \ x(0)=0, y(0)=1. \end{cases}$

10-variant

1. $f(t)=\dfrac{sh2t}{t}.$

2. $F(p)=\dfrac{p^3+13p+3}{(p+3)(p^2+2p+2)}.$

2. $y''-2y'=e^t(t-3),\ \ y(0)=2, y'(0)=2.$

4. $\begin{cases} x'-x-4y=1, \\ y'-2x+3y=0,\ \ x(0)=0, y(0)=1. \end{cases}$

11-variant

1. $f(t)=7\cos 4t-6sh2t.$

2. $F(p)=\dfrac{p^2+3p-6}{(p+1)(p^2+6p+13)}.$

3. $y''-y=\cos t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-3x-y=0, \\ y'-x-3y=0,\ \ x(0)=1, y(0)=-5. \end{cases}$

12-variant

1. $f(t)=7t^5+10e^{-3t}+15.$

2. $F(p)=\dfrac{12-6p}{(p+1)(p^2-4p+13)}.$

3. $y''+4y=\sin t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-2x-3y=0, \\ y'-5x+4y=0,\ \ x(0)=5, y(0)=3. \end{cases}$

13-variant

1. $f(t)=2\sin 8t+11ch3t.$

2. $F(p)=\dfrac{2p^2+5p+5}{(p+2)(p^2+2p-3)}.$

3. $y''+3y'-4y=e^{-4t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-2x-2y=2, \\ \quad y'-4y=1, x(0)=0, y(0)=0\ . \end{cases}$

14-variant

1. $f(t)=5t-4e^{-2t}+3.$

2. $F(p)=\dfrac{2p^2+9p+44}{(p-1)(p^2+4p+13)}.$

3. $y''+y=6e^{-t},\ \ y(0)=3, y'(0)=1.$

4. $\begin{cases} \qquad x'+y=0, \\ y'-2x-2y=0, x(0)=1, y(0)=1. \end{cases}$

15-variant

1. $f(t)=e^{4t}\sin^2 4t.$

2. $F(p)=\dfrac{2p^2-3p+12}{(p+1)(p^2-2p+5)}.$

3. $2y''+3y'+y=3e^{t},\ \ y(0)=1, y'(0)=1.$

4. $\begin{cases} \qquad x'+y'=0, \\ x'-2y'+x=0,\ \ x(0)=1, y(0)=-1. \end{cases}$

16-variant

1. $f(t)=2sh5t-4\cos 7t.$

2. $F(p)=\dfrac{2p^2-7p+11}{(p+1)(p^2-2p+2)}.$

2. $y''+6y'+9y=9e^{3t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'+y=2, \\ y'-x=1,\ \ x(0)=-1, y(0)=0. \end{cases}$

17-variant

1. $f(t)=e^{3t}\cos^2 3t.$

2. $F(p)=\dfrac{3p^2+2p+11}{(p+3)(p^2-2p+1)}.$

2. $y''+y=sht,\ \ y(0)=2, y'(0)=1.$

4. $\begin{cases} x'-5x-4y=0, \\ y'-4x-5y=0,\ \ x(0)=-3, y(0)=7. \end{cases}$

18-variant

1. $f(t)=7ch3t-6\sin 10t.$

2. $F(p)=\dfrac{3p^2+8p+17}{(p-2)(p^2+2p+1)}$

3. $y''-y'-2y=\sin t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x-2y=0, \\ y'-4x-3y=0,\ \ x(0)=7, y(0)=-1. \end{cases}$

19-variant

1. $f(t)=7\sin 5t+12ch3t.$

2. $F(p)=\dfrac{p^2+5p+9}{(p+10)(p^2+4p+20)}.$

3. $y''-3y'+2y=e^{-2t},\ \ y(0)=1, y'(0)=2.$

4. $\begin{cases} x'-3x-5y=2, \\ y'-3x-y=1,\ \ x(0)=0, y(0)=2. \end{cases}$

20-variant

1. $f(t)=5t^8+7e^{-3t}+10.$

2. $F(p)=\dfrac{3p^2+3p+7}{(p+6)(p^2+4p-5)}.$

3. $y''+6y'+5y=12e^t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-2x+2y=0, \\ y'+4x=0,\ \ x(0)=3, y(0)=1. \end{cases}$

21-variant

1. $f(t)=6\sin 5t\cos 7t.$

2. $F(p)=\dfrac{4p^2+2p+6}{(p+4)(p^2+6p-7)}.$

3. $y''+y'-2y=e^{-t},\ \ y(0)=-1, y'(0)=0.$

4. $\begin{cases} x'-x+5y=0, \\ y'+x+3y=0,\ \ x(0)=7, y(0)=1. \end{cases}$

22-variant

1. $f(t)=5\sin 4t+9\cos 2t.$

2. $F(p)=\dfrac{5p^2+p+5}{(p+2)(p^2-8p+25)}.$

2. $y''-5y'+6y=e^{3t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x-2y=0, \\ y'-2x-y=0,\ \ x(0)=2, y(0)=5. \end{cases}$

23-variant

1. $f(t)=e^{-2t}\sin^2 4t.$

2. $F(p)=\dfrac{3p-2}{(p-1)(p^2-6p+10)}.$

2. $y''-2y'+2y=e^t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'+7x-y=0, \\ y'+2x+5y=0,\ \ x(0)=1, y(0)=1. \end{cases}$

24-variant

1. $f(t)=\sin 7t\cos 9t.$

2. $F(p)=\dfrac{p^2-6p+8}{(p+2)(p^2-2p+4)}.$

3. $y''+y'=\sin 2t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x-2y=t, \\ y'-2x-y=t,\ \ x(0)=4, y(0)=2. \end{cases}$

25-variant

1. $f(t)=2t^5+9e^{-6t}-13.$

2. $F(p)=\dfrac{p^2+2p-1}{p^3+3p^2+3p+1}.$

3. $y''+y'=e^{2t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-7x-3y=0, \\ y'-x-5y=0,\ \ x(0)=4, y(0)=2. \end{cases}$

26-variant

1. $f(t)=5t^3+8e^{-4t}+15.$

2. $F(p)=\dfrac{2p+3}{p(p^2+4p+5)}.$

3. $y''-2y'-3y=2t,\ \ y(0)=1, y'(0)=1.$

4. $\begin{cases} x'-6x-3y=0, \\ y'+8x+5y=0,\ \ x(0)=-4, y(0)=9. \end{cases}$

27-variant

1. $f(t)=e^{2t}\sin^2\dfrac{t}{4}.$

2. $F(p)=\dfrac{2p^3+p^2+2p+2}{p^3(p^2+2p+2)}$

3. $y''+2y'=2+e^t,\ \ y(0)=1, y'(0)=2.$

4. $\begin{cases} x'-x+4y=0, \\ y'-2x-3y=0,\ \ x(0)=3, y(0)=6. \end{cases}$

28-variant

1. $f(t)=8t-6e^{-2t}+12.$

2. $F(p)=\dfrac{p+2}{(p+1)(p-2)(p^2+4)}.$

2. $y''-4y=\cos 2t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'+4x-y=0, \\ y'+2x+y=0,\ \ x(0)=2, y(0)=3. \end{cases}$

29-variant

1. $f(t)=e^{-3t}\sin^2\dfrac{3t}{2}.$

2. $F(p)=\dfrac{p^3+p^2+p+2}{p(p+1)(p^2+1)}.$

2. $y''-y'=e^{2t},\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x-3y=2, \\ y'-x+y=1,\ \ x(0)=-1, y(0)=2. \end{cases}$

30-variant

1. $f(t)=e^{-t}\cos^2 5t.$

2. $F(p)=\dfrac{3p^2-3p-8}{(p+1)^2(p-3)}.$

3. $y''-3y'+2y=\sin 3t,\ \ y(0)=0, y'(0)=0.$

4. $\begin{cases} x'-x+3y=0, \\ y'-x-y=e^{-t},\ \ x(0)=1, y(0)=1. \end{cases}$

IY bob
MATEMATIK FIZIKA TENGLAMALARI

4.1. MATEMATIK FIZIKA MASALALARINING QO‘YILISHI

Ikkinchi tartibli ikki o‘zgaruvchili chiziqli xususiy hosilali differensial tenglamalar. Matematik fizikaning asosiy tenglamalari. Koshi masalasi, chegaraviy masalalar, aralash masalalaining qo‘yilishi

4.1.1. Erkli o‘zgaruvchilar, noma’lum funksiya va uning xususiy hosilalarini bog‘lovchi menglamaga *xususiy hosilali differensial tenglama* deyiladi.

Xususiy hosilali differensial tenglamaga kiruvchii hosilalarning eng yuqori tartibiga tenglamaning *tartibi* deyiladi.

Xususiy hosilali differensial tenglama noma’lum funksiya va uning barcha hosilalariga nisbatan chiziqli bo‘lsa, bu tenglamaga *chiziqli tenglama*, aks holda *chiziqli bo‘lmagan tenglama* deyiladi.

Fizika va texnikaning ko‘pchilik masalalari xususiy hosilali differensial tenglamalarning nisbatan katta bo‘lmagan sinfiga keltiriladi. Bu sinfda eng ko‘p uchraydigan tenglamalar - ikkinchi tartibli ikki o‘zgaruvchili chiziqli xususiy hosilali differensial tenglamalardir.

Ikkinchi tartibli chiziqli ikki o‘zgaruvchili xususiy hosilali differensial tenglamalar umumiy holda

$$a_{11}u_{xx} + 2a_{12}u_{xy} + a_{13}u_{yy} + b_1u_x + b_2u_y + cu = f(x,y), \qquad (1.1)$$

ko‘rinishda beriladi. Bunda x, y - erkli o‘zgaruvchilar, $u(x,y)$ - noma’lum funksiya, $a_{11}, a_{12}, a_{22}, b_1, b_2, c$ - koeffitsiyentlar (x va y o‘zgaruvchilarning uzluksiz funksiyalari), $f(x,y)$ - berilgan funksiya, $u_x, u_y, u_{xx}, u_{xy}, u_{yy}$ - noma’lum funksiyaning erkli o‘zgaruvchilar bo‘yicha xususiy hosilalari (bundan keyin shu kabi ekvivalent yozuvlardan foydalanamiz).

Ushbu

$$a_{11}dy^2 - 2a_{12}dydx + a_{22}dx^2 = 0 \qquad (1.2)$$

oddiy differensial tenglamaga (1.1) tenglamaning *xarakteristik tenglamasi* deyiladi, uning $\varphi(x,y) = C_1$ va $\psi(x,y) = C_2$ umumiy integrallariga *xarakteristikalar* deyiladi.

(1.1) tenglamaning turi (1.2) xarakteritik tenglama $\Delta(x,y)=a_{12}^2-a_{11}a_{22}$ diskriminantining ishorasiga bog'liq ravishda aniqlanadi. Agar $P(x,y)\in G$ nuqtada:

1) $\Delta>0$ bo'lsa, tenglamaga P nuqtada *giperbolik turdagi* tenglama;

2) $\Delta<0$ bo'lsa, tenglamaga P nuqtada *elliptik turdagi* tenglama;

3) $\Delta=0$ bo'lsa, tenglamaga P nuqtada *parabolik turdagi* tenglama deyiladi.

1- misol. $u_{xx}-2xu_{xy}+yu_{yy}=0$ tenglama giperbolik, elliptik va parabolik turda bo'ladigan sohalarni aniqlang.

Berilgan tenglamada $a_{11}=1,\ a_{12}=-x,\ a_{22}=y.$

U holda

$$\Delta=a_{12}{}^2-a_{11}a_{22}=x^2-y\,.$$

Demak, berilgan tenglama tekislikning: $x^2-y>0$ bo'lgan nuqtalarida, ya'ni $y=x^2$ parabola ickarisida yotgan nuqtalarda giperbolik turda, tashqarisida yotgan nuqtalarda elliptik turda, parabolada yotgan nuqtalarda parabolik turda bo'ladi.

Giperbolik turdagi tenglama uchun (1.2) xarakteristik tenglamaning yechimlari $\varphi(x,y)=C_1$ va $\psi(x,y)=C_2$ funksiyalar bo'ladi. Bunda $\xi=\varphi(x,y)$ va $\eta=\psi(x,y)$ almashtirishlar (1.1) tenglamani

$$u_{\xi\eta}=\frac{1}{2a_{12}}F(\xi,\eta,u,u_\xi,u_\eta) \qquad (1.3)$$

kanonik ko'rinishda keltiradi.

Bunda o'zgaruvchilar $\xi=\frac{1}{2}(\varphi(x,y)+\psi(x,y))$, $\eta=\frac{1}{2}(\varphi(x,y)-\psi(x,y))$ kabi tanlansa, (1.1) tenglama

$$u_{\xi\xi}-u_{\eta\eta}=\frac{1}{2a_{12}}F(\xi,\eta,u,u_\xi,u_\eta) \qquad (1.4)$$

kanonik ko'rinishga o'tadi.

Elliptik turdagi tenglamalar uchun (1.2) xarakteristik tenglama $\varphi(x,y)+i\psi(x,y)=C_1$ va $\varphi(x,y)-i\psi(x,y)=C_2$ qo'shma kompleks yechimlarga ega bo'ladi. Bunda $\xi=\varphi(x,y)$ va $\eta=\psi(x,y)$ almashtirishlar orqali (1.1) tenglamadan

$$u_{\xi\xi}+u_{\eta\eta}=\frac{1}{a_{11}}F(\xi,\eta,u,u_\xi,u_\eta) \qquad (1.5)$$

kanonik tenglama hosil qilinadi.

Parabolik turdagi tenglamalar uchun (1.2) xarakteristik tenglama $\varphi(x,y)=C$ umumiy integralga ega bo‘ladi. Bu holda $\xi=\varphi(x,y)$ va $\eta=\psi(x,y)$ almashtirishlar bajailadi. Bunda $\eta=\psi(x,y)$ funksiya $\varphi(x,y)$ funksiyaga bog‘liq bo‘lmaydi. Bu almashtirishlar yordamida (1.1) tenglama

$$u_{\eta\eta}=\frac{1}{a_{12}}F(\xi,\eta,u,u_{\xi},u_{\eta}) \tag{1.6}$$

kononik tenglamaga keltiriladi.

2- misol. Berilgan tenglamalarni kanonik ko‘rinishga keltiring:

1) $u_{xx}+2u_{xy}-3u_{yy}+2u_x+6u_y=0;$ 2) $u_{xx}-2u_{xy}+2u_{yy}=0.$

1) Berilgan tenglamada $a_{11}=1,\ a_{12}=1,\ a_{22}=-3.$

U holda $\Delta=a_{12}{}^2-a_{11}a_{22}=1-1\cdot(-3)=4>0$.

Demak, berilgan tenglama giperbolik turdagi tenglama.

Uning xarakteristik tenglamasini tuzamiz :

$$dy^2-2dxdy-3dx^2=0 \text{ yoki } \frac{dy}{dx}=1\pm 2.$$

U holda berilgan tenglamaning xarakteristikalari:

$$y+x=C_1,\ \ 3x-y=C_2.$$

$\xi=y+x,\ \eta=3x-y$ almashtirishlar bajaramiz va bu funksiyalarning xususiy hosilalarini topamiz:

$$\xi_x=1,\ \xi_y=1,\ \eta_x=3,\ \eta_y=-1.$$

Berilgan tenglamadagi xususiy hosilalarni aniqlaymiz:

$$u_x=u_\xi\cdot 1+u_\eta\cdot 3=u_\xi+3u_\eta; \qquad u_y=u_\xi\cdot 1+u_\eta\cdot(-1)=u_\xi-u_\eta;$$

$$u_{xx}=u_{\xi\xi}\cdot 1+u_{\xi\eta}\cdot 3+u_{\eta\xi}\cdot 1+3u_{\eta\eta}\cdot 3=u_{\xi\xi}+6u_{\xi\eta}+9u_{\eta\eta};$$

$$u_{xy}=u_{\xi\xi}\cdot 1+u_{\xi\eta}\cdot(-1)+3u_{\eta\xi}\cdot 1+3u_{\eta\eta}\cdot(-1)=u_{\xi\xi}+2u_{\xi\eta}-3u_{\eta\eta};$$

$$u_{yy}=u_{\xi\xi}\cdot 1+u_{\xi\eta}\cdot(-1)-u_{\eta\xi}\cdot 1-u_{\eta\eta}\cdot(-1)=u_{\xi\xi}-2u_{\xi\eta}+u_{\eta\eta}.$$

Topilgan xususiy hosilalarni berilgan tenglamaga qo‘yib, uning kanonik ko‘rinishini topamiz:

$$u_{\xi\xi}+\frac{1}{2}u_\xi=0.$$

2) Berilgan tenglamada $a_{11}=1,\ a_{12}=-1,\ a_{22}=2$

va $\Delta=a_{12}{}^2-a_{11}a_{22}=1-2=-1<0$. Demak, berilgan tenglama elliptik turdagi tenglama. Uning xarakteristik tenglamasini tuzamiz :

$$dy^2+2dxdy+2dx^2=0 \text{ yoki } \frac{dy}{dx}=-1\pm i.$$

Bundan berilgan tenglamaning xarakteristikalarini topamiz:

$$y + x - ix = C_1, \quad y + x + ix = C_2.$$

$\xi = y + x,\ \eta = x$ almashtirishlar bajaramiz. U holda

$$\xi_x = 1,\ \xi_y = 1,\ \eta_x = 1,\ \eta_y = 0.$$

Tegishli xususiy hosilalarni aniqlaymiz:

$$u_x = u_\xi \cdot 1 + u_\eta \cdot 1 = u_\xi + u_\eta; \qquad u_y = u_\xi \cdot 1 + u_\eta \cdot 0 = u_\xi;$$

$$u_{xx} = u_{\xi\xi} \cdot 1 + u_{\xi\eta} \cdot 1 + u_{\eta\xi} \cdot 1 + u_{\eta\eta} \cdot 1 = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta};$$

$$u_{xy} = u_{\xi\xi} \cdot 1 + u_{\xi\eta} \cdot 1 = u_{\xi\xi} + u_{\xi\eta}; \qquad u_{yy} = u_{\xi\xi} \cdot 1 + u_{\xi\eta} \cdot 0 = u_{\xi\xi}.$$

Topilgan xususiy hosilalarni berilgan tenglamaga qo‘yib, uni kanonik ko‘rinishga keltiramiz:

$$u_{\xi\xi} + u_{\eta\eta} = 0.$$

3- misol. $49u_{xx} - 14u_{xy} + u_{yy} + 14u_x - 2u_y = 0$ kanonik ko‘rinishga keltirib, umumiy yechimini toping.

Berilgan tenglamada $\Delta = a_{12}^2 - a_{11}a_{22} = 49 - 49 = 0$. Demak, berilgan tenglama parabolik turdagi tenglama. Uning xarakteristik tenglamasi $7dy + dx = 0$ tenglamaga teng kuchli. Bu tenglama $7y + x = C$ yechimga ega. Demak, yangi $\xi = x + 7y$ o‘zgaruvchi kiritamiz. Ikkinchi o‘zgaruvchini $\eta = x$ kabi tanlaymiz. Bunda $D = \xi_x\eta_y - \eta_x\xi_y = 1 \cdot 0 - 1 \cdot 7 = -7 \neq 0$. Demak, $\xi = x + 7y$, $\eta = x$ almashtirish bajarish mumkin. Bundan $\xi_x = 1,\ \xi_y = 7,\ \eta_x = 1,\ \eta_y = 0$.

Berilgan tenglamadagi xususiy hosilalarni aniqlaymiz:

$$u_x = u_\xi \cdot 1 + u_\eta \cdot 1 = u_\xi + u_\eta; \qquad u_y = u_\xi \cdot 7 + u_\eta \cdot 0 = 7u_\xi;$$

$$u_{xx} = u_{\xi\xi} \cdot 1 + u_{\xi\eta} \cdot 1 + u_{\eta\xi} \cdot 1 + u_{\eta\eta} \cdot 1 = u_{\xi\xi} + 2u_{\xi\eta} + u_{\eta\eta};$$

$$u_{xy} = 7u_{\xi\xi} \cdot 1 + 7u_{\xi\eta} \cdot 1 = 7u_{\xi\xi} + 7u_{\xi\eta}; \qquad u_{yy} = 7u_{\xi\xi} \cdot 7 + u_{\xi\eta} 0 = 49u_{\xi\xi}.$$

Bu hosilalarni berilgan tenglamaga qo‘ysak, tenglama

$$u_{\eta\eta} + \frac{2}{7}u_\eta = 0$$

kanonik ko‘rinishni oladi. Bu tenglama fiksirlangan har qanday ξ da η ning o‘zgarmas koeffitsiyentli chiziqli ikkinchi tartibli oddiy differensial tenglamasi bo‘ladi. Uning xarakteristik tenglamalari $k_1 = 0$ va $k_2 = -\frac{2}{7}$ ildizlarga ega. U holda bu tenglamaning umumiy yechimi

$$u(\xi, \eta) = C_1(\xi) + C_2(\xi)e^{-\frac{7}{2}\eta}$$

bo‘ladi, bu yerda $C_1(\xi), C_2(\xi)$ - ξ o‘zgaruvchining ixtiyoriy funksiyalari.

Bundan x va y o‘zgaruvchilarga qaytib, berilgan tenglamaning umumiy yechimini topamiz:

$$u(x,y) = C_1(7x+y) + C_2(7x+y)e^{-\frac{7}{2}x},$$

bu yerda $C_1(\xi), C_2(\xi)$ - $\xi = x + 7y$ o‘zgaruvchining ikki marta differensiallanuvchi ixtiyoriy funksiyalari.

4.1.2. Har xil tabiatli tebranma jarayonlar (torning ko‘ndalang tebranishi, sterjenning bo‘ylama tebranishi, elektromagnit, gidrodinamik, gazodinamik va akustik tebranishlar) giperbolik turdagi tenglamalar bilan ifodalanadi. Bunday tenglamalardan eng soddasi *to‘lqin tenglamasi* (torning tebranish tenglamasi) deb ataluvchi

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1.8)$$

tenglama hisoblanadi.

Issiqlik tarqalish va diffuziya hodisalari hamda filtratsiya masalalari parabolik turdagi tenglamalarga keltiriladi. Bunday eng sodda tenglamalarga *issiqlik tarqalish tenglamasi* (yoki *Fur’e tenglamasi*)

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1.9)$$

misol bo‘ladi.

Elektr va magnit maydon haqidagi masalalar, statsionar issiqlik holati haqidagi masalalar, siqilmaydigan suyuqlikning potensial harakati haqidagi masalalar elliptik turdagi tenglamalar bilan aniqlanadi. Bunday tenglamalarning tipik vakili sifatida

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (1.10)$$

*Laplas tenglamasi*ni ko‘rsatish mumkin.

4.1.3. U yoki bu fizik jarayonni to‘liq modellashtirish uchun faqat jarayonni ifodalovchi differensial tenglamalarning bo‘lishi yetarli hisoblanmaydi, bunda yana bu jarayonning boshlang‘ich holatini (boshlang‘ich shartlarni) va bu jarayon sodir bo‘layotgan G sohaning S chegarasidagi rejimni (chegaraviy shartlarni) bilish ham kerak bo‘ladi.

Xususiy hosilali differensial tenglamalar uchun asosan boshlang‘ich va chegaraviy shartlarni ifodalovchi uch turdagi masala qo‘yiladi.

1. Ciperbolik va parabolik turdagi tenglamalar uchun *Koshi masalasi*. Bunda tenglamaning berilish sohasi butun fazo bilan ustma-ust tushadi,

faqat boshlang‘ich shartlar beriladi, chegaraviy shartlar qatnashmaydi.

2. Elliptik turdagi tenglamalar uchun *chegaraviy masala.* Bunda tenglama G berilish sohasining S chegarasidagi shartlar beriladi, boshlang‘ich shartlar qatnashmaydi.

3. Ciperbolik va parabolik turdagi tenglamalar uchun *aralash masala.* Bunda boshlang‘ich va aralash shartlar beriladi, G soha fazoning biror qismi bo‘ladi.

Matematik fizika tenglamalarining yechimlari boshlang‘ich va chegaraviy shartlarni ifodalovchi masalaning qo‘yilishiga bog‘liq bo‘ladi. Bunda birinchidan qo‘yilgan masalaning yechimi mavjud va yagona bo‘lishi kerak, ikkinchidan bu yechim turg‘un bo‘lishi (shartlarning ozgina o‘zgarishiga yechimning kichik o‘zgarishi mos kelishi) lozim.

Agar qo‘yilgan masalaning yechimi mavjud, yagona va turg‘un bo‘lsa, u holda bu masalaga to‘g‘ri qo‘yilgan (korrekt) masala deyiladi.

Mashqlar

4.1.1. Berilgan tenglamalar giperbolik, elliptik va parabolik turda bo‘ladigan sohalarni aniqlang:

1) $u_{xx} - yu_{yy} + 2u_x + 6u_y - 3u = 0;$

2) $xu_{xx} + 2xu_{xy} + (x-1)u_{yy} + 4u_x = 0;$

3) $(1-x^2)u_{xx} - 2xyu_{xy} - (1-y^2)u_{yy} + +2xu_x + u_y = 0;$

4) $u_{xx} + 2\sin xu_{xy} - \cos 2xu_{yy} + \cos xu_y = 0.$

4.1.2. Berilgan tenglamalarni kanonik ko‘rinishga keltiring:

1) $u_{xx} - 2u_{xy} + u_{yy} - u_x + u_y = 0;$ 2) $u_{xx} - 6u_{xy} + 9u_{yy} - u_x + 2u_y = 0;$

3) $u_{xx} + 4u_{xy} + 5u_{yy} + u_x + 2u_y = 0;$ 4) $u_{xx} - 2u_{xy} + 10u_{yy} = 0;$

5) $u_{xx} + u_{xy} - 2u_{yy} - 3u_x - 15u_y = 0;$ 6) $u_{xx} - 4u_{xy} + 3u_{yy} - 3u_x + 9u_y = 0;$

7) $(1+x^2)^2 u_{xx} + u_{xy} + 2x(1+x^2)u_x = 0;$ 8) $(1+x^2)^2 u_{xx} + (1+y^2)^2 u_{yy} = 0;$

9) $u_{xx} - 2\sin xu_{xy} + (2-\cos^2 x)u_{yy} - \cos xu_y = 0;$ 10) $x^2 u_{xx} - 2xu_{xy} + u_{yy} = 0.$

4.1.3. Berilgan tenglamalarni kanonik ko‘rinishga keltirib, umumiy yechimini toping:

1) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0, \ x \neq 0;$ 2) $2u_{xx} + 5u_{xy} - 3u_{yy} = 0;$

3) $u_{xx} - 2\cos xu_{xy} - (3+\sin^2 x)u_{yy} + \sin xu_y = 0;$ 4) $u_{xx} - yu_{yy} - \frac{1}{2}u_y = 0, \ y > 0.$

4.2. TO‘LQIN TENGLAMALARINI YECHISH

To‘lqin tenglamalarini yechishning Fure usuli. To‘lqin tenglamalarini yechishning D’alamber usuli

4.2.1. Ushbu

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < l \tag{2.1}$$

to‘lqin tenglamasining

$$u\big|_{t=0} = f(x), \quad \frac{\partial u}{\partial t}\bigg|_{t=0} = F(x), \quad 0 \le x \le l \tag{2.2}$$

boshlang‘ich shartlarni va

$$u\big|_{x=0} = 0, \quad u\big|_{x=l} = 0, \quad t \ge 0 \tag{2.3}$$

chegaraviy shartlarni qanoatlantiruvchi xususiy yechimini topish masalasini qaraylik, bu yerda $f(x)$, $F(x)$ - $[0;l]$ oraliqda aniqlangan funksiyalar bo‘lib, $f(0) = F(0) = 0$.

Agar bu masalada $u = u(x,t)$ torning muvozanat holatidan chetlashishini bildirsa, (2.1) tenglama uzunligi l ga teng va $x = 0$, $x = l$ nuqtalarga mahkamlangan bir jinsli torning erkin tebranishini ifodalaydi, bu yerda $a^2 = \frac{T}{\rho}$, T - taranglik; $\rho = \rho(x)$ - chiziqli zichlik.

Agar masalada $u(x,t)$ deb tok yoki kuchlanish tushunilsa, (2.1) tenglama simdagi elektr tebranishlarni ifodalaydi, bu yerda $a^2 = \frac{1}{CL}$, C - sig‘im, L - induktivlik.

Agar masalada $u(x,t)$ sifatida sterjenning bo‘ylama cho‘zilishi tushunilsa (2.1) tenglama sterjandagi bo‘ylama tebranishlarni ifodalaydi, bu yerda $a^2 = \frac{1}{v^2}$, v - sterjen bo‘ylab dinamik kuchlanishlarning tarqalish tezligi.

Bunda bitta differensial tenglama bilan butunlay boshqa-boshqa fizik jarayonlar ifodalanishiga sabab, bu masalalarda izlanayotgan kattaliklarning bitta xarakterdagi holati kuzatiladi.

To‘lqin tenglamalari giperbolik turdagi tenglama hisoblanadi. Ularni yechishning *Fure* (*o‘zgaruvchilarni ajratish*) usulini aniqlik uchun torning kichik tebranishlari misolida qarab chiqaylik. Fure usulida (2.1) tenglamaning yechimi biri faqat t ga bog‘liq bo‘lgan va boshqasi faqat x ga bog‘liq bo‘lgan ikkita funksiyaning ko‘paytmasi, ya’ni

$$u(x,t)=T(t)X(x) \tag{2.4}$$

ko‘rinishda izlanadi (o‘zgaruvcilari ajratiladi). Bunda ko‘paytuvchilarning har ikkalasi (2.3) chegaraviy shartlarni qanoatlantiradi va aynan nolga teng bo‘lmaydi deb qaraladi.

$u(x,t)$ ning (2.4) shaklini (2.1) tenglamaga qo‘ysak,

$$\frac{T''(t)}{a^2T(t)}=\frac{X''(x)}{X(x)}$$

tenglik kelib chiqadi. O‘ng tomoni faqat t ga bog‘liq bo‘lgan va chap tomoni faqat x ga bog‘liq bo‘lgan bu tenglik har ikkala tomoni bitta o‘zgarmasga teng bo‘lganida bajarilishi mumkin. Bu o‘zgarmasni $(-\lambda)$ bilan belgilab, bu tenglikdan ikkita oddiy differensial tenglama hosil qilinadi:

$$T''(t)+\lambda a^2T(t)=0, \tag{2.5}$$

$$X''(x)+\lambda X(x)=0. \tag{2.6}$$

(2.3) shartlarni qanoatlantiruvchi (2.6) ko‘rinishdagi notrivial (nolga teng bo‘lmagan) yechimni topish uchun

$$X''(x)+\lambda X(x)=0 \tag{2.6}$$

tenglamaning

$$X(0)=0, \quad X(l)=0 \tag{2.7}$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish, ya’ni (2.6)-(2.7) masalani yechish kerak bo‘ladi.

(2.6)-(2.7) masalaning notrival yechimi mavjud bo‘ladigan λ ning qiymatlariga (2.6)-(2.7) masalaning *xos qiymatlari*, ularga mos notrival yechimlarga (2.6)-(2.7) masalaning *xos funksiyalari* deyiladi. Bunday talqin qilingan masala *Shturm-Liuvill muammosi* deb yuritiladi.

(2.6)-(2.7) masalaning xos qiymatlari

$$\lambda_k=\left(\frac{k\pi}{l}\right)^2, \ k\in 1,2,... \tag{2.8}$$

va xos funksiyalari

$$X_k(x)=\sin\frac{k\pi}{l}x, \ k\in 1,2,...$$

kabi topiladi.

$\lambda=\lambda_k$ da (2.5) tenglamaning umumiy yechimi

$$T_k(t)=A_k\cos\frac{k\pi a}{l}t+B_k\sin\frac{k\pi a}{l}t,$$

bo‘ladi, bu yerda A_k, B_k - ixtiyoriy o‘zgarmaslar.

(2.8) xos qiymatlarning

$$u_k(x,t) = X_k(x)T_k(t) = \left(A_k \cos\frac{k\pi a}{l}t + B_k \sin\frac{k\pi a}{l}t \right)\sin\frac{k\pi}{l}x$$

funksiyalari har qanday A_k va B_k $(k=1,2,...)$ o‘zgarmaslarda (2.1) tenglamani (2.3) chegaraviy shartlarda qanoatlantiradi.

(2.1) tenglama chiziqli va bir jinsli bo‘lgani sababli yechimlarning har qanday chekli yig‘indisi va shu kabi

$$u(x,t) = \sum_{k=1}^{\infty}\left(A_k \cos\frac{k\pi a}{l}t + B_k \sin\frac{k\pi a}{l}t \right)\sin\frac{k\pi}{l}x \tag{2.9}$$

qator ham, agar u yaqinlashuvchi, x va t o‘zgaruvchilar bo‘yicha ikki marta differensiallanuvchi bo‘lsa, (2.1) tenglamaning yechimi bo‘ladi.

(2.9) qatorning koeffitsiyentlari

$$A_k = \frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}x dx, \quad B_k = \frac{2}{k\pi a}\int_0^l F(x)\sin\frac{k\pi}{l}x dx \quad (k=1,2,...) \tag{2.10}$$

kabi topiladi.

1- misol. Chetki nuqtalari mahkamlangan, boshlang‘ich $t=0$ vaqtda $hx(l-x)$ $(h>0-const)$ parabola shaklida bo‘lgan va tezlikka ega bo‘lmagan, uzunligi l ga teng bir jinsli torning erkin tebranishini Fure usuli bilan toping (T taranglik va ρ zichlik berilgan).

Bu masala

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}, \quad t>0, \quad 0<x<l$$

tenglamani

$$u\big|_{t=0} = hx(l-x), \quad \frac{\partial u}{\partial t}\bigg|_{t=0} = 0, \quad 0 \le x \le l$$

boshlang‘ich shartlarda va

$$u\big|_{x=0} = 0, \quad u\big|_{x=l} = 0, \quad t \ge 0$$

chegaraviy shartlarda yechishga keltiriladi.

Masalani Fure usuli bilan yechamiz, ya’ni berilgan tenglamaning berilgan chegaraviy shartlarni qanoatlantiruvchi notrival yechimlarini

$$u(x,t) = T(t)X(x)$$

ko‘rinishda izlaymiz.

Ma’lumki, bu yechim

$$u(x,t) = \sum_{k=1}^{\infty}\left(A_k \cos\frac{k\pi a}{l}t + B_k \sin\frac{k\pi a}{l}t \right)\sin\frac{k\pi}{l}x$$

ko‘rinishga keltiriladi.

Bunda tenglamaning koeffitsiyentlari

$$A_k = \frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}xdx, \quad B_k = \frac{2}{k\pi a}\int_0^l F(x)\sin\frac{k\pi}{l}xdx \quad (k=1,2,...)$$

yoki berilgan boshlang‘ich shartlarga ko‘ra

$$A_k = \frac{2}{l}\int_0^l hx(l-x)\sin\frac{k\pi}{l}xdx, \quad B_k = \frac{2}{k\pi a}\int_0^l 0\cdot\sin\frac{k\pi}{l}xdx \quad (k=1,2,...)$$

integrallar bilan aniqlanadi.

Bu integrallarning ikkinchisidan $B_k = 0$ kelib chiqadi.

Integrallarning birinchisini ikki marta bo‘laklab integrallash orqali A_k ni aniqlaymiz:

$$A_{2m+1} = \frac{8l^2h}{\pi^3(2m+1)^3}, \; A_{2m+2} = 0, \; m = 0,1,....$$

A_k va B_k koeffitsiyentlarning topilgan qiymatlarini hisobga olib, berilgan masalaning izlanayotgan yechimini topamiz:

$$u(x,t) = \frac{8l^2h}{\pi^3}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^3}\cos\frac{(2m+1)\pi a}{l}t\sin\frac{(2m+1)\pi}{l}x.$$

Agar torning chap tomoni mahkamlangan, o‘ng tomoni o‘zining muvozanat holati bilan elastik bog‘langan bo‘lsa, (2.1) to‘lqin tenglamasi uchun chegaraviy shartlar

$$u(0,t)\big| = 0, \quad u_x(l,t)\big| = -hu(l,t), \quad t \ge 0, \quad h > 0 - const \qquad (2.11)$$

kabi qo‘yiladi.

Bunda (2.1) tenglamaning (2.11) chegaraviy shartlarni qanoatlantiruvchi notrival yechimi

$$u(x,t) = T(t)X(x)$$

uchun

$$X''(x) + \lambda X(x) = 0$$

differesial tenglamaning

$$X(0) = 0, \qquad X'(l) + hX(l) = 0$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish masalasi kelib chiqadi. Bu masalaning xos qiymatlari

$$\sqrt{\lambda}\cos\sqrt{\lambda}l + h\sin\sqrt{\lambda}l = 0$$

tenglikdan topiladi. Bunda $\lambda = v^2$ belgilash kiritilsa, v ni topish uchun

$$tg(vl) = -\frac{v}{h}. \qquad (2.12)$$

tenglama kelib chiqadi. Bu tenglamani, masalan, grafik usul bilan yechish mumkin.

Har ikki chetki nuqtasida mahkamlangan bir jinsli torning intensivligi $g(x,t)$ bo‘lgan tashqi kuch ta’sirida tebranishi masalasi

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + g(x,t), \quad t > 0, \quad 0 < x < l \tag{2.13}$$

tenglamani

$$u\big|_{t=0} = f(x), \quad \frac{\partial u}{\partial t}\bigg|_{t=0} = F(x), \quad 0 \le x \le l \tag{2.14}$$

boshlang‘ich shartlarda va

$$u\big|_{x=0} = 0, \quad u\big|_{x=l} = 0, \quad t \ge 0 \tag{2.15}$$

chegaraviy shartlarda yechishga keltiriladi.

Bunda $v(x,t)$ yechim torning majburiy tebranishini, ya’ni boshlang‘ich harakatsiz tashqi $g(x,t)$ kuch ta’sirida harakatlanuvchi tebranishlarni ifodalaydi, $w(x,t)$ yechim esa faqat boshlang‘ich harakatlar natijasida hosil bo‘luvchi erkin tebranishlarni ifodalaydi.

Bu masalaning $u(x,t)$ yechimini

$$u(x,t) = v(x,t) + w(x,t) \tag{2.16}$$

yig‘indi ko‘rinishida izlanadi.

Bunda $v(x,t)$

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial x^2} + g(x,t) \tag{2.17}$$

bir jinsli bo‘lmagan tenglamaning

$$v\big|_{t=0} = 0, \quad \frac{\partial v}{\partial t}\bigg|_{t=0} = 0 \tag{2.18}$$

boshlang‘ich shartlarni va

$$v\big|_{x=0} = 0, \quad v\big|_{x=l} = 0 \tag{2.19}$$

chegaraviy shartlarni qanoatlantiruvchi yechimi.

$w(x,t)$ esa

$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2} \tag{2.20}$$

bir jinsli tenglamaning

$$w\big|_{t=0} = f(x), \quad \frac{\partial w}{\partial t}\bigg|_{t=0} = F(x) \tag{2.21}$$

boshlang‘ich shartlarni va

$$w\big|_{x=0} = 0, \quad w\big|_{x=l} = 0 \tag{2.22}$$

chegaraviy shartlarni qanoatlantiruvchi yechimi.

(2.17)-(2.19) masalani yechishda xos funksiyalarni yoyish usuli qo‘llanadi. Bu usulning mohiyati $g(x,t)$ tashqi kuchlarni bir jinsli chegaraviy va boshlang‘ich shartlarga mos $\{X_n(x)\}$ xos funksiyalar bo‘yicha

$$g(x,t)=\sum_{k=1}^{\infty} g_k(t)X_k(x)$$

qatorga yoyishdan va har bir $f_k(t)X_k(x)$ qo‘shiluvchiga ta’sirning $u_k(x,t)$ javoblarini topishdan iborat. Bundan $u_k(x,t)$ javoblarni yig‘ib, berilgan masalaning yechimini topailadi:

$$u(x,t)=\sum_{k=1}^{\infty} u_k(x,t).$$

(2.17)-(2.19) masalaning yechimi

$$v(x,t)=\sum_{k=1}^{\infty} T_k(t)\sin\frac{k\pi}{l}x \qquad (2.23)$$

ko‘rinishda izlanadi, bu nyerda $\sin\frac{k\pi}{l}x$ (2.19) chegaraviy shartlar o‘z-o‘zidan bajariluvchi bir jinsli masalaning xos funksiyalari.

$v(x,t)$ni (2.23) ko‘rinishda (2.17) tenglamaga qo‘yib,

$$\sum_{k=1}^{\infty}\left(T_k''(t)+\frac{k^2\pi^2a^2}{l^2}T_k(t)\right)\sin\frac{k\pi}{l}x=g(x,t) \qquad (2.24)$$

qator hosil qilinadi.

$g(x,t)$ funksiyani $(0;l)$ intervalda Fure qatoriga sinuslar bo‘yicha (xos funksiyalarning) yoyilsa

$$g(x,t)=\sum_{k=1}^{\infty} g_k(t)\sin\frac{k\pi}{l}x, \qquad (2.25)$$

kelib chiqadi, bu yerda

$$g_k(t)=\frac{2}{l}\int_0^l g(\xi,t)\sin\frac{k\pi}{l}\xi d\xi \qquad (2.26)$$

$g(x,t)$ funksiyaning (2.24) va (2.25) yoyilmalarni taqqoslab, $T_k(t)$ noma’lum funksiyaga nisbatan ushbu

$$T_k''(t)+\frac{k^2\pi^2a^2}{l^2}T_k(t)=g_k(t),\ \ k=1,2,,... \qquad (2.27)$$

differeensial tenglama hosil qilinadi.

(2.24) qator bilan aniqlanuvchi $v(x,t)$ yechim (2.18) boshlang‘ich shartlarni qanoatlantirishi uchun $T_k(t)$ funksiya

$$T_k(0)=0, \qquad T_k'(0)=0,\ \ k=1,2,... \qquad (2.28)$$

shartlarni qanoatlantirishi kerak.

O‘zgarmasni variasiyalash usulidan foydalanib (2.27) tenglamaning (2.28) boshlang‘ich shartlardagi yechimini topiladi:

$$T_k(t)=\frac{l}{k\pi a}\int_0^l g_k(t)\sin\frac{k\pi a}{l}(t-\tau)d\tau,\quad k=1,2,... \qquad (2.29)$$

bu yerda $g_k(t)$ (2.26) formula bilan aniqlanadi.

$T_k(t)$ning topilgan qiymatlarini (2.24) qatorga qo‘yib, (2.17)-(2.19) masalaning $v(x,t)$ yechimi topiladi. Bunda (2.24) qator va bu qatordan x va t bo‘yicha ikki marta hadma-had differensiallash natijasida hosil qilingan qator tekis yaqinlashishi kerak.

Agar $g(x,t)$ funksiya va uning x bo‘yicha ikkinchi tartibli hosilalri uzluksiz, har qanday t da $g(0,t)=g(l,t)=0$ bo‘lsa, u holda (2.24) qator yaqinlashuvchi bo‘lishi aniqlangan. Bunda (2.13)-(2.15) masalaning yechimi

$$u_k(x,t)=\sum_{k=1}^{\infty}\left(A_k\cos\frac{k\pi a}{l}t+B_k\sin\frac{k\pi a}{l}t\right)\sin\frac{k\pi}{l}x+\sum_{k=1}^{\infty}T_k(t)\sin\frac{k\pi}{l}x \qquad (2.30)$$

ko‘rinishda bo‘ladi, bu yerda

$$T_k(t)=\frac{l}{k\pi a}\int_0^l g_k(t)\sin\frac{k\pi a}{i}(t-\tau)d\tau,$$

$$A_k=\frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}xdx,\qquad B_k=\frac{2}{k\pi a}\int_0^l F(x)\sin\frac{k\pi}{l}xdx\quad (k=1,2,...).$$

66.2- misol. Chegaralangan torning majburiy tebranishi haqidagi masalani Fure usuli bilan yeching: $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+t\sin x$, $t>0,\ 0<x<\pi,\ u\big|_{t=0}=0,$ $\frac{\partial u}{\partial t}\Big|_{t=0}=0$, $0\le x\le\pi,\ u\big|_{x=0}=0,\quad u\big|_{x=l}=0,\quad t\ge 0.$

⊖ Torda boshlang‘ich chetlashishlar yo‘q. Shu sababli chetki nuqtalarda mahkamlangan bir jinsli torning sof majburiy tebraniashini qaraymiz.

U holda berilgan masalaning yechimi

$$u(x,t)=\sum_{k=1}^{\infty}T_k(t)\sin\frac{k\pi}{l}x$$

ko‘rinishga keladi.

Bunda

$$T_k(t)=\frac{l}{k\pi}\int_0^l g_k(t)\sin\frac{k\pi}{l}(t-\tau)d\tau$$

bo‘ladi, bu yerda

$$g_k(t) = \frac{2}{l}\int_0^l g(\xi,t)\sin\frac{k\pi}{l}\xi d\xi \, .$$

U holda berilgan masala uchun

$$g_k(t) = \frac{2}{\pi}\int_0^\pi t\sin\xi\sin k\xi d\xi = (bundan \;\; k=1) = \frac{t}{\pi}\int_0^\pi (1-\cos 2\xi)d\xi = t,$$

$$T_k(t) = \frac{\pi}{\pi}\int_0^t \tau\sin(t-\tau)d\tau = \tau\cos(t-\tau)\Big|_0^t - \int_0^t \cos(t-\tau)d\tau =$$

$$= t + \sin(t-\tau)\Big|_0^t = t - \sin t.$$

Demak, berilgan masalaning yechimi

$$u(x,t) = (t-\sin t)\sin x.$$

Ikki tomoni mahkamlangan, chetki nuqtalari berilgan qonun bo‘yicha harakatlanuvchi torning majburiy tebranishi masalasi

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + g(x,t)\,, \quad t>0, \quad 0<x<l \qquad (2.31)$$

tenglamani

$$u\Big|_{t=0} = f(x), \quad \frac{\partial u}{\partial t}\Big|_{t=0} = F(x)\,, \quad 0\le x\le l \qquad (2.32)$$

boshlang‘ich shartlarda va

$$u\Big|_{x=0} = \xi_1(t) \quad u\Big|_{x=l} = \xi_2(t)\,, \quad t\ge 0 \qquad (2.33)$$

chegaraviy shartlarda yechishga keltiriladi.

Masalani yechish uchun yordamchi

$$w(x,t) = \xi_1(t) + (\xi_2(t)-\xi_1(t))\frac{x}{l} \qquad (2.34)$$

funksiya kiritiladi, bunda

$$w\Big|_{x=0} = \xi_1(t) \quad w\Big|_{x=l} = \xi_2(t)\,. \qquad (2.35)$$

(2.31)-(2.33) masalaning $u(x,t)$ yechimini

$$u(x,t) = v(x,t) + w(x,t) \qquad (2.36)$$

yig‘indi ko‘rinishida izlanadi.

Bu yerda $v(x,t)$ funksiya

$$\frac{\partial^2 v}{\partial t^2} = a^2\frac{\partial^2 v}{\partial x^2} + g_1(x,t)\,, \quad t>0, \quad 0<x<l \qquad (2.37)$$

tenglamaning

$$v\Big|_{t=0} = \tilde{f}(x), \quad \frac{\partial v}{\partial t}\Big|_{t=0} = \tilde{F}(x)\,, \quad 0\le x\le l \qquad (2.38)$$

boshlang‘ich shartlarni va

$$v\big|_{x=0}=0, \quad u\big|_{x=l}=0, \quad t\ge 0 \qquad (2.39)$$

chegaraviy shartlarni qanoatlantiruvchi yechimi, bu yerda

$$g_1(x,t)=g(x,t)-\xi_1''(t)-(\xi_2''(t)-\xi_1''(t))\frac{x}{l}.$$

3- misol. Aralash masalani Fure usuli bilan yeching: $\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial^2 u}{\partial x^2}, \quad t>0,$

$$0<x<1, \; u\big|_{t=0}=0, \; \frac{\partial u}{\partial t}\bigg|_{t=0}=1+x, \quad 0\le x\le 1, \; u\big|_{x=0}=t, \quad u\big|_{x=1}=2t, \quad t\ge 0.$$

Chegaraviy shartlar bir jinsli emas (torning chetki nuqtalari qo‘zg‘aluvchan). Masalaning shartiga ko‘ra: $\xi_1(t)=t, \; \xi_2(t)=2t.$

Yordamchi funksiya kiritamiz:

$$w(x,t)=\xi_1(t)+(\xi_2(t)-\xi_1(t))\frac{x}{l}=t+tx=t(1+x).$$

Bu funksiya uchun $w\big|_{x=0}=t, \quad w\big|_{x=1}=2t.$

Berilgan masalaning yechimini

$$u(x,t)=v(x,t)+w(x,t)$$

ko‘rinishida izlaymiz, bu yerda $v(x,t)$ - noma’lum funksiya.

$v=u-w$ funksiya uchun

$$\frac{\partial^2 v}{\partial t^2}=\frac{\partial^2 v}{\partial x^2}, \; t>0, \; 0<x<1, \; v\big|_{x=0}=0, \quad v\big|_{x=1}=0,$$

$$v\big|_{t=0}=f(x)-\xi_1(0)-(\xi_2(0)-\xi_1(0))\frac{x}{l}=0,$$

$$\frac{\partial v}{\partial t}\bigg|_{t=0}=F(x)-\xi_1'(0)-(\xi_2'(0)-\xi_1'(0))\frac{x}{l}=1+x-1-x=0$$

bo‘ladi. Bu masala yagona $v(x,t)\equiv 0$ yechimga ega bo‘ladi.

Demak, berilgan masalaning yechimi

$$u(x,t)=t(1+x).$$

4.2.2. Torning tebranish tenglamasini yechshning keng qo‘llaniladigan usullaridan biri *D’alamber* (*xarakteristikalar, yugiruvchi to‘lqinlar*) usulidir. Bu usulning asosida

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2} \qquad (2.40)$$

tenglamani

$$\xi=x-at, \quad \eta=x+at. \qquad (2.41)$$

almashtirishlar orqali

$$\frac{\partial^2 u}{\partial\xi\partial\eta} = 0.$$

tenglamaga keltirish yotadi.

Bu tenglama

$$u = \theta_1(\xi) + \theta_2(\eta)$$

yoki

$$u(x,t) = \theta_1(x-at) + \theta_2(x+at) \tag{2.42}$$

yechimga ega bo‘ladi, bu yerda θ_1, θ_2 - o‘z argumentlari bo‘yicha ikki marta differensiallanuvchi ixtiyoriy funksiyalar.

(2.42) ifoda bilan aniqlanuvchi $u(x,t)$ funksiya (2.40) to‘lqin tenglamasining *umumiy yechimi* (*D’alamber yechimi*) deyiladi. (2.40) tenglamaning har qanday yechimi θ_1 va θ_2 funksiyalarning mos tanlanishida (2.42) ko‘rinishda ifodalanadi. Bunda $u = \theta_1(x-at)$ to‘g‘ri to‘lqinlarni ifodalaydi ($\theta_1(x)$ egri chiziq o‘ng tomonga a tezlik bilan suriladi), $u = \theta_2(x+at)$ teskari to‘lqinlarni ifodalaydi ($\theta_2(x)$ egri chiziq chap tomonga a tezlik bilan suriladi). (2.42) yechim to‘g‘ri va teskari to‘lqinlar yig‘indisidan iborat bo‘ladi.

θ_1 va θ_2 funksiyalarni topish, ya’ni torning tebranish qonunini aniqlash uchun boshlang‘ich shartlardan, ayrim masalalarda chegaraviy shartlardan ham foydalaniladi.

Chegaralanmagan tor uchun bir jinsli Koshi masalasi quyidagicha qo‘yiladi:

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}, \quad t>0, \quad -\infty < x < \infty \tag{2.43}$$

tenglamaning

$$u\big|_{t=0} = f(x), \quad \frac{\partial u}{\partial t}\bigg|_{t=0} = F(x), \quad -\infty \le x \le \infty \tag{2.44}$$

boshlang‘ich shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping. Bunda $t=0$ vaqtda $f(x)$ funksiya torning shaklini, $F(x)$ funksiya esa tor bo‘ylab $\frac{\partial u}{\partial t}$ tezliklarning taqsimotini ifodalaydi.

Koshi masalasining yechimi

$$u(x,t) = \frac{f(x-at)+f(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz. \tag{2.45}$$

D’alamber formulasi bilan topiladi.

4- misol. Koshi masalasini D'alamber usuli bilan yeching:

1) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \ t > 0, \ -\infty < x < +\infty, \ u\Big|_{t=0} = x^2, \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0;$

2) $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \ t > 0, \ -\infty < x < +\infty, \ u\Big|_{t=0} = 0, \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = \cos x;$

3) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \ t > 0, \ -\infty < x < +\infty, \ u\Big|_{t=0} = e^{-x^2}, \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = \frac{x}{1+x^2}.$

1) Masalaning shartiga ko'ra: $a = 1, \ f(x) = x^2, \ F(x) = 0.$

U holda (2.45) formula bilan topamiz:

$$u(x,t) = \frac{(x-t)^2 + (x+t)^2}{2} = x^2 + t^2.$$

2) D'alamber formulasidan $a = 2, \ f(x) = 0, \ F(x) = \cos x$ larda topamiz:

$$u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} \cos z dz = \frac{1}{4}(\sin(x+2t) - \sin(x+2t)) = \frac{1}{2}\cos x \sin 2t.$$

3) Bu masalada $a = 1, \ f(x) = e^{-x^2}, \ F(x) = \frac{x}{1+x^2}.$

U holda D'alamber formulasiga ko'ra

$$u(x,t) = \frac{e^{-(x-t)^2} + e^{-(x+t)^2}}{2} + \frac{1}{2}\int_{x-t}^{x+t}\frac{zdz}{1+z^2} = \frac{e^{-(x^2+t^2)}(e^{2tx} + e^{-2tx})}{2} + \frac{1}{4}\ln(1+x^2)\Big|_{x-t}^{x+t} =$$

$$= e^{-(x^2+t^2)} ch 2xt + \frac{1}{4}\ln\left(\frac{1+(x+t)^2}{1+(x-t)^2}\right).$$

Chegaralanmagan tor uchun bir jinsli bo'lmagan Koshi masalasi quyidagicha qo'yiladi:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + g(x,t), \quad t > 0, \quad -\infty < x < \infty \tag{2.46}$$

to'lqin tenglamasining

$$u\Big|_{t=0} = f(x), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = F(x), \quad -\infty \le x \le \infty \tag{2.47}$$

boshlang'ich shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping.

Bu masalaning $u(x,t)$ yechimi

$$u(x,t) = v(x,t) + w(x,t) \tag{2.48}$$

yig'indi ko'rinishida izlanadi. Bunda $v(x,t)$ yechim torning majburiy tebranishini, ya’ni boshlang‘ich harakatsiz tashqi $g(x,t)$ kuch ta’sirida harakatlanuvchi tebranishlarni ifodalaydi, $w(x,t)$ yechim esa faqat boshlang‘ich harakatlar natijasida hosil bo‘luvchi erkin tebranishlarni ifodalaydi.

Bunda funksiya $v(x,t)$

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial x^2} + g(x,t) \tag{2.49}$$

bir jinsli bo‘lmagan tenglamaning

$$v\big|_{t=0} = 0, \quad \left.\frac{\partial v}{\partial t}\right|_{t=0} = 0, \quad -\infty \le x \le \infty \tag{2.50}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimi.

$w(x,t)$ funksiya esa

$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2} \tag{2.51}$$

bir jinsli tenglamaning

$$w\big|_{t=0} = f(x), \quad \left.\frac{\partial w}{\partial t}\right|_{t=0} = F(x), \quad -\infty \le x \le \infty \tag{2.52}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimi.

Bunda (2.51) - (2.52) masalaning yechimi

$$w(x,t) = \frac{f(x-at) + f(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} F(z)dz. \tag{2.53}$$

D’alamber formulasi bilan topiladi.

(2.49) - (2.50) masalaning yechimini topish ushbu masalani yechishga keltiriladi:

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial x^2} \tag{2.54}$$

bir jinsli tenglamaning

$$v\big|_{t=0} = 0, \quad \left.\frac{\partial v}{\partial t}\right|_{t=\tau} = g(x,\tau), \quad -\infty \le x \le \infty \tag{2.55}$$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini toping.

Bu yerda τ - prametr. Bu masalaning yechimi

$$v(x,t) = \frac{1}{2a}\int_0^t \left(\int_{x-a(t-\tau)}^{x+a(t-\tau)} g(z,\tau)dz \right) d\tau \tag{2.56}$$

kabi topiladi.

U holda (2.46) tenglamaning (2.47) boshlang‘ich shartlarni qanoatlantiruvchi yechimi

$$u(x,t)=\frac{f(x-at)+f(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}F(z)dz+\frac{1}{2a}\int_0^t\left(\int_{x-a(t-\tau)}^{x+a(t-\tau)}g(z,\tau)dz\right)d\tau \qquad (68.85)$$

bo‘ladi. Bu formulaga *Dyuamel formulasi* deyiladi.

5- misol. Tor uchun bir jinsli bo‘lmagan Koshi masalasini yeching:

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+x, \quad t>0, \quad -\infty<x<+\infty, \quad u\big|_{t=0}=\cos x, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=\sin x, \quad t\ge 0.$$

Masalaning shartiga ko‘ra: $a=1,\ f(x)=\cos x,\ F(x)=\sin x,\ g(x,t)=x.$

Masalaning yechimini Dyuamel formulasi bilan topamiz:

$$u(x,t)=\frac{f(x-at)+f(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}F(z)dz+\frac{1}{2a}\int_0^t\left(\int_{x-a(t-\tau)}^{x+a(t-\tau)}g(z,\tau)dz\right)d\tau=$$

$$=\frac{\cos(x-t)+\cos(x+t)}{2}+\frac{1}{2}\int_{x-t}^{x+t}\sin z dz+\frac{1}{2}\int_0^t\left(\int_{x-(t-\tau)}^{x+(t-\tau)}zdz\right)d\tau=$$

$$=\cos x\cos t-\frac{1}{2}\cos z\Big|_{x-t}^{x+t}+\frac{1}{2}\cdot\int_0^t\frac{z^2}{2}\Bigg|_{x-t+\tau}^{x+t+\tau}d\tau=\cos x\cos t-\sin x\sin t+\int_0^t(xt-x\tau)d\tau=$$

$$=\cos(x+t)+\left(xt\tau-\frac{x}{2}\tau^2\right)\Bigg|_0^t=\cos(x+t)+\frac{1}{2}xt^2.$$

Mashqlar

4.2.1. Chegaralangan torning erkin tebranishi haqidagi masalani Fure usuli bilan yeching:

1) $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}, t>0,\ 0<x<l,\ u\big|_{x=0}=0,\ u\big|_{x=l}=0, u\big|_{t=0}=\begin{cases}\dfrac{2hx}{l}, & 0\le x\le\dfrac{l}{2},\\ \dfrac{2h(l-x)}{l}, & \dfrac{l}{2}\le x\le l\end{cases}, \left.\dfrac{\partial u}{\partial t}\right|_{t=0}=0;$

2) $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2},\ t>0,\ 0<x<l,\ u\big|_{x=0}=0,\ u\big|_{x=l}=0,\ u\big|_{t=0}=0,\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0}=\sin\dfrac{\pi}{l}x;$

3) $\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial^2 u}{\partial x^2}, t>0,\ 0<x<\pi,\ u\big|_{x=0}=0,\ u\big|_{x=\pi}=0,\ u\big|_{t=0}=\sin x,\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0}=\sin x;$

4) $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}, t>0,\ 0<x<l,\ u\big|_{x=0}=0,\ u\big|_{x=l}=0, u\big|_{t=0}=\sin\dfrac{5\pi}{l}x,\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0}=0.$

4.2.2. Chegaralangan torning majburiy tebranishi haqidagi masalani Fure usuli bilan yeching:

1) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \sin \pi x, \; t > 0, \; 0 < x < 1, \; u\big|_{t=0} = 0, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 0, \quad u\big|_{x=0} = 0, \quad u\big|_{x=1} = 0;$

2) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + (4t - 8)\sin 2x, \; t > 0, \; 0 < x < \pi, \; u\big|_{t=0} = 0, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 0, u\big|_{x=0} = 0, \; u\big|_{x=\pi} = 0.$

4.2.3. Aralash masalani Fure usuli bilan yeching:

1) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; 0 < x < l, \; u\big|_{t=0} = 0, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 0, \quad u\big|_{x=0} = 0, \quad u\big|_{x=l} = t;$

2) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; 0 < x < l, \; u\big|_{t=0} = 0, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 0, \; u\big|_{x=0} = 0, \; u\big|_{x=l} = C = const.$

4.2.4. Koshi masalasini D'alamber usuli bilan yeching:

1) $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = \sin x, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 1;$

2) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = x, \; \frac{\partial u}{\partial t}\Big|_{t=0} = -x;$

3) $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = x^2, \; \frac{\partial u}{\partial t}\Big|_{t=0} = x;$

4) $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = \sin x, \; \frac{\partial u}{\partial t}\Big|_{t=0} = \cos x;$

5) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = \frac{\sin x}{x}, \; \frac{\partial u}{\partial t}\Big|_{t=0} = 0;$

6) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = e^{-x^2}, \; \frac{\partial u}{\partial t}\Big|_{t=0} = \sin 2x;$

7) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = \frac{x}{1+x^2}, \; \frac{\partial u}{\partial t}\Big|_{t=0} = \sin x;$

8) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; t > 0, \; -\infty < x < +\infty, \; u\big|_{t=0} = \sin x, \; \frac{\partial u}{\partial t}\Big|_{t=0} = \frac{1}{\sin 2x}.$

4.2.5. Tor uchun bir jinsli bo'lmagan Koshi masalasini yeching:

1) $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} + 2x, \quad t > 0, \quad -\infty < x < +\infty, \quad u\big|_{t=0} = x^2, \quad \frac{\partial u}{\partial t}\Big|_{t=0} = \cos x, \quad t \ge 0;$

2) $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 2t, \quad t > 0, \quad -\infty < x < +\infty, \quad u\big|_{t=0} = \sin x, \quad \frac{\partial u}{\partial t}\Big|_{t=0} = 2, \quad t \ge 0.$

4.3. ISSIQLIK O‘TKAZUVCHANLIK TENGLAMALARINI YECHISH

Issiqlik o‘tkazuvchanlik tenglamasi uchun Koshi masalasi. Shekli sterjenda issiqlikning tarqalishi. Issiqlik o‘tkazuvchanlik tenglamalarini Fure usuli bilan yechish

4.3.1. Bir jinsli sterjenning issiqlik o‘tkazuvchanlik tenglamasi uchun Koshi masalasi quyidagicha qo‘yiladi:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2},\ t > 0,\ -\infty < x < +\infty \tag{3.1}$$

tenglamaning

$$u\big|_{t=0} = f(x),\ -\infty < x < +\infty \tag{3.2}$$

boshlang‘ich shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping, bu yerda $a^2 = \frac{k}{c\rho}$, ρ - sterjerning zichligi, c - sterjerning solishtirma issiqlik sig‘imi, k - sterjerning issiqlik o‘tkazuvchanlik koeffitsiyenti.

Bu masala ushbu fizik ma’noga ega: bir jinsli cheksiz sterjenning boshlang‘ich $t = 0$ vaqtdagi ma’lum $f(x)$ temperaturasiga asosan istalgan $t > 0$ vaqtdagi temperaturasini toping.

Masalaning yechimi

$$u(x,t) = \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{+\infty} f(\lambda) e^{-\frac{(x-\lambda)^2}{4a^2 t}} d\lambda,\ t > 0. \tag{3.3}$$

ko‘rinishda topiladi. Bu formula *Puasson integrali* deb ataladi.

1- misol. Koshi masalasini yeching:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ t > 0,\ -\infty < x < +\infty,$$

$$u\big|_{t=0} = e^{-\frac{x^2}{2}},\ -\infty < x < +\infty.$$

Masalaning yechimini Puasson integrali bilan topamiz:

$$u(x,t) = \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{+\infty} f(\lambda) e^{-\frac{(x-\lambda)^2}{4a^2 t}} d\lambda = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{2}} e^{-\frac{(x-\lambda)^2}{4t}} d\lambda =$$

$$= \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2}{2}} e^{-\frac{x^2 - 2x\lambda + \lambda^2}{4t}} d\lambda = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\frac{\lambda^2(1+2t) - 2xt + x^2}{4t}} d\lambda =$$

$$= \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} e^{-\frac{(1+2t)}{4t}\left(\lambda^2 - \frac{2x}{1+2t} + \frac{x^2}{(1+2t)^2}\right) - \frac{x^2}{2(1+2t)}} d\lambda = \frac{1}{2\sqrt{\pi t}} e^{-\frac{x^2}{2(1+2t)}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\frac{(1+2t)}{2t}\left(\lambda - \frac{x}{1+2t}\right)^2} d\lambda.$$

$\frac{\sqrt{1+2t}}{\sqrt{2t}}\left(\lambda - \frac{x}{1+2t}\right) = z$ o‘zgaruvchini almashtirish bajaramiz:

$$u(x,t) = \frac{1}{2\sqrt{\pi t}} e^{-\frac{x^2}{2(1+2t)}} \frac{\sqrt{2t}}{\sqrt{1+2t}} \int_{-\infty}^{+\infty} e^{-\frac{z^2}{2}} dz =$$

$$= \frac{1}{2\sqrt{\pi t}} \cdot \frac{\sqrt{2t}}{\sqrt{1+2t}} \cdot \sqrt{2\pi} \cdot e^{-\frac{x^2}{2(1+2t)}} = \frac{1}{\sqrt{1+2t}} e^{-\frac{x^2}{2(1+2t)}}.$$

Demak, berilgan Koshi masalasining yechimi

$$u(x,t) = \frac{1}{\sqrt{1+2t}} e^{-\frac{x^2}{2(1+2t)}}, \ t > 0.$$

4.3.2. Chekli l uzunlikka ega va Ox o‘qining $0 \le x \le l$ kesmasida joylashgan sterjenda issiqlikning tarqalishi haqidagi masalani qo‘yish uchun

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + g(x,t)$$

tenglama va

$$u\big|_{t=0} = f(x)$$

boslang‘ich shartdan tashqari chegaravish shartlar berilishi kerak bo‘ladi. Bunda shegaraviy shartlarning asosan uch turda beriladi.

Birinchi turdagi chegaraviy shartlarda sterjenning chetlarida

$$u\big|_{x=0} = \xi_1(t), \quad u\big|_{x=l} = \xi_2(t)$$

temperaturalar beriladi, bu yerda $\xi_1(t), \xi_2(t)$- jarayon o‘rganilayotgan $0 \le t \le T$ vaqt oralig‘ida berilgan funksiyalar.

Ikkinchi turdagi chegaraviy shartlarda sterjenning chetlarida

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \eta_1(t), \quad \left.\frac{\partial u}{\partial x}\right|_{x=l} = \eta_2(t)$$

hosilalar beriladi.

Uchinchi turdagi chegaraviy shartlarda sterjenning chetlarida funksiya va uning hosilasi orasidagi chiziqli moslik

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \lambda(u(0,t) - \theta(t)), \quad \left.\frac{\partial u}{\partial x}\right|_{x=l} = -\lambda(u(l,t) - \theta(t))$$

beriladi, bu yerda $\theta(t)$- atrof muhit temperaturasi (aniq funksiya), λ- issiqlik almashinish koeffitsiyenti.

Issiqlik o‘tkazuvchanlik tenglamasi uchun birinchi turdagi chegaraviy shartlar bilan berilgan aralash masala quyidagich qo‘yiladi:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + g(x,t), \ \ 0 < x < l, \ t > 0 \qquad (3.4)$$

tenglamaning

$$u\big|_{t=+0} = f(x),\ 0 \le x \le l \tag{3.5}$$

boshlang‘ich shartni va

$$u\big|_{x=0} = \xi_1(t),\quad u\big|_{x=l} = \xi_2(t),\ \ t \ge 0 \tag{3.6}$$

chegaraviy shartlarni qanoatlantiruvchi, $0 < x < l,\ t > 0$ da ikki marta differensiallanuvchi $u(x,t)$ yechimini toping.

$u(x,t)$ funksiya yopiq $D = \{0 \le x \le l, 0 \le t \le T\}$ sohada uzluksiz, ya’ni $f(x), \xi_1(t), \xi_2(t)$ funksiyalar D sohada uzluksiz va $f(0) = \xi_1(0)$, $f(l) = \xi_2(t)$ moslashish shartlari bajarilsa $u(x,t)$ funksiya uchun quyidagi teoremalar o‘rinli bo‘ladi.

1-Teorema (maksimal qiymat prinsipi). Agar $u(x,t) \in C(D)$ funksiya $\{0 < x < l, 0 < t \le T\}$ sohaning nuqtalarida $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ issiqlik o‘tkazuvchanlik tenglamasini qanoatlantirsa, u holda $u(x,t)$ funksiyaning maksimum va minimum qiymatlariga yoki boshlang‘ich $t = 0$ vaqtda yoki chegaraning $x = 0$ yoki $x = l$ kesmalarida erishiladi.

2-Teorema. (3.4)-(3.6) masalaning yechimi $\{0 < x < l, 0 < t \le T\}$ to‘g‘ri to‘rtburchakda yagona bo‘ladi.

3-Teorema. (3.4)-(3.6) masalaning yechimi boshlang‘ich va chegaraviy shartlarga uzluksiz bog‘liq bo‘ladi.

4.3.3. Issiqlik o‘tkazuvchanlik tengalamasi uchun birinchi turdagi chegaraviy shartlar bilan berilgan aralash masalasini qaraymiz:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + g(x,t), t > 0,\quad 0 < x < l \tag{3.7}$$

bir jinsli bo‘lmagan tenglamaning

$$u\big|_{t=+0} = f(x),\ 0 \le x \le l \tag{3.8}$$

boshlang‘ich shartni va

$$u\big|_{x=0} = \xi_1(t),\quad u\big|_{x=l} = \xi_2(t),\ \ t \ge 0 \tag{3.9}$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping.

1. (3.19)-(3.21) masalaning sodda holini qaraymiz:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2},\ \ t > 0,\quad 0 < x < l \tag{3.10}$$

bir jinsli tenglamaning

$$u\big|_{t=+0} = f(x),\ 0 \le x \le l \tag{3.11}$$

boshlang‘ich shartni va

$$u\big|_{x=0} = 0,\quad u\big|_{x=l} = 0,\ \ t \ge 0 \tag{3.12}$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping.

(3.10) tenglamaning (3.11) chegaraviy shartlarni qanoatlantiruvchi notrival yechimi

$$u(x,t) = T(t)X(x) \qquad (3.13)$$

ko‘rinishda izlanadi. $u(x,t)$ ning (3.13) shaklini (3.10) tenglamaga qo‘yilsa

$$\frac{T'(t)}{a^2T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$

tenglik va bundan ikkita oddiy differensial tenglamani hosil qilinadi:

$$T'(t) + \lambda a^2 T(t) = 0, \qquad (3.14)$$

$$X''(x) + \lambda X(x) = 0. \qquad (3.15)$$

(3.10) tenglama (3.12) chegaraviy shartlarni qanoatlantiruvchi notrival yechimga ega bo‘lishi uchun (3.15) tenglamaning

$$X(0) = 0, \quad X(l) = 0 \qquad (3.16)$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish, ya’ni (3.14)-(3.16) masalani yechish kerak bo‘ladi.

Bu masala

$$\lambda_k = \left(\frac{k\pi}{l}\right)^2, \; k \in 1,2,... \qquad (3.17)$$

xos qiymatlarda

$$X_k(x) = \sin\frac{k\pi}{l}x, \; k \in 1,2,...$$

yechimlarga (xos funksiyalarga) ega bo‘ladi.

$\lambda = \lambda_k$ da (3.14) tenglamaning umumiy yechimi

$$T_k(t) = A_k e^{-\left(\frac{k\pi a}{l}\right)^2 t}$$

ko‘rinishda bo‘ladi, bu yerda A_k - ixtiyoriy o‘zgarmaslar.

3.17) xos qiymatlarning

$$u_k(x,t) = X_k(x)T_k(t) = A_k e^{-\left(\frac{k\pi a}{l}\right)^2 t} \sin\frac{k\pi}{l}x, \; k = 1,2,...$$

funksiyalar har qanday A_k o‘zgarmaslarda (3.14) tenglamani (3.16) chegaraviy shartlarda qanoatlantiradi.

(3.10) tenglama chiziqli va bir jinsli bo‘lgani sababli yechimlarning har qanday chekli yig‘indisi, jumladan

$$u_k(x,t) = \sum_{k=1}^{\infty} A_k e^{-\left(\frac{k\pi a}{l}\right)^2 t} \sin\frac{k\pi}{l}x \qquad (3.18)$$

qator ham (3.10) tenglamaning yechimi bo‘ladi.

Bu qatorning A_k koeffitsiyentlari

$$A_k = \frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}xdx \qquad (k=1,2,...) \tag{3.19}$$

kabi topiladi.

2- misol. Issiqlik o‘tkazuvchanlik masalasini yeching:

$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$, $t>0,\ 0<x<2,\ u\big|_{t=0} = x$, $0\le x\le 2$, $u\big|_{x=0} = 0$, $u\big|_{x=2} = 0$, $t\ge 0$.

Masalaning shartiga ko‘ra: $a=2,\ l=2,\ f(x)=x.$

Izlanayotgan yechim

$$u_k(x,t) = \sum_{k=1}^{\infty} A_k e^{-\left(\frac{k\pi a}{l}\right)^2 t}\sin\frac{k\pi}{l}x = \sum_{k=1}^{\infty} A_k e^{-(k\pi)^2 t}\sin\frac{k\pi}{2}x$$

ko‘rinishda bo‘ladi, bu yerda

$$A_k = \frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}xdx = \int_0^2 x\sin\frac{k\pi}{2}xdx.$$

Bundan bo‘laklab integrallash formulasiga ko‘ra

$$A_k = \int_0^2 x\sin\frac{k\pi}{2}xdx = -\frac{2}{k\pi}x\cos\frac{k\pi}{2}x\Bigg|_0^2 + \frac{2}{k\pi}\int_0^2\cos\frac{k\pi}{2}xdx =$$

$$= -\frac{4}{k\pi}\cos k\pi + \left(\frac{2}{k\pi}\right)^2\sin\frac{k\pi}{2}x\Bigg|_0^2 = -\frac{4}{k\pi}(-1)^k = 4\frac{(-1)^{k+1}}{k\pi}.$$

Demak, berilgan masalaning yechimi

$$u_k(x,t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}e^{-(k\pi)^2 t}\sin\frac{k\pi}{2}x.$$

2. (3.10)-(3.12) masalaning bosqa holini qaraymiz:

$$\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2} + g(x,t),\quad t>0,\quad 0<x<l \tag{3.13}$$

bir jinsli bo‘lmagan tenglamaning

$$u\big|_{t=+0} = f(x),\ 0\le x\le l \tag{3.14}$$

boshlang‘ich shartni va

$$u\big|_{x=0} = 0,\quad u\big|_{x=l} = 0,\quad t\ge 0 \tag{3.15}$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping.

Bu masalaning $u(x,t)$ yechimi

$$u(x,t) = v(x,t) + w(x,t) \tag{3.16}$$

yig‘indi ko‘rinishida izlanadi.

Bunda $v(x,t)$ funksiya

$$\frac{\partial v}{\partial t}=a^2\frac{\partial^2 v}{\partial x^2}+g(x,t) \tag{3.17}$$

bir jinsli bo‘lmagan tenglamaning

$$v\big|_{t=0}=0 \tag{3.18}$$

boshlang‘ich shartni va

$$v\big|_{x=0}=0,\quad v\big|_{x=l}=0 \tag{3.19}$$

chegaraviy shartlarni qanoatlantiruvchi yechimi.

$w(x,t)$ funksiya esa

$$\frac{\partial w}{\partial t}=a^2\frac{\partial^2 w}{\partial x^2} \tag{3.20}$$

bir jinsli tenglamaning

$$w\big|_{t=0}=f(x) \tag{3.21}$$

boshlang‘ich shartni va

$$w\big|_{x=0}=0,\quad w\big|_{x=l}=0 \tag{3.22}$$

chegaraviy shartlarni qanoatlantiruvchi yechimi.

Masalanining $v(x,t)$ yechimi

$$v(x,t)=\sum_{k=1}^{\infty}T_k(t)\sin\frac{k\pi}{l}x \tag{3.23}$$

ko‘rinishda izlanadi.

Bu yerda $\sin\frac{k\pi}{l}x$

$$X''(x)+\lambda X(x)=0,\ \ X(0)=0,\ \ X(l)=0$$

masalaning xos funksiyalari.

$v(x,t)$ ni (3.23) ko‘rinishda (3.27) tenglamaga qo‘yib

$$\sum_{k=1}^{\infty}\left(T_k'(t)+\frac{k^2\pi^2a^2}{l^2}T_k(t)\right)\sin\frac{k\pi}{l}x=g(x,t). \tag{3.24}$$

qator hosil qilanadi.

$g(x,t)$ funksiyani $(0;l)$ intervalda Fure qatoriga sinuslar bo‘yicha (xos funksiyalarning) yoyilsa

$$g(x,t)=\sum_{k=1}^{\infty}g_k(t)\sin\frac{k\pi}{l}x, \tag{3.25}$$

kelib chiqadi, bu yerda

$$g_k(t)=\frac{2}{l}\int_0^l g(\xi,t)\sin\frac{k\pi}{l}\xi d\xi . \tag{3.26}$$

$g(x,t)$ funksiyaning (3.24) va (3.25) yoyilmalarni taqqoslab, $T_k(t)$ noma'lum funksiyaga nisbatan ushbu

$$T_k'(t)+\left(\frac{k\pi a}{l}\right)^2 T_k(t)=g_k(t),\ k=1,2,,... \qquad (3.27)$$

differensial tenglama hosil qilinadi.

(3.27) tenglama bilan aniqlanuvchi $v(x,t)$ yechim (3.18) boshlang'ich shartlarni qanoatlantirishi uchun $T_k(t)$ funksiya

$$T_k(0)=0\,,\ \ k=1,2,... \qquad (3.28)$$

shartlarni qanoatlantirishi kerak.

(3.27) tenglamaning (3.28) boshlang'ich shartlardagi yechimi

$$T_k(t)=\int_0^l g_k(t)e^{-\left(\frac{k\pi a}{t}\right)^2(t-\tau)}d\tau\,,\ \ k=1,2,... \qquad (3.29)$$

kabi topiladi, bu yerda $g_k(t)$ (3.26) formula bilan aniqlanadi.

$T_k(t)$ning bu qiymatlarini (3.23) qatorga qo'yib, (3.17)-(3.19) masalaning $v(x,t)$ yechimini topiladi.

U holda (3.13)-(3.15) masalaning yechimi

$$u_k(x,t)=\sum_{k=1}^{\infty}A_k e^{-\left(\frac{k\pi a}{l}\right)^2 t}\sin\frac{k\pi}{l}x+\sum_{k=1}^{\infty}T_k(t)\sin\frac{k\pi}{l}x \qquad (3.30)$$

ko'rinishda bo'ladi, bu yerda

$$T_k(t)=\int_0^l g_k(t)e^{-\left(\frac{k\pi a}{t}\right)^2(t-\tau)}d\tau\,,\ \ A_k=\frac{2}{l}\int_0^l f(x)\sin\frac{k\pi}{l}xdx \qquad (k=1,2,...).$$

3- misol. Issiqlik o'tkazuvchanlik masalasini yeching:

$$\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2}+2\cos t\sin 4x,\quad u\big|_{t=0}=0\,,\ 0<x\le\pi\,,\ u\big|_{x=0}=0\,,\ u\big|_{x=\pi}=0\,,\ \ t\ge 0\,.$$

Sterjenda boshlang'ich issiqlik taqsimoti yo'q. Shu sababli bu masalaning $u(x,t)$ yechimi

$$u(x,t)=v(x,t)=\sum_{k=1}^{\infty}T_k(t)\sin\frac{k\pi}{l}x$$

ko'rinishga keladi.

Bunda

$$T_k(t)=\int_0^l g_k(t)e^{-\left(\frac{k\pi a}{t}\right)^2(t-\tau)}d\tau$$

bo'ladi, bu yerda

$$g_k(t)=\frac{2}{l}\int_0^l g(\xi,t)\sin\frac{k\pi}{l}\xi d\xi\,.$$

U holda berilgan masala uchun

$$g_k(t)=\frac{2}{\pi}\int_0^{\pi}2\cos t\sin 4\xi\sin k\xi d\xi=(bundan\quad k=4)=\frac{2\cos t}{\pi}\int_0^{\pi}(1-\cos 2\xi)d\xi=2\cos t.$$

$$T_4(t)=2\int_0^t\cos\tau e^{-(t-\tau)}d\tau=2\cos\tau e^{-(t-\tau)}\Big|_0^t+2\int_0^t\sin\tau e^{-(t-\tau)}d\tau=2(\cos t-e^{-t})+$$

$$+2\sin\tau e^{-(t-\tau)}\Big|_0^t-2\int_0^t\cos\tau e^{-(t-\tau)}d\tau=2(\cos t+\sin t-e^{-t})-T_4(t).$$

Bundan

$$T_k(t)=\cos t+\sin t-e^{-t}.$$

Demak, masalaning izlanayotgam yechimi

$$u(x,t)=(\cos t+\sin t-e^{-t})\sin 4x.$$

3. (3.7)-(3.9) masalani qaraymiz:

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}+g(x,t)\,,t>0,\quad 0<x<l \tag{3.31}$$

bir jinsli bo‘lmagan tenglamaning

$$u\big|_{t=+0}=f(x)\,,\ 0\le x\le l \tag{3.32}$$

boshlang‘ich shartni va

$$u\big|_{x=0}=\xi_1(t)\,,\quad u\big|_{x=l}=\xi_2(t)\,,\ \ t\ge 0 \tag{3.33}$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping.

(3.31)-(3.33) masalaning $u(x,t)$ yechimini

$$u(x,t)=v(x,t)+w(x,t) \tag{3.34}$$

yig‘indi ko‘rinishida izlanadi, bu yerda

$$w(x,t)=\xi_1(t)+(\xi_2(t)-\xi_1(t))\frac{x}{l}. \tag{3.35}$$

U holda (3.31)-(3.33) masalaning yechimi $v(x,t)$ funksiya uchun (3.33)-(3.35) masalaning yechimiga keltiriladi.

4- misol. Aralash masalani Fure usuli bilan yeching:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+2\sin t\sin x,\ u\big|_{t=0}=0\,,\ \ 0\le x\le\pi\,,\ \ u\big|_{x=0}=\pi\,,\ \ u\big|_{x=\pi}=2\pi\,,\ \ t\ge 0\,.$$

Masalaning shartiga ko‘ra: $\xi_1(t)=\pi,\ \xi_2(t)=2\pi.$

Yordamchi funksiya kiritamiz:

$$w(x,t)=\xi_1(t)+(\xi_2(t)-\xi_1(t))\frac{x}{l}=\pi+x.$$

Bu funksiya uchun $w\big|_{x=0}=\pi\,,\quad w\big|_{x=\pi}=2\pi\,.$

Berilgan masalaning yechimini

$$u(x,t) = v(x,t) + w(x,t)$$

ko‘rinishida izlaymiz, bu yerda $v(x,t)$- noma’lum funksiya. Bu funksiya uchun

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + 2\sin t \sin x,\ t > 0,\ 0 < x < \pi,\ v\big|_{t=0} = 0,\ 0 \le x \le \pi,\ v\big|_{x=0} = 0,\ v_{x=\pi} = 0,\ t \ge 0.$$

bo‘ladi.

Bu masalaning $v(x,t)$ yechimi

$$v(x,t) = \sum_{k=1}^{\infty} T_k(t) \sin\frac{k\pi}{l}x$$

ko‘rinishga keladi.

Bunda

$$T_k(t) = \int_0^l g_k(t) e^{-\left(\frac{k\pi a}{t}\right)^2 (t-\tau)} d\tau$$

bo‘ladi, bu yerda

$$g_k(t) = \frac{2}{l}\int_0^l g(\xi,t)\sin\frac{k\pi}{l}\xi d\xi .$$

U holda berilgan masala uchun

$$g_k(t) = \frac{2}{\pi}\int_0^{\pi} 2\sin t \sin\xi \sin k\xi d\xi = (bundan\ \ k=1) = \frac{2\sin t}{\pi}\int_0^{\pi}(1-\cos 2\xi)d\xi = 2\sin t.$$

$$T_1(t) = 2\int_0^t \sin\tau e^{-(t-\tau)}d\tau = 2\sin\tau e^{-(t-\tau)}\Big|_0^t - 2\int_0^t \cos\tau e^{-(t-\tau)}d\tau = 2\sin t - 2\cos\tau e^{-(t-\tau)}\Big|_0^t +$$

$$-2\int^t \sin\tau e^{-(t-\tau)}d\tau = 2(\sin t - \cos t + e^{-t}) - T_1(t).$$

Bundan

$$T_1(t) = \sin t - \cos t + e^{-t}.$$

Demak, masalaning izlanayotgam yechimi

$$u(x,t) = (\sin t - \cos t + e^{-t})\sin x + x + \pi.$$

Mashqlar

4.3.1. Koshi masalasini yeching:

1) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ t > 0,\ -\infty < x < +\infty,\ u\big|_{t=0} = e^{-k^2x^2},\ k > 0 - const,\ -\infty < x < +\infty;$

2) $\frac{\partial u}{\partial t} = 9\frac{\partial^2 u}{\partial x^2},\ t > 0,\ -\infty < x < +\infty,\ u\big|_{t=0} = e^{-\frac{x^2}{4}},\ -\infty < x < +\infty.$

4.3.2. Issiqlik o‘tkazuvchanlik masalasini yeching:

1) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2},\quad t\geq 0,\quad 0\leq x\leq l,\quad u\big|_{t=0}=u_0=const,\quad u\big|_{x=0}=0,\quad u\big|_{x=l}=0;$

2) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2},\ t\geq 0,\quad 0\leq x\leq \pi,\quad u\big|_{t=0}=\sin x,\ u\big|_{x=0}=0,\quad u\big|_{x=\pi}=0;$

3) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2},\ t\geq 0,\quad 0\leq x\leq l,\quad u\big|_{t=0}=2\sin 3x,\quad u\big|_{x=0}=0,\quad u\big|_{x=l}=0;$

4) $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2},\ t\geq 0,\ 0\leq x\leq l,\ u\big|_{t=0}=x(l-x),\ u\big|_{x=0}=0,\ u\big|_{x=l}=0;$

5) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad t\geq 0,\ 0\leq x\leq l,\quad u\big|_{t=0}=3\sin\frac{\pi}{l}x-5\sin\frac{2\pi}{l}x,\quad u\big|_{x=0}=0,\quad u\big|_{x=l}=0;$

6) $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2},\ t\geq 0,\ 0\leq x\leq 6,\quad u\big|_{t=0}=3\sin 3\pi x-\sin 4\pi x,\quad u\big|_{x=0}=0,\quad u\big|_{x=6}=0;$

7) $\frac{\partial u}{\partial t}=\frac{1}{36}\frac{\partial^2 u}{\partial x^2}+3\cos t\sin 6x,\ t\geq 0,\ 0\leq x\leq \pi,\ u\big|_{t=0}=0,\ u\big|_{x=0}=0,\ u\big|_{x=\pi}=0;$

8) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\sin\frac{\pi}{4}x,\ t\geq 0,\ 0\leq x\leq 2,\ u\big|_{t=0}=0,\ u\big|_{x=0}=0,\ u\big|_{x=2}=0;$

9) $\frac{\partial u}{\partial t}=\frac{1}{4}\frac{\partial^2 u}{\partial x^2}+10\sin t\sin x,\ t\geq 0,\ 0\leq x\leq \pi,\ u\big|_{t=0}=2\sin 4x,\ u\big|_{x=0}=0,\ u\big|_{x=\pi}=0;$

10) $\frac{\partial u}{\partial t}=\frac{1}{9}\frac{\partial^2 u}{\partial x^2}+t\sin 3x,\ t\geq 0,\ 0\leq x\leq \pi,\ u\big|_{t=0}=2\sin 2x,\ u\big|_{x=0}=0,\ u\big|_{x=\pi}=0;$

11) $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+t\sin x,\ t\geq 0,\ 0\leq x\leq \pi,\ u\big|_{t=0}=0,\ u\big|_{x=0}=0,\ u\big|_{x=\pi}=e^{-t};$

12) $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}+\sin 2x,\ t\geq 0,\ 0\leq x\leq \pi,\ u\big|_{t=0}=\sin 3x,\ u\big|_{x=0}=e^{t},\ u\big|_{x=\pi}=e^{2t}.$

4.4. LAPLAS TENGLAMALARINI YECHISH

Laplas tenglamalari. Dirixle masalasini to‘g‘ti to‘rtburchak uchun yechish. Dirixle masalasini halqa uchun yechish. Dirixle masalasini doira uchun yechish

4.4.1. Fizik tabiatning har xil statsoinar (vaqtga bog‘liq bo‘lmagan) jarayonlarini o‘rganish elliptik tipdagi tenglamalarga keltiriladi. Elliptik tipdagi sodda tenglamalarga

$$\Delta u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$

Laplas tenglamalari kiradi.

Bunda

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

differensial operatorga *Laplas operatori* (yoki *Laplasian*) deyiladi.

Ikki x va y erkli o‘zgaruvchining $u = u(x, y)$ funksiyasi uchun Laplas tenglamasi

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

ko‘rinishda bo‘ladi. Uning yechimi kompleks o‘zgaruvchining biror D sohada analitik funksiyasi $f(z) = u(x, y) + iv(x, y)$ ning haqiqiy va mavhum qismlaridan iborat bo‘ladi.

Bir o‘zgaruvchining $u = u(x)$ funksiyasi uchun

$$\Delta u = \frac{\partial^2 u}{\partial x^2} = 0$$

tenglama kelib chiqadi. Bu tenglamaning yechimi $u = C_1 x + C_2$ funksiya bo‘ladi, bu yerda C_1, C_2 - ixtiyoriy o‘zgarmaslar.

Agar $u = u(x, y, z)$ funksiya $V \subset R^3$ sohada ikkinchi tartibligacha hosilalari bilan uzluksiz bo‘lsa va Laplas tenglamasini qanoatlantirsa, bu funksiyaga *garmonik funksiya* deyiladi.

σ sirt bilan chegaralangan V soha berilgan bo‘lsin. Bunda Laplas tenglamasi uchun tipik masala bunday qo‘yiladi: V sohada garmonik va σ sirtda quyidagi ko‘rinishlardan biri bilan berilgan chegaraviy shartlarni qanoatlantiruvchi $u(M), M \in V$ funksiyani toping.

1. $u\big|_{\sigma} = f_1(M),\ M \in V$ - *birinchi chegaraviy masala* yoki *Dirixle masalasi*;

2. $\left.\frac{\partial u}{\partial n}\right|_{\sigma} = f_2(M),\ M \in V$ - *ikkinchi chegaraviy masala* yoki *Neyman masalasi*;

3. $\left.\left(\frac{\partial u}{\partial n} + hu\right)\right|_{\sigma} = f_3(M),\ M \in V$ - *uchinchi chegaraviy masala*, bu yerda

f_1, f_2, f_3, h- berilgan funksiyalar, $\frac{\partial u}{\partial n}$- σ sirtga tashqi normal yo‘nalishi bo‘yicha hosila.

Masalaning yechimi qayerda, σ sirt bilan chegaralangan V sohaning ichkarisida yoki tashqarisida, izlanishiga qarab, $\Delta u = 0$ tenglama uchun chegaraviy masalalar *ichki va tashqi chegaraviy masalalar*ga bo‘linadi.

Laplas operatori silindrik (qutb) koordinatalarda

$$\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2} \qquad \left(\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}\right)$$

formula bilan, sferik koordinatalarda

$$\Delta u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2}$$

formula bilan aniqlanadi.

Sferik simmetriyaga ega $u = u(r)$ yechim

$$\frac{d}{dr}\left(r^2\frac{\partial u}{\partial r}\right) = 0$$

tenglamadan $u = \frac{C_1}{r} + C_2 \quad (C_1, C_2 - const)$ yoki, masalan, $C_1 = 1$va $C_2 = 0$ da

$$u_0(r) = \frac{1}{r}$$

kabi aniqlanadi. Bu yechimga *Laplas tenglamasining fazodagi fundamental yechimi* deyiladi.

Silindrik yoki doiraviy (ikki o‘zgaruvchi uchun) simmetriya ega yechim

$$\frac{d}{dr}\left(r\frac{\partial u}{\partial r}\right) = 0$$

tenglamadan $u = C_1 \ln r + C_2 \quad (C_1, C_2 - const)$ yoki, masalan, $C_1 = -1$va $C_2 = 0$ da

$$u_0(r) = \ln\frac{1}{r}$$

kabi topiladi. Bu yechimga *Laplas tenglamasining tekislikdagi fundamental yechimi* deyiladi.

4.2. Dirixle masalasini to‘g‘ti to‘rtburchak uchun qaraymiz:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (4.1)$$

Laplas tenglamasining

$$u\big|_{x=0} = \psi_1(y),\ u\big|_{x=a} = \psi_2(y),\ -\frac{b}{2} \le y \le \frac{b}{2},\ u\big|_{y=-\frac{b}{2}} = \varphi_1(x),\ u\big|_{y=\frac{b}{2}} = \varphi_2(x),\ 0 \le x \le a \quad (4.2)$$

chegaraviy shartlarni qanoatlantiruvchi $u(x, y)$ yechimini toping.

4.2.1. (4.1)-(4.2) masalaning xususiy hollaridan birini, ya’ni $\psi_1(y) = \psi_2(y) = 0$ bo‘lgan holini qaraymiz. Bunda chegaraviy shartlar

$$u\big|_{x=0} = 0,\quad u\big|_{x=a} = 0,\quad -\frac{b}{2} \le y \le \frac{b}{2},\quad u\big|_{y=-\frac{b}{2}} = \varphi_1(x),\quad u\big|_{y=\frac{b}{2}} = \varphi_2(x),\ 0 \le x \le a \quad (4.3)$$

kabi qo‘yiladi.

(4.1)-(4.3) masalani Fure usuli bilan yechishda (4.1) tenglamaning (4.3) chegaraviy shartlarni qanoatlantiruvchi notrival yechimi

$$u(x,y) = X(x)Y(y) \tag{4.4}$$

ko‘rinishda izlanadi.

$u(x,y)$ ning (4.4) shaklini (4.1) tenglamaga qo‘yilsa

$$\frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} = -\lambda\,,\ \lambda > 0$$

tenglik yoki bu tenglikdan ikkita oddiy differensial tenglama kelib chiqadi:

$$X''(x) + \lambda X(x) = 0, \tag{4.5}$$

$$Y''(y) - \lambda Y(y) = 0. \tag{4.6}$$

(4.5) tenglamaning umumiy yechimi

$$X(x) = C_1 \cos\lambda x + C_2 \sin\lambda x$$

kabi aniqlanadi. Bunda

$$X(0) = 0, \quad X(a) = 0$$

shartlardan

$$C_1 = 0, \quad C_2 \neq 0\,, \quad \sin\lambda a = 0$$

va

$$\lambda_k = \frac{k\pi}{a},\ k \in 1,2,...\ , \quad X_k(x) = C\sin\frac{k\pi}{a}x,\ k \in 1,2,...$$

kelib chiqadi.

$\lambda = \lambda_k$ da (4.6) tenglamaning umumiy yechimi

$$Y(y) = D_{11}e^{\frac{k\pi}{a}y} + D_{21}e^{-\frac{k\pi}{a}y} = D_1 ch\frac{k\pi}{a}y + D_2 sh\frac{k\pi}{a}y$$

ko‘rinishda bo‘ladi, bu yerda D_1, D_2 - ixtiyoriy o‘zgarmaslar.

Bunda (4.4) tenglamaning umumiy yechimi

$$u_k(x,y) = X_k(x)Y_k(y) = \left(A_k ch\frac{k\pi}{a}y + B_k sh\frac{k\pi}{a}y\right)\sin\frac{k\pi}{a}x\,,\ k = 1,2,... \tag{4.7}$$

kabi aniqlanadi.

(4.4) tenglama chiziqli va bir jinsli bo‘lgani sababli

$$u(x,y) = \sum_{k=1}^{\infty}\left(A_k ch\frac{k\pi}{a}y + B_k sh\frac{k\pi}{a}y\right)\sin\frac{k\pi}{a}x \tag{4.8}$$

qator (4.4) tenglamaning yechimi bo‘ladi.

$u(x,y)$ funksiya Laplas tenglamasini (4.3) shartlarning birinchi ikkitasida qanoatlantiradi. (4.3) shartlarning qolgan ikkitasidan A_k, B_k koeffitsiyentlar topiladi.

A_k va B_k koeffitsiyentlarni (4.8) qatorga qo‘yib, (4.1)-(4.3) masalaning yechimini topiladi:

$$u(x,y)=\sum_{k=1}^{\infty}\left(\frac{a_k+b_k}{2ch\frac{k\pi b}{2a}}ch\frac{k\pi}{a}y+\frac{b_k-a_k}{2sh\frac{k\pi b}{2a}}sh\frac{k\pi}{a}y\right)\sin\frac{k\pi}{a}x, \qquad (4.9)$$

bu yerda

$$a_k=\frac{2}{a}\int_0^a\varphi_1(x)\sin\frac{k\pi}{a}xdx, \quad b_k=\frac{2}{a}\int_0^a\varphi_2(x)\sin\frac{k\pi}{a}xdx. \qquad (4.10)$$

4.2.2. (4.1)-(4.2) masalaning xususiy hollaridan yana birini, ya'ni $\varphi_1(x)=\varphi_2(x)=0$ bo'lgan holini qaraymiz. Bunda chegaraviy shartlar

$$u\big|_{x=0}=\psi_1(y),\ \ u\big|_{x=a}=\psi_2(y),\ -\frac{b}{2}\le y\le\frac{b}{2},\quad u\big|_{y=-\frac{b}{2}}=0,\quad u\big|_{y=\frac{b}{2}}=0,\ 0\le x\le a \quad (4.11)$$

kabi qo'yiladi.

Oldingi (4.1)-(4.3) masala yechimning natijalaridan foydalanish uchun

$$x'=y+\frac{b}{2}, \qquad y'=x-\frac{a}{2}$$

o'zgaruvchilar kiritilsa, (4.11) chegaraviy shartlar (4.3) ko'rinishga keladi, a va b sonlarining o'rni almashadi, (4.10) yechimga φ_1 va φ_2 funksiyalar o'rniga ψ_1 va ψ_2 funksiyalar kiradi. Agar x va y o'zgaruvchilarga qaytilsa, (4.1)-(4.11) masalaning yechimi

$$u(x,y)=\sum_{k=1}^{\infty}\left(\frac{c_k+d_k}{2ch\frac{k\pi a}{2b}}ch\frac{k\pi}{b}\left(x-\frac{a}{2}\right)+\frac{d_k-c_k}{2sh\frac{k\pi a}{2b}}sh\frac{k\pi}{b}\left(x-\frac{a}{2}\right)\right)\sin\frac{k\pi}{b}\left(y+\frac{b}{2}\right) \quad (4.12)$$

ko'rinishda bo'ladi, bu yerda

$$c_k=\frac{2}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}}\psi_1(y)\sin\frac{k\pi}{b}\left(y+\frac{b}{2}\right)dy,\ \ d_k=\frac{2}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}}\psi_2(y)\sin\frac{k\pi}{b}\left(y+\frac{b}{2}\right)dy. \qquad (4.13)$$

Dirixle masalasini to'g'ti to'rtburchak uchun umumiy yechimi, ya'ni (4.1)-(4.2) masalaning yechimi (4.10) va (4.12) yechimlarning yig'indisidan iborat bo'ladi.

1- misol. Laplas tenglamasining to'g'ti to'rtburchakning ichki qismida

$$u\big|_{x=0}=0,\ u\big|_{x=2}=0,\ -1\le y\le 1,\ u\big|_{y=-1}=0,\ u\big|_{y=1}=\sin\frac{k\pi}{2}x,\ 0\le x\le 2$$

chegaraviy shartlarni qanoatlantiruvchi $u(x,y)$ yechimini toping.

⊖ Masala yechimining koeffitsiyentlarini topamiz:

$$a_k=\frac{2}{a}\int_0^a\varphi_1(x)\sin\frac{k\pi}{a}xdx=\frac{2}{2}\int_0^2 0\cdot\sin\frac{k\pi}{2}xdx=0.$$

$$b_k = \frac{2}{a}\int_0^a \varphi_2(x)\sin\frac{k\pi}{a}xdx = \frac{2}{2}\int_0^2 \sin\frac{k\pi}{2}x\sin\frac{k\pi}{2}x =$$
$$= \frac{1}{2}\int_0^2 (1-\cos k\pi x)dx = \frac{1}{2}\left(x\Big|_0^2 - \frac{1}{k\pi}\sin k\pi x\Big|_0^2 \right) = 1.$$

Bundan

$$u(x,y) = \sum_{k=1}^{\infty}\left(\frac{a_k + b_k}{2ch\frac{k\pi b}{2a}} ch\frac{k\pi}{a}y + \frac{b_k - a_k}{2sh\frac{k\pi b}{2a}} sh\frac{k\pi}{a}y \right)\sin\frac{k\pi}{a}x =$$

$$= \sum_{k=1}^{\infty}\left(\frac{0+1}{2ch\frac{k\pi 2}{2\cdot 2}} ch\frac{k\pi}{2}y + \frac{1-0}{2sh\frac{k\pi 2}{2\cdot 2}} sh\frac{k\pi}{2}y \right)\sin\frac{k\pi}{2}x = \sum_{k=1}^{\infty}\left(\frac{ch\frac{k\pi}{2}y}{2ch\frac{k\pi}{2}} + \frac{sh\frac{k\pi}{2}y}{2sh\frac{k\pi}{2}} \right)\sin\frac{k\pi}{2}x.$$

4.3. Dirixle masalasini halqa uchun qaraymiz: qutb koordinatalarida berilgan

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} = 0, \quad R_1 < r < R_2, \quad -\infty < \varphi < +\infty \qquad (4.14)$$

Laplas tenglamasining

$$u(r,\varphi + 2\pi) = u(r,\varphi) \qquad (4.15)$$

davriylik shartini va

$$u(R_1,\varphi) = f_1(\varphi), \quad u(R_2,\varphi) = f_2(\varphi) \qquad (4.16)$$

chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

Avval (4.14)-(4.16) masalaning xususiy holini, ya'ni doiraviy simmetriyaga ega yechimini topamiz. Bunda yechim φ ga bog‘liq bo‘lmaydi va

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) = 0 \qquad (4.17)$$

tenlamaning

$$u(R_1) = u_1, \quad u(R_2) = u_2 \quad (u_1, u_2 = const) \qquad (4.18)$$

chegaraviy shartlarni qanoatlantiruvchi $u(r)$ yechimini topish masalasi kelib chiqadi.

(4.17) tenglamaning umumiy yechimi

$$u = C_1 \ln r + C_2 \qquad (4.19)$$

kabi aniqlanadi, bu yerda C_1, C_2 - ixtiyoriy o‘zgarmaslar.

C_1 va C_2 o‘zgarmaslar (4.18) chegaraviy shartlardan topiladi.

O‘zgarmaslarni (4.19) tenglamaga qo‘yib, (4.17)-(4.19) masalaning yechimi topiladi:

$$u(r)=\frac{u_2-u_1}{\ln R_2-\ln R_1}\ln r+\frac{u_1\ln R_2-u_2\ln R_1}{\ln R_2-\ln R_1}. \qquad (4.20)$$

2- misol. Laplas tenglamasining $1\le r\le 2$ halqaning ichki qismida $u\big|_{r=1}=4,\quad u\big|_{r=2}=6$ chegaraviy shartlarni qanoatlantiruvchi $u(r)$ yechimini toping.

Masala doiraviy simmetriyaga ega. Shu sababli bu masalaning yechimi

$$u(r)=\frac{u_2-u_1}{\ln R_2-\ln R_1}\ln r+\frac{u_1\ln R_2-u_2\ln R_1}{\ln R_2-\ln R_1}=$$

$$=\frac{6-4}{\ln 2-\ln 1}\ln r+\frac{4\ln 2-6\ln 1}{\ln 2-\ln 1}=\frac{2}{\ln 2}\ln r+4.$$

Fure usuliga ko‘ra (4.14)-(4.16) masalaning notrival yechimi

$$u(r,\varphi)=R(r)\Phi(\varphi) \qquad (4.21)$$

ko‘rinishda ifodalanadi va ikkita oddiy differensial tenglamani hosil qilinadi:

$$\Phi''(\varphi)+\lambda\Phi(\varphi)=0, \qquad (4.22)$$

$$r^2R''(r)+rR'(r)-\lambda R(r)=0. \qquad (4.23)$$

$u(r,\varphi+2\pi)=u(r,\varphi)$ davriylikning bajarilishi shartiga ko‘ra (4.22) tenglamadan $\lambda=k^2$ kelib chiqadi.

$\lambda=k^2$ da (4.23) tenglamaning umumiy yechimi

$$\Phi(\varphi)=A\cos k\varphi+B\sin k\varphi \qquad (4.24)$$

kabi aniqlanadi.

$k=0$ da (4.23) tenglama

$$R(r)=A_0\ln r+B_0 \qquad (4.25)$$

yechimga ega bo‘ladi. $k>0$ da (4.23) tenglamaning yechimi

$$R(r)=Cr^k+\frac{D}{r^k} \qquad (4.26)$$

kabi aniqlanadi.

(4.24),(4.25),(4.26) yechimlar umumlashtirilib,

$$u(r,\varphi)=\sum_{k=1}^{\infty}\left(\left(A_kr^k+\frac{B_k}{r^k}\right)\cos k\varphi+\left(C_kr^k+\frac{D_k}{r^k}\right)\sin k\varphi\right)+A_0\ln r+B_0 \qquad (4.27)$$

qator hosil qilinadi.

Tenglamaning noma’lum koeffitsiyentlari (4.16) chegaraviy shartlardan topiladi va ularni (4.27) tenglamaga qo‘yib, (4.14)-(4.16) masalaning yechimi topiladi:

$$u(r,\varphi)=\frac{c_0-a_0}{2(\ln R_2-\ln R_1)}\ln r+\frac{a_0\ln R_2-c_0\ln R_1}{2(\ln R_2-\ln R_1)}+$$

$$+\sum_{k=1}^{\infty}\left(\frac{(c_kR_2^k-a_kR_1^k)r^{2k}-(c_kR_1^k-a_kR_2^k)R_1^kR_2^k}{(R_2^{2k}-R_1^{2k})r^k}\cos k\varphi+\right.$$

$$\left.+\frac{(d_kR_2^k-b_kR_1^k)r^{2k}-(d_kR_1^k-b_kR_2^k)R_1^kR_2^k}{(R_2^{2k}-R_1^{2k})r^k}\sin k\varphi\right), \qquad (4.28)$$

bu yerda

$$a_0=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)d\tau,\quad a_k=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\cos k\tau d\tau,\quad b_k=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\sin k\tau d\tau,$$

$$c_0=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)d\tau,\quad c_k=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\cos k\tau d\tau,\quad d_k=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\sin k\tau d\tau. \qquad (4.29)$$

3- misol. Laplas tenglamasining $1\le r\le 2$ halqaning ichki qismida $u\big|_{R_1=1}=1-\cos\varphi,\quad u\big|_{R_2=2}=\sin 2\varphi$ chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

Masala yechimining koeffitsiyentlarini topamiz.

$$a_0=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)d\tau=\frac{1}{\pi}\int_0^{2\pi}(1-\cos\tau)d\tau=\frac{1}{\pi}(\tau-\sin\tau)\Big|_0^{2\pi}=2.$$

$$a_k=\frac{1}{\pi}\int_0^{2\pi}f_1(\tau)\cos k\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}(1-\cos\tau)\cos k\tau d\tau(k=1)=$$

$$=\frac{1}{\pi}\left(\sin\tau\Big|_0^{2\pi}-\frac{1}{2}\int_0^{2\pi}(1+\cos 2\tau)d\tau\right)=-1.\quad b_k=0.\ c_0=0.\ c_k=0.$$

$$d_k=\frac{1}{\pi}\int_0^{2\pi}f_2(\tau)\sin k\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}\sin 2\tau\sin 2\tau d\tau=(k=2)=\frac{1}{2\pi}\int_0^{2\pi}(1-\cos 4\tau)d\tau=1.$$

U holda

$$u(r,\varphi)=\frac{c_0-a_0}{2(\ln R_2-\ln R_1)}+\frac{a_0\ln R_2-c_0\ln R_1}{2(\ln R_2-\ln R_1)}+$$

$$+\sum_{k=1}^{\infty}\left(\frac{(c_kR_2^k-a_kR_1^k)r^{2k}-(c_kR_1^k-a_kR_2^k)R_1^kR_2^k}{(R_2^{2k}-R_1^{2k})r^k}\cos k\varphi+\right.$$

$$\left.+\frac{(d_kR_2^k-b_kR_1^k)r^{2k}-(d_kR_1^k-b_kR_2^k)R_1^kR_2^k}{(R_2^{2k}-R_1^{2k})r^k}\sin k\varphi\right)=\frac{0-2}{2(\ln 2-\ln 1)}+\frac{2\ln 2-0\ln 1}{2(\ln 2-\ln 1}+$$

$$+\frac{(0\cdot 2+1\cdot 1)r^2-(0\cdot 1+1\cdot 2)\cdot 1\cdot 2}{(2^2-1)r}\cos\varphi+\frac{(1\cdot 2^2-0\cdot 1)r^4-(1\cdot 1+0\cdot 2^2)\cdot 1\cdot 2^2}{(2^4-1)r^2}\sin 2\varphi=$$

$$=1-\frac{\ln r}{\ln 2}+\frac{r^2-4}{3r}\cos\varphi+\frac{4r^4-4}{15r^2}\sin 2\varphi.$$

4.4. Dirixle masalasini doira uchun qaraymiz: qutb koordinatalarida berilgan

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}=0,\quad 0<r<R,\quad -\infty<\varphi<+\infty \tag{4.30}$$

Laplas tenglamasining

$$u(r,\varphi+2\pi)=u(r,\varphi) \tag{4.31}$$

davriylik shartini va

$$u(R,\varphi)=f(\varphi) \tag{4.32}$$

chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

Doira uchun (4.25) va (4.26) yechimlarda $A_0=0$ va $D=0$ bo‘ladi, chunki aks holda $r=0$ nuqtada funksiya uzilishga ega bo‘ladi. $A_0=0$ da $B_0=\frac{a_0}{2}=\frac{c_0}{2}$ kelib chiqadi.

U holda (4.27) yechim

$$u(r,\varphi)=\frac{a_0}{2}+\sum_{k=1}^{\infty}(A_k\cos k\varphi+B_k\sin k\varphi)r^k \tag{4.33}$$

ko‘rinishni oladi, bu yerda A_k, B_k koeffitsiyentlar (4.32) shartdan topiladi.

Bunda (4.31)-(4.33) masalaning yechimi

$$u(r,\varphi)=\frac{a_0}{2}+\sum_{k=1}^{\infty}(a_k\cos k\varphi+b_k\sin k\varphi)\left(\frac{r}{R}\right)^k \tag{4.34}$$

kabi aniqlanadi, bu yerda

$$a_0=\frac{1}{\pi}\int_0^{2\pi}f(\tau)d\tau,\quad a_k=\frac{1}{\pi}\int_0^{2\pi}f(\tau)\cos k\tau d\tau,\quad b_k=\frac{1}{\pi}\int_0^{2\pi}f(\tau)\sin k\tau d\tau. \tag{4.35}$$

4-misol. Laplas tenglamasining $2\le r$ doiraning ichki qismida $u\big|_{r=2}=\sin\varphi+\cos\varphi$ chtgaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

⊜ Masala yechimining koeffitsiyentlarini topamiz.

$$a_0=\frac{1}{\pi}\int_0^{2\pi}f(\tau)d\tau=\frac{1}{\pi}\int_0^{2\pi}(\sin\tau+\cos\tau)d\tau=\frac{1}{\pi}(-\cos\tau+\sin\tau)\Big|_0^{2\pi}=0.$$

$$a_k=\frac{1}{\pi}\int_0^{2\pi}f(\tau)\cos k\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}(\sin\tau+\cos\tau)\cos k\tau d\tau(k=1)=$$

$$=\frac{1}{\pi}\left(\frac{1}{2}\sin^2\tau\Big|_0^{2\pi}+\frac{1}{2}\int_0^{2\pi}(1+\cos 2\tau)d\tau\right)=\frac{1}{2\pi}\left(\tau+\frac{1}{2}\sin 2\tau\right)\Bigg|_0^{2\pi}=1.$$

$$b_k=\frac{1}{\pi}\int_0^{2\pi}f(\tau)\cos k\tau d\tau=\frac{1}{\pi}\int_0^{2\pi}(\sin\tau+\cos\tau)\sin k\tau d\tau(k=1)=$$

$$=\frac{1}{\pi}\left(\frac{1}{2}\sin^2\tau\Big|_0^{2\pi}+\frac{1}{2}\int_0^{2\pi}(1-\cos 2\tau)d\tau\right)=\frac{1}{2\pi}\left(\tau-\frac{1}{2}\sin 2\tau\right)\Bigg|_0^{2\pi}=1.$$

U holda

$$u(r,\varphi)=\frac{a_0}{2}+\sum_{k=1}^{\infty}(a_k\cos k\varphi+b_k\sin k\varphi)\left(\frac{r}{R}\right)^k=(\cos\varphi+\sin\varphi)\left(\frac{r}{2}\right).$$

(4.36) formulaga Fure koeffitsiyentlarining qiymatlarini qo‘yib, integrallash tartibini o‘zgartirilsa, uning soddaroq ko‘rinishi hosil qilinadi:

$$u(r,\varphi)=\frac{1}{2\pi}\int_0^{2\pi}f(\tau)\left(\frac{R^2-r^2}{R^2-2rR\cos(\varphi-\tau)+r^2}\right)d\tau. \qquad (4.36)$$

Bu integralga *Puasson integrali* deyiladi.

5-misol. Laplas tenglamasining $1\le r$ doiraning ichki qismida $u\big|_{r=1}=5\sin\varphi$ chtgaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

Masalaning yechimini Puasson integrali bilan topamiz:

$$u(r,\varphi)=\frac{1}{2\pi}\int_0^{2\pi}f(\tau)\left(\frac{R^2-r^2}{R^2-2rR\cos(\varphi-\tau)+r^2}\right)d\tau=$$

$$=\frac{1}{2\pi}\int_0^{2\pi}5\sin\tau\left(\frac{1-r^2}{1-2r\cos(\varphi-\tau)+r^2}\right)d\tau=-\frac{5(1-r^2)}{2\pi}\int_0^{2\pi}\frac{\sin((\varphi-\tau)-\varphi)}{1-2r\cos(\varphi-\tau)+r^2}d\tau=$$

$$=-\frac{5(1-r^2)}{2\pi}\int_0^{2\pi}\frac{\sin(\varphi-\tau)\cos\varphi-\cos(\varphi-\tau)\sin\varphi}{1-2r\cos(\varphi-\tau)+r^2}d\tau=$$

$$=-\frac{5(1-r^2)\cos\varphi}{4r\pi}\int_0^{2\pi}\frac{2r\sin(\varphi-\tau)}{1-2r\cos(\varphi-\tau)+r^2}d\tau+$$

$$+\frac{5(1-r^2)\sin\varphi}{4r\pi}\int_0^{2\pi}\frac{2r\cos(\varphi-\tau)}{1-2r\cos(\varphi-\tau)+r^2}d\tau=$$

$$=-\frac{5(1-r^2)\cos\varphi}{4r\pi}\ln|1-2r\cos(\varphi-\tau)+r^2|\Big|_0^{2\pi}-$$

$$-\frac{5(1-r^2)\sin\varphi}{4r\pi}\int_0^{2\pi}\frac{(1-2r\cos(\varphi-\tau)+r^2)-(1+r^2)}{1-2r\cos(\varphi-\tau)+r^2}d\tau=$$

$$=-\frac{5(1-r^2)\cos\varphi}{4r\pi}(\ln|1-2r\cos(\varphi-2\pi)+r^2|-\ln|1-2r\cos\varphi+r^2|)-$$

$$-\frac{5(1-r^2)\sin\varphi}{4r\pi}\left(\int_0^{2\pi}d\tau-(1+r^2)\int_0^{2\pi}\frac{d\tau}{1-2r\cos(\varphi-\tau)+r^2}\right)=$$

$$=-\frac{5(1-r^2)\sin\varphi}{4r\pi}\left(\tau\Big|_0^{2\pi}-(1+r^2)\int_0^{2\pi}\frac{d\tau}{1-2r\cos(\varphi-\tau)+r^2}\right)=$$

$$=-\frac{5(1-r^2)\sin\varphi}{4r\pi}\left(2\pi-(1+r^2)\int_0^{2\pi}\frac{d\tau}{1-2r\cos(\varphi-\tau)+r^2}\right).$$

Oxirgi integral uchun

$$\int_0^{2\pi}\frac{dx}{a^2\pm 2ab\cos x+b^2}=\frac{2\pi}{|a^2-b^2|}$$

formula o‘rinli bo‘ladi.

U holda

$$u(r,\varphi)=-\frac{5(1-r^2)\sin\varphi}{4r\pi}\left(2\pi-\frac{2\pi(1+r^2)}{1-r^2}\right)=5r\sin\varphi.$$

Mashqlar

4.4.1. Laplas tenglamasining to‘g‘ti to‘rtburchakning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(x,y)$ yechimini toping:

1) $u\big|_{x=0}=0,\quad u\big|_{x=\pi}=0,\quad -1\le y\le 1,\quad u\big|_{y=-1}=0,\quad u\big|_{y=1}=\sin 3x,\ 0\le x\le\pi$;

2) $u\big|_{x=0}=0,\quad u\big|_{x=2}=\sin 4y,\quad -\pi\le y\le\pi,\quad u\big|_{y=-\pi}=0,\quad u\big|_{y=\pi}=0,\ 0\le x\le 2$;

3) $u\big|_{x=0}=\sin y,\quad u\big|_{x=\pi}=0,\quad -\pi\le y\le\pi,\quad u\big|_{y=-\pi}=0,\quad u\big|_{y=\pi}=\sin 2x,\ 0\le x\le\pi$;

4) $u\big|_{x=0}=0,\quad u\big|_{x=\pi}=\sin 2y,\quad -\pi\le y\le\pi,\quad u\big|_{y=-\pi}=\sin 3x,\quad u\big|_{y=\pi}=0,\ 0\le x\le\pi$.

4.4.2. Laplas tenglamasining halqaning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(r)$ yechimini toping.

1) $u\big|_{r=2}=4,\quad u\big|_{r=3}=8,\quad 2\le r\le 3$; 2) $u\big|_{r=3}=7,\quad u\big|_{r=5}=10,\quad 3\le r\le 5$.

4.4.3. Laplas tenglamasining halqaning ichki qismida berilgan chegaraviy shartlarni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

1) $u\big|_{r=1}=\sin\varphi,\ u\big|_{r=3}=\cos\varphi,\ 1\le r\le 3$; 2) $u\big|_{r=2}=\cos 2\varphi,\ u\big|_{r=5}=\sin 3\varphi,$ $2\le r\le 3$;

3) $u\big|_{r=1}=4,\ u\big|_{r=3}=\sin\varphi,\ 1\le r\le 2$; 4) $u\big|_{r=2}=\sin\varphi,\ u\big|_{r=3}=\sin 2\varphi,\ 2\le r\le 3$.

4.4.4. Laplas tenglamasining doiraning ichki qismida berilgan chtgaraviy shartni qanoatlantiruvchi $u(r,\varphi)$ yechimini toping.

1) $u\big|_{r=3}=3+5\cos\varphi,\ r\le 3$; 2) 1) $u\big|_{r=2}=2+3\sin\varphi,\ r\le 2$;

3) $u\big|_{r=3}=\sin^2\varphi,\ r\le 3$; 4) 1) $u\big|_{r=2}=\cos^2\varphi,\ r\le 2$.

5-NAZORAT ISHI

1. Berilgan ikki o'zgaruvchining ikkinchi tartibli tenglamalarini kanonik ko'rinishga keltiring.

2. $\dfrac{\partial^2 u}{\partial t^2} = a^2 \dfrac{\partial^2 u}{\partial x^2}$ tor tebranish tenglamasining $u\big|_{t=0} = \varphi(x),\quad \dfrac{\partial u}{\partial t}\Big|_{t=0} = \psi(x)$ boshlang'ich shartlarni qanoatlantiruvchi yechimini D'alamber usuli bilan yeching.

1-variant

1. $u_{xx} + 5u_{xy} + 4u_{yy} = 0.$ **2.** $a = 9,\ \varphi(x) = e^{-x},\ \psi(x) = \sin x + \cos x.$

2-variant

1. $u_{xx} - 2u_{xy} + u_{yy} + 7u_x + 9u_y + u = 0.$ **2.** $a = 4,\ \varphi(x) = e^{-7x},\ \psi(x) = \sin 4x.$

3-variant

1. $u_{xx} - 4u_{xy} + 13u_{yy} = 0.$ **2.** $a = 4,\ \varphi(x) = x^4,\ \psi(x) = \cos 2x.$

4-variant

1. $u_{xx} + 4u_{xy} + 5u_{yy} + u_x + 2u_y = 0.$ **2.** $a = 9,\ \varphi(x) = e^{-2x},\ \psi(x) = \cos 3x.$

5-variant

1. $u_{xx} + 2u_{xy} - 3u_{yy} + 2u_x + 6u_y = 0.$ **2.** $a = 16,\ \varphi(x) = \sin 5x,\ \psi(x) = \cos 2x.$

6-variant

1. $12u_{xx} + 13u_{xy} + u_{yy} = 0.$ **2.** $a = 4,\ \varphi(x) = x^3,\ \psi(x) = \sin 2x.$

7-variant

1. $u_{xx} + 5u_{xy} - 6u_{yy} + 10u = 0.$ **2.** $a = 9,\ \varphi(x) = x^3,\ \psi(x) = \cos 4x.$

8-variant

1. $u_{xx} + 2u_{xy} + u_{yy} - 5u_x = 0.$ **2.** $a = 16,\ \varphi(x) = e^{5x},\ \psi(x) = \cos 2x.$

9-variant

1. $u_{xx}+4u_{xy}+4u_{yy}+3u_x+6u_y=0.$ **2.** $a=4,\ \varphi(x)=e^{-6x},\ \psi(x)=\cos 3x.$

10-variant

1. $u_{xx}-6u_{xy}+13u_{yy}=0.$ **2.** $a=16,\ \varphi(x)=e^{5x},\ \psi(x)=\sin 2x.$

11-variant

1. $y^2u_{xx}-2xyu_{xy}+x^2u_{yy}=0.$ **2.** $a=4,\ \varphi(x)=e^{-4x},\ \psi(x)=\sin 3x.$

12-variant

1. $y^2u_{xx}-x^2u_{yy}-2xu_x=0.$ **2.** $a=4,\ \varphi(x)=e^{-5x},\ \psi(x)=\cos 2x.$

13-variant

1. $u_{xx}+xu_{yy}=0,\ x>0.$ **2.** $a=16,\ \varphi(x)=\sin 5x,\ \psi(x)=\sin 2x.$

14-variant

1. $u_{xx}+4u_{xy}+20u_{yy}=0.$ **2.** $a=16,\ \varphi(x)=\sin 4x,\ \psi(x)=\cos x.$

15-variant

1. $u_{xx}+20u_{xy}+36u_{yy}+u_x=0.$ **2.** $a=16,\ \varphi(x)=\sin 3x,\ \psi(x)=e^{4x}.$

16-variant

1. $u_{xx}+13u_{xy}+36u_{yy}-5u_y=0.$ **2.** $a=9,\ \varphi(x)=\cos 5x,\ \psi(x)=e^{5x}.$

17-variant

1. $u_{xx}+3u_{xy}-4u_{yy}=0.$ **2.** $a=16,\ \varphi(x)=\cos 3x,\ \psi(x)=e^{9x}.$

18-variant

1. $u_{xx}+x^2u_{yy}=0.$ **2.** $a=9,\ \varphi(x)=\sin 4x,\ \psi(x)=e^{8x}.$

19-variant

1. $u_{xx}+6u_{xy}-16u_{yy}+8u_x=0.$ **2.** $a=9,\ \varphi(x)=\sin 5x,\ \psi(x)=e^{6x}.$

20-variant

1. $u_{xx}-12u_{xy}+20u_{yy}+5u_x=0.$ **2.** $a=4,\ \varphi(x)=x^2,\ \psi(x)=\cos x.$

21-variant

1. $8u_{xx} - 13u_{xy} + 5u_{yy} = 0.$ **2.** $a = 9,\ \varphi(x) = x,\ \psi(x) = \cos 3x.$

22-variant

1. $u_{xx} + 9u_{xy} - 10u_{yy} = 0.$ **2.** $a = 4,\ \varphi(x) = x^2,\ \psi(x) = \sin x.$

23-variant

1. $20u_{xx} - u_{xy} - u_{yy} + 5u_x = 0.$ **2.** $a = 9,\ \varphi(x) = x^2,\ \psi(x) = \sin 3x.$

24-variant

1. $u_{xx} - 2xu_{xy} + u_{yy} = 0.$ **2.** $a = 9,\ \varphi(x) = x^4,\ \psi(x) = \sin 4x.$

25-variant

1. $u_{xx} - 16u_{xy} - 17u_{yy} = 0.$ **2.** $a = 16,\ \varphi(x) = \sin 4x,\ \psi(x) = \cos x.$

26-variant

1. $u_{xx} + 10u_{xy} + 29u_{yy} = 0.$ **2.** $a = 16,\ \varphi(x) = \sin 4x,\ \psi(x) = \sin x.$

27-variant

1. $u_{xx} + 20u_{xy} - 21u_{yy} = 0.$ **2.** $a = 4,\ \varphi(x) = \cos 5x,\ \psi(x) = \cos 2x.$

28-variant

1. $u_{xx} - 12u_{xy} + 20u_{yy} = 0.$ **2.** $a = 4,\ \varphi(x) = \cos 5x,\ \psi(x) = \sin 2x.$

29-variant

1. $u_{xx} + 9u_{xy} - 10u_{yy} = 0.$ **2.** $a = 9,\ \varphi(x) = \cos 4x,\ \psi(x) = \cos x.$

30-variant

1. $u_{xx} - 4xu_{xy} + 4u_{yy} + u_x = 0.$ **2.** $a = 9,\ \varphi(x) = \cos 3x,\ \psi(x) = \sin 5x.$

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JAVOBLAR

1.1. Ehtimollarni bevosita hisoblash

1.1.1. $\Omega=\{(ggg),(rgg),(grg),(ggr),(grr),(rgr),(rrg),(rrr)\}$, $A=\{(grr),(rgr),(rrg)\}$, $B=\{(ggg)\}$, $C=\{(ggg),(rgg),(grg),(ggr)\}$, $D=\{(ggg),(ggr),(rgg)\}$. **1.1.2.** $D=\{(\bar{a}\bar{b}c),(\bar{a}b\bar{c}),(\bar{a}\bar{b}\bar{c})\}$; $\overline{D}=\{(abc),(a\bar{b}c),(ab\bar{c}),(a\bar{b}\bar{c}),(\bar{a}bc)\}$. **1.1.3.** $B\setminus A-$ olingan son 5 bilan tugaydi; $A\cap B-$ olingan son 0 bilan tugaydi. **1.1.5.** 1140. **1.1.6.** 20. **1.1.7.** 6. **1.1.8.** 60. **1.1.9.** $\frac{5}{6}$.
1.1.10. 1) 0; 2) $\frac{1}{3}$. **1.1.11.** 1) $\frac{1}{3}$; 2) $\frac{1}{5}$; 3) 0; 4) $\frac{2}{3}$. **1.1.12.** 1) $\frac{1}{4}$; 2) $\frac{13}{28}$; 3) $\frac{3}{28}$. **1.1.3.** 1) 1; 2) $\frac{1}{5}$; 3) $\frac{3}{5}$. **1.1.14.** $\frac{1}{720}$. **1.1.15.** 1) $\frac{1}{9}$; 2) $\frac{1}{12}$; 3) $\frac{11}{36}$. **1.1.16.** 1) $\frac{1}{45}$; 2) $\frac{7}{45}$; 3) $\frac{1}{15}$. **1.1.17.** 1) $\frac{1}{12}$; 2) $\frac{1}{24}$; 3) $\frac{1}{12}$. **1.1.18.** 1) $\frac{1}{75600}$; 2) $\frac{1}{7560}$; 3) $\frac{1}{30240}$. **1.1.19.** 1) $\frac{1}{4096}$; 2) $\frac{1}{512}$; 3) $\frac{35}{2048}$.
1.1.20. 1) $\frac{1}{406}$; 2) $\frac{115}{203}$. **1.1.21.** 1) 0,0000005; 2) 0,00335. **1.1.22.** 1) $\frac{1}{6}$; 2) $\frac{10}{21}$; 3) $\frac{8}{15}$.
1.1.23. 0,3. **1.1.24.** 0,36. **1.1.25.** 1) $\frac{3}{44}$; 2) $\frac{3}{11}$; 3) $\frac{3}{22}$. **1.1.26.** 1) $\frac{1}{34}$; 2) $\frac{12}{17}$. **1.1.27.** 1) 0,39; 2) 0,11; 3) 0,5. **1.1.28.** 0,6. **1.1.29.** 0,05. **1.1.30.** 1) $\frac{3\sqrt{3}}{4\pi}$; 2) $\frac{2}{\pi}$; 3) $\frac{3\sqrt{3}}{2\pi}$. **1.1.31.** $\frac{2\sqrt{3}}{3\pi}$.

1.2. Ehtimollarni topishning asosiy formulalari

1.2.1. $\frac{5}{11}$. **1.2.2.** 0,6. **1.2.3.** 1) $\frac{5}{7}$; 2) $\frac{5}{6}$ yoki $\frac{2}{3}$. **1.2.4.** $\frac{3}{28}$. **1.2.5.** 0,75. **1.2.6.** 0,45. **1.2.7.** $\frac{11}{14}$.
1.2.8. 1) 0,098; 2) 0,188. **1.2.9.** 1) $\frac{7}{9}$. **1.2.10.** 1) 0,105; 2) 0,094. **1.2.11.** $\frac{57}{115}$. **1.2.12.** $\frac{2}{51}$.
1.2.13. 1) 0,018; 2) 0,044; 3) 0,648; 4) 0,954; 5) 0,998. **1.2.14.** 1) 0,096; 2) 0,188; 3) 0,336; 4) 0,788; 5) 0,976. **1.2.15.** 0,988. **1.2.16.** 0,7. **1.2.17.** 2. **1.2.18.** 5. **1.2.19.** $\frac{11}{36}$. **1.2.20.** 0,94.
1.2.21. $P(B)=0{,}7$, $P_A(B)=\frac{5}{6}$, $P_B(A)=\frac{5}{7}$. **1.2.22.** $P(B)=\frac{2}{3}$, $P(A\cdot B)=0.4$, $P(A+B)=\frac{23}{30}$.
1.2.23. 0,824. **1.2.24.** $\frac{3}{5}$. **1.2.25.** $\frac{3}{4}$. **1.2.26.** 0,93. **1.2.27.** 0,274. **1.2.28.** $\frac{10}{31}$. **1.2.29.** 1) 0,91; 2) 0,09; 3) 2– ta'minotchi. **1.2.30.** 1) 0,1725; 2) 0,8275; 3) 0,3172.

1.3.Sinashlarning takrorlanishi

1.3.1. 1) 0,3125; 2) 0.8125. **1.3.2.** 1) 0,246; 2) 0,738. **1.3.3.** 1) 0,0204; 2) 0,217.
1.3.4. $P_4(2)>P_6(3)>P_8(4)$. **1.3.5.** 14 va 15. **1.3.6.** $0{,}6\le p\le 0{,}62$. **1.3.7.** 55.
1.3.8. $99\le n\le 102$. **1.3.9.** 1) 0,0064; 2) 0,2624; 3) 0,73728; 4) 0,9776; 5) 0,26272; 6) 0,4096.
1.3.10. 1) 0,219; 2) 0,811; 3) 0,101; 4) 0,692; 5) 0,911, 6) 0,329. **1.3.11.** 1) 0,0054; 2) 0,9772.
1.3.12. 1) 0,002; 2) 0,8944. **1.3.13.** 1) 0,0113; 2) 0,0252; 3) 0,8962; 4) 0,7924.
1.3.14. 1) 0,0022; 2) 0,9938; 3) 1; 4) 0,0499. **1.3.15.** 0,1404. **1.3.16.** 0,0902. **1.3.17.** 8 va 0,1396. **1.3.18.** 1) 0,8647; 2) 0,2707. **1.3.19.** 1) 0,1805; 2) 0,3233; 3) 0,1805; 4) 0,8572.

1.3.20. 1) 0,224; 2) 0,1992; 3) 0,5768; 4) 0,9502.

1.4. Tasodifiy miqdorlar

1.4.1. $X=\begin{cases} 0 & 1 & 2 & 3 \\ 0{,}125 & 0{,}375 & 0{,}375 & 0{,}125 \end{cases}.$ **1.4.2.** $X=\begin{cases} 0 & 1 & 2 & 3 \\ 0{,}024 & 0{,}188 & 0{,}452 & 0{,}336 \end{cases}.$

1.4.3. $X=\begin{cases} 0 & 1 & 2 \\ 0{,}3 & 0{,}6 & 0{,}1. \end{cases}$ **1.4.4.** $X=\begin{cases} 1 & 2 & 3 & 4 \\ 0{,}571 & 0{,}286 & 0{,}114 & 0{,}029 \end{cases}.$

1.4.5. $F(x)=\begin{cases} 0, \ agar\ x\le 1, \\ 0{,}2, \ agar\ 1<x\le 2, \\ 0{,}8, \ agar\ 2<x\le 3, \\ 1, \ agar\ x>3. \end{cases}$ **1.4.6.** $F(x)=\begin{cases} 0, \ agar\ x\le 0, \\ 0{,}25, \ agar\ 0<x\le 1, \\ 0{,}75, \ agar\ 1<x\le 2, \\ 1, \ agar\ x>2. \end{cases}$

1.4.7. $F(x)=\begin{cases} 0, \ agar\ x\le 0, \\ 0{,}024, \ agar\ 0<x\le 1, \\ 0{,}212, \ agar\ 1<x\le 2, \\ 0{,}664, \ agar\ 2<x\le 3, \\ 1, \ agar\ x>3. \end{cases}$ **1.4.8.** $F(x)=\begin{cases} 0, \ agar\ x\le 1, \\ 0{,}4, \ agar\ 1<x\le 2, \\ 0{,}64, \ agar\ 2<x\le 3, \\ 0{,}784, \ agar\ 3<x\le 4, \\ 1, \ agar\ x>4. \end{cases}$

1.4.9. 1) $F(x)=\begin{cases} 0, \ agar\ x\le -2, \\ 0{,}1, \ agar\ -2<x\le 1, \\ 0{,}4, \ agar\ 1<x\le 2, \\ 0{,}8, \ agar\ 2<x\le 3, \\ 1, \ agar\ x>3. \end{cases}$ **1.4.10.** 1) $F(x)=\begin{cases} 0, \ agar\ x\le -1, \\ 0{,}3, \ agar\ -1<x\le 0, \\ 0{,}68, \ agar\ 0<x\le 1, \\ 0{,}8, \ agar\ 1<x\le 2, \\ 1, \ agar\ x>2. \end{cases}$

2) $P(X<2)=0{,}4;\ \ P(1\le X<3)=0{,}6.$ 2) $P(X<-1)=0{,}3;\ \ P(1\le X<2)=0{,}12.$

1.4.11. 1) $a=\frac{1}{2},\ b=\frac{1}{\pi}$; 2) 0; 3) $\frac{2}{3}$; 4) $f(x)=\begin{cases} 0, \ agar\ x\le -2, \\ \frac{1}{\pi\sqrt{4-x^2}}, \ agar\ -2<x\le 2, \\ 0, \ agar\ x>2. \end{cases}$

1.4.12.) $a=\frac{1}{2},\ b=1$; 2) 0; 3) $\frac{1}{4}$; 4) $f(x)=\begin{cases} 0, \ agar\ x<-\pi, \\ -\frac{1}{2}\sin x, \ agar\ -\pi\le x\le 0, \\ 0, \ agar\ x>2. \end{cases}$ **1.4.13.** 1) $c=2$;

2) $F(x)=\begin{cases} 0, \ agar\ x\le 1, \\ (x-1)^2, \ agar\ 1<x\le 2, \\ 1, \ agar\ x>2; \end{cases}$ 3) $P(1{,}4\le< X<1{,}9)=0{,}65.$ **1.4.14.** 1) $\frac{1}{\pi}$; 2) $F(x)=\frac{2}{\pi}arctg(e^x)$;

3) $\frac{2}{3}$. **1.4.15.** 1) $F(x)=1-e^{-\frac{x}{T}},\ x\geq 0;$ 2) $\frac{e-1}{e^2}$. **1.4.16.** 1) $F(x)=\begin{cases} 1-\left(\frac{x_0}{x}\right)^a, & agar\ x\geq x_0, \\ 0, & agar\ x<x_0, \end{cases}$ $a>0;$

2) $1-2^{-a}$. **1.4.17.** $Z=\begin{cases} 0 & 1 & 2 & 3 & 4 \\ 0{,}05 & 0{,}30 & 0{,}20 & 0{,}30 & 0{,}15 \end{cases}.$

1.4.18. $Y=\begin{cases} 0 & 1 & 2 \\ 0{,}3 & 0{,}5 & 0{,}2 \end{cases};$ $Z=\begin{cases} -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ 0{,}03 & 0{,}11 & 0{,}21 & 0{,}28 & 0{,}24 & 0{,}11 & 0{,}02 \end{cases}.$

1.4.19. $Z=\begin{cases} 0 & 1 & 2 & 3 & 4 \\ 0{,}0144 & 0{,}1104 & 0{,}3124 & 0{,}3864 & 0{,}1764 \end{cases}.$

1.4.20. $f(z)=a^2ze^{-az},\ z\geq 0.$ **1.4.21.** $f(z)=\frac{1}{3}e^{-\frac{z}{7}}\cdot\left(1-e^{-\frac{3z}{28}}\right).$ **1.4.22.** $g(y)=\frac{1}{5}f\left(\frac{y}{5}\right).$

1.4.23. $g(y)=\begin{cases} 0, & agar\ y\leq 0, \\ 1, & agar\ 0<y\leq 1, \\ 0, & agar\ y>1. \end{cases};$ $G(y)=\begin{cases} 0, & agar\ y\leq 0, \\ y, & agar\ 0<y\leq 1, \\ 1, & agar\ y>1. \end{cases}.$

1.5.Tasodifiy miqdning sonli xarakteristikalari

1.5.1. $M(X)=0{,}9,\ D(X)=1{,}29,\ \sigma(X)=1{,}14.$ **1.5.2.** $M(X)=1{,}5,\ D(X)=1{,}4,\ \sigma(X)=1{,}18.$

1.5.3. $X=\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ \frac{32}{243} & \frac{80}{243} & \frac{80}{243} & \frac{40}{243} & \frac{10}{243} & \frac{1}{243} \end{cases},$ $M(X)=1{,}67,\ D(X)=1{,}11.$

1.5.4. $X=\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \\ 0{,}01024 & 0{,}0768 & 0{,}2304 & 0{,}3456 & 0{,}2592 & 0{,}07778 \end{cases},$ $M(X)=3,\ D(X)=1{,}2.$

1.5.5. $M(X)=2{,}65,\ D(X)=72{,}13.$ **1.5.6.** $M(X)=11{,}4,\ D(X)=107{,}44.$

1.5.7. 1) $M(X)=p,\ D(X)=pq;$ 2) $M(X)=np,\ D(X)=npq.$

1.5.8. $F(x)=\begin{cases} 0, & agar\ x\leq 3, \\ 0{,}8, & agar\ 3<x\leq 4, \\ 1, & agar\ x>4. \end{cases}$ $F(x)=\begin{cases} 0, & agar\ x\leq 2{,}4, \\ 0{,}2, & agar\ 2{,}4<x\leq 3{,}4, \\ 1, & agar\ x>3{,}4. \end{cases}$

1.5.9. $F(x)=\begin{cases} 0, & agar\ x\leq 1, \\ 0{,}6, & agar\ 1<x\leq 2, \\ 1, & agar\ x>2. \end{cases}$ $F(x)=\begin{cases} 0, & agar\ x\leq 0{,}8, \\ 0{,}4, & agar\ 0{,}8<x\leq 1{,}8, \\ 1, & agar\ x>1{,}8. \end{cases}$

1.5.10. 1) $F(x)=\begin{cases} 0, & agar\ x\leq 0, \\ 0{,}2, & agar\ 0<x\leq 1, \\ 0{,}6, & agar\ 1<x\leq 3, \\ 1, & agar\ x>3. \end{cases};$ 2) $F(x)=\begin{cases} 0, & agar\ x\leq 2, \\ 0{,}5, & agar\ 2<x\leq 3, \\ 1, & agar\ x>3. \end{cases};$

3) $F(x)=\begin{cases} 0, & agar\ x\le 0,\\ 0{,}4, & agar\ 0<x\le 2,\\ 0{,}7, & agar\ 2<x\le 4,\\ 1, & agar\ x>4.\end{cases}$; 4) $F(x)=\begin{cases} 0, & agar\ x\le -1,\\ 0{,}3, & agar\ -1<x\le 0,\\ 0{,}7, & agar\ 0<x\le 1,\\ 1, & agar\ x>1.\end{cases}$.

1.5.11. 1) $M(X)=0{,}667,\ D(X)=0{,}056,\ \sigma(X)=0{,}236$; 2) $M(X)=2{,}57,\ D(X)=0{,}6,\ \sigma(X)=0{,}77$; 3) $M(X)=1{,}33,\ D(X)=0{,}22,\ \sigma(X)=0{,}47$; 4) $M(X)=1{,}5,\ D(X)=0{,}15,\ \sigma(X)=0{,}39$. **1.5.12.** 1) $M(X)=\frac{5}{3}$, 2) $D(X)=\frac{1}{18}$; 2) $M(X)=\frac{\pi}{2},\ D(X)=\frac{\pi^2}{4}-2$; 3) $M(X)=-\frac{1}{\ln 3},\ D(X)=\frac{1}{\ln^2 3}$; 4) $M(X)=2,\ D(X)=2$.

1.6. Tasodifiy miqdning taqsimot qonunlarii

1.6.1. $X=\begin{cases} 0 & 1 & 2 & 3 & 4 & 5\\ 0{,}32768 & 0{,}4096 & 0{,}2048 & 0{,}0512 & 0{,}0064 & 0{,}00032\end{cases}$, $M(X)=1,\ D(X)=0{,}8$.

1.6.2. $X=\begin{cases} 0 & 1 & 2 & 3 & 4\\ 0{,}0576 & 0{,}2400 & 0{,}3747 & 0{,}2600 & 0{,}0677\end{cases}$, $M(X)=2{,}04,\ D(X)=1$.

1.6.3. $M(X)=8,\ D(X)=4{,}8,\ \sigma(X)=2{,}19$. **1.6.4.** $M(X)=3,\ D(X)=2{,}55,\ \sigma(X)=1{,}6$.

1.6.5. $X=\begin{cases} 0 & 1 & 2 & \dots & m & \dots\\ 0{,}3679 & 0{,}3679 & 0{,}1839 & \dots & \frac{e}{m!} & \dots\end{cases}$, $M(X)=1;\ D(X)=1$.

1.6.6. $X=\begin{cases} 0 & 1 & 2 & \dots & m & \dots\\ 0{,}1353 & 0{,}2707 & 0{,}2707 & \dots & \frac{2^{-m}e^{-2}}{m!} & \dots\end{cases}$, $M(X)=2;\ D(X)=2$.

1.6.7. $M(X)=D(X)=2,\ P(5\le X\le 10)=0{,}053$. **1.6. 8.** 1) 0,1637; 2) 0,0012.

1.6.9. $X=\begin{cases} 1 & 2 & 3 & \dots & m & \dots\\ 0{,}05 & 0{,}0475 & 0{,}045125 & \dots & 0{,}05\cdot 0{,}95^{m-1} & \dots\end{cases}$, $M(X)=20;\ D(X)=380$, $P(X\ge 5)=0{,}8145$. **1.6.10.** $X=\begin{cases} 0 & 1 & 2 & \dots & m & \dots\\ 0{,}15 & 0{,}1275 & 0{,}108375 & \dots & 0{,}15\cdot 0{,}85^{m-1} & \dots\end{cases}$, $M(X)=6{,}67$; $D(X)=37{,}78$. **1.6.11.** 1000. **1.6.12.** 4. **1.6.13.** $X=\begin{cases} 0 & 1 & 2 & 3\\ \frac{4}{91} & \frac{27}{91} & \frac{216}{455} & \frac{12}{65}\end{cases}$, $M(X)=\frac{9}{5}, D(X)=\frac{108}{175}$.

1.6.14. $X=\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 & 6\\ 0{,}40056 & 0{,}42423 & 0{,}15147 & 0{,}02244 & 0{,}00137 & 0{,}00003 & 0{,}0000001\end{cases}$, $M(X)=0{,}8$, $D(X)=0{,}6145,\ P(3\le X\le 6)=0{,}024$. **1.6.15.** 1) $P(X\le 0{,}5)=\frac{1}{6}$. 1) $f(x)=\begin{cases}\frac{1}{3}, & agar\ 0\le x\le 2,\\ 0, & agar\ x<0,\ x>2.\end{cases}$;

$M(X)=\frac{3}{2},\ \sigma(X)=\frac{\sqrt{3}}{2}$. **1.6.16.** 1) $f(x)=\begin{cases}0{,}5, & agar\ 0\le x\le 2,\\ 0, & agar\ x<0,\ x>2.\end{cases}$; 2) 1; 2) 0,5774.

1.6.17. 1) $M(X)=50,\ D(X)=2500$; 2) 0,3679. **1.6.18.** 1) $M(X)=\frac{1}{3},\ D(X)=\frac{1}{9}$; 2) 0,53.

1.6.19. 0,4043. **1.6.20.** 0,8664. **1.6.21.** 0,733. **1.6.22.** 92.

1.7. Ehtimollar nazariyasining limit teoremalari

1.7.1. $P\ge 0{,}96$. **1.7.2.** $P\ge 0{,}94$. **1.7.3.** $P\ge 0{,}808$. **1.7.4.** $P\ge 0{,}264$. **1.7.5.** $P>0{,}79$.

1.7.6. $P>0{,}796$. **1.7.7.** $P\ge 0{,}64$. **1.7.8.** $P\ge 0{,}432$. **1.7.9.** $f(Y)=\frac{3}{5\sqrt{6\pi}}e^{-\frac{3(y-50)^2}{50}}$, $P=0{,}04$.

1.7.10. $P=0{,}0281$.

1.8. Tanlanmaning xarakteristikalari

1.8.3. 1) $\overline{X}=3{,}7,\ \overline{D}=1{,}81,\ \sigma=1{,}35$. 2) $\overline{X}=3{,}1,\ \overline{D}=2{,}49,\ \sigma=1{,}58$. **1.8.4.** 1) $\overline{X}=41{,}88$, $\overline{D}=2{,}92$. **1.8.5.** $\overline{X}=220,\ \overline{D}=7{,}93$. **1.8.6.** $\overline{X}=2621,\ \overline{D}=919$. **1.8.7.** $\overline{X}=166,\ \overline{D}=33{,}44$.

1.9.Taqsimot noma'lum parametrlarining statistik baholari

1.9.1. 1) $\overline{X}=7{,}63$; 2) $\overline{X}=6{,}51$. **1.9.2.** 1) $S^2=8{,}4$; 2) $S^2=7{,}8$. **1.9.5.** 1,03; 1600. **1.9.6.** 22,5; 1,28.
1.9.7. 0,8883. **1.9.8.** 0,9544. **1.9.9.** $\overline{X}$. **1.9.10.** $\overline{X}-\sqrt{3\overline{D}}$, $\overline{X}+\sqrt{3\overline{D}}$, **1.9.1.** $\overline{X}$. **1.9.12.** 0,6.
1.9.13. 1) (13,72;18,88); 2) (9,82;14,98); 3) (7,52;12,68); 4) (11,92;17,08). **1.9.14.** (1033,2;1166,8).
1.9.15. 864. **1.9.16.** 423. **1.9.17.** 1) (0,53;3,47); 2) (0,71;3,29). **1.9.18.** (34,66;50,94).
1.9.19. 1) (0,0324;0,2076); 2) (0,0432;0,2768); 3) (0,0648;0,4452); 4) (0,0513;0,3287). **1.9.20.** (0;0,595).

1.11. Korrelyatsion tahlil

1.11.1. 1) $y_x=5{,}34x-1{,}36$; 2) $y_x=-0{,}4x^2+12{,}16x-10{,}57$.

2.1. Kompleks sonlar

2.1.1. $x=2,\ y=-1$. **2.1.2.** $x=5,\ y=10$. **2.1.3.** $x=-1,\ y=21$. **2.1.4.** $x=\frac{1}{5},\ y=\frac{1}{3}$.

2.1.5. 1) $z_1=-3+4i,\ z_2=-3-4i$; 2) $z_1=\frac{1}{2}i,\ z_2=-i$; 3) $z_1=i,\ z_2=-3i$; 4) $z_1=5i,\ z_2=i$.

2.1.6. 1) $x=a$ to'g'ri chiziq; 2) $y=b$ to'g'ri chiziq; 3) markazi $O(0;0)$ nuqtada bo'lgan va $r,\ R$ radiusli aylanalar orasidagi halqa; 4) φ va ψ nurlar orasidgi sektor; 5) 3) markazi $O(0;0)$ nuqtada bo'lgan va $r,\ R$ radiusli aylanalar orasidagi halqaning φ va ψ nurlar orasidgi bo'lagi.

2.1.7. 1) $-2+2\sqrt{3}i=4\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)=4e^{\frac{2\pi}{3}i}$; 2) $\sqrt{3}-i=2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)=2e^{-\frac{\pi}{6}i}$;

3) $-\frac{\sqrt{6}}{2}+\frac{\sqrt{6}}{2}i=\sqrt{3}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt{3}e^{\frac{3\pi}{4}i}$; 4) $2+2i=2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=2\sqrt{2}e^{\frac{\pi}{4}i}$;

5) $1-i=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)=\sqrt{2}e^{-\frac{\pi}{4}i}$;

6) $-3-2i=\sqrt{13}\left(\cos\left(arctg\frac{2}{3}-\pi\right)+i\sin\left(arctg\frac{2}{3}-\pi\right)\right)=\sqrt{13}e^{i\left(arctg\frac{2}{3}-\pi\right)}$;

7) $1-\sqrt{3}i=2\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)=2e^{-\frac{\pi}{3}i}$; 8) $-\sqrt{2}-\sqrt{2}i=2\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)=2e^{-\frac{3\pi}{4}i}$.

2.1.8. 1) $z_1+z_2=-3-i,\ z_1-z_2=-7+7i,\ z_1\cdot z_2=2+26i,\ \frac{z_1}{z_2}=-\frac{11}{10}-\frac{7}{10}i;$ 2) $z_1+z_2=-1-i,$ $z_1-z_2=-5-7i,\ z_1\cdot z_2=6-17i,\ \frac{z_1}{z_2}=-\frac{18}{13}+\frac{1}{13}i.$ **2.1.9.** 1) $5\sqrt{2}$; 2) $5\sqrt{2}$; 3) $\sqrt{5}$; 4) 10.

2.1.10. 1) $\frac{6}{5}-\frac{17}{5}i$; 2) $-\frac{9}{5}-\frac{2}{5}i$; 3) $24i$; 4) $48i$. **2.1.11.** 1) $\operatorname{Re}z=\frac{1}{2},\operatorname{Im}z=\frac{\sqrt{3}}{2}$; 2) $\operatorname{Re}z=0,$ $\operatorname{Im}z=\frac{1}{8}$; 3) $\operatorname{Re}z=\frac{4}{5},\operatorname{Im}z=\frac{3}{5}$; 4) $\operatorname{Re}z=-\frac{37}{5},\operatorname{Im}z=-\frac{29}{5}$. **2.1.12.** 1) $z_1\cdot z_2=-4+4\sqrt{3}i,\ \frac{z_1}{z_2}=\sqrt{3}-i$;

2) $z_1\cdot z_2=3\sqrt{3}+3i,\ \frac{z_1}{z_2}=-6i$; 3) $z_1\cdot z_2=-16,\ \frac{z_1}{z_2}=4i$; 4) $z_1\cdot z_2=-4+4\sqrt{3}i,\ \frac{z_1}{z_2}=4+4\sqrt{3}i.$

2.1.13. 1) $16(1+i)$; 2) $-32(1+\sqrt{3}i)$; 3) $2^{13}(1-i)$; 4) -1. **2.1.14.** 1) $\pm(\sqrt{3}-i)$; 2) $i,\ -\frac{\sqrt{3}}{2}-\frac{1}{2}i,$ $\frac{\sqrt{3}}{2}-\frac{1}{2}i$; 3) $\pm(\sqrt{3}+i),\pm(1-\sqrt{3}i)$; 4) $\sqrt[10]{2}\left(\cos\frac{\pi+8k\pi}{20}+i\sin\frac{\pi+8k\pi}{20}\right),\ k=0,1,2,3,4.$

2.1.15. 1) $\pm\frac{\sqrt{2}}{2}(1-i)$; 2) $\frac{\sqrt{3}}{2}(1+i),\ \frac{\sqrt{3}}{2}(-1+i),\ -i$; 3) $-2,\ 1\pm\sqrt{3}i$; 4) $\pm(1\pm i)$; 5) $-2\pm7i$;

6) $\pm i,\ \pm5i$; 7) $-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$; 8) $\cos\left(\frac{\pi+4k\pi}{8}\right)+i\sin\left(\frac{\pi+4k\pi}{8}\right),k=0,1,2,3,4$; 9) $\pm1,\ \pm\pm2i.$

2.2. Kompleks o'zgaruvchining funksiyasi

2.2.1. 1) $w=-3+i$; 2) $w=3-3i$; 3) $w=1$; 4) $w=\frac{\sqrt{2}+1}{2}+i\frac{\sqrt{2}-1}{2}$. **2.2.2.** 1) $f(z_1)=\frac{1+i}{2},$ $f(z_2)=\frac{3-2i}{13}$; 2) $f(z_1)=\frac{1-5i}{2},\ f(z_2)=0$. **2.2.3.** 1) $u=x^3-3xy^2-1,\ v=y^3-3x^2y+2$;

2) $u=-x-2xy,\ v=x^2-y^2-y+2$; 3) $u=x^2-y^2,\ v=-2xy$; 4) $u=x^2-y^2,\ v=2xy-1$.

2.2.4. 1) $f(z)=\bar{z}$; 2) $f(z)=(1+i)\bar{z}$; 3) $f(z)=\frac{1}{\bar{z}}$; 4) $f(z)=\frac{4\bar{z}}{\bar{z}^2-z^2}$.

2.2.5. 1) $2i|z^2|+(2i-1)z+(2i-1)\bar{z}=2i$; 2) $z^2-\bar{z}^2+2(1+i)z-2(1-i)\bar{z}=4i$.

2.2.6. 1) $w_0=(\sqrt{2}+1)i,\ w_1=(-\sqrt{2}+1)i$; 2) $w_0=0;\ w_1=1+i;\ w_2=-1-i$. **2.2.7.** 1) $w_0=\frac{5}{13}-i\frac{12}{13},$

2) $w_0=-\frac{1}{4}+i\frac{1}{4}$. **2.2.8.** $u=\frac{1}{16}v^2-4$. **2.2.9.** $u=1-\frac{1}{4}v^2$. **2.2.10.** 1) $5i$; 2) $-4i$; 3) mavjud emas; 4) 0. **2.2.13.** 1) $e^{1-x}(\cos y-i\sin y)$; 2) $e^x(\cos y-i\sin y)$; 3) $e^{x^2-(1-y)^2}(\cos2x(1-y)+i\sin2x(1-y))$;

4) $\ln5+i\pi(1+2k),\ k\in Z$; 5) $2\ln2+2i\pi\left(\frac{1}{3}+k,\right)\ k\in Z$; 6) $i\pi\left(\frac{1}{2}+2k,\right)\ k\in Z$; 7) $e^{-(1+2k)\pi},\ k\in Z$;

8) $e^{(1+2k)\pi}(\cos\ln 3 - i\sin\ln 3),\ k\in Z;$ 9) $5e^{arctg\frac{4}{3}-2k\pi}\left(\cos\left(\ln 5-\arg tg\frac{4}{3}\right)+i\sin\left(\ln 5 - arctg\frac{4}{3}\right)\right), k\in Z;$

10) $ch2\sin 1 + ish2\cos 1;$ 11) $ish1;$ 12) $-icth\pi;$ 13) $\cos 1;$ 14) $-\cos 1sh2 + i\sin 1ch2;$ 15) 0;

16) $\pi\left(\frac{1}{2}+2k\right)-i\ln(2+\sqrt{3}),\ k\in Z;$ 17) $\frac{\pi}{2}(1+2k)+\frac{i}{2}\ln 2,\ k\in Z;$ 18) $\pi\left(\frac{1}{6}-k\right),\ k\in Z;$

19) $i\pi\left(\frac{1}{2}+2k\right),\ k\in Z;$ 20) $i\pi(1+2k),\ k\in Z.$

2.3. Kompleks o'zgaruvchi funksiyasini differensiallash

2.3.1. 1) $z_0=0, w'(z_0)=0;$ 2) $z_0=0, w'(z_0)=0;$ 3) $z_0=0, w'(z_0)=0;$ 4) hosila mavjud emas; 5) $z\neq 1,\ w'(z)=-\frac{1}{(z-1)^2};$ 6) hosila mavjud emas; 7) $z\in G,\ w'(z)=4z^3;$ 8) $z\neq 0,\ w'(z)=\frac{2}{z};$ 9) $z\in G,\ w'(z)=i\cos iz;$ 10) $z_0=i\left(\frac{\pi}{2}+k\pi\right), w'(z_0)=0.$ **2.3.2.** 1) analitik; 2) analitik emas; 3) analitik; 4) analitik; 5) analitik; 6) analitik. **2.3.3.** 1) $w=2iz^2+iz+C;$ 2) $w=e^z+C.$

2.3.4. 1) $w=z^2-z+Ci;$ 2) $w=chz+Ci.$ **2.3.5.** 1) $k=6, \varphi=\frac{\pi}{2};$ 2) $k=\sqrt{5}, \varphi=-arctg2;$ 3) $k=1, \varphi=0;$ 4) $k=2, \varphi=\frac{\pi}{2}.$ **2.3.6.** 1) $|z+1|<\frac{1}{2}$ soha siqiladi, $|z+1|>\frac{1}{2}$ soha cho'ziladi; 2) $|z|>1$ soha siqiladi, $|z|<1$ soha cho'ziladi. **2.3.7.** 1) $\left|z-\frac{i}{2}\right|=\frac{1}{2};$ 2) $|z|=\frac{1}{\sqrt{3}}.$

2.3.8. 1) $0<x<\infty,\ y=-\frac{1}{2}$ nur; 2) $1<x<\infty,\ y=0$ nur.

2.4. Kompleks o'zgaruvchi funksiyasini integrallash

2.4.1. 1) $2(i-1);$ 2) $5(5-3i);$ 3) $\frac{3-i}{3};$ 4) $-\frac{1-8i}{3};$ 5) $\pi i;$ 6) $2\pi i$ 7) $-\frac{2}{15}+\frac{3}{2}i;$ 8) $-\frac{5+14i}{6};$ 9) $i(1-ch1);$ 10) $-i(1+e^{\pi}).$ **2.4.2.** 1) 0; 2) $-\pi i;$ 3) $-\frac{4}{3}i.$ **2.4.3.** 1) 0; 2) $-\frac{\pi}{2};$ 3) $-\frac{4}{3}.$

2.4.4. 0. **2.4.5.** 0. **2.4.6.** 1) $3+15i;$ 2) $\frac{2-10i}{3};$ 3) $ch1;$ 4) $-\frac{1}{2}+ish1;$ 5) $-sh\pi;$ 6) $-sh1;$ 7) $\frac{1}{4}(sh2-6)i;$ 8) $(sh2-2ch2)i.$ **2.4.7.** 1) $\pi;$ 2) $2\pi i;$ 3) $\frac{e^{16}-1}{2}\pi i;$ 4) $2\pi sh1;$ 5) $2\pi i;$ 6) $\pi i;$ 7) 0; 8) 0.

2.4.8. 1) 0; 2) $\frac{2\pi i}{27};$ 3) $-\frac{2\pi i}{3}.$ **2.4.9.** 1) $2\pi;$ 2) 0; 3) $2\pi.$ **2.4.10.** 1) $\frac{3\pi i}{8};$ 2) $-\frac{3\pi i}{8};$ 3) 0.

2.4.11. 1) $\frac{4\pi i}{e^6};$ 2) $2\pi i(\cos 2-\sin 2);$ 3) $2\pi i;$ 4) $-\frac{\pi i}{4};$ 5) $-\frac{i}{8}\pi(\pi+2)\sqrt{2};$ 6) $-\pi sh1;$ 7) $\pi^3 i;$ 8) $2\pi i.$

2.5. Kompleks hadli qatorlar

2.5.1. 1) yaqinlashuvchi; 2) uzoqlashuvchi; 3) yaqinlashuvchi; 4) uzoqlashuvchi; 5) yaqinlashuvchi; 6) uzoqlashuvchi. **2.5.2.** 1) shartli yaqinlashuvchi; 2) absolut yaqinlashuvchi; 3) shartli yaqinlashuvchi; 4) absolut yaqinlashuvchi. **2.5.3.** 1) $R=\infty;$ 2) $R=0;$ 3) $R=\frac{1}{e};$ 4) $R=\infty;$ 5) $R=\frac{1}{e};$ 6) $R=1;$ 7) $R=1;$ 8) $R=\infty.$ **2.5.4.** 1) $|z|<2;$ 2) $|z|<1;$

3) $|z-2|<1$; 4) $|z-1|<\frac{1}{e}$; 5) $|z|<1$; 6) markazi $O(0;0)$ nuqtada bo‘lgan har qanday doira; 7) $|z|<\sqrt{2}$; 8) $|z|<1$. **2.5.5.** 1) $\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}$; 2) $\sum_{n=0}^{\infty}(-1)^n z^{2n}$; 3) $\sum_{n=0}^{\infty}\frac{3^n}{2^{n+1}}(z-1)^n$; 4) $\sum_{n=1}^{\infty}(n+1)z^n$; 5) $\sum_{n=0}^{\infty}z^{3n+2}$; 6) $\sum_{n=0}^{\infty}(2^n+(-1)^n)(z+1)^n$; 7) $\frac{1}{4}\sum_{n=0}^{\infty}(3^{-n}+(-1)^n)z^n$; 8) $-\sum_{n=1}^{\infty}\frac{i^n}{2^n}z^n$; 9) $1+\sum_{n=1}^{\infty}\frac{(-1)^n 4^{2n-1}}{(2n)!}z^{2n}$; 10) $\frac{1}{2}\sum_{n=1}^{\infty}\frac{z^{2n}}{(2n)!}$; 11) $e^3\sum_{n=1}^{\infty}\frac{(z-3)^n}{n!}$; 12) $\frac{1}{\sqrt{2}}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(1+\frac{1}{2n+1}\left(z+\frac{\pi}{4}\right)\right)\left(z+\frac{\pi}{4}\right)^{2n}$; 13) $\ln 2-\sum_{n=1}^{\infty}\frac{z^n}{n2^n}$; 14) $\ln 17-\sum_{n=1}^{\infty}\left(\frac{5}{17}\right)^n\frac{(z+3)^n}{n}$. **2.5.6.** 1) $z_n=(2n+1)\pi$ $(n\in Z)$, 2-tartibli; 2) $z_1=1$, oddiy, $z_2=-i$, 3-tartibli; 3) $z_n=2n\pi i$ $(n\in Z)$, oddiy; 4) $z_{1n}=n\pi i$ $(n\in Z)$, oddiy, $z_2=-\pi i$, 2-tartibli.

2.5.7. 1) oddiy; 2) 2-tartibli; 3) 4-tartibli; 4) 3-tartibli. **2.5.8.** 1) $\sum_{n=-1}^{\infty}(-1)^{n+2}(z-2)^n$; 2) $\sum_{n=-1}^{\infty}\frac{2z^n}{3^{n+2}}$; 3) $|z|<1$ da $\sum_{n=0}^{\infty}\left(1-\frac{1}{2n+1}\right)z^n$, $1<|z|<2$ da $-\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}-\sum_{n\to 0}^{\infty}\frac{z^n}{2^{n+1}}$, $|z|>2$ da $\sum_{n=1}^{\infty}\frac{2^n-1}{z^{n+1}}$; 4) $|z|<2$ da $-\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{1}{3^{n+1}}+(-1)^n\frac{1}{2^{n+1}}\right)z^n$, $2<|z|<3$ da $-\sum_{n=1}^{\infty}\frac{(-1)^{n-1}2^{n-1}}{z^n}$, $|z|>3$ da $\frac{1}{5}\sum_{n=0}^{\infty}(3^n+(-1)^n 2^n)\frac{1}{z^{n+1}}$. 5) $\sum_{n=1}^{\infty}\frac{(-1)^n z^{n-3}}{n!}$; 6) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!z^{2n-3}}$; 7) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n-2}$; 8) $\sum_{n=0}^{\infty}\frac{(-1)^{n-1}2z^n}{i^{n+1}}$.; **2.5.9.** 1) $-\sum_{n=-1}^{\infty}(z-1)^n$; 2) $-\sum_{n=-1}^{\infty}z^n$; 3) $-\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}-\sum_{n=1}^{\infty}\frac{1}{z^{n+1}}$; 4) $-\sum_{n=0}^{\infty}\frac{z^n}{3^{n+1}}-\sum_{n=1}^{\infty}\frac{2^{n-1}}{z^n}$.
2.5.10. 1) bartaraf qilinadigan nuqta; 2) bartaraf qilinadigan nuqta; 3) $z=0-5$-tartibli qutb, $z=2n\pi, n\in Z, n\neq 0$-oddiy qutb; 4) $z=0-3$-tartibli qutb; 5) muhim nuqta; 6) muhim nuqta.

2.6. Qoldiqlar nazariyasi va uning tatbiqi

2.6.1. 1) $\mathrm{Res}f(0)=0$; 2) $\mathrm{Res}f(0)=0$; 3) $\mathrm{Res}f(\pi)=\frac{\sin^2\pi}{\pi^2}$; 4) $\mathrm{Res}f(4)=2$; 5) $\mathrm{Res}f(2)=\frac{e^2}{27}$; 6) $\mathrm{Res}f(1)=4$; 7) $\mathrm{Res}f(1)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n(2n+1)!}$; 8) $\mathrm{Res}f(1)=-1$; 9) $\mathrm{Res}f(\infty)=1$; 10) $\mathrm{Res}f(\infty)=-1$. **2.6.2.** 1) $\mathrm{Res}f(1)=1$, $\mathrm{Res}f(\infty)=-1$; 2) $\mathrm{Res}f(0)=-1$, $\mathrm{Res}f(1)=1$, $\mathrm{Res}f(\infty)=0$; 3) $\mathrm{Res}f(0)=e-1$, $\mathrm{Res}f(1)=e$, $\mathrm{Res}f(\infty)=1$; 4) $\mathrm{Res}f(0)=-\frac{4}{\pi^2}$, $\mathrm{Res}f\left(\frac{\pi}{2}\right)=0$, $\mathrm{Res}f(\infty)=\frac{4}{\pi^2}$. **2.6.3.** 1) $-\frac{\pi i}{4}$; 2) 0; 3) $-2\pi i$; 4) 0; 5) $\frac{\pi\sqrt{2}}{2}$; 6) $\frac{\pi}{4}$; 7) 4π; 8) $\frac{2\sqrt{3}\pi}{3}$; 9) $\frac{3\pi}{8}$; 10) $\frac{\pi}{16}$; 11) $\frac{2\pi}{e^m}$; 12) $\frac{\pi(m+1)}{2e^m}$; 13) $\frac{\pi(\cos 2+\sin 2)}{e^2}$; 14) $\frac{\pi}{8}(1-e^{-4})$.

3.1. Laplas almashtirishlari

3.1.1. 1) $\frac{2}{p^3}$; 2) $\frac{3}{p^2+9}$; 3) $\frac{1}{p-3}$; 4) $\frac{2}{p^2-4}$. **3.1.2.** 1) $\frac{1-p}{p^2+1}$; 2) $\frac{3p+2}{p^2}$; 3) $\frac{2}{p(p^2+4)}$;

4) $\dfrac{p(p^2+7)}{(p^2+1)(p^2+9)}$; 5) $\dfrac{2e^{-p}}{p^3}$; 6) $\dfrac{(p^2+2)e^{-p}}{p(p^2+4)}$; 7) $\dfrac{p+1}{p^2+2p+5}$; 8) $\dfrac{6}{(p+1)^4}$; 9) $\dfrac{6}{(p^2+1)(p^2+9)}$;

10) $\dfrac{p^2-\omega^2}{(p^2+\omega^2)^2}$; 11) $\dfrac{2}{(p-1)^3}$; 12) $\dfrac{2p^3-6p}{(p^2+1)^3}$; 13) $\dfrac{1}{p(p^2+1)}$; 14) $\dfrac{p^2+2\omega^2}{p^2(p^2+4\omega^2)}$; 15) $\ln\dfrac{p}{p-1}$;

16) $\ln\dfrac{\sqrt{p^2+1}}{p}$; 17) $\dfrac{2p\omega}{(p^2-\omega^2)^2}$; 18) $\dfrac{1}{(p-\omega)^2}$; 19) $\dfrac{1-e^{-p}+e^{-2p}}{p^2}$; 20) $\dfrac{1-e^{-p}+pe^{-2p}}{p^2}$;

21) $\dfrac{e^{-ap}-e^{-bp}}{p^2}$; 22) $\dfrac{1-e^{-ap}}{p^2}$. **3.1.3.** 1) $\dfrac{3}{p+1}+\dfrac{p-1}{(p-1)^2+9}$; 2) $\dfrac{1}{p+\ln 2}+\dfrac{1}{p}$; 3) $\dfrac{p^2-2p+3}{(p-1)(p^2-2p+5)}$;

4) $\dfrac{p}{2}\left(\dfrac{1}{p^2+4}+\dfrac{1}{p^2+16}\right)$; 5) $\dfrac{1}{2}\left(\dfrac{5}{(p+4)^2+25}+\dfrac{1}{(p+4)^2+1}\right)$; 6) $\dfrac{2(5p^2+4)}{p^2-16}$;

7) $\dfrac{2pab}{((p-a)^2+b^2)((p+a)^2+b^2)}$; 8) $\dfrac{p^2+b^2}{(p^2-b^2)^2}$; 9) $\dfrac{120}{p^6}+\dfrac{9}{p}+\dfrac{9p}{p^2+100}$; 10) $\dfrac{p+3}{p^2+6p-16}$.

3.2. Tasvir bo'yicha originalni topish

3.2.1. 1) $(t-1)^2\eta(t-1)$; 2) $e^{-4(t-3)}\eta(t-3)$; 3) $3ch2t-sh2t$; 4) $e^{-t}(\cos t-\sin t)$; 5) $1-e^{-t}(1+t)$;
6) $t-\sin t$; 7) $\dfrac{1}{5}(3-3e^{-2t}\cos t+4e^{-2t}\sin t)$; 8) $-\dfrac{1}{3}e^{t}+\dfrac{1}{4}e^{2t}+\dfrac{1}{12}e^{-2t}$; 9) $\dfrac{1}{2}t^2+2e^{-t}\sin t$;
10) $-\dfrac{1}{3}e^{-t}+\dfrac{1}{3}e^{\frac{1}{2}t}\left(\cos\dfrac{\sqrt{3}}{2}t+\sqrt{3}\sin\dfrac{\sqrt{3}}{2}t\right)$; 11) $e^{-3(t-3)}\sin(t-3)\eta(t-3)$;
12) $e^{2(t-1)}(t-1)\eta(t-1)+e^{2(t-2)}(t-2)\eta(t-2)$; 13) $e^{t}+e^{-t}+\sin t$; 14) $t+cht$; 15) $e^{t}(1-t^2)$;
16) $\dfrac{1}{5}(ch2t-\cos t)$. **3.2.2.** 1) $\dfrac{1}{2}(cht-\cos t)$; 2) $e^{t}-\dfrac{1}{2}t^2-t-1$; 3) $\dfrac{1}{24}(3\sin t-\sin 3t)$;
4) $\dfrac{1}{2}t^2+\cos t-1$. **3.2.3.** 1) $\cos t-\dfrac{1}{2}t\sin t$; 2) $\dfrac{1}{3}(\cos t-\cos 2t)$; 3) $\dfrac{1}{2}(\sin t-\cos t+e^{t})$;
4) $e^{-t}(\sin t+\cos t-1)$. **3.2.4.** 1) $\sum\limits_{n=0}^{\infty}(-1)^{n-1}\dfrac{t^n}{(n!)^2}$; 2) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\dfrac{t^n}{(4n)!}$; 3) $\sum\limits_{n=0}^{\infty}(-1)^{n}\dfrac{t^{2n}}{(2n+1)!(2n!)}$;
4) cost; 5) $\dfrac{1}{t}(1-e^{t})$; 6) $\sum\limits_{n=0}^{\infty}\dfrac{t^{2n}}{(n!)(2n!)}$. **3.2.5.** 1) $-\dfrac{1}{2}e^{t}+\dfrac{1}{2}e^{3t}$; 2) $-\dfrac{1}{3}+\dfrac{11}{15}e^{3t}+\dfrac{3}{5}e^{-2t}$;
3) $-\dfrac{1}{3}e^{t}+\dfrac{1}{4}e^{2t}+\dfrac{1}{12}e^{-2t}$; 4) $-\dfrac{1}{3}cht+\dfrac{1}{3}ch2t$; 5) $-\dfrac{1}{6}+e^{t}-\dfrac{3}{2}e^{2t}+\dfrac{2}{3}e^{3t}$; 6) $1-2e^{t}+e^{3t}$;
7) $e^{t}-\dfrac{1}{2}t^2-t-1$; 8) $t+2+(t-2)e^{t}$.

3.3. Operatsion hisobning tatbiqlari

3.3.1. 1) sht; 2) $\cos t+\sin t-e^{-t}$; 3) $-\dfrac{1}{2}+e^{t}-\dfrac{1}{2}e^{-2t}$; 4) $-\dfrac{2}{3}+\dfrac{2}{3}e^{3t}+e^{-2t}$; 5) sht; 6) $e^{t}t\left(\dfrac{1}{2}t+1\right)$;
7) $\dfrac{1}{2}t\sin t-\cos t+\sin t$; 8) $\dfrac{1}{2}(e^{-t}-te^{-t}-\cos t)$; 9) $tcht$; 10) $\dfrac{1}{2}sht-\dfrac{1}{2}te^{-t}$; 11) $3+t+(t-2)e^{t}$;

12) $\left(1-t+\frac{1}{2}t^2\right)e^t$. **3.3.2.** 1) $C_1+\left(C_2-\frac{t^2+t}{4}\right)e^{-2t}$; 2) $C_1+C_2e^{-t}+\frac{1}{2}e^{-t}(\cos t-\sin t)$. **3.3.3** 1) $\frac{1}{2}t\sin t$; 2) $\frac{1}{2}(sht-\sin t)$; 3) $e^t\left(\frac{1}{2}t^2-t+1\right)-1$; 4) $(e^t+2)\ln\left(\frac{2+e^t}{3}\right)+1-e^t$. **3.3.4.** 1) $x=\frac{6}{5}e^{5t}-\frac{1}{5}e^{-5t}$, $y=\frac{3}{5}e^{5t}+\frac{2}{5}e^{-5t}$; 2) $x=e^{-2t}(1+2t)$, $y=e^{-2t}(1-2t)$, 3) $x=y=e^t$; 4) $x=y=e^t$; 5) $x=\frac{1}{2}t^2+\frac{1}{6}t^3$, $y=1+t+\frac{1}{2}t^2-e^t$; 6) $x=t\cos t$, $y=-t\sin t$; 7) $x=-e^{-t}$, $y=e^{-t}$, $z=0$; 8) $x=1$, $y=t$, $z=t^2$.

3.3.5. $\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$. **3.3.6.** $\frac{1}{R}e^{-\frac{t}{CR}}+\frac{\mu C}{1+\mu CR}\left(e^{\eta t}-e^{-\frac{t}{CR}}\right)$.

4.1. Matematik fizika masalalarining qo'yilishi

4.1.1. 1) $y>0$ da giperbolik, $y<0$ da elliptik, $y=0$ da parabolik; 2) $x>0$ da giperbolik, $x<0$ da elliptik, $x=0$ da parabolik; 3) $x^2+y^2>1$ da giperbolik, $x^2+y^2<1$ da elliptik, $x^2+y^2=1$ da parabolik; 4) $x\neq\frac{\pi}{2}+k\pi, k\in Z$ da giperbolik, $x=\frac{\pi}{2}+k\pi, k\in Z$ da parabolik.

4.1.2. 1) $u_{\eta\eta}+u_\eta=0$; 2) $u_{\eta\eta}-u_\xi-u_\eta=0$; 3) $u_{\xi\xi}+u_{\eta\eta}+u_\eta=0$; 4) $u_{\xi\xi}+u_{\eta\eta}=0$; 5) $u_{\xi\eta}-2u_\xi+u_\eta=0$; 6) $2u_{\xi\eta}-3u_\xi=0$; 7) $u_{\xi\xi}+u_{\eta\eta}=0$; 8) $u_{\xi\xi}+u_{\eta\eta}-2tg\eta u_\eta-2tg\xi u_\xi=0$; 9) $u_{\xi\xi}+u_{\eta\eta}=0$; 10) $u_{\eta\eta}-\xi u_\xi=0$. **4.1.3.** 1) $u(x,y)=yC_1\left(\frac{y}{x}\right)+C_2\left(\frac{y}{x}\right)$;

2) $u(x,y)=C_1\left(\frac{1}{2}x+y\right)+C_2(3x-y)$; 3) $u(x,y)=C_1(2x+\sin x+y)+C_2(2x-\sin x-y)$;

4) $u(x,y)=C_1(x+2\sqrt{y})+C_2(x-2\sqrt{y})$.

4.2. To'lqin tenglamalarini yechish

4.2.1. 1) $u(x,t)=\frac{8h}{\pi^2}\sum_{m=0}^{\infty}\frac{(-1)^m}{(2m+1)^2}\cos\frac{(2m+1)\pi a}{l}t\sin\frac{(2m+1)\pi}{l}x$; 2) $u(x,t)=\frac{l}{\pi a}\sin\frac{\pi a}{l}t\sin\frac{\pi}{l}x$;

3) $u(x,t)=(\sin t+\cos t)\sin x$; 4) $u(x,t)=\cos\frac{5\pi a}{l}t\sin\frac{5\pi}{l}x$. **4.2.2.** 1) $u(x,t)=\frac{1}{\pi^2}(1-\cos\pi t)\sin\pi x$;

4) $u(x,t)=\left(2\cos 2t-\frac{1}{2}\sin 2t+t-2\right)\sin 2x$. **4.2.3.** 1) $u(x,t)=\frac{tx}{l}+\frac{2l}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\sin\frac{k\pi}{l}t\sin\frac{k\pi}{l}x$;

2) $u(x,t)=\frac{Cx}{l}+\frac{2C}{\pi l}\sum_{k=1}^{\infty}\frac{(-1)^k}{k}\cos\frac{k\pi}{l}t\sin\frac{k\pi}{l}x$. **4.2.4.** 1) $u(x,t)=t+\sin x\cos 3t$; 2) $u(x,t)=x(1-t)$;

3) $u(x,t)=x^2+t+4t^2$; 4) $u(x,t)=\sin x\cos 2t-\frac{1}{2}\cos x\sin 2t$; 5) $u(x,t)=\frac{x\sin x\cos t-t\cos x\sin t}{x^2-t^2}$;

6) $u(x,t)=e^{-(x^2+t^2)}ch2xt+\frac{1}{2}\sin 2x\sin 2t$; 7) $u(x,t)=\frac{x(1+x^2-t^2)}{(x^2+t^2+1)^2-4x^2t^2}+\sin x\sin t$;

8) $u(x,t)=\sin x\cos t+\frac{1}{4}\ln\left|\frac{tg(x+t)}{tg(x-t)}\right|$. **4.2.5.** 1) $x^2+4t^2+\frac{1}{2}\cos x\sin 2t+xt^2$; 2) $\sin x\cos 2t+\frac{2}{3}t^3$.

4.3. Issiqlik o'tkazuvchanlik tenglamalarini yechish

4.3.1. 1) $u(x,t)=\dfrac{u_0}{\sqrt{1+4k^2t}}e^{-\frac{k^2x^2}{1+4k^2t}}$; 2) $u(x,t)=\dfrac{1}{\sqrt{1+9t}}e^{-\frac{x^2}{4+36t}}$.

4.3.2. 1) $u(x,t)=\dfrac{4u_0}{\pi}\sum\limits_{m=0}^{\infty}\dfrac{1}{2m+1}e^{-\left(\frac{(2m+1)\pi a}{l}\right)^2 t}\sin\dfrac{(2m+1)\pi}{l}x$; 2) $u(x,t)=e^{-a^2t}\sin x$;

3) $u(x,t)=2e^{-9a^2t}\sin 3x$; 4) $u(x,t)=\dfrac{8l^2}{\pi^3}\sum\limits_{m=0}^{\infty}\dfrac{1}{(2m+1)^3}e^{-\left(\frac{(2m+1)\pi a}{l}\right)^2 t}\sin\dfrac{(2m+1)\pi}{l}x$;

5) $u(x,t)=3e^{-\left(\frac{\pi}{2}\right)^2 t}\sin\dfrac{\pi}{l}x-5e^{-\left(\frac{2\pi}{l}\right)^2 t}\sin\dfrac{2\pi}{l}x$; 6) $u(x,t)=3e^{-36\pi^2t}\sin 3\pi x+e^{-64\pi^2t}\sin 4\pi$;;

7) $u(x,t)=\dfrac{3}{2}(\cos t+\sin t-e^{-t})\sin 6x$; 8) $u(x,t)=\dfrac{16}{\pi^2}\left(1-e^{-\frac{\pi^2}{16}}\right)\sin\dfrac{\pi}{4}x$;

9) $u(x,t)=\dfrac{40}{17}\left(\sin t-4\cos t+4e^{-\frac{1}{4}t}\right)\sin x+2e^{-4t}\sin 4x$; 10) $u(x,t)=(t+e^{-t}-1)\sin 3x+e^{-\frac{2}{3}t}\sin 2x$;

11) $u(x,t)=(t+e^{-t}-1)\sin x+\dfrac{x}{\pi}e^{-t}$; 12) $u(x,t)=\dfrac{1}{4}(1-e^{-4t})\sin 2x+e^{-6t}\sin 3x+e^{t}+\dfrac{x}{\pi}(e^{2t}-e^{t})$.

4.4. Laplas tenglamalarini yechish

4.4.1. 1) $u(x,y)=\left(\dfrac{ch3y}{2ch3}+\dfrac{sh3y}{2sh3}\right)\sin 3x$; 2) $u(x,y)=\left(\dfrac{ch4(x-1)}{2ch4}+\dfrac{sh4(x-1)}{2sh4}\right)\sin 4y$;

3) $u(x,y)=\left(\dfrac{ch2y}{2ch2\pi}+\dfrac{sh2y}{2sh2\pi}\right)\sin 2x+\left(\dfrac{ch\left(x-\frac{\pi}{2}\right)}{2ch\frac{\pi}{2}}-\dfrac{sh\left(x-\frac{\pi}{2}\right)}{2sh\frac{\pi}{2}}\right)\sin y$;

4) $u(x,y)=\left(\dfrac{ch3y}{2ch3\pi}-\dfrac{sh3y}{2sh3\pi}\right)\sin 3x+\left(\dfrac{ch2\pi\left(x-\frac{\pi}{2}\right)}{2ch\pi}-\dfrac{sh2\pi\left(x-\frac{\pi}{2}\right)}{2sh\pi}\right)\sin 2y$.

4.4.2. 1) $u(r)=\dfrac{4}{\ln 3-\ln 2}\ln\dfrac{3r}{4}$; 2) $u(r)=\dfrac{1}{\ln 5-\ln 3}(3\ln r+7\ln 5-10\ln 3)$.

4.4.3. 1) $u(r,\varphi)=\dfrac{1}{8r}((3r^2-3)\cos\varphi-(r^2-9)\sin\varphi)$;

2) $u(r,\varphi)=\dfrac{4}{65r^2}(81-r^4)\cos 2\varphi-\dfrac{27}{665r^3}(r^6-64)\sin 3\varphi$; 3) $u(r,\varphi)=4\left(1-\dfrac{\ln r}{\ln 2}\right)+\dfrac{2}{3r}(r^2-1)\sin\varphi$;

4) $u(r,\varphi)=\dfrac{2}{5r}(9-r^2)\sin\varphi+\dfrac{9}{65r^2}(r^4-16)\sin 2\varphi$. **4.4.4.** 1) $u(r,\varphi)=3+\dfrac{5}{3}r\cos\varphi$;

2) $u(r,\varphi)=2+\dfrac{3}{2}r\sin\varphi$; 3) $u(r,\varphi)=\dfrac{1}{2}-\dfrac{1}{18}r^2\cos 2\varphi$; 4) $u(r,\varphi)=\dfrac{1}{2}+\dfrac{1}{8}r^2\cos 2\varphi$.

ILOVALAR

1- i lo v a

$$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$ funksiya qiymatlarining jadvali

	0	1	2	3	4	5	6	7	8	9
0,0	0,3989	3989	3989	3988	3986	3984	3982	3980	3977	3973
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0,2	3910	3902	3894	3885	3876	3867	3857	3847	3836	3825
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0,4	3683	3668	3652	3637	3621	3605	3589	3572	3555	3538
0,5	3521	3503	3485	3467	3448	3429	3410	3391	3372	3352
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0,7	3123	3101	3079	3056	3034	3011	2989	2966	2943	2920
0,8	2897	2874	2850	2827	2803	2780	2756	2732	2709	2685
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1,0	0,2420	2396	2371	2347	2323	2299	2275	2251	2227	2203
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1,2	1942	1919	1895	1872	1849	1826	1804	1781	1758	1736
1,3	1714	1691	1669	1647	1626	1604	1582	1561	1539	1518
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973	0957
1,7	0940	0925	0909	0893	0878	0863	0848	0833	0818	0804
1,8	0790	0775	0761	0748	0734	0721	0707	0694	0681	0669
1,9	0656	0644	0632	0620	0608	0596	0584	0573	0562	0551
2,0	0,0540	0529	0519	0508	0498	0488	0478	0468	0459	0449
2,1	0440	0431	0422	0413	0404	0396	0387	0379	0371	0363
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2,3	0283	0277	0270	0264	0258	0252	0246	0241	0235	0229
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0139
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2,7	0104	0101	0099	0096	0093	0091	0088	0086	0084	0081
2,8	0079	0077	0075	0073	0071	0069	0067	0065	0063	0061
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3,0	0,0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3,1	0033	0032	0031	0030	0029	0028	0027	0026	0025	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3,4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3,5	0009	0008	0008	0008	0008	0007	0007	0007	0007	0006
3,6	0006	0006	0006	0005	0005	0005	0005	0005	0005	0004
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3,8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

2- i lo v a

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}} dz$$ funksiya qiymatlarining jadvali

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,00	0,0000	**0,43**	0,1664	**0,86**	0,3051	**1,29**	0,4015	**1,72**	0,4573	**2,30**	0,4893
0,01	0,0040	**0,44**	0,1700	**0,87**	0,3078	**1,30**	0,4032	**1,73**	0,4582	**2,32**	0,4898
0,02	0,0080	**0,45**	0,1736	**0,88**	0,3106	**1,31**	0,4049	**1,74**	0,4591	**2,34**	0,4904
0,03	0,0120	**0,46**	0,1772	**0,89**	0,3133	**1,32**	0,4066	**1,75**	0,4599	**2,36**	0,4909
0,04	0,0160	**0,47**	0,1808	**0,90**	0,3159	**1,33**	0,4082	**1,76**	0,4608	**2,38**	0,4913
0,05	0,0199	**0,48**	0,1844	**0,91**	0,3186	**1,34**	0,4099	**1,77**	0,4616	**2,40**	0,4918
0,06	0,0239	**0.49**	0,1879	**0,92**	0,3212	**1,35**	0,4115	**1,78**	0,4625	**2,42**	0,4922
0,07	0,0279	**0,50**	0,1915	**0,93**	0,3228	**1,36**	0,4131	**1,79**	0,4633	**2,44**	0,4927
0,08	0,0319	**0,51**	0,1950	**0,94**	0,3264	**1,37**	0,4147	**1,80**	0,4641	**2,46**	0,4931
0,09	0,0359	**0,52**	0,1985	**0,95**	0,3289	**1,38**	0,4162	**1,81**	0,4649	**2,48**	0,4934
0,10	0,0398	**0,53**	0,2019	**0,96**	0,3315	**1,39**	0,4177	**1,82**	0,4556	**2,50**	0,4938
0,11	0,0438	**0,54**	0,2054	**0,97**	0,3340	**1,40**	0,4192	**1,83**	0,4664	**2,52**	0,4941
0,12	0,0478	**0,55**	0,2088	**0,98**	0,3365	**1,41**	0,4207	**1,84**	0,4671	**2,54**	0,4945
0,13	0,0517	**0,56**	0,2123	**0,99**	0,3389	**1,42**	0,4222	**1,85**	0,4678	**2,56**	0,4948
0,14	0,0557	**0,57**	0,2157	**1,00**	0,3413	**1,43**	0,4236	**1,86**	0,4686	**2,58**	0,4951
0,15	0,0596	**0,58**	0,2190	**1,01**	0,3438	**1,44**	0,4251	**1,87**	0,4693	**2,60**	0,4953
0,16	0,0636	**0,59**	0,2224	**1,02**	0,3461	**1,45**	0,4265	**1,88**	0,4699	**2,62**	0,4956
0,17	0,0675	**0,60**	0,2257	**1,03**	0,3485	**1,46**	0,4279	**1,89**	0,4706	**2,64**	0,4959
0,18	0,0714	**0,61**	0,2291	**1,04**	0,3508	**1,47**	0,4292	**1,90**	0,4713	**2,66**	0,4961
0,19	0,0753	**0,62**	0,2324	**1,05**	0,3531	**1,48**	0,4306	**1,91**	0,4719	**2,68**	0,4963
0,20	0,0793	**0,63**	0,2357	**1,06**	0,3554	**1,49**	0,4319	**1,92**	0,4726	**2,70**	0,4965
0,21	0,0832	**0,64**	0,2389	**1,07**	0,3577	**1,50**	0,4332	**1,93**	0,4732	**2,72**	0,4967
0,22	0,0871	**0,65**	0,2422	**1,08**	0,3599	**1,51**	0,4345	**1,94**	0,4738	**2,74**	0,4969
0,23	0,0910	**0,66**	0,2454	**1,09**	0,3621	**1,52**	0,4357	**1,95**	0,4744	**2,76**	0,4971
0,24	0,0948	**0,67**	0,2486	**1,10**	0,3643	**1,53**	0,4370	**1,96**	0,4750	**2,78**	0,4973
0,25	0,0987	**0,68**	0,2517	**1,11**	0,3665	**1,54**	0,4382	**1,97**	0,4756	**2,80**	0,4974
0,26	0,1026	**0,69**	0,2549	**1,12**	0,3686	**1,55**	0,4394	**1,98**	0,4761	**2,82**	0,4976
0,27	0,1064	**0,70**	0,2580	**1,13**	0,3708	**1,56**	0,4406	**1,99**	0,4767	**2,84**	0,4977
0,28	0,1103	**0,71**	0,2611	**1,14**	0,3729	**1,57**	0,4418	**2,00**	0,4772	**2,86**	0,4979
0,29	0,1141	**0,72**	0,2642	**1,15**	0,3749	**1,58**	0,4429	**2,02**	0,4783	**2,88**	0,4980
0,30	0,1179	**0,73**	0,2673	**1,16**	0,3770	**1,59**	0,4441	**2,04**	0,4793	**2,90**	0,4981
0,31	0,1217	**0,74**	0,2703	**1,17**	0,3790	**1,60**	0,4452	**2,06**	0,4803	**2,92**	0,4982
0,32	0,1255	**0,75**	0,2734	**1,18**	0,3810	**1,61**	0,4463	**2,08**	0,4812	**2,94**	0,4984
0,33	0,1293	**0,76**	0,2764	**1,19**	0,3830	**1,62**	0,4474	**2,10**	0,4821	**2,96**	0,4985
0,34	0,1331	**0,77**	0,2794	**1,20**	0,3859	**1,63**	0,4484	**2,12**	0,4830	**2,98**	0,4986
0,35	0,1368	**0,78**	0,2823	**1,21**	0,3869	**1,64**	0,4495	**2,14**	0,4838	**3,00**	0,49865
0,36	0,1406	**0,79**	0,2852	**1,22**	0,3883	**1,65**	0,4505	**2,16**	0,4836	**3,20**	0,49931
0,37	0,1443	**0,80**	0,2881	**1,23**	0,3907	**1,66**	0,4515	**2,18**	0,4854	**3,40**	0,49966
0,38	0,1480	**0,81**	0,2910	**1,24**	0,3925	**1,67**	0,4525	**2,20**	0,4861	**3,60**	0,499841
0,39	0,1517	**0,82**	0,2939	**1,25**	0,3944	**1,68**	0,4535	**2,22**	0,4868	**3,80**	0,499928
0,40	0,1554	**0,83**	0,2967	**1,26**	0,3962	**1,69**	0,4545	**2,24**	0,4875	**4,00**	0,499868
0,41	0,1591	**0,84**	0,2995	**1,27**	0,3980	**1,70**	0,4554	**2,26**	0,4881	**4,50**	0,499997
0,42	0,1628	**0,85**	0,3023	**1,28**	0,3397	**1,71**	0,4564	**2,28**	0,4887	**5,00**	0,499997

3- i lo v a

$P(m)=\frac{\lambda^m}{m!}e^{-\lambda}$ funksiya qiymatlarining jadvali

m \ λ	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0	0,9048	0,8187	0,7408	0,6703	0,6065	0,5488	0,4966	0,4493	0,4066	0,3679
1	0,0905	0,1637	0,2223	0,2681	0,3033	0,3293	0,3476	0,3595	0,3659	0,3679
2	0,0045	0,0164	0,0333	0,0536	0,0758	0,0988	0,1216	0,1433	0,1647	0,1839
3	0,0002	0,0011	0,0033	0,0072	0,0126	0,0198	0,0284	0,0383	0,0494	0,0613
4	0,0000	0.0001	0,0003	0,0007	0,0016	0,0030	0,0050	0,0077	0,0111	0,0153
5	0,0000	0,0000	0,0000	0.0001	0,0002	0,0003	0,0007	0,0012	0,0020	0,0031
6	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001	0,0002	0,0003	0,0005
7	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001

m \ λ	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0
0	0,1353	0,0498	0,0183	0,0067	0,0025	0,0009	0,0003	0,0001	0,0001
1	0,2707	0,1494	0,0733	0,0337	0,0149	0,0064	0,0027	0,0011	0,0005
2	0,2707	0,2240	0,1465	0,0842	0,0446	0,0223	0,0107	0,0050	0,0023
3	0,1805	0,2240	0,1964	0,1404	0,0892	0,0521	0,0286	0,0150	0,0076
4	0,0902	0,1681	0,1954	0,1755	0,1339	0,0912	0,0572	0,0337	0,0189
5	0,0361	0,1008	0,1563	0,1755	0,1606	0,1277	0,0916	0,0607	0,0378
6	0,0120	0,1504	0,1042	0,1462	0,1606	0,1490	0,1221	0,0911	0,0631
7	0,0034	0,0216	0,0595	0,1045	0,1377	0,1490	0,1396	0,1171	0,0901
8	0,0009	0,0081	0,0298	0,0653	0,1033	0,1304	0,1396	0,1318	0,1126
9	0,0002	0,0027	0,0132	0,0363	0,0689	0,1014	0,1241	0,1318	0,1251
10	0,0000	0,0008	0,0053	0,0181	0,0413	0,0710	0,0993	0,1186	0,1251
11	0,0000	0,0002	0,0019	00082	0,0225	0,0452	0,0722	0,0970	0,1137
12	0,0000	0.0001	0,0006	0,0034	0,0113	0,0264	0,0481	0,0728	0,0948
13	0,0000	0,0000	0,0002	0,0013	0,0052	0,0142	0,0296	0,0504	0,0729
14	0,0000	0,0000	0.0001	0,0005	0,0022	0,0071	0,0169	0,0324	0,0521
15	0,0000	0,0000	0,0000	0,0002	0,0009	0,0033	0,0090	0,0194	0,0347
16	0,0000	0,0000	0,0000	0,0000	0,0003	0,0015	0,0045	0,0109	0,0217
17	0,0000	0,0000	0,0000	0,0000	0.0001	0,0006	0,0021	0,0058	0,0128
18	0,0000	0,0000	0,0000	0,0000	0,0000	0,0002	0,0009	0,0029	0,0071
19	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001	0,0004	0,0014	0,0037
20	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0002	0,0006	0,0019
21	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001	0,0003	0,0009
22	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001	0,0004
23	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0002
24	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0.0001
25	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

4- i lo v a

$t_\gamma = t(\gamma, n)$ ning qiymatlari jadvali

γ / n	**0,95**	**0,99**	**0,999**	γ / n	**0,95**	**0,99**	**0,999**
5	2,78	4,60	8,61	**20**	2,093	2,861	3,883
6	2,57	4,03	6,86	**25**	2,064	2,797	3,745
7	2,45	3,71	5,96	**30**	2,045	2,756	3,659
8	2,37	3,50	5,41	**35**	2,032	2,720	3,600
9	2,31	3,36	5,04	**40**	2,023	2,708	3,558
10	2,26	3,25	4,78	**45**	2,016	2,692	3,527
11	2,23	3,17	4,59	**50**	2,009	2,679	3,502
12	2,20	3,11	4,44	**60**	2,001	2,662	3,464
13	2,18	3,06	4,32	**70**	1,996	2,649	3,439
14	2,16	3,01	4,22	**80**	1,991	2,640	3,418
15	2,15	2,98	4,14	**90**	1,987	2,633	3,403
16	2,13	2,95	4,07	**100**	1,984	2,627	3,392
17	2,12	2,92	4,02	**120**	1,980	2,617	3,374
18	2,11	2,90	3,97	∞	1,960	2,576	3,291
19	2,10	2,88	3,92				

5- i lo v a

$q = q(\gamma, n)$ ning qiymatlari jadvali

γ / n	**0,95**	**0,99**	**0,999**	γ / n	**0,95**	**0,99**	**0,999**
5	1,37	2,67	5,64	**20**	0,37	0,58	0,88
6	1,09	2,01	3,88	**25**	0,32	0,49	0,73
7	0,92	1,621	2,98	**30**	0,28	0,43	0,63
8	0,80	1,38	2,42	**35**	0,26	0,38	0,56
9	0,71	1,20	2,06	**40**	0,24	0,35	0,59
10	0,65	1,08	1,80	**45**	0,22	0,32	0,46
11	0,59	0,98	1,60	**50**	0,21	0,30	0,43
12	0,55	0,90	1,45	**60**	0,188	0,269	0,38
13	0,52	0,83	1,33	**70**	0,174	0,245	0,34
14	0,48	0,78	1,23	**80**	0,161	0,226	0,31
15	0,46	0,73	1,15	**90**	0,151	0,211	0,29
16	0,44	0,70	1,07	**100**	0,143	0,198	0,27
17	0,42	0,66	1,01	**150**	0,115	0,160	0,211
18	0,40	0,63	0,96	**200**	0,099	0,136	0,185
19	0,39	0,60	092	**250**	0,089	0,120	0,162

6- i lo v a

χ^2 taqsimotning kritik nuqtalari

k **ozodlik darajalari soni**	α **qiymatdorlik darajasi**					
	0,01	**0,025**	**0,05**	**0,95**	**0,975**	**0,99**
1	6,6	5,0	3,8	0,0039	0,00098	0,00016
2	9,2	7,4	6,0	0,103	0,051	0,020
3	11,3	9,4	7,8	0,352	0,216	0,115
4	13,3	11,1	9,5	0,711	0,484	0,297
5	15,1	12,8	11,1	1,15	0,831	0,554
6	16,8	14,4	12,6	1,64	1,24	0,872
7	18,5	16,0	14,1	2,17	1,69	1,24
8	20,1	17,5	15,5	2,73	2,18	1,65
9	21,7	19,0	16,9	3,33	2,70	2,09
10	23,2	20,5	18,3	3,94	3,25	2,56
11	24,7	21,9	19,7	4,57	3,82	3,05
12	26,2	23,3	21,0	5,23	4,40	3,57
13	27,7	24,7	22,4	5,89	5,01	4,11
14	29,1	26,1	23,7	6,57	5,63	4,68
15	30,6	27,5	25,0	7,25	6,26	5,23
16	32,0	28,8	26,3	7,96	6,91	5,81
17	33,4	30,2	27,6	8,67	7,56	6,41
18	34,8	31,5	28,9	9,39	8,23	7,01
19	36,2	32,9	30,1	10,1	8,91	7,63
20	37,6	34,2	31,4	10,9	9,59	8,26
21	38,9	35,5	32,7	11,6	10,3	8,90
22	40,3	36,8	33,9	12,3	11,0	9,54
23	41,6	38,1	35,2	13,1	11,7	10,2
24	43,0	39,4	36,4	13,8	12,4	10,9
25	44,3	40,6	37,7	14,6	13,1	11,5
26	45,6	41,9	38,9	15,4	13,8	12,2
27	47,0	43,2	40,1	16,2	14,6	12,9
28	48,3	44,5	41,3	16,9	15,3	13,6
29	49,6	45,7	42,6	17,7	16,0	14,3
30	50,9	47,0	43,8	18,5	16,8	15,0

Styudent taqsimotning kritik nuqtalari

k **ozodlik darajalari soni**	α **qiymatdorlik darajasi (ikki tomonli kritik soha)**					
	0,10	**0,05**	**0,02**	**0,01**	**0,002**	**0,001**
1	6,31	12,7	31,82	63,7	318,3	637,0
2	2,92	4,30	6,97	9,92	22,33	31,6
3	2,35	3,18	4,54	5,84	10,22	12,9
4	2,13	2,78	3,75	4,60	7,17	8,61
5	2,01	2,57	3,37	4,03	5,89	6,86
6	1,94	2,45	3,14	3,71	5,21	5,96
7	1,89	2,36	3,00	3,50	4,79	5,40
8	1,86	2,31	2,90	3,36	4,50	5,04
9	1,83	2,26	2,82	3,25	4,30	4,78
10	1,81	2,23	2,76	3,17	4,14	4,59
11	1,80	2,20	2,72	3,11	4,03	4,44
12	1,78	2,18	2,68	3,05	3,93	4,32
13	1,77	2,16	2,65	3,01	3,85	4,22
14	1,76	2,14	2,62	2,98	3,79	4,14
15	1,75	2,13	2,60	2,95	3,73	4,07
16	1,75	2,12	2,58	2,92	3,69	4,01
17	1,74	2,11	2,57	2,90	3,65	3,96
18	1,73	2,10	2,55	2,88	3,61	3,92
19	1,73	2,09	2,54	2,86	3,58	3,88
20	1,73	2,09	2,53	2,85	3,55	3,85
21	1,72	2,08	2,52	2,83	3,53	3,82
22	1,72	2,07	2,51	2,82	3,51	3,79
23	1,71	2,07	2,50	2,81	3,49	3,77
24	1,71	2,06	2,49	2,80	3,47	3,74
25	1,71	2,06	2,49	2,79	3,45	3,72
26	1,71	2,06	2,48	2,78	3,44	3,71
27	1,71	2,05	2,47	2,77	3,42	3,69
28	1,70	2,05	2,46	2,76	3,40	3,66
29	1,70	2,05	2,46	2,76	3,40	3,66
3,6630	1,70	2,04	2,46	2,75	3,39	3,65
403,65	1,68	2,02	2,42	2,70	3,31	3,55
603,65	1,67	2,00	2,39	2,66	3,23	3,46
1203,46	1,66	1,98	2,36	2,62	3,17	3,37
∞ **3,37**	1,64	1,96	2,33	2,58	3,09	3,29
	0,05	**0,025**	**0,01**	**0,005**	**0,001**	**0,0005**
α **qiymatdorlik darajasi (bir tomonli kritik soha)**						

MUNDARIJA

IV bob. MATEMATIK FIZIKA TENGLAMALARI