O’ZBEKİSTON RESPUBLİKASİ ALOQA, AXBOROTLASHTİRİSH VA TELEKOMMUNİKATSİYA TEXNOLOGİYALARİ DAVLAT QO’MİTASİ

TOSHKENT AXBOROT TEXNOLOGİYALARİ UNİVERSİTETİ

NUKUS FİLİALİ

KOMP`YUTER INJINIRING FAKUL`TETI

AXBOROT TEXNOLOGIYALARI KAFEDRASI

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«XUSUSIY XOSILALI DIFFERENTSIAL TENGLAMALARNI ECHISHDA MATHCAD NING QULLANILISHI» mavzusidagi

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TACDIQLAYMAN
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§2. Ikki qatlamli iteratsiion sxemalar
§3. Uch qatlamli iteratsion sxemalar
§4. Xususiy xosilali differentsial tenglamalarni MathCad tizimida echish
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АННОТАЦИЯ

В данной выпускной квалификационной работе рассмотрены численные методы решения дифференциальных уравнении в частных производных. Кроме того приведены приближенные методы решения дифференциальных уравнении в частных производных с граничными условиями в системе MathCad.

ANNOTATSIYA

Mazkur bitiruv malakaviy ishida xususiy xosilali differentsial tenglamalarni sonli echish usullari qaralgan . Bundan tashqari xususiy hosilali differentsial tenglamaga qoyilgan chegaraviy shartlarda taqriybiy echish usullari MathCad tizimida keltirilgan.

SUMMARY

In the given final qualifying work the numerical methods of the decision the differential equation in private derivative are considered.

Besides the approached methods of the decision the differential equation in private derivative with boundary conditions in system MathCad are given.

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Kirish

Ta`limni tarbiyadan, tarbiyani esa ta`limdan ajratib bo'lmaydi-bu sharqona qarash, sharqona haet falsafasi.

Bu haqda fikr yuritganda, men Abdulla Avloniyning ''Tarbiya biz uchun yo haet-yo mamot, yo najot-yo halokat, yo saodat-yo falokat masalasidir'' degan chuqur ma`noli so'zlarini eslayman.

Shuning uchun ham mustaqillikning dastlabki yillaridanoq butun mamlakat miqesida ta`lim va tarbiya, ilm-fan, kasb-hunar o'rgatish tizimlarini tubdan isloh qilishga nihoyatda kata zarurat sezila boshladi. Kadrlar tayerlash milliy dasturini ishlab chiqish bilan bog`lik jaraen uzoq yillar mobaynida bu sohada talay muommalar yig`ilib qolganini ko'rsatdi. Shuning uchun ham bu og`ir, mas`uliyatli, ammo hal qilishni aslo paysalga solib bo'lmaydigan ishni qadam-baqadam, izchillik bilan bajarishga bel bog`ladik. [1]

Mavzuning dolzarbligi: Bitiruvchi kurs talabalariga va magistrantlarga xususiy xosilali differentsial tenglamalarni echishning har xil usillarini aniqrog`i ikki va uch qatlamli iteratsion sxemalar erdamida echish usillarini va MathCad tizimida echishni urgatish asosiy dolzarb masalalarning biri bulib qoladi.

Matematik fizika tenglamalari kursini urganishda va fizika texnikaning juda kup masalalarini ikkinchi tartibli xususiy xosilali differentsial tenglamalar va ularga quyilgan chegaraviy shartlarda masalalarni echish usullari urgatiladi [2,14-16]. Ko'p xollarda xususiy xosilali differentsial tenglamalarning echimlarini analitik usulda olish mumkin emas. Shuning uchun echimni sonli usilda olishga harakat etamiz. Chegirmalar usuli erdamida $\Delta u=-f$ Puasson tenglamasi uchun chegarviy masalalar, algebrik tenglamalar sistemasiga keltiriladi sistemaning tartibi to'rning ichki tugunlar soniga teng bulip u to'rning odimi kichirgan sari ortadi. Ikkinchi tartibli xususiy hosilali differentsial tenglama chekli chegirmalar usuli erdamida uch nuqtali to'r tenglamalariga keltiriladi. U tenglamani oddiy iteratsiya, Zeydel va matritsaviy haydash usulidan foydalanib echish mumkin [3,4].

Mavzuning maqsadi: Xususiy xosilali differentsial tenglamalarining echimini har xil usillarda, oddiy iteratsiya, Zeydel va matritsaviy xaydash usilida olish va bunday masalalarni studentlarning oson qabul qilishi uchun matonat va bilimni urgatish bulib topiladi.

**Tadqiqot ob`ekti va predmeti:** Matematik fizika tenglamalari fani va undagi chegirmalik tenglamalarga bog`lik aniq masalalarni echishning va uning usillarini urgatish bulib topiladi.

Ishning tuzilishi va tarkibi: Mazkur bitiruv malakaviy ishi kirish, turt paragraftan, xulosa va foydalanilgan adabiyotlar ruyxatidan iborat bo'lib jami 47 saxifa.

Biz ushbu bitiruv malakaviy ishda ikkinchi tartibli xususiy hosilali differentsial tenglamalarga quyilgan chegaraviy masalalarni matritsaviy haydash usilidan foydalanib echishga batafsil tuxtab uttik [7-9]. Bu bitiruv malakaviy ish uch paragraftan yakunlash va foydalanilgan adabietlar tizimidan iborat bulib, birinchi paragrafta ikkinchi tartibli xususiy xosilali differentsial tenglamalarni sonli echishning tugri va iteratsiya usillari keltirilgan yani Puasson tenglamasi uchin chegaraviy shartlarda tugriburchakli oblastta echish kurib utilgan. Ikki metod uchun ham arifmetik operatsiyalar soni $Q=0\ (N^2 \log_2 N)$, bunda N-bitta yunalish buyicha tuginlar soni.

Ikkinchi paragrafta ikki qatlamli iteratsion sxemalarga tuxtab utildi. Iteratsion metod qandaydir dastlabki $y_0 \in H$ yaqinlashishdan boshlab tenglamaning echimini ketma-ket topishga erdam beradi $y_1, y_2, \ldots y_k, y_{k+1} \ldots$ unda, k-iteratsiya nomeri. y_{k+1} ning qiymati uzidan oldingi $y_k, y_{k+1} \ldots$ iteratsiyalar erdamida topiladi. Agar y_{k+1} ni xisoblashda faqat olding y_k iteratsiya foydalanilsa, unda iteratsion metod bir qadamli, agar olding ikki iteratsiya foydalanilsa, unda iteratsiya metodi iki qadamli deb ataladi.

Bitiruv malakaviy ishning uchinchi paragrafida tur tenglamalarini echishning haydash usilining ayrim turlari keltirilgan. Turtinchi paragrafta esa xususiy xosilali differentsial tenglamalarni echishda MathCad ning qullanilishi keltirilgan. 4.1 bandda Laplas tenglamasi va 4.2 bandda esa giperbolik tiptagi tenglama MathCad tizimida echilgan.

§1. Xususiy xosilali differentsial tenglamalarni echish usullari

1.1 Tug`ri va iteratsiya usullari

Biz elliptik tenglamalar uchun chegarviy masalalarni chegirmali approksimatsiyalash natijasida chiziqli algebraik tenglamalar sistemasiga ega bulgan edik. (chegirmali yaki tur tenglamalariga)

$$A(x)\cdot y(x)=\sum_{\xi\in V(x)}\hat{A}(\tilde{o},\xi)\cdot \acute{o}(\xi)+F(x), \qquad x\in\omega_h \tag{1}$$

Bu erda *V(x),* (2r+1) krest shablonining 2p tuginlar tuplami. Tuginning uzi kirmaydi $\xi\neq x$.

V(x)-tuplamining x tuguni atrofi deb ataymiz. A(x) va V(x, ξ)-berilgan tenglama koefitsientlari (1) tenglamaga

$$y|_{\gamma_h}=\mu(x) \tag{2}$$

chegaraviy shart qushib eziladi [3]. Ushbu sistemaning matritsasi A ning tartibi, turning tugunlar soni N ga teng. Misol uchun har bir g`arazsiz uzgaruvchi $x_1$, $x_2$, …$x_p$ ($h_1=h_2=\ldots=h_p=h$) buyicha h odim bilan turga N=0 $\left(\frac{1}{h^p}\right)$ tuginlar soni kerak, bunda p-ulcham. Ikki va uch ulchamda tenglamalar soni katta buladi, $N\approx10^4$-$10^6$ (Mis $h=\frac{1}{100}$) sistemaning matritsasi kup nul`lik elementka ega buladi, va qiyin uzlashtiriladigan matritsaga ega buladi, ya`ni matritsaning eng katta xususiy qiymatining eng kichik xususiy qiymatiga nisbati judayam katta (10^{-3}~10^{-4}).

Elliptik tur tenglamalarining ushbu uzgachaliklariga uxshagan sonli echish uchin tejamli algoritmlar ishlab chiqishni talab etadi. Tug`ri tejamli usuli juda tor, lekin ahamiyatli tur tenglamalar klasini echish uchun qullaniladi. Bundan tashqari tug`ri metodlar iteratsion usilda operatorni yuqori qatlamda turlandirish uchun ham qullaniladi. Hozirgi paytta Puasson tenglamasini Dekart, polyar, tsilindr, sferiq, koordinatalar sistemasida chegirmalik chegaraviy masalani echish uchun ikki tejamkor tug`ri metod bor, ularning biri dekompozitsiya-ya`ni Gaussning ketma-ket yoq etish usilining modifikatsiyasi. Ikkinchisi uzgaruvchini ajratish metodi ya`ni Fur`e turlandirishiga asoslangan. Ikki metod uchun ham arifmetik operatsiyalar soni Q=0 ($N^2 \log_2 N$), bunda N-bir yunalish buyicha tugunlar soni.

Yuqori ikki metodtan tashqari yana bir tug`ri metod-matritsaviy haydash usuli kurib utiladi. U elliptik chegirmali tenglamalarni soha murakkab bulgan holatni tekshiradi. Lekin matritsaviy haydash usili Q=O(N^4) arifmetik amalni bajarishni va katta xotirani talab etadi.

Ketma-ket yaqinlashish metodining iteratsiyasi soha erikli bulgan holda, uzgaruvchini koefitsientlar bilan birga umumiy tenglamalar uchun qullaniladi. Puasson tenglamasi uchun Dirixle masalasini $\overline{G}=\{0\le x_\alpha \le l_\alpha\ ,\ \alpha=1,2\}$ tug`ri burchakta qurib utamiz. G-chegarasi.

$$\Delta u=\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}=-f(x),\ x=(x_1,x_2)\in G,\ u|_\Gamma=\mu(x) \quad (3)$$

$\overline{G}$ da h_1, h_2 qadam bilan tug`ri burchakli tur tuzamiz.

$$\omega_h=\left\{x_{i_1 i_2}=(i_1 h_q, i_2 h_2)\in\overline{G}, \quad \alpha=0,1,...N_\alpha, \quad h_\alpha N_\alpha=l_\alpha, \quad \alpha=1,2\right\}$$

Mayli $\gamma_h=\{x_{i1,i2} \in G\}$ turning chegarasi. (3) tenglamaga mos keluvchi Dirixle masalasinin`g chegirmali masalasi

$$\Delta u=-f(x), \quad x\in\omega_h, \quad u|_{\gamma n}=\mu(x)$$

$$\Delta=\Delta_1+\Delta_2, \qquad \Delta_\alpha u=y_{i_\alpha j_\alpha}, \quad \alpha=1,2 \quad (4)$$

$y_{i_1 i_2}=y(i_1 h_1, i_2 h_2)$ kurinishda buladi.

1.2 Uzgaruvchini ajratish usuli

(4) masalaning echimini birjinsli chegaraviy shartlarda kurib utamiz.

$$\wedge u=-\varphi, \qquad u|_{\gamma h}=0 \quad (5)$$

bu erda φ, (4) masaladagi f tan faqat chegaradagi tugunlarda $\frac{1}{h_1^2}\mu,\ i_1=1,\ i_1=N_1-1$ qiymatga va $\frac{1}{h_2^2}\mu,\ i_2=1,\ i_2=N_2-1$ qiymatga uzgaradi.

Mayli $\mu_k(jh_2)$ va λ_k-xos funktsiya va xos qiymat bulsin.

$$\wedge_2\mu_k+\lambda_k\mu_k=0, \qquad h_2\leq x_2 \leq l_2-h_2 \tag{6}$$

$\mu_k(0)= \mu_k(l_2)=0$

$$\mu_k(jh_2)=\sqrt{\frac{2}{l_2}}\sin\frac{k\pi j}{N_2}, \quad \lambda_k=\frac{4}{h_2^2\sin^2\frac{k\pi h_2}{2l_2}} \quad \lambda_k=\frac{4}{h_2^2}\sin^2\frac{k\pi h_2}{2l_2} \qquad k=1,2,3\ldots N_2-$$

ekanligini bila utirib (5) masalaning echimini quydagicha qator kurinishda qidiramiz [3,10].

$$y_{ij}=\sum_{k=1}^{N_2-1}C_k(ih_1)\mu_k(jh_2)\ i=1,2,3\ldots.N_1-1, j=1,2,3\ldots N_2-1 \tag{7}$$

bu erda S_k-Fur\`e koeffitsienti $x_1=ih_1$ dan qaram. (7) ni (5) tenglamga olib borib quygandan sung

$$\wedge y=\wedge_1 y+\wedge_2 y=$$

$$\sum_{k=1}^{N_2-1}\left[\mu_k(jh_2)\wedge_1 C_k(ih_1)+C_k(ih_1)\wedge_2\ \mu_k(jh_2)\right]=-\sum_{k=1}^{N_2-1}\varphi_k(ih_1)\mu_k(jh_2) \tag{8}$$

ga ega bulamiz. Bu erda $\varphi_k(ih_1)$, $\varphi(x)$ funktsiyasining Fur\`e koeffitsientlari.

$$\varphi_k(ih_1)=\sum_{i=1}^{N_2-1}\varphi(ih_1, jh_2)\cdot\mu_k(jh_2)\cdot h_2$$

(6) ni hisobga olib va μ_k - funktsiyalarining ortogonalligi (8) dan S_k ni aniqlash uchin quydagi masalaga ega bulamiz.

$$\wedge_1 c_k-\lambda_k c_k=-\varphi_k, \qquad h_1\leq x_1 \leq l_1-h_1$$

$$c_k(0)= c_k(l_1)=0, \qquad k=1,2,\ldots N_2-1 \tag{9}$$

kurinib turganidek $c_k(ih_1)$, $x_1=ih_1$ ning funktsiyasi sifatida har bir k uchun haydash metodi bilan topiladi. Hammasi bulib N_2-1 marta haydash algoritmidan foydalanish kerak. (7) formula buyicha $c_k(ih_1)$ ni bila utirib (5) masalaning echimini topamiz.\\рет

φ_k -Fur\`e koeffitsientlaridan hisoblash va u_{ij} - echimni topish umumiy bir formula bilan hisoblab topiladi. Bu erda

$$v_j=\sum_{k=1}^{N-1}z_k\sin\frac{k\pi j}{N}, \qquad j=1,2,3\ldots N-1$$

katorning yig`indisini topish mumkin. Uning uchun Fur`e turlandirishining maxsus algoritmi qullaniladi, ya`ni barcha kursatilgan yig`indi $q=5N\log_2 N$ arifmetik operatsiyada ($N=2^n$) oddiy holda $0(N^2)$ arifmetik operatsiyada hisoblanadi.

Shu bilan birga (4) chegirmalik Dirixle masalasining echimi tug`riburchakda $0(N_1 N_2 \log_2 N_2)$ arifmetik operatsiyada topiladi. Xususiy holda, uzgaruvchini ajratish metodi dekompozitsiya metodi bilan birgalikta qullanilishi mumkin.

§2. Ikki qatlamli iteratsion sxemalar

2.1. Masalaning quyilishi

Mayli birinchi tur

$$Au=f \tag{10}$$

bir jinsli tenglama berilsin. Bunda $A:H\to H$ chiziqli operator, haqiyqiy fazoda skalyar kupaytma va norma $||y||=\sqrt{(y,y)}$ aniqlangan, quydagicha faraz qilamiz $A=A^{*}>0$, $f\in H$-hohlagan vektor. Dastlab iteratsion metodning umumiy xarakteristikasiga tuxtaymiz. Iteratsion metod qandaydir dastlabki $y_0\in H$ yaqinlashishdan boshlab tenglamaning echimini ketma-ket topishga yardam beradi $y_1,y_2,\dots y_k,y_{k+1}\dots$ u holda, k-iteratsiya nomeri. y_{k+1} ning qiymati uzidan oldingi $y_k,y_{k+1}\dots$ iteratsiyalar yardamida topiladi. Agar y_{k+1} ni hisoblashda faqat gina oldingi y_k iteratsiya foydalanilsa, u holda iteratsion metod bir qadamli, agar oldingi ikki iteratsiya foydalanilsa, unda iteratsiya metodi ikki odimli deb ataladi. Biz quydagi bir qadamli iteratsion metod formasi buyicha ikki qatlamli sxema bilan mos kelishini kursatamiz [11].

Hohlagan chiziqli bir qadamli iteratsion metod (10) tenglamani echish uchun

$$B_k y_{k+1}=c_k y_k+F_k, \qquad k=0,1,2,\dots.. \tag{11}$$

Kurinishida yazilishi mumkin. Bu erda B_k va c_k lar N tan olingan chiziqli operatorlar, ular iteratsiya nomeri k-dan g`arazli, ol $F_k\in H$-berilgan funktsiya $y_k$ - k-chi iteratsiya. (10) tenglamanin`g k-dan g`arazli oniq echimi (11) tenglamani qanoatlandirishi shart.

$$(B_k-c_k)u=F_k \qquad \text{al ol} \qquad (B_k-c_k)A^{-1}f=F_k$$

urinli bulganda bajariladi. Bundan kelib chiqadi

1) $(B_k-c_k)^{-1}$ -teskari operator bor

2) $f=A(B_k-c_k)^{-1}F_k$.

Hamma vaqt $\tau^{-1}_{k+1}(B_k-c_k)=A$, $F_k=f\tau_{k+1}$, $k=0,1,2,\dots$ deb olish mumkin, bunda $\tau_{k+1}>0$-sonli parametr.

Natijada ikki qatlamli iteratsion sxemaning kanonik formasini olamiz.

$$B_k \frac{y_{k+1} - y_k}{\tau_{k+1}} + Ay_k = f, \qquad k = 0,1,2,..... \tag{12}$$

k=0 bulganda erkli dastlabki yaqinlashishni olamiz (nul`lik iteratsiya) $y_0 \in H$. B^{-1}_k -teskari operator bor bulganliktan (12) dan

$$y_{k+1}=y_k-\tau_{k+1}B^{-1}_k(Ay_k-f) \tag{13}$$

yaki $y_{k+1}=y_k-\tau_{k+1}B^{-1}_k r_k= y_k-\tau_{k+1}\,\omega_k$, bu erda $r_k= Ay_k-f$, $\omega_k=B^{-1}_k r_k$ tuzatish. Agar y_k -iteratsiya belgili bulsa, u holda y_{k+1} (13) tenglamadan topiladi. y_0- ni bila utirib ketma-ket $y_1,y_2,\ldots$ larni topamiz.

Iteratsion metod qiymatga ega agar u yaqinlashuvchi bulsa ya`ni

$$\| y_k-u\|\to 0, \ k\to\infty \ \text{ da} \tag{14}$$

odatta qandaydir $\varepsilon>0$ kichkina qiymat berib (10) masalaning taqribiy echimini topamiz.

$$\| y_k-u\|\le\varepsilon\| y_0-u\| \tag{15}$$

sharti urinli bulsa hisoblashni tuxtatamiz. Bu shart amalda tekshirish uchun noquloy, sababi u belgisiz vektor, va

$$\|A y_k-f\|\le\varepsilon\| Ay_0-f\| \tag{16}$$

sharti bilan olmashtirilishi mumkin, bu erda

$$r_k=Ay_k-f=Ay_k-Au$$

umumiy holda (15) ning urniga

$$\| y_k-u\|_D\le\varepsilon\| y_0-u\|_D \tag{17}$$

bunda $D=D^*>0$ - qandaydir operator.

$D=A^2$ deb olib (17 tengsizliktan (16) ni olamiz $z_k= y_k-u$ qoldiq uchun tenglama yozamiz. Au=f bulganliktan

$$B_k \frac{z_{k+1} - z_k}{\tau_{k+1}} + Az_k = 0, \qquad k = 0,1,2,..... \tag{18}$$

$z_0\in H$ berilgan. Agar $B_k=B$, k-dan g`arazli bulmasa, unda $\omega_k=B^{-1}r_k$ va

$$B \frac{\omega_{k+1} - \omega_k}{\tau_{k+1}} + A\omega_k = 0, \tag{19}$$

birjinsli tenglamani qanoatlantiradi. haqiqattan ham (13) tan

$$y_{k+1}-y_k=-\tau_{k+1}B^{-1}r_k= -\tau_{k+1}\,\omega_k$$

Bu tenglikning ikki tarafini A operatorini tasir ettirip, va

$$Ay_{k+1}-Ay_k= (Ay_{k+1}-f)-(Ay_k -f)=r_{k+1}- r_k$$

$$r_{k+1}- r_k =B(B^{-1}r_{k+1}-B^{-1} r_k)=B(\omega_{k+1} -\omega_k)$$

ekanligini hisobka olib (19) tenglamani hosil qilamiz [8]. (18) dan kurinib turganidek

$$z_{k+1}=S_{k+1}z_k,\ S_{k+1}=E-\tau_{k+1}B^{-1}A \qquad (20)$$

bu erda S_{k+1} k-qatlamdan k+1 qatlamga utuvchi operator.

z_k, z_{k-1},…z_1 larni ketma-ket yuq qilib k=n-1 da

$$z_n=T_nz_0,\quad T_n=S_nS_{n-1}\ldots S_2S_1 \qquad (21)$$

ga ega bulamiz. Bu erda T_n - (18) sxemaning echuvchi operatori (21) dan

$$\|z_n\|_D=\|T_n z_0\|_D\le\|T_n\|_D\,\|z_0\|_D \quad \text{yoki}$$

$$\|z_n\|_D\le q_n\,\|z_0\|_D\,, \qquad q_n= \|T_n\|_D \qquad (22)$$

ni topamiz. Agar $q_n\le\varepsilon$ bulsa iteratsiya urinlanish sharti tuxtatiladi. Shunday qilib, iteratsiyasi yaqinlashuvchi bulishini tekshirish uchun, echuvchi T_k operatorining normasini baholash kerak. Au=f tenglamaning hohlagan $\{V_k\}$ operatorda va hohlagan $\{\tau_{k+1}\}$ parametrda echish uchun (20) sxema aniq approksimatsiyaga ega. Lekin q_n sxemasi $\{V_k\}$ va $\{\tau_{k+1}\}$ lardan g\`arazli. Shuning uchun V_k va τ_{k+1} larni shunday qilib saylab olish kerak, (20) sxemaning echuvchi operatori T_n ning normasi $\|T_n\|_D=q_n$ eng kichkina bulishi kerak. Shu bilan birga V_k ni saylab olishda, berilgan y_k da u_{k+1} larni

$$V_k\, u_{k+1}=G`_k\,,\ G`_k=V_k\, u_k-\tau_{k+1}(Ay_k -f),$$

tenglaemadan aniqlash uchun arifmetik amallarni boricha azaytish kerak. Iteratsion metodning asosiy masalasi shundan iborat. (12) kurinishidagi hohlagan iteratsion protsesni formal\` kurinishda

$$B\frac{du}{dt}+Au=f,$$

nostatsionar tenglamani echishda ikki qatlamli sxema deb qarash mumkin. Bul erda t_{k+1} parametrni

$$t_{k+1} = \sum_{m=1}^{k+1} \tau_m$$

Vaqtning qadami deb qarash mumkin.

Iteratsion sxema va nostatsionar masala uchun sxema orasidagi farq quydagidan iborat.

a) (12) iteratsion sxema (10) tenglamani aniq approksimatsiya-laydi. Sabebi (10) tenglamaning echimi u hohlagan V_k va τ_{k+1} larda (12) tenglamani qanoatlantiradi.

b) Aniq echimni olish τ_{k+1} parametrni va V_k operatorni saylab olish iteratsiyaning yig`ilish shartiga va arifmetik amallarning ozligiga bog`lik. (nostatsionar masalada qadamni saylab olish, approkcimatsiya talabiga bog`inadi) Mayli Q(ε)-arifmetik amallarning umumiy soni bulsin, yani (12) metod erdamida (10) tenglama echimini berilgan ε>o aniqliqta olish kerak bulsin. Sxemanii yani $\{\tau_k\}$ va $\{V_k\}$ larni shunday qilib saylab olish kerak yani Q(ε) eng kichik bulsin. Agar n=n(ε) ε-aniqlikta oladigan iteratsiyaning minimal soni bulsa u holda

$$Q(\varepsilon) = \sum_{k=1}^{n(\varepsilon)} Q_n = \overline{Q}_n n,$$

bunda Q_k k-chi iteratsiyani topish uchun ketadigan amallar soni. Q_k ning minimumin topish, iteratsiya soni n(ε), va Q_n sonining minimumini tapishga olib keladi.

Agar V_k=E-birlik operator bulsa unda (12) aniq iteratsion sxema deb ataladi.

$$\frac{y_{k+1} - y_k}{\tau_{k+1}} + Ay_k = f, \qquad k = 0,1,2,.....$$

hohlagan $y_0 \in H$ uchun. Agar V_k=E| bulsa (12) sxema noaniq.

2.2 Oddiy iteratsiya usuli

$$\tau_0 = \frac{2}{\gamma_1 + \gamma_2} \qquad (23)$$

parametrda oddiy iteratsiya metodi quydagi kurinishda buladi.

$$\frac{y_{k+1}-y_k}{\tau_0}+Ay_k=f, \qquad (24)$$

Sababi $t_1=\cos\frac{\pi}{2}=\mathbf{0}, \quad \tau_1=\tau_0$.

Bu erda

$$q_1=\frac{2\rho_1}{1+\rho_1^2}=\rho_0 \qquad (25)$$

Bog`lanish formulasi $r_k=Ay_k–f$ uchin $y_{k+1}=Sy_k$, $S=E-\tau_0A$ tenglamaga ega bulamiz. $T_1=S$ bulganliktan\бунда\\\\\\\\\\(wt (25) tan utish operatorining normasi $\|S\|=\rho_0=\frac{1-\xi}{1+\xi}$ buladi. Oddiy iteratsiya metodi buyicha n- iteratsiyadan keyin $r_n=S^nr_0$, $\|r_n\|\le\rho^n_0\|r_0\|$ ga ega bulamiz. $\rho^n_0\le\varepsilon$ sharti urinli buladi agar

$$n\ge\frac{\ln\left(\frac{1}{\varepsilon}\right)}{\ln\left(\frac{1}{\rho_0}\right)} \qquad \text{bulsa va} \quad n\ge n_0(\varepsilon)$$

$$n_0(\varepsilon)=\frac{\ln(\frac{1}{\varepsilon})}{2\xi} \qquad (26)$$

urinli.

2.3. Maksimum qiymat printsipi

Biz endilikda doymiy koeffitsentli tenglamani qarab o’tamiz

$\vartheta_t=a^2\vartheta_{xx}+\beta\vartheta_x+\gamma\vartheta$

Bu tenglama $\vartheta=e^{\mu x+\lambda t}\cdot u$

bu erda

$\mu=-\frac{\beta}{2a^2}, \qquad \lambda=\gamma-\frac{\beta^2}{4a^2}$

urniga quyish yordamida

$$u_t=a^2u_{xx} \qquad (*)$$

kurinishiga keltiradi. Tenglama echimining maksimal qiymat printsipi deb ataluvchi quydagi hossasini keltiramiz [3,4].

Agar u(x,t) funktsiya $0 \le t \le T$ va $0 \le x \le \iota$ yopiq sohada aniqlangan va uzluksiz bulsa va soxaning $0 < x < \iota$ va $0 < t \le T$ nuqtalarida $u_t = a^2 u_{xx}$ harorat

chegara nuqtalarida erishadi. u(x,t)=const bulsa funktsiya harorat utkazuvchanlik tenglamasini qanoatlantiradi va uzining maksimal va minimal qiymatiga hohlagan nuqtada erishadi.

Bu teoremaga qarshi kelmaydi, sababi uning sharti buyicha agar maksimal (minimal`) qiymat oblast ichida erishilsa u holda t=0 yaki x=0, x=l da erishiladi.

Bu teoremaning fizik ma`nosi tushinarlik agar temperatura chegarada va boshlangich vaqt momentida qandaydir M sonidan oshib ketmasa, u holda jism ichida harorat manbbasi bo'lmasa M dan katta temperatura faydo bo'lishi mumkin emas.

Maksimal qiymat uchun teoremaning isbotini ko'rib utamiz.

Teorema isbotini qarama-qarshidan boshlaymiz u(x,t) funktsiyasining t=0 ($0 \le x \le l$)yoki x=0, x=l da maksimal qiymatini M orqali elgilaymiz, va qandaydir (x_0, t_0) $(0 < x_0 < l, 0 < t_0 < T)$ nuqtada u(x, t) funktsiyasi uzining

$$u(x_0, t_0) = M + \varepsilon$$

Maksimal qiymatiga erishsin. (x_0, t_0) nuqtada (13) tenglamaning chap va ung taraflaridagi belgilarini tekshiramiz. (x_0, t_0) nuqtada funktsiya uzining maksimal qiymaticha erishodigan bulsa u holda

$$\frac{\partial u}{\partial x}(x_0, t_0) = 0 \quad \text{Ba} \quad \frac{\partial^2 u}{\partial x^2}(x_0, t_0) \le 0$$

Bulishi shart sung $u(x_0, t_0)$ da $t = t_0$ maksimal qiymatga erishadigan bulsa

(*)

ning ung va chap taraflaridan belgilarini solishtirib ularnin har xil ekanligini qo'ramiz. Bu hali teorema isbotlandi degani emas. Tuliq isbotlash uchun (x_1, t_1) nuqtani topamiz u shartini qanoatlanlantirsin. Uning uchun qushimcha

$$\vartheta(x,t) = u(x,t) + k(t_0 - t) \qquad (**)$$

Erdamga funktsiya kiritamiz k -qandaydir son

$$\vartheta(x_0, t_0)=u(x_0, t_0)=M+\varepsilon$$

va $k(t_0-t)\leq kT$

k>0 sonini $kT<\frac{\varepsilon}{2}$ **eki** $k<\frac{\varepsilon}{2T}$ buladiganday etib saylab olamiz.

U holda $\vartheta(x,t)$ **ning** $t=0$ **yaki** $x=0, x=l$ **diga maksimal qiymati** $M+\frac{\varepsilon}{2}$ dan ortib ketmaydi ya`ni

$$\vartheta(x,t)\leq M+\frac{\varepsilon}{2} \qquad (t=0,\ x=0,\ x=l)$$

(**) formulaning birinchi qushiluvchisi M dan ikkinchisi ega - $\frac{\varepsilon}{2}$ dan ortib ketmaydi. Agar u(x,t) ni chegaralangan $0\leq x\leq l$, $0\leq t\leq T$ sohada uzliksiz deb olmasak u holda u(x,t) hech bir nuqtada uzining maksimum nuqotiga ega bo’la olmaydi. Har qanday uzluksiz funktsiya uzining maksimal qiymatiga tuyiq sohada erishadi degan teorema kuchiga ko’ra

1) u(x,t) o’zining maksimal qiymatiga tuo’griburchakning pastin yoki yon tarafida erishadi va M ga teng bo’ladi.
2) Agar u(x,t) ning qiymati kamida bita nuqtada M dan kata bo’lsa, u holda
3) (x_0, t_0) nuqta bor bulib u(x,t) M dan ortib ketmaydigan maksimal qiymatga ega bo’ladi

$$u(x_0, t_0)=M+\varepsilon, \quad (\varepsilon>0)$$

Bu erda $0<x_0<\ell$, $0<t_0\leq T$ haqiqattan xam analizdan biz bilamiz f(x) funktsiya x_0 nuqtada (0,) interval ichida min ga ega bo’lishi uchun

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0}=0, \qquad \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x_0}>0$$ sharti urinli bo’lishi zarur.

Shunday etib x_0 nuqtada f(x) funktsiya max ga ega bo’lishi uchun

1) $f`(x_0)=0$ 2) $f``(x_0)\leq 0$ bulishi kerak.

Bundan $t_0<T$ bo’lganda $\frac{\partial u}{\partial T}=0$ **agar** $t0=T$ **bo'lsa** $\frac{\partial u}{\partial T}\geq 0$

v(x,t) ning uzluksizligidan u qandaydir (x_1, t_1) nuqtada uzining maksimal qiymatiga erishadi. Haqiyqattan ham

$$\vartheta(x1, t1)(\vartheta\ (x_0, t_0)=M+\varepsilon,$$

Shuning uchun $t_1>0$ va $0<x_1<\ell$ $t=0$ yoki $x=0, x=\ell$ da (17) formula o'rinli buladi (x_1, t_1) nuqtada (14) va (15) ga o'xshash $v_{xx}(x_1, t_1)\le 0$, $v_t(x_1, t_1)\ge 0$ bo'lishi shart (16)ni hisobiga olib

$$u_{xx}(x_1, t_1)= {}_{xx}(x_1, t_1)\le 0,$$

$$u_t(x_1, t_1)= \vartheta_t(x_1, t_1)+k\ge k>0$$

Bundan $u_t(x_1, t_1)-a^2 u_{xx}(x_1, t_1)\ge k>0$ ekanligi kelib chiqadi. Demak $u_t=a^2 u_{xx}$ tenglama (x_1, t_1) ichki nuqtada qanoatlandirilmaydi. Shuningdek (13) tenglamaning u(x,t) echimi soxa ichidagi qiymati u(x,t) ning chegaradigi qiymatidan oshib ketmas ekan ($t=0$, $x=0$, $x=\ell$) Endi birinchi chegaraviy shart uchun birdan birlik teoremasini isbotlaymiz. Birdan birlik teoremasi

$0\le x\le\ell$, $0\le t\le T$ soxada aniqlangan va uzluksiz $u_1(x,t)$ va $u_2(x,t)$ ikkita funktsiya

$$u_t=a^2 u_{xx}+f(x,t) \quad (0<x<\ell,\ t>0)$$

Harorat utkazgishlik tenglamasini qanoatlantirsa va bir hil boshlangich va chegaraviy shartlarini qanoatlandirsa

$$u_1(x,0)=u_2(x,0)=\varphi(x)$$

$$u_1(0,t)=u_2(0,t)=\mu_1(t)$$

$$u_1(\ell, t) = u2(\ell,t)=)=\mu_2(t)$$

U holda $u_1(x,t)=u_2(x,t)$

Bu teoremani isbotlash uchun $\vartheta(x,t)=u_2(x,t)-u_1(x,t)$

funktsiya kiritamiz. $u_1(x,t)$ va $u_2(x,t)$ funktsiyalari $0\le x\le\iota$ va

ℓ **va 0(t(T da uzluksiz bo'lgani uchun ushbu** soxada $\vartheta(x,t)$ham uzluksizdir.

sohada harorat utkazuvchanlik tenglamasining echimi ham bo'ladi. Shunday **qilib maksimum qiymat printsipi ushbu funk**tsiyaga ham tegishli ya'ni u uzining **maksimal yoki minimal qiymat**iga $t=0$ yoki $x=0$ yoki $x=\ell$ chegarada erishadi. Lekin shart buyicha

$\vartheta(x,0)=0, \qquad \vartheta(0,t)=0, \qquad \vartheta(\ell,t)=0$

Shuning uchun

$\vartheta(x,t)\equiv 0,$

ya`ni $u_1(x,t)\equiv u_2(x,t)$.

Bundan birinchi chegaraviy masalaning echimi yagona.

Xossalari.

1. Harorat utkazuvchanlikning ikkita echimi $u_1(x,t)$ va $u_2(x,t)$ lar

   $u_1(x,0)\leq u_2(x,0)$

$u_1(0,t)\leq u_2(0,t)$, $u_1(\ell,t)\leq u_2(\ell,t)$

shartlarini qanoatlantirsa u holda barcha $0\leq x\leq \ell$ va $0\leq t\leq T$ lar uchun

$u_1\leq u_2(x,t)$ bo'ladi.

Haqiqattan ham $\vartheta(x,t)=u_2(x,t)-u_1(x,t)$ ayirma maksimum qiymat printsipi shartlarini qanoatlandirsa va undan tashqari

$\vartheta(x,0)\geq 0, \qquad \vartheta(0,t)\geq 0, \qquad \vartheta(\ell,t)\geq 0$ **shuning uchun** $\vartheta(x,t)\geq 0,$

bo'ladi $0<x<\ell$ **va** $0<t\leq T$. **Teskari holda** $\vartheta(x,t)$ **funktsiya** $0<x<\ell$ va $0<t\leq T$ sohada manfiy minimal qiymatni qabul qilar edi.

2. Harorat utkazuvchanlikning tenglamasining uchta echimi

   $\underline{u}(x,t)$, $u(x,t)$, $\overline{u}(x,t)$ **lar** $\underline{u}(x,t)\leq u(x,t)\leq \overline{u}(x,t)$

3. shartni t=0, x=0, va

x=

ℓ **da qanoatlantirsa u holda bu tengsizlik barcha x, va t lar uchun** $0\leq x\leq \ell$

va $0\leq t\leq T$ sohada urinli bo'ladi.

Harorat utkazuvchanlikning tenglamasining $u_1(x,t)$ va $u_2(x,t)$ echimlari uchun

$$|u_1(x,t)-u_2(x,t)|\leq \varepsilon,$$

$t=0, \qquad x=0, x=\ell$ **da tengsizligi bajarilsa u xolda** $0\leq x\leq \ell$ **va** $0\leq t\leq T$ **shartini barcha x va t lar uchun** $|u_1(x,t)-u_2(x,t)|\leq \varepsilon$

sharti bajariladi.

§3. Uch qatlamli iteratsion sxemalar

3.1 Zeydel metodi

Noaniq sxemalar, aniq sxemalarga solishtirganda doimiy buladi. Oddiy noaniq sxemaga Zeydel metodi kiradi. Quydagi chiziqli algebraik tenglamalar sistemasini qaraymiz.

$$Au=f \qquad (26)$$

yaki $$\sum_{j=1}^{N} a_{ij}u_j = f_i, \qquad i = 1,2,\ldots,N \ .$$

A matritsasining diogonal elementlari $A=(a_{ij})$ nol`dan farqli dep $a_{ij}\neq 0$, quydagi iteratsion metodni yozamiz.

$$\sum_{j=1}^{i} a_{ij}y_j^{k+1} + \sum_{j=i+1}^{N} a_{ij}y_j^{k} = f_i, \qquad a_{ij}\neq 0 \qquad (27)$$

Bu erda \\номердеги y^k_j k-nomerdagi iteratsiya (k+1)- iteratsiyani i=1 dan boshlaymiz.

$$a_{11}y_1^{k+1} + \sum_{j=2}^{N} a_{1j}y_j^{k} = f_1, \qquad a_{11}\neq 0$$

Bundan y_1^{k+1} ni topamiz. i=2 uchun

$$a_{21}y_1^{k+1} + a_{22}y_2^{k+1} + \sum_{j=3}^{N} a_{2j}y_j^{k} = f_2, \qquad a_{22}\neq 0$$

Bizga y_1^{k+1} ning qiymati belgili bulganliktan va $a_{22}\neq 0$, bulgani uchun y_2^{k+1} ni topamiz va h.z. A matritsasini quydagicha yig`indi kurinishida yozamiz.

$$A= A^-+A^++D \qquad (28)$$

bu erda $A=(a_{ij})$, $a_{ij} = a_{ij}$ agar va j<i va $a_{ij}=0$ agar j≥i bosh diogonaldagi quydagi uchburchakli nollik matritsa $A^+=(a^+_{ij})$,

$a^+_{ij}= a_{ij}$ va $j>i$ va $a^+_{ij}=0$ agar $j\le i$ bosh diogonaldagi yoqargi uchburchakli nollik matritsa. $D=(a_{ii}\,\delta_{ij})$, $\delta_{ij}=1$ agar $j=i$ $\delta_{ij}=0$ agar $j\ne i$ diogonallik matritsa Ushbu belgilashlardan foydalanip Zeydel metodini quydagicha yozamiz

$$(A^-+D)y^{k+1}+A^+y^k=f, \qquad y=(y_1,y_2,\dots y_N) \tag{29}$$

Bu ikki qatlamli sxemani kanonik turga keltiramiz.

$$(A+D)(\overline{y^{k+1}}-y^k)+(A+D+A^+)\overline{y^k}=f$$

yaki $$(A^-+D)(y^{k+1}-y^k)+Ay^k=f \tag{30}$$

uni kanonik forma bilan solishtirib

$$B\frac{y^{k+1}-y^k}{\tau_{k+1}}+Ay^k=f \qquad k=0,1,..n-1 \tag{31}$$

hohlagan $y_0\in H$ uchun $B=A^-+D$, $\tau_k=1$ ekanligini topamiz. Sxema noaniq, V-matritsasi uchburchakli va simmetriyali emas. ($B\ne B^*$-uz-uziga tuginlash emas).

$\wedge u=-\varphi$, $x\in\omega_h$ $u|_{\gamma h}=0$

Zeydel metodi quydagicha buladi.

$$y^{k+1}_{i_1-1}+y^{k+1}_{i_2-1}-4y^{k+1}+y^k_{i_1+1}+y^k_{i_2+1}=-h^2\varphi,$$

$$demak \quad y^{k+1}=\frac{y^{k+1}_{i_1-1}+y^{k+1}_{i_2-1}+y^k_{i_1+1}+y^k_{i_2+1}+h^2\varphi}{4} \tag{32}$$

Hisoblash $i_1=1$, $i_2=1$ tuginlardan boshlanadi. (0,1) va (1,0) tuginlari chegarada etganliktan, $y^{k+1}_{i_1-1}$ $\hat{a}\grave{a}$ $y^{k+1}_{i_2-1}$ qiymatlari belgili buladi va (32) ning ung tarafining barchasi belgili buladi. y^{k+1} ning qiymati $i_1=1$, $i_2=1$ tuginda topiladi. Undan sung $i_2=2,3,\dots$ larni $i_1=1$ deb olib y^{k+1} ni quydagi qatorda topamiz. Sung $i_1=2,3,\dots$ larga utamiz. Natijada y^{k+1} turning barcha tuginlarida topiladi.

Agar $A=A^*>0$ operatori uz-uziga tuginlash va ung aniqlangan bulsa (ung aniqlanadi, sababi $A\ge\gamma_1E$, $\gamma_1=\min\lambda_k(A)>0$) Zeydel metodi yaqinlashuvchi. Zeydel metodining yaqinlashuv tezligini baholash uchun har hil fikr yuritamiz. Masalan agar

$$\sum_{j\ne i}|a_{ij}|\le q\,|a_{ii}|, \qquad i=1,2,\dots.N. \tag{33}$$

Bulsa, unda q<1 bulganda Zeydel metodi geometrik progressiya tezligi bilan yaqinlashadi haqiqattan ham

$$a_{ii} z_i^{k+1} = -\sum_{j<i} a_{ij} z_j^{k+1} - \sum_{j>i} a_{ij} z_j^k$$

Formuladan $z^{k+1}=y^{k+1}-u$ hatolik uchun

$$|\,|a_{ii}\,||\,z_i^{k+1}| \le \sum_{j<i} |a_{ij}\,||\,z_j^{k+1}| + \sum_{j>i} |a_{ij}\,|||\,z_j^k\,||_c$$

bu erda $||z||_c = \max_{1\le i\le N} |z_i|$ Mayli $\max_i |z_i^{k+1}|$, qandaydir $i=i_0$ da erishgan u holda $||z^{k+1}||_c=|z_{i0}^{k+1}|$ va

$$||z^{k+1}||_c \le \left[\sum_{j>j_0} |a_{i_0 j}| / \left(|a_{i_0 i_0}| - \sum_{j<j_0} |a_{i_0 j}|\right)\right] || z^k ||_c$$

(33) shart buyicha (a_{ij}) matritsasi dioganal bulib topiladi. Bu shart laplasning chegirmali operatori uchun urinli emas. Lekin Zeydel metodining yaqinlashuvchiligi bu holda uz-uziga tuginlashliktan va A operatorining ung aniqlanganligidan kelib chiqadi.

Yuqori relaksatsiya metodi

Iteratsion protsessni tezlatish uchun Zeydel metodini turlandiramiz yani (30) formulaga ω-parametrini shunday qilib kiritamiz, na`tiyjada

$$(A+\frac{1}{\omega}D)(y^{k+1}-y)+Ay^k=f \qquad (34)$$

Formulaga ega bulamiz. Bu metod relaksatsiya metodi deb ataladi. ω=1 bulganda Zeydel metodi kelib chiqadi. Agar parametr ω>1 bulsa u holda (34) iteratsion protsess yuqori relaksatsiya metodi deb ataladi. (34) ni (31) bilan solishtirib

$B=(A+\frac{1}{\omega}D)$, $\tau_k=1$ yaki

$V=\omega A+D$, $\tau_k=\omega$ ga ega bulamiz.

V-operatori uz-uziga tuginlash emas. y^{k+1} ni hisoblash algoritmi ushburchakli matritsani turlandirishga olib keladi. Agar Zeydel metodi barcha $A=A^9>0$ lar uchun urinli bulsa, relaksatsiya metodining yaqilashuvchi bulishi uchun qushimcha

$0<\omega<\omega'$ shartini talab etamiz. Yaqinlashish tezligi ω parametrdan g`arazli buladi. ω uchun nazariy baho va yaqinlashish tezligi urinli, lekin ularni qullanish $D^{-1}(A+A^{+})$ operatorining spektrini bilishni talab etadi. Uni topish hamma vaqt mumkin emas. Shuning uchun ω-parametrni amalietta iteratsiya sonini azaytish uchun qulayli etib saylab olamiz. Bir xil tiptagi masalalar klasini echishda bu metod juda qulayli.

Yuqori relaksatsiya metodini iteratsiya soni buyicha Chebishevning aniq metodi bilan solishtirish mumkin.

$$n \geq n_0(\varepsilon), \quad n_0(\varepsilon)=0\left(\frac{1}{h}\ln\frac{1}{\varepsilon}\right)$$

3.2. Noaniq iteratsion sxemalar

Biz hohlagan $y_0 \in H$ uchun

$$\frac{y^{k+1}-y^{k}}{\tau_{k+1}}+Ay^{k} = f \tag{35}$$

Kurinishdagi aniq iteratsion sxemani tadqiq qildik.

Bu erda $y_{k+1}=y_k-\tau_{k+1}(Ay_k-f)$ formulasi buyicha hisoblandi. Yuqori relaksatsiya va Zeydel metodi $V\neq E$ noaniq sxemaga (31) misol buladi. Noaniq metodni qullashga toza y_{k+1} iteratsiyani aniqlashda quydagi tenglamani echish kerak.

$$By_{k+1}=F_k, \quad F_k=By_k-\tau_{k+1}(Ay_k-f) \tag{36}$$

F_k- oldindan berilgan. Zeydel metodi holinda $V=\bar{A}+D$

uchburchakli matritsa, yuqori relaksatsiya metodinda $B=(\bar{A}+\frac{1}{\omega}D)$ uchburchakli matritsa.Shuning uchun y_{k+1} ni aniqlash minimal sondagi harakatni, modellik masalada esa y_{k+1} ni aniqlash uchun amallar soni turning tuginlariga proportsional buladi. V-operatoriga quyiladigan talab quydagicha

1) iteratsiyaning minimum soni.

2) V-operatorining tejamkorligi: yani ikkinchi tartibli chegirmali elliptik tenglamalar uchun $By_{k+1}=F_k$

tenglamasining echimi turning tugunlar soniga proportsional arifmetik amalda topilishi kerak. $\{\tau_k\}$ parametrini optimal saylab olish V≠E bulgan holda noaniq sxemaga utqazilishi mumkin.

$$B\frac{y^{k+1}-y^k}{\tau_{k+1}}+Ay^k=f, \quad k=0,1,2,\ldots n-1. \quad \forall y_0\in H \tag{37}$$

$B=B^*>0, \quad A=A^*>0$

$$\gamma_1 B\le A\le\gamma_2 B, \qquad \gamma_1>0 \tag{38}$$

deb olamiz. Bu shartlar (31) iteratsion metodning semeystvosini aniqlaydi. Zeydel va yuqori relaksatsiya metodlari uchun $V\neq B^*$ uz-uziga tuginlash emas operator, unda ular (31) va (38) semeystvoga kirmaydi. $r_k=Ay_k-f$ uchun quydagi bir jinsli tenglamaga ega bulamiz. $r_0=Ay_0-f\in H$ berilgan

$$B\frac{r_{k+1}-r_k}{\tau_{k+1}}+Ar_k=0, \qquad k=0,1,2,\ldots.n-1 \tag{39}$$

Bu sxema quydagi aniq sxemaga ekvivalent

$$\frac{x_{k+1}-x_k}{\tau_{k+1}}+cx_k=0, \qquad k=0,1,2,\ldots.n-1 \tag{40}$$

$x_0\in H$

Bu erda $x_k=B^{1/2}r_k$, $c=B^{-1/2}AB^{-1/2}$ haqiqattan ham V uz-uziga tuginlash ung operator bulganliktan $B=B^*>0$, u holda V-operatorining koreni $B^{1/2}$ bor bulib

$(B^{1/2})^*=B^{1/2}>0$

(39) tenglamaga $B^{-1/2}$ operatori erdamida tasir etqazip va $r_k=B^{-1/2}r_k$ ni almashtirib (40) tenglamani olamiz.

Lemma 1. Mayli

$A=A^*>0,\ B=B^*>0,\ c=B^{-1/2}AB^{-1/2}$

operatorlari berilsin. Unda quydagi operatorliq tengsizliklar

$\gamma_1 B\le A\le\gamma_2 B, \qquad \gamma_1>0$

$$\gamma_1 E\le C\le\gamma_2 E, \tag{41}$$

ekvivalent.

Isboti.

$J=((A-\gamma B)y,y)=(Ay,y)-\gamma(By,y)=(AB^{-1/2}(B^{1/2}y),B^{-1/2}(B^{1/2}y))-$

$-\gamma(B^{1/2}y,B^{1/2}y)=(cx,x)-\gamma(x,x)=((c-\gamma E)x,x)$ bu erda $x=B^{1/2}y$.

y (shuningdek-x), N dan olingan erkli vektor bulganliktan

$$J=((A-\gamma B)y,y)=((c-\gamma E)x,x) \qquad (42)$$

deb $A-\gamma B$ va $c-\gamma E$ operatorlari birdek belgiga ega ekanligi kelib chiqadi.

Mayli $A-\gamma_1 B\geq 0$ unda (42) da $\gamma=\gamma_1$ dep olib

$J=((c-\gamma_1 E)x,x)\geq 0$ ega bulamiz $c\geq\gamma_1 E$ lemma isbotlandi.

Bu muzokaraning natijasi quydagicha $Au=f$ tenglamasini echish uchun (31) noaniq sxemani qullash, $c\nu=\varphi$ tenglamasini

$$\frac{x_{k+1}-x_k}{\tau_{k+1}}+cx_k=\varphi, \qquad k=0,1,2,\ldots n \quad x_0\in H \qquad (43)$$

aniq sxema bilan echish ekvivalent agar

$$S=B^{-1/2}AB^{-1/2}, \ \varphi=B^{-1/2}f \qquad \text{bulsa.}$$

Shuning uchun aniq sxema bilan olingan barcha natijalarni, noaniq sxemaga utqazishimiz mumkin.

Uch qatlamli iteratsion sxemalar

Biz ilgari $Au=f$ operatorlik tenglamani (A tuginlash operator bilan) γ_1 va γ_2 A operatorining chegaralari N_β da (bunda $B=B^* >0$) echish uchun ikki qatlamli sxemalarni kurib chiqan edik.

Endi uch qatlamli iteratsion sxemalardan kurib utamiz. Mayli

$$Au=f \quad A:H\rightarrow H \qquad (44)$$

Tenglamasini uz-uziga tuginlash, musbat aniqlangan operatorda echish talab etilsin. Uning spektorining chegaralari belgili bulsin

$$A=A^*, \quad \gamma_1 E\leq A_0\leq\gamma_2 E, \quad \gamma_1>0 \qquad (45)$$

uch qatlamli iteratsion sxema uch iteratsiyani y_{k-1}, y_k va y_{k+1} di bog`laydi. y_{k+1} iteratsiya y_{k-1}, y_k iteratsiyalar orqali anglatiladi. Aniq sxema odatta quydagicha yoziladi.

$$y_{k+1}=(1+\alpha)Sy_k-\alpha y_{k-1}+(1+\alpha)\tau_0 f \qquad (46)$$

k=1,2…. $y_k=Sy_0+\tau_0 f$, hohlagan $u_0\in H$ uchun bunda $S=E+\tau_0 A$ ikki qatlamli oddiy iteratsion sxema uchun τ_0 optimal parametr bilan utish operatori.

$$\tau_0=\frac{2}{\gamma_1+\gamma_2},\quad \rho_1=\frac{1-\xi}{1+\xi},\qquad \xi=\frac{\gamma_1}{\gamma_2} \tag{47}$$

y_1-dastlabki iteratsiyani ikki qatlamli oddiy itertsiyadan topamiz. (46) sxemani kuydagisha usulda olamiz.

(44) tenglamani taer holida yozib olib τ parametrni $||S||$ minimal` bulganday etib saylab olamiz.

$$u=u-\tau Au+\tau f=S(\tau)u+\tau f,\quad S(\tau)=E-\tau A,$$

Uning uchun $\tau=\tau_0$ deb olib

$$u=S(\tau_0)u+\tau_0 f \tag{48}$$

ga ega bulamiz. Bu tenglamani quydagicha yozish mu`mkin. $(1+\alpha)u=(1+\alpha)Su+(1+\alpha)\tau_0 f$, va sungi tenglamaga $(1+\alpha)Su$ di $(1+\alpha)Sy_k$ ga, αu di αy_k ga almashtirib noaniq sxemani qullaymiz. α-parametr iteratsion minimumlik shartidan olinadi. Biz (46) sxemaning yaqinlashish tezligiga va α-parametrni saylab olishga tuqtalib utirmaymiz. Faqat oqirgi natijani keltiramiz. (46) tenglamaga A operatorini qullaymiz, u $r_k=(Ay_k-f)$ birjinsli tenglamani qanoatlandiradi.

$$r_{k+1}=(1+\alpha)S\,r_k+\alpha\, r_{k-1},\quad k=1,2\ldots \tag{49}$$

$r_1=Sr_0$ bunda $r_0=Ay_0-f\in H$ Bu masala uchun $\alpha=\rho_1^2$ deb olsak

$$||\,Ay_n-f\,||\le q_n||\,Ay_0-f\,|| \tag{50}$$

baholash urinli. Bunda

$$q_n=\rho_1^n\left(1+\frac{1-\rho_1^2}{1+\rho_1^2}\cdot n\right) \tag{51}$$

iteratsiya soni uchun quydagicha baholash urinli.

$$n\ge\frac{\ln\frac{1}{\varepsilon}+\ln\left(1+\frac{1-\rho_1^2}{1+\rho_1^2}\cdot n\right)}{\ln\frac{1}{\rho_1}},\ \text{îё}\qquad n\ge\frac{\ln\frac{1}{\varepsilon}+\ln\left(1+\frac{2\sqrt{\xi}}{1+\xi}\cdot n\right)}{2\sqrt{\xi}}$$

da urinli. Uch qatlamli sxemada iteratsiya soni birqancha ko'b buladi va ulkan xotirani talab etadi (y_{k+1} ni aniqlashda y_k va y_{k-1} vektorlarni xotirada saqlash kerak) γ_1, γ_2 larning aniq berilishiga kuproq g`arazli. Shining uchun γ_1 va γ_2 berilsa amalda uch qatlamli sxemadan kura ikki qatlamli sxemadan foydalangan afzal. Eslatma. Aniq sxemadan noaniq uch qatlamli sxemaga o'tish (46) tenglamada A ni va $B^{-1}A$ va f va $B^{-1}f$ ni almashtirishga olib keladi.

Demak

$$y_{k+1}=(1+\alpha)(E-\tau_0 B^{-1}A)\, y_k -\alpha\, y_{k-1}+(1+\alpha)\tau_0 B^{-1}Af \qquad \text{ya`ni}$$

$$By_{k+1}=(1+\alpha)(B-\tau_0 A)\, y_k -\alpha B\, y_{k-1}+(1+\alpha)\tau_0 f \qquad (52)$$

$By_1=B\, y_0-\tau_0 A\, y_0 +\tau_0 f$ $k=1,2,\dots,$ $y_0\in H$ berilgan. Agar (48) tenglamaning o'rniga

$$Bu=(1+\alpha)(Bu-\tau_0 Au) -\alpha Bu+(1+\alpha)\tau_0 f$$

tenglikni quysaq (52) ni olish mu`mkin. $\tau_0$, $\alpha$- lar uchun (47) formula uz ku`chida qoladi, lekin A-operatorining chegarasi N ta emas N_B da bo'ladi.

$$\gamma_1 B \le A\le \gamma_2 B, \qquad \gamma_1>0 \ , \ B=B^*>0 \qquad (53)$$

Unda (52) masalani echish uchun (50) tengsizlik o'rniga

$$\| Ay_n-f \|_{B^{-1}} \le q_n \| Ay_0-f \|_{B^{-1}} \qquad (54)$$

baholash o'rinli. Bu joyda q_n avvalgidek (51) formuladan aniqlanadi.

Eng tez tusish usuli

Aniq sxema uchun eng tez tusish usuli

$y_{k+1}=y_k-\tau_{k+1}(Ay_k-f)$, $k=1,2,\dots,$ $\forall y_0\in H$ kurinishida bo'ladi. τ_{k+1} parametr quydagi formuladan aniqlanadi.

$$\tau_{k+1} = \frac{(r_k, r_k)}{(Ar_k, r_k)}, \qquad r_k = Ay_k - f, \qquad k = 0,1,2,.... \qquad (55)$$

Bu formula N_A da $z_k=y_k-u$ qoldiqni minimizatsiyalash shartidan olinishi mu`mkin. ya`ni

$$\max_{\{\tau_{k+1}\}} \| z_{k+1} \|_A; \qquad \| z \|_A = \sqrt{(Az, z)} \qquad z_k=y_k-u$$

qoldiq uchun $z_{k+1}=z_k-\tau_{k+1}Az_k$ tenglamaga ega bulamiz. $v_k=A^{1/2}z_k$ deb olib

$$v_{k+1}=v_k-\tau_{k+1}Av_k \tag{56}$$

ga ega bo’lamiz. Normaning kvadratini hisoblb topamiz.

$$||v_{k+1}||^2=||v_k||^2-2\tau_{k+1}(v_k ,Av_k)+ \tau^2_{k+1}||Av_k||^2 \tag{57}$$

$\max\limits_{\{\tau_{k+1}\}} || z_{k+1} ||^2$ shartidan

$$\tau_{k+1} = \frac{(Av_k, v_k)}{|| Av_k ||^2}, \tag{58}$$

ga ega bo’lamiz. Quydagi baholash o’rinli.

$$|| v_n || \leq \rho^n_0 || v_0|| \tag{59}$$

Endi v_k dan $z_k= A^{-1/2}v_k$ ga utish qoldi.

$Az_k=A(y_k-u)=Ay_k-f= r_k$, $(Av_k ,v_k)=||Az_k||^2=||r_k||^2$, $||Av_k||^2=(A r_k, r_k)$ ekanligini hisobga olib (58) ni (55) ko’rinishiga olib kelamiz. (59) tengsizliktan

$$||y_k-u||_A\leq\rho^n_0 || y_0-u||_A \tag{60}$$

sababi $||v_n||^2=(v_n ,v_n)=(A z_n, z_n)=||z_n||^2_A$ Shunday qilib N_A da eng tez tusish usuli.,

$$B\frac{y_{k+1}-y_k}{\tau_{k+1}} + Ay_k = f, \qquad k = 0,1,2,.... \qquad \forall y_0\in H \tag{61}$$

ko’rinishta bo’ladi. Yuqoridagi usul bo’icha

$$\tau_{k+1} = \frac{(r_k, w_k)}{(Aw_k, w_k)}, \qquad w_k = B^{-1}r_k, \qquad r_k = Ay_k - f \tag{62}$$

ega bo’lamiz. Agar $\gamma_1 B \leq A\leq\gamma_2 B$, $\gamma_1>0$, sharti o’rinlansa

$|| Ay_n-f ||_{B^{-1}} \leq q_n || Ay_0-f ||_{B^{-1}}$ baholash o’rinli bo’ladi.

O’z-o’ziga tuginlash emas operator yordamida tenglamani eshish

$Au=f$, $A:H\rightarrow H$ tenglamasini qo’rib o’tamiz. Bu joyda A-musbat aniqlangan o’z-o’ziga tuginlash emas chiziqli operator. Dastlabki ikki qatlamli doimiy parametrga ega sxemadan foydalanamiz.

$$\frac{y_{k+1}-y_k}{\tau}+Ay_k=f, \qquad k=0,1,2,.... \quad \forall y_0\in H \qquad (63)$$

Iteratsiyaning yaqinlashish tezligini baholash uchun $z_{k+1}=Sz_k$, $S=E-\tau A$, $k=0,1,2,\ldots$, $z_0\in H$ birjinsli tenglamani qo'rib o'tamiz. Bu joyda qoldiq $z_k=y_k-u$ bundan $\|z_{k+1}\|\le\|S\|\,\|z_k\|$

τ-parametrini $\max_{\tau}\|S(\tau)\|$ shartidan saylab olamiz. Mayli A va A^{-1} operatorlarning quydagi chegaralari berilsin.

$A\le\gamma_1 E$ ya`ni $(Ay,y)\ge\gamma_1\|y\|^2$ $\quad\gamma_1>0$

$$A^{-1}\ge\frac{1}{\gamma_2}E \text{ ya`ni } \|Ay\|^2\le\gamma_2\|y\|^2, \quad \gamma_2>0 \qquad (64)$$

$B=B^*$ bo'lganda ikkinchi shart $A\le\gamma_2 E$ ge teng kuchli $2-\tau\gamma_2\ge 0$ deb olib.

$\|Sy\|^2=\|y-\tau Ay\|^2=\|y\|^2-2\tau(Ay,y)+\tau^2\|Ay\|^2\le\|y\|^2-2(Ay,y)+\tau^2\gamma_2(Ay,y)=\|y\|^2-\tau(2-\tau\gamma_2)(Ay,y)\le\|y\|^2-\tau(2-\tau\gamma_2)\gamma_1\|y\|^2=(1-2\tau\gamma_1+\tau^2\gamma_1\gamma_2)\|y\|^2$

ya`ni $\|S\|^2\le 1-2\tau\gamma_1+\tau^2\gamma_1\gamma_2$ uchxadning minimumlik shartidan τ ni topamiz.

$$\tau=\frac{1}{\gamma_2}, \quad \|S\|^2\le(1-\gamma_1/\gamma_2), \quad \|S\|\le\sqrt{1-\xi}, \ \tau=\frac{1}{\gamma_2}, \ \xi=\frac{\gamma_1}{\gamma_2} \qquad (65)$$

γ_1 va γ_2 parametr o'rnina $\gamma_1, \gamma_2, \gamma_3$ uch parametr berilgan holatni qo'ramiz. Endi A ni A_0 simmetrik operatorlar yig`indisi ko'rinishida va $A_1$ kosimmetrik operatorlarning yig`indisi ko'rinishida ko'ramiz.

$$A=A_0+A_1, \quad A_0=1/2(A+A^*), \ A_1=1/2(A-A^*) \qquad (66)$$

$A_0^*=A_0$, $A_1^*=-A_1$, $(A_1x,x)=-(x,A_1x)=0$ ya`ni $(Ax,x)=(A_0x,x)$

A-operatori

$$\gamma_1E\le A_0\le\gamma_2E, \quad \|A_1\|\le\gamma_3 \qquad (67)$$

shartin qanoatlantiradi. Bunda $\gamma_2>\gamma_1>0$ va $\gamma_3\ge 0$ berilgan sonlar. $z_{k+1}=(E-\tau A)z_k$ tenglamasini $z_{k+1}=y_{k+1}-u$ qoldiq uchun.

$$z_{k+1}=(E-\tau A_0-\tau A_1)z_k=(\theta E-\tau A_0)z_k+[(1-\theta)E-\tau A_1]z_k \qquad (68)$$

bu joyda $0<\theta<1$ hohlagan son. τ va θ ni shunday etib soylab olamiz, ya`ni $\|S\|=\|E-\tau(A_0-A_1)\|$ minimal bo'lsin. Uchburchak tezligidan

$$\|z_{k+1}\|\le\theta\|E-\tau/\theta\, A_0\|\,\|z_k\|+\|(1-\theta)z_k-\tau A_1)z_k\| \qquad (69)$$

A_0-operatori o'z-o'ziga tuginlash va $\gamma_1 E \le A_0 \le \gamma_2 E$ shuning uchun

$$\max_{\frac{\tau}{\theta}} \| E - \frac{\tau}{\theta} A_0 \| = \rho_{0,} \qquad àãàð \quad \frac{\tau}{\theta} = \tau_0 = \frac{2}{\gamma_1 + \gamma_2} \tag{70}$$

Bu joyda $\rho_0 = \frac{1-\xi}{1+\xi}$, $\xi = \frac{\gamma_1}{\gamma_2}$, unda $\tau = \tau_0 \theta$ (69) ning ung tomanidagi ikkinchi qushiluvchini ko'rib o'tamiz.

$\|(1-\theta)y - \tau A_1 y\|^2 = (1-\theta)^2 \|y\|^2 - 2\tau(1-\theta)(A_1 y, y) + \tau^2 \|A_1 y\|^2 = (1-\theta)^2 \|y\|^2 + \tau^2 \|A_1 y\|^2 \le [(1-\theta)^2 + \tau^2 \gamma_3^2] \|y\|^2 = [(1-\theta)^2 + \tau^2 \theta^2 \gamma_3^2] \|y\|^2$

Shunday etib $\tau = \tau_0 \theta$ bo'lganda

$$\|z_{k+1}\| \le \|S\| \, \|z_k|, \quad \|S\| \le f(\theta)$$

$$f(\theta) = \theta\rho_0 + \sqrt{(1-\theta)^2 + \theta^2 a^2}, \qquad a^2 = \tau_0^2 \cdot \gamma_3^2 \tag{71}$$

tengsizlik o'rinli. Endi $f(\theta)$ funktsiyasining minimumini topamiz.

$$f'(\theta) = \rho_0 - \frac{1 - \theta - a^2\theta}{\sqrt{(1-\theta)^2 + a^2\theta^2}} = \rho_0 - \frac{\alpha - a^2}{\sqrt{\alpha^2 + a^2}}, \qquad \alpha = \frac{1-\theta}{\theta}$$

$f'(\theta)=0$ sharti $\rho_0 \sqrt{\alpha^2 + a^2} = \alpha - a^2$ tenglikni beradi. Undan α-uchun kvadrat tenglama paydo etamiz. $(1-\rho_0^2)\alpha^2 - 2a^2\alpha - a^2(a^2 - \rho_0^2) = 0$ uni echib

$$\alpha = a \frac{a + \rho_0 \sqrt{1 - \rho_0^2 + a^2}}{1 - \rho_0^2}$$

(ikkinchi koren` a va ρ_0 parametrlarining qandaydir qiymatlarida manfiy bo'lishi mumkin)

$$\aleph = \frac{\gamma_3}{\sqrt{\gamma_1\gamma_2 + \gamma_3^2}}$$ belgilash kiritamiz

$$\gamma_3^2 = \frac{\aleph^2}{1-\aleph^2}\gamma_1\gamma_2, \quad a^2 = \tau_0^2 \cdot \gamma_3^2 = \frac{4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} \cdot \frac{\aleph^2}{1-\aleph^2} = \frac{(1-\rho_0^2)\aleph^2}{1-\aleph^2} \cdot \frac{a^2}{\aleph^2} = \frac{1-\rho_0^2}{1-\aleph^2}$$

Ildiz osti ifodalanib

$$1 - \rho_0^2 + a^2 = 1 - \rho_0^2 + \frac{\aleph^2(1-\rho_0^2)}{1-\aleph^2} = \frac{1-\rho_0^2}{1-\aleph^2} = \frac{a^2}{\aleph^2}$$ bunnan

$$\alpha = \frac{a^2(\aleph - \rho_0)}{\aleph(1-\rho_0^2)} = \frac{\aleph(\aleph+\rho_0)}{1-\aleph^2}, \qquad 1+\alpha = \frac{1+\aleph\rho_0}{1-\aleph^2}, \quad \theta = \frac{1}{1+\alpha} = \frac{1-\aleph^2}{1+\aleph\rho_0}.$$

Endi

$$f(\theta) = \frac{1}{1+\alpha}\rho_0 + \frac{1}{1+\alpha}\sqrt{\alpha^2+a^2} = \frac{\rho_0^2+\alpha-a^2}{(1+\alpha)\rho_0}.$$

$$\rho_0^2+\alpha-a^2 = (1+\alpha)-(1-\rho_0^2+a^2) = 1+\alpha-\frac{a^2}{\aleph^2} = \frac{1+\aleph\rho_0}{1-\aleph^2} - \frac{1-\rho_0^2}{1-\aleph^2} = \rho_0\frac{\rho_0+\aleph}{1-\aleph^2}$$

ekanligini hisobga olib

$$\| S \| \le \frac{\aleph+\rho_0}{1+\aleph\rho_0}, \quad \text{àãàð} \quad \tau = \tau_0\frac{1-\aleph^2}{1+\aleph\rho_0}$$

bo’lganda. Shunday etib (63) masalani echish uchun agar A operatori (67) shartni qanoatlantirsa $\|y_k-u\| \le \rho^n\| y_k-u\|$ baho o’rinli, bunda

$$\rho = \frac{\aleph+\rho_0}{1+\aleph\rho_0}, \qquad \aleph = \frac{\gamma_3}{\sqrt{\gamma_1\gamma_2+\gamma_3^2}}, \qquad \tau = \bar{\tau} = \frac{\tau_0(1-\aleph^2)}{1+\aleph\rho_0} \qquad (72)$$

iteratsiya soni $n \ge \dfrac{\ln\frac{1}{\varepsilon}}{\ln\frac{1}{\rho}}$. Endi o’z-o’ziga tuginlash $B=B^*>0$

operatori bilan birga quydagicha noaniq sxemani qo’rib o’taylik.

$$B\frac{y_{k+1}-y_k}{\tau_{k+1}} + Ay_k = f, \qquad k = 0,1,2,.... \quad \forall y_0 \in H \qquad (73)$$

Bu xolda quydagi aniq sxemaga o’tamiz.

$x_{k+1}=x_k-\tau(cx_k-\varphi)$

$x_k=B^{1/2} y_k$, $c=B^{-1/2}AB^{-1/2}$, $\varphi= B^{-1/2}f$ va (67) shart S uchun quydagicha yoziladi.

$$\gamma_1 B \le A_0 \le \gamma_2 B$$

$$(B^{-1}A_1y, A_1y) \le \gamma_3^2(By,y) \qquad (74)$$

ya`ni $\|A_1y\|_{B^{-1}} \le \gamma_3\|y\|_B$ Unda (72) tenglamaning o’rniga

$\|y_k-u\|_B \le \rho^n\| y_k-u\|_B$ ga ega bo’lamiz. Au=f tenglamasin o’z-o’ziga tuginlash emas A operatori bilan echish uchun eng kam tuzatishlar usulidan foydalansa ham bo’ladi.

$\tau = \bar{\tau}$ da u va (63) sxemaning tezligindek tezlik bilan yaqinlashuvchi bo'ladi. Aniq sxema uchun

$$\frac{y_{k+1} - y_k}{\tau_{k+1}} + Ay_k = f, \qquad k = 0,1,2,.... \quad \forall y_0 \in H \qquad (75)$$

τ_{k+1} parametr

$$\tau_{k+1} = \frac{(Av_k, v_k)}{\| Av_k \|^2}, \qquad r_k = Ay_k - f, \qquad k = 0,1,2,.... \qquad (76)$$

$\|r_{k+1}\|^2$ ing minimumlik shartidan formula bo'yicha xisoblanadi va bunda A-ning o'z-o'ziga tuginlashligi hech bir joyda qo'llanilmaydi. Eng kam tuzatishlar usulinda (67) shartning o'rinlanishinda (75) va (76) formulalar uchun $\|Ay_k - f\| \leq \rho^n \|Ay_0 - f\|$ baholash o'rinli. Bu joyda y_k- (75) masalaning echimi, ρ- (72) formula bo'yicha aniqlanadi. Eng tez tusish usuli bu xolda qo'llanilmaydi. Sababi A-o'z-o'ziga tuginlash operator. Noaniq eng kam tuzatishlar metodi uchun $\|A_1 y - f\|_B^{-1} \leq \rho^n \|y\|_B^{-1}$ baho o'rinli.

3.3 Matritsaviy haydash usulining ayrim turlari

1. Ku`chli o'zgarivchili koefitsentlardagi chegirmalik tenglamani echishning oqim usuli

Haydash usulining ku`chli o'zgaruvchi koefitsentlarida qo'llanilip echiladigan variantini qo'rib o'tamiz. Bunga harorat o'tkazuvchilik, gidrodinamika vam magnitlik gidrodinamika masalalari kiradi. Bu joyda harorat o'tkazuvchilik va elektr utkazuvchilik koefitsentlari atrofning termodinamik parametridan g`arazli harorat masalalarida izotermik uchastkalar bo'lishi mu`mkin. U joyda harorat o'tkazuvchilik bulmasligi mu`mkin va izotermik uchastkalar bulishi mu`mkin, u joyda harorat o'tkazgichlik cheksiz katta. Ikkinchi tartibli chegirmali tenglamalarni echishda, bu masalalarni oddiy haydash usuli bilan approksimatsiyalaganda aytarlikday darajada aniqlik yuqoladi. Bunday holda haydash usulini oqim usulidan foydalanib qutilish mumkin. Haydash usulining bu varianti oddiy haydash usulini turlandirish natijasida olish mumkin. Shunday qilib quydagi chegirmali masalani ko'ramiz.

$$a_i y_{i-1} - c_i y_i + a_{i+1} y_{i+1} = -f_i \qquad i=1,2,..,N-1 \qquad (76)$$

$$y_0 = \aleph_1 y_1 + \gamma_1 \ , \qquad y_N = \aleph_2 y_{N-1} + \gamma_2 \qquad (77)$$

bunda $\quad c_i = a_{i+1} + a_i + d_i \ , \quad d_i > 0, \quad 0 \le a_i \le \infty \qquad (78)$

$1 \ge \aleph_1$, $\quad \aleph_2 \ge 0, \quad \aleph_1 + \aleph_2 < 2$ Oddiy haydash formulalari (76)- (77) masala uchun (78) ni xisobga olsak

$$y_i = \alpha_{i+1} y_{i+1} + \beta_{i+1} \ , \quad i=0,1,2,\ldots.N-1,$$

$$\alpha_{i+1} = \frac{a_{i+1}}{a_{i+1} + a_i(1-\alpha_i) + d_i} \qquad (79)$$

$$\beta_{i+1} = \frac{a_{i+1}}{a_{i+1}}(a_i \beta_i + f_i), \qquad i = 1,2,\ldots N-1$$

Yangi nomalum chegirmali funktsiyani kiritamiz.

$$\omega_i = a_i(y_{i-1} - y_i) \qquad (80)$$

va (76) tenglamani (77)-chegaralik shartini

$$\omega_i - \omega_{i+1} - d_i y_i = -f_i \ , \qquad i=1,2,\ldots,N-1 \qquad (81)$$

$$a_1(1-\aleph_1)y_1 + \omega_1 = a_1 v_1, \qquad a_N(1-\aleph_2)y_N - \aleph_2 \omega_N = a_N v_2 \qquad (82)$$

(79) ning birinchi formulasiga (80) ni y_i ning qiymatiga quyamiz.

$y_i = y_{i+1} + \dfrac{\omega_{i+1}}{a_{i+1}}$ u xolda quydagiga ega bulamiz.

$$a_{i+1}(1-\alpha_{i+1})y_{i+1} + \omega_{i+1} = a_{i+1}\beta_{i+1} \qquad (83)$$

$\alpha_i = a_i(1-\alpha_i)$, $\gamma_i = a_i \beta_i$ belgilash kiritib (83)ni quydagi ko'rinishta yozamiz.

$$\alpha_i y_i + \omega_i = \gamma_i \qquad (84)$$

(81) dan va (84) dan y_i ni yuqotib

$$\omega_i = \frac{\alpha_i}{\alpha_i + d_i}\omega_{i+1} + \frac{d_i \gamma_i - \alpha_i f_i}{\alpha_i + d_i} \qquad (85)$$

α_I va γ_i larni aniqlash uchun rekkurent formulalarni yozamiz.

$$\alpha_{i+1} = a_{i+1}(1-\alpha_{i+1}) = \frac{a_{i+1}[a_i(1-\alpha_i) + d_i]}{a_{i+1} + a_i(1-\alpha_i) + d_i} \qquad \text{ya`ni}$$

$$\alpha_{i+1} = \frac{\alpha_i + d_i}{1 + (\alpha_i + d_i)/a_{i+1}} \quad (86)$$

$$\gamma_{i+1} = a_{i+1}\beta_{i+1} = \alpha_{i+1}(a_i\beta_i + f_i) = \alpha_{i+1}(\gamma_i + f_i) = \frac{a_{i+1}(\gamma_i + f_i)}{a_{i+1} + a_i(1-\alpha_i) + d_i}$$

ya`ni

$$\gamma_{i+1} = \frac{\gamma_i + f_i}{1 + (\alpha_i + d_i)/a_{i+1}} \quad (87)$$

i=1 bo'lganda (82) chegaralik shartning birinchisini (84) bilan solishtirib

$\alpha_1 = a_1(1-\aleph_1)$, $\gamma_1 = a_1\gamma_1$

(86) va (87) formulalar $a_i \geq 1$ bo'lganda qulayli. Agar $a_i \leq 1$ bo'lganda (86) va (87) formulalarni quydagi ko'rinishta qo'llagan maqul.

$$\alpha_{i+1} = \frac{a_{i+1}(\alpha_i + d_i)}{a_{i+1} + (\alpha_i + d_i)} \quad (86)^l$$

$$\gamma_{i+1} = \frac{a_{i+1}(\gamma_i + f_i)}{a_{i+1} + (\alpha_i + d_i)} \quad (87)^l$$

(78) shart urinlanganda (86) va $(86)^l$ formulalardan $\alpha_1 \geq 0$ ekanligi kelib chikadi. Shunda (85) formuladagi $\frac{\alpha_i}{\alpha_i + d_i}$ koefitsient hamisha kichkina va ω_i ni xisoblashda echimning o'rinliligini ta`minlaydi. $y_i$ ni aniqlash uchun quydagi formulalardan foydalanish mu`mkin. $a_i \geq 1$ bo'lganda

$$y_i = \alpha_{i+1}y_{i+1} + \beta_{i+1} = \left(1 - \frac{\alpha_{i+1}}{a_{i+1}}\right)y_{i+1} + \frac{\gamma_{i+1}}{a_{i+1}} \quad (88)$$

va $a_i \leq 1$ bo'lganda

$$y_i = \frac{a_{i+1}}{a_{i+1} + \alpha_i + d_i}\, y_{i+1} + \frac{\gamma_i + f_i}{a_{i+1} + \alpha_i + d_i} \quad (89)$$

(88) va (89) dan ko'rinib turganidek haydash doimiy. (85) va (88) (89) formula buyicha xisoblash uchun ω_N, y_N larni xisoblash kerak. Ular (82) chegaraviy shartning ikkinchi tenglamasidan i=N bo'lganda (84) sharttan aniqlanadi.

$$y_N = \frac{a_N \nu_2 + \nu_N \mathrm{N}_2}{a_N(1-\mathrm{N}_2) + N_2 d_N}, \qquad \omega_n = \frac{\gamma_N a_N(1-\mathrm{N}_2) + \alpha_N a_N \nu^2}{a_N(1-\mathrm{N}_2) + N_2 d_N}$$

(78) dan bu ikki ifodaning bo'limlari xomisha nol`dan katta. Endi oqimlik haydashning asosiy formulasini keltiramiz.

1) $\alpha_1 = a_1(1-\aleph_1)$, $\gamma_1 = a_1\nu_1$ ni xisoblymiz

2) $\alpha_{i+1} = \dfrac{\alpha_i + d_i}{1 + (\alpha_i + d_i)/a_{i+1}}$ agar $a_{i+1} \geq 1$ larning qiymatini i=1,2,…,N-1 uchun ketma-ket xisoblab topamiz. Ya`ni

$a_{i+1} < 1$ bo'lganda $\alpha_{i+1} = \dfrac{a_{i+1}(\alpha_i + d_i)}{a_{i+1} + (\alpha_i + d_i)}$,

$a_{i+1} \geq 1$ bo'lganda $\gamma_{i+1} = \dfrac{\gamma_i + f_i}{1 + (\alpha_i + d_i)/a_{i+1}}$

$a_{i+1} < 1$ bo'lganda $\gamma_{i+1} = \dfrac{a_{i+1}(\gamma_i + f_i)}{a_{i+1} + (\alpha_i + d_i)}$

3) $a_N \geq 1$ bo'lg`anda $y_N = \dfrac{\nu_2 + \gamma_N \aleph_2 / a_N}{1 - \aleph_2 + \aleph_2 \alpha_N / a_N}$,

agar $a_N < 1$ bo'lsa $y_N = \dfrac{a_N \nu_2 + \gamma_N \aleph_2}{a_N(1-\aleph_2) + \aleph_2 \alpha_N}$,

agar $a_N \geq 1$ bo'lsa $\omega_n = \dfrac{\gamma_N(1-\aleph_2) + \alpha_N \nu_2}{1 - \aleph_2 + \aleph \alpha_N / a_N}$

agar $a_N < 1$ bo'lsa $\omega_n = \dfrac{\gamma_N(1-\aleph_2) a_N + \alpha_N \nu_2 a_N}{a_N(1-\aleph_2) + \aleph_2 \alpha_N}$

4) i=N-1, N-2,…0 lar uchun $\omega_i = \dfrac{\alpha_i}{\alpha_i + d_i}\omega_{i+1} + \dfrac{d_i \nu_i - \alpha_i f_i}{\alpha_i + d_i}$

agar $a_{i+1} \geq 1$ bo'lsa $y_i = \left(1 - \dfrac{\alpha_{i+1}}{a_{i+1}}\right) y_{i+1} + \dfrac{\gamma_{i+1}}{a_{i+1}}$,

$a_{i+1} < 1$ bo'lganda $y_i = \dfrac{a_{i+1}}{a_{i+1} + \alpha_i + d_i} y_{i+1} + \dfrac{\gamma_i + f_i}{a_{i+1} + \alpha_i + d_i}$ qiymatlarni xisoblaymiz.

Eslatma 1.

Yuqorida faqatgina y_i funktsiyalarini aniqlash uchun gina emas ω_i oqimni xisoblash uchun ham formulalar keltirildi. a_i – koeffitsentining katta qiymatlarida oqimni $\omega_i= a_i(y_{i-1}-y_i)$ formula buyicha xisoblash aniqlikni yuqotishga olib keladi. Shuning uchun ham qo'shimcha izlanuvchi funktsiya sifatida ω_i oqim kiritildi va u (85) rekkurent formula buyicha xisoblanadi.

2. Tsikllik haydash usuli

Bu usul chegirmali tenglamaning davriy echimlarini topish uchun qullaniladi. Bunday masalalar xususiy xosilali tenglamalarni tsilindrlik va sferalik koordinatalar sistemasida taqribiy echishda hosil buladi.

$$a_1y_N-c_1y_1+b_1y_2=-f_1$$

$$a_iy_{i-1}-c_iy_i+a_{i+1}y_{i+1}=-f_i,\quad i=0,1,2,\dots N-1, \tag{90}$$

$$a_Ny_{N-1}-c_Ny_N+b_Ny_1=-f_N$$

tenglamalar sistemasini qurib utaylik. Bunday algebraik masala $a_iy_{i-1}-c_iy_i+a_{i+1}y_{i+1}=-f_i$, uch xadli tenglamani $a_{i+N}=a_i$, $b_{i+N}=b_i$, $c_{i+N}=c_i$, $f_{i+N}=f_i$, shartlarda $y_{i+N}=y_i$, davriy echimni izlashda hosil buladi.

(76) koefitsientlari buyicha $a_i>0$, $b_i>0$, $c_i> a_i+b_i$ o'rinli.

(90) masalani echish formulasini ya`ni tsikllik haydash formulasini keltirimiz.

$$\alpha_{i+1}=\frac{b_i}{c_i-a_i\alpha_i},\qquad \beta_{i+1}=\frac{f_i+a_i\beta_i}{c_i-a_i\alpha_i},\qquad \gamma_{i+1}=\frac{a_i\gamma_i}{c_i-a_i\alpha_i} \tag{91}$$

$$i=0,1,2,\dots N,$$

$$\alpha_2=\frac{b_2}{c_1},\qquad \beta_2=\frac{\beta_2}{c_1},\qquad \gamma_2=\frac{a_1}{c_1}$$

$$\rho_i=\alpha_{i+1}\rho_{i+1}+\beta_{i+1},\qquad q_i=\alpha_{i+1}q_{i+1}+\gamma_{i+1}$$

$$i=N-2,\dots,1.\qquad \rho_{N-1}=\beta_N\ ,\quad q_{N-1}=\alpha_N+\gamma_N \tag{92}$$

$$y_N=\frac{\beta_{N+1}+\alpha_{N+1}\rho_1}{1-\alpha_{N+1}q_1-\gamma_{N+1}},\qquad y_i=\rho_i+y_Nq_i \tag{93}$$

$$i=0,1,2,\dots N-1,$$

tsiklllik haydash usuli urinli, sababi (93) masalaning echimi (91) shartlarning bajarilishida o'rniqli, maraji, $1-\alpha_{N+1}q_1-\gamma_{N+1}$ ifoda y_N uchun nol`ga aylanmaydi.

Haqiqattan ham (91),(92) dan ko'rinib turganidek

$\alpha_i<1$, $\gamma_i>0$, $\alpha_2+\gamma_2<1$. $\alpha_i+\gamma_i<1$ deb olib

$$\alpha_{i+1}+\gamma_{i+1}=\frac{b_i+a_i\gamma_i}{c_i-a_i\alpha_i}<\frac{b_i+a_i-a_i\alpha_i}{c_i-a_i\alpha_i}<1 \qquad (94)$$

(93) va (94) ni xisobga olib $q_{N-1}<1$, $q_i<1$ ekanligini topamiz. Barcha aytilganlardan $1-\alpha_{N+1}q_1-\gamma_{N+1}>0$ ekanligi kelib chiqadi

3. Chegirmali tenglamani faktorizatsiyalash usuli

Chegirmalik chegaraviy masala uchun haydash formulasi

$$Ly_k=A_ky_{k-1}-C_ky_k+B_ky_{k+1}=-F_k, \qquad k=1,2,\ldots N-1,$$

$$y_0=N_1y_1+v_1,\ y_N=N_2y_{N-1}+v_2 \qquad (95)$$

chegirmalik tenglamani faktorizatsiyalash natijasida olinishi mumkin. Siljitish operatori T ni $Ty_k=y_{k+1}$ deb olib, (95) tenglamaning chap tomonini kupaytma kurinishida hosil qilamiz.

$(b_kT-A_k)(\alpha_kT-E)\,y_{k-1}=(b_kT-A_k)\,(\alpha_ky_k-y_{k-1})=$

$=b_k\alpha_ky_{k-1}-(A_k\alpha_k-b_k)\,y_k+A_k\,y_{k-1}$

bu joyda E-birlik operator, $E\,y_k=y_k$. Bu ifodani (95) bilan solishtirib $\alpha_{k+1}\,b_k=B_k$, $A_k\,\alpha_k+b_k=C_k$ ni olamiz. Bundan

$b_k=C_k-A_k\,\alpha_k$ ni yuqotib

$$\alpha_{k+1}=\frac{B_k}{C_k-A_k\alpha_k} \qquad (96)$$

ga ega bo'lamiz. Faktorizatsiyalangan $(b_kT-A_k)\,(\alpha_ky_k-y_{k-1})=-F_k$ tenglamani echish quydagicha buladi. Dastlabki

$$(b_kT-A_k)\beta_k=b_k\,\beta_{k+1}-A_k\,\beta_k=F_k$$

tenglamaning ya`ni

$$\beta_{k+1} = \frac{A_k \beta_k + F_k}{C_k - A_k \alpha_k} \quad k=1,2,\ldots N-1 \qquad (97)$$

deb β_k -funktsiyasi aniqlanadi. Sung

$\alpha_k y_k - y_{k-1} = -\beta_k$ ya`ni

$$y_k=\alpha_{k+1} y_{k+1}+\beta_{k+1} \qquad k=0,1,2,\ldots N-1 \qquad (98)$$

formulasidan y_k aniqlanadi.

Natijada oddiy haydash formulasini xosil qilamiz. (97)-(98) formulalarga

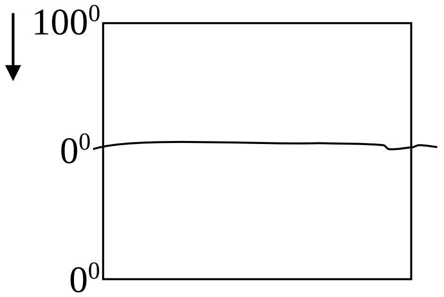
$$\alpha_1=N_1 ,\ \beta_1=\gamma_1 , \qquad y_N=\frac{N_2 \beta_N + \nu_2}{1- N_2 \alpha_N} \qquad (99)$$

boshlang`ish shartlarni qushish kerek. Agar $\Delta v_k=v_{k+1}-v_k$, o'ng va $\nabla v_k= v_k-v_{k-1}$ chap chegirmali operatorlarini kiritsak, unda faktorizatsiya operatorini $Ly_k=A_k y_{k-1}-C_k y_k+B_k y_{k+1}$, $L=L_1L_2$ deb $L_1=b_k \Delta+v_k$, $L_2=\nabla+(\alpha_k-1)$ turlandirish mu`mkin. $L_1L_2y_k= Ly_k$ tengligi urinli bo'ladi. Agar $v_k=b_k-A_k$, $\alpha_{k+1}b_k=B_k$, $A_k\alpha_k+ B_k/\alpha_{k+1}=C_k$ deb olsak. v_k va b_k koefitsentlarni yuqotgandan sung yana (98)-(99) haydash formulalariga aylanib kelamiz.

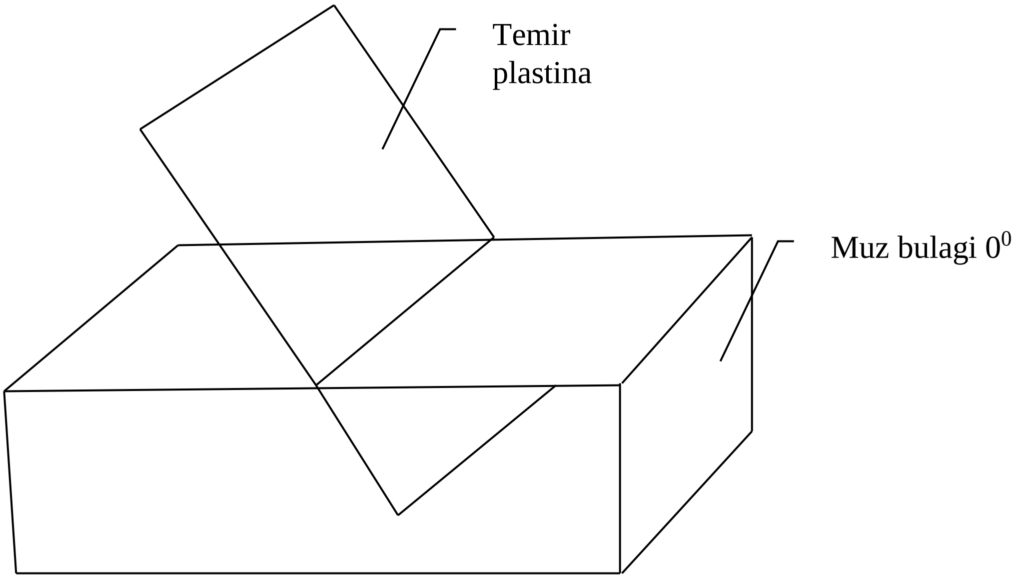
§4. Xususiy xosilali differentsial tenglamalarni MathCad tizimida echish

4.1 MathCad tizimida Laplas tenglamasini echish

Statsionar issiqlik almashuv jarayoni uchun quydagi Laplas masalasini echamiz. Tomonlari **L** bulgan yarimi burchak ostida muz ostida yotgan kvadrat temir plastinani olamiz [5,6]. Plastinaning tepa qismidan 100^0 temperaturadan qaynoq suv quyiladi. Muz bilan qaynoq suv orasidagi plastina harorati chiziqli nizom buyicha uzgaradi. Plastinada haroratning uzgarishini toping surat-1.



Surat-1



Surat-2

$nx := 10 \qquad ny := 10$

$L1 := 1 \qquad L2 := 1$

```
h(u, ε, n) := | b[nx, ny] ← 0
              | for k ∈ 0..n
              |   | u ← b  if  k > 0
              |   | for i ∈ 1..nx − 1
              |   |   for j ∈ 1..ny − 1
              |   |     u[i, j] ← 1/4 · (u[i+1, j] + u[i−1, j] + u[i, j−1] + u[i, j+1])
              |   | a ← norm1(b − u)
              |   | b ← u
              |   | break  if  a < ε
              | u
```

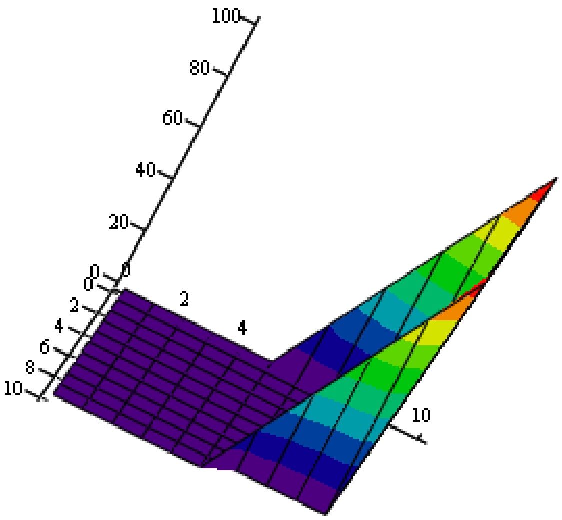
$$f(y) := \begin{cases} 0 & \text{if } y \le \frac{L1}{2} \\ \left[200\left(y - \frac{L1}{2}\right)\right] & \text{otherwise} \end{cases}$$

$i := 1 .. nx - 1 \qquad j := 1 .. ny \qquad hx := \frac{2}{nx} \qquad hy := \frac{L1}{ny} \qquad \varepsilon := 0.1$

$u_{i,0} := 0 \qquad u_{0,j} := f(j \cdot hy) \qquad u_{nx,j} := f(j \cdot hy) \qquad u_{i,ny} := 100 \qquad n := 100$

u =

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	20	40	60	80
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	20	40	60	80



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3.5 MathCad tizimida to'lqin tenglamasini echish

x=0 va x=L nuqtalarda bekitilgan torning tebranish tenglamasi quydagi formula erdamida beriladi [6,12,13].

$$u_{xx} = a^2 u_{tt}$$

Bu erda $a^2 = \frac{\omega}{Tg}$, $\omega -$ uzunlik birligidagi torning og`irligi, g- erkin tusish tezlanishi. T- torning tortilishi. Oddiylik uchun $a^2 = 1$ deb olamiz.

U holda tenglama

$$u_{xx} = u_{tt}, \quad 0 < x < L, \quad t > 0. \tag{100}$$

kurinishga keladi. Torning ikki tarafi berkitilgan bo'lsa chegaraviy shartlar

$$u(0,t) = u(L,t) = 0 \tag{101}$$

Boshlang`ich shart

$$u(x,0) = f(x) \tag{102}$$

Xar bir nuqtadagi boshlang`ich tezlik

$$u_t(x,0) = v(x) \tag{103}$$

Chekli ayirmalarda tenglama olish uchun x o'qini h- odim bilan t- o'qini k-odim bilan torlarga bo'lamiz. (100) xosila chekli ayirmalarda quydagicha bo'ladi.

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2}$$

Bu tenglamani $u_{i,j+1}$ ga qarata echib

$$u_{i,j+1} = 2(1-\lambda^2)u_{i,j} + \lambda^2(u_{i+1,j} + u_{i-1,j}) - u_{i,j-1}, \qquad \lambda = \frac{k}{h} \tag{104}$$

(101) chegaraviy shartlar quydagicha bo'ladi

$$u_{0,j} = u_{n,j} = 0, \quad n = \frac{L}{h}, \; j = 0,1,2,..., \frac{T}{k}. \tag{105}$$

(102) va (103) boshlang`ich shartlar turda quydagicha bo'ladi

$$u_{i,0} = f(ih), \quad i = 1,2,...,n-1,$$

$$\frac{u_{i,1} - u_{i,0}}{k} = v(ih), \tag{106}$$

(106) dan $u_{i,1} = f(ih) + kv(ih), \quad i = 1,2,....,n-1.$ (107)

(106) dan t=0 va (107) dan t=k vaqt momenti uchun bizga torning holati ma`lum , u xolda (104) formulaga j=1 ni quyib

$$u_{i,2} = 2(1-\lambda^2)u_{i,1} + \lambda^2(u_{i+1,1} + u_{i-1,1}) - u_{i,0} \tag{108}$$

ga ega bo'lamiz. Bu formulaning ung tarafining xammasi bizga ma`lum unda uchinchi qatlamni biz hisoblay olamiz. Xuddi shunday qilib ikkinchi va uchinchi qatlamning natijalaridan foydalanib uchinchi qatlam uchun echim olish mumkin. Jaraenni shunday davom ettirib barcha t=2k qatlam uchun echim olinadi. Bu giperbolik tenglamani echishning aniq usuli bo'lib topiladi. Yaqinlashuvchilik sharti shundagina bajariladi agar vaqt buyicha odim koordinata tekisligi odimidan kichik bo'lsa. $\lambda = \frac{k}{h} < 1$

Misol №2

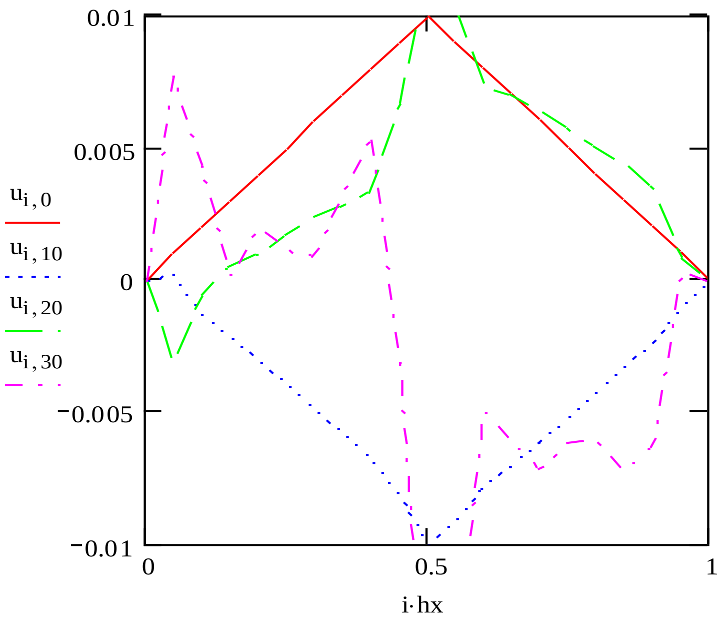
$$u_{xx} = u_{tt}$$

tenglamasini quydagi chegaraviy va boshlang`ich shartlarda eching.

$u(0,t) = 0, \quad u(L,t) = 0,$

$u(x,0) = f(x), \quad u(x,1) = f(x) + kv(x); \; v(x) = 0$

Bunda L=1, x - buyicha nuktalar soni n=20, t- vaqt buyicha m=200 , T=2 U xolda odim h=1/20; vaqt buyicha k=2/200, $\delta = 0,01$ torning burilishi.



Surat-3

$nx := 20 \quad nt := 200 \quad tk := 2 \quad v(x) := 0 \quad \delta := 0.01$

$$f(x) := \begin{cases} (2\cdot\delta\cdot x) & \text{if } x \le \frac{1}{2} \\ [2\cdot\delta\cdot(1-x)] & \text{otherwise} \end{cases}$$

$i := 1 .. nx - 1 \quad j := 1 .. nt \quad hx := \frac{1}{nx} \quad ht := \frac{tk}{nt}$

$\lambda := \frac{ht}{hx} \quad \lambda = 0.2$

$u_{0,j} := 0 \quad u_{nx,j} := 0 \quad u_{i,0} := f(i\cdot hx) \quad u_{i,1} := f(i\cdot hx) + ht\cdot v(i\cdot hx)$

$i := 0 .. nx$

$$p(u) := \left| \begin{array}{l} \text{for } j \in 1 .. nt \\ \quad \text{for } i \in 1 .. nx - 1 \\ \quad\quad u_{i,j+1} \leftarrow 2\cdot\left(1-\lambda^2\right)\cdot u_{i,j} + \lambda^2\left(u_{i+1,j} - u_{i-1,j}\right) - u_{i,j-1} \\ u \end{array} \right.$$

$u := p(u)$

u =

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$1\cdot10^{-3}$	$9.968\cdot10^{-4}$	$9.843\cdot10^{-4}$	$9.538\cdot10^{-4}$
2	$2\cdot10^{-3}$	$2\cdot10^{-3}$	$1.92\cdot10^{-3}$	$1.76\cdot10^{-3}$	$1.52\cdot10^{-3}$	$1.203\cdot10^{-3}$
3	$3\cdot10^{-3}$	$3\cdot10^{-3}$	$2.84\cdot10^{-3}$	$2.526\cdot10^{-3}$	$2.072\cdot10^{-3}$	$1.496\cdot10^{-3}$
4	$4\cdot10^{-3}$	$4\cdot10^{-3}$	$3.76\cdot10^{-3}$	$3.293\cdot10^{-3}$	$2.623\cdot10^{-3}$	$1.788\cdot10^{-3}$
5	$5\cdot10^{-3}$	$5\cdot10^{-3}$	$4.68\cdot10^{-3}$	$4.059\cdot10^{-3}$	$3.175\cdot10^{-3}$	$2.081\cdot10^{-3}$
6	$6\cdot10^{-3}$	$6\cdot10^{-3}$	$5.6\cdot10^{-3}$	$4.826\cdot10^{-3}$	$3.726\cdot10^{-3}$	$2.373\cdot10^{-3}$
7	$7\cdot10^{-3}$	$7\cdot10^{-3}$	$6.52\cdot10^{-3}$	$5.592\cdot10^{-3}$	$4.278\cdot10^{-3}$	$2.666\cdot10^{-3}$
8	$8\cdot10^{-3}$	$8\cdot10^{-3}$	$7.44\cdot10^{-3}$	$6.358\cdot10^{-3}$	$4.829\cdot10^{-3}$	$2.957\cdot10^{-3}$
9	$9\cdot10^{-3}$	$9\cdot10^{-3}$	$8.36\cdot10^{-3}$	$7.122\cdot10^{-3}$	$5.365\cdot10^{-3}$	$3.206\cdot10^{-3}$
10	0.01	0.01	$9.2\cdot10^{-3}$	$7.658\cdot10^{-3}$	$5.484\cdot10^{-3}$	$2.839\cdot10^{-3}$
11	$9\cdot10^{-3}$	$9\cdot10^{-3}$	$8.2\cdot10^{-3}$	$6.667\cdot10^{-3}$	$4.531\cdot10^{-3}$	$1.972\cdot10^{-3}$
12	$8\cdot10^{-3}$	$8\cdot10^{-3}$	$7.28\cdot10^{-3}$	$5.904\cdot10^{-3}$	$3.994\cdot10^{-3}$	$1.722\cdot10^{-3}$
13	$7\cdot10^{-3}$	$7\cdot10^{-3}$	$6.36\cdot10^{-3}$	$5.138\cdot10^{-3}$	$3.443\cdot10^{-3}$	$1.429\cdot10^{-3}$
14	$6\cdot10^{-3}$	$6\cdot10^{-3}$	$5.44\cdot10^{-3}$	$4.371\cdot10^{-3}$	$2.891\cdot10^{-3}$	$1.136\cdot10^{-3}$
15	$5\cdot10^{-3}$	$5\cdot10^{-3}$	$4.52\cdot10^{-3}$	$3.605\cdot10^{-3}$	$2.34\cdot10^{-3}$	$8.437\cdot10^{-4}$

Xulosa

Yuqori o'quv muassosalarida talabalar va magistrantlar «Differentsial tenglamalar», «Matematik fizika tenglamalari» fanini o'rganishda ikkinshi tartibli xususiy xosilali differentsial tenglamalar ya`ni parabolik, giperbolik va elliptik tenglamalar, ularga quyilgan chegaraviy masalalar ko'p uchraydi. Echimni ko'p hollarda analitik usulda olish judayam qiyin shuning uchun biz sonli usullardan foydalanamiz.

Bu bitiruv malakaviy ishda xususiy xosilali differentsial tenglamalarni echish usullarini kurib uttik.

Mazkur ish to'rt paragraftan iborat bo'lib birinchi paragrafta xususiy xosilali differentsial tenglamalarni echish usullaridan tug'ri va iteratsiya usullari, o'zgaruvchini ajratish usuli keltirilgan, ikkinchi paragrafta esa ikki qatlamli iteratsion sxemalarga tuxtalgan. Bu erda oddiy iteratsia va maksimum qiymat printsipi keltirilgan. Ishning uchinchi paragrafida uch qatlamli iteratsion sxemalar haqida aytilgan va undan tashqari to'r tenglamalarini matritsaviy haydash va zeydel usullaridan foydalanib echish yullari ko'rsatilgan.

Bu usul uch diagonalli matritsalarga ega sistemalar uchun qo'llaniladi. (Ular ikkinchi tartibli differentsial tenglamalarni chegaraviy masalalar bilan echishda uchraydi). Matritsaviy haydash usuli chegaraviy tenglamalarni soha murakkab bulganda tadqiq qiladi. Lekin matritsaviy haydash usuli $Q=O(N^k)$ arifmetik amalda bajariladi va katta xotirani talab qiladi. Shuning bilan birga chegaraviy shartlar bilan haydash usulidan foydalanib masalani echkanda haydash matritsasini $O(N^3)$ gacha qisqartish mu`mkin. Xulosa qilib aytganda matritsaviy haydash usulining boshqa xisoblash usullariga qo'ra ancha yutiqga ega ekanligi misollar yordamida ko'rsatildi. Turtinchi paragrafta esa xususiy xosilali differentsial tenglamalarni echishda MathCad ning qullanilishi keltirilgan. 4.1 bandda Laplas tenglamasi va 4.2 bandda esa giperbolik tiptagi tenglama MathCad tizimida echilgan.

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Texnika xavfsizligi qoidalari

1.Informatika xonasini beton polli xonalarga yoki binoning erto'lalariga joylashtirish qat'iyan ta'qiqlanadi va sinf xonasining poli elektr tokini o'tkazmaydigan qilib (masalan taxtadan) yasalishi va barcha kompyuterlarning korpuslari yerga ulanishi (yoki elektr ta'minotining yevroandozalarga moslab ulanishi) talab qilinadi.

2. Yuqori kuchlanish (220 Volt) yuradigan simlarning barchasi, shu jumladan uzaytirgichlarning, kompyuter va boshqa elektr qurilmalarining elektr ta'minotiga ulanish simlarining ikki marta izolyasiya qilinganligi talab qilinadi.

3. Xonadagi kompyuterlarni devor bo'ylab yoki xonaning o'rtasiga ikki qator qilib joylashtirilishi kerak

4. Xonadagi barcha kompyuterlarni elektr tarmog'idan uzuvchi yagona uzgich ham bo'lishi kerak.

5. Kompyuter monitori o'tirgan o'quvchilarning ko'zlari darajasida bo'lib, o'quvchilar undan 40 sm dan 80 sm gacha bo'lgan masofada o'tirishlari imkoniyatiga ega bo'lishlari kerak.

6. Kompyuterning klaviaturasi o'tirgan o'quvchilarning bukilgan tirsaklari darajasida bo'lishi kerak. Sichqon uchun klaviaturaning ikkala tomonidan yetarlicha joy qoldirilishi va ular bir xil balandlikda bo'lishlari kerak.

7. Kompyuterda muttasil ishlash vaqti o'quvchilar uchun 60 minutdan ortmasligi kerak.

8. Kompyuter xonasining kvadrat metrlardagi sathi unga joylangan kompyuterlar sonidan kamida 6 marta ko'p bo'lishi kerak.

9. Kompyuter xonalari yetarli quvvatga ega ventilyasiya tizimiga ega bo'lishi kerak.

10. Klaviaturaning kompyuter ishlamayotgan paytda uzoq muddatga ochiq holda qolishi va unda chang yig'ilib qolishining oldini olish lozim