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1 - bob. Haqiqiy sonlar

1.1 Natural va butun sonlar

O'nta sanoq sistemasidagi sonlarni yozish uchun o'nta belgidan foydalaniladi. Bular 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 dir. Bu belgilar *raqamlar deyiladi*. Masalan: 8, 18, 524, 2815, 62703. 8 - bu ham raqam ham son. 18 - bu raqam emas, u 1 (bir) va 8 (sakkiz) raqamlaridan tashkil topgan son. Yuqorida keltirilgan sonlarning yozuvidagi har bir raqam o'zi egallagan o'rniga qarab turli qiymatga egadir. Xususan, 524 (besh yuz yigirma to'rt) yozuvida 4 raqami bu sonda to'rtta bir borligini, 2 raqami bu sonda ikkita o'n borligini, 5 raqami bu sonda beshta yuz borligini bildiradi. Ya'ni $524 = 5 \cdot 100 + 2 \cdot 10 + 4 \cdot 1$, $62703 = 6 \cdot 10000 + 2 \cdot 1000 + 7 \cdot 100 + 0 \cdot 10 + 3 \cdot 1$.

Sonlarni o'nta raqam yordamida yozishning bunday usuli "o'nta sanoq sistemasi" deyiladi. Predmetlarni sanashda ishlatiladigan sonlar natural sonlar deyiladi. Natural sonlar to'plami N harfi bilan belgilanadi, ya'ni $N = \{1, 2, 3, \dots, n, \dots\}$. Butun sonlar to'plami $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$ harfi bilan belgilanadi. Butun sonlar to'plamida qo'shish, ayirish va ko'paytirish amallari aniqlangan. Ular quyidagi xossalarga ega.

- $a + (-b) = a - b$.

- $-(-a) = a$.

- $(a + b) + c = a + (b + c)$.

- $-(a + b - c) = -a - b + c$.

- $a \cdot (-b) = (-a) \cdot b = -ab$.

- $(ab) \cdot c = a \cdot (bc) = b \cdot (ac)$.

- $(a + b) \cdot c = a \cdot c + b \cdot c$.

$a \in Z$ sonining darajalari quyidagicha aniqlanadi:
 $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$ va hokazo $a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_n$.

Bu yerda a asos, n daraja ko'rsatkich deyiladi. Natural ko'rsatkichli darajaning ba'zi xossalari keltiramiz. Ixtiyoriy $a, b \in Z$ va n, m natural sonlar uchun quyidagi tengliklar o'rinli:

- $a^n \cdot a^m = a^{n+m}$.

- $(ab)^n = a^n \cdot b^n$.

- $\frac{a^n}{a^m} = a^{n-m}$.

- $a^0 = 1$.

Qo'shish va ayirish birinchi bosqichli amallar, ko'paytirish va bo'lish ikkinchi bosqichli amallar, darajaga ko'tarish va ildiz chiqarish uchinchi bosqichli amallar deyiladi. Yuqori bosqichli amallar oldin bajariladi. Bir xil bosqichli amallar qatnashgan ifodalarda birinchi kelgan amal avval bajariladi. Agar ifodada qavslar bo'lsa, dastlab qavs ichidagi amallar bajariladi.

1.1.1 Hisoblashga oid misollar

- Amallarning bajarilish tartibiga e'tibor qilib ifodaning qiymatini hisoblang.

$$18 - 6 : 2 + 3 \cdot 4$$

- A) 3 B) 27 C) 18 D) 33

Yechish: Ifodada birinchi bosqichli va ikkinchi bosqichli amallar qatnashgan. Dastlab biz ikkinchi bosqichli (ko'paytirish va bo'lish) amallarini bajaramiz.

$$18 - 6 : 2 + 3 \cdot 4 = 18 - 3 + 12.$$

Endi faqat birinchi bosqichli amallar qoldi. Ularni navbati bilan bajaramiz.

$$18 - 3 + 12 = 15 + 12 = 27.$$

Javob: 27 (B).

- (96-3-1) Ifodaning qiymatini toping:

$$12 - 6 : 3 + 2 \cdot 4$$

- A) 16 B) 10 C) 18 D) 48

- (96-11-1) Ifodaning qiymatini toping:

$$15 - 9 : 3 + 4 \cdot 3$$

- A) 24 B) 18 C) 48 D) 12

- (96-12-1) Ifodaning qiymatini toping:

$$18 - 12 : 2 + 5 \cdot 3$$

- A) 18 B) 24 C) 4 D) 27

- Ifodaning qiymatini toping:

$$24 - 6 : 3 + 5 \cdot 2$$

- A) 16 B) -3 C) 32 D) 22

- Ifodaning qiymatini toping:

$$8 : 2^2 + 4 \cdot 3 - 10$$

- A) 6 B) 4 C) 18 D) 12

Yechish: Dastlab uchinchi bosqich amali, darajaga ko'tarish amalini, keyin esa ikkinchi bosqich amallari bo'lish va ko'paytirishni bajaramiz:

$$8 : 2^2 + 4 \cdot 3 - 10 = 8 : 4 + 4 \cdot 3 - 10 = 2 + 12 - 10.$$

Endi birinchi bosqich amallarini bajarib $2 + 12 - 10 = 14 - 10 = 4$ ni olamiz. **Javob:** 4 (B).

- Ifodaning qiymatini toping:

$$4^2 : 2 + 3 \cdot 4 - 5$$

- A) 11 B) 39 C) 15 D) 12

- Ifodaning qiymatini toping:

$$3 \cdot 5 + 3 \cdot 2^3 - 25$$

- A) 206 B) 14 C) 119 D) 12

- Ifodaning qiymatini toping:

$$24 : 2 \cdot 5 - 3 \cdot 2^4 - 7$$

- A) 19 B) -52,6 C) -1243 D) 5

- Ifodaning qiymatini toping:

$$27 : 3^3 + 2 \cdot 3^2 - 15$$

- A) 22 B) 4 C) 15 D) 732

11. $(2 \cdot 6 + 8) \cdot 2 - 2$ ifodaning qiymatini toping:
A) 16 B) 0 C) 54 D) 38

Yechish: Dastlab qavs ichidagi ifodaning qiymatini hisoblaymiz: $2 \cdot 6 + 8 = 12 + 8 = 20$. Endi ko'paytirishni keyin ayirishni bajaramiz: $20 \cdot 2 - 2 = 40 - 2 = 38$. **Javob:** 38 (D).

12. $(2 + 8 \cdot 6) \cdot 2 - 2 \cdot 7$ ifodaning qiymatini toping:
A) 86 B) 0 C) 54 D) 38
13. $(5 + 3 \cdot 6) \cdot 2 - 2 \cdot 23$ ifodaning qiymatini toping:
A) 18 B) 0 C) 50 D) 13

14. $3 + 3 \cdot 2(7 \cdot 2 - 4) : 3$ ifodaning qiymatini toping:
A) 40 B) 20 C) 23 D) 35

15. (96-7-1) Hisoblang.
 $21 \cdot 18 - 19 \cdot 18 + 18 \cdot 17 - 17 \cdot 16 + 16 \cdot 15 - 15 \cdot 14$
A) 50 B) 100 C) 98 D) 24

Yechish: Umumiy ko'paytuvchini qavsdan tashqariga chiqarish yordamida hisoblaymiz:

$$\begin{aligned} 21 \cdot 18 - 19 \cdot 18 + 18 \cdot 17 - 17 \cdot 16 + 16 \cdot 15 - 15 \cdot 14 &= \\ = 18(21 - 19) + 17(18 - 16) + 15(16 - 14) &= \\ = 18 \cdot 2 + 17 \cdot 2 + 15 \cdot 2 = 2(18 + 17 + 15) &= \\ = 2 \cdot 50 = 100. \quad \text{Javob: } 100 \text{ (B)}. \end{aligned}$$

16. (97-7-1) Hisoblang.
 $36 \cdot 24 - 33 \cdot 24 + 17 \cdot 11 - 14 \cdot 11 + 18 \cdot 16 - 15 \cdot 16$
A) 166 B) 155 C) 180 D) 153

17. (97-10-1) Hisoblang.
 $27 \cdot 23 - 24 \cdot 23 + 21 \cdot 19 - 18 \cdot 19 + 17 \cdot 11 - 14 \cdot 11$
A) 165 B) 159 C) 143 D) 203

18. (00-5-4) Hisoblang.
 $139 \cdot 15 + 18 \cdot 139 + 15 \cdot 261 + 18 \cdot 261$
A) 13200 B) 14500 C) 15100 D) 16200

19. (96-1-1) Ifodaning qiymatini toping:
 $26 \cdot 25 - 25 \cdot 24 + 24 \cdot 23 - 23 \cdot 22 - 12 \cdot 8$
A) 106 B) 1 C) 54 D) 0

1.1.2 Tub va murakkab sonlar

Agar a va b natural sonlari uchun $a : b = c$ ham natural son bo'lsa, u holda a soni b songa *bo'linadi* yoki a soni b soniga *karrali* deyiladi. b soni a sonining *bo'luvchisi*, c soni esa *bo'linma* deyiladi. Masalan, 24 soni 1,2,3,4,6,8,12 va 24 sonlariga bo'linadi. 1 va o'zidan boshqa bo'luvchisi bo'lmagan natural sonlar *tub sonlar* deyiladi. 1 va o'zidan boshqa bo'luvchilarga ega bo'lgan natural sonlar *murakkab sonlar* deyiladi. 1 soni tub ham murakkab ham emas. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... sonlari – tub sonlardir. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, ... sonlari – murakkab sonlardir.

1. (99-7-10) 30 dan kichik tub sonlar nechta?
A) 11 B) 9 C) 10 D) 12

Yechish: 30 dan kichik tub sonlar 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Ular 10 ta. **Javob:** 10 (C).

2. (98-5-8) 50 dan kichik tub sonlar nechta?
A) 10 B) 15 C) 17 D) 9

3. 30 va 50 sonlari orasida nechta tub son bor?
A) 4 B) 3 C) 5 D) 7

4. (02-5-4) 1; 2; 3; 15; 17; 23; 24; 169; 289; 361 sonlar ketma-ketligida nechta tub son bor?
A) 3 B) 4 C) 5 D) 7

5. 2; 3; 15; 17; 21; 23; 29; 39; 51; 57 sonlar ketma-ketligida nechta murakkab son bor?
A) 3 B) 4 C) 5 D) 7

6. (99-3-7) Quyidagi sonlar guruhidan qaysilari faqat tub sonlardan tashkil topgan?
1) 0; 3; 5; 7; 11; 2) 1; 3; 5; 7; 13; 3) 3; 5; 7; 9; 11; 4) 2; 3; 5; 7; 17; 5) 3; 5; 17; 19; 381
A) 1; 2 B) 2; 4 C) 5 D) 4

1.1.3 Eng katta umumiy bo'luvchi – EKUB va eng kichik umumiy karrali – EKUK

a va b natural sonlarining *eng katta umumiy bo'luvchisi*, *ya'ni EKUBi* deb, ularning har ikkisini bo'luvchi sonlarning eng kattasiga aytiladi va $B(a; b)$ shaklda yoziladi. a va b natural sonlarining *eng kichik umumiy karralisi*, *ya'ni EKUKi* deb ularning har ikkisiga karrali natural sonlarning eng kichigiga aytiladi va $K(a; b)$ shaklda yoziladi. Agar a va b natural sonlarining 1 dan boshqa umumiy bo'luvchisi bo'lmasa, ular *o'zaro tub sonlar* deyiladi. Ikki a va b sonlarning minimumi deb, ularning kichigiga aytiladi va $\min\{a, b\}$ shaklda yoziladi. Ikki a va b sonlarning maksimumi deb, ularning kattasiga aytiladi va $\max\{a, b\}$ shaklda yoziladi. Masalan, $\min\{0, 2\} = 0$, $\max\{1, 3\} = 3$.

1. $a = 2^m \cdot 3^n \cdot \dots \cdot p^k$ tub ko'paytuvchilarga ajratilgan a sonining natural bo'luvchilari soni $(m+1)(n+1) \cdot \dots \cdot (k+1)$ ga teng.

2. $a = 2^m \cdot 3^n \cdot \dots \cdot p^k$ tub ko'paytuvchilarga ajratilgan a sonining bo'luvchilari yig'indisi:

$$Y(a) = \frac{2^{m+1} - 1}{2 - 1} \cdot \frac{3^{n+1} - 1}{3 - 1} \cdot \dots \cdot \frac{p^{k+1} - 1}{p - 1}.$$

3. a va b sonlarining umumiy bo'luvchilari soni ular EKUBining bo'luvchilari soniga teng.

4. $B(a; b) \cdot K(a; b) = a \cdot b$ tenglik o'rinli.

Agar a va b natural sonlari $a = 2^{m_1} \cdot 3^{n_1} \cdot \dots \cdot p^{k_1}$ va $b = 2^{m_2} \cdot 3^{n_2} \cdot \dots \cdot p^{k_2}$ tub ko'paytuvchilarga ajratilgan bo'lsa, u holda

5. $B(a; b) = 2^{\min\{m_1, m_2\}} \cdot 3^{\min\{n_1, n_2\}} \cdot \dots \cdot p^{\min\{k_1, k_2\}}$.

6. $K(a; b) = 2^{\max\{m_1, m_2\}} \cdot 3^{\max\{n_1, n_2\}} \cdot \dots \cdot p^{\max\{k_1, k_2\}}$.

7. O'zaro tub a va b sonlar uchun $B(a; b) = 1$ va aksincha.

1. (02-5-5) 36 ning natural bo'luvchilari nechta?
A) 5 B) 7 C) 8 D) 9

Yechish: 36 ni tub ko'paytuvchilarga ajratamiz. $36 = 2^2 \cdot 3^2$. 1-qoidaga ko'ra, 36 ning bo'luvchilari soni $(2+1) \cdot (2+1) = 9$. **Javob:** 9 (D).

2. 100 ning natural bo'luvchilari sonini toping.
A) 4 B) 6 C) 9 D) 8
3. 480 ning natural bo'luvchilari sonini toping.
A) 14 B) 24 C) 48 D) 32
4. 900 ning natural bo'luvchilari sonini toping.
A) 27 B) 36 C) 49 D) 28
5. 1000 ning natural bo'luvchilari nechta?
A) 24 B) 16 C) 28 D) 32
6. (03-10-11) $8^{n+2} \cdot 12^{n-3}$ ko'paytmaning natural bo'luvchilari soni 42 ga teng bo'lsa, n nechga teng bo'ladi?
A) 4 B) 3 C) 2 D) 5
7. (00-5-6) 48 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 123 B) 100 C) 108 D) 124
- Yechish.** 48 sonini tub ko'paytuvchilarga ajratamiz. $48 = 2^4 \cdot 3$. 2-qoidaga ko'ra 48 sonining barcha natural bo'luvchilari yig'indisi $Y(48) = (2^{4+1} - 1) : (2 - 1) \cdot (3^{1+1} - 1) : (3 - 1) = 31 \cdot 4 = 124$. **Javob:** 124 (D).
8. 12 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 12 B) 28 C) 32 D) 24
9. 24 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 48 B) 58 C) 60 D) 54
10. 20 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 48 B) 42 C) 38 D) 58
11. 28 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 44 B) 58 C) 62 D) 56
12. (96-3-2) 8 va 6 sonlarining eng kichik umumiy karralisi, ya'ni EKUKini toping.
A) 16 B) 24 C) 12 D) 8
- Yechish:** Bu sonlarni tub ko'paytuvchilarga ajratamiz. $8 = 2^3 \cdot 3^0$; $6 = 2 \cdot 3$. 6-qoidaga ko'ra $K(8; 6) = 2^3 \cdot 3 = 24$. **Javob:** 24 (B).
13. (96-12-2) 6 va 4 sonlarining EKUKini toping.
A) 6 B) 14 C) 24 D) 12
14. (96-11-2) 10 va 8 sonlarining EKUKini toping.
A) 80 B) 10 C) 18 D) 40
15. 36 va 48 sonlarining EKUBini toping.
A) 36 B) 14 C) 24 D) 12
16. 480 va 600 sonlarining EKUBini toping.
A) 160 B) 300 C) 240 D) 120
17. (96-9-1) 594 va 378 ning umumiy bo'luvchilari nechta?
A) 8 B) 7 C) 9 D) 5

Yechish: Ikki sonning umumiy bo'luvchilari soni 3-qoidaga ko'ra ular EKUBining umumiy bo'luvchilari soniga teng.

$$594 = 2 \cdot 3^3 \cdot 7^0 \cdot 11, \quad 378 = 2 \cdot 3^3 \cdot 7 \cdot 11^0$$

bo'lgani uchun 594 va 378 sonlarining EKUBi 5-qoidaga ko'ra $B(594; 378) = 2^1 \cdot 3^3 \cdot 7^0 \cdot 11^0 = 2^1 \cdot 3^3$ ga teng. 1-qoidaga ko'ra bu sonning umumiy bo'luvchilari soni $n = (1+1)(3+1)(0+1)(0+1) = 8$ ga teng. **Javob:** 8 (A).

18. (99-7-1) 56 va 16 sonlarining umumiy bo'luvchilari nechta?
A) 4 B) 3 C) 2 D) 5
19. (96-3-61) 630 va 198 ning umumiy bo'luvchilari nechta?
A) 5 B) 6 C) 4 D) 7
20. (96-13-1) 420 va 156 ning umumiy bo'luvchilari nechta?
A) 7 B) 5 C) 6 D) 4
21. (98-2-2) 8 va 12 sonlari eng kichik umumiy karralising natural bo'luvchilari nechta?
A) 6 B) 7 C) 8 D) 9
22. (98-10-1) 21 va 35 sonlarining EKUKi va EKUBining yig'indisini toping.
A) 108 B) 110 C) 112 D) 109
- Yechish:** 21 va 35 sonlarining EKUBi va EKUKi 5 va 6-qoidalarga ko'ra
- $$21 = 3 \cdot 5^0 \cdot 7, \quad 35 = 3^0 \cdot 5 \cdot 7$$
- tenglikdan $B(21; 35) = 3^0 \cdot 5^0 \cdot 7 = 7$, $K(21; 35) = 3 \cdot 5 \cdot 7 = 105$. Ularning yig'indisi $7 + 105 = 112$. **Javob:** 112 (C).
23. (00-3-5) 72 va 96 sonlari EKUKining EKUBiga nisbatini toping?
A) 10 B) 0,1 C) 9 D) 12
24. (98-11-2) 270 va 300 sonlari EKUKining 4 va 6 sonlari EKUKiga nisbatini toping.
A) 25 B) 45 C) 225 D) 95
25. (00-10-2) 108 va 135 sonlari EKUKini 12 va 54 sonlari EKUKiga nisbatini toping?
A) 8 B) 5 C) 12 D) 6
26. (99-2-4) 24, 18 va 30 sonlari EKUKining EKUBiga nisbatini toping.
A) 90 B) 72 C) 48 D) 60
27. (00-7-7) 9, 10, 22 va 25 sonlari orasida nechta o'zaro tub sonlar jufti bor?
A) 4 B) 3 C) 2 D) 6

Yechish. 7-qoidaga ko'ra o'zaro tub sonlarning EKUBi 1 ga teng. $B(9; 10) = 1$, $B(9; 22) = 1$, $B(9; 25) = 1$, $B(10; 22) = 2$, $B(10; 25) = 5$, $B(22; 25) = 1$. Demak, o'zaro tub sonlar jufti 4 ta ekan. **Javob:** 4 (A).

28. (03-4-3) [4; 8] kesmada nechta o'zaro tub sonlar jufti bor?
A) 5 B) 6 C) 4 D) 7
29. (01-12-1) Dastlabki 30 ta natural sonlar ichida 6 soni bilan o'zaro tub bo'lgan sonlar nechta?
A) 7 B) 8 C) 9 D) 10
30. (97-5-10) Qaysi juftlik o'zaro tub sonlardan iborat?
A) (8;14) B) (11;22) C) (12;35) D) (12;34)
31. (97-9-10) Qaysi juftlik o'zaro tub sonlardan iborat?
A) (21;14) B) (21;10) C) (12;15) D) (10;15)
32. (99-8-19) Ikki sonning ko'paytmasi 294 ga, ularning eng katta umumiy bo'luvchisi 7 ga teng. Bu sonlarning eng kichik umumiy karralisini toping.
A) 42 B) 52 C) 56 D) 49
- Yechish.** 4-qoidaga ko'ra $a \cdot b = B(a; b) \cdot K(a; b)$. Mos qiymatlarni qo'yib $294 = 7 \cdot K(a; b)$ ni olamiz. Bu yerdan $K(a; b) = 42$. **Javob:** 42 (A).
33. Ikki sonning ko'paytmasi 192 ga, ularning EKUKi 48 ga teng. Bu sonlarning EKUBini toping.
A) 4 B) 6 C) 5 D) 8
34. (00-7-2) 18 va 12 sonlari EKUKi va EKUBi ko'paytmasini toping?
A) 220 B) 218 C) 214 D) 216
35. 7 va a sonlarining EKUKi va EKUBining ko'paytmasi 126 ga teng. a ni toping.
A) 18 B) 16 C) 25 D) 36
36. 12 va 15 sonlari EKUKi va EKUBi ko'paytmasini toping.
A) 180 B) 160 C) 250 D) 360

1.1.4 Bo'linish belgilari

Raqam - bu sonlarni yozishda ishlatiladigan shartli belgilardir. Ular 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 lardir. Nol barcha sonlarga bo'linadi deb hisoblanadi. a_1, a_2, \dots, a_n raqamlaridan tashkil topgan n xonali sonni $\overline{a_1 a_2 \dots a_n}$ ko'rinishda yozamiz. Bo'linish alomatlarini keltiramiz.

- Oxirgi raqami 0, 2, 4, 6, 8 bo'lgan sonlar va faqat ular 2 ga bo'linadi. 2 ga bo'linadigan sonlar juft sonlar deyiladi. Ular 2, 4, ..., 2n, ... 2 ga bo'lganda 1 qoldiq beradigan sonlar toq sonlar deyiladi. Ular 1, 3, 5, ..., 2n-1, ...**
- a sonining oxirgi ikki raqami ifodalagan son 4 ga (25 ga) bo'linsa, a sonining o'zi ham 4 ga (25 ga) bo'linadi va aksincha.**
- a sonining oxirgi uchta raqami ifodalagan son 8 ga bo'linsa, a sonining o'zi ham 8 ga bo'linadi va aksincha.**
- Oxirgi raqami 0 yoki 5 bo'lgan sonlar va faqat ular 5 ga bo'linadi.**

- Raqamlari yig'indisi 3 ga bo'linadigan sonlar 3 ga bo'linadi va aksincha.**
- Raqamlari yig'indisi 9 ga bo'linadigan sonlar 9 ga bo'linadi va aksincha.**
- Agar $A = \overline{a_1 a_2 \dots a_n}$ ko'rinishdagi natural son uchun $\overline{a_1 a_2 \dots a_{n-1}}$ va $2 \cdot a_n$ sonlarining farqi 7 ga bo'linsa berilgan A soni ham 7 ga bo'linadi.**
- Agar sonning juft o'rinda turgan raqamlar yig'indisi bilan toq o'rinda turgan raqamlar yig'indisining farqi 11 ga bo'linsa, bunday sonlar 11 ga bo'linadi.**
- Agar $A = \overline{a_1 a_2 \dots a_n}$ ko'rinishdagi natural son uchun $\overline{a_1 a_2 \dots a_{n-1}} + 2 \cdot a_n$ soni 19 ga bo'linsa berilgan A natural soni ham 19 ga bo'linadi.**
- Agar n natural son uchun $n = kl$ (k va l o'zaro tub sonlar) tenglik o'rinli bo'lsa, u holda k va l ga bo'lingan sonlar n ga bo'linadi va aksincha. Bu belgini murakkab sonlarga bo'linish belgisi sifatida qabul qilish mumkin. Masalan, $45 = 5 \cdot 9$ va $B(5; 9) = 1$ bo'lgani uchun 5 ga va 9 ga bo'lingan sonlar 45 ga ham bo'linadi.**
- Agar A soni n ga, B soni m ga bo'linsa, u holda $A \cdot B$ soni $n \cdot m$ ga bo'linadi.**
- Agar A va B sonlarning har biri n ga bo'linsa, u holda $A + B$ ham n ga bo'linadi.**
- Agar A va B sonlardan birortasi n ga bo'linsa, u holda $A \cdot B$ ham n ga bo'linadi.**

- 2 ga bo'linadigan sonni toping.
A) 1205 B) 7806 C) 9321 D) 6843

Yechish. 1-alomatga ko'ra, oxirgi raqami 6 bo'lgan 7806 soni 2 ga bo'linadi. Qolgan 1205, 9321, 6843 sonlar 2 ga bo'linmaydi. **Javob:** 7806 (B).

- 2 ga bo'linmaydigan sonni toping.
A) 3456 B) 5842 C) 7648 D) 8641

- 5 ga bo'linadigan sonni toping.
A) 6348 B) 8951 C) 3965 D) 5554

- 5 ga bo'linmaydigan sonni toping.
A) 6665 B) 3335 C) 4440 D) 5554

- 2 ga ham 5 ga ham bo'linadigan sonni toping.
A) 5522 B) 2255 C) 3840 D) 5258

- 4 ga bo'linmaydigan sonni toping.
A) 1100 B) 1520 C) 130 D) 1008

Yechish. 2-alomatga ko'ra, berilgan sonlarning oxirgi ikki raqami ifodalagan sonni 4 ga bo'linishini tekshirishimiz kerak. C) javobdagi 130 sonining oxirgi ikki raqami ifodalagan son 30 bo'lib, u 4 ga bo'linmaydi. Demak, 130 soni ham 4 ga bo'linmaydi. **Javob:** 130 (C).

7. 4 ga bo'linadigan sonni toping.
A) 582 B) 674 C) 804 D) 442
8. 25 ga bo'linadigan sonni toping.
A) 2540 B) 8800 C) 2552 D) 4520
9. 25 ga bo'linmaydigan sonni toping.
A) 6300 B) 8975 C) 6850 D) 9855
10. 3 ga bo'linadigan sonni toping.
A) 326 B) 213 C) 475 D) 739
- Yechish.** 5-alomatga ko'ra, raqamlar yig'indisining 3 ga bo'linishini tekshiramiz. A) javobdagi sonning raqamlari yig'indisi $3 + 2 + 6 = 11$ bo'lib, u 3 ga bo'linmaydi, B) javobda $2 + 1 + 3 = 6$ soni 3 ga bo'linadi. Demak, 213 soni ham 3 ga bo'linadi. **Javob:** 213 (B).
11. 3 ga bo'linmaydigan sonni toping.
A) 6825 B) 8937 C) 5841 D) 3133
12. 9 ga bo'linadigan sonni toping.
A) 881 B) 672 C) 432 D) 763
13. 9 ga bo'linmaydigan sonni toping.
A) 8082 B) 4365 C) 1791 D) 2654
14. 7 ga bo'linadigan sonni toping.
A) 114 B) 235 C) 315 D) 370
- Yechish.** Misolni yechishda 7-alomatdan foydalanamiz. A) javobda $11 - 2 \cdot 4 = 3$ soni 7 ga bo'linmaydi, B) javobda $23 - 2 \cdot 5 = 13$ soni 7 ga bo'linmaydi. C) javobda $31 - 2 \cdot 5 = 21$ soni 7 ga bo'linadi. Demak, 315 soni 7 ga bo'linadi. **Javob:** 315 (C).
15. 7 ga bo'linadigan sonni toping.
A) 514 B) 635 C) 828 D) 546
16. 8 ga bo'linadigan sonni toping.
A) 1140 B) 2350 C) 3700 D) 3152
17. 8 ga bo'linmaydigan sonni toping.
A) 5408 B) 3600 C) 7000 D) 8148
18. 11 ga bo'linadigan sonni toping.
A) 1540 B) 2350 C) 3712 D) 8152
- Yechish.** Misolni yechishda 8-alomatdan foydalanamiz. A) javobda keltirilgan sonning juft o'ringdagi raqamlar yig'indisi $5 + 0 = 5$, toq o'ringdagi raqamlar yig'indisi $1 + 4 = 5$, ularning ayirmasi $5 - 5 = 0$ soni 11 ga bo'linadi. Demak, 1540 soni ham 11 ga bo'linadi. **Javob:** 1540 (A).
19. 11 ga bo'linmaydigan sonni toping.
A) 2332 B) 4554 C) 6798 D) 1011
20. 19 ga bo'linmaydigan sonni toping.
A) 323 B) 266 C) 456 D) 319
21. 19 ga bo'linadigan sonni toping.
A) 2140 B) 1653 C) 3751 D) 5152

22. (97-9-61) n raqamining qanday qiymatlarida $50 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
A) 3 B) 5 C) 3; 9 D) 1; 9
- Yechish.** Ma'lumki, tub sonlar eng kam tub ko'paytuvchilarga ajraladi. n raqam bo'lganligi uchun, u 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 qiymatlarni qabul qiladi. U holda $50 + n$ ifoda 50, 51, ..., 59 qiymatlarni qabul qiladi. Bu sonlar ichida ikkitasi, ya'ni 53 va 59 lar tub sonlardir. Demak, $n = 3$ yoki $n = 9$ bo'lsa, $50 + n$ ifoda eng kam tub ko'paytuvchilarga ajraydi. **Javob:** 3; 9 (C).
23. n raqamining qanday qiymatlarida $40 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
A) 1; 3; 7 B) 1; 5 C) 3; 9 D) 1; 9
24. n raqamining qanday qiymatlarida $30 + n$ soni eng kam tub ko'paytuvchilarga ajraladi?
A) 1; 3; 7 B) 1; 7 C) 3; 9 D) 1; 9
25. n raqamining nechta qiymatida $25 + n$ tub son bo'ladi?
A) 1 B) 7 C) 3 D) 2
26. n raqamining nechta qiymatida $39 + n$ tub son bo'ladi?
A) 1 B) 2 C) 3 D) 4
27. (97-9-72) n raqamining qanday qiymatlarida $\overline{7851n}$ soni 9 ga qoldiqsiz bo'linadi?
A) 2 B) 4 C) 6 D) 9
- Yechish:** 6-alomatga ko'ra biror son 9 ga qoldiqsiz bo'linishi uchun, uning raqamlari yig'indisi 9 ga bo'linishi zarur va yetarli. $7 + 8 + 5 + 1 + n = 21 + n$ soni 9 ga bo'linishi uchun $n = 6$ bo'lishi kerak. **Javob:** 6 (C).
28. (97-4-12) n raqamining qanday qiymatlarida $\overline{6134n}$ soni 3 ga qoldiqsiz bo'linadi?
A) 1 B) 4 C) 2 D) 1; 4; 7
29. (02-4-28) $246^*013579$ soni 9 ga bo'linishi uchun yulduzchaning o'rniga qanday raqam qo'yilishi kerak?
A) 0 B) 4 C) 7 D) 8
30. n raqamining qanday qiymatlarida $\overline{27n}$ soni 7 ga qoldiqsiz bo'linadi?
A) 3 B) 4 C) 2 D) 7
31. n raqamining qanday qiymatlarida $\overline{6134n}$ besh xonali son 11 ga qoldiqsiz bo'linadi?
A) 6 B) 4 C) 2 D) 7
32. (98-10-2) Quyidagi sonlardan qaysi biri 36 ga qoldiqsiz bo'linmaydi?
A) 2016 B) 3924 C) 1782 D) 8244
- Yechish:** 4 va 9 o'zaro tub sonlar bo'lib, ularning ko'paytmasi 36 ga teng. 10-alomatga ko'ra, 36 ga qoldiqsiz bo'lingan sonlar 4 va 9 ga ham bo'linadi. 1782 sonining oxirgi ikki raqamidan tashkil topgan son 82 bo'lib, u (2-alomatga qarang) 4 ga bo'linmaydi. Shuning uchun 1782 soni 36 ga bo'linmaydi. **Javob:** 1782 (C).

33. (00-7-3) 752 sonining o'ng tomoniga qanday raqam yozilsa, hosil bo'lgan son 36 ga qoldiqsiz bo'linadi?
A) 0 B) 2 C) 6 D) 4
34. (97-4-2) 17827516 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?
A) 3 B) 10 C) 4 D) 5
35. (97-9-62) 41582637 quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?
A) 4 B) 9 C) 5 D) 10
36. (98-2-3) Quyidagi sonlardan qaysi biri 15 ga karrali emas?
A) 6525 B) 3105 C) 4620 D) 6145
37. (98-9-4) Quyidagi sonlardan qaysi biri 12 ga qoldiqsiz bo'linmaydi?
A) 9216 B) 13626 C) 12024 D) 18312
38. 19 ga karrali sonni toping.
A) 297 B) 399 C) 405 D) 810
39. (96-7-2) Berilgan sonlardan qaysilari 15 ga qoldiqsiz bo'linadi?
 $x = 220350, y = 321000, z = 1024145$.
A) faqat x B) faqat z
C) y va z D) x va y

Yechish: 10-alomatga ko'ra biror son 15 ga qoldiqsiz bo'linishi uchun u 3 ga ham 5 ga ham bo'linishi kerak. 4-alomatga ko'ra berilgan sonlarning uch-alasi ham 5 ga bo'linadi. x sonining raqamlari yig'indisi: $2 + 2 + 0 + 3 + 5 + 0 = 12$, y sonining raqamlari yig'indisi: $3 + 2 + 1 + 0 = 6$, z sonining raqamlari yig'indisi: $1 + 0 + 2 + 4 + 1 + 4 + 5 = 17$. 5-alomatga ko'ra z soni 3 ga bo'linmaydi, x va y lar esa 3 ga bo'linadi. Shuning uchun x va y sonlari 15 ga bo'linadi. **Javob:** x va y (D).

40. (98-10-51) Berilgan $p = 10189144, q = 396715256$ va $r = 78901644$ sonlardan qaysilari 8 ga qoldiqsiz bo'linadi?
A) hech qaysisi B) p va q C) p va r D) p
41. (97-3-2) $x = 30112, y = 330000$ va $z = 102588$ sonlardan qaysilari 12 ga qoldiqsiz bo'linadi?
A) faqat y B) faqat x
C) x va z D) y va z
42. (97-7-2) Quyidagi sonlardan qaysilari 18 ga qoldiqsiz bo'linadi?
 $x = 10842, y = 54900, z = 306198$
A) faqat x B) faqat y
C) x va y D) y va z
43. (97-10-2) Quyidagi sonlardan qaysilari 6 ga qoldiqsiz bo'linadi?
 $x = 123386, y = 402108, z = 261000$
A) faqat x B) faqat y C) faqat z D) y va z
44. (01-11-2) Quyidagi ko'paytmalardan qaysi biri 45 ga qoldiqsiz bo'linadi?
A) $42 \cdot 85$ B) $35 \cdot 61$ C) $80 \cdot 123$ D) $36 \cdot 20$

Yechish: 36 soni 9 ga, 20 soni esa 5 ga bo'linadi. 11-alomatga ko'ra, $36 \cdot 20$ ko'paytma $9 \cdot 5 = 45$ ga bo'linadi. **Javob:** $36 \cdot 20$ (D).

45. (02-12-21) Quyidagi ko'paytmalardan qaysi biri 45 ga qoldiqsiz bo'linadi?
A) $42 \cdot 85$ B) $35 \cdot 61$ C) $80 \cdot 123$ D) $243 \cdot 80$
46. Quyidagi ko'paytmalardan qaysi biri 18 ga qoldiqsiz bo'linadi?
A) $42 \cdot 15$ B) $25 \cdot 61$ C) $80 \cdot 23$ D) $43 \cdot 20$
47. Quyidagi ko'paytmalardan qaysi biri 12 ga qoldiqsiz bo'linadi?
A) $11 \cdot 15$ B) $25 \cdot 30$ C) $80 \cdot 21$ D) $43 \cdot 20$
48. Quyidagi ko'paytmalardan qaysi biri 21 ga qoldiqsiz bo'linadi?
A) $11 \cdot 15$ B) $14 \cdot 30$ C) $20 \cdot 27$ D) $31 \cdot 20$
49. Quyidagi ko'paytmalardan qaysi biri 35 ga qoldiqsiz bo'linadi?
A) $18 \cdot 15$ B) $28 \cdot 40$ C) $50 \cdot 27$ D) $49 \cdot 56$

1.1.5 Qoldiqli bo'lish

- Qoldiqli bo'lish formulasi** $a = n \cdot m + r$, $0 \leq r < n$. Bu yerda a – bo'linuvchi, n – bo'luvchi, m – bo'linma, r – qoldiq.
- Agar $A = n \cdot m_1 + r_1$ va $B = n \cdot m_2 + r_2$ sonlarining n ga bo'lgandagi qoldiqlari yig'indisi $r_1 + r_2 = n$ bo'lsa, u holda $A + B$ soni n ga bo'linadi.
- $A = n \cdot m_1$ va $B = n \cdot m_2 + r$ ko'rinisdagi sonlar bo'lsa, u holda $A + B$ va B sonlarining n ga bo'lgandagi qoldiqlari teng bo'ladi.

1. (98-7-3) Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
1) $43 = 9 \cdot 5 - 2$ 2) $43 = 8 \cdot 5 + 3$
3) $43 = 7 \cdot 5 + 8$ 4) $43 = 21 \cdot 2 + 1$
A) 1; 2; 4 B) 2; 3; 4 C) 2; 4 D) 3; 4
- Yechish:** Qoldiqli bo'lish (1 ga qarang) formulasiga ko'ra qoldiq r bo'luvchi n dan kichik va $u \cdot n \cdot m$ ga qo'shiladi. 1) da 2 ayrilgan, 3) da esa qoldiq 8 bo'luvchi 7 dan katta. 2) va 4) lar qoldiqli bo'lishni ifodalaydi. **Javob:** 2; 4 (C).
2. (98-12-3) Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
1) $47 = 4 \cdot 11 + 3$ 2) $47 = 6 \cdot 6 + 11$
3) $47 = 9 \cdot 5 + 2$ 4) $47 = 7 \cdot 7 - 2$
A) 1; 3 B) 1; 2; 3 C) 1; 4 D) 2; 3
3. Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
1) $45 = 2 \cdot 23 - 1$ 2) $45 = 8 \cdot 6 - 3$
3) $45 = 7 \cdot 6 + 3$ 4) $45 = 11 \cdot 4 + 1$
A) 1; 2; 4 B) 2; 3; 4 C) 2; 4 D) 3; 4
4. Qaysi tengliklar qoldiqli bo'lishni ifodalaydi?
1) $25 = 2 \cdot 12 + 1$ 2) $25 = 8 \cdot 3 + 1$
3) $25 = 4 \cdot 6 + 1$ 4) $25 = 3 \cdot 9 - 2$
A) 1; 2; 3 B) 3; 4 C) 2; 4 D) 1; 3

5. 723 ni 6 ga bo'lgandagi qoldiqni toping.
A) 4 B) 3 C) 1 D) 2
Yechish: 723 sonini $723 = 6 \cdot 120 + 3$ ko'rinishda yozish mumkin. 1-qoidaga ko'ra, $r = 3$. Demak, qoldiq 3 ga teng ekan. **Javob:** 3 (B).
6. 500 ni 7 ga bo'lgandagi qoldiqni toping.
A) 6 B) 3 C) 1 D) 2
7. 790 ni 8 ga bo'lgandagi qoldiqni toping.
A) 4 B) 3 C) 6 D) 2
8. 893 ni 9 ga bo'lgandagi qoldiqni toping.
A) 6 B) 3 C) 7 D) 2
9. Quyidagi yig'indilarning qaysi biri 6 ga bo'linadi?
A) $86+628$ B) $75+412$ C) $83+623$ D) $76+214$
Yechish: $86+628$ sonini 6 ga bo'linishini 2-qoida yordamida tekshiramiz: $86 = 6 \cdot 14 + 2$, $628 = 6 \cdot 104 + 4$. Birinchisida qoldiq $r_1 = 2$, ikkinchisida qoldiq $r_2 = 4$. 2-qoidaga ko'ra $r_1 + r_2 = 2 + 4 = 6$ soni 6 ga bo'linadi. **Javob:** $86+628$ (A).
10. Quyidagi yig'indilarning qaysi biri 7 ga bo'linadi?
A) $47+701$ B) $64+218$ C) $76+189$ D) $85+216$
11. Quyidagi yig'indilarning qaysi biri 8 ga bo'linadi?
A) $58+794$ B) $68+215$ C) $76+316$ D) $91+217$
12. Quyidagi yig'indilarning qaysi biri 9 ga bo'linadi?
A) $48+368$ B) $60+543$ C) $84+692$ D) $78+216$
13. $(98-6-7) 3^{20}$ ni 7 ga bo'lgandagi qoldiqni toping.
A) 6 B) 3 C) 1 D) 2
Yechish: 3-qoidaga ko'ra $(mk+r)^n$ sonni k ga bo'lgandagi qoldiq r^n ni k ga bo'lgandagi qoldiqqa teng, ya'ni $(mk+r)^n = m_1k+r^n$ (m_1 - bo'linma) bo'lgani uchun $3^{20} = 9^{10} = (7+2)^{10} = 7n+2^{10} = 7n+32^2 = 7n+(4 \cdot 7+4)^2 = 7n+7n_1+4^2 = 7(n+n_1)+16 = 7(n+n_1+2)+2$ ekanini hosil qilamiz. Demak, qoldiq 2 ga teng ekan. **Javob:** 2 (D).
14. $(98-11-57) 9^{10}$ ni 7 ga bo'lgandagi qoldiqni toping.
A) 1 B) 3 C) 2 D) 6
15. $(99-3-6) 4^{12}$ ni 9 ga bo'lganda qoldiq necha bo'ladi?
A) 1 B) 2 C) 4 D) 7
16. 5^{40} ni 8 ga bo'lganda qoldiq necha bo'ladi?
A) 1 B) 2 C) 4 D) 5
17. 13^9 ni 5 ga bo'lganda qoldiq necha bo'ladi?
A) 3 B) 2 C) 4 D) 1
18. 2002^{2002} sonini 4 ga bo'lganda qoldiq nimaga teng?
A) 0 B) 1 C) 2 D) 4
19. 2011^{2010} sonini 5 ga bo'lgandagi qoldiqni toping.
A) 0 B) 1 C) 2 D) 4
20. $(96-6-2) 243$ ni qandaydir songa bo'lganda bo'linma 15 ga, qoldiq 3 ga teng chiqdi. Bo'luvchini toping.
A) 17 B) 16 C) 18 D) 19
Yechish: 1-qoidaga ko'ra $243 = x \cdot 15 + 3$ tenglik o'rinli. Bu yerdan $15x = 243 - 3 \iff x = 240 : 15 = 16$. **Javob:** 16 (B).
21. $(97-2-2) 215$ ni 19 ga bo'lganda, qoldiq 6 bo'ladi. Bo'linma nechaga teng?
A) 13 B) 12 C) 9 D) 11
22. $(97-8-2) 358$ ni qanday songa bo'lganda bo'linma 17 va qoldiq 1 bo'ladi?
A) 19 B) 21 C) 22 D) 20
23. $(00-7-4) 624$ ni qanday songa bo'lganda bo'linma 41 ga qoldiq esa 9 ga teng bo'ladi?
A) 16 B) 17 C) 13 D) 15
24. $(97-12-2)$ Natural sonni 18 ga bo'lganda, bo'linma 15 ga, qoldiq 3 ga teng bo'ldi. Bo'linuvchini toping.
A) 173 B) 243 C) 253 D) 273
25. $(99-1-3) 7 + 69 + 671 + 6673 + 66675$ sonni 6 ga bo'lgandagi qoldiqni toping.
A) 1 B) 4 C) 3 D) 5
Yechish: 3-qoidaga ko'ra $7 + 69 + 671 + 6673 + 66675 = 7+60+9+660+11+6660+13+66660+15$ son bilan $7 + 9 + 11 + 13 + 15 = 55$ sonning 6 ga bo'lgandagi qoldig'i teng. 55 ni 6 ga bo'lgandagi qoldig'i esa 1 ga teng. **Javob:** 1 (A).
26. $1 + 93 + 995 + 9997 + 9999 + 999901$ sonni 9 ga bo'lgandagi qoldiqni toping.
A) 1 B) 4 C) 8 D) 5
27. $27 + 1029 + 10031 + 100033 + 1000035$ sonni 25 ga bo'lgandagi qoldiqni toping.
A) 1 B) 4 C) 8 D) 5
28. $(99-3-5) 36455478354$ sonni 2, 4, 5, 9, 10 va 25 ga bo'lganda hosil bo'ladigan qoldiqlar yig'indisini toping?
A) 18 B) 16 C) 15 D) 14
29. 36455468 sonni 2, 4, 5, 10 va 25 ga bo'lganda hosil bo'ladigan qoldiqlar yig'indisini toping?
A) 18 B) 29 C) 15 D) 14
30. $(00-2-10) 3$ ga bo'linmaydigan natural sonning kubini 9 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
A) 1 yoki 8 B) 0 yoki 1
C) 0 yoki 8 D) 3 yoki 6
Yechish: Berilgan natural son 3 ga bo'linmaganligi uchun u quyidagi ko'rinishda $n = 3k+r$, $r = 1$ yoki $r = 2$ bo'ladi. Shuning uchun $n^3 = (3k+r)^3 = 27k^3+3 \cdot 9k^2 \cdot r+3 \cdot 3k \cdot r^2+r^3$. 3-qoidaga ko'ra bu son bilan r^3 ning 9 ga bo'lgandagi qoldiqlari teng. $r = 1$ yoki $r = 2$ bo'lgani uchun $r^3 = 1$ yoki $r^3 = 8$ bo'ladi. **Javob:** 1 yoki 8 (A).

31. (99-8-25) a va b natural sonlarini 5 ga bo'lganda, mos ravishda 1 va 3 qoldiq hosil bo'ladi. Bu sonlar kvadratlarining yig'indisini 5 ga bo'lganda, qoldiq nechaga teng bo'ladi?
A) 4 B) 1 C) 2 D) 0
32. Toq natural sonning kubini 4 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
A) 1 B) 2 C) 1 yoki 3 D) 3
33. Natural sonni 3 ga bo'lganda 1 qoldiq hosil bo'ladi. Bu sonning kvadratini 6 ga bo'lganda, qoldiq qanday sonlar bo'lishi mumkin?
A) 1 yoki 4 B) 1 yoki 3 C) 2 D) 5
34. (03-4-5) Nechta ikki xonali son 15 ga qoldiqsiz bo'linadi?
A) 4 B) 5 C) 7 D) 6
- Yechish:** Ma'lumki, ikki xonali son 10 dan boshlanib 99 bilan tugaydi. Demak, $15 = 1 \cdot 15$, $30 = 2 \cdot 15$, $45 = 3 \cdot 15$, ..., $90 = 6 \cdot 15$. 6 ta ikki xonali son 15 ga qoldiqsiz bo'linadi. **Javob:** 6 (D).
35. Nechta ikki xonali son 9 ga qoldiqsiz bo'linadi?
A) 9 B) 10 C) 11 D) 8
36. Nechta uch xonali son 50 ga qoldiqsiz bo'linadi?
A) 19 B) 20 C) 17 D) 18
37. (01-6-2) Barcha uch xonali sonlar ichida 45 ga qoldiqsiz bo'linadiganlari nechta?
A) 19 B) 20 C) 18 D) 21
38. (99-2-2) 821 ga qanday eng kichik musbat sonni qo'shganda, yig'indi 6 ga qoldiqsiz bo'linadi?
A) 4 B) 1 C) 5 D) 7

1.1.6 Oxirgi raqam

- $10^n = \dots 0$, $850^n = \dots 0$
- $1^n = 1$, $271^n = \dots 1$
- $2^{4k+1} = \dots 2$, $2^{4k+2} = \dots 4$,
 $2^{4k+3} = \dots 8$, $2^{4k} = \dots 6$.
- $3^{4k+1} = \dots 3$, $3^{4k+2} = \dots 9$,
 $3^{4k+3} = \dots 7$, $3^{4k} = \dots 1$.
- $4^{2k} = \dots 6$, $4^{2k+1} = \dots 4$.
- $5^n = \dots 5$, $275^n = \dots 5$
- $6^n = \dots 6$, $2756^n = \dots 6$
- $7^{4k+1} = \dots 7$, $7^{4k+2} = \dots 9$,
 $7^{4k+3} = \dots 3$, $7^{4k} = \dots 1$.
- $8^{4k+1} = \dots 8$, $8^{4k+2} = \dots 4$,
 $8^{4k+3} = \dots 2$, $8^{4k} = \dots 6$.
- $9^{2k} = \dots 1$, $9^{2k+1} = \dots 9$.

1. (96-13-11) 2^{100} ning oxirgi raqamini toping?
A) 2 B) 0 C) 4 D) 6

Yechish: 2 soni darajalarining oxirgi raqamlari har 4 – darajadan keyin takrorlanib keladi. 3-ga qarang. 2 ning daraja ko'rsatkichi 4 ga bo'linsa, u son 6 raqami bilan tugaydi. **Javob:** 6 (D).

2. 21^{1964} ning oxirgi raqamini toping.
A) 3 B) 1 C) 7 D) 9
3. 15^{1994} ning oxirgi raqamini toping.
A) 3 B) 1 C) 7 D) 5
4. (96-3-71) 8^{99} ning oxirgi raqamini toping.
A) 0 B) 2 C) 4 D) 6
5. 3^{2010} ning oxirgi raqamini toping.
A) 3 B) 1 C) 7 D) 9
6. 6^{1991} ning oxirgi raqamini toping.
A) 2 B) 6 C) 8 D) 4
7. 7^{1971} ning oxirgi raqamini toping.
A) 7 B) 9 C) 3 D) 1
8. 9^{2009} soni qanday raqam bilan tugaydi?
A) 0 B) 1 C) 3 D) 9
9. 24^{2011} soni qanday raqam bilan tugaydi?
A) 0 B) 6 C) 4 D) 8
10. (97-11-2) Yig'indining oxirgi raqamini toping.

$$15 \cdot 25 \cdot 37 \cdot 43 + 34 \cdot 48 \cdot 77$$

- A) 4 B) 9 C) 0 D) 5

Yechish: Birinchi ko'paytmaning oxirgi raqami $5 \cdot 5 \cdot 7 \cdot 3 = 525$ ko'paytmaning oxirgi raqami bilan bir xil, ya'ni 5 ga teng. Ikkinchi ko'paytmaning oxirgi raqami $4 \cdot 8 \cdot 7 = 224$ esa 4 raqami bilan tugaydi. Yig'indining oxirgi raqami $5 + 4 = 9$, ya'ni 9 bilan tugaydi. **Javob:** 9 (B).

11. (97-1-2) Ayirma qanday raqam bilan tugaydi?

$$17 \cdot 28 \cdot 41 \cdot 35 - 24 \cdot 12 \cdot 87$$

- A) 0 B) 2 C) 4 D) 6

12. (97-6-2) Yig'indining oxirgi raqamini toping.

$$16 \cdot 27 \cdot 38 \cdot 19 + 22 \cdot 43 \cdot 98$$

- A) 8 B) 6 C) 4 D) 2

13. (99-6-7) Ifodaning qiymati qanday raqam bilan tugaydi?

$$11^6 + 14^6 - 13^3 - 8$$

- A) 1 B) 2 C) 3 D) 4

14. (99-6-11) Yig'indi qanday raqam bilan tugaydi?

$$9^{1996} + 9^{1997}$$

- A) 0 B) 1 C) 2 D) 3

15. (01-2-4) $43^{43} - 17^{17}$ ayirmaning oxirgi raqamini toping.
A) 5 B) 2 C) 1 D) 0
16. $41^{14} + 56^{65} + 75^{57}$ ning oxirgi raqamini toping.
A) 5 B) 2 C) 1 D) 0

1.1.7 Butun sonlar

Butun sonlar to'plami $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$ harfi bilan belgilanadi. Quyidagi tengliklar o'rinli.

- $(-1)^{2n} = 1$, $(-1)^{2n-1} = -1$
- $m(-n) = -m \cdot n$, $m : (-n) = -m : n$.
- $-m(-n) = m \cdot n$, $-m : (-n) = m : n$.

1. (97-12-7) Quyidagi ifodalarning qaysi biri 1 ga teng?
A) $(-(-1)^2)^3$ B) $((-1)^3)^3$
C) $(-(-1)^4)^5$ D) $((-1)^3)^4$

Yechish: 1-ga ko'ra -1 ning juft darajasi 1 ga teng. Shuning uchun $((-1)^3)^4 = (-1)^{12} = 1$.
Javob: (D).

2. (96-6-9) Quyidagi ifodalardan qaysi biri -1 ga teng?
A) $((-1)^2)^3$ B) $(-(-1)^2)^3$
C) $((-1)^3)^2$ D) $(-(-1)^3)^3$
3. (97-2-9) Quyidagi ifodalardan qaysi biri 1 ga teng?
A) $(-(-1)^2)^3$ B) $((-1)^3)^5$
C) $-((-1)^5)^4$ D) $((-1)^3)^4$
4. (97-8-9) Quyidagilardan qaysi biri -1 ga teng?
A) $((-1)^3)^2$ B) $(-(-1)^3)^6$
C) $(-(-1)^2)^4$ D) $-((-1)^3)^4$

5. (96-12-6) $8 + 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
A) 99 B) 15 C) 33 D) 27

Yechish: Dastlab ikkinchi bosqichli amallarni, ya'ni ko'paytirish va bo'lishni 2 va 3-qoidalar asosida bajaramiz. $8+6 : (-2) - 2 \cdot (-11) = 8-3+22 = 5+22 = 27$. **Javob:** 27 (D).

6. (96-3-6) $-8 + 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
A) 23 B) 11 C) -11 D) -10
7. (96-11-6) $-8 - 6 : (-2) - 2 \cdot (-11)$ ni hisoblang.
A) 17 B) -55 C) 55 D) 77

8. Hisoblang.

$$13 \cdot 58 - 83 \cdot 42 - 58 \cdot 15 + 42 \cdot 81$$

- A) -100 B) -200 C) 100 D) -10

9. (99-3-2)* Hisoblang.

$$1 - 3 + 5 - 7 + 9 - 11 + \dots + 97 - 99$$

- A) -46 B) -48 C) -50 D) -52

10. (01-1-2)* Hisoblang.

$$4 - 7 + 8 - 11 + 12 - 15 + \dots + 96 - 99$$

- A) -75 B) -80 C) -72 D) -63

11. Hisoblang.

$$199 - 198 + 197 - 196 + \dots + 3 - 2 + 1$$

- A) 75 B) 80 C) 100 D) 99

1.2 Ratsional sonlar. Kasrlar

Bir butunning bir yoki bir nechta teng qismlarini ifodalovchi son kasr deyiladi. Ixtiyoriy $n \in N$, $m \in Z$ uchun $\frac{m}{n}$ ifoda oddiy kasr deyiladi. Bu yerda m - kasrning surati, n - kasrning maxraji deyiladi, o'rtadagi chiziq esa kasr chizig'i deyiladi. Agar $n = 1$ bo'lsa, $\frac{m}{1} = m$ bo'ladi. Demak, har qanday butun sonni maxraji bir bo'lgan oddiy kasr ko'rinishida yozish mumkin. Oddiy kasrga ratsional son deyiladi. Ratsional sonlar to'plami Q harfi bilan belgilanadi. Demak, $Q = \{\frac{m}{n} : n \in N, m \in Z\}$. Agar $\frac{a}{b}$ va $\frac{c}{d}$ kasrlar uchun $ad = bc$ tenglik bajarilsa, berilgan kasrlar teng kasrlar deyiladi. Masalan, $\frac{1}{2} = \frac{3}{6}$, chunki $1 \cdot 6 = 2 \cdot 3$ yoki $\frac{1}{3} = \frac{2}{6}$, chunki $1 \cdot 6 = 2 \cdot 3$. Umumiy holda

$$\frac{a}{b} \quad \text{va} \quad \frac{an}{bn}$$

kasrlar tengdir, chunki $abn = ban$ tenglik o'rinlidir. Demak, kasrning surat va maxrajini bir xil natural songa ko'paytirsak yoki bo'lsak berilgan kasrga teng kasr hosil bo'ladi. Bu xossa kasrlarning asosiy xossasi deyiladi. Agar kasrning surat va maxraji o'zaro tub bo'lsa, bunday kasr qisqarmas kasr deyiladi. Kasrlarning asosiy xossasidan foydalanib, maxrajleri har xil bo'lgan kasrlarning maxrajlarini bir xil qilish mumkin. Kasrlarning maxrajlarini tenglashtirishdagi bunday almashtirishga kasrlarni umumiy maxrajga keltirish deyiladi. Masalan, $\frac{1}{2}$ va $\frac{1}{3}$ kasrlarini olaylik, ularga teng kasrlar $\frac{3}{6}$ va $\frac{2}{6}$ kasrlardir. Bu kasrlar umumiy maxrajga (bir xil maxrajga) keltirildi. Agar kasrning surati maxrajidan kichik ($a < b$) bo'lsa, u to'g'ri kasr deyiladi. Aksincha, surati maxrajidan katta yoki teng ($a \geq b$) bo'lsa, kasr noto'g'ri kasr deyiladi. Masalan, $\frac{1}{2}$ va $\frac{1}{3}$ to'g'ri kasrlardir. $\frac{3}{3}$ va $\frac{8}{5}$ noto'g'ri kasrlardir.

1.2.1 Oddiy kasrlar

- $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ bir xil maxrajli kasrlarni qo'shish
- $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ kasrlarni qo'shish
- $\frac{a+b-c}{m} = \frac{a}{m} + \frac{b}{m} - \frac{c}{m}$ kasrni kasrlarga ajratish

4. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ kasrni kasrga ko'paytirish
5. $a \cdot \frac{b}{c} = \frac{ab}{c}$ butun sonni kasrga ko'paytirish
6. $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$ ishoralar bilan ishlash
7. $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ kasrni kasrga bo'lish
8. $c : \frac{a}{b} = \frac{cb}{a}$ butun sonni kasrga bo'lish
9. $\frac{a}{b} : c = \frac{a}{bc}$ kasrni butun songa bo'lish
10. $\frac{1}{a \cdot b} = \frac{1}{b-a} \left(\frac{1}{a} - \frac{1}{b} \right)$ kasrni yoyish

1. (96-3-12) Hisoblang. $-\frac{1}{2} - \frac{1}{3}$
 A) $\frac{5}{6}$ B) $-\frac{2}{5}$ C) $\frac{2}{5}$ D) $-\frac{5}{6}$

Yechish: Kasrlarni umumiy maxrajga keltiramiz, keyin qo'shamiz.

$$-\frac{1}{2} - \frac{1}{3} = -\frac{3}{6} - \frac{2}{6} = -\left(\frac{3}{6} + \frac{2}{6}\right) = -\frac{3+2}{6} = -\frac{5}{6}.$$

Javob: $-\frac{5}{6}$ (D).

2. (96-3-13) Ayirmani toping. $\frac{1}{2} - \frac{2}{3}$
 A) $\frac{1}{6}$ B) 1 C) $-\frac{1}{3}$ D) $-\frac{1}{6}$

3. (96-11-13) Hisoblang. $-\frac{1}{4} - \frac{1}{5}$
 A) $-\frac{9}{20}$ B) $-\frac{2}{9}$ C) $-\frac{1}{10}$ D) $\frac{9}{20}$

4. (96-11-14) Ayirmani toping. $\frac{1}{4} - \frac{4}{5}$
 A) $-\frac{11}{20}$ B) -1 C) $-\frac{7}{20}$ D) $\frac{13}{20}$

5. (96-12-13) Hisoblang. $-\frac{1}{3} - \frac{1}{4}$
 A) $-\frac{2}{7}$ B) $-\frac{7}{12}$ C) $\frac{1}{6}$ D) $-\frac{1}{6}$

6. (96-3-9) Hisoblang. $-\frac{3}{15} + \frac{1}{5} - \frac{1}{3}$
 A) $-\frac{19}{30}$ B) $-\frac{1}{3}$ C) $\frac{19}{30}$ D) $\frac{1}{3}$

Yechish: Birinchi kasrning surat va maxrajini 3 ga qisqartiramiz. $-\frac{1}{5} + \frac{1}{5} - \frac{1}{3} = -\frac{1}{3}$. **Javob:** $-\frac{1}{3}$ (B).

7. (96-11-9) Hisoblang. $-\frac{3}{15} + \frac{1}{5} + \frac{1}{3}$
 A) $-\frac{1}{3}$ B) $\frac{2}{15}$ C) $\frac{7}{15}$ D) $\frac{1}{3}$

8. (96-12-9) Hisoblang. $\frac{3}{15} - \frac{1}{5} - \frac{1}{3}$
 A) $\frac{1}{3}$ B) $-\frac{3}{10}$ C) $\frac{3}{10}$ D) $-\frac{1}{3}$

9. Hisoblang. $-\frac{5}{15} + \frac{1}{5} + \frac{1}{3}$
 A) $-\frac{1}{3}$ B) $\frac{2}{15}$ C) $\frac{1}{5}$ D) $\frac{1}{3}$

10. Hisoblang. $\frac{5}{15} - \frac{1}{5} - \frac{1}{3}$
 A) $\frac{1}{3}$ B) $-\frac{1}{5}$ C) $\frac{3}{10}$ D) $-\frac{1}{3}$

11. (00-6-16) Hisoblang.

$$\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \frac{1}{11 \cdot 14} + \frac{1}{14 \cdot 17}$$

- A) $\frac{15}{34}$ B) $\frac{5}{17}$ C) $\frac{5}{34}$ D) $\frac{16}{173}$

Yechish: Har bir kasr maxrajidagi ko'paytuvchilar 3 ga farq qiladi. 10-xossadan foydalanib, berilgan ifodaning qiymatini hisoblaymiz:

$$\begin{aligned} & \frac{1}{3} \cdot \left(\frac{1}{2} - \frac{1}{5} \right) + \frac{1}{3} \cdot \left(\frac{1}{5} - \frac{1}{8} \right) + \frac{1}{3} \cdot \left(\frac{1}{8} - \frac{1}{11} \right) + \frac{1}{3} \cdot \left(\frac{1}{11} - \frac{1}{14} \right) + \\ & + \frac{1}{3} \cdot \left(\frac{1}{14} - \frac{1}{17} \right) = \frac{1}{3} \cdot \left(\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \frac{1}{11} - \right. \\ & \left. - \frac{1}{14} + \frac{1}{14} - \frac{1}{17} \right) = \frac{1}{3} \left(\frac{1}{2} - \frac{1}{17} \right) = \frac{5}{34}. \end{aligned}$$

Javob: $\frac{5}{34}$ (C).

12. Hisoblang.

$$\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11} + \frac{1}{11 \cdot 13}$$

- A) $\frac{5}{34}$ B) $\frac{5}{39}$ C) $\frac{5}{33}$ D) $\frac{5}{78}$

13. Hisoblang.

$$\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \frac{1}{15 \cdot 19} + \frac{1}{19 \cdot 23}$$

- A) $\frac{5}{69}$ B) $\frac{7}{96}$ C) $\frac{1}{12}$ D) $\frac{7}{94}$

14. Hisoblang.

$$\frac{1}{3 \cdot 8} + \frac{1}{8 \cdot 13} + \frac{1}{13 \cdot 18} + \frac{1}{18 \cdot 23} + \frac{1}{23 \cdot 28}$$

- A) $\frac{5}{84}$ B) $\frac{7}{96}$ C) $\frac{25}{84}$ D) $\frac{15}{84}$

15. Hisoblang.

$$\frac{1}{3 \cdot 10} + \frac{1}{10 \cdot 17} + \frac{1}{17 \cdot 24} + \frac{1}{24 \cdot 31} + \frac{1}{31 \cdot 38}$$

- A) $\frac{5}{114}$ B) $\frac{7}{104}$ C) $\frac{5}{104}$ D) $\frac{7}{114}$

16. (98-12-62)* Hisoblang.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{999 \cdot 1000}$$

- A) 0,750 B) 1,125 C) 0,998 D) 0,999

Yechish: Misolni yechishda 10-xossadan foydalanamiz. Etibor qilgan bo'lsangiz yig'indida birinchi qo'shiluvchi minus oxirigisi qoladi. 10-xossaga ko'ra

$$(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \dots + (\frac{1}{999} - \frac{1}{1000}) =$$

$$= 1 - \frac{1}{1000} = \frac{999}{1000} = 0,999$$

ni olamiz. **Javob:** 0,999 (D).

17. (00-3-15)* Hisoblang.

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{13 \cdot 15}$$

A) $\frac{11}{15}$ B) $\frac{7}{30}$ C) $\frac{8}{15}$ D) $\frac{7}{15}$

18. (00-8-57)* Hisoblang.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{99 \cdot 100}$$

A) $\frac{1}{9}$ B) $\frac{1}{10}$ C) $\frac{1}{100}$ D) $\frac{99}{100}$

19. (03-7-43)* Hisoblang.

$$\frac{2}{5 \cdot 7} + \frac{2}{7 \cdot 9} + \frac{2}{9 \cdot 11} + \dots + \frac{2}{73 \cdot 75}$$

A) $\frac{16}{75}$ B) $\frac{28}{75}$ C) $\frac{1}{5}$ D) $\frac{14}{75}$

20. (03-8-29)* Hisoblang.

$$1 + \frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} + \frac{1}{12 \cdot 13} + \frac{1}{13 \cdot 14} + \frac{1}{14 \cdot 15}$$

A) $1\frac{3}{80}$ B) 1,16 C) $1\frac{1}{30}$ D) $1\frac{7}{80}$

21. (03-1-9)* Hisoblang.

$$\frac{1}{8} + \frac{1}{24} + \frac{1}{48} + \frac{1}{80}$$

A) 0,1 B) 0,2 C) 0,4 D) 0,6

Yechish: Bu misolni yechishda ham 10-xossadan foydalanamiz: $\frac{1}{8} + \frac{1}{24} + \frac{1}{48} + \frac{1}{80} = \frac{1}{2 \cdot 4} + \frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \frac{1}{8 \cdot 10} = \frac{1}{2} (\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \frac{1}{6} - \frac{1}{8} + \frac{1}{8} - \frac{1}{10}) = \frac{1}{2} (\frac{1}{2} - \frac{1}{10}) = \frac{1}{2} \cdot \frac{4}{10} = \frac{1}{5} = 0,2$.

Javob: 0,2 (B).

22. (00-2-4)* Hisoblang.

$$\frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} + \frac{1}{195}$$

A) $\frac{4}{15}$ B) $\frac{7}{15}$ C) $\frac{17}{45}$ D) $\frac{2}{15}$

23. (99-5-1)* Hisoblang.

$$\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \dots + \frac{1}{182}$$

A) $\frac{11}{42}$ B) $\frac{10}{33}$ C) $\frac{1}{4}$ D) $\frac{12}{35}$

24. (00-9-10)* Hisoblang.

$$\frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \dots + \frac{1}{255}$$

A) $\frac{7}{51}$ B) $\frac{2}{15}$ C) $\frac{2}{25}$ D) $\frac{3}{35}$

25. (96-11-11) Hisoblang.

$$(-\frac{1}{3}) \cdot \frac{2}{7} : \frac{5}{42}$$

A) $\frac{4}{5}$ B) $\frac{5}{441}$ C) $-\frac{4}{5}$ D) $\frac{10}{882}$

Yechish: Ko'paytirish (4-ga qarang) va bo'lish (7-ga qarang) qoidalaridan foydalanamiz:

$$(-\frac{1}{3}) \cdot \frac{2}{7} : \frac{5}{42} = -\frac{1}{3} \cdot \frac{2}{7} \cdot \frac{42}{5} = -\frac{1}{3} \cdot \frac{2 \cdot 3 \cdot 7}{5} = -\frac{4}{5}$$

Javob: $-\frac{4}{5}$ (C).

26. (96-12-10) Hisoblang.

$$-\frac{1}{3} \cdot (-\frac{2}{7}) : \frac{5}{42}$$

A) $\frac{5}{441}$ B) $\frac{4}{5}$ C) $-\frac{5}{441}$ D) $-\frac{4}{5}$

27. Hisoblang.

$$\frac{1}{17} : (-\frac{5}{51}) \cdot \frac{4}{15}$$

A) $\frac{5}{441}$ B) $\frac{4}{25}$ C) $-\frac{5}{24}$ D) $-\frac{4}{25}$

28. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} (-\frac{2}{5}) : \frac{1}{20}$$

A) $\frac{5}{4}$ B) 4 C) -4 D) $-\frac{4}{25}$

29. (98-7-5) Hisoblang. $243 : (9 : 11)$

A) 27 B) $2\frac{5}{11}$ C) $\frac{11}{27}$ D) 297

30. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} - \frac{1}{3} \cdot (-\frac{2}{7}) : (-\frac{5}{42})$$

A) $\frac{7}{10}$ B) $-\frac{3}{10}$ C) $\frac{5}{42}$ D) $\frac{3}{35}$

Yechish: Ko'paytirish (4-ga qarang) va bo'lish (7-ga qarang) qoidalaridan foydalanamiz: $\frac{3}{8} : \frac{3}{4} - \frac{1}{3} \cdot (-\frac{2}{7}) : (-\frac{5}{42}) = \frac{3}{8} \cdot \frac{4}{3} - \frac{1}{3} \cdot \frac{2}{7} \cdot \frac{42}{5} = \frac{1}{2} - \frac{4}{5} = \frac{5-8}{10} = -\frac{3}{10}$. **Javob:** $-\frac{3}{10}$ (B).

31. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} - \frac{1}{3} : \left(-\frac{2}{9}\right)$$

- A) 10 B) -1 C) 2 D) 3,5

32. Hisoblang.

$$\frac{3}{8} : \frac{3}{4} + \frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{5}{42}\right)$$

- A) $1\frac{3}{10}$ B) $-\frac{3}{10}$ C) $\frac{5}{42}$ D) $\frac{3}{35}$

33. Hisoblang.

$$\frac{7}{8} : \frac{7}{4} + \frac{1}{5} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{4}{35}\right)$$

- A) 1 B) 0 C) $\frac{3}{35}$ D) 3

34. Hisoblang.

$$\frac{5}{8} : \frac{5}{4} + \frac{1}{3} \cdot \left(-\frac{2}{7}\right) : \left(-\frac{3}{42}\right)$$

- A) $1\frac{5}{6}$ B) $\frac{9}{10}$ C) $\frac{5}{6}$ D) $\frac{7}{42}$

35. (96-9-58) $\frac{3}{4}$ va $\frac{8}{9}$ sonlari orasida maxraji 36 ga teng bo'lgan nechta kasr son bor?

- A) 1 B) 2 C) 3 D) 4

Yechish: Berilgan kasrlarni 36 maxrajga keltiramiz. $\frac{3}{4} = \frac{27}{36}$ va $\frac{8}{9} = \frac{32}{36}$. Bu sonlar orasida maxraji 36 ga teng bo'lgan $\frac{28}{36}, \frac{29}{36}, \frac{30}{36}, \frac{31}{36}$ sonlar bor. **Javob:** 4 (D).

36. $\frac{3}{5}$ va $\frac{11}{15}$ sonlari orasida maxraji 45 ga teng bo'lgan nechta kasr son bor?

- A) 5 B) 2 C) 3 D) 4

37. $\frac{3}{4}$ va $\frac{9}{10}$ sonlari orasida maxraji 40 ga teng bo'lgan nechta kasr son bor?

- A) 6 B) 5 C) 3 D) 4

38. (96-1-8) $\frac{9}{11}$ va 1 sonlari orasida maxraji 33 ga teng bo'lgan nechta kasr son bor?

- A) 2 B) 1 C) 5 D) 6

39. (96-10-8) $\frac{2}{3}$ dan katta va $\frac{5}{6}$ dan kichik bo'lgan, maxraji 30 ga teng bo'lgan nechta kasr mavjud?

- A) 1 B) 2 C) 4 D) 5

1.2.2 Aralash kasrlar

Bizga $\frac{a}{n}$, $a \geq n$ noto'g'ri kasr berilgan bo'lsin. Qoldiqli bo'lish (1.1.5-dagi 1-ga qarang) qoidasiga ko'ra, shunday m natural va r ($0 \leq r < n$) butun sonlar mavjud bo'lib, bular uchun $a = nm + r$ tenglik o'rinli bo'ladi. U holda $\frac{a}{n} = \frac{nm+r}{n} = \frac{nm}{n} + \frac{r}{n} = m + \frac{r}{n}$. Demak, $\frac{a}{n}$

noto'g'ri kasrni natural son m bilan to'g'ri kasr $\frac{r}{n}$ ning yig'indisi shaklida yozish mumkin. Bunday qoida noto'g'ri kasrdan butun ajratish deyiladi. Masalan,

$$\frac{12}{5} = \frac{10+2}{5} = \frac{10}{5} + \frac{2}{5} = 2 + \frac{2}{5}.$$

Natural son bilan to'g'ri kasrning yig'indisini qo'shish amalisiz yozish qabul qilingan, ya'ni $2 + \frac{2}{5}$ o'rniga $2\frac{2}{5}$ ko'rinishda yoziladi. Bu son aralash kasr deyiladi. Endi musbat kasrlarni taqqoslash haqida qoidalar beramiz.

1. Bir xil maxrajli $\frac{a}{b}$ va $\frac{c}{b}$ kasrlarning surati kattasi katta bo'ladi, ya'ni $a > c \iff \frac{a}{b} > \frac{c}{b}$.

2. Suratleri bir xil $\frac{a}{b}$ va $\frac{a}{c}$ kasrlarning maxraji kattasi kichik bo'ladi, ya'ni $b < c \iff \frac{a}{b} > \frac{a}{c}$.

3. Ikki aralash kasrning butun qismi kattasi katta bo'ladi. Agar ularning butun qismlari teng bo'lsa, u holda kasr qismi kattasi katta bo'ladi.

4. $0 < a < b < c \iff \frac{1}{a} > \frac{1}{b} > \frac{1}{c}$.

5. a soniga teskari son $\frac{1}{a}$. Masalan, $a = 0,8$ bo'lsa, unga teskari son $\frac{1}{0,8} = 1,25$.

6. a soniga qarama-qarshi son $-a$. Masalan, $a = 0,8$ bo'lsa, unga qarama-qarshi son $-0,8$.

1. (97-5-9) Amalni bajaring. $1\frac{3}{5} - 3\frac{1}{5}$
A) $-1\frac{2}{5}$ B) $1\frac{2}{5}$ C) $1\frac{3}{5}$ D) $-1\frac{3}{5}$

Yechish: Aralash kasrlarni ayirishda butun qismidan butun qismi, kasr qismidan kasr qismi ayiriladi. Demak, $1\frac{3}{5} - 3\frac{1}{5} = (1-3) + \left(\frac{3}{5} - \frac{1}{5}\right) = -2 + \frac{2}{5} = -\left(2 - \frac{2}{5}\right) = -\frac{10-2}{5} = -\frac{8}{5} = -1\frac{3}{5}$.

Javob: $-1\frac{3}{5}$. (D).

2. (97-9-9) Amalni bajaring. $3\frac{4}{7} - 5\frac{2}{7}$
A) $-1\frac{5}{7}$ B) $1\frac{4}{7}$ C) $1\frac{5}{7}$ D) $-\frac{4}{7}$

3. Amalni bajaring. $3\frac{4}{9} - 2\frac{5}{18}$
A) $-1\frac{1}{6}$ B) $1\frac{5}{6}$ C) $1\frac{1}{6}$ D) $-1\frac{1}{18}$

4. Amalni bajaring. $2011\frac{1}{9} - 2010\frac{11}{18}$
A) $-0,1$ B) $0,5$ C) $0,6$ D) $-0,5$

5. (96-1-7) $0,6$ ga teskari sonni toping.
A) $-0,6$ B) $1\frac{2}{3}$ C) $0,4$ D) -6

- Yechish:** Teskari son ta'rifiga ko'ra (5-ga qarang), $0,6 = \frac{6}{10}$ ga teskari son $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$ dir. **Javob:** $1\frac{2}{3}$ (B).
6. (96-9-57) 0,8 songa teskari sonni toping.
A) $-0,8$ B) 8 C) $-\frac{5}{4}$ D) 1,25
7. (96-10-7) $-1,5$ soniga teskari sonni toping.
A) 1,5 B) $-0,75$ C) $-\frac{2}{3}$ D) $\frac{2}{3}$
8. (00-2-6) $\frac{11}{25}$ va $4\frac{6}{11}$ sonlariga teskari sonlar ko'paytmasini toping.
A) $\frac{1}{2}$ B) 1 C) $\frac{3}{4}$ D) 2
9. (06-111-15) O'zaro teskari sonlarni aniqlang:
1) $3 - \sqrt{2}$ va $3 + \sqrt{2}$; 2) $\frac{\sqrt{5}}{3}$ va $\frac{3\sqrt{5}}{5}$;
3) $\frac{2\sqrt{3}}{5}$ va $\frac{5\sqrt{3}}{6}$; 4) $\sqrt{2} + 1$ va $\sqrt{2} - 1$;
A) 1; 2; 3 B) 1; 3; 4 C) 1; 3 D) 2; 3; 4
10. 2,5 ga qarama-qarshi sonni toping.
A) 0,4 B) $-0,4$ C) $-2,5$ D) 2
Yechish: Qarama-qarshi son ta'rifiga ko'ra (6-ga qarang), 2,5 ga qarama-qarshi son $-2,5$ dir. **Javob:** $-2,5$ (C).
11. $-1,6$ ga qarama-qarshi sonni toping.
A) 0,125 B) 1,6 C) 0,8 D) 16
12. 0,4 ga qarama-qarshi bo'lgan songa teskari sonni toping.
A) 0,4 B) $-0,4$ C) $-2,5$ D) 2
13. (03-1-56) 0,8 ga teskari bo'lgan songa qarama-qarshi sonni toping.
A) $-0,8$ B) 1,25 C) $-1,25$ D) $-1,2$
14. (98-3-5) Agar $a = \frac{5}{11}$, $b = \frac{3}{7}$, $c = \frac{6}{13}$ bo'lsa, a, b va c ni o'sish tartibida joylashtiring.
A) $a < b < c$ B) $b < a < c$
C) $b < c < a$ D) $c < b < a$
Yechish: Berilgan sonlarga teskari sonlarni taqqoslaymiz. $\frac{1}{a} = \frac{11}{5} = 2\frac{1}{5}$, $\frac{1}{b} = \frac{7}{3} = 2\frac{1}{3}$, $\frac{1}{c} = \frac{13}{6} = 2\frac{1}{6}$. Bu kasrlarning butun qismlari (2 ga) teng. Ularning kasr qismlari 2-qoida bo'yicha taqqoslanadi va biz $\frac{1}{c} < \frac{1}{a} < \frac{1}{b}$ ekanligini olamiz. 4-qoidaga ko'ra, $c > a > b$ ekan. **Javob:** $b < a < c$ (B).
15. (98-10-7) Sonlarni o'sish tartibida joylashtiring?
 $a = \frac{49}{150}$; $b = \frac{102}{300}$; $c = \frac{22}{75}$
A) $a < c < b$ B) $b < c < a$
C) $c < a < b$ D) $b < c < a$
16. (98-10-53) Sonlarni o'sish tartibida joylashtiring?
 $a = \frac{5}{11}$; $b = \frac{6}{13}$; $c = \frac{7}{15}$
A) $a < b < c$ B) $b < a < c$
C) $b < c < a$ D) $c < b < a$
17. (99-4-10) Sonlarni kamayish tartibida joylashtiring?
 $a = \frac{7}{36}$; $b = \frac{11}{34}$; $c = \frac{7}{32}$; $d = \frac{9}{25}$
A) $a > b > c > d$ B) $b > a > d > c$
C) $d > a > b > c$ D) $d > b > c > a$
18. (02-3-3) $\frac{3}{7}$, $\frac{4}{17}$, $\frac{21}{23}$ sonlariga bo'lganda, butun son chiqadigan eng kichik natural sonni toping.
A) 84 B) 36 C) 42 D) 63
Yechish: $\frac{3}{7}$, $\frac{4}{17}$, $\frac{21}{23}$ sonlariga bo'lganda, butun son chiqadigan eng kichik natural sonni n bilan belgilaymiz. Demak, $n : \frac{3}{7} = n \cdot \frac{7}{3}$, $n : \frac{4}{17} = n \cdot \frac{17}{4}$, $n : \frac{21}{23} = n \cdot \frac{23}{21}$ sonlari butun bo'lishi uchun n soni 3, 4 va 21 sonlariga bo'linishi kerak. Minimallik shartidan n ularning EKUKi ekanligi kelib chiqadi. EKUK topish qoidasiga ko'ra, $K(3; 4; 21) = 84$. **Javob:** 84 (A).
19. (03-7-6) $\frac{2}{7}$, $\frac{4}{11}$, $\frac{6}{13}$ sonlariga bo'linganda, bo'linma butun son chiqadigan eng kichik natural sonni toping.
A) 6 B) 12 C) 18 D) 24
20. (00-5-7) $\frac{1}{30}$ va $\frac{1}{45}$ kasr umumiy maxrajining barcha natural bo'luvchilari soni nechta?
A) 11 B) 7 C) 12 D) 13
21. (03-6-5) Agar $\frac{29}{31} + \frac{38}{41} + \frac{47}{51} = a$ bo'lsa,
 $\frac{2}{31} + \frac{3}{41} + \frac{4}{51}$
quyidagilardan qaysi biriga teng?
A) $3 - a$ B) $4 - a$ C) $5 - a$ D) $3 - \frac{a}{2}$
Yechish: Izlanayotgan sonni x bilan belgilaymiz. $\frac{29}{31} + \frac{38}{41} + \frac{47}{51} = a$ va $\frac{2}{31} + \frac{3}{41} + \frac{4}{51} = x$ sonlarni (chap tomoniga chap tomonini, o'ng tomoniga o'ng tomonini) qo'shamiz. Natijada $1 + 1 + 1 = a + x$ ni olamiz. Bu yerdan $x = 3 - a$. **Javob:** $3 - a$ (A).
22. Agar $\frac{3}{5} + \frac{7}{15} + \frac{21}{25} = a$ bo'lsa,
 $\frac{2}{5} + \frac{8}{15} + \frac{4}{25}$
quyidagilardan qaysi biriga teng?
A) $3 - a$ B) $4 - a$ C) $5 - a$ D) $3 - \frac{a}{2}$

23. (03-7-8) Agar $\frac{29}{31} + \frac{38}{41} + \frac{47}{51} + \frac{56}{61} = a$ bo'lsa,
 $\frac{2}{31} + \frac{3}{41} + \frac{4}{51} + \frac{5}{61}$

quyidagilardan qaysi biriga teng?

A) $3 - a$ B) $4 - a$ C) $5 - a$ D) $3 - \frac{a}{2}$

24. Agar $\frac{3}{10} + \frac{5}{20} + \frac{7}{30} + \frac{9}{40} = a$ bo'lsa,
 $\frac{2}{10} + \frac{5}{20} + \frac{8}{30} + \frac{11}{40}$

quyidagilardan qaysi biriga teng?

A) $3 - a$ B) $4 - a$ C) $2 - a$ D) $3 - 2a$

25. (97-10-6) Hisoblang.

$$1 \frac{8}{17} \cdot 3 \frac{2}{5} : \frac{11}{12} \cdot 2 \frac{1}{5} : \frac{4}{9}$$

A) 2,7 B) $24 \frac{3}{17}$ C) 27 D) $29 \frac{1}{9}$

Yechish: Aralash kasrlarni noto'g'ri kasrga keltirib so'ngra bo'lish amalini 1.2.1-dagi 7-qoidaga ko'ra ko'paytmaga almashtiramiz, keyin ularning surat va maxrajlarini qisqartiramiz:

$$1 \frac{8}{17} \cdot 3 \frac{2}{5} : \frac{11}{12} \cdot 2 \frac{1}{5} : \frac{4}{9} = \frac{25}{17} \cdot \frac{17}{5} \cdot \frac{12}{11} \cdot \frac{11}{5} \cdot \frac{9}{4} = 27.$$

Javob: 27 (C).

26. (96-7-6) Hisoblang.

$$5 \frac{5}{7} : 2 \frac{2}{5} \cdot 5 \frac{1}{4} : 1 \frac{1}{6} \cdot \frac{2}{3}$$

A) $7 \frac{1}{7}$ B) $8 \frac{1}{7}$ C) $6 \frac{6}{7}$ D) $5 \frac{5}{7}$

27. (97-3-6) Hisoblang.

$$\frac{3}{4} \cdot 1 \frac{1}{7} : \frac{2}{15} \cdot 12 \frac{1}{4} : 7 \frac{1}{2}$$

A) $10 \frac{1}{2}$ B) 11 C) $9 \frac{1}{4}$ D) $7 \frac{1}{2}$

28. (97-7-6) Hisoblang.

$$\frac{42}{95} \cdot 1 \frac{3}{14} : \frac{3}{5} : 2 \cdot 4 \frac{3}{4}$$

A) $\frac{13}{8}$ B) $1 \frac{3}{8}$ C) $2 \frac{1}{8}$ D) $1 \frac{5}{7}$

29. (98-3-8) Hisoblang.

$$3 \frac{1}{3} \cdot 2 \frac{1}{4} \cdot \left(-\frac{1}{2}\right) \cdot \frac{4}{5}$$

A) 3 B) -3 C) 2,5 D) -2,5

30. (98-10-56) Hisoblang.

$$2 \frac{2}{3} : 1 \frac{1}{7} \cdot 3 \frac{3}{7} \cdot \left(-\frac{1}{4}\right)$$

A) 4 B) 3 C) -2 D) $\frac{2}{7}$

31. (07-108-1) Hisoblang.

$$\frac{15}{56} \cdot 1 \frac{1}{7} : \frac{2}{15} \cdot 24 \frac{1}{2} : 7 \frac{1}{2}$$

A) 11 B) $10 \frac{1}{2}$ C) $7 \frac{1}{2}$ D) 21

32. (96-7-9) Hisoblang.

$$\left(7 \frac{1}{3} - 6 \frac{7}{8}\right) : \frac{3}{4} + 8 \frac{8}{9} \cdot 2 \frac{1}{80}$$

A) $17 \frac{2}{3}$ B) $18 \frac{1}{2}$ C) $21 \frac{1}{2}$ D) $16 \frac{1}{3}$

Yechish: Dastlab qavs ichidagi ayirmaning hisoblaymiz.

$$7 \frac{1}{3} - 6 \frac{7}{8} = 7 - 6 + \frac{1}{3} - \frac{7}{8} = 1 + \frac{8 \cdot 1 - 3 \cdot 7}{24} =$$

$$= 1 - \frac{13}{24} = \frac{24 - 13}{24} = \frac{11}{24}.$$

Aralash kasrlarni noto'g'ri kasrlarga aylantirib, amallarni bajaramiz:

$$\frac{11}{24} \cdot \frac{4}{3} + \frac{80}{9} \cdot \frac{161}{80} = \frac{11}{18} + \frac{161}{9} = \frac{11 + 322}{18} =$$

$$\frac{333}{18} = \frac{37}{2} = 18 \frac{1}{2}. \quad \text{Javob: } 18 \frac{1}{2} \text{ (B).}$$

33. (97-1-3) Hisoblang.

$$1 \frac{1}{4} + \frac{5}{12} : \left(\frac{1}{3} \cdot 2 \frac{1}{2} - \frac{7}{8}\right)$$

A) $11 \frac{1}{4}$ B) $-1 \frac{1}{4}$ C) $9 \frac{1}{4}$ D) $-8 \frac{3}{4}$

34. (97-3-9) Hisoblang.

$$\left(5 \frac{3}{4} - 4 \frac{8}{9}\right) \cdot 2 + 67 \frac{1}{2} : 2 \frac{1}{7}$$

A) $24 \frac{1}{3}$ B) $33 \frac{2}{9}$ C) $36 \frac{1}{9}$ D) $31 \frac{1}{3}$

35. (97-6-3) Hisoblang.

$$\left(\frac{5}{9} - 1 \frac{1}{6} \cdot \frac{1}{2}\right) : \frac{5}{9} + \frac{1}{3}$$

A) $\frac{3}{20}$ B) $\frac{17}{60}$ C) $\frac{7}{30}$ D) $-\frac{7}{60}$

36. (97-7-9) Hisoblang.

$$\left(4 \frac{1}{10} - 3 \frac{4}{15}\right) \cdot \frac{5}{6} + 4 \frac{1}{10} : 1 \frac{1}{5}$$

A) $3 \frac{5}{9}$ B) $4 \frac{1}{9}$ C) $5 \frac{2}{3}$ D) $2 \frac{3}{5}$

37. (97-10-9) Hisoblang.

$$\left(12 \frac{1}{9} - 10 \frac{2}{5}\right) : 38 \frac{1}{2} + 2 \frac{8}{9} \cdot 18$$

A) $24 \frac{1}{15}$ B) $32 \frac{7}{45}$ C) $38 \frac{3}{5}$ D) $52 \frac{2}{45}$

38. (07-107-1) Hisoblang.

$$8 \frac{3}{4} + \frac{5}{12} : \left(\frac{1}{3} \cdot 2 \frac{1}{2} - \frac{7}{8}\right)$$

A) $-1 \frac{1}{4}$ B) $-6 \frac{3}{4}$ C) $-8 \frac{3}{4}$ D) $9 \frac{1}{4}$

1.2.3 O'nli kasrlar

1. Agar kasrning maxrajini 10 va uning darajalari ko'rinishida tasvirlash mumkin bo'lsa, bunday kasrga o'nli kasr deyiladi.

2. $a \cdot 10^{-n} = \frac{a}{10^n}$ ni o'nli kasrga aylantirish uchun a sonida chapdan o'ngga tomon n ta raqamdan so'ng vergul qo'yiladi. a da raqamlar soni n tadan kam bo'lsa, oldiga nol raqamlari qo'yiladi.

Masalan, $\frac{2345}{10^3} = 2,345$, $\frac{23}{10^4} = 0,0023$.

1. (96-3-64) Hisoblang.

$$2,701 \cdot 10^{-4} + 3,205 \cdot 10^{-3}.$$

- A) $5,906 \cdot 10^{-3}$ B) $5,906 \cdot 10^{-4}$
C) $3,4751 \cdot 10^{-3}$ D) $3,0215 \cdot 10^{-4}$

Yechish: Umumiy ko'paytuvchi 10^{-3} ni qavsdan tashqariga chiqaramiz.

$$2,701 \cdot 10^{-4} + 3,205 \cdot 10^{-3} = 10^{-3}(2,701 \cdot 10^{-1} + 3,205) = 10^{-3}(0,2701 + 3,205) = 3,4751 \cdot 10^{-3}.$$

Javob: $3,4751 \cdot 10^{-3}$ (C).

2. (06-111-2) $2,014 : 0,19 + 2,5 \cdot 0,3$ ni hisoblang.
A) 11,35 B) 9,35 C) 12,85 D) 8,85

3. (96-13-4) Ushbu

$$3,104 \cdot 10^{-2} + 1,81 \cdot 10^{-3}$$

yig'indi quyidagi sonlarning qaysi biriga teng?

- A) $3,285 \cdot 10^{-3}$ B) $3,285 \cdot 10^{-2}$
C) $4,914 \cdot 10^{-2}$ D) $4,914 \cdot 10^{-3}$

4. (96-9-4) $1,011 \cdot 10^{-3} + 2,1 \cdot 10^{-4}$ ni hisoblang.

- A) $3,111 \cdot 10^{-3}$ B) $3,111 \cdot 10^{-4}$
C) $3,111 \cdot 10^{-7}$ D) $1,221 \cdot 10^{-3}$

5. (96-12-62) Hisoblang.

$$1,015 \cdot 10^{-4} + 3,14 \cdot 10^{-5}$$

- A) $4,155 \cdot 10^{-4}$ B) $4,155 \cdot 10^{-5}$
C) $4,155 \cdot 10^{-9}$ D) $1,329 \cdot 10^{-4}$

6. (98-12-8) Hisoblang.

$$\frac{3,21 \cdot 5,95 - 4,44}{2,21 \cdot 5,95 + 1,51}$$

- A) 1 B) 2 C) $\frac{1}{2}$ D) $1\frac{1}{2}$

Yechish: Kasr suratida quyidagicha almashtirish bajaramiz.

$$3,21 \cdot 5,95 - 4,44 = (2,21 + 1) \cdot 5,95 - 4,44 = 2,21 \cdot 5,95 + 5,95 - 4,44 = 2,21 \cdot 5,95 + 1,51$$

ekanidan

$$\frac{3,21 \cdot 5,95 - 4,44}{2,21 \cdot 5,95 + 1,51} = \frac{2,21 \cdot 5,95 + 1,51}{2,21 \cdot 5,95 + 1,51} = 1$$

ni hosil qilamiz. **Javob:** 1 (A).

7. (98-7-9) Hisoblang.

$$\frac{2,21 \cdot 5,95 + 1,51}{6,42 \cdot 5,95 - 8,88}$$

- A) 1 B) $\frac{1}{2}$ C) $1\frac{1}{2}$ D) $-\frac{62}{41}$

8. Hisoblang.

$$\frac{6,86 \cdot 4,75 - 4,62}{2,44 + 4,75 \cdot 2,43}$$

- A) 1 B) $\frac{1}{2}$ C) 2 D) -2

9. Hisoblang.

$$\frac{1,27 \cdot 3,45 + 2,25}{4,54 \cdot 3,45 - 2,4}$$

- A) 1 B) $\frac{1}{2}$ C) $1\frac{1}{2}$ D) $-\frac{1}{2}$

10. (01-8-17) Hisoblang.

$$0,21 : (0,05 + \frac{3}{20}) - 2,5 \cdot 1,4$$

- A) -2,45 B) -2,55 C) -2 D) -3,35

Yechish: Dastlab qavs ichini hisoblaymiz:

$$0,05 + \frac{3}{20} = 0,05 + \frac{15}{100} = 0,05 + 0,15 = 0,20.$$

Ikkinchi, keyin birinchi bosqichli amallarni bajarib

$$0,21 : 0,20 - 2,5 \cdot 1,4 = 1,05 - 3,50 = -2,45$$

ni olamiz. **Javob:** -2,45 (A).

11. (01-2-11) Hisoblang.

$$4 - 3,3 : \left(2\frac{1}{7} - 1\frac{1}{5}\right)$$

- A) 3,5 B) 2,5 C) -1,5 D) 0,5

12. (00-5-17) Ifodaning qiymatini toping.

$$-2,4 + 3\frac{1}{3} - (-2,6)$$

- A) -10,6 B) 12,5 C) $3\frac{8}{15}$ D) -12,5

13. (00-5-18) Hisoblang.

$$\left(-\frac{3}{8}\right) \cdot (-32) + 0,5 \cdot (-8)$$

- A) 8 B) 4 C) 6 D) 7

14. (96-1-5) Hisoblang.

$$\left(2,5 - 2\frac{1}{3}\right) \cdot 5,2 : 2\frac{3}{5}$$

- A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) 3 D) $\frac{3}{7}$

15. (96-6-1) Hisoblang.

$$1,75 - \left(-1\frac{2}{7}\right) \cdot 6,5 \cdot \frac{7}{9}$$

A) $-4,75$ B) $2,15$ C) $8,25$ D) $4,75$

16. (96-9-56) Hisoblang.

$$6\frac{3}{8} - \left(2,5 - 2\frac{1}{3}\right) : 1\frac{1}{3}$$

A) $5\frac{2}{3}$ B) $6\frac{1}{4}$ C) $4\frac{1}{2}$ D) $2\frac{1}{3}$

Yechish: Dastlab qavs ichini hisoblaymiz:

$$2,5 - 2\frac{1}{3} = 2\frac{1}{2} - 2\frac{1}{3} = (2 - 2) + \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

$1\frac{1}{3}$ aralash kasrni noto'g'ri kasrga aylantirib, amallarni bajaramiz

$$6\frac{3}{8} - \frac{1}{6} : \frac{4}{3} = 6\frac{3}{8} - \frac{1}{6} \cdot \frac{3}{4} = 6 + \frac{3}{8} - \frac{1}{8} = 6\frac{1}{4}$$

ni olamiz. **Javob:** $6\frac{1}{4}$ (B).

17. (97-1-7) Hisoblang.

$$\left(\frac{1}{6} - 1\frac{1}{15} + \frac{1}{10}\right) : 0,6 + 0,4$$

A) $1\frac{11}{15}$ B) $0,88$ C) $-1\frac{1}{3}$ D) $-\frac{14}{15}$

18. (97-2-1) Hisoblang.

$$-1\frac{3}{4} \cdot 6,5 \cdot \left(-\frac{4}{7}\right) - 3,75$$

A) $-2,75$ B) $-10,25$ C) $2,75$ D) $10,25$

19. (97-8-1) Hisoblang.

$$5,8 - \frac{3}{7} \cdot 2,2 \cdot \left(-2\frac{1}{3}\right)$$

A) $3,6$ B) -8 C) 8 D) $-3,6$

20. (97-11-7) Hisoblang.

$$0,2 + 1,8 \cdot \left(\frac{4}{9} - 1\frac{1}{2} + \frac{1}{6}\right)$$

A) $-1,4$ B) $1,8$ C) $0,04$ D) $-0,36$

21. (98-8-5) Hisoblang.

$$\frac{3}{16} + \frac{1}{16} \cdot (0,312 : 0,3 - 3,15 \cdot 1,6)$$

A) $\frac{1}{4}$ B) $\frac{3}{16}$ C) $-\frac{1}{16}$ D) $-\frac{1}{8}$

22. (98-1-3) Hisoblang.

$$19,9 \cdot 18 - 19,9 \cdot 16 + 30,1 \cdot 18 - 30,1 \cdot 16$$

A) 98 B) 100 C) 10 D) 110

Yechish: Birinchi va uchinchi qo'shiluvchilardan umumiy ko'paytuvchi 18 ni, ikkinchi va to'rtinchi qo'shiluvchilardan 16 ni qavsdan tashqariga chiqaramiz. Natijada $18(19,9 + 30,1) - 16(19,9 + 30,1) = 18 \cdot 50 - 16 \cdot 50 = 50(18 - 16) = 50 \cdot 2 = 100$.

Javob: 100 (B).

23. (99-6-2) Hisoblang.

$$13,5 \cdot 5,8 - 8,3 \cdot 4,2 - 5,8 \cdot 8,3 + 4,2 \cdot 13,5$$

A) 42 B) 52 C) 50 D) 48

24. (00-2-1) Ifodaning qiymatini toping.

$$12,7 \cdot 64 + 173 \cdot 3,6 + 12,7 \cdot 36 + 17,3 \cdot 64$$

A) 3000 B) 1800 C) 2000 D) 3600

25. (98-8-3) Hisoblang.

$$109 \cdot 9,17 - 5,37 \cdot 72 - 37 \cdot 9,17 + 1,2 \cdot 72$$

A) 360 B) 350 C) 290 D) 380

26. (99-8-7) Ifodaning qiymatini toping.

$$79,9 - 79,8 + 79,7 - 79,6 + 79,5 - 79,4 + \dots +$$

$$+ 60,3 - 60,2 + 60,1 - 60$$

A) 100 B) 20 C) 10 D) $18,8$

27. (98-8-7) Hisoblang.

$$\left(\frac{5}{6} \cdot 5 - 5\right) : \frac{2}{3} - 0,5^2$$

A) 1 B) -1 C) $0,5$ D) $-1,5$

Yechish: Dastlab qavs ichidagi amallarni bajaramiz.

$$\frac{5}{6} \cdot 5 - 5 = \frac{25}{6} - 5 = \frac{25 - 30}{6} = -\frac{5}{6}.$$

Agar $0,5 = \frac{1}{2}$ ekanligini hisobga olsak, $0,5^2 = \frac{1}{4}$ ekanligini olamiz. Natijada,

$$-\frac{5}{6} : \frac{2}{3} - \frac{1}{4} = -\frac{5}{6} \cdot \frac{3}{2} - \frac{1}{4} = -\frac{5}{4} - \frac{1}{4} = -\frac{6}{4} = -1,5.$$

Javob: $-1,5$ (D).

28. (98-1-7) Hisoblang.

$$\left(\frac{2}{3} : 3 - 1\right) \cdot 1,5^2 - 0,25$$

A) $1,5$ B) -2 C) -5 D) $-0,2$

29. (98-4-1) Hisoblang.

$$(1,6^2 - 2,2 \cdot \frac{3}{11}) : 1,4$$

A) $1,4$ B) $1,2$ C) $1,5$ D) $1,6$

30. (99-4-4) Hisoblang.

$$2,8 \cdot \left(2\frac{1}{3} : 2,8 - 1\right) + 2\frac{4}{5}$$

A) $5,6$ B) $2\frac{2}{3}$ C) $2\frac{1}{3}$ D) $2,8$

31. (00-6-2) Hisoblang.

$$(0,2 \cdot 0,1 - 0,1) : 0,25 + 0,75$$

- A) 1,07 B) -2,45 C) 3,95 D) 0,43

32. (00-6-3) Ifodaning qiymatini toping.

$$\left(1\frac{2}{3} \cdot 2,2 + 1\right) : 2\frac{1}{5} - \frac{5}{11}$$

- A) 1 B) 1,6 C) $2\frac{1}{3}$ D) $1\frac{2}{3}$

33. (07-109-1) Hisoblang.

$$\left(3,5 - 3\frac{1}{3}\right) \cdot 10,4 : 5\frac{1}{5}$$

- A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{3}{7}$ D) $\frac{1}{12}$

34. (96-1-3) Ifodaning qiymatini toping.

$$\frac{6,8 \cdot 0,04 \cdot 1,65}{3,3 \cdot 5,1 \cdot 0,16}$$

- A) 6 B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{6}$

Yechish: Kasr suratida verguldan keyingi raqamlarni sanaymiz. Ular 5 ta. Endi kasr maxrajida verguldan keyingi raqamlarni sanaymiz. Ular 4 ta. Kasr maxrajidagi 5,1 ni, unga teng bo'lgan 5,10 bilan almashtiramiz, natijada kasr maxraji va suratida verguldan keyingi raqamlar soni tenglashadi. Kasr surati va maxrajini 10^5 ga ko'paytiramiz

$$\frac{6,8 \cdot 0,04 \cdot 1,65 \cdot 10^5}{3,3 \cdot 5,10 \cdot 0,16 \cdot 10^5} = \frac{68 \cdot 4 \cdot 165}{33 \cdot 510 \cdot 16}$$

Endi kasrni qisqatiramiz

$$\frac{68 \cdot 4 \cdot 165}{33 \cdot 510 \cdot 16} = \frac{17 \cdot 4 \cdot 4 \cdot 33 \cdot 5}{33 \cdot 17 \cdot 30 \cdot 16} = \frac{5}{30} = \frac{1}{6}$$

Javob: $\frac{1}{6}$ (D).

35. (96-9-54) Ifodaning qiymatini toping.

$$\frac{0,7 \cdot 1,8 \cdot 2,6}{7,2 \cdot 7,8 \cdot 1,4}$$

- A) $\frac{1}{24}$ B) $\frac{2}{5}$ C) 0,04 D) $\frac{1}{12}$

36. (96-10-3) Ifodaning qiymati nechaga teng?

$$\frac{0,15 \cdot 1,6 \cdot 4,6}{9,2 \cdot 0,03 \cdot 6,4}$$

- A) $\frac{5}{8}$ B) $\frac{2}{5}$ C) 2 D) 0,2

37. (99-4-3) Ifodaning qiymatini toping.

$$\frac{3,2 \cdot 0,027 \cdot 0,005}{0,09 \cdot 0,0025 \cdot 0,64}$$

- A) 3 B) 0,3 C) 30 D) 2

38. (03-5-1) Hisoblang.

$$\frac{0,13}{0,00013} + \frac{0,02}{0,0005} - \frac{0,7}{0,0014}$$

- A) 540 B) 580 C) 620 D) 1400

39. (03-10-3) Ifodaning qiymatini toping.

$$\frac{0,07}{0,21} + \frac{0,4}{0,06} + \frac{0,9}{0,05}$$

- A) 25 B) 20 C) 15 D) 30

40. (01-6-1) Hisoblang.

$$\frac{400 - 21,5 \cdot 18,5}{1,5 \cdot 2\frac{1}{5} + 2,8 \cdot 1\frac{1}{2}}$$

- A) $\frac{2}{7}$ B) $\frac{3}{5}$ C) $\frac{3}{7}$ D) $\frac{3}{10}$

Yechish: Kasr suratini hisoblaymiz, $400 - 21,5 \cdot 18,5 = 400 - 397,75 = 2,25$. Endi kasr maxrajini $1,5 \cdot 2,2 + 2,8 \cdot 1,5 = 1,5(2,2 + 2,8) = 1,5 \cdot 5 = 7,5$. Natijada, $2,25 : 7,5 = 0,3 = \frac{3}{10}$. **Javob:** $\frac{3}{10}$ (D).

41. (96-10-5) Hisoblang.

$$\left(5\frac{1}{3} - 3,2\right) : 2\frac{2}{3} + 1\frac{2}{5}$$

- A) $2\frac{1}{2}$ B) 2,2 C) 3,2 D) 2

42. (98-6-4) Hisoblang.

$$\frac{[(1,2 : 36) + 0,3] \cdot 9}{0,2}$$

- A) 148,5 B) 1,5 C) 150 D) 15

43. (01-5-1) Hisoblang.

$$\frac{(6\frac{3}{5} - 3\frac{3}{14}) \cdot 5\frac{5}{6}}{(21 - 1,25) : 2,5}$$

- A) 2,5 B) 3 C) -2,5 D) 4

44. (99-2-1) Hisoblang.

$$\frac{7,4 + \frac{13}{17} \cdot 0,15 \cdot 1\frac{4}{13} \cdot 6\frac{2}{3}}{0,2 \cdot 5 - 0,16}$$

- A) 10 B) 8 C) 12 D) 6

45. (00-1-1) Hisoblang.

$$\frac{\frac{5}{11} \cdot 0,006 \cdot 2\frac{1}{5} + 1\frac{1}{8} \cdot 0,004 \cdot \frac{8}{9}}{0,5 \cdot 0,0009 + 0,0001 \cdot 0,5}$$

- A) 10 B) 0,4 C) 20 D) 2

46. (02-4-1) Hisoblang.

$$\left(2\frac{3}{4} - 0,25\right) \cdot 0,8 - 1\frac{2}{3} \cdot 1,8$$

- A) 1 B) 1,5 C) -1 D) -1,5

47. (02-6-1) Hisoblang.

$$32 \cdot 0,99 \cdot 25 \cdot 1,25 + 411 + 57 \cdot 5 \cdot 0,4 \cdot 25 \cdot \frac{4}{19}$$

- A) 2001 B) 2000 C) 1999 D) 2002

1.2.4 Cheksiz davriy o'nli kasrlar

Cheksiz o'nli kasrlarning kasr qismidagi bir yoki bir necha raqamlari bir xil tartibda ketma-ket takrorlansa, bunday kasrlar *cheksiz davriy o'nli kasrlar* deb, takrorlanadigan raqamlar guruhiga shu kasrning *davri* deb ataladi. Davr qavsga olib yoziladi. Masalan, $0,5555\dots = 0,(5)$; $2,1232323\dots = 2,1(23)$. Cheksiz davriy o'nli kasr ko'rinishida tasvirlash mumkin bo'lgan sonlar *rational sonlar* deyiladi.

1. Agar qisqarmas kasrning maxrajini tub ko'paytuvchilarga ajratganda 2 va 5 sonlaridan boshqa tub ko'paytuvchilar uchrasa, bunday kasr cheksiz davriy o'nli kasr bo'ladi.

Misollar: $\frac{3}{48} = \frac{1}{16} = \frac{1}{2^4}$ - chekli o'nli kasr.

$\frac{5}{12} = \frac{5}{2^2 \cdot 3}$ - cheksiz davriy o'nli kasr.

2. Davriy kasrlar ikki xil bo'ladi. a) agar davr verguldan keyin darhol boshlansa, bunday davriy kasr sof davriy kasr deyiladi. Misol: $0,333\dots = 0,(3)$; $2,161616\dots = 2,(16)$. b) agar davr verguldan keyin darhol boshlanmasa, bunday davriy kasr aralash davriy kasr deyiladi. Misol: $0,377\dots = 0,3(7)$; $2,81212\dots = 2,8(12)$.

3. Sof davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo'lsa shuncha 9 dan, surati esa davrning o'zidan iborat.

Masalan, $0,(3) = \frac{3}{9} = \frac{1}{3}$, $2,(16) = 2\frac{16}{99}$.

4. Aralash davriy kasr shunday oddiy kasrga tengki, uning maxraji davrda nechta raqam bo'lsa shuncha 9 va verguldan keyin davrgacha nechta raqam bo'lsa shuncha 0 dan tuzilgan sondan, surati esa verguldan keyingi ikkinchi davrgacha bo'lgan raqamlardan tuzilgan sondan birinchi davrgacha bo'lgan raqamlardan tuzilgan son ayirmasidan iborat.

Masalan, $0,3(7) = \frac{37-3}{90} = \frac{34}{90} = \frac{17}{45}$,

$2,8(12) = 2\frac{812-8}{990} = 2\frac{804}{990} = 2\frac{134}{165}$.

1. (96-1-12) Quyidagi sonlardan qaysi biri 0,(2) ga teng?

A) $\frac{1}{9}$ B) $\frac{2}{9}$ C) $\frac{2}{3}$ D) 0,22

Yechish: 0,(2) sof davriy kasr, 3-qoidaga ko'ra $0,(2) = \frac{2}{9}$ dir. **Javob:** $\frac{2}{9}$ (B).

2. (96-9-62) Quyidagi sonlardan qaysi biri 0,(5) ga teng?

A) $\frac{1}{2}$ B) $\frac{5}{9}$ C) 0,555 D) $\frac{1}{5}$

3. (97-9-71) 8,(5) ni oddiy kasrga aylantiring.

A) $8\frac{4}{9}$ B) $8\frac{5}{8}$ C) $8\frac{7}{8}$ D) $8\frac{5}{9}$

4. 0,(18) ni oddiy kasr shaklida yozing.

A) $\frac{2}{11}$ B) $\frac{18}{90}$ C) $\frac{8}{99}$ D) $\frac{18}{900}$

5. (99-4-27) 0,5(6) soni quyidagilardan qaysi biriga teng?

A) $\frac{56}{99}$ B) $\frac{1}{18}$ C) $\frac{17}{30}$ D) $\frac{28}{45}$

6. (01-6-22) 0,2(3) ni oddiy kasrga aylantiring.

A) $\frac{7}{30}$ B) $\frac{4}{15}$ C) $\frac{3}{8}$ D) $\frac{2}{7}$

7. (03-8-27) 0,2(18) ni oddiy kasr shaklida yozing.

A) $\frac{12}{55}$ B) $\frac{13}{55}$ C) $\frac{28}{99}$ D) $\frac{218}{900}$

8. (02-11-2) $3\frac{127}{495}$ ni cheksiz davriy o'nli kasr ko'rinishida yozing.

A) 3,(127) B) 3,(254) C) 3,2(54) D) 3,2(56)

9. (99-7-6) Hisoblang. $0,(5) + 0,(1)$

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) 1,5 D) $\frac{1}{4}$

Yechish: 0,(5) va 0,(1) sof davriy kasrlardir. Ularni 3-qoidaga ko'ra oddiy kasrlarga aylantiramiz. $0,(5) = \frac{5}{9}$; $0,(1) = \frac{1}{9}$. Endi ularni qo'shamiz. $\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$. **Javob:** $\frac{2}{3}$ (A).

10. (98-5-4) $0,(8) + 0,(7)$ ni hisoblang.

A) 0,(15) B) 1,(6) C) 1,(5) D) 1,(15)

11. (01-3-39) $0,(8) + 0,(3)$ ni hisoblang.

A) $1\frac{1}{9}$ B) $1\frac{2}{9}$ C) 1,(11) D) 1,(1)

12. (02-5-2) $0,5(6) + 0,(8)$ ni hisoblang.

A) 0,6(4) B) 1,3(6) C) 1,4(5) D) 1,36

13. $0,(5) + 0,(6) + 0,(7)$ ni hisoblang.

A) 1,(8) B) 1,3(6) C) 2 D) 1,(18)

14. $3,(7) + 6,(2)$ ni hisoblang.

A) 9,(9) B) $\frac{80}{9}$ C) 10 D) $\frac{89}{9}$

15. (98-11-3) Hisoblang.

$$\frac{0,8(3) - 0,4(6)}{0,(3)}$$

A) 1,1 B) $1\frac{1}{3}$ C) 3 D) 0,3

16. (96-3-68) Ushbu

$$a = 0,5(3), \quad b = \frac{47}{90}, \quad c = 1 - 0,48(1)$$

sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?

A) $a < b < c$ B) $b < c < a$
C) $c < b < a$ D) $b < a < c$

Yechish: Berilgan sonlarni oddiy kasrlarga aylantiramiz. $a = \frac{53-5}{90} = \frac{48}{90} = \frac{480}{900}$, $b = \frac{470}{900}$,

$c = 1 - \frac{481 - 48}{900} = 1 - \frac{433}{900} = \frac{467}{900}$. Bir xil maxrajli kasrlarni taqqoslash qoidasiga ko'ra $c < b < a$ munosabat o'rinni. **Javob:** $c < b < a$ (C).

17. (96-12-66) a, b va c sonlar uchun quyidagi munosabatlardan qaysi biri o'rinni?

$$a = 0,6(4), \quad b = \frac{59}{90}, \quad c = 1 - 0,36(9)$$

- A) $a < c < b$ B) $a < b < c$
C) $b < a < c$ D) $c < a < b$

18. (98-1-10) Sonlarni kamayish tartibida joylashtiring.

$$a = 2, (4), \quad b = 2,5 - \frac{1}{8}, \quad c = 1,2 : 0,5$$

- A) $a > b > c$ B) $a > c > b$
C) $b > a > c$ D) $c > a > b$

19. Sonlarni o'sish tartibida joylashtiring.

$$a = 0,8(87), \quad b = \frac{87}{99}, \quad c = 1 - 0, (13)$$

- A) $a < c < b$ B) $a < b < c$
C) $b < a < c$ D) $c < b < a$

20. Sonlarni o'sish tartibida joylashtiring.

$$a = 0, (6) + 0, (7), \quad b = 1, (3), \quad c = 2 - \frac{7}{9}$$

- A) $a < c < b$ B) $a < b < c$
C) $b < a < c$ D) $c < b < a$

21. Sonlarni o'sish tartibida joylashtiring.

$$a = -0,1(3), \quad b = -0,13(5), \quad c = -0,103(5)$$

- A) $a < c < b$ B) $a < b < c$
C) $b < a < c$ D) $c < b < a$

22. Sonlarni o'sish tartibida joylashtiring.

$$a = \frac{10}{7}, \quad b = \frac{100}{77}, \quad c = \frac{1000}{777}$$

- A) $a < c < b$ B) $a < b < c$
C) $b < a < c$ D) $c < b < a$

23. (96-9-3) Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilarini chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi?

$$1) \frac{7}{32}; \quad 2) \frac{11}{160}; \quad 3) \frac{5}{48}; \quad 4) \frac{5}{14};$$

- A) 2;3 B) 3;4 C) 4;1 D) 1;2

Yechish: Berilgan kasrlarning maxrajlarini tub ko'paytuvchilarga ajratamiz. $32 = 2^5$; $160 = 2^5 \cdot 5$; $48 = 2^4 \cdot 3$ va $14 = 2 \cdot 7$. 48 va 14 sonlarining tub ko'paytuvchilari ichida 2 va 5 dan farqli 3 va 7 tub sonlari qatnashyapti. 1-qoidaga ko'ra $\frac{5}{48}$ va $\frac{5}{14}$ ni chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi. **Javob:** 3;4 (B).

24. (96-13-3) Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilarini chekli o'nli kasr ko'rinishiga keltirib bo'lmaydi?

$$1) \frac{14}{625}; \quad 2) \frac{3}{64}; \quad 3) \frac{32}{75}; \quad 4) \frac{11}{375};$$

- A) 1;2 B) 2;3 C) 3;4 D) 4;1

25. Quyidagi oddiy kasr ko'rinishida berilgan sonlardan qaysilari chekli o'nli kasr bo'ladi?

$$1) \frac{3}{48}; \quad 2) \frac{7}{120}; \quad 3) \frac{7}{112}; \quad 4) \frac{3}{96};$$

- A) 1;2 B) 2;3 C) 1;3;4 D) 1;2;4

26. Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$$m = 2\frac{5}{17}, \quad n = \frac{7}{32}, \quad p = \frac{2}{333}$$

- A) m, n B) faqat m C) n D) m, p

27. (98-12-5) Davri 0 yoki 9 dan farqli bo'lgan cheksiz davriy o'nli kasrlarni ko'rsating:

$$m = 2,32666\dots, \quad n = \frac{7}{99}, \quad p = \frac{5}{16},$$

$$q = 7,145222\dots, \quad l = 3,222$$

- A) m, n B) m, q C) m, n, q D) m, n, p

28. (01-11-1) Hisoblang.

$$\left(6\frac{1}{3} \cdot 0, (5) + 0, (4) : \frac{3}{19}\right) \cdot 4\frac{5}{19}$$

- A) 28 B) 27,5 C) 27 D) 26,5

Yechish: 0, (5) va 0, (4) davriy kasrlarni, $\frac{5}{9}$ va $\frac{4}{9}$ shaklda yozib, qavs ichidagi amallarni bajaramiz:

$$\frac{19}{3} \cdot \frac{5}{9} + \frac{4}{9} \cdot \frac{19}{3} = \frac{19}{3} \left(\frac{5}{9} + \frac{4}{9}\right) = \frac{19}{3} \cdot 1 = \frac{19}{3}.$$

Endi ko'paytirish amalini bajaramiz

$$\frac{19}{3} \cdot 4\frac{5}{19} = \frac{19}{3} \cdot \frac{81}{19} = 27. \quad \text{Javob: } 27 \text{ (C).}$$

29. (99-10-1) Hisoblang.

$$\frac{0,48 \cdot 0,75 + 0,52 : 1\frac{1}{3}}{(0, (3) + 0, (6)) : 0,012}$$

- A) 1 B) 0,08 C) 0,008 D) 0,009

30. (02-12-20) Hisoblang.

$$\left(\frac{81 \cdot 3}{567} + \frac{22}{77}\right) \cdot 24,5 - \frac{2}{3} : 0, (3)$$

- A) 16,5 B) 14,5 C) 15,5 D) 16,5

31. (03-6-2) Hisoblang.

$$\frac{0, (4) + 0, (41) + 0, (42) + 0, (43)}{0, (5) + 0, (51) + 0, (52) + 0, (53)}$$

- A) $\frac{170}{211}$ B) $\frac{83}{103}$ C) $\frac{63}{107}$ D) $\frac{65}{106}$

32. (03-7-4) Hisoblang.

$$\frac{0, (40) + 0, (41) + 0, (42) + 0, (43)}{0, (50) + 0, (51) + 0, (52) + 0, (53)}$$

- A) $\frac{170}{211}$ B) $\frac{83}{103}$ C) $\frac{63}{107}$ D) $\frac{65}{106}$

33. Hisoblang.

$$\left(2011\frac{1}{5} - 2010\frac{1}{6}\right) \cdot 1\frac{29}{31}$$

- A) $2\frac{28}{29}$ B) $2\frac{29}{31}$ C) $3\frac{1}{29}$ D) 2

34. Hisoblang.

$$\frac{0,202 - 0,004}{\frac{8}{9} \cdot 81 \cdot 0,125}$$

- A) 0,99 B) 0,099 C) 0,022 D) 0,0099

1.2.5 Protsent va proporsiya

Turmushda ko'p qo'llaniladigan kasrlar maxsus nomlarga ega. Masalan, $\frac{1}{2}$ va $\frac{1}{4}$ kasrlari yarim va chorak

deb yuritiladi. $\frac{1}{100}$ kasr yoki yuzdan bir ulush tushunchasi keng qo'llaniladi. Bu kasrga maxsus nom berilgan u *protsent yoki foiz* deb ataladi. Protsent deb biror sonning yuzdan bir ulushiga aytiladi. Protsent odatda % belgi bilan ifodalanadi. $n\%$ yozuvi $\frac{n}{100}$ ni bildiradi. n protsent $\frac{n}{100}$ oddiy kasrning boshqacha ko'rinishidir. a sonining $n\%$ ni topish uchun a ni $\frac{n}{100}$ kasrga ko'paytirish kerak. Masalan, a sonining 10% i $a \cdot \frac{10}{100} = 0,1a$ ga, a ning 25% i esa $0,25a$ ga teng.

$\frac{a}{b}$ yoki $a : b$ ga a ning b ga nisbati deyiladi. Ikkita nisbatning tengligi *proporsiya* deyiladi. Proporsiya umumiy holda

$$a : b = c : d \quad \text{yoki} \quad \frac{a}{b} = \frac{c}{d}$$

ko'rinishda yoziladi. a va d lar proporsiyaning chetki hadlari, b va c lar proporsiyaning o'rta hadlari deyiladi. Proporsiya quyidagi xossalarga ega.

1. $a : b = c : d \iff ad = bc.$
2. $a : b = c : d \iff na : b = nc : d.$
3. $a : b = c : d \iff a : c = b : d.$
4. $a : b = c : d \iff d : b = c : a.$

1. Maktab kutubxonasida 40000 ta kitob bor. Ularning 2% i matematikaga oid kitoblardir. Kutubxonada matematikaga oid nechta kitob bor?

- A) 400 B) 200 C) 800 D) 1000

Yechish: Sonning protsentini topish formulasiga asosan $\frac{40000 \cdot 2}{100} = 800$. **Javob:** 800 (C).

2. Maktab bog'ida 9652 tup mevali daraxt bo'lib, ularning 75% olma daraxti. Maktab bog'ida necha tup olma daraxti bor?

- A) 7237 B) 7239 C) 7300 D) 7229

3. Matematika fakultetida 80 ta a'lochi talaba bo'lib, bu fakultetdagi barcha talabalarning 20% ni tashkil qiladi. Fakultetdagi jami talabalar sonini toping.

- A) 400 B) 320 C) 500 D) 360

4. Viloyat olimpiadasida 80 ta o'quvchi qatnashdi. Ulardan 16 tasi barcha test savollarini yechdi. Test savollarining barchasini to'g'ri yechgan o'quvchilar olimpiada ishtirokchilarining necha foizini tashkil qiladi.

- A) 40 B) 20 C) 80 D) 10

5. SamDU matematika yo'nalishiga 70 ta talaba qabul qilinadi. Bu yo'nalishga 20 ta harbiy tavsiyanomali abituriyent hujjat topshirgan. Harbiy tavsiyanomali abituriyentlar uchun yo'nalish bo'yicha ajratilgan qabulning 20% miqdorida qo'shimcha joy ajratilgan. Ko'pi bilan nechta harbiy tavsiyanomali abituriyent talabalikka tavsiya qilinmay qolishi mumkin.

- A) 14 B) 6 C) 4 D) 0

6. Bir kilogramm asal 10000 so'm turadi. Moliyaviy krizis tufayli uning narxi 12% ga arzonlashdi. Endi bir kilogramm asal qancha turadi.

- A) 9100 B) 9200 C) 8800 D) 8200

7. Quyidagi sonlar guruhlaridan 1) 7; 8; 14; 16; 2) 1; 2; 3; 4; 3) 3; 4; 15; 20; qaysilari proporsiya tashkil qiladi?

- A) 1; 2 B) 1; 3 C) hammasi D) 2; 3

Yechish: 1) da $7 \cdot 16 = 8 \cdot 14$ tenglik o'rinli. Demak, 7,8,14,16; sonlar guruhi proporsiya tashkil qiladi. 2) da $1 \cdot 4 \neq 2 \cdot 3$, $1 \cdot 3 \neq 2 \cdot 4$, $1 \cdot 2 \neq 3 \cdot 4$ bo'lgani uchun bu sonlar guruhi proporsiya tashkil qilmaydi. 3) da $3 \cdot 20 = 4 \cdot 15$ tenglik o'rinli. Demak, 3,4,15,20; sonlar guruhi proporsiya tashkil qiladi. **Javob:** 1, 3 (B).

8. Piyoda 2,5 saotda 14 km yo'l bosdi. U shunday tezlik bilan yursa, 4,2 km yo'lni necha soatda bosadi.

- A) 0,7 B) 0,5 C) 0,75 D) 0,6

9. Proporsiyaning chetki hadlari 14 va 20 ga, o'rta hadlaridan biri 35 ga teng. Proporsiyaning ikkinchi o'rta hadini toping.

- A) 2 B) 8 C) 10 D) 7

10. 4, 8, 12, a sonlari ko'rsatilgan tartibda proporsiya tashkil qilsa, a ni toping.

- A) 20 B) 24 C) 28 D) 32

11. $21 : x = 7 : 8$ proporsiyaning noma'lum hadini toping.

- A) 21 B) 24 C) 22 D) 28

1.3 Irratsional sonlar

Cheksiz davriy o'nli kasrlar bilan bir qatorda cheksiz davriy bo'lmagan o'nli kasrlar ham mavjud. Masalan, $0,10110111011110\dots$ son cheksiz davriy bo'lmagan o'nli kasrga misol bo'ladi. Bu sonni tashkil qiluvchi raqamlar ma'lum bir qonuniyat asosida joylashgan, lekin hech bir raqamlar guruhi davriy emas. *Davriy bo'lmagan cheksiz o'nli kasrlarga irratsional sonlar deyiladi.* Irratsional sonlarga misol qilib quyidagilarni ko'rsatish mumkin.

$$\sqrt{2} = 1,4142135\dots, \quad \sqrt{3} = 1,7320508\dots,$$

$$\pi = 3,1415926535\dots, \quad e = 2,718281828459\dots$$

Irratsional sonlar musbat va manfiy bo'lishi mumkin. *Barcha ratsional va irratsional sonlar haqiqiy sonlar to'plamini tashkil qiladi.* Ma'lumki, ratsional sonlar to'plami Q harfi bilan belgilanadi. Ratsional va irratsional sonlar to'plami o'zaro kesishmaydi. Shuning uchun irratsional sonlar to'plamini $R \setminus Q$ orqali belgilash mumkin.

1. a va b ratsional sonlar yig'indisi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan

Yechish: a va b ratsional sonlar bo'lganligi uchun ularni oddiy kasr ko'rinishida yozish mumkin. Oddiy kasrlar yig'indisi yana oddiy kasr, ya'ni ratsional sonidir. Demak, ratsional sonlar yig'indisi doim ratsional son bo'ladi. **Javob:** doim ratsional (A).
2. a va b ratsional sonlar ayirmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
3. a va b ratsional sonlar ko'paytmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
4. α va β irratsional sonlar yig'indisi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
5. α va β irratsional sonlar ayirmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
6. a ratsional son, α irratsional son bo'lsa, ularning yig'indisi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
7. a ratsional son, α irratsional son bo'lsa, ularning ayirmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
8. a noldan farqli ratsional son, α irratsional son bo'lsa, ularning ko'paytmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
9. α va β irratsional sonlar. Ularning nisbati qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan

Yechish: α va β sonlar sifatida $\alpha = 2\pi$ va $\beta = \pi$ irratsional sonlarini olsak, u holda $\alpha : \beta = 2$ ratsional sonni olamiz. Agar biz $\alpha = \sqrt{6}$ va $\beta = \sqrt{3}$ irratsional sonlarini olsak, u holda $\alpha : \beta = \sqrt{2}$ irratsional sonni olamiz. **Javob:** ratsional ham irratsional ham bo'lishi mumkin (C).
10. a noldan farqli ratsional son, α irratsional son bo'lsa, $\alpha : a$ (bo'linma) qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
11. α va β irratsional sonlar. Ularning ko'paytmasi qanday son bo'ladi?
 - A) doim ratsional
 - B) doim irratsional
 - C) ratsional ham irratsional ham bo'lishi mumkin
 - D) to'g'ri javob keltirilmagan
12. α va β irratsional sonlar bo'lib, ularning yig'indisi $\alpha + \beta$ ratsional son bo'lsin. Quyidagilardan qaysi biri doim ratsional son bo'ladi?
 - A) $\alpha \cdot \beta$
 - B) $\alpha + 2\beta$
 - C) $\alpha^2 + \beta^2 + 2\alpha\beta$
 - D) $\alpha - \beta$
13. Quyidagi sonlardan qaysilari irratsional sonlar: $a = 0, (123456789)$; $b = 3, 12(61)$; $\alpha = \pi^2$; $\beta = 2, 101001000100001\dots$;
 - A) α ; b
 - B) a ; α
 - C) α ; β
 - D) a ; b

1.4 Haqiqiy sonlar

Yuqorida ta'kidlaganimizdek barcha ratsional va irratsional sonlar to'plami birgalikda haqiqiy sonlar to'plami tashkil qiladi. Ma'lumki, haqiqiy sonlar to'plami R harfi bilan belgilanadi. Gorizontaal ℓ to'g'ri chiziq olamiz (1.1-chizma). Unda ixtiyoriy O nuqta olamiz va uni koordinata boshi deb ataymiz. O nuqtaga nol sonini mos qo'yamiz. O nuqtadan o'ngda E nuqta tanlaymiz. OE kesma *masshtab birligi* deyiladi. E nuqtaga 1 (bir) sonini mos qo'yamiz. ℓ to'g'ri chiziq bo'ylab, O nuqtadan E nuqtaga borish yo'nalishi musbat yo'nalish hisoblanadi. E nuqtadan bir masshtab o'ngdagi nuqtaga 2 (ikki) soni mos qo'yiladi va hokazo. O nuqtadan bir masshtab chapdagi E' nuqtaga -1 (minus bir) soni mos qo'yiladi, E' nuqtadan bir masshtab chapdagi nuqtaga -2 (minus ikki) soni mos qo'yiladi va hokazo. Shunday qilib R to'plamning elementlari bilan ℓ to'g'ri chiziqda joylashgan nuqtalar o'rtasida o'zaro bir qiymatli moslik o'rnatiladi. Bu holda ℓ koordinatalar to'g'ri chizig'i berilgan deyiladi. Ma'lumki, ixtiyoriy $r \in R$ soni uchun, ℓ koordinata to'g'ri chizig'ida unga mos keluvchi yagona M nuqta mavjud. r soni M nuqtaning koordinatasi deyiladi va $M(r)$ ko'rinishda yoziladi. ℓ to'g'ri chiziqda koordinata boshi O dan o'ng tomonda joylashgan nuqталarga mos kelgan sonlar musbat, O dan chap tomonda joylashgan nuqталarga mos kelgan sonlar manfiy sonlar deyiladi. Nol soni musbat ham manfiy ham hisoblanmaydi. Musbat sonlar "plyus" (+) ishorasi, manfiy sonlar "minus" (-) ishorasi orqali yoziladi. Masalan, $+1, +2, 5, +5, 8, \dots, -1, -2, 8, -8, 7, \dots$. Musbat sonlar oldidagi + ishorasi ni yozmaslikka kelishilgan, ya'ni $+1 = 1, +2, 5 = 2, 5$.

Haqiqiy sonning *moduli* deb koordinata boshidan shu songa mos keluvchi nuqttagacha bo'lgan masofaga aytiladi. a sonining moduli $|a|$ ko'rinishida yoziladi. Sonning moduli shu sonning absolyut qiymati deb ham ataladi. Har qanday musbat sonning moduli shu sonning o'ziga teng, manfiy sonning moduli shu sonning qarama-qarshisiga teng. Sonning modulini quyidagi formula shaklida ham yozish mumkin:

$$|a| = \begin{cases} a, & \text{agar } a \geq 0, \\ -a, & \text{agar } a \leq 0. \end{cases} \quad (1.1)$$

$|a - b|$ miqdor a va b sonlariga mos keluvchi nuqtalar orasidagi masofaga teng. Agar a va b sonlariga mos keluvchi nuqtalar A va B bo'lib, A nuqta B dan chapda joylashgan bo'lsa, u holda a soni b sonidan kichik bo'ladi va aksincha. Haqiqiy sonning butun va kasr qismlari tushunchalarini keltiramiz. Butun bo'lmagan $a \in R$ sonining *butun qismi* deb sonlar o'qida a sonidan chapda yotuvchi birinchi butun songa aytiladi va $[a]$ shaklda yoziladi. $a \in R$ sonining *kasr qismi* deb $a - [a]$ miqdorga aytiladi va $\{a\}$ shaklda yoziladi. Ma'lumki, butun sonning butun qismi o'ziga teng,

kasr qismi esa nolga teng. Misol uchun $a = 2,34$ va $b = -2,71$ sonlarining butun va kasr qismlarini hisoblaymiz. Ta'rifga ko'ra $2,34$ dan chapda yotuvchi birinchi butun son bu 2 dir. Uning kasr qismi $a - [a] = 2,34 - 2 = 0,34$. Xuddi shunday $[b] = [-2,71] = -3$ va $\{b\} = \{-2,71\} = -2,71 - (-3) = 0,29$.

Agar $b > 0$ soni uchun $b = a \cdot 10^n$ ($1 \leq a < 10, n \in Z$) tenglik bajarilsa, $a \cdot 10^n$ ifoda b sonining *standart shakli* deyiladi. Masalan, $543,26 = 5,4326 \cdot 10^2$; $0,000026 = 2,6 \cdot 10^{-5}$.

n faktorial deb 1 dan n gacha bo'lgan natural sonlarning ko'paytmasiga aytiladi va $n! = 1 \cdot 2 \cdot \dots \cdot n$ ko'rinishda yoziladi. Agar $n \geq 5$ bo'lsa, $n! = 1 \cdot 2 \cdot \dots \cdot n$ ko'paytma k ta 0 raqami bilan tugaydi, bu yerda k soni quyidagicha aniqlanadi

$$k = \left[\frac{n}{5} \right] + \left[\frac{n}{5^2} \right] + \left[\frac{n}{5^3} \right] + \dots \quad (1.2)$$

Haqiqiy son modulining asosiy xossalarini keltiramiz.

- $|a| \geq 0$.
- $|-a| = |a|$.
- $|a| = |b| \iff a = \pm b$.
- $|a \cdot b| = |a| \cdot |b|$.
- $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}, (b \neq 0)$.
- $|a|^2 = a^2$.
- $|a + b| \leq |a| + |b|$.
- $|a| - |b| \leq |a - b|$.
- $|a| < c, (c > 0) \iff -c < a < c$.
- $|a| > c, (c > 0) \iff \begin{cases} a > c \\ a < -c \end{cases}$.

- (97-12-13) Agar $m > n > k > 0$ bo'lsa,

$$|n - m| + |n + k| - |m - k|$$

ni soddalashtiring.

- A) $2k - 2m$ B) $2k - 2n$ C) $2k$ D) $2m - 2k$

Yechish: Ma'lumki, $|x| = \begin{cases} x, & \text{agar } x \geq 0 \\ -x, & \text{agar } x \leq 0 \end{cases}$
 $m > n > k > 0$ bo'lgani uchun $n - m < 0$ shu sababli $|n - m| = -(n - m)$, $n + k > 0$ shu sababli $|n + k| = n + k$; $m - k > 0$ shu sababli $|m - k| = m - k$ bo'ladi. U holda

$$|n - m| + |n + k| - |m - k| = -(n - m) + n + k - (m - k) = -n + m + n + k - m + k = 2k.$$

Javob: $2k$ (C).

- (98-5-9) Hisoblang.

$$\frac{|4 - 5| |4 - 6| + 4|3 - 6|}{|3 - 4| |7 - 5|}$$

- A) 1 B) $\frac{1}{2}$ C) $1\frac{2}{5}$ D) $1\frac{1}{5}$

3. (99-7-11) Hisoblang.

$$\frac{|4 - 4 \cdot |3 - 6| - 8|}{|4 - |3 - 8| - 7|}$$

- A) 2 B) 1 C) 3 D) 2, 5

4. (96-6-14) Agar $a > b > c$ bo'lsa,

$$|a - b| + |c - a| - |b - c|$$

ni soddalashtiring.

- A) $a - 2b$ B) $2c$ C) $2a$ D) $2a - 2b$

5. (97-2-14) Agar $x > y > z$ bo'lsa,

$$|x - y| - |z - y| - |z - x|$$

ni soddalashtiring.

- A) $2x$ B) $2y - 2x$ C) $2z - 2y$ D) $2y$

6. (97-8-14) Agar $p > q > k > 0$ bo'lsa,

$$|p + q| - |k - q| + |k - p|$$

ni soddalashtiring.

- A) $2p$ B) $2p + 2q - 2k$
C) $2p + 2q + 2k$ D) $2p + 2k$

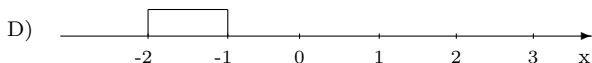
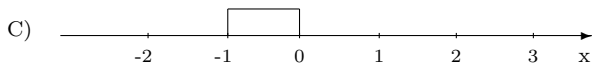
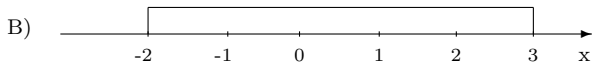
7. Agar $x > y > 0$ bo'lsa,

$$\left| xy - \frac{x^2 + y^2}{2} \right| + \left| \frac{x^2 + y^2}{2} + xy \right| - 2y^2$$

ni soddalashtiring.

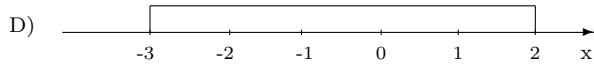
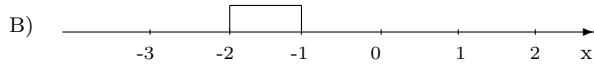
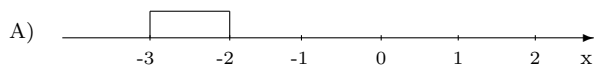
- A) $x^2 - y^2$ B) $x^2 + y^2$ C) y^2 D) x^2

8. (96-3-7) Agar $a = -2$ va $b = 3$ bo'lsa, chizmada $|a - b|$ ga mos to'g'ri javobni ko'rsating.



Yechish: Ma'lumki, $|a - b|$ miqdor a va b sonlarga mos keluvchi nuqtalar orasidagi masofadir. Demak, -2 va 3 nuqtalar orasidagi masofa B) javobda keltirilgan. **Javob:** (B).

9. (96-12-7) Agar $a = -3$ va $b = 2$ bo'lsa, chizmada $|a - b|$ ga mos to'g'ri javobni ko'rsating.



10. (97-4-9) Sonlarni kamayish tartibida yozing.

$$m = |4, 8|; \quad n = |-4, (8)|; \quad p = |4\frac{3}{5}| \text{ va } q = |-3, 2|$$

- A) $n > m > p > q$ B) $m > n > p > q$
C) $m > p > q > n$ D) $p > m > q > n$

11. (97-9-69) Sonlarni kamayish tartibida yozing.

$$m = |8, (8)|; \quad n = |-8, 8|; \quad p = |8\frac{7}{9}| \text{ va } q = |-8\frac{6}{7}|$$

- A) $n > m > p > q$ B) $m > n > p > q$
C) $m > q > n > p$ D) $q > m > n > p$

12. (03-2-63) $a > 0$; $b < 0$; $|a| \neq |b|$. Quyidagi ifodalardan qaysi birining qiymati musbat bo'lmasligi mumkin?

- A) $a - b$ B) $|a + b|$ C) $a^3 b^2$ D) $|a| - |b|$

13. (98-5-6) $-5, 2$ bilan $10, 4$ orasida nechta butun son bor?

- A) 16 B) 10 C) 15 D) 12

Yechish: Sonlar o'qi chiziladi va $-5, 2$ hamda $10, 4$ nuqtalar belgilanadi. Endi ular orasidagi butun sonlarni sanaymiz: Manfiylari $-5, -4, -3, -2, -1$ ya'ni 5 ta, $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ musbatlari 10 ta, nol ham butun son u bilan jami 16 ta. **Javob:** 16 (A).

14. (99-7-8) Koordinatalari $-3, 2$ va $4, 2$ bo'lgan sonlar orasida nechta butun son bor?

- A) 7 B) 6 C) 9 D) 8

15. (98-7-11) Son o'qida -4 dan $2, 3$ birlik masofada joylashgan sonlarni aniqlang.

- A) $-6, 3$ B) $-6, 3$ va $1, 7$
C) $6, 3$ va $1, 7$ D) $-6, 3$ va $-1, 7$

16. (02-10-40) $-\frac{21}{6} + 2$, (2) ni butun qismini toping.

- A) -2 B) -1 C) 0 D) 1

17. $0, (4) + 1, (5) - 2, (3)$ ni kasr qismini toping.

- A) $0, (5)$ B) $0, (6)$ C) $0, 6$ D) $0, 56$

18. $0, 00003602$ sonini standart shaklda yozing.

- A) $3, 6 \cdot 10^{-5}$ B) $0, 36 \cdot 10^{-4}$
C) $36, 02 \cdot 10^{-6}$ D) $3, 602 \cdot 10^{-5}$

1.4.1 Aralash tipdagi masalalar

1. $60! - 50!$ ayirmaning oxirida nechta nol bo'ladi?
A) 14 B) 12 C) 10 D) 8

Yechish: Agar biz (1.2) formuladan foydalansak, $50!$ oxirida 12 ta nol, $60!$ oxirida esa 14 ta nol raqami bo'ladi. Bu sonlarni tagma-tag yozib ayir-sangiz, ayirmaning oxirida 12 ta nol bo'lishiga ishonch hosil qilasiz. **Javob:** 12 (B).

2. $150!$ sonining oxirida nechta nol bo'ladi?
A) 30 B) 33 C) 36 D) 37
3. $3! + 6! + 9! + \dots + 33!$ yig'indining oxirgi raqamini toping.
A) 0 B) 1 C) 2 D) 6
4. $(1! + 2! + 3! + 4! + \dots + 33!)^{33}$ yig'indining oxirgi raqamini toping.
A) 9 B) 1 C) 2 D) 3
5. $10! + 11! + 12!$ yig'indi quyidagilarning qaysi biriga bo'linmaydi?
A) 144 B) 350 C) 800 D) 500

6. $15!$ sonini 1001 ga bo'lgandagi qoldiqni toping.
A) 0 B) 1 C) 11 D) 7

Yechish: 11 ga bo'linish belgisi (1.1.4-band 8-alomatga qarang) ga ko'ra, 1001 ning toq o'rindagi raqamlari yig'indisi $1 + 0 = 1$ bilan, juft o'rindagi raqamlari yig'indisi $0 + 1 = 1$ teng, shuning uchun 1001 soni 11 ga bo'linadi. Bo'lishni bajarib $1001 = 11 \cdot 91$ ekanligini olamiz. Ma'lumki, 91 soni 7 ga bo'linadi va bo'linma 13 ga teng. Demak, 1001 soni quyidagi $1001 = 7 \cdot 11 \cdot 13$ ko'paytma shaklida ifodalanadi. Faktorial ta'rifiga ko'ra $15!$, $1001 = 7 \cdot 11 \cdot 13$ ga qoldiqsiz bo'linadi, ya'ni qoldiq nol bo'ladi. **Javob:** 0 (A).

7. 48 va 60 sonlarining nechta tub bo'lmagan, umumiy bo'luvchilari bor.
A) 4 B) 6 C) 3 D) 5

8. $4 \cdot 45^n$ sonining 198 ta natural bo'luvchisi bo'lsa, n nechtaga teng.
A) 2 B) 3 C) 4 D) 5

9. $\frac{80!}{8^n}$ ifoda butun son bo'lsa, n ning eng katta qiymati nimaga teng bo'ladi?
A) 10 B) 18 C) 20 D) 26

10. 5200 000 sonining nechta natural bo'luvchisi bor?
A) 48 B) 56 C) 64 D) 96

11. 1440 sonining barcha natural bo'luvchilari yig'indisini toping.
A) 5225 B) 4914 C) 2317 D) 198

12. $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytma necha xonali son bo'ladi?
A) 21 B) 18 C) 19 D) 20

Yechish: $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytmani quyidagicha $4^{10} \cdot 3^3 \cdot 5^3 \cdot 25^8$ yozib olamiz. Natural ko'rsatkichli darajaning xossaligidan foydalanib, uni

$$4^{10} \cdot 3^3 \cdot 5^3 \cdot 25^8 = 27 \cdot 5 \cdot 4^{10} \cdot 25^9 = 27 \cdot 5 \cdot 4 \cdot 100^9 =$$

$$= 54 \cdot 10 \cdot (10^2)^9 = 54 \cdot 10 \cdot 10^{18} = 54 \cdot 10^{19}$$

shaklga keltiramiz. Berilgan ko'paytma 54 va uning orqasida 19 ta nol bo'lgan sondan iborat. Demak, $4^{10} \cdot 15^3 \cdot 25^8$ ko'paytma 21 xonali sondan iborat bo'lar ekan. **Javob:** 21 (A).

13. $8^{18} \cdot 5^{55}$ ko'paytma necha xonali son bo'ladi?
A) 36 B) 54 C) 55 D) 73

14. $2^{10} \cdot 5^9 \cdot 4^6 \cdot 25^4$ ko'paytma necha xonali son bo'ladi?
A) 21 B) 18 C) 19 D) 20

15. (02-1-3) Sonning uchdan bir qismini toping.

$$\frac{(-2) \cdot (-3)^{17} - (-3)^{16}}{9^7 \cdot 15}$$

- A) 1 B) 3 C) 2 D) 9

16. a va b natural sonlar $\frac{5a-b}{b} = 11$ munosabatni qanoatlantirsa, $a+b$ ifodaning eng kichik qiymati nimaga teng bo'ladi?
A) 17 B) 16 C) 14 D) 13

17. Uch xonali uchta turli natural sonning yig'indisi 349 bo'lsa, ular ichidan eng kattasini toping.
A) 101 B) 146 C) 148 D) 147

18. a soni 2 dan farqli tub son bo'lsa, quyidagilardan qaysi biri juft son bo'ladi?
A) a B) $2a - 3$ C) $a^2 + a + 1$ D) $a^3 - 3a$

19. Hisoblang.

$$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right) \dots \left(1 + \frac{1}{2n}\right)$$

- A) $\frac{1}{2n}$ B) $\frac{n+1}{2}$ C) $2n+1$ D) $\frac{2n+1}{2}$

20. Hisoblang.

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{100}\right)$$

- A) $\frac{1}{2}$ B) $\frac{1}{10}$ C) 2 D) $\frac{1}{100}$

21. Hisoblang.

$$\frac{0,(3)}{0,44} + \frac{19}{10}$$

$$1,9 + \frac{0,(3)}{0,44}$$

- A) $\frac{4}{9}$ B) 1 C) 3 D) $\frac{11}{9}$

22. $(1, (3) + 5) : (5 + 1, (333))$ ni hisoblang.

- A) $\frac{1}{9}$ B) 1 C) 3 D) $\frac{5}{9}$

23. (98-7-2) Hisoblang.

$$\frac{488 \cdot 475 - 462}{244 + 475 \cdot 243}$$

- A) 3 B) 1 C) 0,5 D) 2

24. $2\frac{5}{9} - 3,2(7) + 0,55$ ni butun qismini toping.

- A) -2 B) -1 C) 0 D) 1

25. $1 - 5, (8) - 6, (5)$ son modulining kasr qismini toping.

- A) 0, (5) B) 0, (4) C) 0,4 D) 0,44

2 - bob. Algebraik ifodalar

Agar sonli ifodaning ayrim sonlari yoki barcha sonlari harflar bilan almashtirilsa, u harfiy ifoda deyiladi. Algebrada harfiy ifodalar o'rganiladi. *Shu sababdan bunday ifodalar algebraik ifodalar deyiladi.* Odatda harfiy ifodalar orasiga ko'paytirish belgisi qo'yilmaydi. Masalan, $5 \cdot a \cdot b \cdot c^2 = 5abc^2$; $4 \cdot x \cdot y \cdot z = 4xyz$.

2.1 Natural ko'rsatkichli daraja

Haqiqiy a sonining *natural ko'rsatkichli darajasi* quyidagicha aniqlanadi: $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$ va hokazo $a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_n$. Bu yerda a asos, n daraja ko'rsatkich dey-

iladi. Natural ko'rsatkichli darajaning ayrim xossalari keltiramiz. Haqiqiy $a > 0$, $b > 0$ va $n, m \in N$ sonlar uchun quyidagi tengliklar o'rinli:

1. $a^n \cdot a^m = a^{n+m}$.

2. $a^n : a^m = a^{n-m}$.

3. $(ab)^n = a^n \cdot b^n$.

4. $(a^n)^m = a^{nm}$.

5. $a^0 = 1$.

6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

7. $a^{-n} = \frac{1}{a^n}$.

8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

1. 2^5 ni hisoblang.

- A) 10 B) 7 C) 32 D) 16

Yechish: Daraja ta'rifiga ko'ra, $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 4 \cdot 2 \cdot 2 \cdot 2 = 8 \cdot 2 \cdot 2 = 16 \cdot 2 = 32$. **Javob:** 32 (C).

2. 3^5 ni hisoblang.

- A) 81 B) 15 C) 243 D) 27

3. 5^4 ni hisoblang.

- A) 20 B) 125 C) 625 D) 25

4. 4^3 ni hisoblang.

- A) 12 B) 16 C) 64 D) 32

5. 6^3 ni hisoblang.

- A) 12 B) 216 C) 36 D) 18

6. $2^2 \cdot 2^3$ ni 2 ning darajasi shaklida yozing.

- A) 2^4 B) 2^3 C) 2^5 D) 2^6

Yechish: 1-xossaga ko'ra, $2^2 \cdot 2^3 = 2^{2+3} = 2^5$. **Javob:** 2^5 (C).

7. $3 \cdot 3^2 \cdot 3^5$ ni 3 ning darajasi shaklida yozing.

- A) 3^7 B) 3^8 C) 3^5 D) 3^6

8. $4 \cdot 4 \cdot 4^5$ ni 4 ning darajasi shaklida yozing.

- A) 4^7 B) 4^8 C) 4^5 D) 4^6

9. $5 \cdot 5 \cdot 5^2 \cdot 5^5$ ni 5 ning darajasi shaklida yozing.

- A) 5^7 B) 5^8 C) 5^9 D) 5^6

10. $7 \cdot 7 \cdot 7^2 \cdot 7^3$ ni 7 ning darajasi shaklida yozing.

- A) 7^7 B) 7^8 C) 7^9 D) 7^6

11. $2^{30} \cdot 4^{20} \cdot 8^{10}$ ni 2 ning darajasi shaklida yozing.

- A) 2^{60} B) 2^{100} C) 2^{80} D) 2^{90}

12. 30^4 darajani tub sonlar darajasi shaklida yozing.

- A) $2^4 \cdot 3^4 \cdot 5^4$ B) $6^4 \cdot 5^4$ C) $3^4 \cdot 10^4$ D) $2^4 \cdot 15^4$

Yechish: 30 ni tub ko'paytuvchilarga ajratamiz $30 = 2 \cdot 3 \cdot 5$. Demak, $30^4 = (2 \cdot 3 \cdot 5)^4$. 3-xossaga ko'ra, $30^4 = 2^4 \cdot 3^4 \cdot 5^4$. **Javob:** $2^4 \cdot 3^4 \cdot 5^4$ (A).

13. 6^8 darajani tub sonlar darajasi shaklida yozing.

- A) $2^8 \cdot 3^8$ B) $6^4 \cdot 6^4$ C) $1^8 \cdot 6^8$ D) $2^4 \cdot 3^4$

14. 15^5 darajani tub sonlar darajasi shaklida yozing.

- A) $3^8 \cdot 5^3$ B) $3^3 \cdot 5^2$ C) $3^5 \cdot 5^5$ D) $2^5 \cdot 3^5$

15. 18^9 darajani tub sonlar darajasi shaklida yozing.

- A) $2^9 \cdot 3^9$ B) $2^9 \cdot 3^{18}$ C) $2^{18} \cdot 3^9$ D) $2^9 \cdot 9^9$

16. 20^7 darajani tub sonlar darajasi shaklida yozing.

- A) $2^7 \cdot 10^7$ B) $2^{14} \cdot 5^7$ C) $4^7 \cdot 5^7$ D) $2^5 \cdot 5^2$

17. 21^6 darajani tub sonlar darajasi shaklida yozing.

- A) $3^3 \cdot 7^3$ B) $3^4 \cdot 7^2$ C) $3^6 \cdot 7^6$ D) $3^5 \cdot 7^5$

18. $2^6 : 2^3$ bo'linmani daraja shaklida yozing.

- A) 2^3 B) 2^2 C) 2^6 D) 2^1

Yechish: 2-xossaga ko'ra, $2^6 : 2^3 = 2^{6-3} = 2^3$. **Javob:** 2^3 (A).

19. $3^8 : 3^5$ bo'linmani daraja shaklida yozing.

- A) 3^3 B) 3^2 C) 3^6 D) 3^1

20. $5^{12} : 5^7$ bo'linmani daraja shaklida yozing.

- A) 5^3 B) 5^2 C) 5^5 D) 5^8

21. $7^{12} : 7^4$ bo'linmani daraja shaklida yozing.

- A) 7^3 B) 7^4 C) 7^5 D) 7^8

22. $6^5 : 6$ bo'linmani daraja shaklida yozing.

- A) 6^3 B) 6^4 C) 6^5 D) 6^2

23. $(3^4)^5$ ni daraja shaklida yozing.

- A) 3^9 B) 3^{11} C) 3^{20} D) 3^{15}

Yechish: 4-xossaga ko'ra, $(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$. **Javob:** 3^{20} (C).

24. $(2^5)^3$ ni daraja shaklida yozing.

- A) 2^8 B) 2^{11} C) 2^{20} D) 2^{15}

25. $(3^4)^8$ ni daraja shaklida yozing.

- A) 3^{12} B) 3^{11} C) 3^{24} D) 3^{32}

26. $(5^2)^7$ ni daraja shaklida yozing.

- A) 5^9 B) 5^{11} C) 5^{49} D) 5^{14}

27. $(7^4)^6$ ni daraja shaklida yozing.

- A) 7^9 B) 7^{10} C) 7^{24} D) 7^{12}

28. $(\frac{2}{3})^2$ kasrni darajaga ko'taring.

- A) $\frac{4}{6}$ B) $\frac{4}{9}$ C) $\frac{2}{9}$ D) $\frac{4}{3}$

Yechish: 6-xossaga ko'ra, $(\frac{2}{3})^2 = \frac{2^2}{3^2} = \frac{4}{9}$.

Javob: $\frac{4}{9}$ (B).

29. $(\frac{2}{5})^3$ kasrni darajaga ko'taring.

- A) $\frac{6}{15}$ B) $\frac{8}{5}$ C) $\frac{8}{125}$ D) $\frac{8}{25}$

30. $(\frac{3}{7})^4$ kasrni darajaga ko'taring.

- A) $\frac{24}{56}$ B) $\frac{81}{2401}$ C) $\frac{81}{28}$ D) $\frac{81}{343}$

31. $(\frac{2}{7})^3$ kasrni darajaga ko'taring.

- A) $\frac{6}{21}$ B) $\frac{8}{21}$ C) $\frac{8}{343}$ D) $\frac{8}{49}$

32. $(\frac{2}{3})^{-7}$ ni natural ko'rsatkichli daraja shaklida yozing.

- A) $(\frac{3}{2})^7$ B) $(\frac{2}{3})^7$ C) $(\frac{2}{3})^5$ D) $(\frac{3}{2})^{14}$

Yechish: 8-xossaga ko'ra, $(\frac{2}{3})^{-7} = (\frac{3}{2})^7$. **Javob:**

$(\frac{3}{2})^7$ (A).

33. $(\frac{2}{5})^{-3}$ ni natural ko'rsatkichli daraja shaklida yozing.

- A) $(\frac{5}{2})^3$ B) $(\frac{2}{5})^3$ C) $(\frac{5}{3})^3$ D) $(\frac{5}{2})^5$

34. $(\frac{3}{7})^{-5}$ ni natural ko'rsatkichli daraja shaklida yozing.

- A) $(\frac{7}{5})^3$ B) $(\frac{7}{3})^5$ C) $(\frac{7}{3})^8$ D) $(\frac{5}{3})^7$

35. $(\frac{2}{9})^{-1}$ ni natural ko'rsatkichli daraja shaklida yozing.

- A) 9^2 B) $\frac{9}{2}$ C) $(\frac{1}{2})^9$ D) 2^9

36. (99-8-20) Soddalashtiring.

$$5 \cdot 4^{2n-3} - 20 \cdot (2^{n-2})^4$$

- A) 2 B) 4^{2n} C) 4 D) 0

Yechish: 4 ni 2^2 shaklda, 20 esa $5 \cdot 2^2$ ko'rinishda yozamiz, 3 va 4-xossalardan foydalanib

$$5 \cdot 4^{2n-3} - 20 \cdot (2^{n-2})^4 = 5 \cdot (2^2)^{2n-3} - 5 \cdot 2^2 \cdot 2^{4n-8} = 5 \cdot 2^{4n-6} - 5 \cdot 2^{4n-6} = 0$$

ni olamiz. **Javob:** 0 (D).

37. (98-7-25) Soddalashtiring.

$$\frac{2^{5n-3} \cdot 2^{3n+2}}{2^{4n-1}}$$

- A) 2^{3n} B) 2^{4n+1} C) 2^{4n+2} D) 2^{4n}

38. (98-12-24) Soddalashtiring.

$$\frac{3^{4n+3} \cdot 3^{3n-2}}{3^{2n-1}}$$

- A) 3^{5n+2} B) 3^{5n+3} C) 3^{5n+1} D) 3^{5n-1}

39. (01-3-30) Soddalashtiring.

$$\frac{2^{5n+3} \cdot 2^{3n-4}}{2^{4n+1}}$$

- A) 2^{4n-1} B) 2^{n-2} C) 2^{2n-2} D) 2^{4n-2}

40. (96-10-25) Hisoblang.

$$\frac{0,5^5 \cdot 32^2}{4^3}$$

- A) 2 B) $\frac{1}{2}$ C) 4 D) $\frac{1}{4}$

Yechish: 0,5 ni $\frac{1}{2}$ shaklda 32 ni 2^5 , 4 esa 2^2 ko'rinishda yozamiz, 4 va 6-xossalardan foydalanib

$$0,5^5 \cdot \frac{32^2}{4^3} = (\frac{1}{2})^5 \cdot \frac{(2^5)^2}{(2^2)^3} = \frac{1^5}{2^5} \cdot \frac{2^{10}}{2^6} = \frac{2^{10}}{2^{11}} = \frac{1}{2}$$

ni olamiz. **Javob:** $\frac{1}{2}$ (B).

41. (96-1-24) Hisoblang. $\frac{9^2 \cdot 3^5}{81^2}$

- A) 1 B) 3 C) $\frac{1}{81}$ D) 9

42. (96-9-65) Hisoblang. $\frac{27^3}{3^4 \cdot 9^2}$

- A) 3 B) $\frac{1}{3}$ C) 1 D) 9

43. (98-7-24) Hisoblang.

$$\frac{5(3 \cdot 7^{15} - 19 \cdot 7^{14})}{7^{16} + 3 \cdot 7^{15}}$$

- A) 7 B) 49 C) $\frac{1}{7}$ D) $\frac{1}{49}$

44. (98-12-23) Hisoblang.

$$\frac{5 \cdot 2^{32} - 4 \cdot 2^{30}}{4^{16}}$$

- A) 4 B) 2 C) 5 D) 16

45. (99-6-1) Hisoblang.

$$\frac{10^9 \cdot 3^5}{3^3 \cdot 10^{11}}$$

- A) 0,09 B) 0,9 C) 9 D) 0,03

46. (97-9-78) Hisoblang.

$$\frac{72^6 \cdot 24^4}{36^8 \cdot 8^3}$$

- A) 24 B) 32 C) 16 D) 36

47. (99-7-7) Hisoblang.

$$\frac{100^5}{(80+20)^{10}} \cdot 50^5$$

- A) $\frac{1}{32}$ B) 16 C) 8 D) $\frac{1}{64}$

2.2 Birhad va uning xossalari

Birhad deb sonlar, o'zgaruvchilarning natural darajalari va ularning ko'paytmalari qatnashgan hamda sonlar va o'zgaruvchilar ustida boshqa amallarni o'z ichiga olmaydigan ifodaga aytiladi. Son yoki bitta harf ham birhad sanaladi. Son va harflar birhadning ko'paytuvchilari deyiladi. Birhad quyidagi xossalarga ega.

1. **Birhadning uning ko'paytuvchilari o'rinarini almashtirish mumkin.** Masalan, $ab \cdot 5xy = 5abxy$.
2. **Birhadning bir necha sonli ko'paytuvchilarni ularning ko'paytmasi bilan almashtirish mumkin.** Masalan, $5ab \cdot 3xy \cdot 4tz = 60abtxyz$.
3. **Birhadning bir xil harfiy ko'paytuvchilarni mos darajali ko'paytmaga almashtirish mumkin.** Masalan, $5ab \cdot a^2b^3 = 5a^{1+2}b^{1+3} = 5a^3b^4$.
4. **Birhadning ko'paytuvchilaridan biri nolga teng bo'lsa, bunday ko'paytma nolga teng.** Masalan, $5ab \cdot 0 \cdot 8xy = 0$.
5. **Birhadning 1 ga teng ko'paytuvchini tashlab yuborish mumkin.** Masalan, $4ab \cdot 0,25x^2y = 1 \cdot abx^2y = abx^2y$.
6. **Agar birhad oldiga " + " belgisini qo'ysak, berilgan birhadga teng birhad hosil bo'ladi.** Masalan, $+abc = abc$.
7. **Agar birhad oldiga " - " belgisini qo'ysak, berilgan birhad -1 ga ko'paytirilgan hisoblanadi.** Masalan, $-a(-5)c = (-1)(-5)ac = 5ac$.

Faqat ishoralari bilan farq qiluvchi birhadlar *qarama-qarshi birhadlar* deyiladi. Masalan, $5xyz$ va $-5xyz$ yoki $4x^2y^3$ va $-4x^2y^3$. Nolga teng bo'lmagan birhadning birgina sonli ko'paytuvchi birinchi o'rinda, birhadning harfiy ko'paytuvchilar alfavit tartibida daraja ko'rsatkichi orqali bir marta yozilgan bo'lsa, birhad *standart shaklda* deyiladi. Masalan, $15a^2b^3x^5y^8$ birhad standart shaklda. Standart shakldagi birhadning sonli ko'paytuvchisiga *birhadning koeffitsiyenti* deyiladi. Masalan, $5a^2b^3x^5y^8$ birhadning koeffitsiyenti 5 ga teng. Teng birhadlar yoki faqat koeffitsiyenti bilan farq qiluvchi birhadlar *o'xshash* deyiladi. Masalan, $5a^2bc$ va $-3a^2bc$ birhadlar o'xshash. Standart shakldagi birhadning darajasi deb, birhadning harfiy ko'paytuvchilar darajalarining yig'indisiga aytiladi. Masalan, $a^2b^3x^5y^8z$ birhadning darajasi $2 + 3 + 5 + 8 + 1 = 19$ dir.

1. Quyidagi ifodalarning qaysilari birhad.
 - 1) $3ab^{-2}$, 2) $a + b$, 3) $\frac{1}{2}abc$, 4) $1 : (2c)$
 A) 1 B) 2 C) 3 D) 4

Yechish: 1) da manfiy daraja qatnashyapti, demak, u birhad emas. 2) da " + " ishora qatnashyapti, u birhad emas. 4) da " : " amali qatnashyapti, u ham birhad emas. 3) da son va harflar faqat ko'paytirish amali bilan bog'langan, demak u birhad. **Javob:** 3 (C).

2. Quyidagi ifodalarning qaysilari birhad.
 - 1) ab^{-1} , 2) $a - b$, 3) $\frac{1}{2}a^3b^2$, 4) $2c : 3$
 A) 1; 2 B) 2; 3 C) 3; 4 D) 2; 4
3. $3ab^2 \cdot 2xy^3$ birhadning koeffitsiyentini toping.
 - A) 3 B) 2 C) 6 D) 5
4. $4ab^2 \cdot 0,25xy^3$ birhadning koeffitsiyentini toping.
 - A) 3 B) 2 C) 1 D) 5
5. Birinchi darajali birhadni toping.
 - A) abc B) 6^3a C) $2^{-2}a^3$ D) $5xyz$
6. Ikkinchi darajali birhadni toping.
 - A) $2abc$ B) 6^3a^2 C) $2^2a^2b^2$ D) $2xyz$
7. Uchinchi darajali birhadni toping.
 - A) abc B) 3^3a C) $2a^3b$ D) $3x^3y^3$
8. $3^2a^3b^2xy^3$ birhadning darajasini toping.
 - A) 11 B) 9 C) 8 D) 10
9. $3ab^2 \cdot 2a^3b^5$ birhadni standart shaklga keltiring.
 - A) $6ab^2 \cdot a^3b^5$ B) $6a^3b^{10}$ C) $6a^4b^7$ D) $6a^3b^7$
10. $3x^2 \cdot 2x^3y^5$ birhadni standart shaklga keltiring.
 - A) $6x^6y^6$ B) $6x^5y^9$ C) $6x^5y^5$ D) $5x^5y^5$
11. Standart shakldagi birhadlarni ajrating.
 - 1) $3x^2 \cdot 2x^3y^5$; 2) $6ab^2x^3z^5$; 3) $7x^6y^6$
 A) 1; 2 B) 1; 3 C) 2; 3 D) 1; 2; 3
12. n ning qanday qiymatida $8a^2x^3y^n$ birhadning darajasi uning koeffitsiyentiga teng bo'ladi?
 - A) 6 B) 5 C) 3 D) 4

Yechish: Birhadning koeffitsiyenti 8 ga, uning darajasi esa $2 + 3 + n$ ga teng. Ularni tenglashtiramiz $2 + 3 + n = 8$. Bu yerdan $n = 3$ ni olamiz. **Javob:** 3 (C).
13. n ning qanday qiymatida $5a^2x^3y^n$ birhadning darajasi uning koeffitsiyentidan 5 marta katta bo'ladi?
 - A) 16 B) 15 C) 23 D) 20
14. Quyidagilar ichidan qaysi biri $10a^3b^5$ teng.
 - A) $2ab \cdot 5a^2b$ B) $a \cdot 10a^2b^5$
 - C) $\frac{1}{2}ab^2 \cdot 20ab^3$ D) $ab \cdot 5ab^2 \cdot 2ab^3$
15. Qarama-qarshi birhadlarni toping.
 - 1) $3ab$ va $\frac{1}{3ab}$ 2) a va a^{-1}
 - 3) $\frac{1}{2}ab^2$ va $-0,5ab^2$ 4) $a + b$ va $a - b$
 A) 1 B) 2 C) 3 D) 4
16. $-0,5ab^2x^3$ ga o'xshash birhadni toping.
 - A) $3abx$ B) ba^2x^3 C) ab^2x D) $2ab^2x^3$
17. O'xshash birhadlarni ajrating.
 - 1) $3ab$ va $\frac{ab}{3}$ 2) ab va $-ab$
 - 3) $\frac{1}{2}ab^2$ va $-0,5ab^2$ 4) $7a^2b$ va $7^{-1}a^2b$
 A) 1; 2 B) 2; 3 C) 1; 2; 3 D) 1; 2; 3; 4

2.3 Ko'phad va uning xossalari

Bir necha birhadning algebraik yig'indisiga *ko'phad* deyiladi. Ko'phadni tashkil etuvchi birhadlar shu ko'phadning *hadlari* deyiladi. Ko'phadlar ikkita, uchta va hokazo n ta birhadlar yig'indisidan iborat bo'lishi mumkin. Masalan, $x^2 + 2xy + y^2$; $x^4 - y^4$; $a^2 + b - c^2 + d$ ifodalarda, birinchisi - uchhad, ikkinchisi - ikkihada, uchinchisi - to'rt had. Ko'phadni tashkil etuvchi birhadlar standart shaklda va ular ixchamlangan bo'lsa, *ko'phad standart shaklda* berilgan deyiladi. Ko'phad quyidagi xossalarga ega.

1. **Ko'phadning hadlari o'rinlarini almashtirish mumkin.** $x^2 + y^2 = y^2 + x^2$, $x^4 - y^4 = -y^4 + x^4$.

2. **Ko'phadga noldan iborat birhad qo'shsak, berilgan ko'phad o'zgarmaydi.**
 $x^2 + y^2 + 0 = x^2 + y^2$, $x^4 + 0 - y^4 = x^4 - y^4$.

3. **Ko'phadda o'xshash hadlarni ixchamlash mumkin.** $x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$,
 $3x^2 + x - x + y^2 = 3x^2 + 0 + y^2 = 3x^2 + y^2$.

4. **Birhadni ko'phadga ko'paytirish uchun, birhad ko'phadning har bir hadiga ko'paytiriladi va ko'paytmalar qo'shiladi.** $x^2(x - xy + y) = x^2x - x^2xy + x^2y = x^3 - x^3y + x^2y$.

5. **Ko'phadni ko'phadga ko'paytirish uchun, ko'phadlardan birining har bir hadi ikkinchi ko'phadning barcha hadlariga ko'paytirilib, o'xshash hadlar ixchamlanadi.** Masalan,
 $(x + y)(x - y) = x(x - y) + y(x - y) = x^2 - xy + xy - y^2 = x^2 + 0 - y^2 = x^2 - y^2$.

1. (97-10-5) Soddalashtiring.

$$2\frac{2}{3} \cdot \left(1\frac{1}{2}a - 2\frac{1}{4}\right) + 1\frac{1}{5} \cdot \left(2\frac{1}{2}a - \frac{5}{6}\right)$$

A) $a + 5$ B) $7a - 7$ C) 7 D) $3a - 5$

Yechish: Aralash kasrlarni noto'g'ri kasrga keltirib, qavslarni ochamiz:

$$\begin{aligned} & 2\frac{2}{3} \cdot \left(1\frac{1}{2}a - 2\frac{1}{4}\right) + 1\frac{1}{5} \cdot \left(2\frac{1}{2}a - \frac{5}{6}\right) = \\ & = \frac{8}{3} \cdot \left(\frac{3}{2}a - \frac{9}{4}\right) + \frac{6}{5} \cdot \left(\frac{5}{2}a - \frac{5}{6}\right) = \\ & = 4a - 6 + 3a - 1 = 7a - 7 \end{aligned}$$

Javob: $7a - 7$ (B).

2. (97-5-2) $4a - 13a + 5a$ ni soddalashtiring.

A) $4a$ B) $-4a$ C) $6a$ D) $-6a$

3. (97-9-2) $7x - 14x + 6x$ ni soddalashtiring.

A) x B) $-2x$ C) $2x$ D) $-x$

4. (97-9-6) $-8 - 2(1 - b) - 2b + 1$ ni soddalashtiring.

A) 9 B) $9 - 4b$ C) $9 + 4b$ D) -9

5. (98-1-14) Soddalashtiring.

$$a(b - c) + b(c - a) - c(b - a)$$

A) $-2ac$ B) $2ab$ C) 0 D) 2

6. (97-3-5) Soddalashtiring.

$$2\frac{1}{3} \cdot \left(\frac{6}{7}m + 3\right) - 1\frac{2}{3} \cdot \left(\frac{3}{5}m - 3\right)$$

A) $m - 2$ B) 4 C) $m + 12$ D) $4 + m$

7. (99-4-13) Soddalashtiring.

$$\frac{4}{9} \cdot \left(4\frac{1}{2}y - 1\frac{1}{2}\right) - \frac{2}{7} \cdot \left(1\frac{1}{6} - 3\frac{1}{2}y\right)$$

A) $0, 2y - 1$ B) $2y + 1$ C) $3y - 1$ D) $y - 1$

8. (96-1-25) Ifodani ko'phadning standart shakliga keltiring.

$$2x(x - 1) - (2x - 1) \cdot (x + 1)$$

A) $4x^2 - 1$ B) $2x^2 - 3x$

C) $3x + 1$ D) $-3x + 1$

Yechish: Dastlab qavslarni ochamiz, keyin o'xshash hadlarni ixchamlaymiz. $2x(x - 1) - (2x - 1) \cdot (x + 1) = 2x^2 - 2x - 2x \cdot (x + 1) + 1 \cdot (x + 1) = 2x^2 - 2x - 2x^2 - 2x + x + 1 = -3x + 1$. **Javob:** $-3x + 1$ (D).

9. (99-8-24) P va Q ko'phadlar ayirmasini toping.

$$P = \frac{1}{3}x - \frac{1}{3}y - (x + 2y), \quad Q = \frac{1}{3}x + \frac{1}{3}y - (x - y)$$

A) $-\frac{11}{3}y$ B) $4y$ C) $-4y$ D) $\frac{13}{3}y$

10. Ko'phadlarni ko'paytiring. $(a - b)(a + b)$

A) $a^2 - b$ B) $a^2 - 2b$ C) $a^2 + b^2$ D) $a^2 - b^2$

11. Ko'phadlarni ko'paytiring. $(a - b)(a^2 + ab + b^2)$

A) $a^3 - b^3$ B) $a^2 - b^3$ C) $a^3 + b^3$ D) $a^2 - b^2$

12. Ko'phadlarni ko'paytiring. $(a + b)(a^2 - ab + b^2)$

A) $a^3 - b^3$ B) $a^2 - b^3$ C) $a^3 + 3b^3$ D) $a^3 + b^3$

13. (01-8-12) Ushbu

$$(a + 3b)(a + b + 2) - (a + b)(a + 3b + 2)$$

ko'phadni standart shaklga keltiring.

A) $2a - b$ B) $a - 2b$ C) $4a + 2b$ D) $4b$

14. (99-8-10) Agar $a + b + 3 = 10$ bo'lsa,

$$3, 8a + 7, 7 + 1, 7b + 2, 5a + 11, 2 + 4, 6b$$

ifodaning qiymatini toping.

A) 53 B) 58 C) 72 D) 63

15. (06-121-4) Ifodani soddalashtirgandan keyin hosil bo'lgan ko'phadning nechta hadi bo'ladi?

$$(y^4 - y^2 + 1)(y^2 + 1) - (y - 1)(y + 2) + y^4 + y^3$$

A) 4 B) 3 C) 5 D) 6

16. $(a^4 + a^2)(a^4 - a^2)$ ifoda standart shaklga keltirilsa, u nechta haddan iborat bo'ladi?

A) 4 B) 3 C) 5 D) 2

17. $(2a - b)(4a^2 + 2ab + b^2) - b^3$ ifoda standart shaklga keltirilsa, u nechta haddan iborat bo'ladi?

A) 4 B) 3 C) 5 D) 2

2.4 Qisqa ko'paytirish formulalari

1. Ikki son yig'indisining kvadrati

$$(a + b)^2 = a^2 + 2ab + b^2.$$

2. Ikki son ayirmasining kvadrati

$$(a - b)^2 = a^2 - 2ab + b^2.$$

3. Ikki son kvadratlarining ayirmasi

$$a^2 - b^2 = (a - b)(a + b).$$

4. Ikki son yig'indisining kubi

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

5. Ikki son ayirmasining kubi

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$

6. Ikki son kublarining ayirmasi

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

7. Ikki son kublarining yig'indisi

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

1. $a^2 + 4a + 4$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2a + 1)^2$ B) $(a + 2)^2$
C) $(2a + 3)^2$ D) $(a + 0, 5)^2$

Yechish: Berilgan ko'phadni quyidagicha yozib olamiz $a^2 + 2 \cdot a \cdot 2 + 2^2$. Bu esa 1-ga ko'ra $(a + 2)^2$ dir. **Javob:** $(a + 2)^2$ (B).

2. $4a^2 + 12a + 9$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2a + 1)^2$ B) $(a + 2)^2$
C) $(2a + 3)^2$ D) $(a + 0, 5)^2$

3. $4x^4 + 20x^2 + 25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2x^2 + 5)^2$ B) $(x^2 + 2)^2$
C) $(2x^2 + 3)^2$ D) $(2x^2 + 0, 5)^2$

4. $9x^6 + 12x^3 + 4$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2x^3 + 5)^2$ B) $(x^2 + 2)^2$
C) $(3x^3 + 2)^2$ D) $(3x^3 + 0, 5)^2$

5. $4x^2 + 2x + 0, 25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2x + 5)^2$ B) $(x + 2)^2$
C) $(2x + 3)^2$ D) $(2x + 0, 5)^2$

6. $25x^6 - 10x^3 + 1$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.

- A) $(5x^3 - 1)^2$ B) $(x^2 - 2)^2$
C) $(3x^3 - 2)^2$ D) $(3x^3 - 0, 5)^2$

Yechish: Berilgan ko'phadni quyidagicha yozib olamiz $25x^6 - 10x^3 + 1 = (5x^3)^2 - 2 \cdot 5x^3 \cdot 1 + 1^2$. Bu esa 2-ga ko'ra $(5x^3 - 1)^2$ ga teng. **Javob:** $(5x^3 - 1)^2$ (A).

7. $x^2 - 6x + 9$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.

- A) $(x - 1)^2$ B) $(x^2 - 3)^2$
C) $(x - 3)^2$ D) $(3x - 0, 5)^2$

8. $2x^2 - 4x + 2$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.

- A) $(\sqrt{2}x - 1)^2$ B) $(2(x - 1))^2$
C) $(x - \sqrt{2})^2$ D) $2(x - 1)^2$

9. $4x^4 - 2x^2 + 0, 25$ ko'phadni ikki had yig'indisining kvadrati shaklida yozing.

- A) $(2x^2 - 5)^2$ B) $(x^2 - 0, 2)^2$
C) $(2x - 0, 5)^2$ D) $(2x^2 - 0, 5)^2$

10. $9x^4 - 12x^2 + 4$ ko'phadni ikki had ayirmasining kvadrati shaklida yozing.

- A) $(3x^2 - 1)^2$ B) $(3x^2 - 2)^2$
C) $(3x - 2)^2$ D) $(3x^2 - 1)^2$

11. $27a^3 + 27a^2b + 9ab^2 + b^3$ ko'phadni ikki had yig'indisining kubi shaklida yozing.

- A) $(2a + b)^3$ B) $(a + 3b)^3$
C) $(3a + b)^3$ D) $(3a - b)^3$

Yechish: Berilgan ko'phadni quyidagicha yozish mumkin $(3a)^3 + 3 \cdot (3a)^2 \cdot b + 3 \cdot 3a \cdot b^2 + b^3$. Bu esa 4-ga ko'ra $(3a + b)^3$ dir. **Javob:** $(3a + b)^3$ (C).

12. $1 + 3a + 3a^2 + a^3$ ko'phadni ikki had yig'indisining kubi shaklida yozing.

- A) $(a + 1)^3$ B) $(1 + a^2)^3$
C) $(3a + 1)^3$ D) $(1 - a)^3$

13. $x^3 + 6x^2 + 12x + 8$ ko'phadni ikki had yig'indisining kubi shaklida yozing.

- A) $(a + 3)^3$ B) $(x + 2)^3$
C) $(x - 2)^3$ D) $(a + 2)^3$

14. $8x^3 + 36x^2 + 54x + 27$ ko'phadni ikki had yig'indisining kubi shaklida yozing.

- A) $(2x + 3)^3$ B) $(3x + 2)^3$
C) $(2x - 3)^3$ D) $(2a + 3)^3$

15. $y^6 + 3y^4 + 3y^2 + 1$ ko'phadni ikki had yig'indisining kubi shaklida yozing.

- A) $(x^2 + 1)^3$ B) $(y^2 + 1)^3$
C) $(y^2 - 1)^3$ D) $(a^2 + 1)^3$

16. $1 - 3a^2 + 3a^4 - a^6$ ko'phadni ikki had ayirmasining kubi shaklida yozing.

- A) $(1 - a^2)^3$ B) $(1 + a^2)^3$
C) $(2a + 1)^3$ D) $(1 - a)^3$

Yechish: Berilgan ko'phadni quyidagicha yozish mumkin $1^3 - 3 \cdot 1 \cdot a^2 + 3 \cdot 1 \cdot (a^2)^2 - (a^2)^3$. Bu esa 5-ga ko'ra $(1 - a^2)^3$ dir. **Javob:** $(1 - a^2)^3$ (A).

17. $1 - 3b + 3b^2 - b^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.

- A) $(a - 1)^3$ B) $(1 - b^2)^3$
C) $(3b + 1)^3$ D) $(1 - b)^3$

18. $125z^3 - 75z^2 + 15z - 1$ ko'phadni ikki had ayirmasining kubi shaklida yozing.

- A) $(5a - 1)^3$ B) $(1 - 5z)^3$
C) $(5z + 1)^3$ D) $(5z - 1)^3$

19. $1 - 3y + 3y^2 - y^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.

- A) $(1 - y^2)^3$ B) $(1 + y)^3$
C) $(2y + 1)^3$ D) $(1 - y)^3$

20. $125 - 75z + 15z^2 - z^3$ ko'phadni ikki had ayirmasining kubi shaklida yozing.

- A) $(5a - 1)^3$ B) $(1 - 5z)^3$
C) $(5z + 1)^3$ D) $(5 - z)^3$

21. (96-1-17) Soddashtiring.

$$(2a - b)^2 - (2a + b)^2$$

- A) 0 B) $-2b^2$ C) $-8ab$ D) $-4ab + 2b^2$

Yechish: 1-usul. Ikki son ayirmasining kvadrati va ikki son yig'indisining kvadrati (2 va 1 ga qarang) formulalaridan foydalanamiz:

$$(2a-b)^2 - (2a+b)^2 = 4a^2 - 4ab + b^2 - (4a^2 + 4ab + b^2) = 4a^2 - 4a^2 - 4ab - 4ab + b^2 - b^2 = -8ab.$$

2-usul. Ikki son kvadratlarining ayirmasi (3 ga qarang) formulasidan foydalanamiz:

$$(2a-b)^2 - (2a+b)^2 = (2a-b-2a-b)(2a-b+2a+b) = -2b \cdot 4a = -8ab. \quad \text{Javob: } -8ab \text{ (C).}$$

22. (96-9-68) Soddashtiring.

$$(a - 3b)^2 - (a + b)^2$$

- A) $8b^2 - 8ab$ B) $8b^2$ C) $2b^2 - 8ab$ D) $-8b^2$

23. (96-9-76) Ushbu $(4x-3)^2 - x(4x+1)$ ifodani ko'phadning standart shakliga keltiring.

- A) $2x^2 + x - 9$ B) $12x^2 - 25x + 9$
C) $4x^2 - 13x$ D) $8x^2 - x + 7$

24. (96-10-18) Soddashtiring.

$$(1 - 2a)^2 + (1 + 2a)(2a - 1)$$

- A) $8a^2 - 4a$ B) $-2a$ C) $-2a + 2$ D) $8a^2$

25. (98-11-8) Soddashtiring.

$$12^2 - (x + 7)^2 - (5 - x) \cdot (19 + x)$$

- A) 0 B) 50 C) 140 D) 90

26. (96-11-20) $(b - c)(b^2 + bc + c^2)$ ifodaning $b = -2$ va $c = 1$ bo'lgandagi qiymatini hisoblang.

- A) 7 B) 5 C) -9 D) -7

Yechish: 6-qoidaga (ikki son kublarining ayirmasi) ko'ra $(b - c)(b^2 + bc + c^2) = b^3 - c^3$. Endi $b = -2$ va $c = 1$ deymiz. $(-2)^3 - 1^3 = -8 - 1 = -9$. **Javob:** -9 (C).

27. (96-12-20) $(x^2 + xy + y^2)(x - y)$ ifodaning $x = 1$ va $y = -2$ bo'lgandagi qiymatini hisoblang.

- A) 5 B) -9 C) 7 D) 9

28. (00-5-21) $(2a + 3b)(4a^2 - 6ab + 9b^2)$ ifodaning $a = 2$ va $b = 1$ dagi qiymatini toping.

- A) 91 B) 93 C) 96 D) 99

29. (00-5-23) $(x + 3)(x^2 - 3x + 9)$ ifodaning $x = \frac{1}{2}$ dagi qiymatini hisoblang.

- A) $-26,875$ B) $\frac{343}{27}$ C) $27\frac{1}{2}$ D) 27, 125

30. (97-12-9) Ifodani soddashtirgandan keyin nechta haddan iborat bo'ladi?

$$(y^3 - 1)^2 + (y^2 + 1)(y^4 - y^2 + 1)$$

- A) 4 B) 5 C) 6 D) 3

Yechish: 2 va 7-formulardan foydalansak, berilgan ifodani quyidagicha yozish mumkin $y^6 - 2y^3 + 1 + y^6 + 1^3$. O'xshash hadlarni ixchamlab $2y^6 - 2y^3 + 2$ ni olamiz. Demak, ifoda 3 ta haddan iborat bo'lar ekan. **Javob:** 3 (D).

31. Ifodani soddashtirgandan keyin unda nechta had qoladi?

$$(a - 5)(a + 5) - a^2$$

- A) 4 B) 3 C) 2 D) 1

32. Ifodani soddashtirgandan keyin unda nechta had qoladi?

$$(x^3 - 1)(x^3 + 1) + x^6$$

- A) 4 B) 3 C) 2 D) 1

33. Ifodani soddashtirgandan keyin unda nechta had qoladi?

$$(1 - 2a)(2a + 1) + 4a^2$$

- A) 4 B) 3 C) 2 D) 1

34. (03-8-44)* Agar $a + a^{-1} = 3$ bo'lsa, $a^2 + a^{-2}$ ni hisoblang?

- A) 7 B) 4 C) 9 D) 13

Yechish: $a + a^{-1} = 3$ tenglikning ikkala qismini kvadratga ko'taramiz $(a + a^{-1})^2 = 3^2$. Endi 1-qoidaga (ikki son yig'indisining kvadrati) ko'ra $a^2 + 2a \cdot a^{-1} + a^{-2} = a^2 + 2 + a^{-2} = 9$. Bu yerdan $a^2 + a^{-2} = 9 - 2 = 7$ ni olamiz. **Javob:** 7 (A).

35. (00-6-7)* Agar $a - \frac{1}{a} = \frac{2}{3}$ bo'lsa, $\frac{a^4 + 1}{a^2}$ ning qiymatini toping?

- A) $2\frac{4}{9}$ B) $1\frac{1}{3}$ C) $1\frac{5}{9}$ D) $2\frac{5}{9}$

36. (99-6-40)* $a^2 + \frac{9}{a^2} = 22$ bo'lsa, $a - \frac{3}{a}$ nimaga teng.

- A) 3 B) -3 C) 2 D) ± 4

37. (01-8-7)* Agar $a + \frac{1}{a} = 3$ bo'lsa, $\frac{a^6 + 1}{a^3}$ ning qiymatini toping.

- A) 27 B) 24 C) 18 D) $21\frac{1}{3}$

38. Agar $a - \frac{1}{a} = 2$ bo'lsa, $a^4 + \frac{1}{a^4}$ ning qiymatini hisoblang.

- A) 36 B) 34 C) 49 D) 63

39. (02-9-6)* Agar $a + \frac{1}{a} = 3$ bo'lsa, $\frac{a^4 + 1}{2a^2}$ ning qiymati nimaga teng?

- A) 3,5 B) 4 C) 5,5 D) 7

2.5 Ko'phadni ko'paytuvchilarga ajratish

Ko'phadni ko'paytuvchilarga ajratish shunday shakl almashtirishki, bunda ko'phad ikki yoki undan ortiq ko'phad yoki birhadidan iborat ko'paytmaga almashtiriladi. Ko'phadlarni ko'paytuvchilarga ajratishning eng ko'p ishlatiladigan sodda usullaridan ikkitasiga to'xtalamiz.

1) *Umumiy ko'paytuvchini qavsdan tashqariga chiqarish usuli.* Bu usulni quyidagi ikki misolda ko'pib chiqamiz.

1-misol. Ko'phadni ko'paytuvchilarga ajrating:

$$4x^4y^2 - 8x^3y^2 + 12x^2yz^2.$$

Yechish: Berilgan ko'phadning barcha hadlari uchun umumiy ko'paytuvchi $4x^2y$ ni qavsdan tashqariga chiqarsak,

$$4x^4y^2 - 8x^3y^2 + 12x^2yz^2 = 4x^2y(x^2y - 2xy + 3z^2)$$

ga ega bo'lamiz. Shunday qilib, berilgan ko'phad ikkita ko'paytuvchiga ega bo'ldi.

2-misol. Ko'phadni ko'paytuvchilarga ajrating:

$$3x^2(a - b) - 8x^3y^2(a - b) + 5xy^2(a - b).$$

Yechish: Ko'phaddagi hamma hadlar uchun $x(a - b)$ umumiy ko'paytuvchi bo'ladi. Shu sababli berilgan ko'phad quyidagicha ko'paytuvchilarga ajraladi:

$$x(a - b)(3x - 8x^2y^2 + 5y^2).$$

2) *Guruhlash usuli.* Bu usul ko'phadning barcha hadlari uchun umumiy ko'paytuvchi mavjud bo'lmagan holda qo'llaniladi.

3-misol. Ko'phadni ko'paytuvchilarga ajrating:

$$2ac + bc - 3b - 6a.$$

Yechish: Ko'phadning barcha hadlari umumiy ko'paytuvchiga ega emas. Birinchi va oxirgi hadlarining umumiy ko'paytuvchisi $2a$ ga, ikkinchi va uchinchi hadlarining umumiy ko'paytuvchisi b ga teng. Birinchi va oxirgi hadlaridan $2a$ ni, ikkinchi va uchinchi hadlaridan b ni qavs oldiga chiqaramiz. Natijada quyidagini olamiz:

$$2ac + bc - 3b - 6a = 2a(c - 3) + b(c - 3) = (c - 3)(2a + b).$$

1. Ko'phadni ko'paytuvchilarga ajrating.

$$2x^2y - 2xy^2$$

- A) $2xy(y - x^2)$ B) $2xy(x - y)$
C) $2xy(x - y^2)$ D) $2xy(y - x)$

Yechish: Misolni yechishda 1-usuldan foydalanamiz. Berilgan ko'phadning barcha hadlari uchun umumiy ko'paytuvchi $2xy$ ni qavsdan tashqariga chiqaramiz

$$2x^2y - 2xy^2 = 2xy(x - y).$$

Javob: $2xy(x - y)$ (B).

2. Ko'phadni ko'paytuvchilarga ajrating.

$$2x^2y^7 - 8x^5y^5$$

- A) $2x^2y^5(y^2 - x^3)$ B) $2x^2y^5(x^2 - 4y^3)$
C) $2x^2y^5(x^2 - y^3)$ D) $2x^2y^5(y^2 - 4x^3)$

3. Ko'phadni ko'paytuvchilarga ajrating.

$$x^2y^7 - x^5y^5 + 2x^3y^5$$

- A) $x^2y^5(y^2 - x^3 + xy)$ B) $x^2y^5(x^2 - y^3 + 2xy)$
C) $x^2y^5(x^2 - y^3 + 2x)$ D) $x^2y^5(y^2 - x^3 + 2x)$

4. Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b^3 - 6ab^2$$

- A) $2ab^2(ab - 5)$ B) $2ab^2(ab - 3)$
C) $2ab(3 - ab)$ D) $2a^2b(ab - 3)$

5. Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b^3 - 6ab^2 + 8a^2b^2$$

- A) $2ab^2(ab - 5 + 4a)$ B) $2ab^2(ab - 3 + 4a)$
C) $2ab(3 - ab + 4a)$ D) $2a^2b(ab - 3 + 4a)$

6. (98-1-18) Ko'phadni ko'paytuvchilarga ajrating.

$$2a^2b - 3a + 10ab^2 - 15b$$

- A) $(2ab + 3)(a - 5b)$ B) $(a + 5b)(2ab - 3)$
C) $(3 + ab)(2a - 5b)$ D) $(2a^2 + b)(b - 5a)$

Yechish: Misolni yechishda 2-usul, guruhlash usulidan foydalanamiz. Berilgan ko'phadda ikkinchi va uchinchi hadlari o'rinlarini almashiramiz:

$$2a^2b - 3a + 10ab^2 - 15b = 2a^2b + 10ab^2 - 3a - 15b.$$

Birinchi va ikkinchi hadlaridan $2ab$ ni, uchinchi va oxirgi hadlaridan -3 ni qavs oldiga chiqaramiz. Natijada quyidagini olamiz:

$$2a^2b + 10ab^2 - 3a - 15b = 2ab(a + 5b) - 3(a + 5b).$$

Endi umumiy ko'paytuvchi $a + 5b$ ni qavs oldiga chiqaramiz $(a + 5b)(2ab - 3)$.

Javob: $(a + 5b)(2ab - 3)$ (B).

7. (98-8-18) Ushbu $2n^2 - 3an - 10n + 15a$ ko'phadni ko'paytuvchilarga ajrating.

- A) $(5 - n)(3a - 2n)$ B) $(5 + n)(2n - 3a)$
C) $(3a - n)(5 - 2n)$ D) $(2n + 3a)(n + 5)$

8. (00-6-18) $4y(5x - y) - (5x - 2)(5x + 2)$ ning eng katta qiymatini toping.

- A) 10 B) 5 C) 4 D) 2

9. (97-1-13) Ushbu $1 - (2x - 3)^2$ ifodani ko'paytuvchilarga ajrating.

- A) $2(x + 2)(x + 1)$ B) $3(x - 2)(x + 1)$
C) $4(2 - x)(x - 1)$ D) $2(1 - x)(x - 2)$

Yechish: Misolni yechishda qisqa ko'paytirish formulasining 3-dan foydalanamiz.

$$1 - (2x - 3)^2 = 1^2 - (2x - 3)^2 = (1 - 2x + 3)(1 + 2x - 3) = (4 - 2x)(2x - 2) = 2(2 - x) \cdot 2(x - 1) = 4(2 - x)(x - 1).$$

Javob: $4(2 - x)(x - 1)$ (C).

10. (97-6-13) Ushbu $9 - (2c - 1)^2$ ifodani ko'paytuvchilarga ajrating.
 A) $2(c - 1)(c + 2)$ B) $4(c - 2)(c + 1)$
 C) $(3c - 1)(c + 4)$ D) $4(c + 1)(2 - c)$

11. (97-10-18) Ko'paytuvchilarga ajrating.

$$(x^2 + 9)^2 - 36x^2$$

- A) $(x^2 - 5)(x^2 + 4)$ B) $(x - 3)^2 \cdot (x + 3)^2$
 C) $(x - 6)^2 \cdot (x + 6)^2$ D) $x^2(x^2 - 6)$

12. (97-11-13) Ifodani ko'paytuvchilarga ajrating.

$$1 - (8a - 3)^2$$

- A) $8(4a + 1) \cdot (1 - 2a)$ B) $(16a - 1) \cdot (4a - 3)$
 C) $4(2a + 1) \cdot (4a - 1)$ D) $8(1 - 2a) \cdot (4a - 1)$

13. (96-7-18) Ko'paytuvchilarga ajrating.

$$(a^2 + 16)^2 - 64a^2$$

- A) $(a^2 - 8) \cdot (a^2 + 4)$ B) $(a - 2)^2 \cdot (a + 2)^2$
 C) $(a - 4)^2 \cdot (a + 4)^2$ D) $a^2 \cdot (a^2 - 60)$

14. $9a^4 - 1$ ni ko'paytuvchilarga ajrating.

- A) $(3a^2 - 1)(3a^2 + 1)$ B) $(9a^2 - 1)(a^2 + 1)$
 C) $(3a - 1)(3a + 1)$ D) $(9a^2 - 1)(a^2 - 1)$

Yechish: Berilgan $9a^4 - 1$ ko'phadni quyidagicha yozib olamiz $9a^4 - 1 = (3a^2)^2 - 1^2$. Endi 3-formuladan foydalansak, $(3a^2)^2 - 1^2 = (3a^2 - 1)(3a^2 + 1)$ ni olamiz. **Javob:** $(3a^2 - 1)(3a^2 + 1)$ (A).

15. $25a^4 - 9b^2$ ni ko'paytuvchilarga ajrating.

- A) $(5a^2 - 3)(5a^2 + 3b)$ B) $(5a^2 - 3b)(5a^2 + 3b)$
 C) $(5a - 3b)(5a + 3b)$ D) $(25a^2 - b)(a^2 - 9b)$

16. (01-8-8) Ko'paytuvchilarga ajrating.

$$(a + b)(a + b + 2) - (a - b)(a - b - 2)$$

- A) $2(a + b)(b + 1)$ B) $4a(b + 1)$
 C) $2a(b - 1)$ D) $4a(b - 1)$

17. $y^4 - 9$ ni ko'paytuvchilarga ajrating.

- A) $(y^2 - 1)(y^2 + 9)$ B) $(y^2 - 9)(y^2 + 1)$
 C) $(y^2 - 3)(y^2 + 3)$ D) $(y^2 - 3)(y^2 - 3)$

18. $a^6 - b^6c^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

Yechish: Berilgan $a^6 - b^6c^6$ ko'phadni quyidagicha yozib olamiz $a^6 - b^6c^6 = (a^3)^2 - (b^3c^3)^2$. Endi 3-formuladan foydalansak, $a^6 - b^6c^6 = (a^3 - b^3c^3)(a^3 + b^3c^3)$ ni olamiz. Biribchi qavsga 6-formulani, ikkinchi qavsga 7-formulani qo'llaymiz. Natijada $a^6 - b^6c^6 = (a - bc)(a^2 + abc + b^2c^2)(a + bc)(a^2 - abc + b^2c^2)$. **Javob:** 4 (C).

19. $a^6 + b^6c^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

20. $y^6 - 64$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

21. $27^2x^6 - y^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

22. $9^3x^6 - y^6$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

23. $x^6y^6 - 4^3$ ko'phad nechta ratsional koeffitsiyentli ko'paytuvchiga ajraladi?

- A) 2 B) 3 C) 4 D) 6

24. $(x + 3y)^3 + (x - 3y)^3 - 52xy^2$ ko'phadni ko'paytuvchilarga ajrating.

- A) $2x(x^2 + y^2)$ B) $2y(x^2 + y^2)$
 C) $2x(x^2 - y^2)$ D) $x(x^2 - y^2)$

25. Quyidagilardan qaysi biri $x^5 - 16x$ ko'phadning ko'paytuvchisi emas.

- A) x B) $x - 2$ C) $x + 2$ D) $x + 1$

Yechish: Berilgan $x^5 - 16x$ ko'phadni ko'paytuvchilarga ajratish uchun umumiy ko'paytuvchi x ni qavs oldiga chiqaramiz, x^4 ni $(x^2)^2$ deb, 16 ni esa 4^2 deb yozib olamiz. Natijada, $x^5 - 16x = x((x^2)^2 - 4^2)$ ga ega bo'lamiz. $(x^2)^2 - 4^2$ ifodaga ikki son kvadratlarining ayirmasi uchun formulasini qo'llab $x^5 - 16x = x(x^2 - 4)(x^2 + 4) = x(x - 2)(x + 2)(x^2 + 4)$ ni olamiz. Demak, x , $x - 2$ va $x + 2$ lar $x^5 - 16x$ ko'phadning ko'paytuvchilari ekan. **Javob:** $x + 1$ (D).

26. Quyidagilardan qaysi biri $x^5 + x^3 + x$ ko'phadning ko'paytuvchisi emas.

- A) x B) $x^2 - x + 1$ C) $x + 2$ D) $x^2 + x + 1$

27. (99-4-16)* Ko'paytuvchilarga ajrating.

$$(a + b + 2) \cdot (a + b) - (a - b)^2 + 1$$

- A) $(a + b)(2a - 1)$ B) $(a + 1)(b + 1)$
 C) $2b(a + 1)$ D) $(2b + 1)(2a + 1)$

28. (99-10-7)* Ko'paytuvchilarga ajrating.

$$a^5 + a^4 - 2a^3 - 2a^2 + a + 1$$

- A) $(a + 1)^2 \cdot (a - 1)^3$ B) $(a + 1)^3 \cdot (a - 1)^2$
 C) $(a + 1)^4 \cdot (a - 1)$ D) $(a + 1) \cdot (a - 1)^4$

29. (00-6-9) Ko'paytuvchilarga ajrating.

$$b^2 + ab - 2a^2 - b + a$$

- A) $(a - b)(2a - b)$ B) $(a + b)(2a - b - 1)$
 C) $(a - b)(2a - b - 1)$ D) $(b - a)(2a + b - 1)$

30. (00-10-77)* Ko'phadni ko'paytuvchilarga ajrating.

$$(x - y)^3 - (z - y)^3 + (z - x)^3$$

- A) $3(x - y)(y - z)(x - z)$ B) $-3(x - y)(z - y)(x - z)$
 C) $3(y - x)(y - z)(z - x)$ D) $-3(x - y)(z - y)(z - x)$

2.6 Algebraik kasrlar

Bizga noldan farqli P va Q ko'phadlar berilgan bo'lsin. Algebraik kasr deb P ko'phadni nolga teng bo'lmagan Q ko'phadga bo'lishdan hosil bo'lgan $\frac{P}{Q}$ bo'linmaga aytiladi. P ko'phad $\frac{P}{Q}$ algebraik kasrning surati, Q ko'phad esa maxraji deyiladi. Algebraik kasrlarga misollar keltiramiz:

$$\frac{3a^2 - 2a}{a^2 + 12a + 36}; \frac{3a + 16}{a^2 - 36}; \frac{6(a^2 - 36)}{a + 6}; \frac{a^2 + b}{7}.$$

$\frac{P}{Q}$ va $\frac{M}{N}$ algebraik kasrlar ustida arifmetik amallar quyidagicha bajariladi:

$$1. \frac{P}{Q} + \frac{M}{N} = \frac{PN + QM}{QN}.$$

$$2. \frac{P}{Q} - \frac{M}{N} = \frac{PN - QM}{QN}.$$

$$3. \frac{P}{Q} \cdot \frac{M}{N} = \frac{PM}{QN}.$$

$$4. \frac{P}{Q} : \frac{M}{N} = \frac{PN}{QM}.$$

1. (96-12-72) Soddashtiring.

$$\frac{x^3 + x^2 + x + 1}{x^2 + 1}$$

- A) $x - 1$ B) x C) $2x$ D) $x + 1$

Yechish: Berilgan kasrning suratida quyidagicha shakl almashtirish qilamiz $x^3 + x^2 + x + 1 = x^3 + x + x^2 + 1 = x(x^2 + 1) + (x^2 + 1)$. Umumiy ko'paytuvchi sifatida $x^2 + 1$ ni qavs oldiga chiqaramiz, natijada

$$\frac{x(x^2 + 1) + (x^2 + 1)}{x^2 + 1} = \frac{(x^2 + 1)(x + 1)}{x^2 + 1} = x + 1.$$

Javob: $x + 1$ (D).

2. (96-9-15) Soddashtiring.

$$\frac{1 - x^{-1} + x^{-2}}{1 - x + x^2}$$

- A) 1 B) x^2 C) $\frac{1}{x^2}$ D) $1 - \frac{1}{x}$

3. (97-4-21) Soddashtiring.

$$\frac{a^{-3} + b^{-3}}{a^2 - ab + b^2} \cdot a^3 \cdot b^3$$

- A) $(a + b)^2$ B) 1 C) ab D) $a + b$

4. (96-3-21) Kasrni qisqartiring:

$$\frac{x^2 - 3xy}{9y^2 - x^2}$$

- A) $\frac{x}{x + 3y}$ B) $\frac{-x}{x + 3y}$ C) $\frac{x}{x - 3y}$ D) $\frac{-x}{x - 3y}$

5. (96-3-74) Soddashtiring:

$$\frac{x^3 + 2x^2 + x}{(x + 1)^2}$$

- A) $2x$ B) $x + 1$ C) $x + 2$ D) x

6. (98-11-9) Soddashtiring.

$$\frac{x^6 - x^4}{x^3 + x^2}$$

- A) $x^3 - x^2 + 1$ B) $x^3 + x^2 + 1$
C) $x^3 - x^2$ D) $x^3 + x^2$

7. (00-8-54) Soddashtiring.

$$\frac{a^8 - a^4}{a^4 + a^2}$$

- A) a^6 B) $a^4 - a^2$ C) $a^4 - 1$ D) $a^4 + a^2$

Yechish: Berilgan kasrning suratini quyidagicha yozib olamiz

$$a^8 - a^4 = (a^4)^2 - (a^2)^2 = (a^4 - a^2)(a^4 + a^2).$$

Bu yerdan

$$\frac{a^8 - a^4}{a^4 + a^2} = \frac{(a^4 - a^2)(a^4 + a^2)}{a^4 + a^2} = a^4 - a^2.$$

Javob: $a^4 - a^2$ (B).

8. (99-1-10) Soddashtiring.

$$\frac{p - q}{p^3 \cdot q^2} - \frac{p + q}{p^2 \cdot q^3}$$

- A) $-\frac{p^2 + q^2}{p^3 \cdot q^3}$ B) $\frac{2pq - p^2 - q^2}{p^3 \cdot q^3}$
C) $-\frac{2}{p^3 \cdot q^2}$ D) $-\frac{2}{p^3 \cdot q - p^2 \cdot q^2}$

9. (99-6-5) Soddashtiring.

$$\left(\frac{-16x^{31}}{9y^3}\right)^3 : \left(\frac{8x^{23}}{3y^2}\right)^4$$

- A) $\frac{-y}{x}$ B) $\frac{-x}{y}$ C) $\frac{x}{9y}$ D) $\frac{-x}{9y}$

10. (01-2-14) Soddashtiring.

$$\frac{a^2 + \frac{1}{a}}{a + \frac{1}{a} - 1}$$

- A) $a - 1$ B) $a^2 - a + 1$
C) $a^2 + a + 1$ D) $a + 1$

11. (02-8-2) Soddashtiring.

$$\frac{1 - b^{-1} + b^{-2}}{1 - b + b^2}$$

- A) b^{-1} B) b^{-2} C) b^2 D) $b + 1$

2.7 Ratsional ifodalar

Ratsional ifoda deb shunday ifodaga aytiladiki, unda bir nechta algebraik kasrlar arifmetik amal belgilari bilan bog'langan bo'ladi va ifodada nolga teng ko'phadga bo'lish ishtirok etmaydi. Masalan,

$$\left(\frac{5m}{m+3} - \frac{14m}{m^2+6m+9}\right) : \frac{5m+1}{m^2-9} + \frac{3 \cdot (m-3)}{m+3}.$$

1. (97-10-19) Soddashtiring:

$$\left(\frac{3a}{a+6} - \frac{2a}{a^2+12a+36}\right) : \frac{3a+16}{a^2-36} + \frac{6(a-6)}{a+6}$$

A) 6 B) $\frac{6}{a+6}$ C) $\frac{1}{a-6}$ D) $a-6$

Yechish: Qavs ichidagi kasrlarni umumiy max-rajga keltiramiz:

$$\begin{aligned} \frac{3a}{a+6} - \frac{2a}{a^2+12a+36} &= \frac{3a}{a+6} - \frac{2a}{(a+6)^2} = \\ &= \frac{3a^2+18a-2a}{(a+6)^2} = \frac{3a^2+16a}{(a+6)^2}. \end{aligned}$$

Birinchi kasrning suratidan umumiy ko'paytuvchi a ni qavs oldiga chiqarib, keyingi amallarni bajaramiz:

$$\begin{aligned} \frac{a(3a+16)}{(a+6)^2} \cdot \frac{(a-6)(a+6)}{3a+16} + \frac{6(a-6)}{a+6} &= \\ = \frac{a(a-6)}{a+6} + \frac{6(a-6)}{a+6} &= \frac{(a-6)(a+6)}{a+6} = a-6. \end{aligned}$$

Javob: $a-6$ (D).

2. (96-7-19) Soddashtiring:

$$\left(\frac{5m}{m+3} - \frac{14m}{m^2+6m+9}\right) : \frac{5m+1}{m^2-9} + \frac{3 \cdot (m-3)}{m+3}$$

A) $\frac{3}{m+3}$ B) 3 C) $m-3$ D) 1

3. (97-7-19) Soddashtiring.

$$\left(\frac{2x}{x-5} + \frac{x}{x^2-10x+25}\right) : \frac{2x-9}{x^2-25} - \frac{5(x+5)}{x-5}$$

A) 5 B) $\frac{x+5}{x-5}$ C) $\frac{5}{x+5}$ D) $5+x$

4. (98-1-21) Soddashtiring.

$$\left(\frac{4a}{4-a^2} - \frac{a-2}{4+2a}\right) \cdot \frac{4}{a+2} - \frac{a}{2-a}$$

A) -1 B) $\frac{2a}{2-a}$ C) $\frac{3+a}{2-a}$ D) 1

5. (98-2-8) Soddashtiring.

$$\frac{x^3-8}{x^2+2x+4} - \frac{x^2-4}{x-2}$$

A) 4 B) $2x$ C) $-2x$ D) -4

6. (98-2-29) Ushbu

$$\frac{x^{-3}+8}{x^{-2}-2x^{-1}+4}$$

ifodaning $x=0,5$ dagi qiymatini hisoblang.

A) 4,5 B) 3 C) 4 D) 5

Yechish: $x^{-1} = y$ almashtirish olsak, berilgan ifoda quyidagi ko'rinishga keladi:

$$\frac{y^3+2^3}{y^2-2y+2^2} = \frac{(y+2)(y^2-2y+2^2)}{y^2-2y+2^2} = y+2.$$

Agar $x=0,5$ bo'lsa, u holda $y=2$ bo'ladi va natijada biz $y+2=2+2=4$ ni olamiz. **Javob:** 4 (C).

7. (01-11-6) Ushbu

$$\frac{a^3+b^3}{a^2-ab+b^2} \cdot (a-b) \frac{a^3-b^3}{a^2+ab+b^2} \cdot (a+b)$$

ifodaning $a=3$ va $b=2$ bo'lgandagi qiymatini hisoblang.

A) 24 B) 25 C) 30 D) 32

8. (98-10-12) Soddashtiring.

$$\frac{x^3+y^3}{x^2-xy+y^2} - \frac{x^2-y^2}{x+y}$$

A) $2x$ B) $2y$ C) $-2y$ D) $-2x$

9. (99-4-26) Ifodani soddashtiring.

$$\frac{5x+6}{x^2-4} - \frac{x}{x^2-4} : \frac{x}{x-2} - \frac{x+2}{x-2}$$

A) 1 B) -1 C) $\frac{x-2}{x+2}$ D) $\frac{x^2+4}{4-x^2}$

10. (99-9-19) Soddashtiring.

$$\left(\frac{1}{a(a+1)} + \frac{1}{(a+1)(a+2)}\right) \cdot \frac{a^2+2a}{8}$$

A) $\frac{1}{6}$ B) $\frac{1}{8}$ C) $\frac{3}{4}$ D) $\frac{1}{4}$

11. (00-3-16) Soddashtiring.

$$\left(\frac{a^2-4}{a^2+4}\right)^2 + \left(\frac{4a}{a^2+4}\right)^2$$

A) $a-4$ B) 2 C) $\frac{a^2-4}{a^2+4}$ D) 1

12. (00-7-13) Soddashtiring.

$$(a^3-3a^2b+3ab^2-b^3) \cdot (a+b) : \left(\frac{a^3+b^3}{a+b} - ab\right)$$

A) b^2-a^2 B) a^2-b^2 C) $(a-b)^2$ D) $(a+b)^2$

13. (01-5-6) Soddashtiring.

$$\frac{x}{x^2+y^2} - \frac{y \cdot (x-y)^2}{x^4-y^4}$$

A) $\frac{1}{x+y}$ B) $\frac{1}{x-y}$ C) $x+y$ D) $x-y$

14. (01-6-10) Soddalashtiring.

$$\left(2a + \frac{2ab}{a-b}\right) \left(\frac{ab}{a+b} - a\right) : \frac{4,5a^2}{a^2 - b^2}$$

A) $\frac{4a^2}{9}$ B) $-\frac{2a^2}{9}$ C) $\frac{2a^2}{9}$ D) $-\frac{4a^2}{9}$

15. (01-8-18) Soddalashtiring.

$$\frac{a^2}{a^2 - 1} + \frac{1}{a + 1} : \left(\frac{1}{2 - a} + \frac{2}{a^2 - 2a}\right)$$

A) $\frac{a}{a^2 - 1}$ B) $\frac{1}{a - 1}$ C) $\frac{2a^2 - a}{a^2 - 1}$ D) 1

16. (02-9-14) Soddalashtiring.

$$\left(\frac{2}{1 - x^2} - \frac{2}{(x - 1)^2}\right) \cdot (1 - x)^2 - \frac{4}{1 + x}$$

A) 4 B) -4 C) 0 D) $\frac{1 - x}{1 + x}$

17. (03-4-10) Soddalashtiring.

$$\left(\frac{a + x}{a} - \frac{x - y}{x}\right) \cdot \frac{a^2}{x^2 + ay} : \frac{a}{8x}$$

A) 10 B) 6 C) 7 D) 8

18. (03-6-7) Soddalashtiring.

$$\frac{x^3y + 2x^2y - 3xy}{x^3 + 5x^2 + 6x} : \frac{1 - x^2}{x^2 + 3x + 2}$$

A) $\frac{y}{x}$ B) $-x$ C) $-y$ D) x

19. (03-7-10) Soddalashtiring.

$$\frac{x^3y + 2x^2y - 3xy}{x^3 + 5x^2 + 6x} : \frac{x^2 - 1}{x^2 + 3x + 2}$$

A) $\frac{y}{x}$ B) $-x$ C) $-y$ D) y

2.8 Aralash tipdagi masalalar

1. (97-9-80) Hisoblang.

$$\frac{1000^3 + 3 \cdot 1000 \cdot 995 \cdot 1995 + 995^3}{1000^2 + 2 \cdot 1000 \cdot 995 + 995^2}$$

A) 1995 B) 195 C) 995 D) 2195

Yechish: Kasr suratidagi 1995 ni $1000 + 995$ shaklda yozib, qisqa ko'paytirish formulasining 7-dan foydalansak,

$$1000^3 + 3 \cdot 1000^2 \cdot 995 + 3 \cdot 1000 \cdot 995^2 + 995^3 = \\ = (1000 + 995)^3$$

ni olamiz. Kasrning maxraji esa qisqa ko'paytirish formulasining 1-ga ko'ra $(1000 + 995)^2$ ga teng. Shunday ekan

$$\frac{(1000 + 995)^3}{(1000 + 995)^2} = 1995.$$

Javob: 1995 (A).

2. (99-7-2) Hisoblang.

$$889^3 + 3000 \cdot 889 \cdot 111 + 111^3 + 889 + 111$$

A) 10001000 B) 1001000
C) 1001001000 D) 1000001000

3. (96-1-13) Hisoblang.

$$\frac{1^2 - 0,4^2}{2,8 \cdot 0,4 - 2,8}$$

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) -5 D) 5

4. (00-6-5) Soddalashtiring.

$$\frac{1,6^2 - 1,6 \cdot 0,8 + 0,4^2}{1,4^2 - 0,2^2}$$

A) 1,6 B) 0,375 C) 1,2 D) 0,75

5. (96-10-13) Hisoblang.

$$\frac{4,5^2 - 1,5^2}{0,3 \cdot 0,7 - 0,3}$$

A) -20 B) 20 C) 200 D) -200

6. Hisoblang.

$$\frac{10^{45} + 10^{46} + 10^{50}}{10^{49} + 10^{45} + 10^{44}}$$

A) 10 B) 30 C) 100 D) 1000

7. (98-7-10) Hisoblang.

$$\frac{(3,7^2 - 6,3^2) \cdot (13^2 - 12,6^2)}{(4,2^2 - 5,8^2) \cdot (2,3^2 - 0,3^2)}$$

A) 32 B) 0,32 C) 3,2 D) $\frac{1}{32}$

8. (98-8-9) Hisoblang.

$$\frac{0,5^2 - 0,5}{0,4^2 + 2 \cdot 0,04 + 0,1^2}$$

A) 1 B) -1 C) -0,1 D) 10

9. (98-1-9) Hisoblang.

$$\frac{0,2^2 - 2 \cdot 0,06 + 0,3^2}{0,5 \cdot 0,9 - 0,5}$$

A) 0,2 B) -2 C) -0,2 D) 0,25

10. (99-6-6) Hisoblang.

$$(202^2 - 54^2 + 256 \cdot 352) : (4^4 \cdot 10^2)$$

A) 4 B) 1 C) 2 D) 5

Yechish: $202^2 - 54^2$ ayirmani ikki son kvadratlarining ayirmasi uchun 2.4 ning 3-formulasiga ko'ra $202^2 - 54^2 = 148 \cdot 256$. Endi birinchi qavs ichini hisoblaymiz $148 \cdot 256 + 256 \cdot 352 = 256(148 + 352) = 256 \cdot 500$. Endi bo'linmani hisoblaymiz $256 \cdot 500 : (256 \cdot 100) = 5$. **Javob:** 5 (D).

11. (97-4-14) Agar $x = 4,5$ va $y = 3,5$ bo'lsa,

$$x^3 - x^2y - xy^2 + y^3$$

ni hisoblang.

- A) 10 B) 9,5 C) 8 D) 7,2

12. (01-8-5) Hisoblang.

$$\frac{0,6 \cdot 0,8 + 0,6 \cdot 1,2}{0,2^2 - 0,4^2}$$

- A) -10 B) 10 C) -0,1 D) -100

13. (02-5-6) Hisoblang.

$$\frac{2,7(1,7^3 - 1,5^3)}{5,1^2 + 5,1 \cdot 4,5 + 4,5^2}$$

- A) 0,45 B) 0,27 C) 0,3 D) 0,06

14. (03-11-61) Hisoblang.

$$\frac{0,6^2 - 0,6 \cdot 0,2 + 0,1^2}{1,5 - 1,5^2}$$

- A) -0,5 B) $-\frac{1}{3}$ C) -3 D) $-\frac{2}{3}$

15. (00-10-12) Soddashtiring.

$$\frac{5 \cdot 2^{k-2} + 10 \cdot 2^{k-1}}{10^{k+2}}$$

- A) $4^{-1} \cdot 5^{-k}$ B) $4^{-2} \cdot 5^{-k}$
C) $4 \cdot 5^{-k}$ D) $2^{-1} \cdot 5^{-k}$

16. (00-10-74) Kasrni qisqartiring.

$$\frac{2^{m+1} + 2^{-m+1}}{(4^m + 1)(3^{m+2} + 3^{m+1})}$$

- A) $0,5 \cdot 6^{-m}$ B) $\left(\frac{2}{3}\right)^m$ C) 6^{-m-1} D) 3^m

17. (97-2-6) Ushbu $\frac{12-3n}{n}$ ifoda n ning nechta natural qiymatida natural son bo'ladi?

- A) 6 B) 3 C) 5 D) 4

Yechish: Ushbu $\frac{12-3n}{n} = \frac{12}{n} - \frac{3n}{n} = \frac{12}{n} - 3$ tenglik o'rinli. Bu ayirma natural bo'lishi uchun $\frac{12}{n}$ bo'linma 3 dan katta natural son bo'lishi kerak. Bulardan $n = 1, 2, 3$ ekanligini olamiz. Ular 3 ta. **Javob:** 3 (B).

18. (97-8-6) $\frac{10n-24}{n}$ ifoda natural son bo'ladigan n ning natural qiymatlari nechta?

- A) 4 B) 7 C) 6 D) 5

19. (97-12-5) Ifoda natural son bo'ladigan n ning barcha natural qiymatlari nechta?

$$\frac{16n^2 - 128}{n^2}$$

- A) 5 B) 3 C) 2 D) 6

20. (98-8-11) Ushbu $\frac{3n-1}{n+2}$ ifoda n ning nechta butun qiymatida natural son bo'ladi?

- A) 1 B) 3 C) 4 D) 2

21. (96-6-6) $\frac{6n-12}{n}$ ifoda n ning nechta natural qiymatida natural son bo'ladi?

- A) 6 B) 5 C) 3 D) 4

22. (03-10-10) a ning nechta butun qiymatida

$$\frac{a^4 - 9}{a^3 - 3a} : \frac{a^3 + 3a}{a - 5a^2}$$

ifodaning qiymati butun son bo'ladi?

- A) 2 B) 3 C) 1 D) 4

23. (01-7-7) n ning nechta butun qiymatida $\frac{n^2 - n + 3}{n + 1}$ kasr butun son bo'ladi?

- A) 1 B) 2 C) 3 D) 4

24. (01-11-30) Ushbu

$$\frac{4^{a+1} - 2^{2a-1}}{2^{2a}}$$

ifodaning qiymati 9 dan qancha kam?

- A) 4 B) 3,5 C) 3 D) 5,5

Yechish: $4^{a+1} = (2^2)^{a+1} = 2^{2a+2}$ desak, u holda berilgan kasr quyidagiga teng bo'ladi:

$$\frac{2^{2a+2} - 2^{2a-1}}{2^{2a}} = \frac{2^{2a}(2^2 - 2^{-1})}{2^{2a}} = 4 - \frac{1}{2} = 3,5.$$

Bu qiymatni 9 dan ayirib, ya'ni $9 - 3,5 = 5,5$ ni olamiz. **Javob:** 5,5 (D).

25. (02-1-39) Agar $\frac{x}{y} = 2$ bo'lsa, $x^2 - 4y^2$ nimaga teng?

- A) 4 B) 8 C) 0 D) -8

26. (02-6-2) Agar $a + b + c = 0$ bo'lsa, $a^3 + a^2c - abc + b^2c + b^3$ ifodaning qiymatini toping.

- A) 0 B) 1 C) 2 D) -1

27. Soddashtiring.

$$\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-d}{cd} + \frac{d-a}{ad}$$

- A) 1 B) $abcd$ C) $(abcd)^{-1}$ D) 0

28. m ning qanday butun qiymatida quyidagi ifodani qisqartirish mumkin:

$$\frac{x^2 + mx + 36}{x^2 + 8x + 7}$$

- A) -37 B) -36 C) -35 D) 37

29. Kasr qisqarishi mumkin bo'lsa, m ning eng katta va eng kichik qiymatlari farqini toping.

$$\frac{x^3 - x^2 - 4x + 4}{x^2 + mx + 6}$$

- A) 12 B) 5 C) 7 D) 10

3 - bob. Ildizlar

3.1 Arifmetik ildiz. Kvadrat ildiz

Kvadrati a ga teng bo'lgan b son a sonning kvadrat ildizi deyiladi. Kvadrat ildizdan b sonni topish a son dan kvadrat ildiz chiqarish deyiladi. $a \geq 0$ sonning kvadrat ildizi quyidagicha belgilanadi:

$$\sqrt{a} = b. \quad (3.1)$$

Bu yerda a ildiz ostidagi ifoda deyiladi. (3.1) tenglik a va b sonlari orasida $b^2 = a$ bog'lanish borligini bildiradi. Agar $a < 0$ bo'lsa, u holda $b^2 = a$ tenglikni qanoatlantiruvchi haqiqiy b soni mavjud emas. Shuning uchun kvadrat ildiz haqida so'z borganda, kvadrat ildiz ostidagi ifodaning manfiy maslasi nazarda tutiladi. Ixtiyoriy $a \geq 0$ son uchun $b^2 = a$ tenglikni qanoatlantiruvchi yagona $b \geq 0$ son mavjud. Bu son a sonining arifmetik ildizi deyiladi. Masalan, $\sqrt{16} = 4$, ya'ni 4 soni 16 ning arifmetik ildizidir. Ko'pincha ildiz atamasi o'rniga radikal termini ham ishlatiladi. Kvadrat ildiz quyidagi xossalarga ega.

1. $(\sqrt{a})^2 = a, \quad a \geq 0.$
2. $\sqrt{ab} = \sqrt{a}\sqrt{b}, \quad a \geq 0, b \geq 0.$
3. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad a \geq 0, b > 0.$
4. $a\sqrt{b} = \sqrt{a^2b}, \quad a \geq 0.$
5. $a\sqrt{b} = -\sqrt{a^2b}, \quad a \leq 0.$
6. $\sqrt{a^2} = |a|, \quad a \in R.$

1. (97-11-8) Soddashtiring.

$$2\sqrt{5\frac{1}{2}} + \frac{1}{3}\sqrt{99} - 2\sqrt{2\frac{3}{4}}$$

- A) $3\sqrt{11}$ B) $2\sqrt{22}$ C) $\sqrt{22}$ D) 2

Yechish: 4-formuladan foydalanib,

$$2\sqrt{5\frac{1}{2}} + \frac{1}{3}\sqrt{99} - 2\sqrt{2\frac{3}{4}} = \sqrt{4 \cdot \frac{11}{2}} + \sqrt{\frac{1}{9} \cdot 11} -$$

$$-\sqrt{4 \cdot \frac{11}{4}} = \sqrt{22} + \sqrt{11} - \sqrt{11} = \sqrt{22}$$

ekanini hosil qilamiz. **Javob:** $\sqrt{22}$ (C).

2. (02-2-6) Hisoblang.

$$3\sqrt{\frac{1}{5}} + \frac{1}{2}\sqrt{20} + \sqrt{\frac{4}{5}}$$

- A) $2\sqrt{5}$ B) $\sqrt{5}$ C) $3\sqrt{5}$ D) $\frac{6}{\sqrt{5}}$

3. (02-9-10) Hisoblang.

$$3\sqrt{3\frac{2}{3}} - \sqrt{132} + 4\sqrt{2\frac{1}{16}}$$

- A) 0 B) $2\sqrt{33}$ C) $3\sqrt{3}$ D) $4\sqrt{11}$

4. (97-1-8) Hisoblang.

$$4\sqrt{3\frac{1}{2}} - 0,5\sqrt{56} - 3\sqrt{1\frac{5}{9}}$$

- A) $2\sqrt{14}$ B) $2\sqrt{7}$ C) 0 D) 2

5. (97-6-8) Hisoblang.

$$15\sqrt{\frac{3}{5}} - 0,5\sqrt{60} + 2\sqrt{3\frac{3}{4}}$$

- A) 0 B) $\sqrt{15}$ C) $5\sqrt{3}$ D) $3\sqrt{15}$

6. (98-6-9) Hisoblang.

$$\frac{\sqrt{32} + \sqrt{98} - \sqrt{50}}{\sqrt{72}}$$

- A) 2 B) 1 C) $\sqrt{2}$ D) $2\sqrt{2}$

7. (99-6-36) Hisoblang.

$$\sqrt{192} - \sqrt{108} + \frac{\sqrt{243}}{3}$$

- A) $5\sqrt{3}$ B) $5\sqrt{2}$ C) $3\sqrt{5}$ D) $3\sqrt{3}$

8. (01-1-6) Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{3\sqrt{5} - 2\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$$

- A) $\frac{1}{2}(\sqrt{5} + 3\sqrt{2})$ B) $\frac{1}{2}(3\sqrt{5} - 2\sqrt{2})$
 C) $9 + 2,5\sqrt{10}$ D) $2,5\sqrt{10} - 9$

Yechish: Kasr maxrajini irratsionallikdan qutqarish deganda shu narsa tushuniladiki, bu kasr o'ziga teng bo'lgan boshqa bir kasr bilan almashtiriladi, bu kasrning maxrajida ildiz amali ishtirok etmaydi. Bu kasr surat va maxrajini $2\sqrt{5} + 3\sqrt{2}$ ga ko'paytiramiz va amallarni bajaramiz

$$\frac{3\sqrt{5} - 2\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}} \cdot \frac{2\sqrt{5} + 3\sqrt{2}}{2\sqrt{5} + 3\sqrt{2}} = \frac{18 + 5\sqrt{10}}{4 \cdot 5 - 9 \cdot 2}$$

Bu kasrning maxraji 2 ga teng. Bulardan $9 + 2,5\sqrt{10}$ ni olamiz. **Javob:** $9 + 2,5\sqrt{10}$ (C).

9. (96-7-24) Hisoblang.

$$\frac{3 - \sqrt{5}}{3 + \sqrt{5}} + \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$$

- A) 2 B) $3\frac{\sqrt{5}}{2}$ C) 4,5 D) 7

10. (97-7-24) Ifodaning qiymatini toping.

$$\frac{3 + \sqrt{7}}{3 - \sqrt{7}} - \frac{3 - \sqrt{7}}{3 + \sqrt{7}}$$

- A) $4 + \sqrt{7}$ B) $-3\sqrt{7}$ C) $6\sqrt{7}$ D) 3

11. Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{4}{1 - \sqrt{5} + \sqrt{20}}$$

- A) $\sqrt{5} + 1$ B) $\sqrt{5} - 1$ C) $\sqrt{10}$ D) $1 - \sqrt{5}$

12. (01-6-23) Hisoblang.

$$\left(\frac{\sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}} + \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}} \right)^2$$

- A) 12 B) 14 C) 18 D) 16

13. (02-6-26) Hisoblang.

$$\frac{1}{\sqrt{7} - \sqrt{6}} - \frac{3}{\sqrt{6} - \sqrt{3}} - \frac{4}{\sqrt{7} + \sqrt{3}}$$

- A) 0 B) 1 C) 2 D) 3

14. (03-8-24) Hisoblang.

$$\left(\frac{15}{\sqrt{6} + 1} + \frac{4}{\sqrt{6} - 2} - \frac{12}{3 - \sqrt{6}} \right) \cdot (\sqrt{6} + 11)$$

- A) -115 B) 127 C) 100 D) -116

15. (03-10-17) Hisoblang.

$$\frac{\sqrt{2} + 1}{3 + 2\sqrt{2}} - \frac{\sqrt{2} - 1}{3 - 2\sqrt{2}}$$

- A) 1 B) -1 C) 2 D) -2

16. (96-6-50) Hisoblang.

$$\frac{1}{3 - \sqrt{8}} - 2\sqrt{2} + 6$$

- A) 8 B) 7 C) 9 D) 10

Yechish: Dastlab birinchi kasr maxrajini irratsionallikdan qutqaramiz. Buning uchun kasrning surat va maxrajini $3 + \sqrt{8}$ ga ko'paytiramiz

$$\frac{1}{3 - \sqrt{8}} \cdot \frac{3 + \sqrt{8}}{3 + \sqrt{8}} - 2\sqrt{2} + 6 = \frac{3 + \sqrt{8}}{9 - 8} - 2\sqrt{2} + 6.$$

Endi $9 - 8 = 1$ va $\sqrt{8} = 2\sqrt{2}$ ekanligini hisobga olsak, $3 + 2\sqrt{2} - 2\sqrt{2} + 6 = 9$ ni olamiz. **Javob:** 9 (C).

17. (97-2-50) Hisoblang.

$$2\sqrt{3} - 5 - \frac{11}{\sqrt{12} - 1}$$

- A) $2\sqrt{3} - 4$ B) 4 C) -4 D) -6

18. (97-8-50) Hisoblang.

$$\frac{19}{\sqrt{20} - 1} - 2\sqrt{5} + 3$$

- A) $4\sqrt{5} + 4$ B) $4\sqrt{5} - 4$
C) $2\sqrt{5} + 4$ D) 4

19. (97-12-49) Hisoblang.

$$\frac{19}{\sqrt{20} + 1} + 6 - 2\sqrt{5}$$

- A) 6 B) 5 C) $4\sqrt{5} - 7$ D) $4\sqrt{5} - 6$

20. (98-2-20) Hisoblang.

$$\frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{3} - 1}$$

- A) 2 B) 3 C) 4 D) $\sqrt{3}$

Yechish: Birinchi kasr surat va maxrajini $2 - \sqrt{3}$ ga ikkinchi kasr surat va maxrajini $\sqrt{3} + 1$ ko'paytiramiz hamda $(2 - \sqrt{3})(2 + \sqrt{3}) = 1$ va $(\sqrt{3} + 1) \cdot (\sqrt{3} - 1) = 2$ ekanligini hisobga olsak,

$$\frac{1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} + \frac{2}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} =$$

$$= 2 - \sqrt{3} + \sqrt{3} + 1 = 3 \text{ ni olamiz. } \mathbf{Javob: 3 (B).}$$

21. (99-6-38) Hisoblang.

$$\frac{\sqrt{5}}{\sqrt{5} - 2} - \frac{10}{\sqrt{5}}$$

- A) 1 B) 4 C) 3 D) 5

22. (99-10-15) Hisoblang.

$$\left(\frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{2 + \sqrt{3}} \right) \cdot (2 + \sqrt{2})$$

- A) $2\sqrt{2}$ B) $2\sqrt{3}$ C) 2 D) $3\sqrt{2}$

23. (00-10-47) Amallarni bajaring.

$$\frac{9}{5 - \sqrt{7}} + \frac{22}{7 + \sqrt{5}} - \frac{1}{\sqrt{7} + \sqrt{5}}$$

- A) 1 B) 6 C) $\frac{1}{5}$ D) 5

24. (98-7-17) O'zaro teskari sonlarni aniqlang.

1) $\frac{\sqrt{7}}{2}$ va $\frac{2\sqrt{7}}{7}$ 2) $\sqrt{6} - \sqrt{5}$ va $\sqrt{6} + \sqrt{5}$

3) $\frac{2\sqrt{5}}{9}$ va $\frac{9\sqrt{5}}{10}$ 4) $\sqrt{3} - 1$ va $\sqrt{3} + 1$.

- A) hammasi B) 2; 3; 4
C) 1; 3; 4 D) 1; 2; 3

25. (01-7-4) Soddalashtiring.

$$4 + 5\sqrt{2} + \frac{\sqrt{75}}{\sqrt{3} - \sqrt{6}}$$

- A) $2\sqrt{2} + 1$ B) 3 C) 2 D) -1

26. (01-11-8) Hisoblang.

$$\left(\sqrt{18} + \sqrt{72} - \sqrt{12} \right) \cdot \left(\sqrt{18} + \sqrt{72} + \sqrt{12} \right)$$

- A) 148 B) 149 C) 147 D) 150

27. (99-2-13) Ushbu

$$\sqrt{9 + \sqrt{65}} \cdot \sqrt{9 - \sqrt{65}}$$

son 14 dan qanchaga kam?

A) 8 B) 9 C) 10 D) 11

Yechish: 2-xossadan foydalanib

$$\begin{aligned} \sqrt{9 + \sqrt{65}} \cdot \sqrt{9 - \sqrt{65}} &= \sqrt{(9 + \sqrt{65}) \cdot (9 - \sqrt{65})} \\ &= \sqrt{81 - 65} = \sqrt{16} = 4 \text{ ni olamiz. Ifodaning} \\ &\text{qiymati 14 dan 10 ta kam ekan. } \mathbf{Javob:} \text{ 10 (C).} \end{aligned}$$

28. (99-7-4) Hisoblang.

$$\sqrt{9 + \sqrt{77}} \cdot \sqrt{9 - \sqrt{77}}$$

A) 3 B) 12 C) 2 D) 4

29. (00-1-48) Hisoblang.

$$\sqrt{7 + 2\sqrt{10}} \cdot \sqrt{7 - 2\sqrt{10}}$$

A) 2 B) 3,2 C) 3 D) 2,5

30. (03-5-2) Hisoblang.

$$\frac{\sqrt{196} \cdot \sqrt{19,6}}{\sqrt{0,196} \cdot \sqrt{1,96}}$$

A) 1000 B) 100 C) 196 D) 10

31. (96-7-14) Hisoblang.

$$\sqrt{\frac{65^3 + 35^3}{100} - 35 \cdot 65}$$

A) 100 B) 30 C) 10 D) 45

Yechish: Ikki son kublari yig'indisidan foydalansak, $65^3 + 35^3 = 100 \cdot (65^2 - 65 \cdot 35 + 35^2)$ ni olamiz. Buni ildiz ostidagi ifodaga qo'yib

$$\sqrt{\frac{100 \cdot (65^2 - 65 \cdot 35 + 35^2)}{100} - 35 \cdot 65 =$$

$$= \sqrt{65^2 - 2 \cdot 65 \cdot 35 + 35^2} = \sqrt{(65 - 35)^2} = 30$$

ni olamiz. **Javob:** 30 (B).

32. (97-3-14) Ifodaning qiymatini toping.

$$\sqrt{\frac{68^3 - 32^3}{36} + 68 \cdot 32}$$

A) $16\frac{2}{3}$ B) 85 C) 100 D) $25\frac{5}{6}$

33. (99-5-11) Agar $\sqrt{t^5 + 3} - \sqrt{t^5 - 2} = 1$ bo'lsa,

$$\sqrt{t^5 + 3} + \sqrt{t^5 - 2}$$

ning qiymati nechga teng bo'ladi?

A) 2 B) 3 C) 4 D) 5

34. (98-7-20) Agar $a : b = -\sqrt{5}$ bo'lsa, $a^2 - 5b^2$ ni hisoblang.

A) 0 B) $\sqrt{5}$ C) 5 D) -5

35. (03-5-12) Agar $\sqrt{5} = m$, $\sqrt{7} = n$ bo'lsa, $\sqrt{560}$ ni m va n orqali ifodalang.

A) $4mn$ B) $2mn$ C) $6mn$ D) $8mn$

3.1.1 Hisoblashga oid misollar

Ildiz qatnashgan ifodalarni hisoblashda (soddalashtirishda) quyidagi formulalardan foydalaniladi.

$$1. \sqrt{a + b\sqrt{c}} = \sqrt{\frac{a+A}{2}} + \sqrt{\frac{a-A}{2}}, \quad A = \sqrt{a^2 - b^2c}.$$

$$2. \sqrt{a - b\sqrt{c}} = \sqrt{\frac{a+A}{2}} - \sqrt{\frac{a-A}{2}}, \quad A = \sqrt{a^2 - b^2c}.$$

$$3. \sqrt{a + b + 2\sqrt{a \cdot b}} = \sqrt{a} + \sqrt{b}, \quad a > 0; b \geq 0.$$

$$4. \sqrt{a + b - 2\sqrt{a \cdot b}} = \sqrt{a} - \sqrt{b}, \quad a > b \geq 0.$$

1. (03-4-11) Hisoblang.

$$\sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$$

A) $\sqrt{3}$ B) $2\sqrt{3}$ C) $2\sqrt{2}$ D) $\sqrt{2}$

Yechish: Misolni yechishda 1 va 2-formulalardan foydalanamiz. Demak, $a = 2$, $b = 1$, $c = 3$ bunga ko'ra ko'ra $A = \sqrt{2^2 - 1^2 \cdot 3} = \sqrt{4 - 3} = 1$. 1-formulaga ko'ra

$$\sqrt{2 + \sqrt{3}} = \sqrt{\frac{2+1}{2}} + \sqrt{\frac{2-1}{2}} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}.$$

Ikkinchi ifoda $\sqrt{2 - \sqrt{3}}$ ga 2-formulani qo'llab

$$\sqrt{2 - \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

ni olamiz. Ularni ayirib $2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$ ga ega bo'lamiz. **Javob:** $\sqrt{2}$ (D).

2. (01-1-4) Hisoblang.

$$\sqrt{5 - 2\sqrt{6}} - \sqrt{5 + 2\sqrt{6}}$$

A) $2\sqrt{2}$ B) $-4\sqrt{6}$ C) $\sqrt{2}$ D) $-2\sqrt{2}$

3. (01-11-36) Hisoblang.

$$\sqrt{6 + 4\sqrt{2}} + \sqrt{6 - 4\sqrt{2}}$$

A) 3,8 B) 3,6 C) 4 D) 4,2

4. (02-10-43) Hisoblang.

$$\sqrt{52 - 30\sqrt{3}} - \sqrt{52 + 30\sqrt{3}}$$

A) -10 B) 10 C) -8 D) 8

5. (03-11-74) Hisoblang.

$$\sqrt{17 - 12\sqrt{2}} \cdot (6 + 4\sqrt{2})$$

A) $\sqrt{2}$ B) $-\sqrt{2}$ C) $\sqrt{3 + \sqrt{8}}$ D) 2

6. Hisoblang.

$$\sqrt{15 - \sqrt{56}} - \sqrt{14}$$

A) $\sqrt{3}$ B) $-2\sqrt{14}$ C) -1 D) 1

7. (96-3-73) Ayirmaning qiymatini toping.

$$\sqrt{9 - 2\sqrt{20}} - \sqrt{9 + 2\sqrt{20}}$$

- A) -3 B) -6 C) -4 D) -5

Yechish: 1-usul. $x = \sqrt{9 - 2\sqrt{20}} - \sqrt{9 + 2\sqrt{20}}$ deb belgilaymiz. $x < 0$ ekani ravshan. U holda

$$x^2 = 9 - 2\sqrt{20} - 2\sqrt{(9 - 2\sqrt{20})(9 + 2\sqrt{20})} + 9 + 2\sqrt{20} = 18 - 2\sqrt{81 - 80} = 18 - 2 = 16.$$

$x < 0$ va $x^2 = 16$ dan $x = -4$ ni olamiz.

2-usul. Misolni yechishda 3 va 4-formulalardan foydalanamiz. Ildiz ostidagi $9 - 2\sqrt{20}$ ni $5 + 4 - 2\sqrt{5 \cdot 4}$ deb yozib olamiz. U holda 3-ga ko'ra u $\sqrt{5} - \sqrt{4} = \sqrt{5} - 2$ ga teng bo'ladi. Ikkinchi qo'shiluvchi $\sqrt{9 + 2\sqrt{20}}$ esa $\sqrt{5} + \sqrt{4} = \sqrt{5} + 2$ ga teng. Ularni ayirib $\sqrt{5} - 2 - \sqrt{5} - 2 = -4$ ni olamiz. **Javob:** -4 (C).

8. (96-6-28) Hisoblang.

$$\sqrt{23 - 8\sqrt{7}} + \sqrt{23 + 8\sqrt{7}}$$

- A) 7 B) 6 C) 8 D) 9

9. (96-9-13) Yig'indining qiymatini toping.

$$\sqrt{11 + 6\sqrt{2}} + \sqrt{11 - 6\sqrt{2}}$$

- A) 6 B) 4 C) 8 D) 5

10. (96-12-71) Ayirmaning qiymatini toping.

$$\sqrt{9 + 2\sqrt{20}} - \sqrt{9 - 2\sqrt{20}}$$

- A) 4 B) 5 C) 6 D) 3

11. (97-2-28) Hisoblang.

$$\sqrt{19 + 8\sqrt{3}} + \sqrt{19 - 8\sqrt{3}}$$

- A) 6 B) 7 C) 9 D) 8

12. (96-13-13) Yig'indining qiymatini toping.

$$\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}$$

- A) 3 B) 5 C) 4 D) 6

13. (00-10-4) Hisoblang.

$$\sqrt{21 - 2\sqrt{21 + 2\sqrt{19 - 6\sqrt{2}}}}$$

- A)
- $3\sqrt{2} + 1$
- B)
- $3\sqrt{2} + 2$
- C)
- $3\sqrt{2} - 2$
- D)
- $3\sqrt{2} - 1$

14. (01-7-6) Ushbu
- $\sqrt{45 \cdot 10 \cdot 18}$
- va
- $\sqrt{21 \cdot 56 \cdot 6}$
- sonlarning eng katta umumiy bo'luvchisini toping.

- A) 9 B) 10 C) 18 D) 6

15. (97-5-21) Soddalashtiring.

$$\sqrt{7 - 4\sqrt{3}}$$

- A)
- $2 + \sqrt{3}$
- B)
- $\sqrt{3} - 2$
-
- C)
- $3 + \sqrt{3}$
- D)
- $2 - \sqrt{3}$

Yechish: Misolni yechishda 3 va 4-formulalardan foydalanamiz. Ildiz ostidagi $7 - 4\sqrt{3}$ ni $4 + 3 - 2\sqrt{4 \cdot 3}$ deb yozib olamiz. U holda 3-ga ko'ra u $\sqrt{4} - \sqrt{3} = 2 - \sqrt{3}$ ga teng bo'ladi. **Javob:** $2 - \sqrt{3}$ (D).

16. (98-5-10) Hisoblang.

$$\sqrt{19 - 8\sqrt{3}}$$

- A)
- $4 - \sqrt{3}$
- B)
- $4 + \sqrt{3}$
- C)
- $3 + \sqrt{3}$
- D)
- $3\sqrt{3}$

17. (00-8-28) Hisoblang.

$$\sqrt{6 - 2\sqrt{5}}$$

- A)
- $\sqrt{5} - 1$
- B)
- $1 - \sqrt{5}$
- C)
- $\sqrt{3}$
- D)
- $1 + \sqrt{5}$

18. (98-10-41) Hisoblang.

$$\sqrt{19 - 8\sqrt{3}} + \sqrt{3}$$

- A) -4 B) 4 C)
- $4 + 2\sqrt{3}$
- D)
- $2\sqrt{3} - 4$

19. (03-7-50) Hisoblang.

$$\sqrt{11 - 4\sqrt{7}}$$

- A)
- $\sqrt{7} + 2$
- B)
- $\sqrt{7} - 2$
- C)
- $\sqrt{7}$
- D)
- $2 - \sqrt{7}$

20. (97-8-27) Hisoblang.

$$\sqrt{11 - 6\sqrt{2}} + \sqrt{11 + 6\sqrt{2}}$$

- A) 8 B) 4 C) 3 D) 6

21. (97-12-27) Hisoblang.

$$\sqrt{9 - 4\sqrt{2}} - \sqrt{9 + 4\sqrt{2}}$$

- A) 2 B) 3 C) -3 D) -2

22. (98-7-14) Hisoblang.

$$\left(\sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}\right)^2 \cdot 0,5^{-2}$$

- A) 38 B) 30 C) 40 D) 44

23. (99-10-14) Hisoblang.

$$\frac{\sqrt{3 + 2\sqrt{2}} + \sqrt{3 - 2\sqrt{2}}}{4\sqrt{2}}$$

- A)
- $\frac{\sqrt{2}}{4}$
- B) 0,5 C)
- $\frac{\sqrt{2}}{2}$
- D) 0,75

24. (03-1-1) Hisoblang.

$$\sqrt{\left(\frac{\pi}{2} - \sqrt{3}\right)^2} + \sqrt{\left(\frac{\pi}{3} - \sqrt{2}\right)^2} - \sqrt{5 + 2\sqrt{6}}$$

- A)
- $\sqrt{3} + \sqrt{2}$
- B)
- $\frac{5\pi}{6} - 2(\sqrt{2} + \sqrt{3})$
-
- C)
- $\frac{5\pi}{6}$
- D)
- $\frac{-5\pi}{6}$

3.1.2 Ildizli ifodalarni soddalashtirish

Biz bu bandda arifmetik ildizning xossalari hamda qisqa ko'paytirish formulalaridan foydalanib misollar yechamiz.

1. (01-1-62) Ifodani $a \geq 0,5$ da soddalashtiring.

$$\sqrt{a^2} - \sqrt{a^2 + a + 0,25} + \sqrt{a^2 - a + 0,25}$$

- A) $a - 0,25$ B) $a - 0,5$ C) $a - 0,75$ D) $a - 1$

Yechish: $a \geq 0,5$ shartdan hamda $a^2 + a + 0,25 = (a + 0,5)^2$, $a^2 - a + 0,25 = (a - 0,5)^2$ tengliklardan, 3.1-dagi 6-xossaga ko'ra

$$\begin{aligned} \sqrt{a^2} - \sqrt{a^2 + a + 0,25} + \sqrt{a^2 - a + 0,25} &= \\ &= a - (a + 0,5) + (a - 0,5) = a - 1 \end{aligned}$$

tenglikni olamiz. **Javob:** $a - 1$ (D).

2. (97-6-56) Quyidagi ifodani $m = 15$ va $n = 3\sqrt{2}$ bo'lgandagi qiymatini hisoblang.

$$\frac{(\sqrt{m} + n)\sqrt{m - 2\sqrt{m} \cdot n + n^2}}{m - n^2}$$

- A) 1 B) -1 C) -3 D) 0

3. (01-7-14) Agar $a = (2 + \sqrt{3})^{-1}$ va $b = (2 - \sqrt{3})^{-1}$ bo'lsa,

$$(a + 1)^{-1} + (b + 1)^{-1}$$

ning qiymatini hisoblang.

- A) 2 B) 0,5 C) $2\sqrt{3}$ D) 1

4. (01-10-1) Agar $a = \sqrt{2}$ va $b = \sqrt[3]{3}$ bo'lsa,

$$\sqrt{a^2 - 2ab + b^2} + \sqrt{a^2 + 2ab + b^2}$$

ning qiymatini hisoblang.

- A) $\sqrt{8}$ B) $\sqrt[3]{12}$ C) $\sqrt{18}$ D) $\sqrt[3]{24}$

5. (02-6-8) $x = 5\sqrt{6}$ va $y = 6\sqrt{5}$ bo'lsa,

$$\sqrt{x^2 + 2xy + y^2} - \sqrt{x^2 - 2xy + y^2}$$

ning qiymatini hisoblang.

- A) $\sqrt{720}$ B) $\sqrt{700}$ C) $\sqrt{640}$ D) $\sqrt{600}$

6. (97-1-57) Ushbu

$$\frac{(x + \sqrt{y}) \cdot \sqrt{y - 2} \cdot \sqrt{y} \cdot x + x^2}{y - x^2}$$

ifodani $x = 2\sqrt{6}$ va $y = 23$ bo'lganda hisoblang.

- A) 1 B) -1 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

7. (03-10-12) Agar $x = \frac{4}{5}m$ bo'lsa,

$$\frac{\sqrt{m+x} + \sqrt{m-x}}{\sqrt{m+x} - \sqrt{m-x}}$$

ning qiymatini toping.

- A) 2 B) $2m$ C) 4 D) -2

8. (03-10-15) Agar $x < 0$ bo'lsa,

$$\sqrt{x^2 - 12x + 36} - \sqrt{x^2}$$

ni soddalashtiring.

- A) 6 B) -6 C) $6 - 2x$ D) $2x - 6$

9. (01-11-7) Soddalashtiring.

$$\frac{3}{a - \sqrt{a^2 - 3}} + \frac{3}{a + \sqrt{a^2 - 3}}$$

- A) $1,5a$ B) $3a$ C) $2a$ D) $2,5a$

Yechish: Birinchi kasr surat va maxraji $a + \sqrt{a^2 - 3}$ ga, ikkinchisini esa $a - \sqrt{a^2 - 3}$ ga ko'paytiramiz, natijada kasrlarning maxraji irratsionallikdan qutiladi va biz

$$a + \sqrt{a^2 - 3} + a - \sqrt{a^2 - 3} = 2a$$

ni olamiz. **Javob:** $2a$ (C).

10. (02-12-25) Soddalashtiring.

$$\frac{3}{a - \sqrt{a^2 - 3}} - \frac{3}{a + \sqrt{a^2 - 3}}$$

- A) $3a$ B) $3\sqrt{a^2 - 3}$ C) $6a$ D) $2\sqrt{a^2 - 3}$

11. (02-11-12) Soddalashtiring.

$$\left(\frac{1 + \sqrt{x} + x}{x\sqrt{x} - 1}\right)^{-1} - x^{\frac{1}{2}}$$

- A) $\sqrt{x} + 1$ B) 1 C) $\sqrt{x} - 1$ D) -1

12. (02-12-13) Soddalashtiring.

$$\frac{\sqrt{x} + 1}{x\sqrt{x} + x + \sqrt{x}} : \frac{1}{\sqrt{x} - x^2} + x$$

- A) $2x$ B) 2 C) 1 D) $2x - 1$

13. (03-8-41) Kasrni qisqartiring.

$$\frac{c - 2\sqrt{c} + 1}{\sqrt{c} - 1}$$

- A) $\sqrt{c} - 1$ B) $c - 1$ C) $c + 1$ D) $\sqrt{c} + 1$

14. (03-11-5) Soddalashtiring.

$$a \cdot \left(\frac{\sqrt{a} + \sqrt{b}}{2b\sqrt{a}}\right)^{-1} + b \cdot \left(\frac{\sqrt{a} + \sqrt{b}}{2a\sqrt{b}}\right)^{-1}$$

- A) $2ab$ B) ab C) $4ab$ D) $0,5ab$

15. (99-1-15) Soddalashtiring.

$$\left(\frac{1}{\sqrt{a} + \sqrt{b}} - \frac{\sqrt{a} + \sqrt{b}}{a - b}\right) \cdot \frac{a - 2\sqrt{a}\sqrt{b} + b}{2\sqrt{b}}$$

- A) $\frac{\sqrt{b} - \sqrt{a}}{\sqrt{a} + \sqrt{b}}$ B) $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$
C) $\frac{\sqrt{b} + \sqrt{a}}{\sqrt{a} - \sqrt{b}}$ D) $\frac{\sqrt{a} - \sqrt{b}}{a + b}$

16. (00-1-20) Soddalashtiring.

$$\left(\frac{1}{\sqrt{a+1} + \sqrt{a}} + \frac{1}{\sqrt{a} - \sqrt{a-1}}\right)(\sqrt{a+1} - \sqrt{a-1})$$

- A) 1 B) 2 C) $2\sqrt{a}$ D) $2\sqrt{a-1}$

3.2 n-darajali ildiz

n natural son uchun n – darajasi a ga teng bo'lgan b son a sonning n – darajali ildizi deyiladi. n – darajali ildizdan b sonni topish a sondan n – darajali ildiz chiqarish deyiladi. a sonning n – darajali ildizi quyidagicha belgilanadi:

$$\sqrt[n]{a} = b. \quad (3.2)$$

Bu yerda a ildiz ostidagi son, n – ildiz ko'rsatkichi deyiladi. (3.2) tenglik a va b sonlari orasida $b^n = a$ bog'lanish borligini bildiradi. Agar $a \geq 0$ ixtiyoriy son va $n > 1$ natural son bo'lsa, $b^n = a$ tenglikni qanoatlantiruvchi manfiy bo'lmagan b son mavjud bo'lib, u yagonadir. Bu son $a \geq 0$ sonning n – darajali arifmetik ildizi deyiladi. Agar $a < 0$ bo'lib, n juft son bo'lsa, u holda $b^n = a$ tenglikni qanoatlantiruvchi haqiqiy b soni mavjud emas. Agar $a < 0$ bo'lib, n toq son bo'lsa, u holda $\sqrt[n]{a} = -\sqrt[n]{|a|}$ tenglik o'rinli. Shu bois $\sqrt[n]{a}$ haqida so'z ketganda $a \geq 0$ va arifmetik ildiz nazarda tutiladi. a sonning n – darajali ildizi $a^{\frac{1}{n}}$ ko'rinishida ham yoziladi. Kasr ko'rinishdagi sonlar ratsional sonlar bo'lganligi uchun $a^{\frac{m}{n}}$ ni *ratsional ko'rsatkichli daraja* ham deyiladi. Ko'pincha ildiz atamasi o'rniga *radikal* termini ham ishlatiladi. $\frac{1}{n} = \frac{k}{kn}$ tenglikdan $\sqrt[n]{a} = a^{\frac{1}{n}} = a^{\frac{k}{kn}} = \sqrt[kn]{a^k}$ tengliklar kelib chiqadi. Bu xossadan foydalanib har xil ildiz ko'rsatkichga ega bo'lgan ildizlarni bir xil ildiz ko'rsatkich orqali ifodalash mumkin. Masalan, $\sqrt[3]{2}$ va $\sqrt[4]{4}$ larni $\sqrt[12]{2^4} = \sqrt[12]{16}$ va $\sqrt[12]{4^3} = \sqrt[12]{64}$ shaklda yozish mumkin.

Shuni ta'kidlaymizki, intiyoriy n – natural soni uchun $\sqrt[n]{0} = 0$, chunki $0^n = 0$. Endi n – darajali arifmetik ildizning asosiy xossalarini keltiramiz. Bu yerda $a > 0$, $b > 0$ va $n, m \in N$ sonlar.

1. $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.
2. $\sqrt[m]{a \cdot b} = \sqrt[m]{a} \cdot \sqrt[m]{b}$.
3. $\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$.
4. $a \sqrt[m]{b} = \sqrt[m]{a^m \cdot b}$.
5. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$.
6. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$.
7. $\sqrt[n]{a} = \sqrt[nm]{a^m}$.
8. $(\sqrt[n]{a})^n = a, \quad a \geq 0$.
9. $\sqrt[2n+1]{-a} = -\sqrt[2n+1]{a}$.
10. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

1. (00-3-17) Soddashtiring.

$$\frac{a - a\sqrt{a}}{\sqrt[3]{a^2} + \sqrt[6]{a^5} + a} + \frac{\sqrt[3]{a^2} - a}{\sqrt[3]{a} + \sqrt{a}} + 2\sqrt{a}$$

- A) $2\sqrt[3]{a}$ B) $2\sqrt{a}$ C) $\sqrt[3]{a} + 2\sqrt{a}$ D) 0

Yechish: $\sqrt[6]{a} = x$ deb belgilaymiz. U holda $a = x^6$, $\sqrt[3]{a^2} = x^4$, $\sqrt{a} = x^3$, $\sqrt[3]{a} = x^2$ bo'ladi. Bu belgilashda berilgan ifoda

$$\frac{x^6 - x^6 \cdot x^3}{x^4 + x^5 + x^6} + \frac{x^4 - x^6}{x^2 + x^3} + 2x^3$$

ko'rinishga keladi. Uni soddashtiramiz:

$$\begin{aligned} & \frac{x^6(1-x^3)}{x^4(1+x+x^2)} + \frac{x^4(1-x^2)}{x^2(1+x)} + 2x^3 = \\ & = \frac{x^2(1-x)(1+x+x^2)}{1+x+x^2} + \frac{x^2(1-x)(1+x)}{1+x} + 2x^3 = \\ & = x^2(1-x) + x^2(1-x) + 2x^3 = x^2 - x^3 + \\ & \quad + x^2 - x^3 + 2x^3 = 2x^2 = 2\sqrt[3]{a}. \end{aligned}$$

Javob: $2\sqrt[3]{a}$ (A).

2. (99-5-5) Soddashtiring.

$$\frac{27a+1}{9a^{\frac{2}{3}} - 3\sqrt[3]{a} + 1} - \frac{27a-1}{9\sqrt[3]{a^2} + 3a^{\frac{1}{3}} + 1}$$

- A) $\sqrt[3]{a} - 1$ B) 1 C) 2 D) $a + 1$

3. (00-8-53) Soddashtiring.

$$\frac{a^{\frac{3}{4}} - 36a^{\frac{1}{4}}}{a^{\frac{1}{2}} - 6a^{\frac{1}{4}}}$$

- A) $\sqrt[4]{a} - 6$ B) $\sqrt[4]{a} + 6$ C) $\sqrt{a} - 6$ D) $\sqrt{a} + 6$

4. (00-9-14) Soddashtiring.

$$\frac{729a+1}{81\sqrt[3]{a^2} - 9a^{\frac{1}{3}} + 1} - \frac{729a-1}{81a^{\frac{2}{3}} + 9\sqrt[3]{a} + 1}$$

- A) 1 B) 2 C) 3 D) 9

5. (97-9-81) Soddashtiring.

$$\frac{\sqrt[3]{x^2} + 2\sqrt[3]{x} + 1}{x + 3\sqrt[3]{x^2} + 3\sqrt[3]{x} + 1} - \frac{1}{\sqrt[3]{x} + 1}$$

- A) 1 B) $\frac{1}{\sqrt[3]{x} + 1}$ C) $\sqrt[3]{x}$ D) 0

6. (98-5-17) Ifodani soddashtiring,

$$(a^{\frac{1}{2}} - b^{\frac{1}{2}})(a + a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} + b)$$

so'ng a va b lar daraja ko'rsatkichlarining yig'in-disini hisoblang.

- A) 2 B) 1 C) 4 D) 3

Yechish: Agar $a^{\frac{1}{2}} = x$ va $b^{\frac{1}{2}} = y$ desak, $a = x^2$, $b = y^2$ bo'ladi. U holda berilgan ifoda qisqa ko'paytirish formulasining 6-ga ko'ra

$$(x - y)(x^2 + xy + y^2) = x^3 - y^3 = a^{\frac{3}{2}} - b^{\frac{3}{2}}$$

ko'rinishga keladi. Endi a va b larning darajalarini qo'shamiz $\frac{3}{2} + \frac{3}{2} = 3$. **Javob:** 3 (D).

7. (99-7-19) Ifodani soddalashtirib,

$$(a^{\frac{1}{2}} + b^{\frac{1}{2}})(a - a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} + b)$$

a va b asosli darajalar ko'rsatkichlarining yig'indisini toping.

- A) 1 B) 4 C) 2 D) 3

8. (98-5-18) Soddalashtiring.

$$\frac{(5b^{\frac{1}{4}} + 10)(b^{\frac{3}{4}} - 2b^{\frac{1}{2}})}{b - 4b^{\frac{1}{2}}}$$

- A) $1\frac{1}{4}$ B) $\frac{1}{5}$ C) 1 D) 5

9. (02-10-7) Soddalashtiring.

$$\left(\frac{9}{a+8} - \frac{a^{\frac{1}{3}} + 2}{a^{\frac{2}{3}} - 2a^{\frac{1}{3}} + 4} \right) \cdot \frac{a^{\frac{4}{3}} + 8a^{\frac{1}{3}}}{1 - a^{\frac{2}{3}}} + \frac{5 - a^{\frac{2}{3}}}{1 + a^{\frac{1}{3}}}$$

- A) 5 B) $\frac{1}{1-a}$ C) $\frac{2}{1-a^{\frac{2}{3}}}$ D) 4

10. (01-5-5) Soddalashtiring.

$$\frac{a-b}{a+b+2\sqrt{ab}} : \frac{a^{-\frac{1}{2}} - b^{-\frac{1}{2}}}{a^{-\frac{1}{2}} + b^{-\frac{1}{2}}}$$

- A) -1 B) $a+b$ C) $\frac{1}{\sqrt{a} + \sqrt{b}}$ D) $\frac{ab}{a+b}$

11. (98-9-18) Agar $n = 81$ bo'lsa, $\sqrt[3]{n\sqrt{n}}$ qiymati qanchaga teng bo'ladi?

- A) 3 B) 6 C) 9 D) 4

Yechish: 4 va 5-xossalardan foydalanib $\sqrt[3]{n\sqrt{n}} = \sqrt[3]{\sqrt{n^2} \cdot n} = \sqrt[6]{n^3} = \sqrt{n}$ ni olamiz. Endi $n = 81$ desak, $\sqrt{81} = 9$. **Javob:** 9 (C).

12. (02-12-44) Agar $a = 729$ bo'lsa,

$$\frac{a^{\frac{4}{3}} - 8a^{\frac{1}{3}}}{a^{\frac{2}{3}} + 2a^{\frac{1}{3}} + 4} : (\sqrt[3]{a} - 2)$$

ning qiymatini toping.

- A) 9 B) 6 C) 12 D) 15

13. (03-4-9) Agar $x = 256$ bo'lsa,

$$\frac{x-1}{x^{\frac{3}{4}} + x^{\frac{1}{2}}} \cdot \frac{x^{\frac{1}{2}} + x^{\frac{1}{4}}}{x^{\frac{1}{2}} + 1} \cdot x^{\frac{1}{4}} + 1$$

ning qiymatini toping.

- A) 14 B) 15 C) 16 D) 13

14. (03-4-28) $a = 64$ bo'lganda,

$$\frac{a^{\frac{4}{3}} - 8a^{\frac{1}{3}}b}{a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}}} : \left(1 - \frac{2b^{\frac{1}{3}}}{a^{\frac{1}{3}}} \right) - 4a^{\frac{2}{3}}$$

ning qiymatini hisoblang.

- A) -46 B) -48 C) -44 D) -50

15. (03-8-14) Quyida berilgan ifodaning $x = \sqrt{3} - \sqrt[3]{2}$ bo'lgandagi qiymatini toping.

$$\frac{x^2 - 2x\sqrt{3} - \sqrt[3]{4} + 3}{x - \sqrt{3}}$$

- A) $\sqrt{3}$ B) $\sqrt[3]{2}$ C) 1 D) 0

Yechish: Dastlab ifodani soddalashtiramiz, keyin x o'rniga berilgan qiymatini qo'yamiz. Berilgan kasr surati $x^2 - 2x\sqrt{3} - \sqrt[3]{4} + 3$ ni $x^2 - 2x\sqrt{3} + (\sqrt{3})^2 - \sqrt[3]{2^2} = (x - \sqrt{3})^2 - (\sqrt[3]{2})^2$ shaklda yozib olamiz. Bu ifodaga ikki son kvadratlarining ayirmasi uchun formulani qo'llab $(x - \sqrt{3} - \sqrt[3]{2})(x - \sqrt{3} + \sqrt[3]{2})$ ni olamiz. Agar x o'rniga $\sqrt{3} - \sqrt[3]{2}$ ni qo'ysak ikkinchi qavs ichi nolga aylanadi. Shuning uchun ko'paytma nol bo'ladi. **Javob:** 0 (D).

16. (99-9-2) Agar $a = 27$ bo'lsa,

$$\left(\frac{a-b}{\sqrt[3]{a} - \sqrt[3]{b}} + \sqrt[3]{ab} \right) : (\sqrt[3]{a} + \sqrt[3]{b}) + (\sqrt[3]{a^2} - \sqrt[3]{b^2}) : (\sqrt[3]{a} + \sqrt[3]{b})$$

ning qiymatini hisoblang.

- A) 4 B) 4,5 C) 5 D) 6

17. (99-2-11) Agar $a = 8$, $b = 2$ bo'lsa,

$$\frac{a^{\frac{3}{2}} - b^{\frac{3}{2}}}{a^{\frac{1}{2}} - b^{\frac{1}{2}}} - \frac{a^{\frac{3}{2}} + b^{\frac{3}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}}$$

ning qiymati nechaga teng?

- A) 10 B) 6 C) 8 D) 12

18. (01-6-32) Ushbu

$$\left(\frac{x^{\frac{3}{2}} - y^{\frac{3}{2}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}} - x - y \right) \cdot x^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$$

ifodani soddalashtiring, keyin $x = 16^{\frac{1}{3}}$ va $y = 4^{\frac{1}{3}}$ bo'lgandagi qiymatini hisoblang.

- A) 2 B) 4 C) $2\sqrt[3]{4}$ D) 3

3.2.1 Hisoblashga oid misollar

1. (99-8-16) 243 sonini 9 asosli daraja shaklida ifodalang.

- A) $9^{5/2}$ B) $9^{3/4}$ C) $9^{5/3}$ D) $9^{3/2}$

Yechish: Ma'lumki, $243 = 3^5$ dir. 3 esa $\sqrt{9} = 9^{1/2}$ ga teng. Bulardan $243 = 3^5 = (9^{1/2})^5 = 9^{5/2}$ ekanligi kelib chiqadi. **Javob:** $9^{5/2}$ (A).

2. $25 \cdot 5^{n+2}$ sonini 25 asosli daraja shaklida ifodalang.

- A) $25^{1+n/2}$ B) $25^{2+n/4}$ C) $25^{2+n/2}$ D) $25^{n/2}$

3. $64^{0.5} \cdot 16^{3n+9}$ sonini 8 asosli daraja shaklida ifodalang.

- A) 8^{12+4n} B) 8^{13+4n} C) 8^{13+3n} D) 8^{4n}

4. (02-1-1) $\sqrt[3]{2\sqrt{2\sqrt[3]{2}}}$ ni 2 asosli daraja shaklida ifodalang.

- A) $2^{5/9}$ B) $2^{4/3}$ C) $2^{2/3}$ D) $2^{3/2}$

5. (99-2-12) $\sqrt{3 \cdot \sqrt[3]{18}} \cdot \sqrt[6]{96}$ ni hisoblang.

- A) 6 B) 18 C) 9 D) 10

6. (98-5-7) Hisoblang.

$$\frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}}$$

- A) 45 B) 15 C) 5 D) 3

Yechish: 15 ni 3-5 shaklda yozamiz va $a^n = \frac{1}{a^{-n}}$ kossadan foydalansak

$$\frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}} = 3^{\frac{2}{3}} \cdot 5^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 3^1 \cdot 5^1 = 15.$$

Javob: 15 (B).

7. (99-7-9) $30^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} : 10^{-\frac{2}{3}}$ ni hisoblang.

- A) 15 B) 20 C) 60 D) 30

8. (00-3-6) Hisoblang.

$$0,027^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} + 256^{\frac{3}{4}} - 3^{-1} + 5,5^0$$

- A) 33 B) 32,97 C) 31 D) 32

9. Hisoblang.

$$\frac{\sqrt[4]{0,0016}}{\sqrt[5]{0,00032}} - \frac{\sqrt[3]{0,027}}{\sqrt[6]{0,00064}}$$

- A) 1,5 B) -1,5 C) -0,5 D) 0,5

10. $\sqrt[5]{(15^{10} - 10^{10})} : (3^{10} - 2^{10})$ ni hisoblang.

- A) 3 B) 5 C) 25 D) 9

11. (98-11-59) Hisoblang.

$$\frac{\sqrt[3]{-24} + \sqrt[3]{81} + \sqrt[3]{192}}{\sqrt[3]{-375}}$$

- A) -1 B) 1 C) $-\frac{83}{125}$ D) $\frac{83}{125}$

Yechish: 4 va 9-xossalardan foydalanib

$$\frac{-\sqrt[3]{2^3 \cdot 3} + \sqrt[3]{3^3 \cdot 3} + \sqrt[3]{4^3 \cdot 3}}{-\sqrt[3]{5^3 \cdot 3}} = \frac{\sqrt[3]{3}(-2 + 3 + 4)}{-5\sqrt[3]{3}}$$

ni olamiz. Bu ifodani soddalashtirsak, uning qiymati -1 chiqadi. **Javob:** -1 (A).

12. (98-5-2) Hisoblang.

$$\sqrt[3]{9 + \sqrt{73}} \cdot \sqrt[3]{9 - \sqrt{73}}$$

- A) 2 B) 3 C) 4 D) 1

13. (98-7-18) Hisoblang.

$$\sqrt{2\sqrt{2} - 1} \cdot \sqrt[4]{9 + 4\sqrt{2}}$$

- A) 7 B) $\sqrt[4]{7}$ C) $2\sqrt{2} + 1$ D) $\sqrt{7}$

14. (98-12-13) Hisoblang.

$$\left(\sqrt[6]{9 + 4\sqrt{5}} + \sqrt[3]{\sqrt{5} + 2}\right) \cdot \sqrt[3]{\sqrt{5} - 2}$$

- A) 2 B) 1 C) 3 D) 4

15. (00-3-2) Hisoblang.

$$\sqrt[3]{216 \cdot 512} + \sqrt[5]{32 \cdot 243}$$

- A) 45 B) 48 C) 49 D) 54

16. (00-7-18) Hisoblang.

$$\frac{\sqrt[4]{3 + 2\sqrt{2}}}{\sqrt{\sqrt{2} + 1}}$$

- A) 2 B) 1,5 C) 0,5 D) 1

17. (00-8-55) Hisoblang.

$$\sqrt[3]{2 - \sqrt{3}} \cdot \sqrt[6]{7 + 4\sqrt{3}}$$

- A) 1 B) -1 C) 0 D) 7

18. (99-10-3) Hisoblang.

$$\sqrt[4]{\frac{4,1^3 - 2,15^3}{1,95}} + 4,1 \cdot 2,15$$

- A) 1,5 B) 1,75 C) 2,25 D) 2,5

19. (01-3-22) Hisoblang.

$$\sqrt{3 + 2\sqrt{2}} \cdot \sqrt[4]{17 - 12\sqrt{2}}$$

- A) 2 B) 1 C) $\sqrt{2}$ D) $2\sqrt{2}$

20. (01-9-7) Hisoblang.

$$\sqrt{4 - 2\sqrt{2}} \cdot \sqrt[4]{6 + 4\sqrt{2}}$$

- A) 2 B) 1 C) 3 D) 4

Yechish: Ikkinchi ildiz $\sqrt[4]{6 + 4\sqrt{2}}$ ni quyidagicha yozib olamiz $\sqrt{\sqrt{4 + 2} + 2\sqrt{4} \cdot 2} = \sqrt{\sqrt{4 + 2} + \sqrt{2}}$ Birinchi ildizdan 2 ni qavs oldiga chiqarib, ularni ko'paytiramiz $\sqrt{2(2 - \sqrt{2})(2 + \sqrt{2})} = \sqrt{2(4 - 2)} = 2$. **Javob:** 2 (A).

21. (02-3-6) Hisoblang.

$$\sqrt[4]{68 + 8\sqrt{72}} \cdot \sqrt[8]{4 - \sqrt{15}} \cdot \sqrt[8]{4 + \sqrt{15}} + 1$$

- A) $3 + \sqrt{2}$ B) $1 + \sqrt{3}$ C) $\sqrt{2} + \sqrt{3}$ D) $2\sqrt{2}$

22. (02-7-44) Hisoblang.

$$\sqrt[3]{2000 \cdot 1998 - 1997 \cdot 2001 + 5}$$

- A) 2 B) 3 C) $\sqrt[3]{17}$ D) 4

23. (03-4-18) Hisoblang.

$$\sqrt[3]{16 + 16\sqrt{2}} \cdot \sqrt[6]{48 - 32\sqrt{2}}$$

- A) 2 B) 6 C) 4 D) 8

24. (03-6-46) Hisoblang.

$$\sqrt[3]{1 - \sqrt{3}} \cdot \sqrt[6]{4 + 2\sqrt{3}}$$

- A) $-\sqrt{2}$ B) $\sqrt[3]{2}$ C) $-\sqrt[3]{2}$ D) $\sqrt{2}$

25. Hisoblang.

$$\sqrt[3]{5 - \sqrt{17}} \cdot \sqrt[3]{5 + \sqrt{17}} \cdot \sqrt{7 + 2\sqrt{12}}$$

- A) $2 + \sqrt{3}$ B) $\sqrt[3]{8}$ C) $4 + \sqrt{3}$ D) $4 + \sqrt{12}$

Yechish: Birinchi va ikkinchi ko'paytuvchilarni 2-xossadan foydalanib, $\sqrt[3]{(5 - \sqrt{17})(5 + \sqrt{17})} = \sqrt[3]{25 - 17}$ shaklda yozamiz. Bu ildizning qiymati 2 ga teng. 3.1.1 ning 3-xossasidan foydalanib, $\sqrt{7 + 2\sqrt{12}} = \sqrt{7 + 2\sqrt{4 \cdot 3}} = 2 + \sqrt{3}$ ni olamiz. Olingan natijalarni ko'paytirsak, $2(2 + \sqrt{3}) = 4 + \sqrt{12}$ bo'ladi. **Javob:** $4 + \sqrt{12}$ (D).

26. Hisoblang.

$$\sqrt[4]{11 + 2\sqrt{18}} \cdot \sqrt[8]{9 - \sqrt{80}} \cdot \sqrt[8]{9 + \sqrt{80}}$$

- A) $\sqrt{3} + 2$ B) $\sqrt{2} + 3$ C) $\sqrt{3 + \sqrt{2}}$ D) $\sqrt{2}$

27. Hisoblang.

$$\sqrt[4]{4 - \sqrt{12}} \cdot \sqrt[6]{(1 + \sqrt{3})^5} \cdot \sqrt[3]{\sqrt{3} - 1}$$

- A) 4 B) $2^{5/6}$ C) $3^{2/3}$ D) $1 + \sqrt{2}$

28. Hisoblang.

$$\sqrt[4]{0,0016} \cdot \sqrt[3]{0,125}$$

- A) 0,4 B) 0,01 C) 0,1 D) 1

29. Hisoblang.

$$\sqrt[6]{5 - 2\sqrt{6}} \cdot \sqrt[3]{5 + \sqrt{24}} \cdot \sqrt[3]{\sqrt{2} - \sqrt{3}}$$

- A) 1 B) 2 C) -1 D) -2

Yechish: Birinchi ildiz $\sqrt[6]{5 - 2\sqrt{6}}$ ni 3.2-dagi 5-xossadan foydalanib, quyidagicha yozib olamiz

$$\sqrt[3]{\sqrt{3 + 2 - 2\sqrt{3}} \cdot 2} = \sqrt[3]{\sqrt{3} - \sqrt{2}}.$$

3-ildizni 9-xossadan foydalanib, $-\sqrt[3]{\sqrt{3} - \sqrt{2}}$ shaklda yozib, uni birinchi ko'paytuvchi bilan ko'paytirib $-\sqrt[3]{5 - \sqrt{24}}$ ni olamiz. Bu qiymatni $\sqrt[3]{5 + \sqrt{24}}$ bilan ko'paytirib $-\sqrt[3]{25 - 24} = -1$ ni olamiz. **Javob:** -1 (C).

30. Hisoblang.

$$\sqrt{7 + \sqrt{1 + \sqrt{7 + \sqrt[3]{8}}}}$$

- A) 1 B) 2 C) 3 D) 4

31. Hisoblang.

$$\frac{\sqrt{72} - \sqrt{108}}{\sqrt{3} - \sqrt{2}}$$

- A) 6 B) 3 C) -3 D) -6

32. Quyidagi ifoda natural son bo'ladigan n ning eng katta qiymatini toping.

$$\sqrt[n]{3^8 + 9^4 + 81^2}$$

- A) 4 B) 6 C) 8 D) 9

33. Hisoblang.

$$\left(\sqrt{\frac{3}{14}} - \sqrt{\frac{2}{21}} \right) \cdot \frac{42}{\sqrt{7}}$$

- A) $\sqrt{6}$ B) $2\sqrt{6}$ C) $-\sqrt{6}$ D) $-2\sqrt{6}$

34. (03-8-9) Kasrning maxrajini irratsionallikdan qutqaring.

$$\frac{2}{2 + \sqrt[3]{2} + \sqrt[3]{4}}$$

- A) $2 - \sqrt[3]{4}$ B) $1 - \sqrt[3]{4}$ C) $1 + \sqrt[3]{4}$ D) $\sqrt[3]{2}$

35. (97-4-3) Eng katta son berilgan javobni toping.

- A) $\sqrt{15}$ B) $\sqrt[3]{65}$ C) $\sqrt[4]{81}$ D) 4

Yechish: Ma'lumki, $0 < a < b$ bo'lsa, $\sqrt[a]{a} < \sqrt[b]{b}$ bo'ladi. $3 = \sqrt[4]{81} = \sqrt{9} < \sqrt{15} < \sqrt{16} = 4$. Berilgan $\sqrt{15}$; $\sqrt[4]{81}$ va 4 sonlari ichida kattasi 4 soni ekan. $4 = \sqrt[3]{64} < \sqrt[3]{65}$. Demak, eng katta son $\sqrt[3]{65}$ ekan. **Javob:** $\sqrt[3]{65}$ (B).

36. (97-9-63) Eng katta sonni toping.

- A) 3 B) $\sqrt[3]{26}$ C) $\sqrt{10}$ D) $\sqrt[4]{82}$

37. (02-5-3) $a = \sqrt{3}$, $b = \sqrt[3]{5}$ va $c = \sqrt[4]{7}$ sonlarni o'sish tartibida joylashtiring.

- A) $a < b < c$ B) $c < b < a$
C) $b < a < c$ D) $b < c < a$

38. (02-10-42) Sonlarni o'sish tartibida joylashtiring.

$$m = \left(\frac{4}{7}\right)^{-\frac{2}{3}}, \quad n = \left(\frac{49}{16}\right)^{\frac{4}{3}}, \quad k = \left(\frac{16}{49}\right)^{-\frac{1}{4}}$$

- A) $k < m < n$ B) $m < k < n$
C) $m < n < k$ D) $k < n < m$

39. (02-12-34) Sonlarni o'sish tartibida joylashtiring.

$$a = \sqrt[3]{2}, \quad b = \sqrt[4]{3}, \quad c = \sqrt[6]{5}$$

- A) $a < b < c$ B) $c < b < a$
C) $a < c < b$ D) $b < a < c$

3.3 O'rta qiymatlar

O'rta qiymatlardan eng ko'p ishlatiladiganlari o'rta arifmetik, o'rta geometrik, o'rta vaznli va o'rta garmonik qiymatlardir. Ularni misollarda tushuntiramiz.

O'rta arifmetik qiymat. Berilgan $a_1, a_2, a_3, \dots, a_n$ sonlarning *o'rta arifmetik qiymati* deb

$$A = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n} \quad (3.3)$$

songa aytiladi. Masalan, 10, -12, 20 sonlarining o'rta arifmetik qiymati $(10 - 12 + 20) : 3 = 18 : 3 = 6$ ga teng.

O'rta geometrik qiymat. Berilgan $b_1, b_2, b_3, \dots, b_n$ sonlarning *o'rta geometrik qiymati* deb

$$B = \sqrt[n]{b_1 \cdot b_2 \cdot b_3 \cdot \dots \cdot b_n} \quad (3.4)$$

songa aytiladi. Masalan, 4, 10, 25 sonlarining o'rta geometrik qiymati $\sqrt[3]{4 \cdot 10 \cdot 25} = \sqrt[3]{1000} = 10$ ga teng. Berilgan miqdorlarning qiymatlari bir-biriga teng bo'lgan holdan boshqa barcha hollarda o'rta geometrik qiymat o'rta arifmetik qiymatdan kichik bo'ladi. Berilgan sonlar teng bo'lganda o'rta geometrik qiymat o'rta arifmetik qiymatga teng bo'ladi. Xususan $n = 2$ da

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

\sqrt{ab} miqdor berilgan a va b sonlarining *o'rta proporsionali* ham deyiladi. Ma'lumki, $a : x = x : b$ proporsiyada, proporsiyaning o'rta hadi $x = \sqrt{ab}$ ham a va b sonlarining o'rta proporsionali deyiladi. Masalan, 4 va 9 sonlarining o'rta proporsionali $\sqrt{4 \cdot 9} = 6$ ga teng.

O'rta vaznli qiymat. Quyidagi masalani qaraymiz. Zargarlik buyumi tayyorlash maqsadida 8 gramm oltin bilan 32 gramm kumush aralashtirildi. Agar 1 gramm oltinning bahosi 2000 so'm, 1 gramm kumushning bahosi 500 so'm bo'lsa, 1 gramm aralashmaning bahosi necha so'm bo'ladi.

Masalani yechish uchun quyidagilarni topamiz.

- 1) Oltinning jami bahosi: $8 \cdot 2000 = 16000$ so'm.
- 2) Kumushning jami bahosi: $32 \cdot 500 = 16000$ so'm.
- 3) Aralashmaning massasi $8 + 32 = 40$ gramm.
- 4) 1 gramm aralashmaning bahosi

$$\frac{16000 + 16000}{40} = \frac{32000}{40} = 800.$$

Javob: 800 so'm.

Berilgan $a_1, a_2, a_3, \dots, a_m$ sonlarning *o'rta vaznli qiymati* deb

$$C = \frac{a_1 \cdot n_1 + a_2 \cdot n_2 + a_3 \cdot n_3 + \dots + a_m \cdot n_m}{n_1 + n_2 + n_3 + \dots + n_m} \quad (3.5)$$

songa aytiladi. Agar $n_1 = n_2 = \dots = n_m$ bo'lsa, o'rta vaznli qiymat o'rta arifmetik qiymatga teng bo'ladi.

O'rta garmonik qiymat. Quyidagi masalani qaraylik. A va B shaharlar orasidagi masofa S km. Poyezd A dan B ga v_1 km/soat, B dan A ga esa v_2 km/soat tezlik bilan yuradi. Borish va kelishdagi yo'lni poyezd o'rtacha necha km/soat tezlik bilan o'tgan.

Masalani yechish uchun quyidagilarni topamiz.

- 1) Poyezd A dan B ga borish uchun $t_1 = S : v_1$ soat,
- 2) B dan A ga borish uchun $t_2 = S : v_2$ soat sarflagan.
- 3) Hammasi bo'lib borib-kelish uchun sarflangan vaqt:

$$t_1 + t_2 = \frac{S}{v_1} + \frac{S}{v_2} = \frac{Sv_1 + Sv_2}{v_1v_2}.$$

- 4) Bosib o'tilgan yo'lning hammasi $2S$ ga teng bo'lganligi uchun poyezdning o'rtacha tezligi

$$\frac{2S}{\frac{Sv_1 + Sv_2}{v_1v_2}} = 2S \cdot \frac{v_1v_2}{Sv_1 + Sv_2} = \frac{2v_1v_2}{v_1 + v_2} \quad \text{km/soat.}$$

Javob: $\frac{2v_1v_2}{v_1 + v_2}$ km/soat.

Berilgan a va b sonlarning *o'rta garmonik qiymati* deb

$$D = \frac{2ab}{a+b} \quad (3.6)$$

songa aytiladi.

To'g'ri va teskari proporsionallik. To'g'ri va teskari proporsional bog'lanishlarga to'xtalamiz. x va y miqdorlar o'rtasidagi $y = kx$, $k > 0$ bog'lanishga *to'g'ri proporsional bog'lanish*, $y = \frac{k}{x}$ ga *teskari proporsional bog'lanish* deyiladi. $y = kx + b$ tenglikdan noma'lum x ni topish uchun, uning ikkala qismidan b soni ayriladi, keyin tenglikning ikkala qismi $k \neq 0$ ga bo'linadi, natijada $x = \frac{y-b}{k}$ tenglik hosil bo'ladi. Quyidagi masalani qaraymiz.

1-misol. 300 sonini 3, 5, 7 sonlariga to'g'ri proporsional (proporsional yoki mutanosib) bo'laklarga bo'ling.

Yechish: Masala shartiga ko'ra, 300 soni $3x$, $5x$ va $7x$ qismlarga bo'linadi. Demak, $3x + 5x + 7x = 300$. Bu yerdan $15x = 300$ ni olamiz. Bu tenglikning har ikkala qismini 15 ga bo'lib, $x = 20$ ni topamiz. x o'rniga 20 qo'ysak, $3x = 60$, $5x = 100$ va $7x = 140$ ni olamiz.

Javob: 60, 100 va 140.

2-misol. 240 sonini 5 va 7 sonlariga teskari proporsional bo'laklarga bo'ling.

Yechish: Masala shartiga ko'ra, 240 soni $x/5$ va $x/7$ qismlarga bo'linadi. Demak, $\frac{x}{5} + \frac{x}{7} = 240$. Bu tenglikning ikkala qismini 35 ga ko'paytirib,

$$7x + 5x = 240 \cdot 35 \iff 12x = 240 \cdot 35$$

ni olamiz. Bu yerdan $x = 700$ kelib chiqadi. Demak, $700 : 5 = 140$ va $700 : 7 = 100$. **Javob:** 140, 100.

1. Agar a , 1, 8 va -5, 6 sonlarining o'rta arifmetigi 1, 2 ga teng bo'lsa, a ning qiymatini toping.
A) 7, 4 B) 7 C) 6, 8 D) 7, (6)

Yechish: Masala sharti va (3.3) formulaga ko'ra

$$\frac{a + 1, 8 - 5, 6}{3} = 1, 2 \iff a - 3, 8 = 3, 6.$$

Bu yerdan $a = 7, 4$ ni olamiz. **Javob:** 7, 4 (A).

2. 0, 32, 0, 28, 0, 4 va 7 sonlarining o'rta arifmetik qiymatini toping.
A) 0, 7 B) 2 C) 1, 8 D) 2, (6)

3. a_1, a_2, a_3 sonlarining o'rtacha arifmetigi 4 ga, a_4, a_5, a_6, a_7, a_8 sonlarining o'rtacha arifmetigi esa 5 ga teng bo'lsa, $a_1 + a_2 + a_3 + \dots + a_8$ ning qiymatini toping.
A) 37,4 B) 37 C) 36,8 D) 37, (6)
4. Real futbol jamoasidagi 11 ta o'yinchining o'rtacha yoshi 21 ga teng. Bir o'yinchi safdan chiqdi. Qolgan 10 o'yinchining o'rtacha yoshi 20,8 ga teng. Safdan chiqqan o'yinchining yoshini toping.
A) 22 B) 23 C) 19 D) 18
5. (96-1-10) x ; $-2, 1$ va $3, 3$ sonlarining o'rtacha arifmetigi 0,2 ga teng. x ni toping.
A) 0,6 B) $-0,6$ C) 0,8 D) 2
6. (96-9-60) $5, 4$; y ; $-2, 2$ sonlarining o'rtacha arifmetigi 1,2 ga teng. y ni toping.
A) 1,2 B) $-0,8$ C) 0,4 D) $-0,4$
7. (98-1-12) Bir son ikkinchi sondan 6 ta ortiq. Ularning o'rtacha arifmetigi 20 ga teng. Shu sonlardan kattasini toping.
A) 23 B) 27 C) 33 D) 26
- Yechish:** Bu sonlar kattasini x desak, u holda kichigi $x - 6$ bo'ladi. Masala shartiga ko'ra, ularning o'rtacha arifmetigi 20, ya'ni
- $$\frac{x + x - 6}{2} = 20. \iff 2x - 6 = 40 \iff 2x = 46.$$
- Bu yerdan $x = 23$ ni olamiz. **Javob:** 23 (A).
8. (98-6-6) Uchta sonning o'rtacha arifmetigi 17,4 ga teng. Agar sonlarning ikkitasi 17,5 va 21,6 bo'lsa, uchinchi sonni toping.
A) 12,1 B) $-0,2$ C) $-8,4$ D) 13,1
9. (98-8-12) Bir son ikkinchisidan 15 ga kichik. Shu sonlarning o'rtacha arifmetigi 11,5 ga teng. Shu sonlardan kichigini toping.
A) 3 B) 3,5 C) 4 D) 7
10. (02-6-11) Uchta sonning o'rtacha arifmetigi 20 ga, boshqa ikkita sonning o'rtacha arifmetigi esa 25 ga teng. Shu beshta sonning o'rtacha arifmetigini toping.
A) 22,5 B) 22,6 C) 24 D) 22
11. (02-8-6) 7 ta sonning o'rtacha arifmetigi 13 ga teng. Bu sonlarga qaysi son qo'shilsa, ularning o'rtacha arifmetigi 18 ga teng bo'ladi?
A) 53 B) 50 C) 45 D) 56
12. (00-7-5) Uchta sonning o'rtacha arifmetigi 30 ga, dastlabki ikkitasini esa 25 ga teng. uchinchi sonni toping.
A) 44 B) 40 C) 45 D) 38
13. (03-12-52) Oltita o'quvchining o'rtacha bo'yi 120 sm, shulardan bir o'quvchining bo'yi 105 sm. Qolgan besh o'quvchining o'rtacha bo'yi qanchaga teng?
A) 122 B) 123 C) 121 D) 124
14. b_1, b_2 sonlarining o'rtacha geometrigi 3 ga teng. b_3 soni qanday tanlansa b_1, b_2, b_3 sonlarining o'rtacha geometrigi 4 ga teng bo'ladi?
A) 7 B) 6,9 C) 7, (1) D) 7,1
- Yechish:** Masala shartiga ko'ra $\sqrt{b_1 \cdot b_2} = 3$ va $\sqrt[3]{b_1 \cdot b_2 \cdot b_3} = 4$. Birinchi tenglikdan $b_1 \cdot b_2 = 9$ ni olamiz, uni ikkinchi tenglikka qo'yib $\sqrt[3]{9 \cdot b_3} = 4$. Bu yerdan $b_3 = 64 : 9$ ni olamiz. **Javob:** 7, (1) (C).
15. 8, 64, 0,027 sonlarining o'rtacha geometrik qiymatini toping.
A) 2,7 B) 2 C) 2,8 D) 2,4
16. 2, 9, 12 sonlarining o'rtacha geometrigini toping.
A) 7 B) 6 C) 6, (1) D) 7,1
17. x , 25 va -5 sonlarining o'rtacha geometrik qiymati -5 ga teng bo'lsa, x ni toping.
A) 1,4 B) 1 C) 1,6 D) -1
18. b_1, b_2 sonlarining o'rtacha geometrigi 2 ga, b_3, b_4 sonlarining o'rtacha geometrigi esa 3 ga teng bo'lsa, $b_1 \cdot b_2 \cdot b_3 \cdot b_4$ ning qiymatini toping.
A) 37 B) 49 C) 36 D) 32
19. (03-4-4) 4 va 64 sonlarining o'rtacha arifmetigi ularning o'rtacha geometrigidan necha marta katta?
A) $2\frac{1}{4}$ B) $2\frac{3}{4}$ C) 2,2 D) $2\frac{1}{8}$
- Yechish:** 4 va 64 sonlarining o'rtacha arifmetigi $(4 + 64) : 2 = 34$, ularning o'rtacha geometrigi $\sqrt{4 \cdot 64} = 2 \cdot 8 = 16$. Ularning nisbati $34 : 16 = 2\frac{1}{8}$. **Javob:** $2\frac{1}{8}$ (D).
20. Ikki sonning o'rtacha arifmetigi 10 ga, o'rtacha geometrigi esa 6 ga teng. Shu sonlarni toping.
A) 2; 8 B) 2; 18 C) 5; 15 D) 6; 14
21. (98-11-56) Uchta sonning o'rtacha geometrigi 6 ga teng bo'lib, ulardan ikkitasi 8 va 9 bo'lsa, uchinchi son necha bo'ladi?
A) 3 B) 7 C) -5 D) -3
22. (01-6-4) $a > 0$ sonining va 4 ning o'rtacha arifmetigi hamda o'rtacha geometrigi a ning qanday qiymatida o'zaro teng bo'ladi?
A) 3 B) 7 C) 5 D) 4
23. (01-9-31) Ikki musbat sonning o'rtacha geometrigi 8 ga, va boshqa ikkita musbat sonning o'rtacha geometrigi 32 ga teng. Shu to'rtta sonning o'rtacha geometrigini toping.
A) 12 B) 16 C) 15 D) 14
24. (01-12-16) Ikki son o'rtacha geometrigining o'rtacha arifmetigiga nisbati 3 : 5 kabi. Shu sonlardan kichigining kattasiga nisbatini toping.
A) 1 : 9 B) 9 : 25 C) 3 : 5 D) 4 : 15
25. 3 va 48 sonlarining o'rtacha proporsionalini toping.
A) 7 B) 12 C) 11 D) 16

26. 180 sonini 3, 5, 7 sonlariga proporsional bo'laklarga bo'lganda kattasi nechaga teng bo'ladi?
A) 84 B) 62 C) 48 D) 92
- Yechish:** Masala shartiga ko'ra, 180 soni $3x$, $5x$ va $7x$ qismlarga bo'linadi. Demak, $3x + 5x + 7x = 180$. Bu yerdan $15x = 180$ ni olamiz. Bu tenglikning har ikkala qismini 15 ga bo'lib, $x = 12$ ni topamiz. Bu sonlarning kattasi $7x$ demak, $7 \cdot 12 = 84$. **Javob:** 84 (A)
27. 120 sonini 5, 7 sonlariga teskari proporsional bo'laklarga bo'ling.
A) 60; 84 B) 70; 50 C) 48; 72 D) 56; 64
28. $\frac{1}{n-1}$ va $\frac{1}{n+1}$ sonlarining o'rta garmonik qiymatini toping.
A) $\frac{1}{n}$ B) $2n$ C) $\frac{1}{2n}$ D) $\frac{2}{n}$
29. Daryo bo'yidagi A va B pristanlar orasidagi masofa 60 km. Daryo oqimining tezligi 5 km/s, motorli qayiqning tezligi 30 km/s. Motorli qayiq A pristanidan B pristanga borib qaytib keldi. Motorli qayiqning bu yo'ldagi o'rtacha tezligini toping.
A) 30 B) 31 C) 29,1(6) D) 29,16
30. (98-4-4) Massasi 300 g va konsentratsiyasi 15% bo'lgan eritma massasi 500 g va konsentratsiyasi 9% bo'lgan eritma bilan aralashtirildi. Hosil bo'lgan aralashmaning konsentratsiyasini toping.
A) 12,75 B) 11,75 C) 12,25 D) 11,25
- Yechish:** Berilgan masala o'rta vaznli qiymat haqidagi masala ekan. (3.5) formulaga ko'ra
- $$\frac{300 \cdot 15 + 500 \cdot 9}{300 + 500} = \frac{100(3 \cdot 15 + 5 \cdot 9)}{100 \cdot 8} = 11,25.$$
- Javob:** 11,25% (D).
31. (98-12-65) Massasi 400 g va konsentratsiyasi 8% bo'lgan eritma massasi 600 g va konsentratsiyasi 13% bo'lgan eritma bilan aralashtirildi. Hosil bo'lgan aralashmaning konsentratsiyasini toping.
A) 11 B) 12 C) 9 D) 10
32. (00-4-29) Yog'liligi 2% bo'lgan 80 l sut bilan yog'liligi 5% bo'lgan necha litr sut aralashtirilsa, yog'liligi 3% bo'lgan sut olish mumkin?
A) 20 B) 30 C) 40 D) 50
33. (97-2-3) Qotishma mis va qo'rg'oshindan iborat. Qotishmaning 60% i mis bo'lib, mis qo'rg'oshindan 2 kg ko'p. Qotishmada qancha mis bor?
A) 5 B) 7 C) 6 D) 5,5
34. (00-4-28) Yig'ilgan 1 t mevaning 82% i suvdan iborat. Ma'lum vaqtdan keyin bu mevadagi suvning miqdori 70% ga tushdi. Endi bu mevaning og'irligi necha kg chiqadi?
A) 810 B) 820 C) 700 D) 600
35. (01-12-35) 800 kg mevaning tarkibida 80% suv bor. Bir necha kundan keyin mevaning og'irligi 500 kg ga tushdi. Endi uning tarkibida necha foiz suv bor?
A) 62 B) 68 C) 66 D) 60
36. Bir idishda 40% li, ikkinchi idishda 35% li eritma bor. Ularni aralashtirib, 37% li 1 litr eritma olish uchun har bir eritmadan necha litrdan olish kerak.
A) 0,3 va 0,7 B) 0,6 va 0,4
C) 0,2 va 0,8 D) 0,4 va 0,6
37. (00-5-15) 140 g suvga 60 g tuz qo'shish natijasida hosil bo'lgan tuzli eritmada necha protsent tuz bor?
A) 20 B) 30 C) 25 D) 35
38. (01-11-4) Kumush va misdan iborat qotishmaning og'irligi 2 kg. Kumushning og'irligi mis og'irligining $\frac{1}{7}$ qismini tashkil etadi. Qotishmada necha gramm kumush bor.
A) 310 g B) 300 g C) 270 g D) 250 g
- Yechish:** Qotishmadagi kumushning massasini x bilan belgilaymiz. Masala shartiga ko'ra, misning massasi $7x$ bo'ladi va biz $x + 7x = 2$ tenglikni olamiz. Bu yerdan $x = 0,25$ kg = 250 g. **Javob:** 250 g. (D).
39. (01-12-14) 15 kg eritmaning 35 foizi tuzdan iborat. Tuzning miqdori 25 foiz bo'lishi uchun eritmaga necha kg chuchuk suv qo'shish kerak?
A) 6 B) 5 C) 5,5 D) 5,25
40. (02-7-52) 20 litr tuzli suvning tarkibida 12% tuz bor. Bu eritmada tuz miqdori 15% bo'lishi uchun necha litr suv bug'lantirilishi kerak?
A) 4 B) 3 C) 5 D) 4,2
41. (03-8-25) Qotishma kumush va oltindan iborat bo'lib, o'zaro 3 : 5 nisbatda. Agar qotishmada 0,45 kg oltin bo'lsa, qotishmaning og'irligini (kg) toping.
A) 0,72 B) 0,21 C) 1,21 D) 0,8
42. (03-5-15) Massasi 36 kg bo'lgan mis va rux qotishmasining tarkibida 45% mis bor. Qotishmaning tarkibida 60% mis bo'lishi uchun unga yana necha kg mis qo'shish kerak?
A) 13,5 B) 14 C) 12 D) 15
43. (03-8-7) Siment va qumdan iborat 30 kg qorishmaning 60%ini siment tashkil etadi. Qorishmaning 40%i simentdan iborat bo'lishi uchun qorishmaga qancha qum qo'shish kerak?
A) 10 B) 12 C) 15 D) 18
44. (03-12-12) A aralashmaning bir kilogrammi 1000 so'm, B aralashmaning bir kilogrammi esa 2000 so'm turadi. B va A aralashmadan 3 : 1 nisbatda tayyorlangan 1 kg aralashma necha so'm turadi?
A) 1500 B) 1750 C) 1650 D) 1800

4 - bob. Tenglamalar

4.1 Ayniyat va tenglama

Agar ifodada qatnashuvchi o'zgaruvchilarning qabul qilishi mumkin bo'lgan barcha qiymatlarida, ifodaning son qiymati nolga teng bo'lsa, bunday ifoda *aynan nol ifoda* deyiladi. Agar ikki ifodaning ayirmasi aynan nol ifoda bo'lsa, ular *aynan teng ifodalar* deyiladi. Chap va o'ng tomonlari aynan teng ifodalar bo'lgan tenglik *ayniyat* deyiladi. Masalan, $(x+1)^2 = x^2 + 2x + 1$, $a+b = b+a$ tengliklarning har biri ayniyatdir. Bir ifodani aynan unga teng bo'lgan boshqa ifoda bilan almashtirish *ayniy almashtirish* deyiladi. Bir yoki bir nechta noma'lum qatnashgan tenglikka *tenglama* deyiladi. Tenglamada qatnashgan noma'lumlar va ularning darajalariga qarab tenglama har xil turlarga bo'linadi. Masalan,

$$3x + 15 = 0; 2 - 7x = 9; ax + b = 0; \quad (4.1)$$

$$3x - y = 15; 0, 2z - 7x = 14; ax + by = c; \quad (4.2)$$

$$5x^2 - 6x = 11; x^2 = 4; ax^2 + bx + c = 0. \quad (4.3)$$

(4.1) dagi tengliklar birinchi darajali bir noma'lumli yoki chiziqli tenglamalardir. (4.2) dagi tengliklar birinchi darajali ikki noma'lumli tenglamalardir. (4.3) dagi tengliklar ikkinchi darajali bir noma'lumli yoki kvadrat tenglamalardir. Tenglamadagi noma'lumlarining tenglamani to'g'ri tenglikka aylantiruvchi qiymatlari *tenglamaning ildizlari yoki yechimlari* deyiladi.

Tenglama bitta, bir nechta yoki cheksiz ko'p yechimga ega bo'lishi yoki tenglama umuman yechimga ega bo'lmasligi mumkin. Masalan, $3x + 15 = 0$ tenglama yagona ildizga, $2(x+3) = 2x + 6$ tenglama esa cheksiz ko'p ildizga ega, $x^2 + 1 = 0$ tenglama haqiqiy ildizlarga ega emas. Tenglamani yechish – bu uning barcha yechimlarini topish yoki yechimi mavjud emasligini ko'rsatishdir. Ikki tenglama aynan bir xil yechimlarga ega bo'lsa yoki ikkalasi ham yechimga ega bo'lmasa, bunday tenglamalar *teng kuchli (ekvivalent) tenglamalar* deyiladi. Teng kuchli tenglamalar quyidagi xossalarga ega:

1. **Tenglamaning ikkala qismini nolmas songa ko'paytirish yoki bo'lishdan hosil bo'lgan tenglama berilgan tenglamaga teng kuchlidir.**
2. **Tenglamaning ikkala qismiga bir xil ifodani qo'shsak yoki ayirsak, berilgan tenglamaga teng kuchli tenglama hosil bo'ladi.**

2-xossadan quyidagilar kelib chiqadi:

1) Agar tenglamaning har ikki qismida bir xil hadlar bo'lsa, ularni tashlab yuborish mumkin. Masalan, $3x+1+6x^2 = 7+6x^2$ tenglama $3x+1 = 7$ tenglamaga teng kuchlidir.

2) Tenglamaning hadlarini tenglikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o'tkazish mumkin. Masalan, $3x+1 = 6x$ va $1 = 6x-3x$ hamda $1 = 3x$ tenglamalar teng kuchlidir.

1. Quyidagilardan qaysi biri ayniyat emas?
A) $(a+b)^2 = a^2 + 2ab + b^2$
B) $(a-b)^2 = a^2 - 2ab + b^2$
C) $a^2 + b^2 = (a-b)(a+b)$
D) $a^2 - b^2 = (a-b)(a+b)$

Yechish: 1-usul. C) javobning o'ng tomonidagi qavsni ochamiz $(a-b)(a+b) = a^2 + ab - ba - b^2 = a^2 - b^2$. Bu ifoda tenglikning chap tomoni $a^2 + b^2$ ga aynan teng emas.

2-usul. C) da keltirilgan tenglikning ayniyat emasligini ko'rsatamiz. Buning uchun $a = 1$, $b = 1$ deymiz. Natijada tenglikning chap tomoni $1^2 + 1^2 = 2$, o'ng tomoni esa $(1-1)(1+1) = 0$ bo'ladi. Demak, C) javobda keltirilgan tenglik ayniyat emas. **Javob:** (C).

2. (97-8-12) Quyidagilardan qaysi biri ayniyat?

A) $\frac{m^3 - n^3}{m + n} = m^2 + mn + n^2$
B) $2mn - n^2 - m^2 = (m + n)^2$
C) $m - (m - n) - (m - n) = 2n - m$
D) $-\frac{m - n}{n} = \frac{-m - n}{n}$

3. Quyidagilardan qaysi biri ayniyat emas?

A) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
B) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
C) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
D) $a^2 - b^2 = (a-b)(a-b)$

4. Quyidagilardan qaysi biri ayniyat emas?

A) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
B) $(a-b)^3 = a^3 + 3a^2b + 3ab^2 - b^3$
C) $-(a-b^2) = b^2 - a$
D) $x^2 = |x|^2$

5. Quyidagilardan qaysi biri ayniyat emas?

A) $a^n \cdot a^m = a^{n+m}$ B) $(ab)^n = a^n \cdot b^n$
C) $(a^n)^m = a^{n+m}$ D) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

6. (98-9-9) Agar $9x^2 + kx + kx^2 - 9x = x^2 - 17x$ ayniyat bo'lsa, k ning qiymati nechaga teng?

A) -6 B) -8 C) -7 D) -9

Yechish: Berilgan ifoda ayniyat bo'lganligi uchun

$$9x^2 + kx + kx^2 - 9x - x^2 + 17x = (8+k)x^2 + (k+8)x \equiv 0.$$

Bu yerdan $8+k = 0$ shart kelib chiqadi. Bu shartdan $k = -8$ ni olamiz. **Javob:** -8 (B).

7. (98-1-19) a va b ning qanday qiymatida quyidagi tenglik ayniyat bo'ladi?

$$\frac{1}{x^2 - 5x - 6} = \frac{a}{x - 6} + \frac{b}{x + 1}$$

A) $a = 7$, $b = -1$ B) $a = \frac{1}{7}$, $b = -\frac{1}{7}$
C) $a = 1$, $b = 1$ D) $a = -\frac{1}{7}$, $b = \frac{1}{7}$

8. (02-3-9) Agar $a(x-1)^2 + b(x-1) + c = 2x^2 - 3x + 5$ ayniyat bo'lsa, $a + b + c$ yig'indi nechaga teng?

A) 7 B) 8 C) 6 D) 4

9. (00-3-14) a va b ning qanday qiymatlarida

$$\frac{1}{4x^2 - 1} = \frac{a}{2x - 1} - \frac{b}{2x + 1}$$

munosabat ayniyat bo'ladi?

- A) $a = -\frac{1}{2}$, $b = \frac{1}{2}$ B) $a = 1$, $b = -1$
 C) $a = -1$, $b = 1$ D) $a = \frac{1}{2}$, $b = \frac{1}{2}$

10. (02-10-6) a , b va c ning qanday qiymatida

$$\frac{1}{(x+1)^2 \cdot (x+2)} = \frac{a}{x+1} + \frac{b}{(x+1)^2} + \frac{c}{x+2}$$

tenglik ayniyat bo'ladi?

- A) -1 ; 1 ; 1 B) 0 ; 1 ; 2
 C) 1 ; -1 ; $\frac{1}{2}$ D) 2 ; -2 ; $\frac{1}{2}$

11. (96-13-12) Ushbu

$$(-3x + \alpha y)(\beta x - 2y) = \gamma x^2 + 7xy + 2y^2$$

ayniyatdagi noma'lum koeffitsiyentlardan biri β ni toping.

- A) 1 B) -1 C) 2 D) -2

12. Teng kuchli bo'lmagan tenglamani toping?

- A) $2x + 1 = 0$ va $2x + 2 = 1$
 B) $x + y = 5$ va $x = 5 - y$
 C) $x^2 + 7 = 0$ va $|x| + 1 = 0$
 D) $2x - x^2 = 1$ va $x^2 - 2x - 1 = 0$

Yechish: 1-usul. D) javobning o'ng tomonida keltirilgan tenglamaga teng kuchli tenglamani topamiz. Tenglamani ikkala qismini -1 ga ko'paytiramiz va birinchi va ikkinchi hadlarining o'rinlarini almashtirib $2x - x^2 + 1 = 0$ ni olamiz. Endi 1 ni tenglikning o'ng tomoniga qarama-qarshi ishora bilan o'tkazib $2x - x^2 = -1$ ga ega bo'lamiz. Bu esa D) ning chap tomoni bilan bir xil emas.

2-usul. $x = 1$ soni D) ning chap tomonidagi tenglamani yechimidir. $x = 1$ o'ng tomondagi tenglamani yechimi emas. Haqiqatan ham, $1^2 - 2 \cdot 1 - 1 = -2 \neq 0$. **Javob:** (D).

13. Teng kuchli tenglamani toping?

- A) $2x + 1 = 0$ va $2x = 1$
 B) $x + y = 5$ va $x = y + 5$
 C) $x^2 - 6x = 7$ va $x^2 - 6x - 7 = 0$
 D) $-x^2 = 1$ va $|x|^2 = 1$

14. Teng kuchli bo'lmagan tenglamani toping?

- A) $x^2 + 1 = 0$ va $x^2 + 5 = 1$
 B) $x + 3 = 5$ va $x(x + 3) = 5x$
 C) $5x^2 - 6x = 7x$ va $5x^2 - 13x = 0$
 D) $-x^2 = -4$ va $|x|^2 = 4$

15. Teng kuchli tenglamani toping?

- A) $x(y^2 + 1) = 0$ va $x(y + 1)(y - 1) = 0$
 B) $(x + 7)(x + y) = 5(x + 7)$ va $x + y = 5$
 C) $7x - 6 = 6x + 7$ va $x - 13 = 0$
 D) $x^2 y^2 = xy$ va $xy(xy + 1) = 0$

4.2 Chiziqli tenglamalar

Bir noma'lumli chiziqli tenglama deb

$$ax + b = 0 \quad (4.4)$$

shakldagi yoki ayniy almashtirishlardan keyin (4.4) ko'rinishga keltirish mumkin bo'lgan tenglamaga aytiladi.

1. Agar $a \neq 0$ bo'lsa, $ax + b = 0$ tenglama yagona $x = -\frac{b}{a}$ yechimga ega.

2. Agar $a = 0$, $b = 0$ bo'lsa, $ax + b = 0$ tenglama cheksiz ko'p yechimga. Ixtiyoriy haqiqiy son bu tenglamaning yechimi bo'ladi.

3. Agar $a = 0$, $b \neq 0$ bo'lsa, $ax + b = 0$ tenglama yechimga ega emas.

1. (97-1-6) Tenglamani yeching.

$$\frac{3x - 11}{4} - \frac{3 - 5x}{8} = \frac{x + 6}{2}$$

- A) 5 B) $-4, 5$ C) $6, 5$ D) 7

Yechish: Tenglamaning ikkala qismini 8 ga ko'paytiramiz:

$$6x - 22 - 3 + 5x = 4x + 24.$$

x qatnashgan hadlarni tenglamaning chap qismiga, sonlarni o'ng qismiga o'tkazamiz:

$$6x + 5x - 4x = 24 + 22 + 3 \iff 7x = 49.$$

Bu tenglikning har ikkala qismini 7 ga bo'lib $x = 7$ ni olamiz. **Javob:** 7 (D).

2. (97-6-6) Tenglamani yeching.

$$6 - \frac{x - 1}{2} = \frac{3 - x}{2} + \frac{x - 2}{3}$$

- A) $4, 5$ B) 8 C) 17 D) 11

3. (97-7-3) Tenglamani yeching.

$$0,9(4x - 2) = 0,5(3x - 4) + 4,4$$

- A) $1, 2$ B) $2, 5$ C) -3 D) 2

4. (98-1-4) Tenglamani yeching.

$$2,8x - 3(2x - 1) = 2,8 - 3,19x$$

- A) -20 B) 20 C) -2 D) 200

5. (98-8-4) Tenglamani yeching.

$$5,6 - 7(0,8x + 1) = 14 - 5,32x$$

- A) $5, 5$ B) 55 C) -55 D) $-5, 5$

6. (98-7-1) x ni toping.

$$420 : (160 - 1000 : x) = 12$$

- A) 8 B) $\frac{1}{8}$ C) 35 D) 36

7. (98-12-1)
- x
- ni toping.

$$(360 + x) \cdot 1002 = 731460$$

- A) 370 B) 270 C) 470 D) 730

8. (97-11-6) Tenglamani yeching.

$$\frac{x-3}{6} + x = \frac{2x-1}{3} - \frac{4-x}{2}$$

- A) 3 B) 2 C) -2 D)
- \emptyset

Yechish: Tenglamani undagi kasrlarning umumiy maxraji bo'lgan 6 ga ko'paytiramiz:

$$x - 3 + 6x = 2(2x - 1) - 3(4 - x).$$

Qavslarni ochamiz:

$$x - 3 + 6x = 4x - 2 - 12 + 3x.$$

x qatnashgan hadlarni tenglamaning chap qismiga, sonlarni esa o'ng qismiga o'tkazamiz:

$$7x - 7x = -14 + 3 \iff 0 \cdot x = -11.$$

Bu tenglik birorta $x \in R$ da ham o'rinni emas. Demak, berilgan tenglama ildizga ega emas.

Javob: \emptyset (D).

9. Tenglamani yeching.

$$1,6 \cdot (2 + 3x) = 6 \cdot (0,8x - 1) + 6,8$$

- A) 5 B) -0,5 C)
- \emptyset
- D) -2

10. (99-4-12) Tenglamani yeching.

$$0, (3)x - 3 = x - 2(0,5 + 0, (3)x)$$

- A) 20 B)
- \emptyset
- C) 0,2 D) 0,5

11. (96-1-20)
- m
- ning qanday qiymatlarida
- $my + 1 = m$
- tenglama yechimga ega bo'lmaydi?

- A)
- $m = 1$
- B)
- $m = 0$
- C)
- $m = -1$
- D)
- $m = 2$

12. (97-10-22)
- n
- ning qanday qiymatlarida
- $nx + 5 = n - 2x$
- tenglamaning ildizi mavjud emas?

- A) 5 B) -2 C) 1 D) -5

13. (99-7-21)
- a
- ning

$$(a^2 - 4)x + 5 = 0$$

tenglama yechimga ega bo'lmaydigan barcha qiymatlari ko'paytmasini toping.

- A) 4 B) -4 C) 0 D) 2

14. (99-8-21) Tenglama
- a
- ning qanday qiymatida yechimga ega emas?

$$6x - a - 6 = (a + 2)(x + 2)$$

- A) 4 B) 2 C) -2 D) 6

15. (00-3-11)
- k
- ning qanday qiymatida

$$k(k + 6)x = k + 7(x + 1)$$

tenglama yechimga ega bo'lmaydi?

- A) 1 va 7 B) 1 C) 7 D) 1 va -7

16. (02-7-7)
- a
- ning qanday qiymatida

$$\frac{3x - a}{5} = \frac{ax - 4}{3}$$

tenglama ildizga ega emas?

- A) 1,8 B) 2 C) 2,2 D) 1

17. (02-11-9) Tenglamani yeching.

$$\frac{2x + 3}{2} + \frac{2 - 3x}{3} = 2,1(6)$$

- A)
- \emptyset
- B) 2 C) -2 D)
- $x \in R$

Yechish: Tenglamaning ikkala qismini 6 ga ko'paytiramiz:

$$6x + 9 + 4 - 6x = 6 \cdot 2,1(6).$$

2,1(6) davriy kasrni oddiy kasrga aylantirsak, u $2\frac{16-1}{90} = 2\frac{1}{6} = \frac{13}{6}$ ko'rinish oladi. Natijada $(6-6)x = 13 - 13$ tenglikka ega bo'lamiz. Bu tenglik barcha $x \in R$ lar uchun o'rinni. **Javob:** $x \in R$ (D).

18. (03-8-11) Tenglamani yeching.

$$\frac{6x + 2}{4} + \frac{2x + 3}{2} - 2,5x + 2 = 4$$

- A)
- \emptyset
- B)
- $x \in R$
- C) 10 D) -10

19. Tenglamani yeching.

$$\frac{x + 2}{3} + \frac{7x - 1}{2} = 3,8(3)x + 0,1(6)$$

- A)
- \emptyset
- B)
- $x \in R$
- C) 10 D) -10

20. (98-1-20)
- m
- ning qanday qiymatlarida

$$m(mx - 1) = 9x + 3$$

tenglama cheksiz ko'p ildizga ega?

- A)
- $m = 0$
- B)
- $m = 3$
-
- C)
- $m = -3$
- D)
- $m = -1$

Yechish: Qavslarni ochamiz.

$$m^2x - m = 9x + 3$$

Uni $(m^2 - 9)x - (m + 3) = 0$ ko'rinishga keltiramiz. Bu tenglama cheksiz ko'p yechimga ega bo'lishi uchun (2-qoidaga qarang)

$$\begin{cases} m^2 - 9 = 0 \\ m + 3 = 0 \end{cases} \text{ bo'lishi kerak. Demak, } m = -3.$$

Javob: -3 (C).

21. (96-7-22)
- a
- ning qanday qiymatlarida
- $ax - a = x - 1$
- tenglama cheksiz ko'p yechimga ega bo'ladi?

- A)
- $a = 1$
- B)
- $a = 2$
- C)
- $a = -1$
- D)
- $a \in R$

22. (96-10-21) n ning qanday qiymatlarida $nx + 1 = n + x$ tenglama cheksiz ko'p yechimga ega bo'ladi?
A) $n = 0$ B) $n = 1$ C) $n = 2$ D) $n \neq 1$
23. (97-7-22) m ning qanday qiymatlarida $m^2x - m = x + 1$ tenglama ildizlari cheksiz ko'p bo'ladi?
A) $m = 1$ B) $m = 0$ C) $m = -1$ D) $m = \pm 1$
24. (98-12-28) Tenglama a ning qanday qiymatlarida cheksiz ko'p yechimga ega?

$$10(ax - 1) = 2a - 5x - 9$$

- A) $-\frac{1}{2}$ B) 2 C) $\frac{1}{2}$ D) -2

25. (01-1-10) a ning qanday qiymatida

$$(a^2 + 2)x = a(x - a) + 2$$

tenglamaning ildizlari cheksiz ko'p bo'ladi?
A) $-\sqrt{2}$ B) $\sqrt{2}$ C) $\sqrt{2}; -\sqrt{2}$ D) \emptyset

26. (03-7-44) Tenglamani yeching.

$$\frac{3 + 25x}{3x + 7} = 5$$

- A) $-3, 2$ B) 1, 5 C) $-1\frac{1}{5}$ D) 3, 2

27. (03-7-48) Tenglamani yeching.

$$\left(1, 7 : \left(1\frac{2}{3} \cdot x - 3, 75\right)\right) : \frac{8}{25} = 1\frac{5}{12}$$

- A) 5, 2 B) $5\frac{3}{4}$ C) 4 D) 4, 5

28. (99-8-11) Tenglamani yeching.

$$\frac{(x - 12) : \frac{3}{8}}{0, 3 \cdot 3\frac{1}{5} + 7} = 1$$

- A) 25 B) 14 C) 15 D) 16

29. (01-8-4) Tenglamani yeching.

$$\left(4\frac{3}{8}x + 5\frac{1}{16}\right) \cdot \frac{4}{15} = \frac{5}{12}x + 2\frac{2}{5}$$

- A) $\frac{1}{15}$ B) $1\frac{2}{5}$ C) $\frac{3}{185}$ D) $2\frac{1}{5}$

30. (02-7-43) $986^2 - 319^2 = 2001 \cdot n$ bo'lsa, n ning qiymatini toping.

- A) 435 B) 443 C) 515 D) 475

31. (03-11-57) Tenglamani yeching.

$$12\left(1\frac{3}{4}x + \frac{5}{8}\right) = -6\frac{1}{2}$$

- A) $-\frac{1}{3}$ B) $-\frac{2}{3}$ C) $\frac{2}{3}$ D) $-\frac{13}{21}$

32. (02-7-6) m ning qanday qiymatida

$$\frac{6x - m}{2} = \frac{7mx + 1}{3}$$

tenglamaning ildizi nolga teng bo'ladi?

- A) $-\frac{2}{3}$ B) $\frac{4}{5}$ C) $-\frac{3}{2}$ D) $\frac{1}{2}$

4.2.1 Proporsiya

Proporsiya xossalari 1.2.5-bandda keltirilgan.

1. (96-9-75) Proporsiyaning noma'lum hadini toping.

$$3\frac{3}{5} : 2\frac{7}{10} = 3\frac{3}{4} : x$$

- A) $2\frac{13}{16}$ B) $2\frac{3}{10}$ C) $3\frac{1}{3}$ D) $1\frac{15}{16}$

Yechish: Proporsiya o'rta hadlarining ko'paytmasi uning chetki hadlari ko'paytmasi tengligidan

$$2\frac{7}{10} \cdot 3\frac{3}{4} = x \cdot 3\frac{3}{5} \iff \frac{27}{10} \cdot \frac{15}{4} : \frac{18}{5} = x$$

ni olamiz. Ko'paytirish va bo'lishni bajarib $x = 2\frac{13}{16}$ ekanligini topamiz. **Javob:** $2\frac{13}{16}$ (A).

2. (00-5-10) Tenglamani yeching.

$$1\frac{1}{12}x : 2\frac{1}{12} = 2\frac{3}{5}$$

- A) 5 B) 3 C) $1\frac{5}{12}$ D) 4

3. (98-12-12) Tenglamani yeching.

$$(12, 5 - x) : 5 = (3, 6 + x) : 6$$

- A) $5\frac{2}{11}$ B) $5\frac{3}{11}$ C) $5\frac{4}{11}$ D) $5\frac{1}{11}$

4. (96-7-12) Proporsiyaning noma'lum hadini toping.

$$6, 9 : 4, 6 = x : 5, 4$$

- A) 7, 1 B) 7, 7 C) 8, 1 D) 8, 4

5. (97-3-12) Proporsiyaning noma'lum hadini toping.

$$3, 5 : x = 0, 8 : 2, 4$$

- A) 10, 5 B) 9, 2 C) 13, 5 D) 7, 8

6. (97-7-12) Proporsiyaning noma'lum hadini toping.

$$5, 4 : 2, 4 = x : 1, 6$$

- A) 3, 6 B) 4 C) 2, 8 D) 4, 6

7. (97-10-12) Proporsiyaning noma'lum hadini toping.

$$0, 25 : 1, 4 = 0, 75 : x$$

- A) 3, 6 B) 2, 4 C) 4, 2 D) 5, 2

8. (98-7-13) Tenglamani yeching.

$$\left(\frac{1}{3} + x\right) : 7 = \left(\frac{3}{4} + x\right) : 9$$

- A) $1\frac{3}{8}$ B) $1\frac{1}{8}$ C) $1\frac{5}{8}$ D) $1\frac{7}{8}$

9. (03-11-55) Proporsiyaning noma'lum hadini toping.

$$12\frac{1}{2} : 2\frac{1}{2} = 16\frac{2}{3} : y$$

- A) $3\frac{1}{3}$ B) $3\frac{2}{3}$ C) $3\frac{1}{6}$ D) $3\frac{5}{6}$

4.3 Kvadrat tenglamalar

$$ax^2 + bx + c = 0 \quad (4.5)$$

ko'rinishdagi tenglamaga yoki ayniy almashtirishlardan keyin (4.5) ko'rinishga keltirish mumkin bo'lgan tenglamaga *bir noma'lumli ikkinchi darajali tenglama* yoki *kvadrat tenglama* deyiladi. Bunda x – noma'lum, a ($a \neq 0$), b va c – lar ixtiyoriy sonlar. (4.5) tenglamaga kvadrat tenglamaning normal shakli deyiladi. a – birinchi koeffitsiyent, b – ikkinchi koeffitsiyent, c esa uchinchi koeffitsiyent yoki ozod had deyiladi. Kvadrat tenglamaning ildizlari quyidagi formula orqali topiladi

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \quad (4.6)$$

$b^2 - 4ac$ ifoda kvadrat tenglamaning *diskriminanti* deyiladi va D harfi bilan belgilanadi: $D = b^2 - 4ac$.

Agar $a = 1$ bo'lsa, (4.5) tenglama *keltirilgan kvadrat tenglama* deyiladi. Keltirilgan kvadrat tenglama odatda

$$x^2 + px + q = 0 \quad (4.7)$$

ko'rinishda yoziladi.

Agar (4.5) da $b = 0$ yoki $c = 0$ yoki $b = c = 0$ bo'lsa, bunday tenglama *chala kvadrat tenglama* deyiladi. Masalan, $ax^2 + bx = 0$ chala kvadrat tenglamaning ildizlari $x_1 = 0$, $x_2 = -b/a$ lardir. $ax^2 + c = 0$ chala kvadrat tenglama $ac < 0$ shartda yechimga ega va uning ildizlari $x_{1,2} = \pm\sqrt{-c/a}$, ko'rinishga ega. $ax^2 = 0$ chala kvadrat tenglamaning ildizi $x_1 = 0$ dir.

1. Viyet teoremasi. Agar x_1, x_2 sonlar (4.7) tenglamaning ildizlari bo'lsa, u holda

$$\begin{cases} x_1 + x_2 = -p \\ x_1 \cdot x_2 = q \end{cases} \text{ tengliklar o'rinli.}$$

2. Agar $D > 0$ bo'lsa, u holda (4.5) kvadrat tenglama ikkita har xil haqiqiy ildizga ega.

3. Agar $D = 0$ bo'lsa, u holda (4.5) kvadrat tenglama bitta haqiqiy ildizga ega.

4. Agar $D < 0$ bo'lsa, u holda (4.5) kvadrat tenglama haqiqiy ildizlarga ega emas.

5. Agar x_1, x_2 sonlar (4.5) kvadrat tenglamaning ildizlari bo'lsa, $ax^2 + bx + c$ kvadrat uchhad a , $x - x_1$ va $x - x_2$ larning ko'paytmasiga teng bo'ladi: $ax^2 + bx + c = a(x - x_1)(x - x_2)$.

6. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ tenglama $x_1 = \alpha$, $x_2 = \beta$ ildizlarga ega.

Keltirilgan (4.7) kvadrat tenglamaning ildizlari quyidagi xossalarga ega.

7. $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = p^2 - 2q$.

8. $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = p^2 - 4q$.

9. $x_1^3 + x_2^3 = 3pq - p^3$.

10. $x_1^4 + x_2^4 = p^4 - 4p^2q + 2q^2$.

1. Kvadrat tenglamani yeching.

$$x^2 + 5x - 6 = 0$$

A) $-6; 1$ B) $-1; 6$ C) $1; 6$ D) $2; 3$

Yechish: Berilgan kvadrat tenglamada $a = 1$, $b = 5$, $c = -6$. Endi kvadrat tenglama yechimga egami yoki yo'qmi shuni aniqlaymiz. Buning uchun uning diskriminantini hisoblaymiz: $D = 5^2 - 4 \cdot 1 \cdot (-6) = 25 + 24 = 49$. Kvadrat tenglama diskriminanti musbat, shuning uchun u ikkita ildizga ega. Ularni (4.6) formula yordamida hisoblaymiz:

$$x_1 = \frac{-5 - \sqrt{49}}{2 \cdot 1} = \frac{-5 - 7}{2} = -6,$$

$$x_2 = \frac{-5 + \sqrt{49}}{2 \cdot 1} = \frac{-5 + 7}{2} = 1.$$

Javob: $-6; 1$ (A).

2. Kvadrat tenglamani yeching.

$$2x^2 + 3x - 14 = 0$$

A) $-7; 2$ B) $-2; 3\frac{1}{2}$ C) $-3\frac{1}{2}; 2$ D) $2; 3\frac{1}{2}$

3. Kvadrat tenglamani yeching.

$$4x^2 + 12x + 9 = 0$$

A) $-7; 2$ B) $-1, 5$ C) -3 D) \emptyset

4. Kvadrat tenglamani yeching.

$$9x^2 + 6x + 3 = 0$$

A) $-1; \frac{1}{3}$ B) $-2; \frac{1}{6}$ C) -3 D) \emptyset

5. (00-8-64) Tenglamani yeching.

$$1998x^2 - 2000x + 2 = 0$$

A) $1; \frac{2}{1998}$ B) $-1; \frac{2}{1998}$
C) $1; -\frac{2}{1998}$ D) $-1; -\frac{2}{1998}$

Yechish: Berilgan kvadrat tenglamaga 1998 ga bo'lib uni

$$x^2 - \left(1 + \frac{2}{1998}\right)x + 1 \cdot \frac{2}{1998} = 0$$

shaklda yozamiz. 6-xossaga ko'ra uning ildizlari $x_1 = 1$, $x_2 = \frac{2}{1998}$ lardir. **Javob:** $1; \frac{2}{1998}$ (A).

6. Tenglamani yeching.

$$x^2 - 97x + 2010 = 0$$

A) $30; 67$ B) $-30; -67$ C) $15; 134$ D) $2; 1005$

7. Chala kvadrat tenglamani yeching.

$$2x^2 - 6x = 0$$

- A) 0; 3 B) -2; 6 C) 3 D)
- \emptyset

Yechish: Berilgan kvadrat tenglamada $a = 2$, $b = -6$, $c = 0$. Bu tenglama diskriminantini $D = (-6)^2 - 4 \cdot 2 \cdot 0 = 36 > 0$. Kvadrat tenglama ikkita ildizga ega. (4.6) formuladan $x_1 = 0$, $x_2 = 3$.

Javob: 0; 3 (A).

8. Chala kvadrat tenglamani yeching.

$$x^2 - 7x = 0$$

- A) 0; 3 B) 7 C) 0; 7 D) 0

9. Chala kvadrat tenglamani yeching.

$$9x^2 - 1 = 0$$

- A) 0;
- $\frac{1}{3}$
- B)
- $-\frac{1}{3}$
- ;
- $\frac{1}{3}$
- C) 0;
- $-\frac{1}{3}$
- D)
- \emptyset

10. Chala kvadrat tenglamani yeching.

$$3x^2 + \frac{1}{3} = 0$$

- A) 0;
- $\frac{1}{3}$
- B)
- $-\frac{1}{3}$
- ; 1 C) 0;
- $-\frac{1}{3}$
- D)
- \emptyset

11. (96-1-18) Tenglamani nechta ildizi bor?

$$3 - x = -\frac{4}{x}$$

- A) 1 B) 2 C) 3 D) ildizi yo'q

Yechish: $x \neq 0$ deb, tenglikning ikkala qismini x ga ko'paytiramiz va $3x - x^2 + 4 = 0$ ni olamiz. Bu kvadrat tenglamaning diskriminanti $D = 3^2 - 4 \cdot (-1) \cdot 4 = 25 > 0$. Shuning uchun u ikkita ildizga ega. **Javob:** 2 (B).

12. (96-9-69) Tenglamani nechta ildizi bor?

$$\frac{2}{x} = x + 2$$

- A) 3 B) 2 C) 1 D) ildizi yo'q

13. Tenglamani nechta ildizi bor?

$$x^2 - 6x + 9 = 0$$

- A) 1 B) 2 C) 3 D) ildizi yo'q

14. Tenglamani nechta haqiqiy ildizi bor?

$$4x^2 + 8x + 7 = 0$$

- A) 1 B) 2 C) 3 D) ildizi yo'q

15. (96-7-13) Agar

$$(3x - 1) \cdot (x - 2) = 0$$

bo'lsa, $3x - 1$ qanday qiymatlarni qabul qilishi mumkin?

- A) faqat
- $\frac{1}{3}$
- B) faqat 0
-
- C)
- $\frac{1}{3}$
- yoki 0 D) 0 yoki 5

Yechish: $(3x-1) \cdot (x-2) = 0$ dan $x_1 = \frac{1}{3}$, $x_2 = 2$ ekanligi kelib chiqadi. Bu qiymatlarni $3x - 1$ ga qo'yib $3x_1 - 1 = 0$ va $3x_2 - 1 = 5$ ni olamiz. **Javob:** 0 yoki 5 (D).

16. (97-3-13) Agar

$$(2x + 1) \cdot (x - 1,5) = 0$$

bo'lsa, $2x + 1$ qanday qiymatlar qabul qiladi?

- A) faqat 0 B) faqat
- $-\frac{1}{2}$
-
- C) 0 yoki
- $-\frac{1}{2}$
- D) 4 yoki 0

17. (97-7-13) Agar
- $(x - 5) \cdot (\frac{1}{5}x + 4) = 0$
- bo'lsa,
- $\frac{1}{5}x + 4$
- qanday qiymatlar qabul qiladi?

- A) faqat 0 B) faqat -20
-
- C) 0 yoki 5 D) 0 yoki 8

18. (97-10-13) Agar
- $(4x + 1) \cdot (x - \frac{1}{4}) = 0$
- bo'lsa,
- $4x + 1$
- qanday qiymatlar qabul qilishi mumkin?

- A) faqat
- $-\frac{1}{4}$
- B) faqat
- $\frac{1}{4}$
-
- C) faqat 0 D) 0 yoki 2

19. (96-3-18) Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$$x^2 - x - 2$$

- A)
- $(x - 1)(x + 2)$
- B)
- $(x - 1)(x - 2)$
-
- C)
- $(x + 1)(x + 2)$
- D)
- $(x + 1)(x - 2)$

Yechish: Berilgan kvadrat uchhadni chiziqli ko'paytuvchilarga ajratish uchun 5-xossadan foydalanamiz. Shu maqsadda kvadrat tenglamaning ildizlarini topamiz. Uning diskriminanti $D = (-1)^2 - 4 \cdot 1 \cdot (-2) = 1 + 8 = 9$ ga teng. $D > 0$ bo'lganligi uchun tenglamaning ikkita ildizi bor. Ularni (4.6) formula yordamida topamiz

$$x_1 = \frac{1 - 3}{2} = -1, \quad x_2 = \frac{1 + 3}{2} = 2.$$

5-xossadan $x^2 - x - 2 = (x + 1)(x - 2)$ ni olamiz.

Javob: $(x + 1)(x - 2)$ (D).

20. (96-11-19) Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating.

$$x^2 - 3x + 2$$

- A)
- $(x - 1)(x + 2)$
- B)
- $(x - 2)(x + 1)$
-
- C)
- $(x - 1)(x - 2)$
- D)
- $(x + 1)(x + 2)$

21. (97-2-27) Kasrni qisqartiring.

$$\frac{x^2 - 16}{x^2 - 5x + 4}$$

- A)
- $\frac{4 + x}{1 - x}$
- B)
- $\frac{4 - x}{x + 1}$
- C)
- $\frac{x + 4}{x + 1}$
- D)
- $\frac{x + 4}{x - 1}$

22. (97-8-26) Kasrni qisqartiring.

$$\frac{y^2 - 3y - 4}{y^2 - 1}$$

- A)
- $\frac{y+4}{y+1}$
- B)
- $\frac{4-y}{y-1}$
- C)
- $\frac{y+4}{y-1}$
- D)
- $\frac{y-4}{y-1}$

23. (00-8-37) Ko'paytuvchilarga ajrating.

$$3x^2 - 6xm - 9m^2$$

- A)
- $3(x+m)(x-3m)$
- B)
- $(x-3m)^2$
-
- C)
- $3(x-m)(x+3m)$
- D)
- $(3x-m)^2$

24. (00-3-18) Agar

$$x^2 - 3x - 6 = 0$$

tenglamaning ildizlari x_1 va x_2 bo'lsa, $\frac{1}{x_1^3} + \frac{1}{x_2^3}$ ni toping.

- A)
- $\frac{1}{3}$
- B) 0,5 C) -0,5 D) -0,375

Yechish: Berilgan tenglama uchun $p = -3$, $q = -6$. Qiymati izlanayotgan ifodani

$$\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{x_1^3 + x_2^3}{(x_1 x_2)^3}$$

shaklda yozib olamiz. Viyet teoremasi va 9-xossaga ko'ra bu kasrning qiymati

$$\frac{x_1^3 + x_2^3}{(x_1 x_2)^3} = \frac{3 \cdot (-3) \cdot (-6) - (-3)^3}{(-6)^3} = -\frac{81}{216}$$

ga teng. Buni o'nli kasrga aylantirib $-0,375$ ni olamiz. **Javob:** $-0,375$ (D).

25. (96-13-18)
- x_1
- va
- x_2
- sonlar

$$x^2 + x - 5 = 0$$

tenglamaning ildizlari ekanligi ma'lum. $x_1^2 + x_2^2$ ning qiymatini toping.

- A) 10 B) 12 C) 11 D) 9

26. (97-4-24)
- a
- va
- b
- sonlari

$$3x^2 - 2x - 6 = 0$$

tenglamaning ildizlari bo'lsa, $a^2 + b^2$ ni hisoblang.

- A) 6 B) 8 C)
- $4\frac{4}{9}$
- D)
- $4\frac{2}{9}$

27. (98-4-25) Agar

$$x^2 + x - 1 = 0$$

tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3 + x_2^3$ ning qiymati qanchaga teng bo'ladi?

- A) 1 B) 3 C) 2 D) -4

28. (98-5-21) Ushbu

$$x^2 + 4x - 5 = 0$$

tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3 \cdot x_2^3$ ni hisoblang.

- A) 124 B) -125 C) 130 D) 5

29. (99-7-23) Agar

$$x^2 + 2x + 1 = 0$$

tenglamaning ildizlari x_1 va x_2 bo'lsa, $x_1^3 - x_2^3$ ni hisoblang.

- A) 1 B) 3 C) 4 D) 0

30. (01-10-2) Agar
- x_1
- va
- x_2
- lar
- $x^2 + x - 5 = 0$
- tenglamaning ildizlari bo'lsa,
- $x_1^2 x_2^4 + x_1^4 x_2^2$
- ning qiymatini hisoblang.

- A) 225 B) 145 C) 125 D) 275

31. (03-1-5) Agar
- x_1
- va
- x_2

$$x^2 + 3x - 3 = 0$$

tenglamaning ildizlari bo'lsa, $x_1^4 + x_2^4$ ning qiymatini hisoblang.

- A) 207 B) 192 C) 243 D) 168

32. (03-8-19) Tenglamaning katta va kichik ildizlari kublarining ayirmasini toping.

$$x^2 - \frac{\sqrt{85}}{4}x + 1\frac{5}{16} = 0$$

- A) -2 B) -1 C) 2 D) 1

33. (01-2-23) Tenglama ildizlarining o'rta proporsionalini toping.

$$x^2 - 13x + 36 = 0$$

- A) 4 B) 9 C) 6,5 D) 6

Yechish: a va b sonlarining o'rta proporsionali deganda \sqrt{ab} qiymat tushuniladi. Berilgan tenglama diskriminanti $D = (-13)^2 - 4 \cdot 36 = 25 > 0$. Demak, bu tenglamaning ikkita ildizi bor. Viyet teoremasiga ko'ra, $\sqrt{x_1 \cdot x_2} = \sqrt{36} = 6$. **Javob:** 6 (D).

34. (00-1-12) Tenglama ildizlarining o'rta proporsionalini toping.

$$2x^2 - 26x + 72 = 0$$

- A) 4 B) 5 C) 7 D) 6

35. (99-10-5) Tenglama ildizlarining o'rta arifmetigi ularning ko'paytmasidan qancha kam?

$$\frac{x^2 + 16}{x} = 10$$

- A) 13 B) 12 C) 14 D) 11

36. (02-11-14) Ildizlaridan biri
- $3 + \frac{\sqrt{2}}{2}$
- ga teng bo'lgan ratsional koeffitsiyentli kvadrat tenglama tuzing.

- A)
- $x^2 - 3x + 9 = 0$
- B)
- $x^2 - 6x + 17 = 0$
-
- C)
- $x^2 - 12x + 9 = 0$
- D)
- $2x^2 - 12x + 17 = 0$

37. (02-1-49) 3 va -2 sonlari qaysi tenglamaning ildizlari ekanligini ko'rsating.

- A)
- $x^2 - x = 6$
- B)
- $x^2 + x = 6$
-
- C)
- $x^2 + 6 = x$
- D)
- $x^2 + 6 = -x$

4.3.1 Parametrli kvadrat tenglamalar

Kvadrat tenglamalarni qo'shimcha xossalarini keltiramiz.

- (4.5) tenglama $D := b^2 - 4ac = 0$ yoki $a = 0$ va $b \neq 0$ da bitta ildizga ega.
- (4.5) kvadrat tenglama $D > 0$ va $ac > 0$ da bir xil ishorali ikkita ildizga ega.
- (4.5) kvadrat tenglama $D > 0$ va $ac < 0$ da turli ishorali ikkita ildizga ega.
- Agar (4.5) kvadrat tenglamaning ildizlaridan biri nol bo'lsa, u holda $c = 0$ va aksincha.
- Agar $a > 0$ va $D = 0$ bo'lsa, $ax^2 + bx + c$ kvadrat uchhad to'la kvadrat bo'ladi:
 $ax^2 + bx + c = \left(\sqrt{ax} + \frac{b}{2\sqrt{a}}\right)^2$.

- (97-2-24) Tenglama ildizlaridan biri 0 bo'ladigan m ning barcha qiymatlari ko'paytmasini toping.

$$x^2 - 9x + (m^2 - 4)(m^2 - 9) = 0$$

- A) 36 B) $4\sqrt{3}$ C) -6 D) 6

Yechish: 1-usul. 4-xossaga ko'ra tenglamaning ildizi 0 bo'lishi uchun, uning ozod hadi $c = 0$ bo'lishi zarur va yetarli. Bu shart $(m^2 - 4)(m^2 - 9) = 0$ ga teng kuchli. Bu yerdan $m_{1,2} = \pm 2$ va $m_{3,4} = \pm 3$ ekanligini olamiz. Ularning ko'paytmasi $m_1 \cdot m_2 \cdot m_3 \cdot m_4 = 36$ ga teng.

2-usul. Nol berilgan tenglamaning ildizi. Tenglamada $x = 0$ deb $(m^2 - 4)(m^2 - 9) = 0$ ni olamiz. Bu tenglamaning ildizlari ± 2 va ± 3 lardir. Ularning ko'paytmasi esa 36 ga teng. **Javob:** 36 (A).

- (00-10-21) p ning qanday qiymatida

$$x^2 + px + 15 = 0$$

tenglamaning ildizlaridan biri 5 ga teng bo'ladi?

- A) -4 B) 4 C) -2 D) -8

- (01-6-13) a ning qanday qiymatida

$$x^2 - (a - 1)x + 36 = 0$$

tenglamaning ildizlaridan biri 4 ga teng bo'ladi?

- A) 13 B) 14 C) 11 D) 10

- (98-10-43) Tenglama ildizlaridan biri 2 ga teng. Ikkinchi ildizni toping.

$$2x^2 + x - a = 0$$

- A) 2,5 B) -2,5 C) 1,5 D) -1,5

- (97-12-24) Ushbu

$$x^2 + px - 12 = 0$$

tenglamaning ildizlaridan biri 2 ga teng. p : (-12) nimaga teng?

- A) $\frac{1}{3}$ B) $-\frac{5}{12}$ C) $\frac{2}{3}$ D) $-\frac{1}{3}$

- (00-8-31) b ning qanday qiymatida

$$x^2 + \frac{2}{3}x + b$$

uchhad to'la kvadrat bo'ladi?

- A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) $\frac{2}{9}$ D) $\frac{2}{3}$

Yechish: Berilgan kvadrat uchhadga $a = 1$. 5-xossaga ko'ra u to'la kvadrat bo'lishi uchun $D = \left(\frac{2}{3}\right)^2 - 4b = 0$ bo'lishi zarur va yetarli. Bu shartdan $b = \frac{1}{9}$ ni olamiz. **Javob:** $\frac{1}{9}$ (A).

- (00-8-34) k ning qanday qiymatlarida

$$x^2 + 2(k - 9)x + k^2 + 3k + 4$$

ifodani to'la kvadrat shaklida tasvirlab bo'ladi?

- A) $\frac{11}{3}$ B) 3 C) 4 D) $\frac{5}{7}$

- (03-3-14) m ning qanday qiymatlarida

$$(m - 1)x^2 + 2mx + 3m - 2$$

kvadrat uchhadni to'la kvadrat shaklida tasvirlash mumkin?

- A) $2; \frac{1}{2}$ B) -2 C) 2 D) $\frac{1}{2}$

- (98-7-35) Ushbu

$$x^2 + px + 6 = 0$$

tenglama ildizlari ayirmasining kvadrati 40 ga teng bo'lsa, ildizlarining yig'indisi qancha bo'lishini toping?

- A) $\sqrt{40}$ B) 8 C) -8 D) -8 va 8

- (98-12-32) m ning qanday qiymatlarida

$$3x^2 + (3m - 15)x - 27 = 0$$

tenglamaning ildizlari qarama-qarshi sonlar bo'ladi?

- A) 5 B) 0 C) -3; 3 D) -5

- (98-12-33) Ushbu

$$x^2 + px + 6 = 0$$

tenglama ildizlari ayirmasining kvadrati 40 ga teng. p ning qiymatini toping.

- A) -8; 8 B) 8 C) -8 D) $4 + \sqrt{10}$

- (99-1-18) $x^2 + px - 35 = 0$ tenglamaning ildizlaridan biri 7 ga teng. Ikkinchi ildiz va p ning qiymatini toping.

- A) -5; -2 B) -5; 2 C) 5; 2 D) 5; -2

Yechish: Viyet teoremasi $x_1 x_2 = 7 x_2 = -35$ dan $x_2 = -5$ ni olamiz. Yana Viyet teoremasi $x_1 + x_2 = -p \iff 7 + (-5) = -p$ dan $p = -2$ ekanligini olamiz. **Javob:** -5; -2 (A).

13. (01-11-9) Ushbu

$$x^2 + 2ax + a = 0$$

tenglamaning ildizlaridan biri 1 ga teng. Tenglamani ikkinchi ildizini toping.

- A) $-\frac{4}{3}$ B) $-\frac{1}{2}$ C) $\frac{1}{3}$ D) $-\frac{1}{3}$

14. (01-1-9)
- k
- ning qanday qiymatida

$$kx^2 + 12x - 3 = 0$$

tenglamaning ildizlaridan biri 0,2 ga teng bo'ladi.

- A) 135 B) 60 C) -135 D) 15

15. (01-12-39)
- k
- ning qanday qiymatlarida

$$(k-2)x^2 + 7x - 2k^2 = 0$$

tenglama $x = 2$ yechimga ega?

- A) 1; 3 B) 1; -3 C) -1; 3 D) -2; 3

16. (02-5-17) Tenglamani ildizlaridan biri 2 ga teng bo'ladigan
- a
- ning barcha qiymatlarini toping.

$$x^2 - 4x - (a-1)(a-5) = 0$$

- A) $(-\infty; -2) \cup (2; +\infty)$ B) $(-\infty; \infty)$
C) $(-\infty; -4) \cup (4; +\infty)$ D) $\{3\}$

17. (00-7-12) Ildizlari
- $x^2 + px + q = 0$
- tenglamani ildizlariga teskari bo'lgan tenglamani ko'rsating.

- A) $px^2 + qx + 1 = 0$ B) $qx^2 + px - 1 = 0$
C) $qx^2 + px + 1 = 0$ D) $qx^2 - px + 1 = 0$

18. (00-7-47)
- m
- ning qanday qiymatlarida

$$x^2 - 4mx + 48 = 0$$

tenglamaning ildizlaridan biri boshqasidan 3 marta katta bo'ladi?

- A) 2 B) ± 4 C) ± 3 D) 4

Yechish: Masala shartidan $x_2 = 3x_1$ ni olamiz. Viyet teotemasiga ko'ra $x_1 + x_2 = 4m \iff x_1 + 3x_1 = 4m$. Bu yerdan $x_1 = m$ kelib chiqadi. Yana Viyet teotemasidan foydalansak,

$$x_1x_2 = 48 \iff x_1 \cdot 3x_1 = 3m^2 = 48 \iff m^2 = 16.$$

Demak, $m = \pm 4$. **Javob:** ± 4 (B).

19. (00-4-9) Ushbu

$$x^2 - 5x + a = 0$$

tenglamaning ildizlaridan biri ikkinchisidan 9 marta katta bo'lsa a ning qiymatini toping.

- A) 2, 5 B) 2, 4 C) 2, 25 D) 3, 5

20. (02-11-15)
- q
- ning qanday qiymatida

$$x^2 - 8x + q = 0$$

tenglamaning ildizlaridan biri boshqasidan uch marta katta bo'ladi?

- A) 6 B) 8 C) 12 D) 16

21. (03-7-78)
- m
- ning qanday qiymatlarida

$$4x^2 - (3+2m)x + 2 = 0$$

tenglamaning ildizlaridan biri ikkinchisidan sakkiz marta kichik bo'ladi?

- A) 3 B) -6 C) -6; 3 D) 3; 5

22. (00-8-9)
- x_1
- va
- x_2
- sonlari

$$3x^2 + 2x + b = 0$$

tenglamaning ildizlari bo'lib, $2x_1 = -3x_2$ ekanligi ma'lum bo'lsa, b ning qiymatini toping.

- A) -8 B) 6 C) 4 D) -3

23. (01-1-15)
- a
- ning qanday qiymatida

$$5(a+4)x^2 - 10x + a = 0$$

tenglamaning ildizlari turli ishorali bo'ladi?

- A) (-1; 5) B) (-4; 0)
C) (-5; 1) D) (-5; -4) \cup (0; 1)

24. (01-2-62)
- a
- ning qanday qiymatlarida

$$x^2 + ax + 12 = 0$$

tenglamaning ildizlari orasidagi masofa 1 ga teng bo'ladi?

- A) ± 5 B) ± 6 C) ± 7 D) ± 8

Yechish: Masala shartiga ko'ra $|x_1 - x_2| = 1$ bo'ladi. Bu tenglikning ikkala qismini kvadratga ko'taramiz va $|x|^2 = x^2$ ayniyatdan foydalanib

$$x_1^2 - 2x_1x_2 + x_2^2 = (x_1 + x_2)^2 - 4x_1x_2 = p^2 - 4q = 1^2$$

ni olamiz. Berilgan tenglamada $p = a$, $q = 12$. Bularni yuqorida hosil qilingan tenglikka qo'yib $a^2 - 4 \cdot 12 = 1 \iff a^2 = 49$ ni hosil qilamiz. Demak, $a = \pm 7$ ekan. **Javob:** ± 7 (C).

25. (01-5-20)
- a
- ning qanday musbat qiymatida

$$8x^2 - 30x + a^3 = 0$$

tenglamaning ildizlaridan biri ikkinchisining kvadratiga teng bo'ladi?

- A) 3 B) 1 C) 2 D) 4

26. (02-1-50)
- a
- ning qanday qiymatlarida

$$ax^2 - 2x + 3 = 0$$

tenglama bitta ildizga ega bo'ladi?

- A) $\frac{1}{3}$ B) 0 va 1 C) 3 va 1, 5 D) $\frac{1}{3}$ va 0

27. (02-7-2) Tenglamani yeching.

$$x^2 - 3ax + 2a^2 - ab - b^2 = 0$$

- A) $a - b; 2a + b$ B) $-a + b; -2a + b$
C) $-a - b; 2a - b$ D) $a + b; 2a + b$

28. (02-7-4)
- n
- ning qanday qiymatlarida

$$x^2 - 12x + n = 0$$

tenglama ildizlaridan biri ikkinchisidan $2\sqrt{5}$ ga ortiq bo'ladi?

- A) 31 B) 30 C) 3 D) 29

29. (02-8-21)
- a
- ning qanday qiymatlarida

$$x^2 + (a + 2)x + a = 0$$

tenglama ildizlari kvadratlarining yig'indisi 3 ga teng bo'ladi?

- A) -1 B) 1 C) -2 D) 3

30. (97-2-25) Ushbu
- $x^2 - 6x + q = 0$
- tenglamaning ildizlaridan biri 2 ga teng. Bu tenglamaning barcha koeffitsiyentlari yig'indisini toping.

- A) 2 B) -6 C) 3 D) -5

Yechish: $x^2 - 6x + q = 0$ tenglamada $x = 2$ deb $4 - 12 + q = 0$ ni olamiz. Bu yerdan $q = 8$ bo'ladi. U holda koeffitsiyentlar yig'indisi $1 + (-6) + 8 = 3$ ga teng. **Javob:** 3 (C).

31. (02-9-13) Tenglama yechimga ega bo'lmaydigan
- k
- ning butun qiymatlari o'rta arifmetigini toping.

$$kx^2 + 3kx + 2k - 1 = 0$$

- A) -3 B) -2 C) -1,5 D) 3

32. (96-3-77)
- x_1
- va
- x_2
- lar

$$x^2 + |a|x + 6 = 0$$

tenglamaning ildizlari bo'lib, $x_1^2 + x_2^2 = 13$ tenglikni qanoatlantirsa, $x_1 + x_2$ nechaga teng?

- A) 5 B) -6 C) 6 D) -5

33. (01-8-22)
- a
- ning nechta qiymatida

$$\frac{3x - a}{3 - x} + \frac{x + a}{x + 1} = 2$$

tenglama bitta yechimga ega?

- A) 4 B) 3 C) 2 D) 1

Yechish: Tenglikning o'ng tomonidagi 2 ni chap tomonga o'tkazib va kasrlarni qo'shib

$$\frac{2x^2 + 6x - 2ax + 2a - 2(2x - x^2 + 3)}{(3 - x)(1 + x)} = 0$$

tenglamaga kelamiz. Kasr suratida qavslarni ochib, o'xshash hadlarni ixchamlab, uni quyidagi shaklda yozamiz

$$4\left(x^2 - \left(1 + \frac{a-3}{2}\right)x + \frac{a-3}{2}\right) = 0.$$

Qavs ichi keltirilgan kvadrat tenglama bo'lib, 4.3-dagi 6-xossadan uning ildizlari $x_1 = 1$ va $x_2 = \frac{a-3}{2}$ ekanligi kelib chiqadi. Berilgan tenglamaning aniqlanish sohasi $R \setminus \{-1; 3\}$ dan iborat. Agar $x_2 = 1$ desak, tenglama yagona $x_1 = x_2 = 1$

yechimga ega bo'ladi. Bu shartdan $a = 5$ kelib chiqadi. Agar $x_2 = -1$ yoki $x_2 = 3$ desak, ular tenglamaning aniqlanish sohasidan tashqarida qoladi va tenglama faqat $x_1 = 1$ yechimga ega bo'ladi. Bu shartlardan $a = 1$ yoki $a = 9$ ni olamiz. Demak, a ning 3 ta qiymatida tenglama bitta yechimga ega bo'lar ekan. **Javob:** 3 (B).

34. (99-4-19)
- a
- ning qanday qiymatlarida

$$ax^2 - (a + 1)x + 2a - 1 = 0$$

tenglama bitta ildizga ega bo'ladi?

- A) -1;
- $\frac{1}{7}$
- B) 0; -1
-
- C) 1;
- $-\frac{1}{7}$
- ; D) 1; 0;
- $-\frac{1}{7}$

35. (03-1-58) Tenglamaning ildizlari bir-biriga teng bo'ladigan
- k
- ning barcha qiymatlari ko'paytmasini toping.

$$9x^2 + kx = 2x - k + 6$$

- A) 100 B) -120 C) 220 D) -196

36. (03-3-11) Agar

$$x^2 - 3x + m = 0$$

tenglamaning x_1 va x_2 ildizlari uchun $3x_1 - 2x_2 = 14$ munosabat o'rinli bo'lsa, m ning qiymatini toping.

- A) -4 B) 4 C) 6 D) -6

37. (03-3-12)
- p
- ning qanday qiymatida

$$x^2 - px + 5 = 0$$

tenglamaning ildizlaridan biri boshqasidan 4 ga katta?

- A) 6 B) 4 C) -4 D)
- ± 6

38. (03-3-25)
- a
- ning qanday qiymatlarida

$$x + 4 = \frac{a}{x}$$

tenglama ikkita turli haqiqiy ildizga ega?

- A)
- $(-4; \infty)$
- B)
- $(-4; 0) \cup (0; \infty)$
-
- C)
- $[-4; \infty)$
- D)
- $[-4; 0) \cup (0; \infty)$

39. (03-4-12)
- a
- ning qanday qiymatlarida

$$x^2 + 3x + a + 0,75 = 0$$

tenglamaning ikkala ildizi ham manfiy bo'ladi?

- A)
- $0,5 < a < 2$
- B)
- $-0,75 < a < 1,5$
-
- C)
- $0,6 < a < 1,8$
- D)
- $0,8 < a < 1,2$

40. (03-5-16)
- a
- ning qanday qiymatida

$$x^2 - (a - 2)x - a - 1 = 0$$

tenglama ildizlari kvadratlarining yig'indisi eng kichik qiymatga ega bo'ladi?

- A) 1 B) 2 C)
- $\frac{1}{2}$
- D) 4

Yechish: Masala shartiga ko'ra $x_1^2 + x_2^2$ ifodaning qiymati eng kichik bo'ladigan a ni topishimiz kerak. 4.3-dagi 7-tenglikka ko'ra $x_1^2 + x_2^2 = p^2 - 2q = (a-2)^2 - 2(-a-1) = a^2 - 2a + 6 = (a-1)^2 + 5$. Haqiqiy sonning kvadrati manfiylikligidan $x_1^2 + x_2^2 = (a-1)^2 + 5 \geq 5$. Bu ifodaning qiymati minimal bo'lishi uchun $a-1 = 0$, ya'ni $a = 1$ bo'lishi kerak. **Javob:** 1 (A).

41. (99-2-16) x_1 va x_2

$$x^2 - px + p - 1 = 0$$

tenglamaning ildizlari, p ning qanday qiymatida $x_1^2 + x_2^2$ yig'indi eng kichik qiymatni qabul qiladi?
A) 2 B) -2 C) 1 D) -1

42. (00-1-13) y_1 va y_2

$$y^2 - by + b - 1 = 0$$

tenglamaning ildizlari bo'lsa, b ning qanday qiymatida $y_1^2 + y_2^2$ ifodaning qiymati eng kichik bo'ladi?
A) 1, 2 B) 0, 85 C) 1 D) 1, 5

43. (01-7-16) m ning qanday qiymatida

$$x^2 + (m-1)x + m^2 - 1,5 = 0$$

tenglama ildizlari kvadratlarining yig'indisi eng katta bo'ladi?
A) 1, 5 B) -1, 5 C) 1 D) -1

44. (01-8-16) m ning qanday qiymatida

$$x^2 + (2-m)x - m - 3 = 0$$

tenglama ildizlari kvadratlarining yig'indisi eng kichik bo'ladi?
A) 2 B) 1 C) -1 D) -3

45. (01-12-25) a ning qanday qiymatida

$$x^2 + (a+2)x + a = 3$$

tenglama ildizlari kvadratlarining yig'indisi eng kichik bo'ladi?
A) 0 B) -1 C) 1 D) 3

46. (03-7-62) q ning qanday qiymatida

$$x^2 - 8x + q = 0$$

tenglama ildizlari kvadratlarining yig'indisi 34 ga teng bo'ladi?
A) 15 B) -12 C) 12 D) -15

Yechish: Masala shartiga ko'ra $x_1^2 + x_2^2 = 34$. 4.3-dagi 7-tenglikka ko'ra $x_1^2 + x_2^2 = p^2 - 2q$:

$$p^2 - 2q = 34 \iff (-8)^2 - 2q = 34 \iff q = 15.$$

Javob: 15 (A).

47. (03-10-14) q ning qanday qiymatida

$$x^2 - x - q = 0$$

tenglama ildizlari kublarining yig'indisi 19 ga teng bo'ladi?

A) 6 B) 5 C) 7 D) 4

48. (03-11-1) a parametrning qanday butun qiymatida

$$2x^2 + 6ax + a = 0$$

tenglama ildizlari kvadratlarining yig'indisi 38 ga teng bo'ladi?

A) -2 B) 2 C) -3 D) -1

49. (03-11-6) Agar m va n sonlar

$$x^2 + 3mx - 5n = 0 \quad (m \cdot n \neq 0)$$

tenglamaning ildizlari bo'lsa, $n - m$ ning qiymati nechaga teng bo'ladi?

A) 25 B) 24 C) 18 D) 12

50. (03-5-29) x_1 va x_2 sonlar

$$x^2 + 3x + k = 0$$

tenglamaning ildizlari va $\frac{x_1}{x_2} = -\frac{2}{5}$ bo'lsa, k ning qiymatini toping.

A) -10 B) -7 C) -12 D) -8

51. (03-9-4) m ning qanday qiymatida

$$3x^2 - 21x + m = 0$$

tenglama ildizlari kvadratlarining yig'indisi 25 ga teng bo'ladi?

A) 36 B) -36 C) 24 D) 42

4.4 Ratsional tenglamalar

Agar $P(x)$ ko'phadda $x - o'zgaruvchining$ 3 va undan yuqori darajalari qatnashsa, $P(x) = 0$ tenglama *yuqori darajali tenglama* deyiladi. Bunday tenglamani yechishda asosan ikki usul qo'llaniladi. Bular "*yangi o'zgaruvchi kiritish*" va "*ko'paytuvchilarga ajratish*" usullaridir.

$$ax^4 + bx^2 + c = 0, \quad a \neq 0 \quad (4.8)$$

ko'rinishdagi tenglamaga *bikvadrat tenglama* deyiladi. Bikvadrat tenglamada $x^2 = y \geq 0$ deb yangi o'zgaruvchi kiritsak, (4.8) tenglama

$$ay^2 + by + c = 0 \quad (4.9)$$

ko'rinishdagi kvadrat tenglamaga keladi. Agar (4.9) tenglama yechimga ega bo'lmasa yoki uning yechimlari manfiy bo'lsa, u holda (4.8) tenglama yechimga ega emas. Agar (4.9) tenglama y_1 va y_2 yechimga ega bo'lib, ular manfiymas bo'lsa, u holda $x_{1,2} = \pm\sqrt{y_1}$ va $x_{3,4} = \pm\sqrt{y_2}$ lar (4.8) tenglamaning yechimlari bo'ladi. Bikvadrat tenglamaning barcha ildizlari yig'indisi nolga teng.

2-usul – ko'paytuvchilarga ajratish usuli.

1-misol. Quyidagi tenglamani yeching.

$$x^4 - 4x^3 - x^2 + 4x = 0. \quad (4.10)$$

Yechish: $x^4 - 4x^3 - x^2 + 4x = (x^2 - 1)(x^2 - 4x)$ ayniyatdan berilgan tenglama $(x^2 - 1)(x^2 - 4x) = 0$ tenglamaga teng kuchli bo'ladi. Ko'paytmaning nolga teng bo'lish shartidan foydalansak,

$$x^2 - 1 = 0, \quad (4.11)$$

$$x^2 - 4x = 0 \quad (4.12)$$

tenglamalar hosil bo'ladi. (4.10) tenglamaning istalgan yechimi yo (4.11) yo (4.12) tenglamadan birining yechimi bo'ladi va aksincha. (4.11) tenglama $x_1 = -1$, $x_2 = 1$ ildizlarga, (4.12) tenglama esa $x_3 = 0$, $x_4 = 4$ ildizlarga ega. Demak, (4.10) tenglama $x_1 = -1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 4$ ildizlarga ega. **Javob:** $\pm 1; 0; 4$.

Agar bizga $P(x) : Q(x) = 0$ ko'pinishdagi ratsional tenglama berilgan bo'lsa, uning ildizlari $P(x) = 0$ tenglama ildizlaridan $Q(x) = 0$ tenglama ildizlarini chiqarib tashlashdan hosil bo'ladi. Quyidagi misolni qaraymiz.

2-misol. Tenglamani yeching.

$$\frac{x^2 + 5x + 6}{x^2 + x - 2} = 0.$$

Yechish: Ma'lumki, kasrning qiymati nol bo'lishi uchun uning surati nolga teng bo'lishi kerak. Shuning uchun berilgan tenglama

$$\begin{cases} x^2 + 5x + 6 = 0 \\ x^2 + x - 2 \neq 0 \end{cases}$$

sistemaga teng kuchli. Birinchi tenglamaning ildizlari $x_1 = -2$, $x_2 = -3$. Topilgan bu qiymatlarni sistemaning ikkinchi shartini qanoatlantirishini tekshiramiz:

$$(-2)^2 + (-2) - 2 = 0; \quad (-3)^2 + (-3) - 2 = 4 \neq 0.$$

$x = -2$ sistemaning 2-shartini qanoatlantirmadi. Shunday qilib, berilgan tenglamaning ildizi $x = -3$ ekan.

Javob: -3 .

1. (00-3-26) Tenglamaning haqiqiy ildizlari yig'indisini toping.

$$(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$$

- A) 3 B) -3 C) 2 D) -5

Yechish: Tenglama $y = x^2 + 5x$ belgilash yordamida $(y + 4)(y + 6) = 120$ tenglamaga keladi. Qavslarni ochamiz

$$y^2 + 10y + 24 - 120 = 0 \iff y^2 + 10y - 96 = 0.$$

Bu kvadrat tenglamaning yechimlari $y_1 = 6$, $y_2 = -16$ lardir. Endi berilgan tenglama ikkita tenglamaga ajraydi.

1) $x^2 + 5x = 6$, $x^2 + 5x - 6 = 0$, $x_1 = -6$, $x_2 = 1$,

2) $x^2 + 5x + 16 = 0$, $D = 25 - 64 = -39 < 0$.

2) tenglama yechimga ega emas. Demak, berilgan tenglama $x_1 = -6$ va $x_2 = 1$ ildizlarga ega. Ildizlar yig'indisi $x_1 + x_2 = -5$. **Javob:** -5 (D).

2. (96-7-15) Tenglama ildizlari yig'indisini toping.

$$x^4 - 13x^2 + 36 = 0$$

- A) 13 B) 5 C) 0 D) 36

3. (97-7-15) Tenglamaning eng katta va eng kichik ildizlari ayirmasini toping.

$$x^4 - 10x^2 + 9 = 0$$

- A) 1 B) 8 C) 2 D) 6

4. (98-4-33) Tenglamaning ildizlari yig'indisini toping.

$$2x^4 - 7x^2 + 2 = 0$$

- A) 7 B) 3,5 C) 0 D) 2

5. (98-6-20) Tenglama ildizlari ko'paytmasini toping.

$$\left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right) - 3 = 0$$

- A) 3 B) -1 C) 4 D) 1

6. (98-11-10) Tenglamaning haqiqiy ildizlari ko'paytmasini toping.

$$y^4 - 2y^2 - 8 = 0$$

- A) 4 B) -16 C) 16 D) -4

7. (00-1-16) Ifoda nechta ratsional koeffitsiyentli ko'paytuvchilarga ajraladi?

$$(x^4 + x^2 + 1) \cdot (x^4 + x^2 + 2) - 12$$

- A) 4 B) 2 C) 3 D) 5

8. (98-6-22) Tenglamani yeching.

$$\frac{2x^2 - 5x + 3}{(10x - 5)(x - 1)} = 0$$

- A) 1 B) $1; \frac{3}{2}$ C) $\frac{3}{2}$ D) 5

Yechish: Berilgan tenglamada kasr suratini nolga tenglashtiramiz. $2x^2 - 5x + 3 = 0$ kvadrat tenglama ildizlari $x_1 = 1$, $x_2 = 1,5$ lardir. Berilgan tenglamadagi kasr maxraji $x = 1$ da nolga aylanadi, $x = 1,5$ da esa noldan farqli. Demak, $x = 1,5$ tenglamaning ildizi bo'ladi. **Javob:** $\frac{3}{2}$ (C).

9. (98-11-18) Tenglamaning yechimlari quyidagi oraliqlarning qaysi birida joylashgan?

$$\frac{x^2 + 1}{x} + \frac{x}{x^2 + 1} = -2,5$$

- A) $(-\infty; -1)$ B) $[-1; 8)$ C) $[2; 8)$ D) $[3; 8)$

10. (98-11-71) Tenglamani yeching.

$$\frac{1 - \frac{1}{x-1}}{1 + \frac{1}{x-1}} = 0$$

- A) -2 B) 0 C) -1 D) 2

11. (98-12-63) Tenglama ildizlari yig'indisini toping.

$$\frac{2}{3-x} + \frac{1}{2} = \frac{6}{x(3-x)}$$

- A) 4 B) 7 C) 3 D) 10

12. (00-5-36) Tenglamaning ildizlari nechta?

$$\frac{x^2 - x - 2}{x^2 + x} = 0$$

- A) 2 B) 4 C) 1 D) 3

13. (01-1-8) Tenglamani yeching.

$$\frac{2}{x-3} = \frac{x+5}{x^2-9}$$

- A) -2 B) 2 C) 1 D) -1

14. (02-3-25) Tenglama ildizlari ko'paytmasini toping.

$$\frac{26}{5(x+x^{-1})} = 1$$

- A) 1 B) 5 C) 2 D) 2, 4

15. (02-4-4) Tenglamaning ildizlari sonini toping.

$$x^4 - (\sqrt{5} + \sqrt{3})x^2 + \sqrt{15} = 0$$

- A) 2 B) 4 C) 1 D) 0

16. (02-7-41) Tenglama ildizlari yig'indisini toping.

$$(x+1)(x+2)(x+4)(x+5) = 40 \quad (x \in R)$$

- A) -6 B) 0 C) -5 D) 6

Yechish: Berilgan tenglamaning chap qismini $(x+1)(x+5) = x^2 + 6x + 5$ va $(x+2)(x+4) = x^2 + 6x + 8$ larning ko'paytmasi shaklida yozamiz. Bunday guruhlash $1 + 5 = 2 + 4$ tenglikka asoslangan bo'lib, u hosil bo'lgan kvadrat uchhadlarning 1- va 2- hadlarining tengligini ta'minlaydi. Natijada, berilgan tenglamaga teng kuchli bo'lgan

$$(x^2 + 6x + 5)(x^2 + 6x + 8) - 40 = 0$$

tenglamaga ega bo'lamiz. Bu tenglamada $x^2 + 6x + 5 = y$ deb belgilash olsak,

$$y(y+3) - 40 = 0 \quad \text{yoki} \quad y^2 + 3y - 40 = 0$$

kvadrat tenglamani hosil qilamiz. Bu tenglamaning ildizlari $y_1 = -8$, $y_2 = 5$ lardir. Topilgan bu qiymatlarni belgilashga qo'yamiz:

$$1) x^2 + 6x + 5 = -8; \quad 2) x^2 + 6x + 5 = 5.$$

1-kvadrat tenglamaning diskriminanti $D = 6^2 - 4 \cdot 13 = -14 < 0$ manfiy, shuning uchun u haqiqiy ildizlarga ega emas. 2-tenglama $x^2 + 6x = 0$ ko'rinishdagi chala kvadrat tenglama bo'lib, uning ildizlari $x_1 = -6$, $x_2 = 0$ lardir. Tenglama ildizlari yig'indisi $-6 + 0 = -6$. **Javob:** -6 (A).

17. (02-11-20) Tenglama ildizlari yig'indisini toping.

$$\frac{3x^2 + 8x - 3}{x+3} = x^2 - x + 2$$

- A) -8 B) -6 C) -4 D) 4

18. (03-3-28) Tenglama ildizlari ko'paytmasini toping.

$$\frac{3x^2 + 8x - 3}{x+3} = x^2 - x + 2$$

- A) 2 B) -2 C) 6 D) 3

19. (03-6-8) Agar

$$\frac{4x^2 - 4xy + 3y^2}{2y^2 + 2xy - 5x^2} = 1$$

bo'lsa, $\frac{x+y}{x-y}$ ning qiymatini toping.

- A) 2 B) -2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

20. (03-7-56) Tenglama ildizlari ayirmasining modulini toping.

$$\frac{x+8}{3} = x - \frac{x-3}{x}$$

- A) 5, 5 B) 5 C) 3, 5 D) 4

21. (03-8-17) Tenglama ildizlari yig'indisini toping.

$$\frac{3x^2 + 4x - 4}{x+2} = x^2 - 4x + 4$$

- A) 10 B) -5 C) -4 D) 7

22. (03-8-42) Tenglama butun ildizlarining yig'indisini toping.

$$x^2 + 3x + \frac{6}{2-3x-x^2} = 1$$

- A) -3 B) 1 C) -5 D) 3

23. (03-11-63) Tenglama ildizlari yig'indisini toping.

$$\frac{x^3 - 8}{x-2} = 6x + 1$$

- A) 6 B) 4 C) -4 D) 3

24. (03-12-2) Tenglamaning eng kichik va eng katta ildizlari ayirmasini toping.

$$3x^4 - 5x^2 + 2 = 0$$

- A) 2 B) $\frac{2\sqrt{6}}{3}$ C) $-\frac{2\sqrt{6}}{3}$ D) -2

4.5 Tenglamalar sistemasi

4.5.1 Chiziqli tenglamalar sistemasi

Ikki noma'lumli chiziqli tenglamalar sistemasi deb

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \quad (4.13)$$

ko'rinishdagi sistemaga aytiladi. Bu yerda $a_1, b_1, c_1, a_2, b_2, c_2$ ma'lum sonlar, x, y lar esa noma'lum sonlardir. (4.13) sistemaning yechimi deb birinchi va ikkinchi tenglamalarni qanoatlantiruvchi $(x; y)$ sonlar juftiga aytiladi. Sistemani yechish uning hamma yechimlarini topish yoki yechimlari yo'qligini isbotlash demakdir. (4.13) ko'rinishdagi sistemani yechishning quyidagi ikki usulini ko'rib o'tamiz.

O'rniqa qo'yish usuli. Tenglamalar sistemasini o'rniqa qo'yish usuli bilan yechish uchun tenglamalarning birortasidan noma'lumlardan birini ikkinchisi orqali ifodalab, ikkinchi tenglamaga olib borib qo'yiladi. Natijada bir noma'lumli chiziqli tenglama hosil bo'ladi. Bu tenglamani yechib, yechimni tenglamalar sistemasining istalgan bir tenglamasiga qo'yib ikkinchi noma'lumning qiymati topiladi. Quyidagi misolni qaraymiz.

1-misol. Sistemani yeching.

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5. \end{cases} \quad (4.14)$$

Yechish: Bu sistemaning ikkinchisidan x ni topamiz (ya'ni $x = 5 - 2y$) va uni sistemaning birinchi tenglamasiga qo'yamiz: $2(5 - 2y) - 3y = 3 \iff 7 = 7y$. Bu tenglama yagona $y = 1$ ildizga ega. Bu yechimni sistemaning ikkinchisiga qo'yib, $x + 2 \cdot 1 = 5$ dan $x = 3$ ni olamiz. Demak, (4.14) sistemaning yechimi $(3; 1)$ dan iborat. **Javob:** $(3; 1)$.

Qo'shish usuli. Tenglamalar sistemasini qo'shish usuli bilan yechish uchun berilgan sistemaga teng kuchli bo'lgan shunday sistema olamizki, unda y (yoki x) oldidagi koeffitsiyentlar qarama-qarshi sonlar bo'ladi. Sistemaning tenglamalari hadma-had qo'shilsa, x ga nisbatan bir noma'lumli chiziqli tenglama hosil bo'ladi. Bu tenglamadan x ni topib, uni tenglamalar sistemasining istalgan bir tenglamasiga qo'yib y noma'lumning qiymati topiladi. Quyidagi misolni qaraymiz.

2-misol. Sistemani yeching.

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1. \end{cases} \quad (4.15)$$

Yechish: Bu sistemaning ikkinchi tenglamasini 2 ga kopaytiramiz va tenglamalarni hadma-had qo'shib $5x = 5$ tenglamani olamiz. Bu yerdan $x = 1$ ni topamiz va uni sistemaning ikkinchi tenglamasiga qo'yib $1 + 2y = 1$ dan $y = 0$ ni olamiz. Demak, (4.15) sistemaning yechimi $(1; 0)$ dan iborat. **Javob:** $(1; 0)$.

1. Agar (4.13) sistemada $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ bo'lsa, u yagona yechimga ega.

2. Agar (4.13) sistemada $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ bo'lsa, u cheksiz ko'p yechimlarga ega.

3. Agar (4.13) sistemada $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ bo'lsa, sistema yechimga ega emas.

1. (96-6-17) Agar

$$\begin{cases} 3x + y = 45 \\ z + 3y = -15 \\ 3z + x = 6 \end{cases}$$

bo'lsa, $x + y + z$ nimaga teng?

A) 12 B) 10 C) 15 D) 9

Yechish: Sistema tenglamalarini qo'shamiz:

$$4x + 4y + 4z = 45 - 15 + 6.$$

Bu yerdan

$$4(x + y + z) = 36 \iff x + y + z = 9$$

ekanini hosil qilamiz. **Javob:** 9 (D).

2. (96-1-21) $(x; y)$ sonlar jufti

$$\begin{cases} 2x - y = 5 \\ 3x + 2y = 4 \end{cases}$$

sistemaning yechimi bo'lsa, $x - y$ ni toping.

A) 1 B) -1 C) 3 D) 0

3. (96-3-24) Tenglamalar sistemasini qanoatlantiruvchi $(x; y)$ sonlar juftini toping.

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases}$$

A) $(2; 3)$ B) $(-2; 3)$ C) $(3; 2)$ D) $(-2; -3)$

4. (96-3-76) Sistemadan x ni toping.

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$$

A) 1 B) 2 C) 3 D) -2

5. (96-9-17) Sistemadan x ni toping.

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1 \end{cases}$$

A) 1 B) 0 C) -1 D) 2

6. (96-9-72) $(x; y)$ sonlar jufti

$$\begin{cases} 3x - 2y = -8 \\ x + 3y = 1 \end{cases}$$

sistemaning yechimi bo'lsa, $y - x$ ni toping.

A) 0 B) -1 C) -2,5 D) 3

7. (96-11-25) Quyidagi juftliklardan qaysi biri tenglamalar sistemasini qanoatlantiradi?

$$\begin{cases} x + y = 5 \\ x - y = -1 \end{cases}$$

A) $(2; 3)$ B) $(1; 4)$ C) $(4; 1)$ D) $(3; 2)$

8. (96-12-74) Sistemani yeching va y ning qiymatini toping.

$$\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$$

- A) 2 B) 1 C) 3 D) 1,5

9. (96-13-17) Sistemadan x ni toping.

$$\begin{cases} 3x - 4y = 3 \\ x + 2y = 1 \end{cases}$$

- A) -1 B) 3 C) 2 D) 1

10. (97-1-11) $(x; y)$ sonlar jufti tenglamalar sistemasining yechimi bo'lsa, $x \cdot y$ ni toping.

$$\begin{cases} 2x + y - 8 = 0 \\ 3x + 4y - 7 = 0 \end{cases}$$

- A) -90 B) 12 C) -10 D) 80

Yechish: Sistemaning birinchi tenglamasidan y ni topamiz $y = 8 - 2x$ va uni ikkinchi tenglamaga qo'yamiz: $3x + 4(8 - 2x) = 7 \iff 25 = 5x$. Bu yerdan $x = 5$ ni olamiz. x ning bu qiymatini $y = 8 - 2x$ ga qo'yib $y = -2$ ni olamiz. Ularning ko'paytmasi $xy = 5 \cdot (-2) = -10$ ekan. **Javob:** -10 (C).

11. (97-3-21) Agar

$$\begin{cases} 5x + 2y = -3 \\ x - 3y = -4 \end{cases}$$

bo'lsa, $x^2 - y^2$ ning qiymatini toping.

- A) 2 B) 1 C) 0 D) 2,5

12. (97-6-11) $(x; y)$ sonlar jufti

$$\begin{cases} x + 2y - 3 = 0 \\ 2x - 3y + 8 = 0 \end{cases}$$

sistemaning yechimi bo'lsa, $x + y$ ni toping.

- A) -1 B) 1 C) 3 D) 4,5

13. (97-10-21) Agar

$$\begin{cases} 3x - 2y = 1 \\ 4x - y = -2 \end{cases}$$

bo'lsa, $y^2 - x^2$ ning qiymatini toping.

- A) -1 B) -3 C) 3 D) 5

14. (98-3-16) Sistemadan x ni toping.

$$\begin{cases} 3x + 4y = 11 \\ 5x - 2y = 1 \end{cases}$$

- A) 2 B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) 1

15. (98-10-64) Sistemadan y ni toping.

$$\begin{cases} 3x + 4y = 11 \\ 5x - 2y = 1 \end{cases}$$

- A) 0 B) 1 C) 2 D) -2

16. Tenglamalar sistemasini yeching.

$$\begin{cases} x + y = 8 \\ x + 2y = 12 \end{cases}$$

- A) (4; 4) B) (-4; -4) C) (-4; 4) D) (4; 6)

17. (97-4-7) $a = 4b$ va $c + 3b = 0$ ($b \neq 0$) bo'lsa, $\frac{a}{c}$ ni toping.

- A) $-\frac{1}{3}$ B) $1\frac{1}{3}$ C) $1\frac{2}{3}$ D) $-\frac{4}{3}$

Yechish: Sistemadan $a = 4b$, $c = -3b$ ekanligi kelib chiqadi. Demak, $\frac{a}{c} = \frac{4b}{-3b} = -\frac{4}{3}$. **Javob:** $-\frac{4}{3}$ (D).

18. (97-8-17) Agar $2m + n = 2$, $2n + p = 6$ va $2p + m = 4$ bo'lsa, $m + n + p$ ni toping.

- A) 6 B) 4 C) 5 D) 3

19. (97-12-16) Agar $2q - 4p = -9$, $2t - 4q = -7$ va $2p - 4t = 2$ bo'lsa, $p + q + t$ ning qiymatini toping.

- A) -7 B) 8 C) 7 D) -8

20. (00-4-39) Agar $3a + 4b = 16$ va $2c - b = 1$ bo'lsa, $3a + 8c$ ning qiymatini toping.

- A) 18 B) 4 C) 20 D) 23

21. (00-1-11) Agar

$$a^2 - 4a + 5 + b^2 - 2b = 0$$

bo'lsa, $(a + b)^3$ ning qiymatini toping.

- A) 26 B) 27 C) 28 D) 25

22. (02-12-2) Agar $x + y = 4$, $y + z = 8$ va $x + z = 6$ bo'lsa, $x - y + 2z$ ning qiymatini hisoblang.

- A) 8 B) 6 C) 7 D) 10

23. (02-12-19) Nechta natural sonlar jufti

$$x^2 - y^2 = 105$$

tenglikni qanoatlantiradi?

- A) 3 B) 4 C) 2 D) 5

4.5.2 Parametrli tenglamalar sistemasini

1. (00-5-27) k ning qanday qiymatida

$$\begin{cases} kx + 4y = 4 \\ 3x + y = 1 \end{cases}$$

tenglamalar sistemasini yagona yechimga ega bo'ladi?

- A) $k \neq 12$ B) $k = 9$ C) $k \neq 19$ D) $k = 12$

Yechish: 4.5.1-ning 1-qoidasiga ko'ra, sistema $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ shartda yagona yechimga ega. Bu shart berilgan sistema uchun $\frac{k}{3} \neq \frac{4}{1}$ ko'rinishini oladi. Bu yerdan $k \neq 12$. **Javob:** $k \neq 12$ (A).

2. (01-7-19)
- a
- ning qanday qiymatida

$$\begin{cases} x + y = a \\ xy = 9 \end{cases}$$

tenglamalar sistemasi yagona yechimga ega?

- A) 3 B) 6 C) -3 D) -6; 6

3. (01-10-28)
- a
- ning qanday qiymatida

$$\begin{cases} 2x + 3y = 5 \\ x - y = 2 \\ x + 4y = a \end{cases}$$

tenglamalar sistemasi yechimga ega?

- A) 0 B) 1 C) 2 D) 3

4. (02-1-46) Agar

$$\begin{cases} ax + by = 3 \\ bx + ay = 2 \end{cases}$$

tenglamalar sistemasi $x = 3, y = 2$ yechimga ega bo'lsa, a ning qiymatini toping.

- A) 4 B) 5 C) 3 D) 1

5. (02-9-15) Agar
- $y - x = 2$
- va
- $a > 0$
- bo'lsa,

$$\begin{cases} y^2 - x^2 = 8a \\ y + x = a^2 \end{cases}$$

tenglamalar sistemasini yeching.

- A) (5; 7) B) (7; 9) C) (4; 6) D) (-6; -4)

6. (99-9-16)
- k
- ning qanday qiymatida

$$\begin{cases} 3x + 6y = k \\ 9x + 18y = k + 1 \end{cases}$$

tenglamalar sistemasi cheksiz ko'p yechimga ega?

- A)
- $\frac{1}{3}$
- B) 1 C)
- $\frac{1}{2}$
- D)
- $\frac{2}{3}$

Yechish: Sistemani cheksiz ko'p yechimga ega bo'lishlik sharti 4.5.1-ning 2-dan $\frac{3}{9} = \frac{6}{18} = \frac{k}{k+1}$ ni olamiz. Bu yerdan $k = \frac{1}{2}$. **Javob:** $\frac{1}{2}$ (C).

7. (01-2-15)
- k
- ning qanday qiymatida

$$\begin{cases} 3x + (k-1)y = k+1 \\ (k+1)x + y = 3 \end{cases}$$

sistema cheksiz ko'p yechimga ega bo'ladi?

- A) -1 B) -2 C) 0 D) 2

8. (98-3-24)
- k
- ning qanday qiymatlarida

$$\begin{cases} (k^2 - k - 1)x + 2,5y = 5 \\ 2x + y = -k \end{cases}$$

sistemani birorta ham yechimi bo'lmaydi?

- A) -2 B) -2 va 3 C) 3 D) 4 va 3

Yechish: Sistema yechimga ega bo'lmaslik sharti 4.5.1-ning 3-dan $\frac{k^2 - k - 1}{2} = \frac{2,5}{1} \neq \frac{5}{-k}$ yoki

$$\begin{cases} k^2 - k - 1 = 5 \\ k \neq -2 \end{cases}$$

ni olamiz. Sistema 1- tenglamasi yechimlari $k_1 = -2$ va $k_2 = 3$ lardir. Ikkinchi munosabat $k \neq -2$ dan yechim faqat $k = 3$ bo'ladi. **Javob:** $k = 3$ (C).

9. (98-5-20)
- a
- ning qanday qiymatlarida

$$\begin{cases} ax - y = 0 \\ x + y = 10 \end{cases}$$

tenglamalar sistemasi yechimga ega bo'lmaydi?

- A) -1 B) 2 C) 1 D) -2

10. (98-10-71)
- k
- ning qanday qiymatlarida

$$\begin{cases} (k^2 + k + 1)x + 3y - 6 = 0 \\ x + y + k = 0 \end{cases}$$

sistema birorta ham yechimga ega bo'lmaydi?

- A) -2 B) 1 C) -2 va 1 D) 2 va 3

11. (02-5-10)
- m
- ning qanday qiymatlarida

$$\begin{cases} mx + 2y + 4 = 0 \\ 2x + my - 8 = 0 \end{cases}$$

tenglamalar sistemasi yechimga ega emas?

- A) 4 B) -4 C) 2 D) -2; 2

12. (99-2-17)
- a
- ning qanday qiymatida

$$\begin{cases} 2x + ay = 2 \\ ax + 2y = 3 \end{cases}$$

tenglamalar sistemasi yechimga ega bo'lmaydi?

- A) 3 B)
- ± 3
- C) 4 D)
- ± 2

13. (99-1-17) Tenglamalar sistemasi
- a
- ning qanday qiymatlarida yechimga ega emas?

$$\begin{cases} a^2x + 3y = 3 \\ 3x + y = 4 \end{cases}$$

- A)
- ± 3
- B)
- ± 1
- C)
- $\pm\sqrt{3}$
- D) 0

14. (03-10-30)
- a
- ning nechta qiymatida

$$\begin{cases} (a-2)x + 3y = 5 \\ 7x - 18y = 1 \end{cases}$$

tenglamalar sistemasi yechimga ega emas?

- A) 1 B) 2 C) 4 D)
- \emptyset

15. (97-4-23) Agar

$$\begin{cases} x + 2y = 2 \\ 2x + y = k \end{cases}$$

bo'lsa, k ning qanday qiymatida $x + y = 2$ tenglik o'rinli bo'ladi?

- A) 2 B) 4 C) 1 D) 5

Yechish: Sistema tenglamalarini qo'shamiz: $3x + 3y = 2 + k \iff 3(x + y) = 2 + k$. Bu yerdan hamda $x + y = 2$ ekanligini hisobga olsak, $3 \cdot 2 = 2 + k$ ni olamiz. Demak, $k = 4$ ekan. **Javob:** 4 (B).

16. (97-9-83) Agar

$$\begin{cases} x + 3y = 6 \\ 2x + ky = 8 \end{cases}$$

bo'lsa, k ning qanday qiymatida $x + y = 2$ tenglik o'rinli bo'ladi?

A) 0 B) 1 C) 2 D) 4

4.5.3 Ikkinchi va undan yuqori darajali tenglamalar sistemasini

1. (97-8-20) Tenglamalar sistemasini yeching.

$$\begin{cases} y + 4 = 2 \\ x^2 y = -2 \end{cases}$$

A) (1; -2) B) (-1; -2)
C) (1; 2) D) (-1; -2) va (1; -2)

Yechish: Sistemaning 1-tenglamasidan $y = -2$ ekanligini olamiz va uni 2-tenglamaga qo'yamiz

$$x^2 \cdot (-2) = -2 \iff x^2 = 1.$$

Bu chala kvadrat tenglamaning yechimlari $x_1 = -1$, $x_2 = 1$ lardir. Shunday qilib, sistema $(x_1; y) = (-1; -2)$ va $(x_2; y) = (1; -2)$ yechimlarga ega.

Javob: (-1; -2) va (1; -2) (D).

2. (96-9-70) Sistemaning yechimini toping.

$$\begin{cases} x^2 + y^2 - 2xy = 16 \\ x + y = -2 \end{cases}$$

A) (1; -3) B) (-3; 1)
C) (0; -2) D) (1; -3) va (-3; 1)

3. (96-10-20) Sistemaning yechimini toping.

$$\begin{cases} x - y = 4 \\ x^2 + y^2 + 2xy = 4 \end{cases}$$

A) (3; 1) B) (3; -1)
C) (3; -1) va (1; -3) D) (2; -2)

4. (97-12-19) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - y^2 + 2x + 4 = 0 \\ x - y = 0 \end{cases}$$

A) (2; 2) B) (-2; -2) C) (-1; -1) D) (1; 1)

5. (97-2-20) Tenglamalar sistemasini yeching.

$$\begin{cases} x + 2 = 0 \\ xy^2 = -8 \end{cases}$$

A) (-2; -2) B) (-2; 2)
C) (-2; 2) va (-2; -2) D) (2; 2)

6. (96-1-19) Sistemaning yechimini toping.

$$\begin{cases} x^2 + y^2 - 2xy = 1 \\ x + y = 3 \end{cases}$$

A) (2; 1) B) (1; 2)
C) (1, 5; 1, 5) D) (2; 1) va (1; 2)

7. (96-6-20) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - y^2 - 3x = 12 \\ x - y = 0 \end{cases}$$

A) (-4; 4) B) (4; -4) C) (4; 4) D) (-4; -4)

8. (01-3-34) Tenglamalar sistemasini yeching.

$$\begin{cases} x + 3 = 0 \\ xy^2 = -12 \end{cases}$$

A) (-3; 2) B) (-3; -2)
C) (-3; -2), (-3; 2) D) \emptyset

9. (96-7-23) Tenglamalar sistemasini nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 9 \\ y - x = -3 \end{cases}$$

A) 1 B) 2 C) 3 D) 4

Yechish: Sistemaning 2-tenglamasidan $y = x - 3$ ni topib, 1-tenglamaga qo'ysak

$$x^2 + (x - 3)^2 = 9 \iff 2x^2 - 6x = 0$$

tenglama hosil bo'ladi. Bu chala kvadrat tenglamaning yechimlari $x_1 = 0$, $x_2 = 3$ lardir. Bu qiymatlarni $y = x - 3$ ga qo'yib $y_1 = -3$, $y_2 = 0$ ni olamiz. Demak, sistema ikkita $(x_1; y_1) = (0; -3)$ va $(x_2; y_2) = (3; 0)$ yechimlarga ega. **Javob:** 2 (B).

10. (97-10-23) Tenglamalar sistemasini nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 4 \\ x - y = -2 \end{cases}$$

A) 4 B) 3 C) 2 D) 1

11. (97-3-23) Tenglamalar sistemasini nechta yechimga ega?

$$\begin{cases} x^2 + y^2 = 25 \\ x - y = 5 \end{cases}$$

A) 4 B) 3 C) 2 D) 1

12. (98-12-19) Agar $a - b = 12$ va $-ab + a^2 = 144$ bo'lsa, a ning qiymati qanchaga teng bo'ladi?

A) 12 B) -12 C) 36 D) 6

Yechish: 2-tenglikni $a(a - b) = 144$ shaklda yozamiz va $a - b = 12$ ekanligidan foydalansak $12a = 144$ ni olamiz. Bu tenglikning ikkala qismini 12 bo'lib $a = 12$ ni olamiz. **Javob:** 12 (A).

13. (98-11-60) Agar $x^2 + y^2 = 281$ va $x - y = 11$ bo'lsa, xy qanchaga teng bo'ladi?

A) 80 B) 160 C) -80 D) 40

14. (02-12-30) Agar $x^2 - 4xy + y^2 = 4 - 2xy$ va $x + y = 12$ bo'lsa, xy ning qiymatini toping.

A) 32 B) 35 C) 30 D) 34

15. (97-9-67) Agar $ab = 9$ va $3b = 8c$ ($b \neq 0$) bo'lsa, ac ni hisoblang.

A) $3\frac{1}{3}$ B) $3\frac{5}{8}$ C) $3\frac{4}{9}$ D) $3\frac{3}{8}$

16. (01-3-33) Sistemadan
- $3xy$
- ni toping.

$$\begin{cases} x^2 + y^2 - xy = 1 \\ x + y = -2 \end{cases}$$

- A) 1 B) -1 C) 3 D) -3

Yechish: Berilgan sistema

$$\begin{cases} x^2 + y^2 = 1 + xy \\ x + y = -2 \end{cases}$$

sistemaga teng kuchli. Bu sistemaning 2-tenglamasini (har ikkala qismini) kvadratga ko'taramiz

$$(x + y)^2 = (-2)^2 \iff x^2 + y^2 = 4 - 2xy.$$

Endi sistemaning 1-tenglamasidan foydalansak $1 + xy = 4 - 2xy$ ni olamiz. Bu tenglikdan $3xy = 3$ ni olamiz. **Javob:** 3 (C).

17. (01-3-32) Sistemadan
- x
- ni toping.

$$\begin{cases} x + y = 6 \\ x^2 - y^2 = 12 \end{cases}$$

- A) 4 B) 2 C) 1 D) 3

18. (96-3-75) Sistemadan
- x
- ni toping.

$$\begin{cases} x + y = 3 \\ x^2 - y^2 = 6 \end{cases}$$

- A) 1,5 B) 2,5 C) 3 D) 1

19. (96-12-73) Sistemani yeching va
- $x \cdot y$
- ning qiymatini toping?

$$\begin{cases} x^2 + y^2 = 3 \\ x - y = 1 \end{cases}$$

- A) 2 B) 3 C) 1,5 D) 1

20. (98-12-64) Agar

$$\begin{cases} x + y = 3 \\ x \cdot y = 1 \end{cases}$$

bo'lsa, $x^5 \cdot y + x \cdot y^5$ ni hisoblang.

- A) 47 B) 29 C) 51 D) 24

Yechish: $x + y = 3$, $xy = 1$ tengliklardan

$$x^2 + y^2 = \underbrace{(x + y)^2}_3 - 2 \underbrace{xy}_1 = 9 - 2 = 7$$

ekani kelib chiqadi. Shuning uchun

$$\begin{aligned} x^5y + xy^5 &= xy(x^4 + y^4) = x^4 + y^4 = \\ &= (x^2 + y^2)^2 - 2(xy)^2 = 7^2 - 2 = 47 \end{aligned}$$

Javob: 47 (A).

21. (01-4-23) Agar

$$\begin{cases} x^2 - y^2 = 6 \\ x + y = 1 \end{cases}$$

bo'lsa, $x - y$ ning qiymatini toping.

- A) 1 B) -1 C) 6 D) -6

22. (03-12-3) Sistemadan
- ab
- ni toping

$$\begin{cases} b + a = 18 \\ a^2 + b^2 = 170 \end{cases}$$

- A) 45 B) 72 C) 77 D) 80

23. (98-10-17) Tenglamalar sistemasini yeching.

$$\begin{cases} x^2 - 1 = 0 \\ xy^2 = -4 \end{cases}$$

- A) (-1; 2) B) (2; -1)
-
- C) (2; 1) D) (-1; -2) va (-1; 2)

Yechish: Sistemaning 1-tenglamasidan $x_1 = -1$, $x_2 = 1$ ni olamiz. Ikki holni alohida qaraymiz:

$$1) x_1y^2 = -4 \iff -y^2 = -4 \iff y^2 = 4.$$

Bu tenglama $y_1 = -2$ va $y_2 = 2$ yechimlarga ega. Demak, $(x_1; y_1) = (-1; -2)$ va $(x_1; y_2) = (-1; 2)$ juftliklar berilgan sistemaning yechimlari bo'ladi.

$$2) x_2y^2 = -4 \iff y^2 = -4.$$

Bu tenglama haqiqiy ildizlarga ega emas. Shunday qilib sistemaning yechimlari $(-1; -2)$ va $(-1; 2)$ juftliklar ekan. **Javob:** $(-1; -2)$ va $(-1; 2)$ (D).

24. (02-9-11) Sistema nechta yechimga ega?

$$\begin{cases} y = \frac{4}{x} \\ y = -x^2 + 6x - 5 \end{cases}$$

- A)
- \emptyset
- B) 1 C) 2 D) 3

25. (03-9-6) Sistemaning yechimlaridan iborat barcha
- x
- va
- y
- larning yig'indisini toping.

$$\begin{cases} x^3 + y^3 = 35 \\ x^2y + xy^2 = 30 \end{cases}$$

- A) 0 B) 2 C) 6 D) 10

26. (02-11-27)
- x
- ning

$$\begin{cases} x^5 \cdot y^7 = 32 \\ x^7 \cdot y^5 = 128 \end{cases}$$

tenglamalar sistemasining yechimidan iborat barcha qiymatlari yig'indisini toping.

- A) 0 B) 4 C) 8 D) 12

27. (97-4-25) Agar

$$\begin{cases} x^3 - y^3 - 3x^2y = 5 \\ xy^2 = 1 \end{cases}$$

bo'lsa, $\frac{x-y}{2}$ ni hisoblang.

- A) 2 B) 1 C) 3 D) 4,5

Yechish: Berilgan tengliklardan foydalanib

$$(x - y)^3 = \underbrace{x^3 - y^3 - 3x^2y}_5 + 3 \underbrace{xy^2}_1 = 5 + 3 \cdot 1 = 8$$

ni hosil qilamiz. U holda $x - y = 2$ va

$$\frac{x - y}{2} = \frac{2}{2} = 1$$

bo'ladi. **Javob:** 1 (B).

28. (98-2-16) Agar $m^2 - mn = 48$ va $n^2 - mn = 52$ bo'lsa, $m - n$ nechaga teng?

A) 10 B) 8 C) ± 10 D) ± 8

29. (98-5-22) Agar

$$\begin{cases} x^2 - 2xy + y^2 = 9 \\ xy = 10 \end{cases}$$

bo'lsa, $|x + y|$ ni hisoblang.

A) 7 B) 6 C) 5 D) 8

30. (98-10-65) $(x + y)^2$ ni toping.

$$\begin{cases} x^2 + y^2 = 10 \\ xy = 3 \end{cases}$$

A) 13 B) 7 C) 16 D) 19

31. (99-6-37) Agar $ab = 18$, $bc = 25$ va $ac = 8$ bo'lsa, \sqrt{abc} nimaga teng.

A) $2\sqrt{15}$ B) $15\sqrt{2}$ C) $6\sqrt{5}$ D) $8\sqrt{3}$

32. (99-10-11) Agar $x^2 \cdot y = 50$, $x \cdot y^2 = 20$ bo'lsa, xy ning qiymatini hisoblang.

A) 8 B) 10 C) 6 D) 12

33. (00-1-23) Agar $a - b = 1$ va $(a^2 - b^2) \cdot (a - b) = 9$ bo'lsa, ab ning qiymatini toping.

A) 19 B) 22 C) 21 D) 20

34. (97-8-11) Agar $(x - 4)^2 + (x - y^2)^2 = 0$ bo'lsa, $x + 2y$ nechaga teng?

A) 0 B) 4 C) 6 D) 0 yoki 8

35. (00-8-14) Agar

$$\begin{cases} xy = 6 \\ yz = 12 \\ zx = 8 \end{cases}$$

bo'lsa, $x + y + z$ ning qiymatini toping.

A) -9 yoki 9 B) 18 C) 0 D) 36

Yechish: 1-usul. Tengliklarni ko'paytirib, $x^2y^2z^2 = 576$ ekanini hosil qilamiz. Ikkita hol bo'lishi mumkin:

1) $xyz = 24$. Bu tenglikni sistemaning birinchi, ikkinchi va uchinchi tengliklariga ketma-ket bo'lib, $z = 4$; $x = 2$; $y = 3$ ni hosil qilamiz. Bu holda $x + y + z = 2 + 3 + 4 = 9$.

2) $xyz = -24$. Bu tenglikni ham sistemaning birinchi, ikkinchi va uchinchi tengliklariga bo'lib, $z = -4$; $x = -2$; $y = -3$ ni hosil qilamiz. Bu holda $x + y + z = -2 + (-3) + (-4) = -9$.

2-usul. Sistemaning 2-tenglamasini 1-siga bo'lib, $\frac{z}{x} = 2$ yoki $z = 2x$ ni olamiz. z ning bu qiymatini x sistemaning 3-tenglamasiga qo'yib, $2x^2 = 8$ yoki $x^2 = 4$ ni olamiz. Bu tenglamaning ildizlari $x_1 = -2$, $x_2 = 2$ lardir. x ning bu qiymatlarini sistemaning 1-tenglamasiga qo'yib, $y_1 = -3$, $y_2 = 3$ ekanligini olamiz. 2-tenglamadan $z_1 = -4$, $z_2 = 4$ larni olamiz. Demak, $x_1 + y_1 + z_1 = -2 - 3 - 4 = -9$ yoki $x_2 + y_2 + z_2 = 2 + 3 + 4 = 9$. **Javob:** -9 yoki 9 (A).

36. (03-8-40) Sistemadan x ni toping.

$$\begin{cases} \frac{xy}{x+y} = \frac{10}{7} \\ \frac{yz}{y+z} = \frac{40}{13} \\ \frac{zx}{z+x} = \frac{5}{8} \end{cases}$$

A) $\frac{80}{79}$ B) $\frac{5}{7}$ C) $\frac{7}{13}$ D) $\frac{79}{80}$

37. (98-6-2) Agar $xy = 6$, $yz = 2$ va $xz = 3$ ($x > 0$) bo'lsa, xyz ni toping.

A) -6 B) 6 C) 5 D) 12

38. (98-11-52) Agar $xy = 4$, $yz = 7$ va $xz = 28$ ($y > 0$) bo'lsa, xyz ni toping.

A) -28 B) 28 C) 27 D) 56

39. (98-6-11) Agar m va n natural sonlar

$$\sqrt{2}(n - 5) + n^2 - 6mn + 5m = 0$$

tenglikni qanoatlantirsa. $n - m$ ni toping.

A) 2 B) 5 C) 6 D) 4

Yechish: Agar $\sqrt{2}(n - 5)$ noldan farqli bo'lsa, u holda $\sqrt{2}(n - 5) + n^2 - 6mn + 5m$ ifoda ham noldan farqli bo'ladi. Chunki irratsional son bilan butun son yig'indisi irratsionaldir (nol irratsional emas). Shuning uchun $\sqrt{2}(n - 5) + n^2 - 6mn + 5m = 0$ tenglikdan $\sqrt{2}(n - 5) = 0$ va $n^2 - 6mn + 5m = 0$ tengliklar kelib chiqadi. 1-tenglikdan $n = 5$ ni olamiz. Uni 2-tenglikka qo'yib $25 - 30m + 5m = 0$ chiziqli tenglamaga ega bo'lamiz. Uning yechim $m = 1$ dir. Demak, $n - m = 5 - 1 = 4$. **Javob:** 4 (D).

40. (01-10-8) Nechta butun x va y sonlar jufti $x^2 - y^2 = 31$ tenglikni qanoatlantiradi?

A) \emptyset B) 1 C) 2 D) 4

41. (99-10-22) Agar $x^2 \cdot y + x \cdot y^2 = 48$ va $x^2 \cdot y - x \cdot y^2 = 16$ bo'lsa, $\frac{x}{y}$ ning qiymatini hisoblang.

A) $\frac{1}{4}$ B) -2 C) 2 D) $-\frac{1}{2}$

42. (02-6-32) Sistemadan $x \cdot y$ ni toping

$$\begin{cases} x^3 + y^3 = 35 \\ x + y = 5 \end{cases}$$

A) 3 B) 4 C) 5 D) 6

43. (02-7-54) Agar $8a^3 - b^3 = 37$ va $ab^2 - 2a^2b = -6$ bo'lsa, $2a - b$ ning qiymatini toping.

A) 1 B) -1 C) 2 D) -2

44. (02-8-11)

$$\begin{cases} xy + x + y = 11 \\ x^2y + y^2x = 30 \end{cases}$$

tenglamalar sistemasi uchun $x + y$ ning eng katta qiymatini toping.

A) 6 B) 5 C) 7 D) 4

Yechish: Sistemaning 1-tenglamasidan $xy = 11 - (x + y)$ ekanligini topamiz. Agar $x + y$ ni t desak, u holda 2-tenglama

$$xy(x + y) = 30 \iff (11 - t)t = 30$$

ko'rinishga keladi. Bu kvadrat tenglamaning ildizlari $t_1 = 5$, $t_2 = 6$ lardir. $x + y = t$ bo'lganligi uchun, uning katta qiymati 6. **Javob:** 6 (A).

45. (02-12-26) Agar $\frac{1}{n} + \frac{1}{m} = \frac{1}{7}$ va $m + n = -4$ bo'lsa, mn ning qiymatini toping.
A) 20,5 B) -20,5 C) 21 D) -28

46. (02-12-29) Agar $x^3 + 3xy^2 = 185$ va $y^3 + 3x^2y = 158$ bo'lsa, $x - y$ ning qiymatini toping.
A) 4 B) 3,5 C) 2 D) 3

47. (03-11-3) Agar

$$\begin{cases} \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \frac{4}{3} \\ xy = 9 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.

- A) 10 B) 9 C) 8 D) 12

Yechish: Sistemaning 2-tenglamasi $xy = 9$ dan foydalanib uning 1-tenglamasini

$$\frac{\sqrt{x} + \sqrt{y}}{\sqrt{9}} = \frac{4}{3} \iff \sqrt{x} + \sqrt{y} = 4$$

ko'rinishda yozib olamiz. Bu tenglikning ikkala qismini kvadratga ko'tarib, yana bir marta $xy = 9$ dan foydalansak,

$$x + y + 2\sqrt{xy} = 16 \iff x + y = 10$$

ni olamiz. **Javob:** 10 (A).

48. (03-1-65) Agar

$$\begin{cases} x^2y + xy^2 = 120 \\ x^2y - xy^2 = 30 \end{cases}$$

bo'lsa, $x^2 - y^2$ ning qiymatini hisoblang.

- A) 16 B) 20 C) 25 D) 34

49. (07-112-29) Ushbu

$$\begin{cases} x^3 + y^3 = 126 \\ x^2y + xy^2 = 30. \end{cases}$$

tenglamalar sistemasining haqiqiy yechimlaridan iborat barcha x va y larning yig'indisini toping.

- A) 2 B) 12 C) 10 D) 6

50. (98-6-10) Agar $x^2 + y^2 = 225$ va $x^2 - y^2 = 63$ bo'lsa, $|x| - |y|$ ni toping.

- A) 3 B) 4 C) 5 D) 6

51. Agar $x^2 - y^2 = 51$ va $x, y \in N$ bo'lsa, $x + y$ ning eng katta qiymatini toping.

- A) 3 B) 17 C) 51 D) 20

5 - bob. Tengsizliklar

Agar ikkita ifoda $>$ yoki $<$ (\geq yoki \leq) belgisi bilan bog'langan bo'lsa, ular tengsizlik hosil qiladi deyiladi. Masalan, $3x + 5 > 0$; $13x + 1 < 5$; $x^2 - 7 < 0$. Agar tengsizlikda $>$ yoki $<$ belgisi bo'lsa, uni qat'iy tengsizlik deymiz. Masalan, $x - 5 < 7$; $x^2 - 3 > 0$. Agar tengsizlikda \geq yoki \leq belgisi bo'lsa, uni qat'iymas tengsizlik deymiz. Masalan, $x - 9 \geq 7$; $x^2 - 5 \leq 0$. Tengsizlik belgisining chap tomonidagi ifodaga tengsizlikning chap qismi, o'ng tomonidagi ifodaga tengsizlikning o'ng qismi deyiladi. Tengsizlikning ikkala qismi ham sonlardan iborat bo'lsa, unga *sonli tengsizlik* deyiladi. $a < b$, $c < d$ tengsizliklar bir xil ma'noli, $a < b$, $c > d$ tengsizliklar esa har xil ma'noli tengsizliklar deyiladi. Sonli tengsizliklar quyidagi xossalarga ega.

1. Agar $a > b$ bo'lsa, u holda $b < a$ bo'ladi.

2. Agar $a > b$ va $b > c$ bo'lsa, u holda $a > c$ bo'ladi.

3. Agar $a > b$ va c - ixtiyoriy son bo'lsa, u holda $a + c > b + c$ va $a - c > b - c$ bo'ladi.

4. Agar $a > b$ va $c > 0$ bo'lsa, u holda $a \cdot c > b \cdot c$ va $a : c > b : c$ bo'ladi. Ya'ni tengsizlikning ikkala qismini musbat songa ko'paytirsak, yoki bo'lsak tengsizlik belgisi saqlanadi.

5. Agar $a > b$ va $c < 0$ bo'lsa, $a \cdot c < b \cdot c$ va $a : c < b : c$ bo'ladi. Ya'ni tengsizlikning ikkala qismini manfiy songa ko'paytirsak, yoki bo'lsak tengsizlik belgisi qarama-qarshisiga o'zgaradi.

6. Bir xil ma'noli tengsizliklarni hadlab qo'shish mumkin, ya'ni $a < b$ va $c < d$ bo'lsa, $a + c < b + d$ bo'ladi.

7. Har xil ma'noli tengsizliklarni hadlab ayirish mumkin, ya'ni $a > b$ va $c < d$ bo'lsa, $a - c > b - d$ bo'ladi. Ayirmada kamayuchi tengsizlikning belgisi saqlanadi.

8. Nomanfiy hadli bir xil ma'noli tengsizliklarni hadlab ko'paytirish mumkin, ya'ni $0 \leq a < b$ va $0 \leq c < d$ bo'lsa, $a \cdot c < b \cdot d$ bo'ladi.

9. Nomanfiy hadli tengsizliklarning har ikkala qismini bir xil natural darajaga ko'tarish mumkin, ya'ni $0 \leq a < b$ va $n \in N$ bo'lsa, $a^n < b^n$ bo'ladi.

Agar bir tengsizlikning har qanday yechimi shu o'zgaruvchilar qatnashgan ikkinchi tengsizlikning ham yechimi bo'lsa va aksincha, ikkinchi tengsizlikning har qanday yechimi birinchi tengsizlikning ham yechimi bo'lsa, bu tengsizliklar *teng kuchli tengsizliklar* deyiladi. Yechimga ega bo'lmagan tengsizliklar ham teng kuchli tengsizliklar hisoblanadi.

5.1 Chiziqli tengsizliklar

$$ax + b > 0, \quad ax + b \geq 0 \quad (5.1)$$

$$ax + b < 0, \quad ax + b \leq 0 \quad (5.2)$$

ko'rinishdagi yoki soddalashtirishdan so'ng shunday ko'rinishga keladigan tengsizlik chiziqli tengsizlik deyiladi. (5.1) va (5.2) da x noma'lum, a va b lar haqiqiy sonlar, a noma'lum oldidagi koeffitsiyent, b ozod had deyiladi. Eng sodda $x > a$, $x \leq b$ yoki $a < x \leq b$, $a \leq x \leq b$ ko'rinishdagi tengsizliklarning yechimlari $(a; \infty)$, $(-\infty; b]$ yoki $(a; b]$, $[a; b]$ shaklda yozish qabul qilingan.

1. $3x + 1 < 10$ tengsizlikni yeching.
A) $(3; \infty)$ B) $(1; \infty)$ C) $(-\infty; 3)$ D) \emptyset

Yechish: Berilgan tengsizlikda 1 ni tengsizlikning o'ng qismiga o'tkazib, keyin tengsizlikning ikkala qismiga 3 ga bo'lib $x < 3$ ni olamiz. **Javob:** $(-\infty; 3)$ (C).

2. $7x + 5 > 19$ tengsizlikni yeching.
A) $(2; \infty)$ B) $(7; \infty)$ C) $(-\infty; 2)$ D) \emptyset
3. $2x + 7 \leq 11$ tengsizlikni yeching.
A) $[2; \infty)$ B) $(7; \infty)$ C) $(-\infty; 2]$ D) \emptyset
4. $5x + 9 \geq 14$ tengsizlikni yeching.
A) $[1; \infty)$ B) $[5; \infty)$ C) $(-\infty; 1]$ D) \emptyset

5. (00-6-10) Tengsizlikni yeching.

$$1 - \frac{17 - 3x}{2} > 1, 5x$$

- A) $(-2, 5; 0)$ B) $(-\infty; -2, 5)$
C) $(-\infty; 0)$ D) \emptyset

Yechish: Berilgan tengsizlikning har ikkala qismini 2 ga ko'paytirsak, tengsizlik saqlanadi

$$2 - 17 + 3x > 3x \iff -15 > 0 \cdot x.$$

Bu tengsizlik yechimga ega emas. **Javob:** \emptyset (D).

6. Tengsizlikni yeching.

$$\frac{1 - x}{2} < \frac{2x - 1}{4}$$

- A) $(\frac{3}{4}; \infty)$ B) $(1; \infty)$ C) $(-\infty; \frac{3}{4})$ D) \emptyset

7. (01-8-10) Tengsizlikni yeching.

$$\frac{1 - x}{2} + 3 < 3x - \frac{2x + 1}{4}$$

- A) $(1\frac{1}{3}; \infty)$ B) $(1\frac{1}{13}; \infty)$ C) $(1\frac{1}{4}; \infty)$ D) \emptyset

8. (98-2-17) Quyida keltirilgan tengsizliklardan qaysi biri

$$3x - a > b - 2x$$

tengsizlikka teng kuchli emas?

- A) $5x - a > b$ B) $6x - 2a > 2b - 4x$
C) $3x > a + b - 2x$ D) $a - 3x > 2x - b$

9. $2x + 5 > a$ ga teng kuchli tengsizlikni toping.
A) $2x + 5 - a \geq 0$ B) $2x + 5 - a > 0$
C) $2x < a - 5$ D) $-2x < -5 - a$

10. (00-8-33) k ning $4y^2 - 3y + k = 0$ tenglama haqiqiy ildizlarga ega bo'lmaydigan eng kichik butun qiymatini toping.
A) 1 B) 3 C) 4 D) 5

Yechish: $4y^2 - 3y + k = 0$ kvadrat tenglama haqiqiy ildizlarga ega bo'lmasligi uchun $D < 0$ bo'lishi kerak:

$$D = 9 - 16k < 0 \iff k > \frac{9}{16}.$$

Bu tengsizlikni qanoatlantiruvchi eng kichik butun son $k = 1$. **Javob:** 1 (A).

11. (02-2-10) Tengsizlikni qanoatlantiruvchi eng katta butun sonni toping.

$$(x + 1)^2 > (x + 2)^2$$

- A) -2 B) -1 C) -3 D) -4

12. (02-3-18) Tengsizlikni qanoatlantiruvchi eng kichik butun manfiy son nechaga teng?

$$8 + \frac{6x - 8}{10} > \frac{x - 2}{6} + \frac{1 - 5x}{8} + \frac{1}{4}$$

- A) -6 B) -7 C) -5 D) -4

13. (03-11-64) Tengsizlikning eng katta butun yechimini ko'rsating.

$$\frac{2x - 7}{6} + \frac{7x - 2}{3} < 3 - \frac{1 - x}{2}$$

- A) 2 B) -1 C) 1 D) 0

5.2 Chiziqli tengsizliklar sistemasi

1. (97-7-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 3x + 7 \geq 5(x + 1) + 6 \\ (x - 2)^2 - 8 < x(x - 2) + 10 \end{cases}$$

- A) $(-11; 2]$ B) $[-2; 7)$ C) $(-7; -2]$ D) $[2; 11)$

Yechish: Qavsrlarni ochamiz:

$$\begin{cases} 3x + 7 \geq 5x + 5 + 6 \\ x^2 - 4x + 4 - 8 < x^2 - 2x + 10 \end{cases}$$

Endi x qatnashgan hadlarni tengsizlikning chap qismiga, sonlarni tengsizlikning o'ng qismiga o'tkazib quyidagi sistemani olamiz:

$$\begin{cases} -2x \geq 4 \\ -2x < 14 \end{cases} \iff \begin{cases} x \leq -2 \\ x > -7 \end{cases} \iff -7 < x \leq -2.$$

Demak, $x \in (-7; -2]$. **Javob:** $(-7; -2]$ (C).

2. (97-3-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 2x - 3(x - 5) > 10 - 3x \\ x(x + 2) - 4 \leq (x - 1)^2 + 7 \end{cases}$$

- A) $[2; 12, 5)$ B) $[2, 5; \infty)$ C) $[-3; 2)$ D) $(-2, 5; 3]$

3. (96-7-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} x(x+1) + 10 > (x+1)^2 + 3 \\ 3x - 4(x-7) \geq 16 - 3x \end{cases}$$

- A) $[-3; 5]$ B) $(2; 4]$ C) $[-6; 6]$ D) $[6; \infty)$

4. (97-10-25) Tengsizliklar sistemasini yeching.

$$\begin{cases} 4(x-3) - 3 > 8x + 1 \\ 2 + x(x+3) \leq (x+2)^2 + 5 \end{cases}$$

- A) $(4; 7]$ B) $(-\infty; -7)$ C) $(-4; \infty)$ D) $[-7; -4)$

5. (01-8-14) Tengsizliklar sistemasini yeching.

$$\begin{cases} \frac{3x-2}{4} > \frac{1-5x}{6} \\ 3x-1 \leq 3-2x \end{cases}$$

- A) $(\frac{8}{19}; \infty)$ B) $(\frac{8}{19}; \frac{4}{5}]$ C) $(-\infty; \frac{4}{5}]$ D) $x \in R$

6. (97-6-14) Tengsizliklar

$$\begin{cases} 7x+3 \leq 9x-1 \\ 20-3x \geq 4x-15 \end{cases}$$

sistemasini butun yechimlarining o'rta arifmetigini toping.

- A) 3, 5 B) 7 C) 4 D) 3

Yechish: Sistemada x qatnashgan hadlarni tengsizlikning chap qismiga, ozod hadlarni tengsizlikning o'ng qismiga o'tkazib, berilgan sistemaga teng kuchli bo'lgan quyidagi sistemani olamiz:

$$\begin{cases} -2x \leq -4 \\ -7x \geq -35 \end{cases} \iff \begin{cases} x \geq 2 \\ x \leq 5 \end{cases} \iff 2 \leq x \leq 5.$$

So'nggi tengsizlikni butun yechimlari 2, 3, 4, 5 laridir. Ularning o'rta arifmetigi $(2+3+4+5) : 4 = 14 : 4 = 3,5$. **Javob:** 3, 5 (A).

7. (97-1-14) Tengsizliklar sistemasini butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 5x-2 \geq 2x+1 \\ 2x+3 \leq 18-3x \end{cases}$$

- A) 3 B) 2,5 C) 2 D) 1,5

8. (97-11-14) Tengsizliklar sistemasini butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 2x-1 \geq 3x-4 \\ 8x+7 > 5x+4 \end{cases}$$

- A) 2 B) 2,5 C) 1,5 D) 0,75

9. (02-2-11) Tengsizliklar sistemasini butun yechimlarining o'rta arifmetigini toping.

$$\begin{cases} 2x-10 > 0 \\ 27-x > 0 \end{cases}$$

- A) 16 B) 18 C) 17 D) 15

10. (96-1-22) Tengsizliklar sistemasini nechta butun yechimga ega?

$$\begin{cases} 3+4x \geq 5 \\ 2x-3(x-1) > -1 \end{cases}$$

- A) 5 B) 3 C) 4 D) 2

11. (96-6-16) Tengsizliklar sistemasining eng katta butun yechimini toping.

$$\begin{cases} -2x < 22 \\ x+4 < 8 \end{cases}$$

- A) 4 B) 3 C) -11 D) -12

12. (96-9-73) Tengsizliklar sistemasini nechta butun yechimga ega?

$$\begin{cases} 3-4x > 5 \\ 2+3(x-1) \leq 4x+3 \end{cases}$$

- A) 1 B) 2 C) 4 D) 6

13. (97-2-16) Tengsizliklar sistemasining eng kichik butun yechimimini toping?

$$\begin{cases} x+8 < 12 \\ -3x < 15 \end{cases}$$

- A) -5 B) -3 C) -6 D) -4

14. (97-8-16) Tengsizliklar sistemasining barcha butun yechimlari ko'paytmasini toping.

$$\begin{cases} -4y < 12 \\ y+6 < 6 \end{cases}$$

- A) 2 B) 6 C) -6 D) -2

15. (97-12-15) Tengsizliklar sistemasining eng katta va eng kichik butun yechimlari yig'indisini toping.

$$\begin{cases} -2x > -26 \\ x-3 > 1 \end{cases}$$

- A) 17 B) 16 C) 18 D) 19

16. (98-3-15) Tengsizliklar sistemasining butun yechimlari yig'indisini toping?

$$\begin{cases} x+1 < 2x-4 \\ 3x+1 < 2x+10 \end{cases}$$

- A) 9 B) 5 C) 20 D) 21

17. (98-1-6) Qo'sh tengsizlikni yeching.

$$-3 < 2-5x < 1$$

- A) $(-1; 0, 2)$ B) $(-1; -0, 2)$
C) $(-0, 2; 1)$ D) $(0, 2; 1)$

Yechish: Sistemaning har bir qismidan 2 ni ayirib $-5 < -5x < -1$ ni olamiz. Bu tengsizlikni -5 ga bo'lamiz (bu holda tengsizliklar qarama-qarshisiga o'zgaradi, chunki -5 manfiy son) va natijada $1 > x > 0,2$ yoki $0,2 < x < 1$ ga ega bo'lamiz. **Javob:** $(0, 2; 1)$ (D).

18. (98-8-6) Qo'sh tengsizlikni yeching.

$$-4 < 2 - 4x < -2$$

- A) $(-1, 5; -1)$ B) $(1; 2)$ C) $(0; 1)$ D) $(1; 1, 5)$

19. (99-8-9) $5 < x < 98$ tengsizlikni qanoatlantiruvchi va bo'luvchisi 12 ga teng bo'lgan nechta natural son mavjud?

- A) 8 B) 10 C) 12 D) 6

20. (99-8-79) Tengsizlik nechta natural yechimga ega?

$$17,556 : 5,7 \leq y < 31,465 : 3,5$$

- A) 1 B) 2 C) 4 D) 5

21. (98-8-1) Tengsizlikning barcha natural yechimlarini toping.

$$1256 : 314 < 9x - 32 \leq 2976 : 96$$

- A) 4; 5; 6 B) 5; 6; 7 C) 6; 7; 8 D) 7; 8

22. (98-1-1) Tengsizlikning barcha natural yechimlarini toping.

$$6798 : 103 < 54 + 6x < 9156 : 109$$

- A) 2; 3; 4 B) 4; 5; 6 C) 3; 4 D) 4; 5

23. (99-9-24) Tengsizlikning eng katta butun yechimi, eng kichik butun yechimidan qanchaga katta?

$$\begin{cases} 2x - 3 \leq 17 \\ 14 + 3x > -13 \end{cases}$$

- A) 17 B) 19 C) 16 D) 18

Yechish: Sistemada ozod hadlarni tengsizlikning o'ng qismiga o'tkazib, berilgan sistemaga teng kuchli bo'lgan quyidagi sistemani olamiz:

$$\begin{cases} 2x \leq 20 \\ 3x > -27 \end{cases} \iff \begin{cases} x \leq 10 \\ x > -9 \end{cases} \iff -9 < x \leq 10.$$

Bu tengsizlikning eng katta butun yechimi 10, eng kichik butun yechimi -8 dir. Ularning farqi $10 - (-8) = 18$. **Javob:** 18 (D).

24. (98-10-40) Sistemaning eng katta butun va eng kichik butun yechimlari yig'indisini toping.

$$\begin{cases} 2x - 3 < 17 \\ 4x + 6 > 8 \end{cases}$$

- A) 8 B) 11 C) 12 D) 10

25. (98-10-63) Tengsizliklar sistemasining butun yechimlari yig'indisini toping.

$$\begin{cases} -x - 5 < -2x - 2 \\ -2x + 2 > 3 - 3x \end{cases}$$

- A) 0 B) 1 C) 2 D) 3

26. (98-11-25) Tengsizliklar sistemasining butun yechimlari yig'indisini toping.

$$\begin{cases} 0,4(2x - 3) > x - 2 \\ 3x - 7 \geq x - 6 \end{cases}$$

- A) 10 B) 5 C) 6 D) 8

5.3 Oraliqlar usuli

Kasr ratsional tengsizliklarni yechishning eng qulay usullaridan biri – *oraliqlar usulidir*. Bizdan

$$\frac{x^2 + 2x - 3}{x^2 + 2x - 8} > 0 \quad (5.3)$$

tengsizlikni yechish talab qilinsin. Ma'lumki, kasr musbat bo'lishi uchun uning maxraji va surati bir xil ishorali bo'lishi kerak. Shunday ekan (5.3) tengsizlik quyidagi ikki tengsizliklar sistemasiga teng kuchlidir:

$$\begin{cases} x^2 + 2x - 3 > 0 \\ x^2 + 2x - 8 > 0 \end{cases} \text{ yoki } \begin{cases} x^2 + 2x - 3 < 0 \\ x^2 + 2x - 8 < 0 \end{cases}$$

Bu sistemani yechish yana qismlarga ajratiladi. Agar

$$\frac{(x-1)(x-3)(x-5)}{(x+2)(x+4)(x+6)} > 0$$

ko'rinishdagi tengsizliklarni yechish talab qilinsa, ish yanada qiyinlashadi. Shuning uchun bunday ko'rinishdagi tengsizliklar odatda oraliqlar usuli bilan yechiladi. Oraliqlar usulining mohiyati quyidagicha: $P(x)$ ko'phad bo'lib, uni

$$P(x) = p(x)(x-x_1)^{r_1}(x-x_2)^{r_2} \cdots (x-x_n)^{r_n} \quad (5.4)$$

ko'rinishda tasvirlash mumkin bo'lsin. Bunda $p(x)$ haqiqiy ildizlarga ega bo'lmagan ko'phad bo'lib, x ning barcha qiymatlarida faqat yo musbat yo manfiy qiymatlar qabul qiladi. Aniqlik uchun $p(x) > 0$ va $x_1 < x_2 < \cdots < x_n$ bo'lsin. Agar $x > x_n$ bo'lsa, (5.4) dagi hamma ko'paytuvchilar musbat bo'lib, $P(x) > 0$ bo'ladi. Agar $(x-x_n)$ chiziqli ko'paytuvchi (5.4) da toq ($r_n = 2m - 1$ toq son) daraja bilan qatnashsa, u holda $x_{n-1} < x < x_n$ bo'lganda (5.4) dagi so'nggi ko'paytuvchi manfiy, boshqa hamma ko'paytuvchilar musbat bo'ladi. Bu holda $P(x) < 0$ bo'ladi. Bunda $P(x)$ ko'phad x_n ildizdan o'tganda ishorasini o'zgartiradi deyiladi. Agar $(x-x_n)$ chiziqli ko'paytuvchi (5.4) da juft ($r_n = 2m$ juft son) daraja bilan qatnashsa, u holda $x_{n-1} < x < x_n$ bo'lganda (5.4) dagi barcha ko'paytuvchilar musbat bo'lib, $P(x) > 0$ bo'ladi. Bunda $P(x)$ ko'phad x_n ildizdan o'tganda ishorasini o'zgartirmaydi deyiladi.

Xuddi shunday muhokama qilish usuli bilan quyidagi xulosaga kelish mumkin: $P(x)$ ko'phad, x o'zgaruvchi x_k ildizdan o'tganda $(x-x_k)$ chiziqli ko'paytuvchi toq darajada bo'lsa, ishorasini o'zgartiradi va juft darajada bo'lsa, ishorasini o'zgartirmaydi. Ko'phadning bu xosasi yuqori darajali tengsizliklarni yechishda foydalaniladi. Quyidagi ikki misolni qaraymiz:

1-misol. Tengsizlikni yeching.

$$P(x) = (x^2 + 2x + 3)(x-1)^2(x-3)^5(x-7)^9 \leq 0. \quad (5.5)$$

Yechish: Bu yerda $p(x) = x^2 + 2x + 3 = (x+1)^2 + 2 > 0$. Ko'phad ildizlari 1; 3 va 7 larni sonlar o'qida belgilab chiqamiz. Natijada sonlar o'qi $(-\infty; 1)$, $(1; 3)$, $(3; 7)$ va $(7; \infty)$ oraliqlarga ajraydi (5.1-chizmaga qarang). Agar $x > 7$ bo'lsa, (5.5) dagi barcha ko'paytuvchilar musbat bo'lib, $P(x) > 0$ bo'ladi. Endi $3 \leq x \leq 7$ bo'lsin, u holda $(x-7)^9 \leq 0$ bo'lib, qolgan ko'paytuvchilar

musbat bo'ladi va $P(x) \leq 0$. Endi $1 < x < 3$ bo'lsin, u holda $(x-3)$ toq daraja bilan qatnashgani uchun $P(x)$ ishorasini o'zgartiradi, ya'ni $P(x) > 0$ bo'ladi. Agar $x < 1$ bo'lsa, $P(x)$ ko'phad ishorasini o'zgartirmaydi, chunki $x-1$ chiziqli ikki had (5.5) da juft daraja bilan qatnashyapti, ya'ni $P(x) > 0$ bo'ladi. $P(1) = 0$ bo'lganligi uchun 1 nuqta ham (5.5) tengsizlikning yechimi bo'ladi. Demak, (5.5) tengsizlikning yechimi $[3; 7] \cup \{1\}$ to'plamdan iborat.

2-misol. Tengsizlikni yeching.

$$\frac{(x-1)^2(x-3)^4(x-5)^3}{(x+2)^5(x+4)^7(x+6)^8} > 0. \quad (5.6)$$

Yechish: Kasr surati va maxrajidagi ko'phadning ildizlari 1, 3, 5 va $-2, -4, -6$ larni sonlar o'qida belgilab chiqamiz va oraliqlarga $+$ va $-$ ishoralarni qo'yib chiqamiz (5.2-chizmaga qarang). Demak, (5.6) tengsizlikning yechimi $(-4; -2) \cup (5; \infty)$ to'plamdan iborat ekan.

1. Oraliqlar usuli.

1. (96-3-25) Tengsizlikni yeching.

$$(x-2)(x+3) > 0$$

- A) $(-\infty; 2) \cup (3; \infty)$ B) $(-\infty; -3) \cup (2; \infty)$
C) $(-\infty; -2) \cup (3; \infty)$ D) $(-\infty; \infty)$

Yechish: Berilgan ifoda nolga aylanadigan $-3, 2$ nuqtalarni sonlar o'qida belgilab olamiz, natijada sonlar o'qi $(-\infty; -3), (-3; 2)$ va $(2; \infty)$ oraliqlarga bo'linadi (5.3-chizmaga qarang). Agar $x > 2$ bo'lsa, ifodadagi barcha ko'paytuvchilar musbat bo'ladi. Ikkinchi $(-3; 2)$ oraliqda ifoda ishorasini o'zgartiradi, ya'ni manfiy qiymatlar qabul qiladi. Uchinchi $(-\infty; -3)$ oraliqda ifoda yana ishorasini o'zgartiradi, ya'ni ifoda musbat qiymatlar qabul qiladi. Shunday qilib tengsizlikning yechimi $(-\infty; -3) \cup (2; \infty)$ to'plamdan iborat bo'ladi. **Javob:** $(-\infty; -3) \cup (2; \infty)$ (B).

2. (96-11-26) Tengsizlikni yeching.

$$(x-1)(x+2) > 0$$

- A) $(-\infty; 1) \cup (2; \infty)$ B) $(0; \infty)$
C) $(-\infty; -2) \cup (1; \infty)$ D) $(-\infty; \infty)$

3. (96-12-26) Tengsizlikni yeching.

$$(x+2)(x-3) > 0$$

- A) $(-\infty; \infty)$ B) $(-\infty; -3) \cup (2; \infty)$
C) $(0; \infty)$ D) $(-\infty; -2) \cup (3; \infty)$

4. (97-5-23) Tengsizlikni yeching.

$$\frac{x-1}{x-2} \geq 0$$

- A) $[1; 2)$ B) $(-\infty; 1) \cup (2; \infty)$
C) $(1; 2)$ D) $(-\infty; 1] \cup (2; \infty)$

5. (97-9-23) Tengsizlikni yeching.

$$\frac{x-2}{x-1} \leq 0$$

- A) $(1; 2]$ B) $[1; 2)$ C) $[1; 2]$ D) $(-\infty; 1)$

6. (97-9-24) Tengsizlikni yeching.

$$\frac{(x+3)(x-5)}{x+1} \geq 0$$

- A) $(-3; -1] \cup [5; \infty)$ B) $(-3; -1) \cup [5; \infty)$
C) $[-3; -1) \cup [5; \infty)$ D) $[-3; -1)$

7. (01-3-35) Tengsizlikni yeching.

$$\frac{x+1}{x-2} \leq 0$$

- A) $(-\infty; 1]$ B) $[-1; 2)$ C) $(-1; 2]$ D) $(2; \infty)$

8. Tengsizlikni yeching.

$$(x-2)(x-4)(x-5)^2 \leq 0$$

- A) $(-\infty; 2]$ B) $[2; 4] \cup \{5\}$ C) $[2; 4]$ D) \emptyset

9. (97-12-22) Tengsizlikning eng katta va eng kichik butun yechimlari yig'indisini toping.

$$\frac{(x+4)(3-x)}{(x-2)^2} > 0$$

- A) 1 B) -1 C) -2 D) 2

Yechish: Berilgan kasrning surati nolga aylanadigan -4 va 3 , maxraji nolga aylanadigan 2 larga mos keluvchi nuqtalarni sonlar o'qida belgilab olamiz va oraliqlar usulini qo'llaymiz (5.4-chizmaga qarang). Chizmadan ma'lum bo'ldiki, bu tengsizlikning yechimi $(-4; 2) \cup (2; 3)$ to'plamdan iborat. Bu to'plamdagi eng katta butun son 1 , eng kichik butun son esa -3 dir. Ularning yig'indisi $1 + (-3) = -2$. **Javob:** -2 (C).

10. (96-6-23) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(y + 6)(y + 2) < 0$$

- A) 12 B) 20 C) -12 D) -20

11. (99-5-13) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(x - 1)(x + 1)^2(x - 3)^3(x - 4)^4 \leq 0$$

- A) 6 B) 7 C) 8 D) 9

12. (00-9-21) Tengsizlikning barcha butun yechimlari yig'indisini toping.

$$(x + 3)(x - 2)^2(x + 1)^3(x - 5)^4 \leq 0$$

- A) 1 B) 2 C) 3 D) 4

13. (01-6-15) Tengsizlikning barcha butun sonlardagi yechimlari yig'indisini toping.

$$\frac{x - 4}{2x + 6} \leq 0$$

- A) 7 B) 6 C) 8 D) 5

14. (02-4-12) Tengsizlikni qanoatlantiruvchi eng katta butun sonni toping.

$$\frac{x + 5}{(x + 6)^2} < 0$$

- A) 6 B) -6 C) 5 D) -7

2. Oraliqlar usuliga keltirib yeching.

15. (98-6-23) Tengsizlikni yeching.

$$\frac{x^2 - 2x + 3}{x - 1} \geq 0$$

- A) $(1; \infty)$ B) $[1; \infty)$ C) $(-\infty; 1)$ D) $(-\infty; 1]$

Yechish: Berilgan kasrning surati $x^2 - 2x + 3 = (x - 1)^2 + 2 > 0$ doim musbat. Shuning uchun uning maxraji ham musbat bo'lishi kerak, ya'ni $x - 1 > 0 \iff x > 1$. **Javob:** $(1; \infty)$ (A).

16. (99-3-13) Tengsizlikni yeching.

$$\frac{x + 2 - x^2}{x^3 + 1} \geq 0$$

- A) $(-\infty; 2]$ B) $[2; \infty)$
C) $(-\infty; -1) \cup (-1; 2]$ D) $(-1; 2)$

17. (01-3-36) Tengsizlikni yeching.

$$\frac{x^2 - 4x + 5}{x - 2} \geq 0$$

- A) $[2; \infty)$ B) $(-\infty; 2)$ C) $(-\infty; 2]$ D) $(2; \infty)$

18. (02-10-48) Tengsizlikni yeching.

$$(9x^2 + 12x + 4)(x - 2) < 0$$

- A) $(-\infty; -\frac{2}{3}) \cup (-\frac{2}{3}; 2)$ B) $(-\infty; -2)$
C) $(2; \infty)$ D) $(-\frac{2}{3}; 2)$

19. (98-10-60) Tengsizlikning butun yechimlari nechta?

$$\frac{(x^2 + x + 1)(x^2 + 2x - 3)}{x^2 + 3x + 2} \leq 0$$

- A) 5 B) 4 C) 3 D) cheksiz ko'p

20. (99-9-7) Tengsizlikning eng katta va eng kichik butun yechimlari ayirmasini toping.

$$\frac{(x + 3)(x - 7)}{2x^2 - x + 4} < 0$$

- A) 10 B) 9 C) 8 D) 7

21. (00-4-33) Tengsizlikning eng katta butun manfiy va eng kichik butun musbat yechimlari ko'paytmasini toping.

$$\frac{x^4 - 3x^3 + 2x^2}{30 - x^2 - x} < 0$$

- A) -30 B) -35 C) -36 D) -42

Yechish: Berilgan kasr surat va maxrajini ko'paytuvchilarga ajratamiz

$$x^4 - 3x^3 + 2x^2 = x^2(x^2 - 3x + 2) = x^2(x - 1)(x - 2),$$

$$30 - x^2 - x = -(x - 5)(x + 6).$$

Endi

$$\frac{x^4 - 3x^3 + 2x^2}{30 - x^2 - x} = \frac{x^2(x - 1)(x - 2)}{-(x - 5)(x + 6)} < 0$$

tengsizlikka oraliqlar usulini qo'llab $(-\infty; -6) \cup (1; 2) \cup (5; \infty)$ yechimni olamiz. Bu to'plamdagi eng katta butun manfiy son -7, eng kichik butun musbat son esa 6 dir. Ularning ko'paytmasi $-7 \cdot 6 = -42$. **Javob:** -42 (D).

22. (96-7-20) Tengsizlikning butun yechimlari ko'paytmasini toping.

$$2x^2 - 9x + 4 < 0$$

- A) 0 B) 4 C) 24 D) 6

23. (97-3-20) Tengsizlikning butun yechimlari yig'indisini toping.

$$2x^2 \leq 5x + 12$$

- A) 4 B) 9 C) 7 D) 5

24. (97-7-20) Tengsizlikning butun yechimlari ko'paytmasini toping.

$$3x^2 \leq 13x - 4$$

- A) 12 B) 6 C) 30 D) 24

25. (97-10-20) Tengsizlikning butun yechimlari yig'indisini toping.

$$2x^2 - 3x \leq 9$$

- A) 3 B) 4 C) 5 D) 6

26. (00-7-46) Tengsizliklar sistemasining eng katta va eng kichik yechimlari yig'indisini toping.

$$\begin{cases} x^2 - 3x - 4 \leq 0 \\ x^2 - 6x + 8 \leq 0 \end{cases}$$

- A) 3 B) 4 C) 5 D) 6

27. (01-10-18) Kasrning qiymati manfiy bo'ladigan x ning barcha qiymatlarini toping.

$$\frac{x^2 - 4x - 5}{2x - 5}$$

- A) (2, 5; 5) B) $(-\infty; -1] \cup (2, 5; 5]$
C) $(-\infty; -1)$ D) $(-\infty; -1) \cup (2, 5; 5)$

28. (02-2-2) Tengsizlikni qanoatlantiruvchi n ning nechta butun qiymati bor?

$$(n^2 - 3)(n^2 - 21) < 0$$

- A) 6 B) 5 C) 3 D) 4

Yechish: Berilgan tengsizlikni

$$(n - \sqrt{3})(n + \sqrt{3})(n - \sqrt{21})(n + \sqrt{21}) < 0$$

shaklda yozib olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(-\sqrt{21}; -\sqrt{3}) \cup (\sqrt{3}; \sqrt{21})$ yechimni olamiz. Agar biz $\sqrt{3} \approx 1,732$ va $\sqrt{21} \approx 4,582$ ekanligini hisobga olsak, $(\sqrt{3}; \sqrt{21})$ oraliqda 2, 3 va 4 butun sonlari yotishiga ishonch hosil qilamiz. Xuddi shunday $(-\sqrt{21}; -\sqrt{3})$ oraliqda $-4, -3, -2$ butun sonlari yotadi. Demak, n ning 6 ta butun qiymati berilgan tengsizlikni qanoatlantiradi.

Javob: 6 (A).

29. (01-7-22) Nechta tub son

$$x^2 - 50 < 0$$

tengsizlikning yechimi bo'ladi?

- A) 2 B) 3 C) 4 D) 5

30. (00-7-19) Tengsizlik o'rinli bo'ladigan n ning barcha natural qiymatlari yig'indisini toping.

$$n^2(n^2 - n - 6) \leq 0$$

- A) 4 B) 2 C) 5 D) 6

31. (03-4-14) Tengsizlikning eng katta va eng kichik yechimlari ayirmasini toping.

$$\frac{x^2 - 13x + 36}{x^4 + 25} \leq 0$$

- A) 6 B) 4 C) 5 D) 7

32. (03-7-63) Tengsizlikning butun sonlardan iborat yechimlaridan eng kattasidan eng kichigining ayirmasini toping.

$$\frac{x^2 - x - 12}{x^2 - 2x - 35} \leq 0$$

- A) 10 B) 12 C) 11 D) 9

33. (03-8-56) Tengsizlikni qanoatlantiruvchi natural sonlar nechta?

$$\frac{x^2 - 5x - 14}{x + 4} \leq 0$$

- A) 7 B) 8 C) 9 D) 5

34. (03-3-24) Ushbu

$$\frac{(x-1)^2 + 2x - 2}{(x-5)^3} \geq 0$$

tengsizlikning $[-3; 8]$ kesmadagi butun sonlardan iborat yechimlari sonini aniqlang.

- A) 3 B) 4 C) 5 D) 6

35. (03-11-75) Tengsizlikning butun sonlardan iborat yechimlaridan nechtasi $[-5; 6]$ kesmada joylashgan?

$$\frac{(x+4)^2 - 8x - 25}{(x-6)^2} > 0$$

- A) 2 B) 3 C) 4 D) 5

3. Kvadrat tengsizliklar.

Dastlab kvadrat uchhad chiziqli ko'paytuvchilarga ajratiladi, keyin unga oraliqlar usuli qo'llaniladi.

36. (97-1-10) Tengsizlikni yeching.

$$(x-2)^2 + 3(x-2) \geq 7-x$$

- A) $[-2; 1]$ B) $[0; 1] \cup [3; \infty)$
C) $[-3; 3]$ D) $(-\infty; -3] \cup [3; \infty)$

Yechish: Berilgan tengsizlikdagi qavslarni ochamiz, tengsizlikning o'ng qismidagi $7-x$ ni tengsizlikning chap qismiga o'tkazamiz va o'xshash hadlarni soddalashtirib, natijada unga teng kuchli bo'lgan

$$x^2 - 9 \geq 0 \iff (x-3)(x+3) \geq 0$$

tengsizlikni olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(-\infty; -3] \cup [3; \infty)$ yechimni olamiz. **Javob:** $(-\infty; -3] \cup [3; \infty)$ (D).

37. (97-6-10) Tengsizlikni yeching.

$$(x+2)(x-2) - 2(x-1) \leq 23 - 2x$$

- A) $(-\infty; 5]$ B) $(0; 25]$
C) $[-5; 5]$ D) $[-\sqrt{21}; \sqrt{21}]$

38. (97-11-10) Tengsizlikni yeching.

$$2 \cdot (x-1) \cdot (x+1) - x(x+3) < 2 - 3x$$

- A) $(-\infty; 2)$ B) $(-2; 2)$ C) $(0; 4)$ D) $(1; \infty)$

39. (01-2-26) $x^2 + 2x - 15 < 0$ tengsizlikning natural yechimlari ko'paytmasini toping.
A) 0 B) 2 C) 4 D) 6

4. Tengsizlikning bir tamoniga o'tkazing.

40. (99-1-20) $\frac{1}{x} > x$ tengsizlikni yeching.
A) $(-\infty; -1) \cup (0; 1)$ B) $[0; 1)$
C) $(0; 1)$ D) \emptyset

Yechish: Tengsizlikning o'ng qismidagi x ni tengsizlikning chap qismiga o'tkazamiz va umumiy maxraj beramiz va

$$\frac{1 - x^2}{x} > 0 \iff \frac{(1 - x)(1 + x)}{x} > 0$$

tengsizlikni olamiz. Bu tengsizlikka oraliqlar usulini qo'llab $(-\infty; -1) \cup (0; 1)$ yechimni olamiz.

Javob: $(-\infty; -1) \cup (0; 1)$ (A).

41. (99-6-30) Tengsizlikni yeching.

$$\frac{x^2}{x + 3} < x - 3$$

- A) $(-\infty; -3)$ B) $(-3; 3)$ C) $(0; 3)$ D) \emptyset

42. (99-6-45) Tengsizlikni yeching.

$$\frac{5x + 8}{4 - x} < 2$$

- A) $(-\infty; 0) \cup (4; \infty)$ B) $(-\infty; -4) \cup (0; 4)$
C) $[-4; 4]$ D) \emptyset

43. (00-4-32) Tengsizlikni yeching.

$$1 - \frac{6}{x} > \frac{2}{1 - x}$$

- A) $(0; 1) \cup (2; 3)$ B) $(-\infty; 0) \cup (1; 2) \cup (3; \infty)$
C) $(0; 1) \cup (3; \infty)$ D) $(-\infty; 1) \cup (2; 3) \cup (5; \infty)$

44. (01-5-22) Tengsizlikni yeching.

$$\frac{1}{x - 1} \leq 2$$

- A) $(-\infty; 1) \cup [1; 5; \infty)$ B) $(1; 2]$
C) $(1; 2)$ D) $(1; 1, 5]$

45. (02-12-12) Tengsizlikni yeching.

$$\frac{x^2 - 5x + 2}{x - 3} > x$$

- A) $(-3; 1)$ B) $(1; 3)$ C) $(-1; 3)$ D) $(-\infty; 1)$

46. (03-1-66) Tengsizlikni yeching.

$$\frac{2}{x^2 - 9} < \frac{3}{x^2 - 16}$$

- A) $(-\infty; \infty)$
B) $(-4; -3) \cup (3; 4)$
C) $(-\infty; -4) \cup (-3; 3) \cup (4; \infty)$
D) $(-\infty; -4) \cup (4; \infty)$

47. (03-5-17) Tengsizlikni yeching.

$$\frac{1}{x - 2002} \leq \frac{x}{x - 2002}$$

- A) $(-\infty; 1] \cup (2002; \infty)$ B) $(-\infty; 1]$
C) $(2002; \infty)$ D) $[1; 2002)$

48. (01-1-72) Ushbu $x \geq \frac{6}{x - 5}$ tengsizlikni qanoatlantiruvchi eng kichik butun musbat yechimning eng kichik butun manfiy yechimga nisbatini toping.

- A) -1 B) -2 C) $-0,5$ D) -4

49. (01-2-68) Tengsizlikning eng katta va eng kichik butun ildizlari ayirmasini toping.

$$(x^2 - x - 1)(x^2 - x - 7) \leq -5$$

- A) 2 B) 3 C) 4 D) 5

Yechish: Tengsizlikda $x^2 - x - 4 = t$ belgilash olsak, berilgan tengsizlik

$$(t + 3)(t - 3) + 5 \leq 0 \iff (t - 2)(t + 2) \leq 0$$

ko'rinishni oladi. Yana eski o'zgaruvchiga qaytib

$$(x^2 - x - 4 - 2)(x^2 - x - 4 + 2) \leq 0$$

ga ega bo'lamiz. Ko'paytmadagi kvadrat uchhadlarni ko'paytuvchilarga ajratamiz

$$x^2 - x - 6 = (x + 2)(x - 3), \quad x^2 - x - 2 = (x + 1)(x - 2)$$

va berilgan tengsizlikka teng kuchli bo'lgan

$$(x + 2)(x - 3)(x + 1)(x - 2) \leq 0$$

tengsizlikka ega bo'lamiz. Bu tengsizlikka oraliqlar usulini qo'llab $[-2; -1] \cup [2; 3]$ yechimni olamiz. Bu to'plamda eng katta butun son 3, eng kichik butun son esa -2 dir. Ularning ayirmasi $3 - (-2) = 5$. **Javob:** 5 (D).

50. (97-1-58) $x^4 < 9x$ tengsizlikning butun sonlardagi yechimi nechta?

- A) 1 B) 2 C) 3 D) 4

51. (01-1-12) Nechta tub son

$$3 < \frac{5x - 1}{2x - 3} < 5$$

tengsizlikning yechimi bo'ladi?

- A) 0 B) 1 C) 2 D) 3

52. (02-1-63) Nechta tub son

$$2 < \frac{x + 7}{2x - 19} < 4$$

tengsizlikning yechimi bo'ladi?

- A) 1 B) 13 C) 7 D) 3

53. (01-10-17) Nechta butun son

$$x^4 - 8x^2 + 7 \leq 0$$

tengsizlikni qanoatlantiradi?

- A) 0 B) 1 C) 2 D) 4

54. (02-1-21)
- $x > 1$
- va
- $x^2 > x$
- tengsizliklar teng kuchli bo'ladigan sonli oraliqni ko'rsating.

- A)
- $(1; \infty)$
- B)
- $(-\infty; 0)$
- C)
- $(-\infty; \infty)$
- D)
- \emptyset

Yechish: 1-tengsizlikni $x > 1$ (musbat songa ko'paytirilganda tengsizlik saqlanadi) ga ko'paytir-sak 2-tengsizlik hosil bo'ladi. Demak, $x > 1$ da tengsizliklar teng kuchli bo'ladi. **Javob:** $(1; \infty)$ (A).

55. (02-8-7) Tengsizlikning eng kichik butun yechimini toping.

$$\frac{x-10}{2-x} > 1$$

- A) 3 B) 4 C) 1 D) -2

56. (02-10-13) Tengsizlikning manfiy bo'lmagan butun yechimlarini toping.

$$\frac{x+3}{x^2-4} - \frac{1}{x+2} < \frac{2x}{2x-x^2}$$

- A) 1 B) 0; 1; 2 C) 1; 2; 3 D) 1; 2

57. (03-1-14) Tengsizlikning butun yechimlari nechta?

$$x^4 - 10x^2 + 9 \leq 0$$

- A) 2 B) 3 C) 4 D) 6

58. (03-3-19) Tengsizlikning butun sonlardan iborat yechimlari nechta?

$$\frac{x^2 - 12x + 23}{(x+1)(x-4)} \leq -\frac{2}{x-4}$$

- A) 2 B) 3 C) 4 D) 5

59. (03-6-42) Tengsizlikni qanoatlantiruvchi
- x
- ning barcha qiymatlarini toping.

$$\frac{x+1}{x} \leq 1$$

- A)
- $-1 \leq x < 0$
- B)
- $x < 0$
-
- C)
- $-1 < x < 0$
- D)
- $x > 0$

5. Tengsizliklarning qo'llanilishi.

60. (97-1-16)
- k
- ning qanday qiymatlarida
- $k(x+1) = 5$
- tenglamaning ildizi musbat bo'ladi?

- A)
- $(0; \infty)$
- B)
- $(0; 5)$
- C)
- $(-5; 0)$
- D)
- $(5; \infty)$

Yechish: Berilgan tenglama $x = 5 : k - 1$ yechimga ega. Masala shartiga ko'ra u musbat bo'lishi kerak, ya'ni

$$\frac{5}{k} - 1 > 0 \iff \frac{5-k}{k} > 0.$$

Oraliqlar usulini qo'llasak, bu tengsizlikning yechimi $(0; 5)$ ekanligini olamiz. **Javob:** $(0; 5)$ (B).

61. (97-11-16)
- b
- ning qanday qiymatlarida
- $b(2-x) = 6$
- tenglamaning ildizi manfiy?

- A)
- $b \in (-\infty; 0)$
- B)
- $b \in (0; 3)$
-
- C)
- $b \in (-3; 0)$
- D)
- $b \in [3; \infty)$

62. (99-10-4) Tenglamaning ildizlari manfiy bo'ladigan
- k
- ning barcha natural qiymatlari yig'indisini toping.

$$(k-2)^2 \cdot y = k^2 - 25$$

- A) 10 B) 13 C) 11 D) 8

63. (00-3-13)
- k
- ning qanday qiymatlarida

$$\frac{4x-1}{x-1} = k+3$$

tenglama manfiy yechimga ega bo'ladi?

- A)
- $(-\infty; -2)$
- B)
- $(-\infty; -2) \cup (1; 4)$
-
- C)
- $(1; \infty)$
- D)
- $(-2; 1)$

64. (02-12-6)
- m
- ning qanday qiymatlarida

$$4-m = \frac{2}{x-1}$$

tenglamaning ildizlari musbat bo'ladi?

- A)
- $(4; 6)$
- B)
- $(-\infty; 1) \cup (1; 4)$
-
- C)
- $(-\infty; 4) \cup (6; \infty)$
- D)
- $(-\infty; 2) \cup (4; \infty)$

65. (02-9-20)
- t
- ning qanday qiymatlarida

$$3x-4 = 2(x-t)$$

tenglama musbat ildizga ega?

- A)
- $t > -2$
- B)
- $t < 2$
- C)
- $t \leq 1$
- D)
- $t \geq 2$

66. (03-3-6)
- k
- ning qanday qiymatlarida

$$\frac{3x+1}{x+1} = k-2$$

tenglama manfiy ildizga ega?

- A)
- $(3; 5)$
- B)
- $(-\infty; 3) \cup (5; \infty)$
-
- C)
- $(2; 4)$
- D)
- $(1; 3)$

67. (99-2-18) Tenglama yechimga ega bo'lmaydigan
- k
- ning eng katta butun qiymatini toping.

$$kz^2 + 2(k-12)z + 2 = 0$$

- A) 16 B) 18 C) 20 D) 17

Yechish: Agar $k = 0$ bo'lsa, bu tenglama $z = 1 : 12$ yechimga ega. Shuning uchun $k \neq 0$ ni qaraymiz. Kvadrat tenglama yechimga ega bo'lmasligi uchun uning diskriminanti $D = (2(k-12))^2 - 4 \cdot 2k < 0$ bo'lishi kerak. Bu kvadrat tengsizlikni yechib, $k \in (8; 18)$ ekanligini olamiz. k ning eng katta butun qiymati 17. **Javob:** 17 (D).

68. (00-9-12) Tenglama ildizga ega bo'lmaydigan
- m
- ning barcha natural qiymatlari yig'indisini toping.

$$\frac{t-6}{m-8} = \frac{m}{t}$$

- A) 20 B) 25 C) 28 D) 30

69. (00-3-19) k ning qanday eng kichik butun qiymatida

$$x^2 - 2(k+2)x + 6 + k^2 = 0$$

tenglama ikkita turli haqiqiy ildizlarga ega bo'ladi?

- A) -2 B) -1 C) 2 D) 1

5.3.1 Parametrli tengsizliklar

1. (96-3-78) Tengsizliklar sistemasi a ning qanday qiymatlarida yechimga ega emas?

$$\begin{cases} ax > 5a - 1 \\ ax < 3a + 1 \end{cases}$$

- A) $\{1\}$ B) $(-\infty; 0) \cup [1; \infty)$
 C) $(-\infty; 0)$ D) $[1; \infty)$

Yechish: Berilgan sistema

$$5a - 1 < ax < 3a + 1 \quad (5.7)$$

qo'sh tengsizlikka teng kuchli. Quyidagi ikki

1) $a = 0$ va 2) $a \neq 0$ holni qaraymiz.

1-holda $-1 < 0 \cdot x < 1$ tengsizlikni hosil qilamiz va u x ning barcha qiymatlarida o'rinli. Demak, bu holda yechim mavjud.

2-hol, $a \neq 0$ bo'lsin. Ma'lumki, $a < x < b$ oraliq bo'sh to'plam bo'lishi uchun $b \leq a$ bo'lishi zarur va yetarli. Demak, (5.7) qo'sh tengsizlik yechimga ega bo'lmasligi $5a - 1 \geq 3a + 1$ ga teng kuchli. Bu yerdan

$$5a - 3a \geq 2 \iff 2a \geq 2 \iff a \geq 1$$

ekani kelib chiqadi. Demak, $[1; \infty)$. **Javob:** (D).

2. (98-10-61) $kx^2 + 2x + k + 2 > 0$ tengsizlik yechimga ega bo'lmaydigan k ning butun qiymatlari orasidan eng kattasini toping.
 A) -1 B) -2 C) eng kattasi yo'q D) -3
3. (00-6-20) a ning qanday qiymatlarida

$$\begin{cases} 3 - 7x < 3x - 7 \\ 1 + 2x < a + x \end{cases}$$

tengsizliklar sistemasi yechimga ega emas?

- A) $a < 4$ B) $a \leq 1$ C) $a < 2$ D) $a \leq 2$

4. (96-9-19) Tengsizliklar sistemasi a ning qanday qiymatlarida yechimga ega bo'lmaydi?

$$\begin{cases} ax > 7a - 3 \\ ax \leq 3a + 3 \end{cases}$$

- A) $(1, 5; \infty)$ B) $[1, 5; \infty)$
 C) $(-\infty; 0)$ D) $(-\infty; 0) \cup (1, 5; \infty)$

5. (96-13-19) Tengsizliklar sistemasi b ning qanday qiymatlarida yechimga ega bo'lmaydi?

$$\begin{cases} bx \geq 5b - 3 \\ bx \leq 4b + 3 \end{cases}$$

- A) $(6; \infty)$ B) $[6; \infty)$
 C) $(-\infty; 0)$ D) $(-\infty; 0) \cup (6; \infty)$

6. (98-3-13) Tengsizlik yechimga ega bo'lmaydigan k ning eng katta butun qiymatini toping.

$$kx^2 + 4x + k + 1 > 0$$

- A) -1 B) eng kattasi yo'q
 C) -3 D) -2

7. (00-5-33) a ning qanday qiymatida

$$a(x-1) > x-2$$

tengsizlik x ning barcha qiymatlarida o'rinli bo'ladi?

- A) 0 B) 1 C) 2 D) 3

Yechish: Berilgan tengsizlik $(a-1)x > a-2$ tengsizlikka teng kuchli. Quyidagi ikki

1) $a = 1$ va 2) $a \neq 1$ holni qaraymiz.

1-holda $0 \cdot x > -1$ tengsizlikni hosil qilamiz va u x ning barcha qiymatlarida bajariladi. Demak, $a = 1$ masala shartini qanoatlantiradi.

2-hol $a \neq 1$ bo'lsin. Dastlab $a > 1$ holni qaraymiz. Bu holda $a-1 > 0$ bo'lganligi uchun $(a-1)x > a-2$ tengsizlikning yechimi $x > (a-2) : (a-1)$ dan iborat. Bu holda tengsizlik x ning $x \leq (a-2) : (a-1)$ qiymatlarida o'rinli emas. Demak, $a > 1$ sonlar masala shartini qanoatlantirmaydi. Endi $a < 1$ holni qaraymiz. Bu holda $a-1 < 0$ bo'lganligi uchun $(a-1)x > a-2$ tengsizlikning yechimi $x < (a-2) : (a-1)$ dan iborat. $a < 1$ bo'lsa, tengsizlik x ning $x \geq (a-2) : (a-1)$ qiymatlarida o'rinli emas. Demak, $a < 1$ sonlar ham masala shartini qanoatlantirmaydi. **Javob:** $a = 1$ (B).

8. (99-9-17) a ning qanday qiymatlarida

$$ax^2 + 8x + a < 0$$

tengsizlik x ning barcha qiymatlarida o'rinli bo'ladi?

- A) $(0; 4)$ B) $(-4; 0)$ C) $(-4; 4)$ D) $(-\infty; -4)$

9. (01-2-78) n ning 10 dan oshmaydigan nechta natural qiymatida $nx^2 + 4x > 1 - 3n$ tengsizlik x ning ixtiyoriy qiymatida o'rinli bo'ladi?
 A) 10 B) 9 C) 8 D) 7
10. (03-8-12) m ning qanday qiymatida

$$\frac{mx+9}{x} \geq -10$$

tengsizlikning eng katta manfiy yechimi -3 ga teng bo'ladi?

- A) -9 B) -8 C) -7 D) -6

5.3.2 Shartli tengsizliklar

1. (97-9-68) Agar $a < 0 < b$ va $|a| > |b|$ bo'lsa,

$$\frac{1}{a^3 + b^3}, \quad \frac{1}{a^4 + b^3}, \quad \frac{1}{a^3}$$

larni taqqoslang.

- A) $\frac{1}{a^3} > \frac{1}{a^3 + b^3} > \frac{1}{a^4 + b^3}$
 B) $\frac{1}{a^4 + b^3} > \frac{1}{a^3} > \frac{1}{a^3 + b^3}$
 C) $\frac{1}{a^4 + b^3} > \frac{1}{a^3 + b^3} > \frac{1}{a^3}$
 D) $\frac{1}{a^3 + b^3} > \frac{1}{a^3} > \frac{1}{a^4 + b^3}$

Yechish: $a < 0 < b$, $|a| > |b|$ ekanidan $a^3 < a^3 + b^3 < 0 < a^4 + b^3$ tengsizliklarni hosil qilamiz. Shuning uchun

$$\frac{1}{a^4 + b^3} > \frac{1}{a^3} > \frac{1}{a^3 + b^3}. \quad \text{Javob: (B).}$$

2. (96-6-11) Quyidagi tengsizliklardan qaysilari a ning barcha qiymatlarida o'rinli?

- 1) $a^2 > 0$, 2) $a^2 - 10 < 0$
 3) $(a - 5)^2 \geq 0$, 4) $\frac{1}{a^2} + a^2 > 2$
 A) 1 B) 2 C) 1; 3 va 4 D) 3

3. (98-12-34) $a > b > 0$ shartni qanoatlantiruvchi a va b sonlar uchun quyidagi munosabatlardan qaysilari o'rinli?

- 1) $a^3 > ab^2$; 2) $a^4 \geq a^2b^2$
 3) $a^2b^2 < b^4$; 4) $\frac{2}{a} > \frac{2}{b}$
 A) 1 B) 1; 2 C) 3 D) 4

4. (99-5-24) Agar x va y sonlari uchun $x \cdot y = 20$ va $0 < x < 0,8$ munosabat o'rinli bo'lsa, quyidagi tengsizliklardan qaysi biri doimo o'rinli bo'ladi?

- A) $\frac{x}{y} < 20$ B) $x + y < 20$
 C) $y < 16$ D) $y > 25$

5. (99-5-34) Agar $2 < a < 3$ va $-3 < b < -2$ bo'lsa, quyidagilarning qaysi biri doim o'rinli bo'ladi?

- A) $a^2b^2 - 50 < 0$ B) $\frac{(a+b)^2 - 2ab}{a-b} < 0$
 C) $b^3a^2 - 5 < 0$ D) $a^3b^2 - 2 < 0$

6. (01-6-16) Agar

$$\begin{cases} p^2 + q^2 < 20 \\ pq < 22 \end{cases}$$

bo'lsa, $|p + q|$ ning butun qiymatlari nechta?

- A) 5 B) 6 C) 9 D) 8

Yechish: Berilgan sistemaning 2-tengsizligini 2 ga ko'paytirib 1-tengsizlikka qo'shsak $p^2 + q^2 + 2pq < 64$ ni olamiz. Agar $|p + q| = t \geq 0$ desak, so'nggi tengsizlik $t^2 < 64$ ko'rinishni oladi. $t \geq 0$ shartdan $0 \leq t < 8$ ni olamiz. $[0; 8)$ oraliqda 8 ta butun son bor. **Javob:** 8 (D).

7. (99-8-15) Agar $-2 < a < -1$ va $-3 < b < -2,5$ bo'lsa, $a - b$ ayirma qaysi sonlar orasida bo'ladi?

- A) $(0, 5; 2)$ B) $(1; 1, 5)$ C) $(-1, 5; -1)$ D) $(-1, 5; 1)$

8. (00-4-31) Agar $a < -1$ bo'lsa, quyidagi keltirilgan ifodalardan qaysi birining qiymati eng katta bo'ladi?

- A) a^{-1} B) a^{-3} C) a^{-5} D) a^3

6 - bob. Modulli ifodalar

6.1 Modulli tenglamalar

Haqiqiy sonning moduli (absolyut qiymati) xossalari 1.4-bandda keltirilgan. Modul qatnashgan tenglamalarni yechishda qo'llaniladigan ba'zi qoidalarini keltiramiz.

1. $|f(x)| = f(x) \iff f(x) \geq 0$.

2. $|f(x)| = -f(x) \iff f(x) \leq 0$.

3. $|f(x)| = |g(x)| \iff \begin{cases} f(x) = g(x) \\ f(x) = -g(x) \end{cases}$.

4. $|f(x)| = a \ (a \geq 0) \iff \begin{cases} f(x) = a \\ f(x) = -a \end{cases}$.

Noma'lumi modul belgisi ostida bo'lgan tenglamalarni yechishning quyidagi uch usulini bayon qilamiz.

1-usul. Modulning ta'rifidan foydalanib yechish.

2-usul. Tenglamani ikkala tomonini kvadratga ko'tarish.

3-usul. Geometrik usul.

Bu usullarni misollarda ko'rib chiqamiz.

1-misol. $x^2 - 3|x| - 40 = 0$ tenglamani 1-usul yordamida yeching.

Yechish: Absolyut qiymat ta'rifiga asosan berilgan tenglama quyidagi sistemalarga teng kuchli:

$$\begin{cases} x \leq 0 \\ x^2 + 3x - 40 = 0 \end{cases} \quad \text{va} \quad \begin{cases} x \geq 0 \\ x^2 - 3x - 40 = 0 \end{cases}$$

Birinchi sistemadagi kvadrat tenglamani yechimlari $x_1 = -8$ va $x_2 = 5$ lardir. Ammo $x_2 = 5$ yechim $x \leq 0$ shartni qanoatlantirmaydi, shuning uchun 1-sistemaning yechimi $x_1 = -8$ dir.

Ikkinchi sistemadagi kvadrat tenglamani yechimlari $x_1 = -5$ va $x_2 = 8$ lardir. Bu yerda $x_1 = -5$ yechim $x \geq 0$ shartni qanoatlantirmaydi, shuning uchun 2-sistemaning yechimi $x_2 = 8$ dir. **Javob:** $x_1 = -8$, $x_2 = 8$.

2-misol. $|x| = |2x - 5|$ tenglamani 2-usul yordamida yeching.

Yechish: $|x|^2 = x^2$ ayniyatga ko'ra, berilgan tenglama

$$x^2 = (2x - 5)^2 \iff 3x^2 - 20x + 25 = 0$$

tenglamaga teng kuchli. Bu kvadrat tenglamani yechimlari $x_1 = 5/3$, $x_2 = 5$ dir. **Javob:** $x_1 = 5/3$, $x_2 = 5$.

Ba'zi misollarni yechishda "geometrik usul" tez natija beradi.

3-misol. $|x - 3| = 5$ tenglamani geometrik usul yordamida yeching.

Yechish: $|x - 3|$ miqdor sonlar o'qida x va 3 nuqtalar orasidagi masofani ifodalaydi. Demak, sonlar o'qida koordinatasi 3 bo'lgan nuqtadan 5 birlik masofada yotuvchi nuqtalarga mos sonlarni topishimiz kerak. Bu sonlar $3 - 5 = -2$, $3 + 5 = 8$ sonlaridir. (6.1-chizmaga qarang). **Javob:** $x_1 = -2$, $x_2 = 8$.

$|x| = ax^2 + bx + c$ ko'rinishdagi tenglamalarni yechishda 1-usul (modul ta'rifidan) foydalanishni tavsiya qilamiz.

$|ax+b| = |cx+d|$ ko'rinishdagi tenglamalarni yechishda 2-usul (kvadratga ko'trish usuli) yaxshi natija beradi.

$|x-a|+|x-b|+|x-c| = d$ ko'rinishdagi tenglamalarni yechishda "geometrik usul" tez natija beradi.

1. (99-6-48) Tenglamani yeching.

$$|2 - 3x| - |5 - 2x| = 0$$

- A) $-3; \frac{7}{5}$ B) $3; \frac{7}{5}$ C) $3; -1$ D) $-3; 0$

Yechish: Berilgan tenglamani

$$|2 - 3x| = |5 - 2x|$$

ko'rinishda yozamiz. Bu tenglama 3-qoidaga ko'ra 2 ta tenglamaga ajraladi:

1) $2 - 3x = 5 - 2x$ uning yechimi $x = -3$.

2) $2 - 3x = -(5 - 2x)$, bu tenglamaning yechimi esa $x = \frac{7}{5}$. **Javob:** $-3; \frac{7}{5}$ (A).

2. (97-1-75) Tenglamaning nechta ildizi bor?

$$|x + 1| = |2x - 1|$$

- A) 4 B) 3 C) 2 D) 1

3. (97-6-71) Tenglamaning nechta ildizi bor?

$$|x| = |2x - 5|$$

- A) 1 B) 2 C) 3 D) cheksiz ko'p

4. (02-10-10) Tenglamani yeching.

$$|x - 2| = 3 \cdot |3 - x|$$

- A) 2, 75; 3, 5 B) 2, 75 C) 2 D) 2, 5

5. (02-11-26) Tenglamaning ildizlari yig'indisini toping.

$$|x + 1| = 2|x - 2|$$

- A) 7 B) 5 C) 4 D) 6

6. (02-12-11) Tenglamaning butun sonlardan iborat ildizi nechta?

$$|x - 1| \cdot |x + 2| = 4$$

- A) 2 B) 3 C) 4 D) 1

7. (98-4-24) Tenglamaning barcha natural yechimlari yig'indisini toping.

$$|x^2 - 8x + 7| = -7 + 8x - x^2$$

- A) 8 B) 40 C) 25 D) 28

Yechish: Berilgan tenglamani yechish 2-tengkuchlilikka ko'ra quyidagi

$$x^2 - 8x + 7 \leq 0 \iff (x - 1)(x - 7) \leq 0$$

tengsizlikni yechishga teng kuchli. Bu tengsizlik oraliqlar usuli yordamida yechiladi, uning yechimi $[1; 7]$ kesmadan iborat. Bu kesmada 7 ta (1, 2, 3, 4, 5, 6, 7) natural son bor. Ularning yig'indisi $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. **Javob:** 28 (D).

8. (00-5-22) Tenglamani yeching.

$$|2x - 3| = 3 - 2x$$

- A) $\frac{3}{2}$ B) $(-\infty; \frac{3}{2}]$ C) $(-\infty; \frac{3}{2})$ D) $(-\infty; \infty)$

9. (99-4-24) Tenglamaning nechta butun ildizi bor?

$$|x^2 - 2x| = 2x - x^2$$

- A) 1 B) 2 C) 3 D) birorta ham ildizi yo'q

10. (98-1-8) m ning qanday qiymatlarida $|m + 1| = m + 1$ tenglik o'rinli bo'ladi?

- A) $m = -1$ B) $m \in R$ C) $m = 0$ D) $m \geq -1$

11. (98-8-8) a ning qanday qiymatlarida

$$|a + 2| = -a - 2$$

tenglik o'rinli bo'ladi?

- A) $a = -2$ B) $a \in \emptyset$ C) $a < -2$ D) $a \leq -2$

12. (99-6-47) Tenglamaning ildizlari yig'indisini toping?

$$|x^2 + 5x| = 6$$

- A) 10 B) -6 C) -3 D) -10

Yechish: Berilgan tenglamani yechish 4-tengkuchlilikka ko'ra quyidagi ikki

$$\begin{cases} x^2 + 5x = 6 \\ x^2 + 5x = -6 \end{cases} \iff \begin{cases} x^2 + 5x - 6 = 0 \\ x^2 + 5x + 6 = 0 \end{cases}$$

tenglama yechimlari bilan ustma-ust tushadi. Birinchi $x^2 + 5x - 6 = 0$ kvadrat tenglamaning ildizlari $x_1 = -6$, $x_2 = 1$ lardir. Ikkinchi $x^2 + 5x + 6 = 0$ kvadrat tenglamaning ildizlari esa $x_1 = -3$, $x_2 = -2$. Ularning yig'indisi $-6 + 1 + (-3) + (-2) = -10$. **Javob:** -10 (D).

13. (99-10-9) Tenglamaning manfiy ildizlari nechta?

$$\left(\frac{y}{6} + \frac{y}{3} + \frac{y}{2}\right)(y^2 - 3|y| + 2) = 0$$

- A) 1 B) 2 C) 3 D) 4

14. (00-4-11) Tenglama ildizlari ko'paytmasini toping.

$$x^2 - 3|x| - 40 = 0$$

- A) -40 B) 40 C) -32 D) -64

15. (99-2-14) Tenglama ildizlari ko'paytmasini toping.

$$(x - 2)^2 - 4|x - 2| + 3 = 0$$

- A) 3 B) 15 C) -3 D) -15

16. (00-6-12) Tenglama ildizlari yig'indisini toping?

$$|1 - |1 - x|| = 0,5$$

- A) 0 B) 4 C) 3 D) 1

31. (02-2-16) Agar $|x - 2| + 3x = -6$ bo'lsa, $|x|$ ni toping.
A) 4 B) 3 C) 2 D) 6
32. (02-5-9) Tenglama ildizlari ko'paytmasini toping.

$$(2|x| - 1)^2 = |x|$$

- A) $\frac{1}{16}$ B) $-\frac{1}{16}$ C) $\frac{1}{4}$ D) $-\frac{1}{4}$

33. (02-8-8) $|5 - x| = 2(2x - 5)$ bo'lsa, $5 + x$ ning qiymati nechaga teng?
A) 8 B) 7 C) 9 D) 11

6.2 Modulli tengsizliklar

Modul qatnashgan tengsizliklarni yechishda quyidagi teng kuchliliklardan foydalaniladi.

1. $|f(x)| < a, (a > 0) \iff -a < f(x) < a.$
2. $|f(x)| > a, (a > 0) \iff \begin{cases} f(x) > a \\ f(x) < -a. \end{cases}$
3. $|f(x)| < |g(x)| \iff f^2(x) < g^2(x) \iff$
 $\iff (f(x) - g(x))(f(x) + g(x)) < 0.$

Noma'lumi modul belgisi ostida bo'lgan tengsizliklarni yechishning bir nechta usullari bor. Bu usullarni misollarda ko'rib chiqamiz.

1-misol. $|x - 1| \leq 3$ tengsizlikni yeching.

Yechish: *1-usul - modul ta'rifi yordamida.* Absolyut qiymat ta'rifiga asosan berilgan tengsizlik quyidagi sistemalarga teng kuchli:

$$\begin{cases} x - 1 \leq 0 \\ -(x - 1) \leq 3 \end{cases} \quad \text{va} \quad \begin{cases} x - 1 \geq 0 \\ x - 1 \leq 3. \end{cases}$$

Birinchi sistemaning 2-tengsizligini -1 ga ko'paytiramiz, natijada $-3 \leq x - 1 \leq 0$ qo'sh tengsizlikni olamiz. Bu tengsizlikning barcha qismlariga 1 ni qo'shib $-2 \leq x \leq 1$ ni olamiz. Demak, birinchi sistemaning yechimi $[-2; 1]$ kesmadan iborat. Ikkinchi sistema $0 \leq x - 1 \leq 3$ qo'sh tengsizlikka teng kuchli. Bu tengsizlikning barcha qismlariga 1 ni qo'shish orqali $1 \leq x \leq 4$ ni olamiz. Bu yechimlarni birlashtirib, berilgan tengsizlikning yechimi bo'lgan $[-2; 1] \cup [1; 4] = [-2; 4]$ ni olamiz. **Javob:** $[-2; 4]$.

2-misol. $|x - 1| \leq 3$ tengsizlikni yeching.

Yechish: *2-usul - kvadratga ko'tarish.* Berilgan tengsizlikning ikkila tomoni ham x ning barcha qiymatlarida nomanfiy bo'lganligi uchun, tengsizlikni kvadratga ko'tarib, o'ziga teng kuchli $|x - 1|^2 \leq 3^2$ tengsizlikni hosil qilamiz. $|a|^2 = a^2$ ayniyatga ko'ra bu tengsizlik

$$x^2 - 2x - 8 \leq 0 \iff (x + 2)(x - 4) \leq 0 \quad (6.1)$$

tengsizlikka teng kuchli. (6.1) tengsizlikka oraliqlar usulini qo'llab $[-2; 4]$ yechimni olamiz. **Javob:** $[-2; 4]$

3-misol. $|x - 1| \leq 2$ tengsizlikni yeching.

Yechish: *3-usul - "geometrik usul".* $|x - 1|$ miqdor sonlar o'qida x va 1 nuqtalar orasidagi masofani ifodalaydi. Demak, berilgan tengsizlikning yechimi sonlar o'qida koordinatasi 1 bo'lgan nuqtadan masofasi

2 birlik va undan kichik x ning barcha qiymatlaridan iborat (6.2-chizmaga qarang). Sonlar o'qida koordinatasi 1 bo'lgan nuqtadan 2 birlik chapda -1 ni, 2 birlik o'ngda yotuvchi 3 ni, topamiz. Demak, yechim $[-1; 3]$ kesmadan iborat. **Javob:** $[-1; 3]$.

1. (99-4-5) Tengsizlik nechta butun yechimga ega?

$$4 \leq |x| \leq 8$$

- A) 12 B) 10 C) 8 D) 6

Yechish: Tengsizlikni yechishda 1-usuldan foydalanamiz. Berilgan tengsizlik yechimi

$$\begin{cases} x \leq 0 \\ 4 \leq -x \leq 8 \end{cases} \quad \text{va} \quad \begin{cases} x \geq 0 \\ 4 \leq x \leq 8 \end{cases}$$

sistema yechimlari birlashmasidan iborat. 1-sistemaning ikkinchi tengsizligini -1 ga ko'paytirib $-8 \leq x \leq -4$ ni olamiz. 2-sistemaning yechimi tayyor shaklda yozilgan, ularni birlashtirib $[-8; -4] \cup [4; 8]$ yechimni olamiz. Bu to'plamda $-8, -7, -6, -5, -4, 4, 5, 6, 7, 8$ butun sonlari yotadi, ular 10 ta. **Javob:** 10 (B).

2. (99-1-7) Tengsizlikni yeching.

$$1 < |x| < 4$$

- A) $(-\infty; -4) \cup (4; \infty)$ B) $(-4; -1) \cup (1; 4)$
C) $(-\infty; -1) \cup (1; \infty)$ D) $(-1; 1)$

3. (03-5-20) Tengsizlikni yeching.

$$1 < |x - 2| < 3$$

- A) $(-1; 1) \cup (3; 5)$ B) $(-1; 1)$
C) $(3; 5)$ D) $(-1; 5)$

4. (00-3-24) Tengsizlik nechta natural yechimga ega?

$$|x - 3| \leq 6 - x$$

- A) 0 B) 1 C) 2 D) 4

5. (00-6-6) Tengsizlikni yeching.

$$|x^2 - 5| < 4$$

- A) $(-3; 0) \cup (0; 3)$ B) $(-3; 3)$
C) $(-3; -1) \cup (1; 3)$ D) $(-3; -1)$

6. (00-10-70) Tengsizlikni yeching.

$$\frac{|x - 3|}{x^2 - 5x + 6} \geq 2$$

- A) $[\frac{3}{2}; 2)$ B) $[\frac{5}{2}; 4)$ C) \emptyset D) $[-10; 10]$

7. (01-3-7) Tengsizlikni qanoatlantiruvchi butun sonlarning yig'indisini toping.

$$x^2 - 3|x| - 4 \leq 0$$

- A) 0 B) 2 C) 3 D) 1

8. (02-11-23) Tengsizlikning butun yechimlari nechta?

$$|x^2 - 3| < 2$$

- A) 2 B) 3 C) 4 D) 5

9. (03-12-69) Tengsizlikning butun yechimlari nechta?

$$x^2 - 2|x| < 3$$

- A) 7 B) 6 C) 5 D) 4

10. (98-10-66) Tengsizlikning butun yechimlari nechta?

$$2 \cdot |x - 1| \leq |x + 3|$$

- A) 6 B) 5 C) cheksiz ko'p D) 0

Yechish: Tengsizlikni yechishda 2-usuldan foydalanamiz. Tengsizlikning har ikkala qismini kvadratga ko'taramiz va hadlarni tengsizlikning chap qismiga o'tkazamiz:

$$(2x - 2)^2 - (x + 3)^2 \leq 0.$$

Qisqa ko'paytirish formulalarining 3-dan foydalanib, ifodani ko'paytuvchilarga ajratamiz:

$$(2x - 2 + x + 3)(2x - 2 - x - 3) \leq 0$$

$$\iff (3x + 1)(x - 5) \leq 0.$$

Bu tengsizlikni oraliqlar usuli bilan yechib, $[-\frac{1}{3}; 5]$ yechimni olamiz. Bu kesmada 0, 1, 2, 3, 4, 5 butun sonlari yotadi, ular 6 ta. **Javob:** 6 (A).

11. (01-12-17) Tengsizlikni yeching.

$$|x + 1| > 2|x + 2|$$

- A) $(-2; -1)$ B) $[-3; -1]$
 C) $(-3; -\frac{5}{3})$ D) $(-3; 0)$

12. (03-7-22) Tengsizlikni yeching.

$$|x - 4| < |x + 4|$$

- A) $(-4; 4)$ B) $(0; 4) \cup (4; \infty)$
 C) $(0; \infty)$ D) $(-\infty; -4) \cup (-4; 0)$

13. (01-5-24) Tengsizlikni butun yechimlari yig'indisini toping.

$$|5 - 2x| \leq 3$$

- A) 10 B) 15 C) 6 D) 3

14. (02-1-47) Tengsizlikni qanoatlantiruvchi butun sonlar nechta?

$$|3x + 8| \leq 2$$

- A) 1 B) 2 C) 3 D) 4

15. (96-3-26) $|x - 1| \geq 2$ tengsizlikni yeching.

- A) $(-\infty; -1]$ B) $[-1; 3]$
 C) $(-\infty; -1] \cup [3; \infty)$ D) $[1; 3]$

Yechish: Misolni 3-usul, ya'ni geometrik usuldan foydalanib yechamiz. $|x - 1|$ miqdor sonlar o'qida x va 1 nuqtalar orasidagi masofani ifodalaydi. Demak, berilgan tengsizlikning yechimi sonlar o'qida koordinatasi 1 bo'lgan nuqtadan masofasi 2 birlik va undan katta x ning barcha qiymatlaridan iborat (6.3-chizmaga qarang). Sonlar o'qida koordinatasi 1 bo'lgan nuqtadan 2 birlik chapda -1 ni, 2 birlik o'ngda yotuvchi 3 ni, topamiz. Shunday qilib tengsizlikning yechimi $(-1; 3)$ intervalning tashqarisi bo'ladi. Demak, yechim $(-\infty; -1] \cup [3; \infty)$ dan iborat. **Javob:** $(-\infty; -1] \cup [3; \infty)$ (C).

16. (96-7-8) Tengsizlik nechta butun yechimga ega?

$$|x - 2| \leq 5$$

- A) 11 B) 10 C) 8 D) 7

17. (96-11-27) Tengsizlikni yeching.

$$|x - 1| \geq 1$$

- A) $[0; 2]$ B) $(-\infty; 0] \cup [2; \infty)$
 C) $[-2; 0]$ D) $[2; \infty)$

18. (96-12-27) Tengsizlikni yeching.

$$|x - 1| \leq 2$$

- A) $(-\infty; 3]$ B) $(-\infty; -1] \cup [3; \infty)$
 C) $[-1; 3]$ D) $[1; 3]$

19. (97-7-8) Tengsizlik nechta butun yechimga ega?

$$|x + 2| \leq 3$$

- A) 5 B) 6 C) 7 D) 4

20. (97-1-73) Tengsizlikning eng katta natural yechimini toping.

$$|3x - 7| < 5$$

- A) 4 B) 3 C) 2 D) 1

21. (97-3-8) Tengsizlik nechta butun yechimga ega?

$$|3 - x| < 4$$

- A) 4 B) 5 C) 6 D) 7

22. (97-10-8) Tengsizlik nechta butun yechimga ega?

$$|4 - x| < 6$$

- A) 3 B) 5 C) 8 D) 11

23. (98-5-23) Tengsizlikning eng kichik natural yechimini toping.

$$|x - 7| \leq 1$$

- A) 5 B) 7 C) 8 D) 6

24. (99-7-24) Tengsizlikning eng kichik natural yechimini toping.

$$|x - 6| \leq 8$$

- A) 2 B) 7 C) 3 D) 1

25. (99-9-18) Ushbu $|x - 4| \leq 12$ tengsizlikning eng kichik va eng katta butun yechimlari yig'indisini toping.

- A) 6 B) 8 C) -6 D) -8

6.3 Modulli tenglamalar va tengsizliklar sistemasi

1. (98-8-25) b ning qanday qiymatlarida tenglamalar sistemasi yagona yechimga ega?

$$\begin{cases} x = 3 - |y| \\ 2x - |y| = b \end{cases}$$

- A) $b = 0$ B) $b > 0$ C) $b < 1$ D) $b = 6$

Yechish: Agar $(x_0; y_0)$ berilgan sistemaning yechimi bo'lsa, u holda $|y| = |-y|$ tenglikka ko'ra, $(x_0; -y_0)$ ham uning yechimi bo'ladi. Demak, sistema yagona yechimga ega bo'lishi uchun $y_0 = 0$ bo'lishi zarur. Uni sistemaning birinchi tenglamasiga qo'yib $x = 3$ ekanini, ikkinchi tenglamadan esa $b = 6$ ekanini hosil qilamiz. Endi $b = 6$ da berilgan sistemaning yagona yechimga ega ekanligini ko'rsatamiz. Sistemaning birinchi tenglamasida $|y|$ ni tenglamaning chap qismiga o'tkazib, keyin sistema tenglamalarini qo'shsak, $3x = 9$ ni olamiz. Bu tenglama yagona $x = 3$ yechimga ega. $x = 3$ ni sistemaning 1-tenglamasiga qo'yib $y = 0$ ni olamiz. Demak, $b = 6$ da sistema yagona $(3; 0)$ yechimga ega ekan. **Javob:** $b = 6$ (D).

2. (98-1-25) a ning qanday qiymatlarida

$$\begin{cases} 3|x| + y = 2 \\ |x| + 2y = a \end{cases}$$

sistema yagona yechimga ega?

- A) $a = 0$ B) $a > 0$ C) $a = 2$ D) $a = 4$

3. (98-8-23) Agar

$$\begin{cases} x + 2|y| = 3 \\ x - 3y = 5 \end{cases}$$

bo'lsa, $x - y$ ning qiymatini toping.

- A) 3 B) 2 C) 1 D) -1

4. (98-1-23) Agar

$$\begin{cases} |x| + y = 2 \\ 3x + y = 4 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.

- A) 3 B) 1 C) 2,5 D) 2

5. (00-1-18) Tengsizliklar sistemasini yeching.

$$\begin{cases} x \geq 3 \\ |x - 3| \leq 1 \end{cases}$$

- A) $2 \leq x \leq 3$ B) $-2 \leq x \leq 4$
C) $3 \leq x \leq 4$ D) $x \leq 4$

Yechish: 6.2-dagi 1-tengkuchlilikka ko'ra berilgan sistemaning 2-tengsizligi $-1 \leq x - 3 \leq 1$ qo'sh tengsizlikka teng kichli. Bu tengsizlikning har bir qismiga 3 ni qo'shib $2 \leq x \leq 4$ ni olamiz. Sistemaning 1-tengsizligi $x \geq 3$ ni hisobga olsak, $3 \leq x \leq 4$ javobni olamiz. **Javob:** $3 \leq x \leq 4$ (C).

6. (02-10-55) Tengsizliklar sistemasini yeching.

$$\begin{cases} |2x - 3| \leq 1 \\ 5 - 0,4x > 0 \end{cases}$$

- A) $[1; 2]$ B) $(-\infty; 2]$
C) $(-\infty; 1] \cup (2; \infty)$ D) $(-0, 4; 2)$

7. (01-7-20) Tenglamalar sistemasi nechta yechimga ega?

$$\begin{cases} |x| + |y| = 1 \\ x^2 + y^2 = 4 \end{cases}$$

- A) 1 B) 2 C) 4 D) \emptyset

8. (02-12-17) Agar

$$\begin{cases} |x - 1| + |y - 5| = 1 \\ y = 5 + |x - 1| \end{cases}$$

bo'lsa, $x + y$ qanday qiymatlar qabul qilishi mumkin?

- A) 6 yoki 8 B) 7 C) 8 yoki 10 D) 6 yoki 7

9. (00-7-20) Agar

$$\begin{cases} (x - 2)^2 + |y| = 4 \\ |x - 2| + |y| = 2 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.

- A) 4 yoki 2 yoki 0 B) 0 yoki 3
C) 2 yoki 4 D) 0 yoki 4

10. (03-10-31) Agar

$$\begin{cases} |x + y| = 5 \\ xy = 4,75 \end{cases}$$

bo'lsa, son o'qida x va y sonlari orasidagi masofani toping.

- A) $\sqrt{6}$ B) $\sqrt{3}$ C) $\sqrt{5}$ D) $\sqrt{7}$

11. (99-1-7) Qo'sh tengsizlikni yeching.

$$1 < |x| < 4$$

- A) $(-\infty; -4) \cup (4; \infty)$ B) $(-4; -1) \cup (1; 4)$
C) $(-\infty; -1) \cup (1; \infty)$ D) $(-1; 1)$

7 -bob. Irratsional tenglama va tengsizliklar

7.1 Irratsional tenglamalar

Noma'lumi ildiz belgisi ostida bo'lgan tenglamalar *irratsional tenglamalar* deyiladi. Masalan,

$$\sqrt[4]{x-3} = 3 - \sqrt{x}; \quad (5-x)\sqrt{x-3} = 3 - \sqrt[3]{2x}$$

tenglamalar irratsional tenglamalardir. Irratsional tenglamalarni yechish ma'lum bir shakl almashtirishlar yordamida ularni ratsional tenglamalarga keltirishga asoslangan. Radikallardan (ildiz belgisidan) qutilish maqsadida tenglamaning ikkala qismi bir xil darajaga ko'tariladi. Lekin darajaga ko'targanda chet ildizlar hosil bo'lishi mumkin. Shuning uchun oxirgi tenglamani yechishda topilgan ildizlarni, tekshirib ko'rish lozim.

Irratsional tenglamalarni yechishda ko'p qo'llaniladigan quyidagi tengkuchliliklarni keltiramiz.

- $\sqrt[2k]{f(x)} = \varphi(x) \iff \begin{cases} f(x) = [\varphi(x)]^{2k} \\ \varphi(x) \geq 0 \end{cases}$
- $\sqrt[2k+1]{f(x)} = \varphi(x) \iff f(x) = [\varphi(x)]^{2k+1}$.

Irratsional tenglamalarni yechish usullarini misollarda namoyish qilamiz.

1-misol. $\sqrt{x+2} + x = 0$ tenglamani yeching.

Yechish: Tenglamada x ni tenglikning o'ng qismiga o'tkazamiz va ikkala qismini kvadratga ko'taramiz, natijada

$$x+2 = (-x)^2 \iff x^2 - x - 2 = 0$$

kvadrat tenglamaga kelamiz. Bu kvadrat tenglamaning ildizlari $x_1 = -1$ va $x_2 = 2$ lardir. Topilgan yechimlarni berilgan tenglamaga qo'yamiz. Dastlab $x_1 = -1$ ni qo'yamiz: $\sqrt{-1+2} + (-1) = 1 - 1 = 0$. Endi $x_2 = 2$ ni $\sqrt{2+2} + 2 = 2 + 2 = 4 \neq 0$. Demak, $x_2 = 2$ chet ildiz ekan, $x_1 = -1$ esa tenglamani qanoatlantiradi.

Javob: $x = -1$.

2-misol. $\sqrt{x+1} = -7$ tenglamani yeching.

Yechish: Arifmetik ildizning manfiymasligidan $\sqrt{x+1} \geq 0$ ekanligi kelib chiqadi. Tenglamaning o'ng qismi -7 esa manfiy son. Shuning uchun tenglama yechimga ega emas. **Javob:** \emptyset .

Xuddi shunday ko'rsatish mumkinki,

$$|x^2 - 1| + \sqrt{2x+1} = -1$$

irratsional tenglama ham yechimga ega emas.

3-misol. Quyidagi tenglamani yeching:

$$x^2 - 3x + \sqrt{x^2 - 3x + 5} = 7. \quad (7.1)$$

Yechish: Tenglamada $x^2 - 3x = t$ belgilash olamiz, natijada berilgan tenglama $t + \sqrt{t+5} = 7$ ko'rinishga ega bo'ladi. Bu tenglamada t ni tenglikning o'ng qismiga o'tkazib, keyin tenglikning har ikkala qismini kvadratga ko'tarib, $t+5 = (7-t)^2$ kvadrat tenglamaga kelamiz. Bu kvadrat tenglamani yechib $t_1 = 4$, $t_2 = 11$ ni olamiz. $t_2 = 11$ ildiz $t + \sqrt{t+5} = 7$ irratsional

tenglamani qanoatlantirmaydi. Shuning uchun faqat $t_1 = 4$ qiymatni $x^2 - 3x = t$ belgilashga qo'yib,

$$x^2 - 3x = t_1 \iff x^2 - 3x = 4$$

kvadrat tenglamaga ega bo'lamiz. Bu kvadrat tenglamaning ildizlari $x_1 = -1$, $x_2 = 4$ lardir. Tekshirish mumkinki, ular (7.1) tenglamani ham qanoatlantiradi. **Javob:** $x_1 = -1$, $x_2 = 4$.

Eslatma. Bu tenglamada $\sqrt{x^2 - 3x + 5} = t$ belgilash olib, uni $t^2 + t - 12 = 0$ tenglamaga keltirib yechish ham mumkin edi.

- (98-9-19) Tenglama ildizlari yig'indisini toping.

$$\sqrt{x^4 + 5x^2} = -3x$$

- A) 0 B) -2 C) -4 D) 2

Yechish: 1-tengkuchlilikka ko'ra berilgan tenglama

$$\begin{cases} x^4 + 5x^2 = 9x^2 \\ -3x \geq 0 \end{cases}$$

sistemaga teng kuchli. Uning birinchi tenglamasini

$$x^4 - 4x^2 = 0 \iff x^2(x^2 - 4) = 0$$

shaklda yozib olamiz. Bu tenglamaning yechimlari $x_1 = 0$, $x_2 = -2$, $x_3 = 2$ lardir. Endi $-3x \geq 0$ shartni tekshiramiz. Uni faqat $x_1 = 0$ va $x_2 = -2$ sonlar qanoatlantiradi. Shunday qilib, berilgan tenglamaning ildizlari $x_1 = 0$ va $x_2 = -2$ ekan. Ular yig'indisi $0 + (-2) = -2$. **Javob:** -2 (B).

- (97-1-72) Tenglamani yeching.

$$\sqrt{x+2} + x = 0$$

- A) -1 B) -2 C) 2 D) 0

- (97-5-39) Tenglamalar sistemasini yeching.

$$\begin{cases} \sqrt{(x+2)^2} = x+2 \\ \sqrt{(x-2)^2} = 2-x \end{cases}$$

- A) $x \geq -2$ B) $x < 2$
C) $x \leq 2$ D) $-2 \leq x \leq 2$

- (97-7-61) Ushbu $\sqrt{3+2x} = -x$ tenglik x ning qanday qiymatlarida o'rinni?
A) -1 B) 1 C) -3 D) 3

- (98-2-21) Agar $\sqrt{x^4 - 9x^2} = -4x$ tenglamaning katta ildizi x_0 bo'lsa, $x_0 + 10$ nechaga teng?
A) 10 B) 12 C) 20 D) 15

Yechish: Berilgan tenglamaning har ikkala tomonini kvadratga ko'tarib, o'ng tomondagi hadni chap tomonga o'tkazib,

$$x^4 - 25x^2 = 0 \iff x^2(x^2 - 25) = 0$$

tenglamani olamiz. Bu tenglamaning ildizlari $x_1 = -5$, $x_2 = 0$, $x_3 = 5$ lardir. Lekin $x_3 = 5$ dastlabki tenglamani qanoatlantirmaydi. Shuning uchun tenglamaning katta ildizi $x_2 = 0$ bo'ladi. $x_0 + 10 = 0 + 10 = 10$. **Javob:** 10 (A).

6. (99-2-19) Tenglamani yig'indisini toping.

$$\sqrt{x^2 - 3x + 5} + x^2 - 3x = 7$$

- A) 4 B) -3 C) 3 D) -4

7. (99-6-41) $\sqrt{a} - \sqrt{b} = 4$ va $a - b = 24$ bo'lsa, $\sqrt{a} + \sqrt{b}$ nimaga teng.

- A) 6 B) 4 C) 5 D) 3

8. (99-5-15) Tenglamani natural ildizlari nechta?

$$\sqrt{(3x - 13)^2} = 13 - 3x$$

- A) 0 B) 1 C) 2 D) 4

9. (99-8-3) Tenglamani yeching.

$$\sqrt{x+1} + \sqrt{2x+3} = 1$$

- A) -1 B) 3 C) -1; 3 D) 1

10. (99-9-11) Tenglama ildizlari o'rta arifmetigini toping.

$$(x^2 - 25)(x - 3)(x - 6)\sqrt{4 - x} = 0$$

- A) $4\frac{1}{3}$ B) $1\frac{1}{3}$ C) $\frac{2}{3}$ D) $4\frac{1}{2}$

11. (99-9-12) Tenglama ildizlari ko'paytmasini toping.

$$\sqrt{x^2 + 77} - 2\sqrt[4]{x^2 + 77} - 3 = 0$$

- A) -3 B) 3 C) 4 D) -4

Yechish: Tenglamada $\sqrt[4]{x^2 + 77} = t$ belgilash olamiz, natijada berilgan tenglama $t^2 - 2t - 3 = 0$ ko'rinishga ega bo'ladi. Bu kvadrat tenglama yechimlari $t_1 = -1$, $t_2 = 3$ lardir. Yechimlarni $\sqrt[4]{x^2 + 77} = t$ belgilashga qo'yib, $\sqrt[4]{x^2 + 77} = -1$ va $\sqrt[4]{x^2 + 77} = 3$ tenglamalarni hosil qilamiz. Ammo $\sqrt[4]{x^2 + 77} = -1$ tenglama yechimga ega emas (chunki tenglamani chap qismi musbat, o'ng qismi esa manfiy). Ikkinchi tenglamani har ikkala qismini 4-darajaga ko'tarib,

$$x^2 + 77 = 3^4 \iff x^2 = 4$$

tenglamaga ega bo'lamiz. Bu tenglamani yechimlari $x_1 = -2$, $x_2 = 2$ lardir. Tekshirish natijasi ko'rsatadiki, $x_1 = -2$, $x_2 = 2$ lar dastlabki tenglamani qanoatlantiradi. Ularning ko'paytmasi $(-2) \cdot 2 = -4$. **Javob:** -4 (D).

12. (00-1-19) Agar

$$\sqrt{1 - \frac{1}{x}} = \frac{x-1}{x} - 6$$

bo'lsa, $6\frac{1}{8} + x$ ning qiymatini hisoblang.

- A) -7 B) 6 C) 7 D) -6

13. (00-2-19) Tenglamani yeching.

$$\sqrt{(2x-1)^2(3-x)} = (2x-1)\sqrt{3-x}$$

- A) $[0, 5; 3]$ B) $[0; 3]$ C) $[1; 3]$ D) $(-\infty; 0, 5]$

Yechish: Tenglama $x \leq 3$ da ma'noga ega. Demak, berilgan tenglama

$$\begin{cases} \sqrt{(2x-1)^2(3-x)} = (2x-1)\sqrt{3-x} \\ x \leq 3 \end{cases}$$

sistemaga teng kuchli. Uning birinchi tenglamasi $x \leq 3$ shartda

$$|2x-1|\sqrt{3-x} - (2x-1)\sqrt{3-x} = 0$$

tenglamaga teng kuchli. Bu tenglama esa

$$\sqrt{3-x} \cdot (|2x-1| - (2x-1)) = 0$$

ga teng kuchli. Bu tenglama yechimlari esa $\sqrt{3-x} = 0$ hamda $|2x-1| = 2x-1$ tenglama yechimlari bilan ustma-ust tushadi. 1-tenglamadan $x = 3$ ni olamiz. 2-tenglama 6.1-ning 1-tengkuhlliligiga ko'ra $2x-1 \geq 0$ ga teng kuchli. Bu tengsizlikning yechimi $x \geq 0,5$ dan iborat. Agar $x \leq 3$ shartni hisobga olsak, $x \in [0, 5; 3]$ ni olamiz. **Javob:** $[0, 5; 3]$ (A).

14. (00-3-10) Tenglamani yeching.

$$3\sqrt{2x} - 5\sqrt{8x} + 7\sqrt{18x} = 28$$

- A) 1 B) 2 C) 3 D) 4

15. (00-3-22) Tenglama ildizlari yig'indisini toping.

$$\sqrt{x+1} + \sqrt{2x+3} = 1$$

- A) 2 B) 3 C) 4 D) -1

16. (00-4-7) Tenglamani yeching.

$$\frac{2\sqrt{x} - \sqrt{2x}}{2} + 3 = \sqrt{x} + 1$$

- A) 8 B) 4 C) 9 D) 1

17. (00-5-29) Tenglamani yeching.

$$\sqrt{x^2 - x - 2} = x - 3$$

- A) 5 B) 2,2 C) 4 D) \emptyset

18. (00-6-33) Agar

$$\sqrt{3x^2 - 6x + 16} = 2x - 1$$

bo'lsa, $x^2 \cdot (x+2)$ ning qiymatini toping.

- A) -75 B) -45 C) 15 D) 45

19. (00-8-5) Tenglamani yeching.

$$(x^2 - 9)\sqrt{x+1} = 0$$

- A) -1; 3 B) ± 3 C) $\pm 3; 1$ D) 2

20. (00-8-25) Agar

$$\sqrt{8-a} + \sqrt{5+a} = 5$$

bo'lsa, $\sqrt{(8-a)(5+a)}$ ning qiymatini toping.

A) 6 B) 20 C) 12 D) 10

Yechish: Tenglamani ikkala qismini kvadratga ko'taramiz, natijada

$$8 - a + 2\sqrt{8-a}\sqrt{5+a} + a + 5 = 25$$

$$\iff 2\sqrt{(8-a)(5+a)} = 12$$

tenglikni olamiz. Bu yerdan $\sqrt{(8-a)(5+a)} = 6$ ni olamiz. **Javob:** 6 (A).

21. (00-8-26) Agar

$$\sqrt{25-x^2} + \sqrt{15-x^2} = 5$$

bo'lsa, $\sqrt{25-x^2} - \sqrt{15-x^2}$ ifodaning qiymatini toping.

A) 2 B) 3 C) 5 D) 6

22. (00-9-31) Agar $\sqrt[4]{ab} = 2\sqrt{3}$ va $a, b \in N$ bo'lsa, $a - b$ quyida keltirilgan qiymatlardan qaysi birini qabul qila olmaydi?

A) -32 B) 10 C) 0 D) 25

23. (97-5-26) Agar

$$\begin{cases} \sqrt{x} + \sqrt{y} = 3 \\ \sqrt{xy} = 2 \end{cases}$$

bo'lsa, $x + y$ ni toping.

A) 2 B) 3 C) 4 D) 5

24. (01-1-19) Agar

$$\begin{cases} x - y = 21 \\ \sqrt{x} - \sqrt{y} = 3 \end{cases}$$

bo'lsa, $x + y$ ning qiymatini toping.

A) 7 B) 12 C) 23 D) 29

25. (01-2-24) Tenglama ildizlari o'rta arifmetigini toping.

$$x - 5\sqrt{x} + 4 = 0$$

A) 16 B) 8,5 C) 3 D) 2

26. (01-5-9) Tenglama ildizlari yig'indisini toping.

$$(x^2 - 4)\sqrt{x+1} = 0$$

A) 1 B) -1 C) 3 D) 2

Yechish: Tenglamani chap qismi $x \geq -1$ da ma'noga ega. Ko'paytmaning nolga tenglik shartidan $x^2 - 4 = 0$ yoki $x + 1 = 0$ shartlar (tenglamalar) kelib chiqadi Ularning yechimlari $x_1 = -2$, $x_2 = 2$ va $x_3 = -1$ lardir. $x_1 = -2$ yechim $x \geq -1$ shartni qanoatlantirmaydi. Demak, $x_2 = 2$ va $x_3 = -1$ lar berilgan tenglamani yechimi bo'ladi. Ularning yig'indisi $2 + (-1) = 1$. **Javob:** 1 (A).

27. (01-6-25) Agar $\sqrt{x+1} + x - 11 = 0$ bo'lsa, $x + 12$ ning qiymatini toping.

A) 15 B) 16 C) 20 D) 19

28. (01-9-12) Ushbu

$$\begin{cases} y = \sqrt{16-x^2} \\ y - x = 4 \end{cases}$$

tenglamalar sistemasining nechta yechimi bor?

A) 2 B) 1 C) 0 D) 3

29. (01-10-20) Tenglama nechta ildizga ega?

$$\frac{x-9}{\sqrt{x+3}} = x-15$$

A) 0 B) 1 C) 2 D) 3

30. (03-7-20) Tenglamani yeching.

$$\sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \dots}}} = 8$$

A) 56 B) 48 C) 60 D) 64

Yechish: Tenglamada ko'paytmalar cheksiz ko'p bo'lganligi uchun uni quyidagicha yozamiz:

$$\sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \sqrt[3]{x \dots}}} = 8 \iff \sqrt[3]{x \cdot 8} = 8.$$

Bu yerdan $8x = 8^3$ yoki $x = 64$ ni olamiz. **Javob:** 64 (D).

31. (01-12-43) Tenglamani yeching.

$$\sqrt{3x-7} - \sqrt{7-3x} = 0$$

A) 2,3 B) $\frac{3}{7}$ C) $\frac{7}{3}$ D) \emptyset

32. (02-1-8) Tenglama nechta ildizga ega?

$$\sqrt{x^2+1} - \sqrt{x^2-1} = 1$$

A) \emptyset B) 1 C) 2 D) 3

33. (03-8-38) Tenglamani yeching.

$$\sqrt{x} + \sqrt[4]{x} - 12 = 0$$

A) 81 B) 16 C) 25 D) 9

7.2 Irratsional tengsizliklar

Irratsional tengsizliklarni yechishda quyidagi tengkuchliklardan foydalaniladi.

$$1. \sqrt[2k]{f(x)} > \varphi(x) \iff \begin{cases} f(x) > [\varphi(x)]^{2k} \\ \varphi(x) \geq 0 \end{cases}$$

$$\cup \begin{cases} f(x) \geq 0 \\ \varphi(x) < 0. \end{cases}$$

$$2. \sqrt[2k+1]{f(x)} > \varphi(x) \iff f(x) > [\varphi(x)]^{2k+1}.$$

$$3. \sqrt[2k]{f(x)} < \varphi(x) \iff \begin{cases} f(x) \geq 0 \\ \varphi(x) \geq 0 \\ f(x) < [\varphi(x)]^{2k}. \end{cases}$$

$$4. \sqrt[2k+1]{f(x)} < \varphi(x) \iff f(x) < [\varphi(x)]^{2k+1}.$$

1. (97-10-34) Tengsizlikning yechimini ko'rsating.

$$(x-1)\sqrt{6+x-x^2} \leq 0$$

- A) $(-\infty; 1]$ B) $[-2; 3]$
 C) $[-2; 1] \cup \{3\}$ D) $[3; \infty)$

Yechish: Berilgan tengsizlikni yechish uchun ikki holni qaraymiz. 1) $6+x-x^2 = 0$. Uning ildizlari $x_1 = -2$, $x_2 = 3$. Bu sonlar berilgan tengsizlikning ham yechimi bo'ladi.

$$2) \begin{cases} 6+x-x^2 > 0 \\ x-1 \leq 0. \end{cases}$$

Birinchi tengsizlikni -1 ga ko'paytirib, uning chap qismini ko'paytuvchilarga ajratamiz:

$$\begin{cases} (x+2)(x-3) < 0 \\ x-1 \leq 0. \end{cases}$$

Bu holda $(-2; 1]$ yechimni hosil qilamiz. Endi 1-holda topilgan $x_1 = -2$, $x_2 = 3$ yechimlarni hisobga olib berilgan tengsizlikning $[-2; 1] \cup \{3\}$ yechimini hosil qilamiz. **Javob:** $[-2; 1] \cup \{3\}$ (C).

2. (96-7-34) Tengsizlikning yechimini ko'rsating.

$$(x+3)\sqrt{x^2-x-2} \geq 0$$

- A) $[-3; \infty)$ B) $[-1; 2]$
 C) $[-3; -1] \cup [2; \infty)$ D) $[2; \infty)$

3. (97-3-34) Quyidagilardan qaysi biri

$$(x-3)\sqrt{x^2+x-2} \leq 0$$

tengsizlikning yechimi?

- A) $(-\infty; 3]$ B) $(-\infty; -2] \cup [1; 3]$
 C) $[-2; 3]$ D) $[-1; 2] \cup [3; \infty)$

4. (97-7-34) Tengsizlikning yechimini ko'rsating.

$$(x-2)\sqrt{3+2x-x^2} \geq 0$$

- A) $[2; \infty)$ B) $[-1; 3]$ C) $[3; \infty)$ D) $[2; 3] \cup \{-1\}$

5. (00-3-21) Tengsizlikni yeching.

$$\sqrt{3x+10} > \sqrt{6-x}$$

- A) $[-1; 6]$ B) $[-\frac{10}{3}; 6]$
 C) $(-1; 6]$ D) $[-\frac{10}{3}; -1) \cup (-1; 6]$

6. (01-10-19) Tengsizlikni yeching.

$$\sqrt{3x-8} > \sqrt{5-x}$$

- A) $(3, 25; \infty)$ B) $(\frac{8}{3}; 5)$
 C) $(3, 25; 5]$ D) $(3, 25; 5)$

7. (01-12-46) Tengsizlikni yeching.

$$\sqrt{5x-2x^2-42} > 3$$

- A) $\{-2\}$ B) $\{1\}$ C) $\{2\}$ D) \emptyset

8. (02-1-48) Tengsizlikni yeching.

$$\sqrt{x+1} < 4$$

- A) $(-\infty; 15)$ B) $[0; 15]$ C) $[0; 15)$ D) $[-1; 15]$

9. (02-1-68) Tengsizlikni yeching.

$$(x+3)\sqrt{10-3x-x^2} \geq 0$$

- A) $[-3; \infty)$ B) $[2; \infty)$
 C) $[-3; 2]$ D) $\{-5\} \cup [-3; 2]$

10. (02-10-12) Tengsizlikni yeching.

$$\frac{\sqrt{6+x-x^2}}{2x+5} \geq \frac{\sqrt{6+x-x^2}}{x+4}$$

- A) $[-2; -1] \cup \{3\}$ B) $[-2; 1]$ C) $[1; 3]$ D) $[-2; 3]$

11. (98-4-23) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x+2} > x$$

- A) 3 B) 2 C) 4 D) 1

Yechish: Berilgan tengsizlikni yechishda 1-teng-kuchlilikdan foydalanamiz:

$$\begin{cases} x+2 > x^2 \\ x \geq 0 \end{cases} \cup \begin{cases} x+2 \geq 0 \\ x < 0. \end{cases}$$

1-sistemani yechamiz. Uning 1-tengsizligi

$$x^2 - x - 2 < 0 \iff (x+1)(x-2) < 0$$

kvadrat tengsizlikka teng kuchli bo'lib, uning yechimi $(-1; 2)$ dan iborat. Agar sistemaning ikkinchi tengsizligi $x \geq 0$ ni hisobga olsak, 1-sistemaning yechimi $[0; 2)$ ekanligini olamiz.

2-sistemani quyidagicha yozib olamiz:

$$\begin{cases} x+2 \geq 0 \\ x < 0 \end{cases} \iff \begin{cases} x \geq -2 \\ x < 0 \end{cases} \iff -2 \leq x < 0.$$

Bu qo'sh tengsizlik 2-sistemaning yechimi $[-2; 0)$ ekanligini bildiradi. Bu yechimlarni birlashtirib, berilgan tengsizlikning yechimi $[-2; 2)$ ekanligini olamiz. Bu yarim intervalda $-2, -1, 0, 1$ butun sonlar bor. Ular 4 ta. **Javob:** 4 (C).

12. (98-12-82) Tengsizlikni qanoatlantiruvchi butun sonlar nechta?

$$\sqrt{5-x^2} > x-1$$

- A) 5 B) 3 C) 4 D) 2

13. (99-2-20) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x^2-6x+9} < 3$$

- A) 4 B) 6 C) 7 D) 5

14. (00-2-15) Tengsizlikning eng kichik butun musbat yechimini toping.

$$\frac{\sqrt{x+5}}{1-x} < 1$$

- A) 6 B) 3 C) 5 D) 2

15. (00-7-23) Tengsizlikning eng katta butun va eng kichik butun yechimlari ayirmasini toping.

$$\sqrt{x^2 - 16} < \sqrt{4x + 16}$$

- A) 4 B) 5 C) 2 D) 3

16. (01-5-23) Tengsizlikning nechta butun yechimi bor?

$$\sqrt{\frac{3x-4}{8-x}} > 1$$

- A) 4 B) 1 C) 2 D) 3

17. (02-4-26) Tengsizlik nechta butun yechimga ega?

$$\sqrt{x-50} \cdot \sqrt{100-x} > 0$$

- A) 43 B) 54 C) 49 D) 51

Yechish: Arifmetik ildizning manfiy masligidan berilgan tengsizlik

$$\begin{cases} x-50 > 0 \\ 100-x > 0 \end{cases} \iff \begin{cases} x > 50 \\ x < 100 \end{cases}$$

sistemaga teng kuchli bo'ladi. Uning yechimi (50; 100) oraliqdan iborat. Bu oraliqda 49 ta butun son bor. **Javob:** 49 (C).

18. (01-6-26) Tengsizlikning butun yechimlari yig'indisini toping.

$$\frac{\sqrt{2x+7}}{6-3x} \geq 0$$

- A) -4 B) -3 C) 4 D) -5

19. (02-9-26) Tengsizlikning eng kichik va eng katta butun yechimlari ayirmasini toping.

$$x - 4\sqrt{x} - 5 \leq 0$$

- A) -25 B) -24 C) -27 D) -5

20. (02-9-28) Tengsizlikning butun yechimlari nechta?

$$\frac{\sqrt{3+2x-x^2}}{x-2} \leq 0$$

- A) 3 B) 4 C) 5 D) 2

21. (02-12-14) Tengsizlikning butun yechimlari nechta?

$$\frac{5-\sqrt{x}}{\sqrt{x}-2} > 0$$

- A) 20 B) 19 C) 21 D) 2

22. (02-12-35) Qanday eng kichik butun son

$$\sqrt{12-x} < 2$$

tengsizlikni qanoatlantiradi?

- A) 8 B) 9 C) 6 D) 10

23. (03-1-8) Tengsizlikning eng kichik butun yechimini toping.

$$\sqrt{\frac{2-3x}{x+4}} > -2$$

- A) 0 B) -1 C) -2 D) -3

Yechish: Arifmetik ildizning manfiy masligini hisobga olsak,

$$\frac{2-3x}{x+4} \geq 0$$

tengsizlikni yechish kifoya. Bu tengsizlikni oraliqlar usuli yordamida yechib, $(-4; 2/3]$ ni olamiz. Bu yarim intervaldagi eng kichik butun son -3 dir. **Javob:** -3 (D).

24. (03-1-30) Tengsizlikni qanoatlantiruvchi butun sonlarning yig'indisini toping.

$$\sqrt{x} \geq x - 6$$

- A) 6 B) 15 C) 28 D) 45

25. (03-3-20) Tengsizlikning butun yechimlari nechta?

$$\sqrt{x-4} - \sqrt{x-7} \geq 1$$

- A) 0 B) 1 C) 2 D) 4

26. (03-3-30) Tengsizlikning butun yechimlari nechta?

$$\sqrt{5-|2x-1|} < 2$$

- A) 2 B) 3 C) 4 D) 6

27. (03-8-37) Tengsizlikning eng kichik natural yechimini toping.

$$\sqrt{x^2 - 3x + 2} \geq 0$$

- A) 1 B) 2 C) 3 D) 5

28. (03-9-9) Tengsizlikning butun yechimlari nechta?

$$\sqrt{\frac{x^2-2}{x}} \leq 1$$

- A) \emptyset B) 1 C) 2 D) 3

29. (03-11-73) Tengsizlikning butun yechimlari nechta?

$$\sqrt{8+2x-x^2} > 6-3x$$

- A) 2 B) 3 C) 4 D) 5

8 -bob. Progressiyalar

Har bir natural $n \in N$ soniga aniq bir haqiqiy a_n soni mos qo'yilgan bo'lsa, $a_1, a_2, a_3, \dots, a_n, \dots$ sonlar ketma-ketligi berilgan deyiladi. Sonlar ketma-ketligi $\{a_n\}$ shaklda ham yoziladi. Agar ixtiyoriy $n \in N$ uchun $a_n < a_{n+1}$ bo'lsa, $\{a_n\}$ ketma-ketlik o'suvchi deyiladi. Agar ixtiyoriy $n \in N$ uchun $a_n > a_{n+1}$ bo'lsa, $\{a_n\}$ ketma-ketlik kamayuvchi deyiladi.

8.1 Arifmetik progressiya

Agar sonlar ketma-ketligida ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadga shu ketma-ketlik uchun o'zgarmas bo'lgan biror d sonni qo'shish natijasida hosil bo'lsa, bunday sonli ketma-ketlik arifmetik progressiya deyiladi. Masalan, 1) 1, 2, 3, 4, ...; 2) 10, 12, 14, 16, ... ketma-ketliklar arifmetik progressiya tashkil qiladi. Chunki ketma-ketliklarning har bir hadi, ikkinchisidan boshlab o'zidan oldingi hadga, mos ravishda 1 va 2 sonlarini qo'shish natijasida hosil bo'ladi.

Arifmetik progressiyani tashkil qiluvchi sonlar uning hadlari deyiladi va umumiy ko'rinishda

$$a_1, a_2, a_3, \dots, a_{n-1}, a_n, \dots \quad (8.1)$$

yoziqladi. Arifmetik progressiyaning keyingi hadini hosil qilish uchun oldingi hadiga qo'shiladigan d son arifmetik progressiya ayirmasi deyiladi. Agar $d > 0$ bo'lsa, progressiya o'suvchi, $d < 0$ bo'lsa, progressiya kamayuvchi bo'ladi. Agar $d = 0$ bo'lsa, arifmetik progressiyaning barcha hadlari o'zaro teng bo'ladi.

Arifmetik progressiyaning n - hadi a_n quyidagi formula yordamida topiladi: $a_n = a_1 + (n - 1)d$.

Arifmetik progressiya hadlarining xossalari.

1-xossa. Arifmetik progressiyaning ikkinchi hadidan boshlab istalgan hadi o'ziga qo'shni bo'lgan ikki hadning o'rta arifmetik qiymatiga teng, ya'ni

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}.$$

2-xossa. Chekli arifmetik progressiyada boshidan va oxiridan teng uzoqlikda to'rgan hadlar yig'indisi chetki hadlar yig'indisiga teng, ya'ni

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots = a_k + a_{n-k+1}.$$

3-xossa. Arifmetik progressiyaning dastlabki n ta hadlari yig'indisi $a_1 + a_2 + \dots + a_{n-1} + a_n$ ni S_n bilan belgilaymiz. Arifmetik progressiyaning dastlabki n ta hadlari yig'indisi S_n chetki hadlar yig'indisining yarmi bilan hadlar soni ko'paytmasiga teng, ya'ni

$$S_n := a_1 + a_2 + \dots + a_{n-1} + a_n = \frac{a_1 + a_n}{2}n.$$

Arifmetik progressiya xossalarini jamlab, ularni quyidagi tartibda keltiramiz.

1. $a_n = a_1 + (n - 1)d$; $a_n = a_{n-1} + d$.

2. $a_n - a_m = (n - m)d$.

3. $a_n = \frac{a_{n-1} + a_{n+1}}{2} = \frac{a_{n-k} + a_{n+k}}{2}$, $1 \leq k < n$.

4. $a_k + a_m = a_p + a_q$, $k + m = p + q$.

5. $S_n = \frac{a_1 + a_n}{2}n$, $S_n = \frac{2a_1 + d(n - 1)}{2}n$.

6. $S_n - S_{n-1} = a_n$.

1. Arifmetik progressiyada $a_1 = 5$, $d = 2$ bo'lsa, a_7 ni toping.

A) 12 B) 18 C) 17 D) 10

Yechish: 1-xossadan $a_7 = a_1 + 6d$ ni olamiz. a_1 va d lar o'rniga berilganlarni qo'yib, $a_7 = 5 + 6 \cdot 2 = 5 + 12 = 17$. **Javob:** 17 (C) .

2. Arifmetik progressiyada $a_1 = 3$, $d = 4$ bo'lsa, a_9 ni toping.

A) 36 B) 35 C) 39 D) 34

3. Arifmetik progressiyada $a_2 = 5$, $a_3 = 8$ bo'lsa, shu progressiyaning ayirmasini toping.

A) 2 B) 3 C) 5 D) 1,6

4. Agar arifmetik progressiyada $a_5 = 16$, $d = 5$ bo'lsa, a_9 ni toping.

A) 36 B) 35 C) 39 D) 34

5. Agar arifmetik progressiyada $a_3 = 5$, $a_9 = 25$ bo'lsa, a_6 ni toping.

A) 16 B) 15 C) 19 D) 14

6. Agar arifmetik progressiyada $a_1 + a_9 = 20$ bo'lsa, $a_7 + a_3$ ni toping.

A) 16 B) 15 C) 20 D) 25

7. Agar arifmetik progressiyada $a_9 - a_1 = 32$ bo'lsa, d ni toping.

A) 6 B) 5 C) 2 D) 4

8. (96-9-78) Arifmetik progressiyada $a_4 - a_2 = 4$ va $a_7 = 14$. Shu progressiyaning beshinchi hadini toping.

A) 12 B) 8 C) 7 D) 10

Yechish: 2-xossadan hamda birinchi shartdan $2d = 4$ ni olamiz. Yana 2-xossadan foydalanamiz:

$$a_7 - a_5 = 2d \iff a_5 = a_7 - 2d = 14 - 4 = 10.$$

Javob: $a_5 = 10$ (D) .

9. (96-1-27) Arifmetik progressiyada $a_2 = 12$ va $a_5 = 3$. Shu progressiyaning o'ninchi hadini toping.

A) -6 B) 0 C) -12 D) -30

10. (98-12-36) Arifmetik progressiya uchun quyidagi formulalardan qaysilari to'g'ri?

1) $a_1 - 2a_2 + a_3 = 0$ 2) $a_1 = a_3 - a_2$

3) $n = \frac{a_n - a_1 + d}{d}$

A) 1; 3 B) 1 C) 2 D) 1; 2

11. (99-1-22) Arifmetik progressiyada $a_{20} = 0$ va $a_{21} = -41$ bo'lsa, a_1 ni toping.

A) 779 B) -779 C) 41 D) -41

12. (99-9-26) Arifmetik progressiyada $a_2 - a_1 = 6$ bo'lsa, $a_8 - a_6$ ni toping.
A) 10 B) 12 C) 9 D) 18
13. (00-5-32) Arifmetik progressiyada $a_2 = 9$ va $a_{26} = 105$ bo'lsa, shu progressiya birinchi hadi va ayirmasining o'rta proporsional qiymatini toping.
A) 20 B) 4,5 C) $2\sqrt{5}$ D) 9
14. (00-10-22) 4; 9; 14; ... arifmetik progressiyaning sakkizinchi hadi to'rtinchi hadidan nechtaga ortiq?
A) 16 B) 18 C) 20 D) 22
15. (02-4-16) Arifmetik progressiyada $a_1 = 3$ va $d = 2$ bo'lsa, $a_1 - a_2 + a_3 - a_4 + \dots + a_{25} - a_{26} + a_{27}$ ning qiymatini hisoblang.
A) 31 B) 30 C) 29 D) 28
16. (02-9-18) $-\frac{1}{4}; -\frac{1}{5}; \dots$ arifmetik progressiyaning nechta hadi manfiy?
A) 10 B) 6 C) 5 D) 7
- Yechish:** Demak, $a_1 = -\frac{1}{4}$, $a_2 = -\frac{1}{5}$. U holda progressiyaning ayirmasi $d = -\frac{1}{5} + \frac{1}{4} = \frac{1}{20}$. 1-xosadan $a_n = -\frac{1}{4} + (n-1)\frac{1}{20}$ ekanligi kelib chiqadi. Masala shartiga ko'ra $a_n = -\frac{1}{4} + (n-1)\frac{1}{20} < 0$ tengsizlikning yechimi bo'lgan eng katta natural sonni topishimiz kerak. Bu chiziqli tengsizlikning yechimi $n < 6$ dir. Demak, arifmetik progressiyaning 5 ta hadi manfiy ekan. **Javob:** 5 (C).
17. (02-11-38) Arifmetik progressiyaning to'rtinchi hadi va o'n birinchi hadlari mos ravishda 2 va 30 ga teng. Shu progressiyaning uchinchi va o'ninchi hadlari yig'indisini toping.
A) 16 B) 18 C) 24 D) 28
18. (03-2-67) Kinoteatrning birinchi qatorida 21 ta o'rin bor. Har bir keyingi qatorda o'rinlar soni oldingi qatordagidan 2 tadan ko'p. 40 - qatorda nechta o'rin bor?
A) 42 B) 80 C) 99 D) 100
19. (03-3-36) Arifmetik progressiyada $a_2 + a_5 - a_3 = 10$ va $a_1 + a_6 = 17$ bo'lsa, uning o'ninchi hadini toping.
A) 24 B) 26 C) 28 D) 29

3-4 xossalarga oid misollar

20. (97-4-27) Arifmetik progressiyaning dastlabki 6 ta hadlari 7, a_2, a_3, a_4, a_5 va 22 bo'lsa, $a_2 + a_3 + a_4 + a_5$ ni hisoblang.
A) 65 B) 60 C) 82 D) 58

Yechish. Shartga ko'ra $a_1 = 7$, $a_6 = 22$ ekan. Arifmetik progressiyaning 4-xossasiga ko'ra

$$a_2 + a_5 = a_3 + a_4 = a_1 + a_6 = 7 + 22 = 29.$$

U holda $a_2 + a_3 + a_4 + a_5 = 29 + 29 = 58$.

Javob: 58 (D).

21. (97-12-36) Ikkinchi, to'rtinchi va oltinchi hadlarining yig'indisi -18 ga teng arifmetik progressiyaning to'rtinchi hadini toping.
A) 6 B) -5 C) -6 D) -4
22. (98-3-20) Birinchi hadi 1 ga, o'n birinchi hadi 13 ga teng bo'lgan arifmetik progressiyaning oltinchi hadini toping.
A) 4 B) 5 C) 6 D) 7
23. (98-10-67) Ikkinchi hadi 5 ga sakkizinchi hadi 15 ga teng bo'lgan arifmetik progressiyaning beshinchi hadini toping.
A) 7,5 B) 12,5 C) 10 D) 8,5

Yechish. Shartga ko'ra $a_2 = 5$, $a_8 = 15$. Arifmetik progressiyaning 3-xossasiga ko'ra

$$\frac{a_2 + a_8}{2} = a_5 \iff \frac{5 + 15}{2} = 10 = a_5.$$

Javob: 10 (C).

24. (02-1-40) Uchta sonning o'rta arifmetigi 2,6 ga, birinchi son esa 2,4 ga teng. Agar keyingi har bir son avvalgisidan ayni bir songa farq qilsa, keyingi sondan oldingisining ayirmasini toping.
A) $\frac{1}{3}$ B) 0,1 C) $\frac{1}{4}$ D) 0,2
25. (02-5-29) Arifmetik progressiyaning birinchi va to'rtinchi hadi yig'indisi 26 ga teng, ikkinchi hadi esa beshinchi hadidan 6 ga ko'p. Shu progressiyaning uchinchi va beshinchi hadi yig'indisini toping.
A) 20 B) 21 C) 22 D) 23
26. (08-103-27) Arifmetik progressiyaning to'rtinchi va o'n birinchi hadlari mos ravishda 15 va 43 ga teng. Shu progressiyaning uchinchi va o'ninchi hadi yig'indisini toping.
A) 68 B) 60 C) 50 D) 24

Dastlabki n ta hadi yig'indisiga oid misollar

27. (96-3-27) Arifmetik progressiya uchinchi va to'qqizinchi hadlarining yig'indisi 8 ga teng. Shu progressiyaning dastlabki 11 ta hadi yig'indisini toping.
A) 22 B) 33 C) 44 D) 55

Yechish. Shartga ko'ra $a_3 + a_9 = 8$. 4-xossaga ko'ra $a_1 + a_{11} = a_3 + a_9 = 8$. 5-xossaga ko'ra

$$S_{11} = \frac{a_1 + a_{11}}{2} \cdot 11 = \frac{8}{2} \cdot 11 = 44.$$

Javob: 44 (C).

28. (96-11-28) Arifmetik progressiyada $a_3 + a_5 = 12$. S_7 ni toping.
A) 18 B) 36 C) 42 D) 48
29. (96-12-28) Arifmetik progressiyada $a_4 + a_6 = 10$. S_9 ni toping.
A) 25 B) 30 C) 35 D) 45

30. (98-10-18) Arifmetik progressiyada $a_2 + a_{19} = 40$. Shu progressiyaning dastlabki 20 ta hadlari yig'indisini toping.
A) 300 B) 360 C) 400 D) 420
31. (96-6-36) Ikkinchi va o'n to'qqizinchi hadlarining yig'indisi 12 ga teng bo'lgan arifmetik progressiyaning dastlabki yigirmata hadining yig'indisini toping.
A) 110 B) 120 C) 130 D) 115
32. (96-10-29) Arifmetik progressiyada $a_2 = 10$ va $a_5 = 22$. Shu progressiyaning dastlabki sakkizta hadining yig'indisini toping.
A) 162 B) 170 C) 115 D) 160
33. (98-11-26) Arifmetik progressiyaning uchinchi hadi va beshinchi hadi, mos ravishda 11 va 19 ga teng bo'lsa, S_{10} ni toping.
A) 210 B) 190 C) 230 D) 220
34. (98-11-75) $\{a_n\}$ arifmetik progressiyada $a_1 = 3$, $a_{60} = 57$ bo'lsa, progressiyaning dastlabki 60 ta hadi yig'indisi qanchaga teng bo'ladi?
A) 1500 B) $\frac{3423}{2}$ C) 1600 D) 1800
35. (00-4-22) Arifmetik progressiyaning beshinchi hadi 6 ga teng. Uning dastlabki to'qqizta hadi yig'indisini toping.
A) 36 B) 48 C) 54 D) 45
- Yechish.** 3-xossaga ko'ra $\frac{a_1 + a_9}{2} = a_5 = 6$.
5-xossaga ko'ra
- $$S_9 = \frac{a_1 + a_9}{2} \cdot 9 = 6 \cdot 9 = 54. \text{ Javob: } 54 \text{ (C).}$$
36. (99-4-28) Arifmetik progressiyaning o'n uchinchi hadi 5 ga teng. Uning dastlabki 25 ta hadlarining yig'indisini toping.
A) 125 B) 100 C) 75 D) 225
37. (03-6-56) Arifmetik progressiyada $a_{10} = 56$ bo'lsa, uning dastlabki 19 ta hadlari yig'indisini toping.
A) 1024 B) 1032 C) 1056 D) 1064
38. (00-7-25) Arifmetik progressiyaning birinchi va to'qqizinchi hadlari yig'indisi 64 ga teng. Shu progressiyaning dastlabki 9 ta hadlari yig'indisi va beshinchi hadi ayirmasini toping.
A) 256 B) 260 C) 270 D) 208
39. (03-4-19) 15 ta haddan iborat arifmetik progressiyaning sakkizinchi hadi 18 ga teng. Shu progressiyaning hadlari yig'indisini toping.
A) 280 B) 270 C) 250 D) 300
40. (08-106-27, 08-123-27) Arifmetik progressiyaning dastlabki 16 ta hadi yig'indisi 528 ga va $a_{16} = 63$. Shu progressiyaning ayirmasini toping.
A) 7 B) 4 C) 5 D) 6
41. (08-122-27) Arifmetik progressiyaning oltinchi hadi 17 ga, dastlabki 16 ta hadi yig'indisi 392 ga teng. Shu progressiyaning 9-hadini toping.
A) 24 B) 26 C) 13 D) 18
42. (97-2-36) (a_n) arifmetik progressiyaning dastlabki n ta hadi yig'indisi 120 ga teng. Agar $a_3 + a_{n-2} = 30$ bo'lsa, yig'indida nechta had qatnashgan?
A) 6 B) 10 C) 8 D) 12
- Yechish:** 4-xossaga ko'ra $a_1 + a_n = a_3 + a_{n-2} = 30$. Masala shartiga ko'ra
- $$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{30}{2} \cdot n = 15n = 120.$$
- Bu yerdan $n = 8$ ni olamiz. **Javob:** 8 (C).
43. (08-126-27) 25 va 4 sonlari orasiga shu sonlar bilan arifmetik progressiya tashkil etadigan bir nechta son joylashtirilgan. Agar joylashtirilgan sonlarning yig'indisi 87 ga teng bo'lsa, nechta had joylashtirilgan.
A) 6 B) 11 C) 12 D) 9
44. (98-8-27) Arifmetik progressiyaning uchinchi hadi 8 ga, to'rtinchi hadi 5 ga va dastlabki bir nechta hadlari yig'indisi 28 ga teng. Yig'indida nechta had qatnashgan?
A) 10 B) 7 C) 11 D) 8
45. (99-6-54) Arifmetik progressiyaning dastlabki n ta hadining yig'indisi 91 ga teng. Agar $a_3 = 9$ va $a_7 - a_2 = 20$ ekanligi ma'lum bo'lsa, n ni toping.
A) 7 B) 5 C) 3 D) 9
46. (03-12-63) 10; 15; 20; ... arifmetik progressiyaning dastlabki nechta hadining yig'indisi 2475 ga teng bo'ladi?
A) 40 B) 25 C) 30 D) 35
47. (97-7-27) 100 dan katta bo'lmagan 4 ga karrali barcha natural sonlarning yig'indisini toping.
A) 1250 B) 1300 C) 1120 D) 1000
- Yechish:** Ma'limki, 4 ga karrali sonlar $4n$, $n = 1, 2, 3, \dots$ shaklda bo'lib, ular arifmetik progressiya tashkil qiladi. Bu progressiya uchun $a_n = 4n$ dir. Demak, $a_1 = 4$, $a_{25} = 100$. 5-xossadan
- $$S_{25} = \frac{4 + 100}{2} \cdot 25 = 52 \cdot 25 = 1300.$$
- Javob:** 1300 (B).
48. (96-7-27) 100 dan katta bo'lmagan 3 ga karrali barcha natural sonlarning yig'indisini toping.
A) 1683 B) 2010 C) 1500 D) 1080
49. (97-6-17) $a_n = 4n - 2$ formula bilan berilgan ketma-ketlikning dastlabki 50 ta hadi yig'indisini toping.
A) 4500 B) 5050 C) 3480 D) 5000
50. (97-11-17) Hadlari $b_n = 3n - 1$ formula bilan berilgan ketma-ketlikning dastlabki 60 ta hadining yig'indisini toping.
A) 4860 B) 4980 C) 5140 D) 5430

51. (02-11-37) 9 ga bo'lganda, qoldig'i 4 ga teng bo'ladigan barcha ikki xonali sonlarning yig'indisini toping.
A) 527 B) 535 C) 536 D) 542
52. (03-9-26) 7 ga bo'lganda, qoldig'i 2 ga teng bo'ladigan barcha ikki xonali sonlarning yig'indisini toping.
A) 640 B) 647 C) 650 D) 654
53. (03-1-70) Dastlabki mingta natural sonlarning o'rta arifmetigini toping.
A) 500 B) 501 C) 501,5 D) 500,5
54. (00-2-11) 25 ta ketma-ket natural sonning yig'indisi 1000 ga teng. Bu sonlarning kichigi nechaga teng bo'ladi?
A) 30 B) 28 C) 26 D) 27
55. (98-1-27) Arifmetik progressiyaning dastlabki 16 ta hadlari yig'indisi 840 ga va $a_{16} = 105$ bo'lsa shu progressiyaning ayirmasini toping.
A) 9 B) 7 C) 15 D) 5
56. (98-2-18) Arifmetik progressiyada $S_{20} - S_{19} = -30$ va $d = -4$ bo'lsa, a_{25} ning qiymatini toping.
A) -40 B) -50 C) -48 D) -56
Yechish: 6-xossaga ko'ra, $a_{20} = S_{20} - S_{19} = -30$. 2-qoidaga ko'ra, $a_{25} - a_{20} = 5d$. Bu ifodaga $a_{20} = -30$ va $d = -4$ larni qo'yib, $a_{25} = -50$ ni olamiz. **Javob:** -50 (B).
57. (00-3-44) Arifmetik progressiyaning dastlabki 13 ta hadi yig'indisi 104 ga teng. Yettinchi hadining kvadratini toping.
A) 25 B) 36 C) 49 D) 64
58. (00-5-1) 1 dan 75 gacha bo'lgan toq sonlar yig'indisi qanday raqam bilan tugaydi?
A) 0 B) 2 C) 3 D) 4
59. (00-9-13) $y; 3y + 5; 5y + 10; \dots$ arifmetik progressiyaning dastlabki 8 ta hadi yig'indisi 396 ga teng. y ning qiymatini toping.
A) 2 B) 3 C) 4 D) 5
60. (01-1-26) Agar soat 1 da bir marta, 2 da ikki marta va hokazo 12 da o'n ikki marta zang ura, bir sutkada necha marta zang uradi?
A) 72 B) 78 C) 108 D) 156
61. (01-5-28) Arifmetik progressiya uchun $a_{17} = 2$ ga teng bo'lsa, $S_{21} - S_{12}$ ni toping
A) 18 B) 15 C) 16 D) 17
62. (02-1-55) Arifmetik progressiya birinchi o'nta hadining yig'indisi 140 ga teng bo'lsa, $a_2 + a_9$ ni aniqlang.
A) 24 B) 26 C) 30 D) 28
63. (02-4-22) Agar arifmetik progressiya hadlari uchun $a_1 + a_3 + \dots + a_{19} = a_2 + a_4 + \dots + a_{20} + 10$ tenglik o'rinli bo'lsa, arifmetik progressiyaning ayirmasini toping.
A) 1 B) -1 C) 0 D) -2

Yechish: Ma'lumki, barcha $n \geq 2$ lar uchun $a_n - a_{n-1} = d$ tenglik o'rinli. Masala shartida berilgan tenglikni

$$-10 = (a_2 - a_1) + (a_4 - a_3) + \dots + (a_{20} - a_{19})$$

shaklda yozib olamiz. Har bir qavs ichidagi ayirma d ga teng va qavslar soni 10 ta. Demak, $-10 = 10d$ tenglik o'rinli. Bu yerdan $d = -1$ ni olamiz.

Javob: -1 (B).

64. (02-4-18) Arifmetik progressiya hadlari uchun $a_1 + a_3 + \dots + a_{21} = a_2 + a_4 + \dots + a_{20} + 15$ tenglik o'rinli bo'lsa, a_{11} ni toping.
A) 11 B) 13 C) 15 D) 17
65. (03-5-27) Arifmetik progressiyaning oltinchi hadi 10 ga, dastlabki 16 ta hadining yig'indisi 200 ga teng. Bu progressiyaning 12-hadini toping.
A) 16 B) 14 C) 18 D) 20

8.2 Geometrik progressiya

Birinchi hadi noldan farqli bo'lib, ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadni shu ketma-ketlik uchun o'zgarmas va noldan farqli bo'lgan biror q songa ko'paytirishdan hosil bo'lgan sonlar *ketma-ketligi geometrik progressiya* deyiladi. Masalan, 1) 1, 3, 9, ...; 2) 20, 10, 5, ... ketma-ketliklar geometrik progressiya tashkil qiladi. Birinchi misolda $q = 3$, ikkinchisida $q = 0,5$.

Geometrik progressiyani tashkil qiluvchi sonlar uning hadlari deyiladi va umumiy ko'rimishda

$$b_1, b_2, b_3, \dots, b_{n-1}, b_n, \dots \quad (8.2)$$

yoziqladi. Geometrik progressiyaning keyingi hadini hosil qilish uchun oldingi hadiga ko'paytiriladigan q son geometrik progressiya *maxraji* deyiladi. Agar $b_1 > 0$ va $q > 1$ bo'lsa, progressiya *o'suvchi*, $q \in (0; 1)$ bo'lsa, progressiya *kamayuvchi* bo'ladi. Agar $q < 0$ bo'lsa, progressiya *ishorasi almashinuvchi (o'zgaruvchi)*, $|q| < 1$ bo'lsa, progressiya *cheksiz kamayuvchi* deyiladi. Geometrik progressiyaning n - hadi b_n quyidagi formula yordamida topiladi: $b_n = b_1 q^{n-1}$.

Geometrik progressiya hadlarining xossalari.

1-xossa. Agar geometrik progressiyaning barcha hadlari musbat bo'lsa, u holda uning ikkinchi hadidan boshlab istalgan hadi o'ziga qo'shni bo'lgan ikki hadning o'rta geometrik qiymatiga teng, ya'ni

$$b_n = \sqrt{b_{n-1} b_{n+1}}.$$

2-xossa. Chekli geometrik progressiyada boshidan va oxiridan teng uzoqlikda to'rgan hadlar ko'paytmasi chetki hadlar ko'paytmasiga teng, ya'ni

$$b_1 b_n = b_2 b_{n-1} = b_3 b_{n-2} = \dots = b_k b_{n-k+1}.$$

3-xossa. Geometrik progressiyaning dastlabki n ta hadlari yig'indisi $S_n = b_1 + b_2 + \dots + b_{n-1} + b_n$ bo'lsin.

Geometrik progressiyaning dastlabki n ta hadlari yig'indisi S_n uchun quyidagi formulalar o'rinli:

$$S_n = \frac{b_1 - b_n q}{1 - q}, \quad S_n = \frac{b_n q - b_1}{q - 1}, \quad S_n = \frac{b_1 (q^n - 1)}{q - 1}.$$

4-xossa. Cheksiz kamayuvchi geometrik progressiya barcha hadlarining yig'indisi S uchun quyidagi formula o'rinli. $S = b_1 + b_2 + \dots + b_n + \dots = \frac{b_1}{1 - q}$.

Geometrik progressiyaning asosiy xossalarini jamlab, ularni quyidagi tartibda keltiramiz.

- $b_n = b_1 \cdot q^{n-1}, \quad b_n = q b_{n-1}.$
- $b_n : b_m = q^{n-m}.$
- $b_n^2 = b_{n-1} b_{n+1}, \quad n \geq 2.$
- $b_k b_m = b_p b_q, \quad k + m = p + q.$
- $S_n = \frac{b_1(1 - q^n)}{1 - q}, \quad S_n = \frac{b_n q - b_1}{q - 1}, \quad (q \neq 1).$
- $S_n - S_{n-1} = b_n.$
- $S = \frac{b_1}{1 - q}, \quad |q| < 1.$

- Agar geometrik progressiyada $b_1 = 2, q = 3$ bo'lsa, b_5 ni toping.
A) 162 B) 158 C) 120 D) 254

Yechish: 1-xossaga ko'ra $b_5 = b_1 q^4$. Endi b_1 va q larning qiymatlari qo'yib $b_5 = 2 \cdot 3^4 = 2 \cdot 81 = 162$ ni olamiz. **Javob:** 162 (A).

- Agar geometrik progressiyada $b_2 = 1, q = 2$ bo'lsa, $b_5 - b_4$ ni hisoblang.
A) 2 B) 5 C) 4 D) 8

- Agar geometrik progressiyada $b_3 = 10, q = 3$ bo'lsa, b_5 ni toping.
A) 90 B) 85 C) 60 D) 270

- Musbat hadli progressiyada $b_5 = 64, b_7 = 16$ bo'lsa, geometrik progressiya maxrajini toping.
A) 0,2 B) 0,5 C) 4 D) 2

- Geometrik progressiyada $b_3 b_5 = 64$ bo'lsa, b_4 ni toping.
A) 8 B) -8 C) ± 8 D) 4

- Agar ishorasi almashinuvchi geometrik progressiyada $b_3 = 4, b_7 = 9$ bo'lsa, b_5 ni toping.
A) 2 B) -6 C) 6 D) 5

- (98-4-21) Nolga teng bo'lmagan x, y, z sonlar ko'rsatilgan tartibda ishorasi o'zgaruvchi geometrik progressiyani, $x + y; y + z; z + x$ sonlar esa arifmetik progressiyani tashkil etadi. Geometrik progressiyaning maxrajini toping.
A) -2 B) -1 C) -3 D) -4

Yechish: Geometrik progressiyaning maxraji q bo'lsin. U holda $y = qx, z = q^2 x$. Endi $x + y, y + z, z + x$ sonlar arifmetik progressiya tashkil etganligi uchun $2(y + z) = x + y + z + x$, ya'ni $y + z =$

$2x$ bo'ladi. y, z larning o'rniga ularning ifodalarini qo'yib $qx + q^2 x - 2x = 0$ tenglamani, bu yerdan esa $x(q^2 + q - 2) = 0$ ekanini topamiz. Masala shartiga ko'ra $x \neq 0$, shuning uchun $q^2 + q - 2 = 0$ bo'ladi. Uning ildizlari $q_1 = 1, q_2 = -2$. Geometrik progressiyaning ishorasi o'zgaruvchiligidan $q = -2$ ekanini kelib chiqadi. **Javob:** -2 (A).

- (08-120-27) O'suvchi geometrik progressiyaning birinchi hadi 2 ga, yettinchi va to'rtinchi hadlarining ayirmasi 1404 ga teng. Shu progressiyaning maxrajini toping.
A) 2 B) 3 C) $2\sqrt{2}$ D) 4

- (97-9-87) Geometrik progressiyaning dastlabki 6 ta hadi 2, b_2, b_3, b_4, b_5 va 486 bo'lsa, $b_2 + b_3 + b_4 + b_5$ ni hisoblang.
A) 200 B) 260 C) 230 D) 240

- (98-7-38) Quyidagi ketma-ketliklardan qaysilari geometrik progressiyani tashkil etadi?

$$1) a_n = 2x^n \quad 2) c_n = ax^n + 1 \quad 3) b_n = \left(\frac{3}{5}\right)^n$$

- A) 1;3 B) 2;3 C) hech biri D) 1;2;3

- (00-2-21) Nechanchi hadidan boshlab $-8; 4; -2; \dots$ geometrik progressiya hadlarining absolyut qiymati 0,001 dan kichik bo'ladi?
A) 16 B) 12 C) 15 D) 14

- (00-10-23) $64; 32; 16; \dots$ geometrik progressiyaning to'qqizinchi hadi oltinchi hadidan nechtaga kam?
A) 1,025 B) 1,5 C) 1,25 D) 1,75

Yechish: Berilgan geometrik progressiyada $b_1 = 64, b_2 = 32$. Bu yerdan $q = 1/2$ ni olamiz. U holda 1-xossadan

$$b_6 = b_1 q^5 = \frac{64}{32} = 2, \quad b_9 = b_1 q^8 = \frac{64}{256} = \frac{1}{4}.$$

Ularning farqi $b_6 - b_9 = 2 - 0,25 = 1,75$. **Javob:** 1,75 (D).

- (02-4-23) Agar geometrik progressiya hadlari uchun $b_1 b_3 \dots b_{13} = b_2 b_4 \dots b_{14} / 128$ tenglik o'rinli bo'lsa, progressiyaning maxrajini toping.
A) 1 B) 2 C) 3 D) 4

- (03-2-5) $\{b_n\}$ geometrik progressiyada $b_4 - b_2 = 24$ va $b_2 + b_3 = 6$ bo'lsa, b_1 ning qiymatini toping.
A) 0,4 B) 1 C) $1\frac{1}{5}$ D) $\frac{1}{5}$

- (03-6-57) Ikkinchi hadi 6 ga, birinchi uchta hadining yig'indisi 26 ga teng o'suvchi geometrik progressiyaning uchinchi va birinchi hadlari ayirmasini toping.
A) 15 B) 16 C) 14 D) 13

- (03-12-66) O'suvchi geometrik progressiyaning birinchi hadi 3 ga, yettinchi va to'rtinchi hadlarining ayirmasi 168 ga teng. Shu progressiyaning

maxrajini toping.

- A) 3 B) $\frac{3}{2}$ C) $\sqrt{7}$ D) 2

3-4-xossalariga oid misollar

17. (00-9-40) x ning qanday qiymatlarida $0, (16)$; x va $0, (25)$ sonlar ishoralari almashinuvchi geometrik progressiyaning ketma-ket keluvchi hadlari bo'ladi?
A) $0, (20)$ B) $\pm 0, (20)$ C) $-0, (20)$ D) $-0, (21)$

Yechish: Geometrik progressiyaning ishora almashinuvchiligidan $x < 0$ ekani, 3-xossasidan esa

$$x^2 = 0, (16) \cdot 0, (25) = \frac{16}{99} \cdot \frac{25}{99}$$

ekani kelib chiqadi. Shuning uchun

$$x = -\frac{4 \cdot 5}{99} = -\frac{20}{99} = -0, (20).$$

Javob: $-0, (20)$ (C).

18. (96-6-37) Geometrik progressiyada $b_2 \cdot b_3 \cdot b_4 = 216$ bo'lsa, uning uchinchi hadini toping.
A) 12 B) 8 C) 4 D) 6
19. (97-8-36) $b_3 \cdot b_4 \cdot b_5 = 64$ ga teng bo'lgan geometrik progressiyaning to'rtinchi hadini toping.
A) 10 B) 12 C) 4 D) 8
20. (00-3-46) Geometrik progressiyada uchinchi va yettinchi hadlarining ko'paytmasi 144 ga teng. Uning beshinchi hadini toping.
A) 6 B) ± 12 C) -8 D) -12
21. (00-3-47) $\frac{1}{3}$ va $\frac{1}{48}$ sonlar orasiga shunday uchta musbat sonni joylashtiringki, natijada geometrik progressiya hosil bo'lsin. O'sha qo'yilgan uchta sonning yig'indisini toping.
A) 0,5 B) $\frac{7}{12}$ C) 0,375 D) $\frac{7}{24}$
22. (00-7-26) Barcha hadlari musbat bo'lgan geometrik progressiyaning birinchi hadi 2 ga, beshinchi hadi 18 ga teng. Shu progressiyaning beshinchi va uchinchi hadlari ayirmasini toping.
A) 10 B) 12 C) 8 D) 11
23. (02-8-9) 3 va 19683 sonlari o'rtasiga 7 ta shunday musbat sonlar joylashtirilganki, hosil bo'lgan to'qqizta son geometrik progressiya tashkil etsin. 5-o'rinda turgan son nechaga teng?
A) 243 B) 343 C) 286 D) 729

Dastlabki n ta hadi yig'indisiga oid misollar

24. Geometrik proggressiyada $b_1 = 3$, $q = 2$. Shu proggressiyaning dastlabki oltita hadining yig'indisini toping.
A) 63 B) 189 C) 126 D) 184

Yechish: 5-xossaga ko'ra

$$S_6 = \frac{b_1(1 - q^6)}{1 - q} = \frac{3(1 - 2^6)}{1 - 2} = 3 \cdot 63 = 189.$$

Javob: 189 (B).

25. Geometrik proggressiyada $b_1 = 3$, $q = 2$. Shu proggressiyaning dastlabki oltita hadining yig'indisini toping.
A) 63 B) 189 C) 126 D) 184
26. 81, 27, 9, ... geometrik progressiyaning nechta hadining yig'indisi 120 ga teng bo'ladi.
A) 3 B) 4 C) 5 D) 6
27. Geometrik progressiyada $b_6 - b_1 = 84$, $q = 3$. Shu progressiyaning dastlabki beshta hadining yig'indisini toping.
A) 63 B) 89 C) 42 D) 21
28. (97-12-35) Dastlabki beshta hadining yig'indisi -62 ga, dastlabki oltita hadining yig'indisi -126 ga va maxraji 2 ga teng geometrik progressiyaning birinchi hadini toping.
A) -1 B) -3 C) -4 D) -2
- Yechish:** $S_5 = -62$, $S_6 = -126$ ekanidan 7-xossaga ko'ra $b_6 = S_6 - S_5 = -126 + 62 = -64$ ni topamiz. $q = 2$ va $b_6 = b_1 \cdot q^5$ ekanidan $b_1 \cdot 2^5 = -64$, bundan esa $b_1 = -2$ ni hosil qilamiz.
- Javob:** -2 (D).
29. (98-2-19) Geometrik progressiyada $S_6 - S_5 = -128$ va $q = -2$. b_8 ning qiymatini toping.
A) 512 B) 256 C) -512 D) -256
30. (98-3-21) Geometrik progressiyaning maxraji 3 ga, dastlabki to'rta hadlari yig'indisi 80 ga teng. Uning to'rtinchi hadini toping.
A) 24 B) 32 C) 54 D) 27
31. (98-8-26) Geometrik progressiyaning birinchi hadi 486 ga, maxraji $\frac{1}{3}$ ga teng. Shu progressiyaning dastlabki to'rtta hadi yig'indisini toping.
A) 680 B) 840 C) 720 D) 760
32. (98-1-26) Geometrik progressiyaning maxraji -2 ga, dastlabki beshta hadining yig'indisi 5,5 ga teng. Progressiyaning beshinchi hadini toping.
A) 4 B) -8 C) 8 D) -16
33. (98-11-27) Agar olti hadli geometrik progressiyaning dastlabki uchta hadining yig'indisi 112 ga va oxiridagi uchta hadining yig'indisi 14 ga teng bo'lsa, birinchi hadi nechaga teng bo'ladi?
A) 72 B) 64 C) 56 D) 63
34. (99-4-29) Geometrik progressiyada $q = 2$ va $S_4 = 5$. b_2 ni toping.
A) 0,4 B) 0,8 C) $1\frac{1}{3}$ D) $\frac{2}{3}$
35. (00-6-25) O'suvchi geometrik progressiyaning dastlabki to'rtta hadi yig'indisi 15 ga, undan keyingi to'rttasiniki esa 240 ga teng. Shu progressiyaning dastlabki oltita hadi yig'indisini toping.
A) 31 B) 48 C) 63 D) 127
- Yechish:** Masala sharti va 1-xossadan
- $$\begin{cases} b_1 + b_2 + b_3 + b_4 = b_1(1 + q + q^2 + q^3) = 15 \\ b_5 + b_6 + b_7 + b_8 = q^4 b_1(1 + q + q^2 + q^3) = 240 \end{cases}$$

ni va $q > 1$ ni olamiz. $b_1(1 + q + q^2 + q^3)$ ning qiymatini sistemaning 2-tenglamasiga qo'yib $q^4 \cdot 15 = 240$ ni, bu yerdan esa $q = 2$ ni olamiz. q ning qiymatini sistemaning 1-tenglamasiga qo'yib $b_1 = 1$ ekanligini topamiz. 5-xossadan $S_6 = 2^6 - 1 = 63$. **Javob:** 63 (C).

36. (01-1-28) Oltita haddan iborat geometrik progressiyaning dastlabki uchta hadining yig'indisi 168 ga, keyingi uchtasiniki esa 21 ga teng. Shu progressiyaning maxrajini toping.
A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 2
37. (02-1-56) Agar hadlari haqiqiy sondan iborat bo'lgan o'suvchi geometrik progressiyaning birinchi uchta hadi yig'indisi 7 ga, ko'paytmasi 8 ga teng bo'lsa, shu progressiyaning beshinchi hadini toping.
A) 6 B) 32 C) 12 D) 16
38. (02-4-21) Geometrik progressiyada $b_1 = 1$ va $q = \sqrt{2}$ bo'lsa, $b_1 + b_3 + b_5 + \dots + b_{15}$ ning qiymatini hisoblang.
A) 253 B) 254 C) 255 D) 256
39. (02-5-28) Geometrik progressiyaning dastlabki oltita hadi yig'indisi 1820 ga, maxraji esa 3 ga teng. Shu progressiyaning birinchi va beshinchi hadlari yig'indisini toping.
A) 164 B) 246 C) 328 D) 410
40. (02-11-39) Geometrik progressiyaning ikkinchi hadi 2 ga, beshinchi hadi 16 ga teng. Shu progressiyaning dastlabki oltita hadi yig'indisini toping.
A) 81 B) 72 C) 65 D) 63
41. (03-9-25) Geometrik progressiyaning birinchi hadi va maxraji 2 ga teng. Shu progressiyaning dastlabki nechta hadlari yig'indisi 1022 ga teng bo'ladi?
A) 5 B) 8 C) 9 D) 10
42. (03-4-20) Geometrik progressiyaning oltinchi va birinchi hadi ayirmasi 1210 ga, maxraji 3 ga teng. Shu progressiyaning dastlabki beshta hadi yig'indisini toping.
A) 610 B) 615 C) 600 D) 605

Cheksiz kamayuvchi geometrik progressiya

43. (99-1-23) Geometrik progressiya barcha hadlari yig'indisini toping.

$$\sqrt{5}, 1, \frac{1}{\sqrt{5}}, \dots$$

- A) $\frac{5}{\sqrt{5}-1}$ B) $\frac{6\sqrt{5}}{5}$ C) $\frac{\sqrt{5}-1}{\sqrt{5}}$ D) 4,16

Yechish: Berilgan ketma-ketlik cheksiz kamayuvchi geometrik progressiya bo'lib, bunda $b_1 = \sqrt{5}$ va $q = \frac{1}{\sqrt{5}}$. U holda 7-formulaga ko'ra

$$S = \frac{\sqrt{5}}{1 - \frac{1}{\sqrt{5}}} = \frac{5}{\sqrt{5}-1}. \quad \text{Javob: } \frac{5}{\sqrt{5}-1} \text{ (A).}$$

44. (99-10-25) Cheksiz kamayuvchi geometrik progressiyaning hadlari yig'indisi 8 ga, dastlabki to'rttasiniki esa $\frac{15}{2}$ ga teng. Agar uning barcha hadlari musbat bo'lsa, b_1 ni toping.
A) 2 B) 4,5 C) 4 D) 3

45. (00-3-48) Cheksiz kamayuvchi geometrik progressiyaning hadlari yig'indisi 9 ga, maxraji esa $\frac{1}{3}$ ga teng. Uning birinchi hamda uchinchi hadlari ayirmasini toping.
A) $5\frac{1}{3}$ B) $4\frac{2}{3}$ C) $5\frac{2}{3}$ D) $2\frac{1}{3}$

46. (01-8-25) Cheksiz kamayuvchi geometrik progressiyaning birinchi hadi ikkinchisidan 8 ga ortiq, hadlarining yig'indisi esa 18 ga teng. Progressiyaning uchinchi hadini toping.
A) $1\frac{1}{3}$ B) $-33\frac{1}{3}$ C) $-1\frac{1}{3}$ D) $2\frac{2}{3}$

47. (02-1-16) Cheksiz kamayuvchi geometrik progressiya hadlari yig'indisi 56 ga, hadlari kvadratlari yig'indisi esa 448 ga teng. Progressiyaning maxrajini toping.
A) 0,75 B) 0,8 C) 0,25 D) 0,5

Yechish: Masala shartiga ko'ra $S = \frac{b_1}{1-q} = 56$ va $b_1^2 + b_2^2 + \dots + b_n^2 + \dots = b_1^2(1 + q^2 + \dots + q^{2(n-1)} + \dots) = 448$. Qavs ichidagi ifoda maxraji q^2 va $b_1 = 1$ bo'lgan cheksiz kamayuvchi geometrik progressiyaning yig'indisini ifodalaydi. Uning yig'indisi $\frac{1}{1-q^2}$ dir. Natijada biz

$$\begin{cases} b_1 : (1-q) = 56 \\ b_1^2 : (1-q^2) = 448 \end{cases}$$

sistemaga ega bo'lamiz. Sistemaning 1-tenglamasidan $b_1 = 56(1-q)$ ni, 2-tenglamaga qo'yib, q ga nisbatan $7(1-q) = 1+q$ tenglamani olamiz. Bu tenglamaning yechimi $q = 6/8 = 0,75$. **Javob:** 0,75 (A).

48. (02-5-30) a ning qanday qiymatida

$$2a + a\sqrt{2} + a + \frac{a}{\sqrt{2}} + \dots$$

cheksiz kamayuvchi geometrik progressiya yig'indisi 8 ga teng bo'ladi?

- A) 1 B) $\frac{4}{\sqrt{2}}$ C) $2 - \sqrt{2}$ D) $2(2 - \sqrt{2})$

49. (02-12-32) Cheksiz kamayuvchi geometrik progressiyaning yig'indisi 243 ga, dastlabki beshta hadiniki esa 275 ga teng. Bu progressiyaning maxraji $\frac{1}{5}$ dan qanchaga kam?
A) $\frac{7}{15}$ B) $\frac{8}{15}$ C) $\frac{3}{5}$ D) $\frac{13}{15}$

9 -bob. Matnli masalalar

9.1 Sonlarga oid masalalar

1. Hech bir uchtasi bitta to'g'ri chiziqda yotmaydigan n ta nuqtadan $\frac{n(n-1)}{2}$ ta to'g'ri chiziq o'tkazish mumkin.

2. $N(A) - A$ to'planning elementlari soni bo'lsa, $N(A \cup B) = N(A) + N(B) - N(A \cap B)$ o'rinli.

1. (96-10-39) Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqta berilgan. Shu 6 ta nuqtalar orqali nechta turlicha to'g'ri chiziq o'tkazish mumkin?

A) 6 B) 12 C) 10 D) 15

Yechish: 1-qoidaga ko'ra, hech qaysi uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqtadan

$$\frac{6(6-1)}{2} = 15$$

to'g'ri chiziq o'tkazish mumkin. **Javob:** 15 (D).

2. (96-1-36) Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 7 ta nuqta berilgan. Shu 7 ta nuqtalar orqali nechta turlicha to'g'ri chiziq o'tkazish mumkin?

A) 28 B) 21 C) 42 D) 35

3. (01-2-43) Istalgan uchtasi bir to'g'ri chiziqda yotmaydigan to'rtta nuqtani juft-juft ravishda tutashirish natijasida nechta kesma hosil bo'ladi?

A) 4 B) 5 C) 6 D) 7

4. (98-12-101) 13 kishi bir-biri bilan salomlashganda, qo'l berib ko'rishishlar soni qancha bo'ladi?

A) 169 B) 156 C) 78 D) 130

5. (96-6-4) 2 soat 30 minut 3 sekund necha sekund bo'ladi?

A) 10203 B) 8203 C) 9003 D) 9803

6. (97-8-4) 1 soat 160 minut 2 sekund necha sekunddan iborat?

A) 106002 B) 12202 C) 14202 D) 13202

7. (97-2-4) $3m^2 + 1dm^2 + 5sm^2$ necha sm^2 ga teng?

A) 3015 B) 3105 C) 30015 D) 30105

8. (97-12-4) $2m^2 + 3dm^2 + 4sm^2$ necha sm^2 ga teng?

A) 2034 B) 20244 C) 21034 D) 20304

9. (97-5-8) Chumoli 5 minutda $15\frac{5}{6}$ m yuradi. U 1 minutda necha metr yuradi?

A) $3\frac{5}{6}$ B) $15\frac{1}{6}$ C) $3\frac{1}{6}$ D) 3

Yechish: Chumoli 1 minutda x metr yursin. U holda $15\frac{5}{6} : 5 = x : 1$ bo'ladi. Bu yerdan $x = 3\frac{1}{6}$.

Javob: $3\frac{1}{6}$ (C).

10. (03-2-64) Toshbaqa 1 minutda 50 sm yo'l bosadi. U 0,1 km masofani qancha soatda o'tadi?

A) $2\frac{2}{3}$ B) $2\frac{1}{2}$ C) $3\frac{1}{3}$ D) $3\frac{1}{2}$

11. (97-9-8) G'ildirak 7 minutda $12\frac{3}{5}$ marta aylanadi. U 1 minutda necha marta aylanadi?

A) $1\frac{4}{5}$ B) 1 C) $1\frac{3}{5}$ D) $1\frac{2}{5}$

12. (98-7-12) Piyoda kishi 1 km yo'lni $\frac{2}{9}$ soatda o'tadi.

U $\frac{3}{4}$ km yo'lni qancha soatda o'tadi?
A) $\frac{1}{5}$ B) $\frac{1}{6}$ C) $\frac{8}{27}$ D) $\frac{1}{4}$

13. (00-5-24) 1 l dengiz suvida o'rtacha 0,00001 mg oltin bor. $1km^3$ dengiz suvida necha kg oltin bor?

A) 0,1 B) 0,01 C) 1 D) 10

14. (01-2-6) Tiko avtomashinasida 100 km yo'lni o'tish uchun 5,8 l yonilg'i sarflanadi. 8,7 l yonilg'i bilan bu avtomashinada necha km yul yurish mumkin?

A) 160 B) 154,8 C) 150 D) 145,4

15. (98-11-21) Xaritada ikki shahar orasidagi masofa 4,5 sm ga teng. Xaritada mashtab 1 : 200000 bo'lsa, shaharlar orasidagi haqiqiy masofa necha km bo'ladi?

A) 0,9 B) 9 C) 90 D) 900

16. (00-5-11) Xaritada 3,6 sm uzunlikdagi kesmaga 72 km masofa mos keladi. Agar xaritada ikki shahar orasidagi masofa 12,6 sm bo'lsa, ular orasidagi masofa necha km?

A) 240 B) 244 C) 246 D) 252

17. Boks tushadigan maydoncha (ring) - tomoni 6 m ga teng kvadrat. Ring uch qator yo'g'on arqon bilan o'raladi. Buning uchun necha metr arqon kerak?

A) 80 B) 72 C) 76 D) 88

18. (97-9-11) Avtomashina bakiga 70 l benzin quyildi. Gulistonga borish uchun benzinning $\frac{2}{5}$ qismi,

Chimiyonga borish uchun esa $\frac{3}{7}$ qismi sarflandi. Bakda necha litr benzin qolgan?

A) 13 B) 15 C) 18 D) 12

19. (98-2-6) Agar kamayuvchini 16 ta va ayriluvchini 20 ta orttirilsa, ayirma qanday o'zgaradi?

A) 4 ta kamayadi B) 36 ta ortadi
C) 36 ta kamayadi D) 4 ta ortadi

20. (98-3-7) Agar $4\frac{3}{5}$ son $2\frac{1}{2}$ marta oshirilgan bo'lsa, u qanchaga ko'paygan?

A) 6,6 B) 6 C) 7 D) 6,9

Yechish: Agar $4\frac{3}{5}$ soni $2\frac{1}{2}$ marta oshirilsa,

$4\frac{3}{5} \cdot 2\frac{1}{2} = \frac{23}{2}$ son hosil bo'ladi. Ularning farqi

$\frac{23}{2} - 4\frac{3}{5} = \frac{69}{10} = 6,9$. **Javob:** 6,9 (D).

21. (99-6-59) $\frac{65}{6}$ va $\frac{39}{8}$ kasrlar butun qismlarining o'rtta arifmetigini toping.
A) 7 B) 6 C) 8 D) 5
22. (99-9-21) [1; 3] oraliqdagi maxraji 3 ga teng bo'lgan barcha qisqarmas kasrlarning yig'indisini toping.
A) $8\frac{1}{3}$ B) $8\frac{2}{3}$ C) $7\frac{1}{3}$ D) 8
23. (00-2-2) $32 < a < 92$ shartni qanoatlantiruvchi ikki xonali a sonning birinchi raqami o'chirilganda u 31 marta kamaydi. O'chirilgan raqam nechaga teng?
A) 5 B) 4 C) 6 D) 7
24. " 9^A sinfdagi barcha o'quvchilar a'lochi" tasdig'ining inkorini toping.
A) " 9^A sinfdagi barcha o'quvchilar ikkichi"
B) " 9^A sinfdagi birorta ham a'lochi o'quvchi yo'q"
C) " 9^A sinfdagi kamida bir o'quvchi a'lochi emas"
D) " 9^A sinfdagi birgina a'lochi o'quvchi bor"
25. "Tenglama yagona yechimga ega" tasdig'ining inkorini toping.
A) "Tenglama yechimga ega emas"
B) "Tenglama cheksiz ko'p yechimga ega"
C) "Tenglama ikkitadan ko'p yechimga ega"
D) "Tenglama yechimga ega emas yoki tenglama bittadan ko'p yechimga ega"
26. " x haqiqiy soni 1 dan katta" tasdig'ining inkorini toping.
A) " x haqiqiy soni 1 dan kichik"
B) " x haqiqiy soni 1 dan kichik yoki teng"
C) " x manfiy bo'lgan haqiqiy son"
D) " x haqiqiy soni 0 yoki undan kichik"
27. (00-2-14) Ikkita toq sonning yig'indisi 5 ga bo'linadi. Bu sonlar kublarining yig'indisi qanday raqam bilan tugaydi?
A) 6 B) 5 C) 4 D) 0
- Yechish:** 1-xulosa: ikkita toq sonning yig'indisi juft sonidir. 2-xulosa: 5 ga bo'linadigan juft son faqat 0 raqami bilan tugaydi. 3-xulosa: bu toq sonlarning oxirgi raqamlari 1 va 9 yo 3 va 7 yo 5 va 5 bo'lishi mumkin. Har uchchala holda ham
- $$\dots 1^3 + \dots 9^3 = \dots 1 + \dots 9 = \dots 0,$$
- $$\dots 3^3 + \dots 7^3 = \dots 7 + \dots 3 = \dots 0,$$
- $$\dots 5^3 + \dots 5^3 = \dots 5 + \dots 5 = \dots 0$$
- raqami bilan tugaydi. **Javob:** 0 (D).
28. (02-1-28) 1, 2 va 3 raqamlari yordamida yozilgan turli raqamli barcha uch xonali sonlar yig'indisi toping.
A) 1233 B) 2133 C) 1332 D) 2331
29. 2 va 0 raqamlari yordamida yoziladigan barcha to'rt xonali sonlarni yozing. Ulardan eng kattasi bilan eng kichigining ko'paytmasini toping.
A) 2222 B) 2000 C) 4222 D) 444000
30. Fermada 43 ta sigir va shuncha buzoq bor. Har bir sigirga bir kunda 8 kg dan, har bir buzoqqa esa 5 kg dan ozuqa beriladi. 30 kunda mollar uchun qancha ozuqa kerak?
A) 28770 B) 12560 C) 16770 D) 3000
31. (02-1-30) Agar a va b ixtiyoriy natural sonlar bo'lsa, u holda $2a + 8b$ ifoda quyidagi sonlarning qaysi biriga qoldiqsiz bo'linadi?
A) 2 B) 3 C) 4 D) 12
32. (01-10-11) Sayohatchilar guruhidagi erkaklarning ayollar soniga nisbati 3 : 4 kabi. Quyida keltirilganlardan qaysi biri guruhdagi sayohatchilar soniga teng bo'la olmaydi?
A) 28 B) 21 C) 23 D) 35
33. (99-5-8) Qishloqda bolalar kattalardan 2 marta ko'p, nafaqaxo'rlar esa qolgan aholidan 3 marta kam. Agar 15 sonning o'ng va chap tomoniga bir xil raqam yozilsa, qishloq aholisining soni hosil bo'ladi. Bu qanday raqam?
A) 2 B) 3 C) 4 D) 6
34. (96-3-62) 1 dan 100 gacha bo'lgan sonlar orasida 2 ga ham, 3 ga ham bo'linmaydiganlari nechta?
A) 33 B) 30 C) 32 D) 21
- Yechish:** Dastlab 1 dan 100 gacha bo'lgan sonlar orasidan 2 ga ham, 3 ga ham bo'linadiganlari sonini topamiz. A bilan 1 dan 100 gacha bo'lgan sonlar orasida 2 ga bo'linadigan sonlar to'plamini, B bilan 1 dan 100 gacha bo'lgan sonlar orasida 3 ga bo'linadigan sonlar to'plamini belgilaymiz. U holda $A \cap B$ to'plam 6 ga bo'linadigan sonlar to'plami bo'ladi. Demak, $N(A) = 50$, $N(B) = 33$ va $N(A \cap B) = 16$ bo'ladi. 1-tenglikka ko'ra $N(A \cup B) = 50 + 33 - 16 = 67$. Shunday qilib 2 ga ham, 3 ga ham bo'linmaydiganlari soni $100 - 67 = 33$ ta ekan. **Javob:** 33 (A).
35. (96-12-60) 1 dan 100 gacha bo'lgan sonlar orasida 2 ga ham, 5 ga ham bo'linmaydiganlari nechta?
A) 35 B) 40 C) 41 D) 32
36. (98-9-3) Sinfdagi 35 ta o'quvchidan 28 tasi suzish seksiyasiga, 14 tasi voleybol seksiyasiga qatnashadi. Agar har bir o'quvchi hech bo'lmaganda bitta seksiyaga qatnashsa, ikkala seksiyaga qatnashadigan o'quvchilar necha foizni tashkil etadi?
A) 20 B) 18 C) 25 D) 15
37. (03-10-34) 30 kishidan 22 tasi raqs to'garagiga, 17 tasi esa xorda ashula aytadi. Necha kishi faqat raqs to'garagiga qatnashadi?
A) 8 B) 10 C) 12 D) 13
38. (03-12-54) 30 ta turistdan 20 tasi ingliz tilini, 15 tasi fransuz tilini bilishadi. Shu turistlardan nechitasi ikkala tilni ham bilishadi?
A) 5 B) 10 C) 15 D) 8

Tenglama yoki tenglamalar sistemasi yordamida yechiladigan masalalar.

39. (98-12-61) Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son ayirmasi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?
A) 5 B) 11 C) 9 D) 4
- Yechish:** Raqamlari a va b bo'lgan kiki xonali sonni $10a + b$ ko'rinishda yozish mumkin. Uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan ikki xonali son $10b + a$ dir. Ularning ayirmasi
- $$10a + b - (10b + a) = 9a - 9b = 9(a - b).$$
- Demak, ayirma 9 ga qoldiqsiz bo'linadi. **Javob:** 9 (C).
40. (00-1-5) Ikki xonali sonning o'ng tomoniga 0 raqami yozilsa, berilgan sonning yarmi bilan 323 ning yig'indisiga teng bo'ldan son hosil bo'ladi. Berilgan sonni toping.
A) 54 B) 14 C) 24 D) 34
41. (98-4-2) Ikki xonali son bilan uning raqamlari o'rinlarini almashtirishdan hosil bo'lgan son yig'indisi quyidagilardan qaysi biriga qoldiqsiz bo'linadi?
A) 3 B) 11 C) 9 D) 4
42. (98-12-67) A, B - raqamlar; AB va $5A$ esa ikki xonali sonlar. Agar $AB \cdot 3 = 5A$ bo'lsa, $A^2 + B^2$ ning qiymati qanchaga teng bo'ladi?
A) 65 B) 13 C) 50 D) 37
43. (99-7-13) Ikki xonali son o'zining raqamlari yig'indisidan 4 marta katta. Raqamlari kvadratlarining yig'indisi 5 ga teng. Shu ikki xonali sonning kvadratini hisoblang.
A) 441 B) 169 C) 121 D) 144
44. (01-2-5) Raqamlari yig'indisining uchlanganiga teng ikki xonali sonni toping.
A) 17 B) 21 C) 13 D) 27
45. (03-1-63) Raqamlari yig'indisiga bo'lganda, bo'linmasi 4 ga va qoldig'i nolga teng bo'ladigan ikki xonali sonlar nechta?
A) 2 B) 3 C) 4 D) 5
46. (96-3-22) Onasi 50, qizi 28 yoshda. Necha yil oldin qizi onasidan 2 marta yosh bo'lgan.
A) 5 yil B) 6 yil C) 8 yil D) 4 yil
- Yechish:** Faraz qilaylik, x yil oldin qizi onasidan 2 marta yosh bo'lgan bo'lsin. x yil oldin onasi 50 - x yoshda, qizi esa 28 - x yoshda bo'ladi. Masala shartiga ko'ra, $2(28 - x) = 50 - x$. Bu yerdan $x = 6$ ni olamiz. **Javob:** 6 yil (B).
47. (96-12-23) Buvisi 100, nabirasi 28 yoshda. Necha yil oldin nabirasi buvisidan 4 marta yosh bo'lgan.
A) 8 yil B) 5 yil C) 4 yil D) 6 yil
48. (02-1-41) Olim otasidan 32 yosh kichik. Otasi esa bobosidan shuncha yosh kichik. Uch yil avval ularning yoshlari yig'indisi 111 ga teng bo'lgan bo'lsa, hozir Oliming bobosi necha yoshda?
A) 69 B) 72 C) 75 D) 80
49. (02-7-50) 36 yoshdagi onaning yoshi 4 ta bolalari yoshlari yig'indisidan 3 marta ortiq. Necha yildan keyin onaning yoshi bolalari yoshlarining yig'indisiga teng bo'ladi?
A) 8 B) 9 C) 10 D) 7
50. (03-1-61) Egizaklar yoshining yig'indisi 10 yilda ikki marta ortdi. Yana 10 yildan keyin ulardan har birining yoshi nechaga teng bo'ladi?
A) 20 B) 30 C) 40 D) 25
51. (96-1-2) Bir nechta natural sonlarning yig'indisi 75 ga teng. Agar shu sonlarning har biridan 2 ni ayirib yig'indi hisoblansa, u 61 ga teng bo'ladi. Yig'indida nechta son qatnashgan?
A) 5 B) 7 C) 14 D) 8
- Yechish:** Faraz qilaylik, yig'indida n ta son qatnashgan bo'lsin. Masala shartiga ko'ra, $a_1 + a_2 + \dots + a_n = 75$ va $a_1 - 2 + a_2 - 2 + \dots + a_n - 2 = 61$. Bulardan
- $$a_1 + a_2 + \dots + a_n - 2n = 61 \iff 75 - 2n = 61.$$
- Bu yerdan $n = 7$ ni olamiz. **Javob:** 7 (B).
52. (98-9-57) Ko'paytmaning har bir hadi 2 ga ko'paytirildi, natijada ko'paytma 1024 marta ortdi. Ko'paytmada nechta had qatnashgan.
A) 8 B) 9 C) 10 D) 11
53. (98-10-49) Berilgan beshta sonning har biri 3 ga ko'paytirilib, so'ngra hosil bo'lgan sonlarning har biriga 2 qo'shildi. Hosil bo'lgan sonlar yig'indisi 70 ga teng bo'lsa, berilgan sonlar yig'indisi nechaga teng bo'lgan?
A) 20 B) 22 C) 15 D) 25
54. (96-3-5) Ikki sonning yig'indisi 51 ga, ayirmasi esa 21 ga teng. Shu sonlarni toping.
A) 36; 15 B) 35; 16 C) 37; 14 D) 33; 18
- Yechish:** Faraz qilaylik, bu sonlarning biri x , ikkinchisi y bo'lsin. Masala shartiga ko'ra,
- $$\begin{cases} x + y = 51 \\ x - y = 21. \end{cases}$$
- Bu sistemani qo'shish usuli yordamida yechamiz. Sistema tenglamalarini qo'shib $2x = 72$ ni olamiz. Bu yerdan $x = 36$ kelib chiqai. Topilgan x ning bu qiymatini sistemaning 1-tenglamasiga qo'yib, $y = 51 - 36 = 15$ ni olamiz. **Javob:** 36; 15 (A).
55. Katakdak quyonlar va tovuqlar bor. Agar oyoqlar 314 ta va boshlar 100 ta bo'lsa, quyonlar nechta?
A) 46 B) 37 C) 57 D) 43
56. Bir qutida 100 so'mlik va 500 so'mlik puldan 90 dona bor. 100 so'mliklarning yig'indisi 500 so'mliklarning yig'indisiga teng bo'lsa, qutidagi necha ming so'm pul bor?
A) 21 B) 25 C) 15 D) 18

57. 60 kishilik harbiy qo'shin 25 kunga yetadigan oziq-ovqat bilan ta'minlangan edi. 5 kundan keyin do'shman bilan tuqnashuvda 10 kishi halok bo'ldi. Qolgan oziq-ovqat harbiy qo'shinga necha kunga yetadi?
A) 28 B) 25 C) 24 D) 20
58. (98-12-30) Ikki sonning yig'indisi 6,5 ga teng. Ulardan biri ikkinchisidan 4 marta kichik. Shu sonlarning kattasini toping.
A) 5,2 B) 5 C) 4 D) 5,3
59. (96-11-5) Ikki sonning yig'indisi 7 ga teng. Ulardan biri ikkinchisidan 4 marta kichik bo'lsa, Shu sonlarning kattasini toping.
A) 5,2 B) 6,2 C) 5,6 D) 5,4
60. (03-3-1) Ikki sonning yig'indisi 6 ga, kvadratlarining ayirmasi esa 48 ga teng. Shu sonlarning ko'paytmasini toping.
A) 8 B) -8 C) 7 D) -7
61. (03-12-5) Ikki natural sonning ko'paytmasi ularning yig'indisidan 29 ga, ayirmasidan 41 ga ortiq. Shu ikki sondan birini toping.
A) 7 B) 8 C) 9 D) 10
62. (00-2-12) 100 shunday ikki natural songa ajratilganki, ulardan biri 7 ga, ikkinchisi 11 ga bo'linadi. Bu sonlar ayirmasining modulini toping.
A) 8 B) 14 C) 10 D) 12
63. (00-4-15) Biror sonni 2 ga bo'lsak, bo'linma berilgan sondan 4 taga katta chiqadi. Berilgan sonni toping.
A) 4 B) 6 C) 8 D) -8
64. (00-8-30) Kasr surati va maxrajining yig'indisi 23 ga teng. Surati maxrajidan 9 ta kam. Kasrni toping.
A) $\frac{7}{16}$ B) $\frac{8}{15}$ C) $\frac{16}{7}$ D) $\frac{10}{13}$
65. (00-10-81) Kasr suratiga 2 qo'shilsa kasr 1 ga, maxrajiga 3 qo'shilsa, u $\frac{1}{2}$ ga teng bo'ladi. Shu kasrning $\frac{3}{5}$ qismini toping.
A) $\frac{3}{7}$ B) $\frac{4}{7}$ C) $\frac{3}{5}$ D) $\frac{3}{4}$
66. (01-3-27) Kasrning maxraji suratidan 4 birlik ortiq. Agar kasrning surati va maxraji 1 birlik ortirilsa, $\frac{1}{2}$ soni hosil bo'ladi. Berilgan kasrning kvadratini toping.
A) $\frac{25}{81}$ B) $\frac{49}{121}$ C) $\frac{9}{49}$ D) $\frac{121}{225}$
67. (01-7-11) Ikki natural sonning yig'indisi 64 ga teng. Shu sonlardan kattasini uning kichigiga bo'lganda, bo'linma 3 ga, qoldiq 4 ga teng chiqdi. Berilgan sonlardan kattasini toping.
A) 54 B) 42 C) 56 D) 49
68. (02-1-37) a ning qanday qiymatida $9 - a$ va $15 - a$ lar qarama-qarshi sonlar bo'ladi?
A) 9 B) 10 C) 12 D) 15
69. (03-10-7) Ikki sonning yig'indisi 6 ga, ko'paytmasi 7 ga teng bo'lsa, bu sonlar kublarining yig'indisini toping.
A) 90 B) 48 C) 64 D) 72
- Yechish:** Izlanayotgan sonlar x va y bo'lsin. Masala shartlaridan
- $$\begin{cases} x + y = 6 \\ xy = 7 \end{cases}$$
- ni olamiz. Sistemaning 1-tenglamasini kvadratga ko'tarib, 2-tenglamasini -3 ga ko'paytirib qo'shib $x^2 - xy + y^2 = 15$ ni olamiz. Ikki son kublarining yig'indisi $x^3 + y^3 = (x + y)(x^2 - xy + y^2) = 6 \cdot 15$ dan $x^3 + y^3 = 90$ kelib chiqadi. **Javob:** 90 (A).
70. (03-11-60) Birinchi son ikkinchi sondan 2,5 ga ortiq. Birinchi sonning $\frac{1}{5}$ qismi ikkinchi sonning $\frac{4}{5}$ qismiga teng. Shu sonlarning yig'indisini toping.
A) 4 B) 6 C) $6\frac{1}{3}$ D) $4\frac{1}{6}$
71. (02-7-47) a sonning b songa nisbati $\frac{2}{3}$ ga, c sonning b songa nisbati $\frac{1}{2}$ ga teng. c sonining a soniga nisbati nechaga teng?
A) $\frac{3}{4}$ B) $\frac{5}{7}$ C) $\frac{5}{6}$ D) $\frac{2}{3}$
72. (02-9-9) a sonini 3 ga bo'lgandagi qoldiq 1 ga, 4 ga bo'lgandagi qoldiq esa 3 ga teng. a sonni 12 ga bo'lgandagi qoldiqni toping.
A) 1 B) 3 C) 5 D) 7
73. (03-5-8) a soni $b^2 - 3$ bilan to'g'ri proporsional. $b = 5$ bo'lganda, $a = 88$ bo'lsa, $b = -3$ bo'lganda, a soni nechaga teng bo'ladi?
A) 24 B) 6 C) 18 D) 12
74. (03-8-21) $25\frac{1}{2}$ sonini 7; 8; 2 sonlariga mutanosib bo'laklarga bo'lgandagi eng kichik sonni toping.
A) 3 B) 4 C) 5 D) 3,5
75. (03-6-30) $\frac{5}{7}$ qismi 4 ga teng bo'lgan sonni toping.
A) $5\frac{6}{7}$ B) $5\frac{1}{5}$ C) $5\frac{2}{5}$ D) $5\frac{3}{5}$
- Yechish:** Izlanayotgan son x bo'lsin. U holda masala shartiga ko'ra, $x \cdot \frac{5}{7} = 4$ bo'ladi. Bu yerdan $x = 5\frac{3}{5}$ kelib chiqadi. **Javob:** $5\frac{3}{5}$ (D).
76. (03-6-33) 0,23 qismi 690 ga teng sonni toping.
A) 3000 B) 2500 C) 2800 D) 3500
77. (03-8-26) 0,4(6) qismi 360 sonining 0,6(4) qismiga teng sonni toping.
A) $497\frac{1}{7}$ B) $506\frac{2}{7}$ C) $400\frac{3}{7}$ D) $497\frac{5}{7}$

78. (97-10-10) Turist butun yo'lining 0,85 qismini o'tganda, ko'zlangan manzilgacha 6,6 km qolgani ma'lum bo'ldi. Butun yo'ning uzunligi necha km?
A) 52 km B) 44 km C) 36,6 km D) 64,4 km
Yechish: Butun yo'ning uzunligi x km bo'lsin. U holda masala shartiga ko'ra, $x \cdot 0,85 + 6,6 = x$ bo'ladi. Bu yerdan $6,6 = 0,15x \iff x = 44$ kelib chiqadi. **Javob:** 44 (B).
79. (96-7-10) Turist butun yo'lining 0,35 qismini o'tganda unga, yo'ning yarmigacha 18,3 km qolgani ma'lum bo'ldi. Butun yo'ning uzunligini toping.
A) 110 km B) 102 km C) 122 km D) 98 km
80. (97-3-10) Velosipedchi bir soatda butun yo'ning 0,65 qismini o'tdi, bu esa yo'ning yarmidan 7,5 km ko'p. Butun yo'ning uzunligini toping.
A) 47,5 km B) 62,5 km C) 50 km D) 65 km
81. (00-1-6) Sayohat uchun ma'lum miqdorda pul yig'ish kerak edi. Agar har bir sayohatchi 750 so'mdan to'lasa, to'lovga 1200 so'm yetmaydi, agar har bir sayohatchi 800 so'mdan to'lasa, keragidan 1200 so'm ortib qoladi. Sayohatga necha kishi qatnashishi kerak edi?
A) 38 B) 48 C) 45 D) 46
82. (01-2-10) Fermada tovuq va qo'ylar bor. Ularning jami soni 170 ta, oyoqlari soni 440 ta. Qo'ylar tovuqlardan nechta kam?
A) 50 B) 60 C) 70 D) 80
83. (01-8-2) Kostyum paltodan 5950 so'm arzon. Agar palto kostyumdan 1,7 marta qimmat bo'lsa, kostyum necha so'm turadi?
A) 8750 B) 7550 C) 3500 D) 8500
84. (01-10-10) 30 so'mlik va 35 so'mlik daftarlardan jami 490 so'mlik xarid qilindi. Quyida keltirilganlardan qaysi biri 30 so'mlik daftarlar soniga teng bo'lishi mumkin?
A) 5 B) 6 C) 7 D) 8
Yechish: 30 so'mlik daftardan x dona, 35 so'mlik daftardan y dona xarid qilingan bo'lsin. U holda masala shartiga ko'ra,
 $30x + 35y = 490 \iff 30x = 7(70 - 5y), x, y \in N$
bo'ladi. Bu tenglikning o'ng tomoni 7 ga karrali va demak, chap tomoni $30x$ ham 7 ga karrali bo'ladi. $30x$ ning 7 ga karrali ekanligidan x ning 7 ga karrali ekanligi kelib chiqadi. Masala shartlarini hisobga olsak x faqat 7 yoki 14 bo'lishi mumkin. **Javob:** 7 (C).
85. (02-6-16) 20 va 25 so'mlik daftarlardan hammasi bo'lib 350 so'mlik xarid qilindi. Quyida keltirilgan sonlardan qaysi biri 25 so'mlik daftarlarning soniga teng bo'lishi mumkin?
A) 4 B) 5 C) 6 D) 7
86. (02-1-34) Do'konga birinchi kuni 5,42 t ikkinchi kuni birinchi kundagiga qaraganda 2,43 t kam, uchinchi kuni esa dastlabki 2 kundagidan 3,21 t kam un keltirildi. Uchinchi kuni qancha un keltirilgan?
A) 13,61 B) 2,99 C) 7,85 D) 5,2
87. (02-3-4) Sinfda o'qiydigan o'g'il bolalar sonining barcha o'quvchilar soniga nisbati $\frac{4}{7}$ ga teng bo'lsa, qiz bolalar sonining o'g'il bolalar soniga nisbati nechaga teng bo'ladi?
A) $\frac{3}{4}$ B) $\frac{3}{5}$ C) $\frac{1}{2}$ D) $\frac{2}{5}$
88. (00-5-12) Sexda 120 ta samovar va 20 ta patnis yasalgan. Sarf qilingan hamma materialning 0,96 qismi samovarga ketgan. Agar har bir samovarning og'irligi 3,2 kg dan bo'lsa, har bir patnis necha kg bo'lgan?
A) 0,8 B) 0,04 C) 7,68 D) 0,768
89. (03-6-29) 2 o'ram bir xil sim xarid qilindi. Birinchi o'ram 3060 so'm, ikkinchi esa 1904 so'm turadi. Agar birinchi o'ram ikkinchi o'ramdan 17 m uzun bo'lsa, birinchi o'ramda necha m sim bor?
A) 40 B) 45 C) 47 D) 28
Yechish: 1 m simning bahosi x so'm, birinchi o'ramda y m uzunlikdagi sim bo'lsin. U holda ikkinchi o'ramda $y - 17$ m sim bo'ladi. Masala shartiga ko'ra,
$$\begin{cases} xy = 3060 \\ x(y - 17) = 1904. \end{cases}$$
Sistema 2-tenglamasini $xy - 17x = 1904$ shaklda yozib, 1-tenglikdan foydalansak, $3060 - 17x = 1904$ ni olamiz. Bu yerdan $x = 68$ kelib chiqadi. x ning bu qiymatini sistemaning 1-tenglamasiga qo'yib $y = 45$ hosil qilamiz. **Javob:** 45 (B).
90. (03-6-32) Binoni 3 ta bo'yoqchi birgalikda bo'yadi. Birinchi binoning $\frac{5}{13}$ qism yuzasini bo'yadi. Ikkinchi esa, uchinchisiga nisbatan 3 marta ko'p yuzani bo'yadi. Uchinchi bo'yoqchi qancha qism yuzani bo'yagan?
A) $\frac{1}{18}$ B) $\frac{1}{13}$ C) $\frac{1}{9}$ D) $\frac{2}{13}$
91. (03-10-19) 7 ta kitob va 4 ta jurnalning birgalikdagi bahosi, 4 ta kitob va 7 ta jurnalning birgalikdagi bahosidan 525 so'm ortiq. Kitob jurnalga qaraganda qancha so'm qimmat turishini aniqlang.
A) 150 B) 175 C) 200 D) 125
92. (00-8-2) Kitob betlarini sahifalab chiqish uchun 1012 ta raqam ishlatildi. Agar sahifalash 3-betdan boshlangan bo'lsa, kitob necha betlik?
A) 374 B) 400 C) 506 D) 421

9.2 Foizga oid masalalar

1. Ishchining maoshi a so'm. Uning maoshi $p\%$ ga oshirilsa, u $(1+p : 100) \cdot a$ so'm maosh oladi.
 2. Mahsulotning bahosi a so'm. Uning bahosi $p\%$ ga kamaytirilsa, u $(1-p : 100) \cdot a$ so'm turadi.
 3. Agar ishchining maoshi avval $p\%$ ga, so'ng-ra $q\%$ ga oshsa, ishchining dastlabki maoshi $p+q+pq : 100$ foizga oshadi.
 4. Agar mahsulotning bahosi avval $p\%$ ga, so'ng-ra $q\%$ ga kamaysa, mahsulotning dastlabki bahosi $p+q-pq : 100$ foizga kamayadi.
1. Talabani stipendiyasi 120 ming so'm. Agar uning stipendiyasi 20% ortsa, u necha ming so'm stipendiya oladi?
A) 140 B) 144 C) 142 D) 148
Yechish: 1-qoidaga ko'ra talabani stipendiyasi $(1+20 : 100)120 = 144$ ming so'm bo'ladi. **Javob:** 144 (B).
 - (96-11-4) Nafaqaxo'ring oylik nafaqasi 450 ming so'm. Agar uning nafaqasi 20% ortsa, u necha ming so'm nafaqa oladi?
A) 540 B) 470 C) 900 D) 490
 - (98-3-4) Ishchining oylik maoshi 350 ming so'm. Agar uning maoshi 30% ortsa, u necha ming so'm maosh oladi?
A) 405 B) 380 C) 1050 D) 455
 - (98-3-6) Ishchining maoshi dastlab 20% ga, so'ngra yana 20% oshirilgan bo'lsa, uning maoshi necha foizga oshgan?
A) 40 B) 50 C) 42 D) 44
Yechish: 3-qoidaga ko'ra ishchining maoshi $20+20+20 \cdot 20 : 100 = 44$ foizga oshadi. **Javob:** 44 (D).
 - (01-10-7) Ishlab chiqarish samaradorligi birinchi yili 15% ga, ikkinchi yili 12% ga ortdi. Shu ikki yil ichida samaradorlik necha foizga ortgan?
A) 27 B) 28 C) 28,6 D) 28,8
 - (01-5-7) Xodimning oylik maoshi ketma-ket ikki marta bir xil foizga oshirilgandan so'ng, u dastlabki maoshdan 69% ga oshgan bo'lsa, maosh har gal necha foizga oshirilgan?
A) 30 B) 34,5 C) 40 D) 35
 - (01-2-72) Xo'jalikda paxta ishlab chiqarish har yili 10% ga ortsa, 3 yilda paxta ishlab chiqarish necha foizga ortadi?
A) 30 B) 32 C) 33 D) 33,1
 - (00-1-3) Inflyasiya natijasida mahsulotning narxi 25% ga oshirildi. Lekin mahsulotga talabning kamligi tufayli uning narxi 10% ga kamaytirildi. Mahsulotning oxirgi narxi dastlabkisiga qaraganda

necha foiz ortdi?

- A) 12,8 B) 11,5 C) 12 D) 12,5
9. (00-7-6) Mahsulotning narxi birinchi marta 25% ga, ikkinchi marta yangi bahosi yana 20% oshirildi. Mahsulotning oxirgi bahosi necha foizga kamaytirilsa, uning narxi dastlabki narxiga teng bo'ladi?
A) 45 B) 48 C) 50 D) $33\frac{1}{3}$
 10. (96-1-4) Do'konga 96 t karam keltirildi. Agar karamning 80% i sotilgan bo'lsa, do'konda qancha karam qolgan.
A) 16 B) 19,2 C) 24 D) 20,2
 11. (96-6-3) Magazinga keltirilgan tarvuzlarning 56% i birinchi kuni, qolgan 132 tasi ikkinchi kuni sotildi. Birinchi kuni qancha tarvuz sotilgan?
A) 168 B) 148 C) 178 D) 138
 12. (96-1-9) Ikki sonning ayirmasi 33 ga teng. Agar shu sonlardan kattasining 30% i kichigining $\frac{2}{3}$ qis-miga teng bo'lsa, shu sonlarni toping.
A) 56; 23 B) 27; 60 C) 17; 50 D) 37; 70
 13. (96-10-9) Ikki sonning ayirmasi 5 ga teng. Agar shu sonlardan kattasining 20% i kichigining $\frac{2}{9}$ qis-miga teng bo'lsa, shu sonlarni toping.
A) 30; 35 B) 36; 41 C) 45; 50 D) 63; 68
 14. (96-9-59) Ikki sonning yig'indisi 24 ga teng. Agar shu sonlardan birining 85% ikkinchisining $\frac{7}{20}$ qis-miga teng bo'lsa, shu sonlarni toping.
A) 18; 6 B) 20; 4 C) 7; 17 D) 8; 16
 15. (02-12-1) Ikki sonning yig'indisi 24 ga teng. Shu sonlardan birining 35% i ikkinchisining 85% iga teng. Shu sonlardan kichigini toping.
A) 3,5 B) 7 C) 6 D) 9
 16. (96-12-63) Go'sht qaynatilganda o'z vaznining 40% ini yuqotadi. 6 kg go'sht qaynatilganda vazni necha kg kamayadi?
A) 2,4 B) 2,2 C) 1,9 D) 2
Yechish: 6 ning 40% ini hisoblaymiz:
$$\frac{6}{100} \cdot 40 = 2,4.$$
Demak, 2,4 kg kamayadi. **Javob:** 2,4 (A).
 17. (96-9-55) Olxo'ri quritilganda 35% olxo'ri qoqisi hosil bo'ladi. 64 kg olxo'ri quritilsa, gancha olxo'ri qoqisi olinadi?
A) 20 B) 18,2 C) 22,4 D) 25
 18. (99-7-5) Bug'doydan 90% un olinadi. 3 t bug'doydan qancha un olish mumkin.
A) 2,5 B) 2,6 C) 2,1 D) 2,7
 19. (98-5-3) Paxtadan 30% foiz tola olinsa, 60 tonna tola olish uchun qancha paxta kerak?
A) 100 B) 400 C) 200 D) 300

20. (07-118-26) 15 kg eritmaning 40 foizi tuzdan iborat. Tuzning miqdori 25 foiz bo'lishi uchun eritmaga necha kg chuchuk suv qo'shish kerak.
A) 6 B) 9 C) 8 D) 10
21. (97-4-5) 30 ta talabadan 25 tasi qishki sinovlarning hammasini topshirdi. Ba'zi sinovlarni topshira olmagan talabalar barcha sinovlarni topshirgan talabalarning necha foizini tashkil etadi?
A) 10% B) 15% C) 20% D) 25%
22. (97-5-5) Qutiga 25 kg massali yuk joylandi. Agar qutining massasi yuk massasining 12% ini tashkil etsa, qutining massasini toping.
A) 3 kg B) 3,5 kg C) 4 kg D) 4,5 kg
23. (97-6-5) Noma'lum sonning 14% i 80 ning 35% iga teng. Noma'lum sonni toping.
A) 120 B) 168 C) 200 D) 280
24. (97-8-3) Kutubxonadagi kitoblarning 55% i o'zbek tilida, qolgan kitoblar rus tilida. Rus tilidagi kitoblar 270 ming ta. Kutubxonada o'zbek tilida necha ming kitob bor?
A) 325 B) 310 C) 320 D) 330
25. (98-1-2) Ushbu $2\frac{3}{5}; \frac{1}{2}$ sonlar ayirmasining 10% ini toping.
A) 0,22 B) 0,3 C) 0,021 D) 0,21
26. (98-10-5) 21 kg shakar va 129 kg boshqa mahsulotlardan muzqaymoq tayyorlandi. Shakar muzqaymoqning necha foizini tashkil etadi?
A) 13 B) 15 C) 16 D) 14
27. (99-1-5) Institutdagi talabalarning 35% ini qizlar tashkil qiladi. Yigitlar qizlardan 252 taga ko'p. Talabalarning umumiy sonini toping.
A) 840 B) 640 C) 546 D) 740
- Yechish:** Masala shartidan talabalarning 65% i yigitlar ekanligi kelib chiqadi. Yigitlar qizlardan 30% ga ko'p. Agar jami talabalar sonini x desak, u holda $\frac{x}{100} \cdot 30 = 252$ bo'ladi. Bu yerdan $x = 840$.
Javob: 840 (A).
28. (99-1-6) 520 sonini shunday ikki bo'lakka bo'lingki, ulardan birining 80% i ikkinchisining 24% ini tashkil qilsin. Bo'laklarning kattasini toping.
A) 400 B) 120 C) 420 D) 460
29. (99-3-3) 200 soni 30 foizga orttirildi, hosil bo'lgan son 20 foizga kamaytirildi. Natijada qanday son hosil bo'ladi?
A) 206 B) 210 C) 208 D) 212
30. (00-9-57) Agar ikkita sondan biri 20% ga, ikkinchisi 12,5% ga kamaytirilsa, ularning ko'paytmasi necha foizga kamayadi?
A) 40 B) 50 C) 45 D) 30
31. (01-1-63) Agar sonning 40% ini 5 ga ko'paytirganda 8 chiqsa, shu sonning o'zini toping.
A) 2 B) 4 C) 6 D) 8
32. (01-2-71) Uy bekasi kilosi 1500 so'mdan yong'oq sotib oldi. Yong'oqlar qobig'idan tozalangach, umumiy og'irligining 60% i qoldi. Uy bekasi bir kilogramm tozalangan yong'oq uchun necha so'm sarflagan?
A) 1900 B) 1800 C) 2200 D) 2500
33. (03-1-64) Kilosi 6000 so'mdan baliq sotib olindi. Tozalangandan keyin baliqning og'irligi dastlabki og'irligining 80% ini tashkil etdi. 1 kg tozalangan baliq necha so'mga tushgan?
A) 7800 B) 6500 C) 6400 D) 7500
34. (01-10-5) 17 ning 17% ini toping.
A) 1 B) 3,24 C) 2,89 D) 10
35. (02-6-12) 19 ning 19% ini toping.
A) 1 B) 2,89 C) 3,69 D) 3,61
36. (01-12-26) Firma mahsulotni 380 so'mga sotib, 4 foiz zarar qildi. Shu mahsulotning tannarxini toping.
A) 400 B) 495 C) $395\frac{5}{6}$ D) $395\frac{1}{6}$
- Yechish:** Mahsulotning tannarxi x bo'lsin, u holda $x - \frac{x}{100} \cdot 4 = 380$ bo'ladi. Bu yerdan $x = 395\frac{5}{6}$. **Javob:** $395\frac{5}{6}$ (C).
37. (02-1-36) Kitob 2000 so'm turadi. Uning narxi 2 marta 5% dan arzonlashtirildi. Kitobning narxi necha so'm bo'ldi?
A) 1800 B) 1802 C) 1803 D) 1805
38. (02-2-7) Mehnat unumdorligi 40% oshgach, korxonada kuniga 560 ta buyum ishlab chiqaradigan bo'ldi. Korxonada oldin kuniga nechta buyum ishlab chiqargan?
A) 400 B) 420 C) 380 D) 440
39. (02-10-44) 6 foizi 30 ning 22 foiziga teng bo'lgan sonni toping.
A) 110 B) 108 C) 96 D) 90
40. (03-2-36) 12% ga arzonlashtirilgandan keyin mahsulotning bahosi 1100 so'm bo'ldi. Mahsulotning dastlabki bahosini aniqlang.
A) 1200 B) 1240 C) 1280 D) 1250
41. (01-6-4) x ($x > 0$) ga teskari bo'lgan son x ning 36% ini tashkil etadi. x ning qiymatini toping.
A) $2\frac{1}{3}$ B) $1\frac{2}{3}$ C) $1\frac{1}{3}$ D) $3\frac{1}{3}$
- Yechish:** Masala shartidan $\frac{1}{x} = \frac{36x}{100}$ tenglikni olamiz. Bu yerdan $x^2 = \frac{100}{36} \iff x = \frac{10}{6} = 1\frac{2}{3}$. **Javob:** $1\frac{2}{3}$ (B).
42. (03-10-23) Biznesmen o'z pulining 50% ini yo'qotdi. Qolgan puliga aksiya sotib olgach, u 40% daromad (foyda) oldi. Uning oxirgi puli dastlabki pulining necha foizini tashkil etadi?
A) 60 B) 70 C) 80 D) 100

9.3 Harakatga oid masalalar

1. S masofa v tezlik bilan t vaqtda bosib o'tilsa, $S = vt$ bo'ladi.

2. A va B punktlar orasidagi masofa S bo'lsin.

a) A va B lardan bir-biriga qarab ikki yo'lovchi v_1 va v_2 tezliklar bilan yo'lga chiqib, t vaqtdan keyin uchrashsa, $(v_1 + v_2)t = S$ bo'ladi.

b) A va B lardan bir tomonga ikki yo'lovchi v_1 va v_2 tezliklar bilan yo'lga chiqib, t vaqtdan keyin 1 - yo'lovchi 2- sga yetib olsa, $v_1t - v_2t = S$ bo'ladi.

3. Agar tezlik $p\%$ oshsa belgilangan masofani bosib o'tish uchun ketadigan vaqt $100p : (100 + p)$ foizga qisqaradi.

1. (97-12-6) Motosiklchi va velosipedchi bir tomonga qarab harakat qilishmoqda. Velosipedchining tezligi 12 km/soat, motosiklchining 30 km/soat va ular orasidagi masofa 72 km bo'lsa, necha soatdan keyin motosiklchi velosipedchini quvib yetadi?
A) 3 B) 4 C) 3,5 D) 2,5

Yechish: Motosiklchi velosipedchini t soatdan keyin quvib yetsin. t soatda motosiklchi $30t$ km, velosipedchi esa $12t$ km masofa bosib o'tadi. Bu yerdan $30t - 12t = 72$ tenglamani hosil qilamiz. Uni yechib $t = 4$ ekanini topamiz. **Javob:** 4 (B).

2. (96-3-3) Passajir va yuk poyezdi bir-biriga tomon harakatlanmoqda. Ular orasidagi masofa 275 km. Yuk poyezdining tezligi 50 km/soat. Passajir poyezdining tezligi yuk poyezdining tezligidan 20% ortiq. Ular necha soatdan keyin uchrashadi?
A) 3 B) 2 C) 2,5 D) 4

3. (96-3-69) Uzunligi 400 m bo'lgan poyezd uzunligi 500 m bo'lgan tunneldan 30 s da o'tib ketdi. Poyezdning tezligini toping.
A) 35 m/s B) 30 m/s C) 40 m/s D) 45 m/s

4. (96-6-7) Ikki shahardan bir-biriga qarab ikki turist yo'lga chiqdi. Birinchisi avtomashinada, tezligi 62 km/soat. Ikkinchisi avtobusda tezligi 48 km/soat. Agar ular 0,6 soatdan keyin uchrashgan bo'lsa, shaharlar orasidagi masofani toping.
A) 70 km B) 64 km C) 62 km D) 66 km

5. (96-9-9) Poyezdning uzunligi 800 m. Poyezdning ustun yonidan 40 s da o'tib ketgani ma'lum bo'lsa, uning tezligini toping.
A) 30 m/s B) 15 m/s C) 25 m/s D) 20 m/s

6. (99-3-9) Yo'lovchilar poyezdining 3 soatda yurgan masofasi yuk poyezdining 4 soatda yurgan masofasidan 10 km ortiq. Yuk poyezdining tezligi yo'lovchilar poyezdining tezligidan 20 km/soat ga kam. Yuk poyezdining tezligini toping.
A) 40 B) 45 C) 48 D) 50

Yechish: Yuk poyezdining tezligi v bo'lsin. U holda yo'lovchilar poyezdining tezligi $v+20$ bo'ladi.

Yo'lovchilar poyezdi 3 soatda $3(v + 20)$ km masofani bosib o'tadi, yuk poyezdi 4 soatda $4v$ km masofani bosib o'tadi. Masala shartiga ko'ra

$$3(v + 20) = 4v + 10.$$

Bu yerdan $v = 50$ ni olamiz. **Javob:** 50 (D).

7. (97-8-7) Oralaridagi masofa 200 km bo'lgan A va B punktlardan bir vaqtning o'zida ikki turist bir-biriga qarab yo'lga chiqdi. Birinchisi avtobusda tezligi 40 km/soat, ikkinchisi avtomobilda. Agar ular 2 soatdan keyin uchrashgan bo'lishsa, avtomobilning tezligini toping.
A) 58 km/soat B) 55 km/soat
C) 65 km/soat D) 60 km/soat

8. (97-10-4) Muayyan masofani bosib o'tish uchun ketadigan vaqtni 25% ga kamaytirish uchun tezlikni necha foiz orttirish kerak?
A) 25 B) 20 C) $33\frac{1}{3}$ D) 30

9. (99-9-4) A va B shaharlar orasidagi masofa 188 km. Bir vaqtning o'zida bir-biriga qarab A shahardan velosipedchi, B shahardan motosiklchi yo'lga tushdi va ular A shahardan 48 km masofada uchrashdi. Agar velosipedchining tezligi 12 km/soat bo'lsa, motosiklchining tezligini toping.
A) 45 B) 42 C) 30 D) 35

10. (01-10-15) Uzunligi 200 m bo'lgan poyezd balandligi 40 m bo'lgan ustun yonidan 50 sekundda o'tib ketdi. Uzunligi 520 m bo'lgan ko'prikdan shu poyezd o'sha tezlik bilan necha minutda o'tib ketadi?
A) 2 B) 2,5 C) 3 D) 4

11. (96-11-3) Kater va teploxod bir-biriga tomon harakatlanmoqda. Ular orasidagi masofa 120 km. Teploxodning tezligi 50 km/soat. Katerning tezligi teploxodning tezligidan 60% kam. Ular necha soatdan keyin uchrashadi?
A) $1\frac{5}{7}$ B) 2 C) $2\frac{1}{4}$ D) $2\frac{1}{3}$

12. (98-2-7) A va B pristanlar orasidagi masofa 96 km. A pristanidan oqim bo'ylab sol jo'natildi. Xuddi shu paytda B pristanidan oqimga qarshi motorli qayiq jo'nadi va 4 soatdan keyin sol bilan uchrashdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligini toping.
A) 20 km/soat B) 19 km/soat
C) 17 km/soat D) 24 km/soat

Yechish: Solning tezligi daryo oqimining tezligi bilan bir xil, ya'ni 3 km/s. Qayiqning turg'un suvdagi tezligi v km/s bo'lsin, u holda uning oqimga qarshi tezligi $v - 3$ km/s bo'ladi. 2-qoidaning a) bandiga ko'ra $(3 + v - 3) \cdot 4 = 96$ ni olamiz. Bu yerdan $v = 24$. **Javob:** 24 (D).

13. (98-9-6) Ikki pristan orasidagi masofa 63 km. Bir vaqtning o'zida oqim bo'ylab birinchi pristanidan

sol, ikkinchisidan motorli qayiq jo'natildi va motorli qayiq solni 3 soatda quvib yetdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligi qanchaga teng bo'ladi?

A) 21 B) 20 C) 22 D) 19

14. (01-9-34) Matorli qayiqning daryo oqimi bo'yicha tezligi 21 km/soat dan ortiq va 23 km/soat dan kam. Oqimga qarshi tezligi esa 19 km/soat dan ortiq va 21 km/soatdan kam. Qayiqning turg'un suvdagi tezligi qanday oraliqda bo'ladi?

A) (18;20) B) (19;21) C) (18;19) D) (20;22)

15. (02-1-2) Katerning daryo oqimi bo'ylab va oqimga qarshi tezliklari yig'indisi 30 km/soat. Katerning turg'un suvdagi tezligi (km/soat)ni toping.

A) 15 B) 16 C) 10 D) 18

16. (03-3-10) Paroxod daryo oqimi bo'ylab 48 km va oqimga qarshi shuncha masofani 5 soatda bosib o'tdi. Agar daryo oqimining tezligi soatiga 4 km bo'lsa, Paroxodning turg'un suvdagi tezligini toping.

A) 12 B) 16 C) 20 D) 24

17. (03-6-10) Avtomobil butun yo'lining $\frac{3}{7}$ qismini 1 soatda, qolgan qismini 1,5 soatda bosib o'tdi. Uning birinchi tezligi ikkinchi tezligidan necha marta katta?

A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{9}{8}$ D) $\frac{8}{9}$

18. (03-7-15) Avtomobil butun yo'lining $\frac{3}{7}$ qismini 1 soatda, qolgan qismini 2 soatda bosib o'tdi. Uning birinchi tezligi ikkinchi tezligidan necha marta katta?

A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) $\frac{9}{8}$ D) $\frac{8}{9}$

9.4 Ishga oid masalalar

1. Agar 1-kombayn hosilni x soatda, 2-kombayn y soatda, ikkala kombayn birgalikda hosilni z soatda yig'ib olsa, u holda

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}. \quad (9.1)$$

1. (98-10-11) Bir kombayn daladagi hosilni 15 soatda, boshqasi esa shu hosilni 10 soatda yig'ib olishi mumkin. Ikkala kombayn birgalikda hosilni qancha soatda yig'ib olishi mumkin?

A) 7 B) 8 C) 5,5 D) 6

Yechish: (9.1) tenglikda $x = 15$, $y = 10$ deb z ni topamiz:

$$\frac{1}{z} = \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}, \quad z = 6.$$

Javob: $z = 6$ (D).

2. (96-3-67) Meshdagi suv Anvarning o'ziga 20 kunga, ukasiga esa 60 kunga yetadi. Meshdagi suv ikkalasiga necha kunga yetadi?

A) 15 B) 14 C) 12 D) 16

3. (96-9-7) Meshdagi suv Anvarning o'ziga 14 kunga, ukasi ikkalasiga esa 10 kunga yetadi. Meshdagi suv Anvarning ukasiga necha kunga yetadi?

A) 35 B) 39 C) 28 D) 26

4. (96-12-8) Birinchi kuni ish normasining $\frac{1}{3}$ qismi bajarildi. Ikkinchi kuni birinchi kunda bajarilgan ishning $\frac{1}{6}$ qismicha ko'p ish bajarildi. Shu ikki kunda qancha ish normasi bajarildi?

A) 0,5 B) $\frac{2}{9}$ C) $\frac{13}{18}$ D) $\frac{5}{6}$

5. (98-12-73) Birinchi quvur hovuzni 3 soatda to'ldiradi, ikkinchisi esa 5 soatda. Ikkala quvur birgalikda hovuzni qancha vaqtda to'ldiradi?

A) $1\frac{7}{8}$ B) $2\frac{1}{2}$ C) $2\frac{1}{5}$ D) $1\frac{4}{5}$

6. (99-2-7) Hovuzga 2 ta quvur o'tkazilgan. Birinchi quvur bo'sh hovuzni 10 soatda to'ldiradi, ikkinchisi esa 15 soatda bo'shatadi. Hovuz bo'sh bo'lgan vaqtda ikkala quvur birdaniga ochilsa, hovuz necha soatdan keyin to'ladi?

A) 25 B) 28 C) 30 D) 32

7. Ali o'rtog'i bilan ishning 20% bajardi. Keyin bir o'zi 4 kun ishlab qolgan ishning 25% ni bajardi. Ali bu ishning hammasini necha kunda qila oladi?

A) 20 B) 25 C) 30 D) 16

Yechish: Ali 4 kunda qolgan 80% ishning 25% ni bajardi. Demak, u 4 kunda jami ishning

$$\frac{0,8 \cdot 25}{100} = \frac{20}{100} = 20\%$$

ni bajardi. Bu yerdan Ali 20 kunda ishning 100% ni bajarishi kelib chiqadi. **Javob:** 20 (A).

8. (00-4-19) Usta muayyan ishni 12 kunda, uning shogirdi esa 30 kunda bajaradi. Agar 3 ta usta va 5 ta shogird birga ishlasalar, o'sha ishni necha kunda bajarishadi?

A) 2,4 B) 3,6 C) 2,5 D) 1,2

9. (00-7-9) Muayyan ishni bajarishga bir ishchi 3 soat, ikkinchi ishchi esa 6 soat vaqt sarflaydi. Birinchi ishchi 1 soat ishlagandan keyin, unga ikkinchi ishchi qo'shildi. Ikkala ishchi birgalikda qolgan ishni necha soatda tugatishadi?

A) 2 soat 30 min B) 1 soat 40 min

C) 1 soat 20 min D) 2 soat

10. (03-9-7) Ikkita ishchi birgalikda ishlab, ma'lum ishni 12 kunda tamomlaydi. Agar ishchilarning bittasi shu ishning yarmini bajargandan keyin, ikkinchi ishchi qolgan yarmini bajarsa, shu ishni 25 kunda tamomlashi mumkin. Ishchilardan biri boshqasiga qaraganda necha marta tez ishlaydi?

A) 1,2 B) 1,5 C) 1,6 D) 1,8

10 -bob. Funktsiyalar

Tabiatda ikki xil miqdorlar, o'zgaruvchi va oz'garmas miqdorlar uchraydi. Masalan, bizga bir nechta aylana (har xil radiusli) berilgan bo'lsin. Barcha aylanalar uchun aylana uzunligining o'z radiusiga nisbati o'zgarmas bo'lib u 2π ga teng, lekin ularning diametrlari, aylana uzunliklari radius o'zgarishi bilan, o'zgarib turadi. Agar bizga har xil (katta, kichik) kvadratlar berilgan bo'lsa, bu kvadratlarining yuzalari tomoni o'zgarishi bilan o'zgarib turadi, lekin ularning burchaklari 90° ligicha o'zgarimasdan qolaveradi. Odatda o'zgarmas miqdorlar a, b, c, d, \dots ; o'zgaruvchi miqdorlar x, y, z, u, v, \dots harflari bilan belgilanadi. Matematikada ko'pincha o'zaro bir-biriga bog'liq ravishda o'zgaradigan miqdorlar qaraladi. Yuqoridagi misolimizda aylananing uzunligi ℓ bilan uning radiusi R orasida $\ell = 2\pi R$ bog'lanish bor. Ma'lumki, kvadratning yuzasi uning tomoni kvadratiga teng, ya'ni kvadratning yuzasini S , uning tomoni uzunligini a desak, u holda ular orasida $S = a^2$ bog'lanish mavjud.

Agar x miqdorning har bir qiymatiga y miqdorning yagona qiymati mos kelsa, y miqdor x miqdorning funksiyasi deyiladi. Bu holda x – argument yoki erkli o'zgaruvchi, y – esa funksiya yoki erksiz o'zgaruvchi deyiladi. x va y miqdorlar o'rtasidagi bog'lanishni o'rnatuvchi moslik f orqali belgilanadi va quyidagicha yoziladi: $y = f(x)$. Argumentning qabul qilishi mumkin bo'lgan qiymatmatlari to'plami funksiyaning aniqlanish sohasi, funksiyaning o'zi qabul qilishi mumkin bo'lgan qiymatmatlari to'plami funksiyaning o'zgarish sohasi yoki qiymatlar to'plami deyiladi. f funksiyaning aniqlanish sohasi $D(f)$ bilan, uning qiymatlar to'plami $E(f)$ bilan belgilanadi. Funksiya analitik, jadval va grafik usullar bilan berilishi mumkin. Agar moslik biror formula yordamida berilgan bo'lsa, funksiya analitik usulda berilgan deyiladi. Masalan,

$$1) y = 3x; \quad 2) y = x^2; \quad 3) y = \sqrt{5-x}; \quad 4) y = \frac{x^3 + 8}{x-2}$$

funksiyalar analitik usulda berilgan. Agar analitik usulda berilgan funksiyaning aniqlanish sohasi to'g'risida alohida shart qo'yilmagan bo'lsa, u holda $y = f(x)$ da o'ng tomonda turuvchi ifoda ma'noga ega bo'ladigan x ning barcha qiymatlari olinadi. Yuqorida keltirilgan 1- va 2-funksiyalarning aniqlanish sohasi haqiqiy sonlar to'plami, ya'ni $D(f) = R$ dir. 3-funksiyaning aniqlanish sohasi $D(f) = (-\infty; 5]$ dan, 4-funksiyaning aniqlanish sohasi $D(f) = (-\infty; 2) \cup (2; \infty)$ dan iborat.

Juft va toq funksiyalar.

Agar $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ dan $-x \in D(f)$ ekanligi kelib chiqsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(-x) = f(x)$ bo'lsa, f ga *juft funksiya* deyiladi.

Agar $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ dan $-x \in D(f)$ ekanligi kelib chiqsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(-x) = -f(x)$ bo'lsa, f ga *toq funksiya* deyiladi. $y = 2x$ – toq, $y = 3x^2$ – juft funksiya misol bo'ladi.

Agar f juft funksiya bo'lsa, uni f_+ bilan, agar f

toq funksiya bo'lsa, uni f_- bilan belgilaymiz. Juft va toq funksiyalar quyidagi xossalarga ega:

- Juft funksiyaning songa ko'paytmasi** $\alpha \cdot f_+ = \varphi_+$ **juft, toq funksiyaning songa ko'paytmasi** $\alpha \cdot f_- = \varphi_-$ **toq funksiya bo'ladi.**
- Juft funksiyalar yig'indisi** $f_+ + g_+ = \varphi_+$ **va ayirmasi** $f_+ - g_+ = \psi_+$ **juft funksiya bo'ladi.**
- Juft funksiyalar ko'paytmasi** $f_+ \cdot g_+ = \varphi_+$ **va nisbati** $f_+ : g_+ = \psi_+$, $g \neq 0$ **juft funksiya bo'ladi.**
- Noldan farqli juft va toq funksiyalar ko'paytmasi** $f_+ \cdot g_- = \varphi_-$ **va nisbati** $f_+ : g_- = \psi_-$ **toq funksiya bo'ladi.**
- Noldan farqli toq va juft funksiyalar ko'paytmasi** $f_- \cdot g_+ = \varphi_-$ **va nisbati** $f_- : g_+ = \psi_-$ **toq funksiya bo'ladi.**
- Juft funksiyaning ixtiyoriy natural darajasi** $f_+^n = \varphi_+$, $n \in N$ **juft funksiya bo'ladi.**
- Toq funksiyalar yig'indisi** $f_- + g_- = \varphi_-$ **va ayirmasi** $f_- - g_- = \psi_-$ **toq funksiya bo'ladi.**
- Noldan farqli toq funksiyalar ko'paytmasi** $f_- \cdot g_- = \varphi_+$ **va nisbati** $f_- : g_- = \psi_+$ **juft funksiya bo'ladi.**
- Toq funksiyaning ixtiyoriy juft natural darajasi** $f_-^{2n} = \varphi_+$, $n \in N$ **juft funksiya bo'ladi.**
- Toq funksiyaning ixtiyoriy toq natural darajasi** $f_-^{2n-1} = \varphi_-$, $n \in N$ **toq funksiya bo'ladi.**
- Noldan farqli juft va toq funksiyalar yig'indisi** $f_+ + g_-$ **na toq na juft funksiya bo'ladi.**
- Noldan farqli juft va toq funksiyalar ayirmasi** $f_+ - g_-$ **na toq na juft funksiya bo'ladi.**

Davriy funksiyalar.

Shunday $T \neq 0$ soni mavjud bo'lib, $y = f(x)$ quyidagi ikki shartni qanoatlantirsa: 1) ixtiyoriy $x \in D(f)$ uchun $T + x \in D(f)$ bo'lsa, 2) ixtiyoriy $x \in D(f)$ uchun $f(T + x) = f(x)$ bo'lsa, f ga *davriy funksiya* deyiladi. Bu holda T soni $y = f(x)$ funksiyaning davri deyiladi. Davriy funksiya misol sifatida x ning kasr qismi, ya'ni $f(x) = \{x\}$ ni olish mumkin. Bu funksiyaning davri $T = 1$. Haqiqatan ham $f(x+1) = \{x+1\} = \{x\} = f(x)$. Agar f va g lar T davrli funksiyalar bo'lsa, u holda ularning yig'indisi, ayirmasi, ko'paytmasi va nisbati ham T davrli funksiyalar bo'ladi. Keyinchalik biz ko'rsatamizki, trigonometrik funksiyalardan $\sin x$ va $\cos x$ funksiyalar 2π davrli, $\operatorname{tg} x$ va $\operatorname{ctg} x$ funksiyalar π davrli funksiyalar bo'ladi. Agar f va g larning davrlari o'lehdosh bo'lmasa, ularning yig'indisi (ayirmasi) davriy funksiya bo'lmaydi. Masalan, $\varphi(x) = \{x\} + \sin x$ davriy funksiya emas. Ammo $f(x) = \{x\}$ davriy funksiya bo'lib, davri $T = 1$, $g(x) = \sin x$ esa 2π davrli funksiya bo'ladi.

Monoton funksiyalar.

$[a; b]$ kesmada aniqlangan $y = f(x)$ funksiya, shu kesmadan olingan har qanday x_1, x_2 lar uchun $x_1 < x_2$ bo'lganda $f(x_1) < f(x_2)$ bo'lsa, f ga $[a; b]$ kesmada *o'suvchi funksiya* deyiladi. $[a; b]$ kesmada aniqlangan $y = f(x)$ funksiya, shu kesmadan olingan har qanday x_1, x_2 lar uchun $x_1 < x_2$ bo'lganda $f(x_1) > f(x_2)$ bo'lsa, f ga $[a; b]$ kesmada *kamayuvchi funksiya* deyiladi. Berilgan kesmada faqat o'suvchi yoki kamayuvchi bo'lgan funksiyalar *monoton funksiyalar* deyiladi. $y = 2x$ funksiya $[-1; 0]$ kesmada o'suvchi, $y = x^2$ funksiya esa $[-1; 0]$ da kamayuvchi funksiyaga misol bo'ladi. Agar f va g funksiyalar $[a; b]$ kesmada o'suvchi bo'lsa, ularning yig'indisi ham $[a; b]$ da o'suvchi bo'ladi. Agar f va g funksiyalar $[a; b]$ kesmada kamayuvchi bo'lsa, ularning yig'indisi ham $[a; b]$ da kamayuvchi funksiya bo'ladi. Ammo o'suvchi funksiyalarning ayirmasi o'suvchi bo'lmashligi mumkin. Masalan, $f(x) = x^2 + 1$ funksiya $[0; 2]$ kesmada o'suvchi, $g(x) = 2x$ ham $[0; 2]$ kesmada o'suvchi. Ularning ayirmasi bo'lgan $\varphi(x) = x^2 + 1 - 2x = (x - 1)^2$ funksiya $[0; 1]$ da kamayuvchi, $[1; 2]$ kesmada o'suvchi. Demak, $\varphi(x)$ funksiya $[0; 2]$ kesmada monoton emas.

1. (96-9-10) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\frac{x(x+1)}{(x-2)(4-x)}}$$

- A) $[-1; 0] \cup (2; 4)$ B) $(-1; 0) \cup [2; 4]$
 C) $(-1; 0] \cup [2; 4)$ D) $(-\infty; -1) \cup (0; 2) \cup (4; \infty)$

Yechish: Kvadrat ildiz ma'noga ega bo'lishi uchun ildiz ostidagi ifoda nomanfiy bo'lishi kerak, ya'ni

$$\frac{x(x+1)}{(x-2)(4-x)} \geq 0.$$

Uni oraliqlar usuli bilan yechib $x \in [-1; 0] \cup (2; 4)$ ekanini hosil qilamiz. **Javob:** $[-1; 0] \cup (2; 4)$ (A).

2. (96-3-16) $f(x) = \frac{x-2}{x^2-1}$ funksiyaning aniqlanish sohasini toping.

- A) $(0; \infty)$ B) $(-\infty; 1) \cup (1; \infty)$
 C) $(-\infty; \infty)$ D) $(-\infty; -1) \cup (-1; 1) \cup (1; \infty)$

3. (96-12-17) $f(x) = \frac{x+2}{x^2-1}$ funksiyaning aniqlanish sohasini toping.

- A) $(-\infty; \infty)$ B) $(-\infty; -1) \cup (-1; 1) \cup (1; \infty)$
 C) $(-\infty; 0)$ D) $(0; \infty)$

4. (99-1-12) $y = \frac{2x-3}{x(x+2)}$ funksiyaning aniqlanish sohasini toping.

- A) $(-\infty; -2) \cup (-2; 0) \cup (0; \infty)$
 B) $(-\infty; 0) \cup (2; \infty)$
 C) $(-\infty; -2) \cup (0; \infty)$
 D) $(-\infty; 1, 5) \cup (1, 5; \infty)$

5. (00-6-21) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\frac{x^2 - 4x + 4}{1 - x^2}}$$

- A) $(-1; 1)$ B) $(-1; 1) \cup \{2\}$
 C) $(-1; 2)$ D) $(-\infty; -1) \cup \{2\}$

6. (96-3-70) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\frac{(x-1)(3-x)}{x(4-x)}}$$

- A) $[0; 1) \cup [3; 4)$ B) $(0; 1] \cup [3; 4)$
 C) $(0; 1] \cup (3; 4)$ D) $(-\infty; 0) \cup [1; 3] \cup (4; \infty)$

7. (96-13-10) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\frac{(x-2)(4-x)}{x(x+1)}}$$

- A) $(-1; 0) \cup [2; 4]$ B) $[-1; 0] \cup (2; 4)$
 C) $(-1; 0] \cup [2; 4)$ D) $(-\infty; -1) \cup (0; 2] \cup [4; \infty)$

8. (99-4-23) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{x^2 - 9} + \frac{2}{\sqrt{-x}}$$

- A) $(0; 3)$ B) $[-3; 0)$ C) $(-\infty; 0)$ D) $(-\infty; -3]$

9. (99-6-46) $y = \sqrt{3x - x^3}$ funksiyaning aniqlanish sohasini toping.

- A) $(-\infty; -\sqrt{3}] \cup [0; \sqrt{3}]$ B) $(-\infty; -\sqrt{3}) \cup (0; \sqrt{3})$
 C) $[0; \sqrt{3})$ D) $(-\infty; \sqrt{3}) \cup (\sqrt{3}; \infty)$

10. $y = [x]$ funksiyaning qiymatlar sohasini toping.

- A) N B) $(-\infty; \infty)$ C) Z D) $0, 1$

Yechish: Berilgan funksiyaning aniqlanish sohasi $R = (-\infty; \infty)$ dan iborat. Har bir $x \in R$ da $f(x) \in Z$ ekanligidan $E(f) \subset Z$ ni, ixtiyoriy $n \in Z$ uchun $f(n) = n$ ekanligidan $Z \subset E(f)$ ni olamiz. Bu ikki munosabatdan $E(f) = Z$ ekanligi kelib chiqadi. **Javob:** Z (C).

11. $y = \{x\}$ funksiyaning qiymatlar sohasini toping.

- A) N B) $[0; 1)$ C) $[0; 1]$ D) $0, 1$

12. $y = x^2$ funksiyaning qiymatlar sohasini toping.

- A) $(0; \infty)$ B) $(-\infty; \infty)$ C) $[0; \infty)$ D) $(2; \infty)$

13. $y = \sqrt{x^2 + 4}$ funksiyaning qiymatlar sohasini toping.

- A) $(0; \infty)$ B) $[2; \infty)$ C) $(2; \infty)$ D) $(-\infty; 2)$

14. $y = 7 - x^2$ funksiyaning qiymatlar sohasini toping.

- A) $(7; \infty)$ B) $(-\infty; 7)$ C) $(-\infty; 7]$ D) $(0; \infty)$

15. (98-11-66) Ushbu $y = \sqrt{9 - x^2}$ funksiyaning qiymatlar sohasini ko'rsating.

- A) $(-\infty; \infty)$ B) $[-3; 3]$ C) $[0; \infty)$ D) $[0; 3]$

16. (96-7-26) Quyidagi funksiyalardan qaysi biri juft?

A) $g(x) = \frac{5x^3}{(x-3)^2}$ B) $g(x) = \frac{x(x-2)(x-4)}{x^2 - 6x + 8}$

C) $g(x) = \frac{9x^2}{x^2 - 25}$ D) $g(x) = x^2 + |x + 1|$

Yechish: $9x^2$ juft funksiya, $x^2 - 25$ ham juft funksiya, ularning nisbati 3-xossaga ko'ra juft funksiya bo'ladi. **Javob:** (C).

17. (97-3-26) Quyidagi funksiyalardan qaysi biri toq?

$$\begin{array}{ll} \text{A) } y = \frac{5x^3}{(x-3)^2} & \text{B) } y = \frac{x(x-4)(x-2)}{x^2-6x+8} \\ \text{C) } y = \frac{9x^2}{x^2-25} & \text{D) } y = \frac{x^4-2x^2}{3x} \end{array}$$

18. Quyidagi funksiyalardan qaysi juft emas.

$$\begin{array}{ll} \text{A) } y = 5x^4 - 7x^8 & \text{B) } y = 2|x| \\ \text{C) } y = \frac{9x^3}{x^3-x} & \text{D) } y = \frac{x^4-8x^2}{7x} \end{array}$$

19. Quyidagi funksiyalardan qaysi toq emas.

$$\begin{array}{ll} \text{A) } y = x + 1 & \text{B) } y = 4x^9 - 8x^7 \\ \text{C) } y = x|x| & \text{D) } y = x(\sqrt{1+x} + \sqrt{1-x}) \end{array}$$

20. Juft ham, toq ham bo'lmagan funksiyani toping.

$$\begin{array}{ll} \text{A) } y = x^8 - 7x^{100} & \text{B) } y = 2x - 5 \\ \text{C) } y = |x| + 5x^{64} & \text{D) } y = x \end{array}$$

21. Juft ham, toq ham bo'lmagan funksiyani toping.

$$\begin{array}{ll} \text{A) } y = x & \text{B) } y = [x] \\ \text{C) } y = |x| & \text{D) } y = x^2 \end{array}$$

22. (99-1-13) $y = x|x|$, $x \in R$ funksiya uchun qaysi tasdiq to'g'ri?

- A) toq funksiya
B) juft funksiya
C) kamayuvchi funksiya
D) juft funksiya ham, toq funksiya ham emas

23. $[0; \infty)$ da o'suvchi funksiyani toping.

$$\begin{array}{ll} \text{A) } y = 1 - x & \text{B) } y = (x-5)^2 \\ \text{C) } y = x|x| & \text{D) } y = 7 - x^3 \end{array}$$

Yechish: Berilgan funksiyalar $[0; \infty)$ da aniqlangan, ya'ni $x \geq 0$. Bu shartda $y = x|x| = x^2$. Bu funksiyani ta'rif yordamida, $[0; \infty)$ da o'suvchi ekanligini ko'rsatamiz. Faraz qilaylik, $x_1, x_2 \in [0; \infty)$ lar $x_1 < x_2$ shartni qanoatlantiruvchi ixtiyoriy sonlar bo'lsin. Nomanfiy hadli tengsizlikni kvadratga ko'tarish (5-bob, 9-xossa) mumkin. Bundan $f(x_1) = x_1^2 < x_2^2 = f(x_2)$ kelib chiqadi. Demak, $y = x|x|$ funksiya $[0; \infty)$ da o'suvchi ekan. **Javob:** $y = x|x|$ (C).

24. $(-\infty; 0)$ da kamayuvchi funksiyani toping.

$$\begin{array}{ll} \text{A) } y = 5 - x^2 & \text{B) } y = x^2 \\ \text{C) } y = x|x| & \text{D) } y = x^3 \end{array}$$

25. $R = (-\infty; \infty)$ da monoton funksiyani ko'rsating.

$$\begin{array}{ll} \text{A) } y = 1 - 7x & \text{B) } y = (x-3)^2 \\ \text{C) } y = 1 + |x| & \text{D) } y = x^2 - 5x - 6 \end{array}$$

26. $[0; 2]$ da monoton bo'lmagan funksiyani toping.

$$\begin{array}{ll} \text{A) } y = (x-1)^2 & \text{B) } y = x^2 \\ \text{C) } y = (x-2)^2 & \text{D) } y = (x-3)^2 \end{array}$$

27. $[0; 2]$ da monoton bo'lmagan funksiyani toping.

$$\begin{array}{ll} \text{A) } y = \{0, 2 \cdot x\} & \text{B) } y = \{0, 3 \cdot x\} \\ \text{C) } y = \{0, 4 \cdot x\} & \text{D) } y = \{0, 5 \cdot x\} \end{array}$$

28. (00-2-8) Agar $f(x) = x^2 - 8x + 7$ bo'lsa, $f(4 - \sqrt{11})$ ni hisoblang.

$$\text{A) } 2 \quad \text{B) } 2 - \sqrt{2} \quad \text{C) } 2 + \sqrt{11} \quad \text{D) } 3$$

Yechish: Berilgan funksiyani $f(x) = (x-4)^2 - 9$ shaklda yozib olamiz va $x = 4 - \sqrt{11}$ deymiz, natijada

$$f(4 - \sqrt{11}) = (4 - \sqrt{11} - 4)^2 - 9 = 11 - 9 = 2$$

bo'ladi. **Javob:** 2 (A).

29. (96-1-16) Agar $f(x) = (1 + \frac{1}{x})(7 + 4x)$ bo'lsa, $f(-\frac{1}{2})$ ni toping.

$$\text{A) } 9 \quad \text{B) } -3 \quad \text{C) } 15 \quad \text{D) } -5$$

30. (02-8-17) Agar

$$f(x) = \sqrt{x^3 - 1} \quad \text{bo'lsa, } f(\sqrt[3]{x^2 + 1}) = ?$$

$$\text{A) } |x| \quad \text{B) } x \quad \text{C) } -x \quad \text{D) } 0$$

31. (03-1-15) Agar

$$f(x) = \begin{cases} |x+1|, & x > -2 \\ 3-4|x|, & x \leq -2 \end{cases}$$

bo'lsa, $f(-1) - f(-3)$ ni hisoblang.

$$\text{A) } 0 \quad \text{B) } 3 \quad \text{C) } 6 \quad \text{D) } 9$$

32. (03-6-13) Agar

$$f\left(\frac{3x-2}{2}\right) = x^2 - x - 1$$

bo'lsa, $f(0)$ ni toping.

$$\text{A) } -\frac{5}{9} \quad \text{B) } -\frac{13}{9} \quad \text{C) } -\frac{7}{9} \quad \text{D) } -\frac{11}{9}$$

33. (03-11-17) Agar $f(x+2) = x^3 + 6x^2 + 12x + 8$ bo'lsa, $f(\sqrt{3})$ ni toping.

$$\text{A) } 3\sqrt{3} \quad \text{B) } 2\sqrt{3} \quad \text{C) } 4\sqrt{3} \quad \text{D) } 12$$

10.1 Tekislikda koordinatalar sistemasi

Tekislikda perpendikulyar ikki to'g'ri chiziq berilgan bo'lsin. Qulaylik uchun ularning birini gorizontol, ikkinchisini vertikal qilib olamiz. Bu to'g'ri chiziqning kesishish nuqtasini O bilan belgilaymiz va uni sanoq boshi deb ataymiz. Gorizontol to'g'ri chiziqni absissalar o'qi yoki Ox o'qi, vertikal to'g'ri chiziqni ordinatalar o'qi yoki Oy o'qi deb ataymiz. Bularning hammasi birgalikda tekislikda to'g'ri burchakli koordinatalar sistemasi deyiladi. Bundan tashqari bu o'qlardagi nuqtalar bilan, haqiqiy sonlar o'rtasida moslik o'rnatilgan bo'lsin. Absissalar o'qida O nuqtadan o'ngdagi nuqtalarga musbat sonlar, chapdagi nuqtalarga manfiy sonlar mos qo'yilgan bo'lsin. Xuddi shunday ordinatalar o'qida O nuqtadan yuqoridagi nuqtalarga musbat sonlar, pastdagi nuqtalarga manfiy sonlar mos qo'yilgan bo'lsin. Absissalar o'qida O nuqtadan o'ng tomonga yo'nalish, musbat yo'nalish hisoblanadi. Ox o'qining musbat yo'nalishidan soat strelkasining aylanishiga teskari yo'nalish to'g'ri yo'nalish hisoblanadi. Bu o'qlar tekislikni olti qismga ajratadi – bular absissalar o'qi, ordinatalar o'qi va 4 chorakdir. Choraklarning tartib raqami 10.1-chizmada ko'rsatilgan.

To'g'ri burchakli koordinatalar sistemasida nuqtaning vaziyati quyidagicha aniqlanadi. Faraz qilaylik, tekislikda M nuqta berilgan bo'lsin. M nuqtadan koordinata o'qlariga perpendikulyarlar tushiramiz. Perpendikulyarning absissalar o'qi bilan kesishgan nuqtasini x bilan, ordinatalar o'qi bilan kesishgan nuqtasini y bilan belgilaymiz (10.1-chizma) va ularni M nuqtaning koordinatalari deb ataymiz. M nuqtaning koordinatalarini ko'rsatish maqsadida $M(x; y)$ shaklda yozamiz. Shunday qilib tekislikdagi M nuqtalar bilan tartiblangan $(x; y)$ ($x, y \in R$) sonlar jufti o'rtasida o'zaro bir qiymatli moslik o'rnatiladi. $y = f(x)$ funksiyaning grafigi deb, tekislikda $\{(x; f(x)) : x \in D(f)\} = Gr(f)$ ko'rinishdagi nuqtalar to'plamiga aytiladi.

1. Agar $x > 0, y > 0$ bo'lsa, $M(x; y)$ nuqta I chorakda yotadi va aksincha.
2. Agar $x < 0, y > 0$ bo'lsa, $M(x; y)$ nuqta II chorakda yotadi va aksincha.
3. Agar $x < 0, y < 0$ bo'lsa, $M(x; y)$ nuqta III chorakda yotadi va aksincha.
4. Agar $x > 0, y < 0$ bo'lsa, $M(x; y)$ nuqta IV chorakda yotadi va aksincha.
5. Berilgan $M(x_0; y_0)$ nuqta $y = f(x)$ funksiyaning grafigiga tegishli bo'lishi uchun $y_0 = f(x_0)$ tenglik o'rinli bo'lishi zarur va yetarli.
6. $\begin{cases} y = f(x) \\ y = g(x) \end{cases}$ sistemaning yechimlari f va g funksiya grafiglarining kesishish nuqtalari bo'ladi va aksincha.
7. $f(x) = g(x)$ tenglamaning yechimlari f va g funksiyalar grafiglarining kesishish nuqtalarining absissasi bo'ladi va aksincha.

1. $M(3; 2)$ nuqta qaysi chorakda yotadi.
A) I B) II C) III D) IV

Yechish: Berilgan M nuqtaning absissasi ($3 > 0$) ham ordinatasi ($2 > 0$) ham musbat. 1-qoidaga ko'ra $M(3; 2)$ nuqta I chorakda yotadi (10.2-chizma qarang). **Javob:** I (A).

2. $M(-1; 2)$ nuqta qaysi chorakda yotadi.
A) I B) II C) III D) IV

3. $M(-3; -2)$ nuqta qaysi chorakda yotadi.
A) I B) II C) III D) IV

4. $M(5; -4)$ nuqta qaysi chorakda yotadi.
A) I B) II C) III D) IV

5. $y = 2x$ funksiya grafigiga tegishli nuqtani toping.
A) (2; 4) B) (1; -3) C) (-1; 2) D) (0; 1)

Yechish: 5-qoidaga ko'ra, qaysi nuqtaning koordinatalari $y = 2x$ tenglikni qanoatlantirsa, shu nuqta funksiya grafigiga tegishli bo'ladi. A) javobdagi (2; 4) nuqta $4 = 2 \cdot 2$ tenglikni qanoatlantiradi. **Javob:** (2; 4) (A).

6. $y = x^3 - 5$ funksiya grafigiga tegishli nuqtani toping.
A) (1; -3) B) (2; 3) C) (1; 4) D) (0; 0)

7. $y = [x] + 3$ funksiya grafigiga tegishli nuqtani toping.
A) (1, 5; 3) B) (2, 2; 3) C) (1, 7; 4) D) (0; 0)

8. $y = \{x\}$ funksiya grafigiga tegishli nuqtani toping.
A) (1; 0) B) (2; 1) C) (1, 7; 1) D) (0; 1)

9. $y = |x + 1|$ funksiya grafigiga tegishli nuqtani toping.
A) (1; 1) B) (2; 1) C) (1; 2) D) (1; 0)

10. (96-3-15) Quyidagi nuqtalarning qaysi biri $f(x) = -3x + 4$ funksiyaning grafigiga tegishli?
A) (3; -5) B) (-3; 5) C) (5; -3) D) (2; 4)

11. (96-12-16) Quyidagi nuqtalarning qaysi biri $f(x) = -4x + 3$ funksiyaning grafigiga tegishli?
A) (-1; 1) B) (2; 5) C) (-5; 2) D) (1; -1)

12. (07-112-3) Quyidagilardan qaysi biri $f(x) = -2x + 7$ funksiya grafigiga tegishli.
A) (2; 1) B) (1; 2) C) (2; 4) D) (3; 1)

13. Grafigi $M(1; 0)$ nuqtadan o'tuvchi funksiyani toping.
A) $y = |x - 3|$ B) $y = x^2 - 1$
C) $y = 3x$ D) $y = [x]$

14. $M(1; 5)$ qaysi funksiyaning grafigiga tegishli?
A) $y = |x + 3|$ B) $y = x^2 + 1$
C) $y = 3x + 1$ D) $y = [x + 4, 5]$

15. $y = x^2 - 2x + 1$ funksiya grafigining Ox o'qi bilan kesishish nuqtasi absissasini toping.
A) -3 B) 3 C) -1 D) 1
- Yechish:** Ox o'qida yotuvchi nuqtalar $(x; 0)$ ko'rinishda bo'ladi. Demak, $0 = x^2 - 2x + 1$ tenglamaning ildizlari masalaning yechimi bo'ladi. Bu yerdan $x = 1$ ni olamiz. **Javob:** 1 (D).
16. (02-4-5) $y = (x + 3)(x^2 + x + 1)$ funksiya grafigining Oy o'qi bilan kesishish nuqtasi ordinatasini toping.
A) -3 B) 3 C) -1 D) 1
17. $y = |x - 1|$ va $y = 1 - x^2$ funksiyalarning grafiglari nechta nuqtada kesishadi?
A) 2 B) 1 C) 3 D) 4
18. $y = |x|$ va $y = x^2$ funksiya grafiglarining kesishish nuqtalarini toping.
A) $(-1; 1)$ B) $(-1; 1)$ va $(1; -1)$
C) $(-1; -1)$ D) $(1; 1)$ va $(-1; 1)$

10.2 Chizikli funksiya

$y = kx + b$ funksiyaga *chizikli funksiya* deyiladi. Bu yerda $k \neq 0$, $b \in R$ haqiqiy sonlar. Bu funksiyaning aniqlanish sohasi $D(f) = R$. Bu funksiyaning qiymatlar sohasi $E(f) = R$. $y = kx + b$ funksiyaning grafigi to'g'ri chiziq bo'ladi (10.3-chizma), k esa to'g'ri chiziqning *burchak koeffitsiyenti* deyiladi.

Chizikli funksiyalar quyidagi xossalarga ega.

- $y = kx + b$ funksiyaning grafigi Oy o'qini $(0; b)$ nuqtada kesib o'tuvchi to'g'ri chiziqdir. Funksiya
 - $k > 0$ da o'suvchi;
 - $k < 0$ da kamayuvchi;
- $y = kx + b$ funksiyaning grafigi Ox o'qining musbat yo'nalishi bilan α burchak tashkil etsa, $\text{tg } \alpha = k$ bo'ladi.
- $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar $k_1 \neq k_2$ da yagona nuqtada kesishadi:
 - $k_1 = k_2$, $b_1 = b_2$ da ustma-ust tushadi;
 - $k_1 = k_2$, $b_1 \neq b_2$ da kesishmaydi.

- Agar $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar orasidagi burchak φ bo'lsa, u holda $\text{tg } \varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2}$ tenglik o'rinli. Xususan:
 - $k_1 = k_2$ da parallel bo'ladi;
 - $k_1 \cdot k_2 = -1$ da perpendikulyar bo'ladi.
- $(x_0; y_0)$ nuqtadan $ax + by + c = 0$ to'g'ri chiziqqacha bo'lgan masofa: $d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.
- Absissalar o'qini $x = a$ nuqtada, ordinatalar o'qini $y = b$ nuqtada kesuvchi to'g'ri chiziq tenglamasi $\frac{x}{a} + \frac{y}{b} = 1$ dir. Bu chiziqning koordinata o'qlari orasidagi kesmasining uzunligi $\sqrt{a^2 + b^2}$ ga teng.
- $y = kx + b$ funksiya grafigiga:
 - $x = a$ ga nisbatan simmetrik bo'lgan to'g'ri chiziq tenglamasi $y = -kx + 2ak + b$;
 - $y = a$ ga nisbatan simmetrik bo'lgan to'g'ri chiziq tenglamasi $y = -kx + 2a - b$.

- (99-1-47) $2y = 2x + 3$ to'g'ri chiziqning Ox o'qi bilan hosil qilgan burchagini toping.
A) 45^0 B) 30^0 C) 60^0 D) 75^0

Yechish: $2y = 2x + 3$ dan y ni topamiz:

$$y = x + \frac{3}{2}.$$

Agar $y = kx + b$ to'g'ri chiziqning Ox o'qi bilan hosil qilgan burchagi α bo'lsa, 2-xossaga ko'ra $\text{tg } \alpha = k$ bo'ladi. Bu holda $k = 1$ bo'lgani uchun $\text{tg } \alpha = 1$, ya'ni $\alpha = 45^0$. **Javob:** $\alpha = 45^0$ (A).

- $y = x + 7$ to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
A) 45^0 B) 30^0 C) 60^0 D) 75^0
- $y = \sqrt{3}x - 9$ to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
A) 45^0 B) 30^0 C) 60^0 D) 75^0
- $3y = \sqrt{3}x - 9$ to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
A) 45^0 B) 30^0 C) 60^0 D) 75^0

- $y = 7 - x$ to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan tashkil qilgan burchagini toping.
A) 145^0 B) 135^0 C) 120^0 D) 75^0
- (98-3-44) k ning qanday qiymatlarida $kx + 3y + 5 = 0$ va $(k + 1) \cdot x - 2y - 1 = 0$ to'g'ri chiziqlar parallel bo'ladi?
A) -3 va 5 B) $\frac{3}{5}$ C) -5 va 3 D) $-\frac{3}{5}$

Yechish: $kx + 3y + 5 = 0$ dan $y = -kx : 3 - 5 : 3$ ni, $(k + 1)x - 2y - 1 = 0$ dan $y = (k + 1)x : 2 - 1 : 2$ ni topamiz. Parallellik sharti 4-xossaning a) bandiga ko'ra, $(k + 1) : 2 = -k : 3$ ni olamiz. Uning yechimi $k = -\frac{3}{5}$. **Javob:** $-\frac{3}{5}$ (D).

7. k ning qanday qiymatlarida $kx + 4y + 7 = 0$ va $x - 2y - 1 = 0$ to'g'ri chiziqlar parallel bo'ladi?
A) -3 B) $-1\frac{1}{4}$ C) -2 D) $-\frac{3}{4}$
8. (01-12-3) $y_1 = \sqrt{3}x + \frac{1}{\sqrt{3}}$ va $y_2 = -\frac{1}{\sqrt{3}}x - \sqrt{3}$ to'g'ri chiziqlar orasidagi burchakni toping.
A) 90° B) 60° C) 80° D) 95°
9. a ning qanday qiymatlarida $ax + 2y = 3$ va $2x - y = -1$ to'g'ri chiziqlar perpendikulyar bo'ladi?
A) $a = -1$ B) $a = 2$ C) $a = 0$ D) $a = 1$
10. (96-1-26) a ning qanday qiymatlarida $ax + 2y = 3$ va $2x - y = -1$ to'g'ri chiziqlar kesishadi?
A) $a \neq 4$ B) $a \neq 2$ C) $a \in R$ D) $a \neq -4$
11. (99-9-13) $y = \sqrt{3}x + 2$ va $y = -x + 2$ to'g'ri chiziqlarning kesishishidan hosil bo'lgan o'tkir burchakni toping.
A) 65° B) 75° C) 60° D) 85°
12. (96-7-16) k ning qanday qiymatida $y = kx + 6$ funksiyaning grafigi $M(0, 5; 4, 5)$ nuqtadan o'tadi?
A) 3 B) -3 C) -2 D) 4
13. (99-6-4) k ning qanday qiymatida $y = kx - 10$ funksiyaning grafigi $A(-4; 14)$ nuqtadan o'tadi?
A) -2 B) -1 C) -6 D) -3
14. (99-3-10) a ning qanday qiymatlarida $2ax + 3y = 3$ va $4x + 3y = 7$ to'g'ri chiziqlar kesishish nuqtasining absissasi manfiy bo'ladi.
A) $a < 3$ B) $a > 3$ C) $a < 2$ D) $a > 2$

Yechish: $a = 2$ da berilgan to'g'ri chiziqlar parallel bo'ladi (4-xossaning a) bandi) va $b_1 \neq b_2$ shartda kesishmaydi. $a \neq 2$ holni qaraymiz. 10.1-ning 6-xossasiga ko'ra, $\begin{cases} 2ax + 3y = 3 \\ 4x + 3y = 7 \end{cases}$ sistemaning yechimlari berilgan to'g'ri chiziqlarning kesishish nuqtasi bo'ladi. Sistemaning 2-tenglamasini -1 ga ko'paytirib, 1-tenglamaga qo'shamiz va $(2a - 4)x = -4$ ni olamiz. Bu tenglamaning yechimi $x = \frac{2}{2-a}$ dir. Hozir biz berilgan to'g'ri chiziqlar kesishish nuqtasining absissasini topdik. Uni manfiy deymiz, ya'ni $\frac{2}{2-a} < 0$. Bu tengsizlik $2-a < 0$ tengsizlikka teng kuchli, ya'ni $2 < a$. **Javob:** $a > 2$ (D).

15. (98-9-15) Koordinata o'qlari $\frac{x}{8} - \frac{y}{6} = 1$ to'g'ri chiziqdan qanday uzunlikdagi kesma ajratadi?
A) 12 B) 14 C) 9 D) 10
16. (98-10-42) n ning qanday qiymatida $2y = 8 + n - (3n + 4)x$ va $3y = 5 - 2n - (4n - 3)x$ tenglamalar bilan berilgan to'g'ri chiziqlarning kesishish nuqtasi Oy o'qida yotadi?
A) 2 B) $1,5$ C) $-1,5$ D) -2
17. (99-8-33) $f(-2) = 3$ va $f(2) = 5$ shartni qanoatlantiruvchi chiziqli funksiyani toping.

$$\begin{array}{ll} \text{A) } f(x) = \frac{1}{2}x + 4 & \text{B) } f(x) = 2x - 1 \\ \text{C) } f(x) = 2x + 1 & \text{D) } f(x) = 3x + 9 \end{array}$$

18. (96-6-13) Agar $k < 0$ va $l > 0$ bo'lsa, $y = kx + l$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
A) I; II B) I; II; III
C) II; I; IV D) I; III; IV

Yechish: $y = kx + l$ to'g'ri chiziqning koordinata o'qlari bilan kesishish nuqtalarini topamiz. Agar $x = 0$ desak, $y = l$ bo'ladi, $y = 0$ desak, $x = -l/k$. Demak, $y = kx + l$ funksiyaning grafigi koordinata o'qlarini $(0; l)$ va $(-l/k; 0)$ nuqtalarda kesib o'tadi. Bu nuqtalar koordinata o'qlarining musbat yo'nalishlarida turibdi (10.4-chizma). Shunday qilib, funksiya grafigi koordinatalar tekisligining II; I va IV choraklari orqali o'tadi. **Javob:** II; I; IV (C).

19. Agar $k > 0$ va $l > 0$ bo'lsa, $y = kx + l$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashadi?
A) I; II va III B) I va II
C) I; III va IV D) I; II va IV
20. (97-8-13) Ushbu $y = kx + l$ ($k < 0$ va $l < 0$) funksiyaning grafigi qaysi choraklarda joylashgan?
A) I; II va III B) I; III va IV
C) II va IV D) II; III va IV
21. (96-12-24) Grafigi chizmada tasvirlangan funksiyaning qiymatlari x ning qanday qiymatlarida -2 dan kichik bo'ladi?

$$\text{A) } x \geq 0 \quad \text{B) } x > 0 \quad \text{C) } x < 0 \quad \text{D) } x \leq 0$$

22. (97-8-60) Chizmada $a = 4$; $b = 3$ va $c = 5$ bo'lsa, OC to'g'ri chiziqning burchak koeffitsiyentini toping.
- A) $\frac{4}{3}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{3}{4}$
23. (98-10-91) k ning qanday qiymatlarida $kx + 3y + 1 = 0$ va $2x + (k + 1)y + 2 = 0$ to'g'ri chiziqlar parallel bo'ladi?
- A) 2 B) -2 C) -3 D) -3 va 2
24. (98-11-14) $y = -\frac{41}{5}x$ funksiyaning grafigi $y = kx + \frac{41}{5}$ funksiyaning grafigiga k ning qaysi qiymatida parallel bo'ladi?
- A) $(\frac{5}{41})^{-1}$ B) $\frac{5}{41}$ C) $-(\frac{5}{41})^{-1}$ D) $-\frac{5}{41}$
25. (99-1-46) $x + y = 1$ tenglama bilan berilgan to'g'ri chiziqqa parallel to'g'ri chiziqni toping.
- A) $2x + 2y + 3 = 0$ B) $y = x - 1$
C) $x - y = 2$ D) $y = x + 1$
26. (98-3-41) $y = 1$ ga nisbatan $y = 2x + 1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
- A) $y = 2x - 1$ B) $y = 2x + 1$
C) $y = 1 - 2x$ D) $y = 2x$
- Yechish:** 7-xossaning b) bandiga ko'ra $y = 1$ ga nisbatan $y = 2x + 1$ ga ($a = 1$, $b = 1$, $k = 2$) simmetrik bo'lgan to'g'ri chiziq tenglamasi $y = -2x + 1$ bo'ladi. **Javob:** $y = 1 - 2x$ (C).
27. (98-10-88) $y = x$ ga nisbatan $y = 2x + 1$ ga simmetrik bo'lgan to'g'ri chiziqning tenglamasini toping.
- A) $y = 2x - 1$ B) $y = \frac{x}{2} - 1$
C) $y = \frac{x}{2} + 1$ D) $y = \frac{x - 1}{2}$
28. (98-12-29) Ox o'qqa nisbatan $y = 2x + 3$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqning tenglamasini ko'rsating.
- A) $y = -2x - 3$ B) $y = 2x - 3$
C) $y = -2x + 3$ D) $y = 3x - 2$
29. (01-3-12) Ushbu
- $$(a + 3)x + (a^2 - 16)y + 2 = 0$$
- to'g'ri chiziq a ning qanday qiymatida absissa o'qiga parallel bo'ladi?
- A) -3 B) 2 C) -2 D) 3
30. (01-12-40) m va n ning qanday qiymatlarida $2xm - 3ny = 12$ va $3xm + 2ny = 44$ to'g'ri chiziqlar (1; 2) nuqtada kesishadi?
- A) $m = 10$, $n = 4$ B) $m = 8$, $n = 6$
C) $m = 4$, $n = 10$ D) $m = 12$, $n = 2$
31. (02-1-45) Agar barcha x lar uchun $f(x) = 6x - 3$ bo'lsa, $y = f(x - 1)$ tenglama bilan aniqlanadigan to'g'ri chiziqning burchak koeffitsiyentini toping.
- A) 6 B) 5 C) 7 D) -6
32. (02-12-5) $y = 2x + 1$ va $y = -2 - x$ funksiyalarning graflari qaysi chorakda kesishadi?
- A) I B) II C) III D) IV
33. (96-11-31) $M(2; 1)$ nuqtadan $y = x + 2$ to'g'ri chiziqqacha bo'lgan masofani toping.
- A) 2,25 B) $1,5\sqrt{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$
- Yechish:** 5-xossadan foydalanib, $M(2; 1)$ nuqtadan $x - y + 2 = 0$ to'g'ri chiziqqacha bo'lgan masofani hisoblaymiz
- $$d = \frac{|2 - 1 + 2|}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = 1,5\sqrt{2}.$$
- Javob:** $1,5\sqrt{2}$ (B).
34. (96-12-31) $M(2; 2)$ nuqtadan $y = x + 1$ to'g'ri chiziqqacha bo'lgan masofani toping.
- A) 1,5 B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{2}$ D) 2,25
35. (03-11-30) Koordinatalar boshidan $5x + 12y = 60$ to'g'ri chiziqqacha bo'lgan masofani aniqlang.
- A) $4\frac{8}{13}$ B) 5 C) $5\frac{3}{13}$ D) $4\frac{7}{13}$

10.3 Kvadrat funksiya

$y = ax^2 + bx + c$ ko'rinishdagi funksiyaga kvadrat funksiya deyiladi. Bu yerda a , b , c lar berilgan sonlar bo'lib, $a \neq 0$. Kvadrat funksiyaning aniqlanish sohasi $D(y) = R$. Kvadrat funksiya quyidagi xossalarga ega.

1. $y = ax^2 + bx + c$ ($a \neq 0$) kvadrat funksiyaning grafigi paraboladan iborat (10.5-chizma):
 a) $a > 0$ da parabola shoxlari yuqoriga yo'nalgan;
 b) $a < 0$ da parabola shoxlari pastga yo'nalgan;
 c) $D > 0$ da parabola Ox o'qi bilan 2 ta umumiy nuqtaga ega – 10.5-chizma;
 d) $D = 0$ da parabola Ox o'qiga urinadi, ya'ni 1 ta umumiy nuqtaga ega;
 e) $D < 0$ da parabola Ox o'qi bilan umumiy nuqtaga ega emas.

2. Parabola uchining koordinatalari $(x_0; y_0)$ quyidagi formula yordamida hisoblanadi:

$$x_0 = -\frac{b}{2a}, \quad y_0 = ax_0^2 + bx_0 + c = -\frac{b^2 - 4ac}{4a}.$$

3. Agar parabola uchi $(x_0; y_0)$ da bo'lsa, $y = ax^2 + bx + c$ kvadrat funksiya $y = a(x - x_0)^2 + y_0$ ko'rinishda tasvirlanadi.

4. $y = ax^2 + bx + c$ kvadrat funksiyaning qiymatlar sohasi:

- a) $a > 0$ da $E(y) = [y_0; \infty)$,
 b) $a < 0$ da $E(y) = (-\infty; y_0]$.

5. Parabola simmetriya o'qining tenglamasi $x = x_0$. Bu yerda x_0 parabola uchining absissasi.

6. $ax^2 + bx + c = 0$ tenglamaning x_1, x_2 ildizlari $y = ax^2 + bx + c$ funksiyaning nollari deyiladi va $\frac{x_1 + x_2}{2} = x_0$ bo'ladi. Bu yerda x_0 parabola uchining absissasi.

1. (98-8-24) Agar $B(-2; -7)$ nuqta $y = kx^2 + 8x + m$ parabolaning uchi bo'lsa, k va m ning qiymatlarini toping.
 A) $k = 1, m = -9$ B) $k = 2, m = -1$
 C) $k = -1, m = -16$ D) $k = 2, m = 1$

Yechish: Masala sharti va 2-xossadan parabola uchining absissasi x_0 uchun $-2 = -\frac{8}{2k}$ ni olamiz. Ya'ni $k = 2$ bo'ladi. Endi $y = 2x^2 + 8x + m$ tenglikka B nuqtaning koordinatlarini qo'yib m ning qiymatini topamiz: $-7 = 8 - 16 + m, m = 1$.
Javob: $k = 2, m = 1$. (D).

2. (96-6-21) $y = x^2 - 4x + 3$ parabolaning uchi koordinatalar tekisligining qayerida joylashgan.
 A) IV chorakda B) Ox o'qida
 C) III chorakda D) II chorakda
3. (97-2-21) $y = x^2 + 4x - 2$ parabolaning uchi koordinatalar tekisligining qayerida joylashgan.
 A) I chorakda B) II chorakda
 C) Oy o'qida D) III chorakda
4. (97-8-21) $y = x^2 - 6x + 10$ parabolaning uchi koordinatalar tekisligini qayerida joylashgan.
 A) II chorakda B) III chorakda
 C) Oy o'qida D) I chorakda

5. (97-3-16) k ning qanday qiymatida $y = kx^2 - 2$ funksiyaning grafigi $A(-1; 1)$ nuqtadan o'tadi?
 A) 4 B) -3 C) 3 D) 2

6. (98-4-45) $y = kx^2 - 2kx + 3$ va $y = 2 - kx$ funksiyalarning grafiglari k ning nechta butun qiymatlarida kesishmaydi?
 A) 3 B) 2 C) cheksiz ko'p D) 4

Yechish: 10.1-ning 6-xossasiga ko'ra, funksiyalarning grafiglari kesishmasa

$$\begin{cases} y = kx^2 - 2kx + 3 \\ y = 2 - kx \end{cases}$$

sistema yechimga ega emas. Bu yerdan $kx^2 - 2kx + 3 = 2 - kx$ tenglamaning yechimga ega emasligi kelib chiqadi. Agar $k = 0$ bo'lsa, $3 = 2$ tenglik hosil bo'ladi. Bu tenglik to'g'ri emas. Demak, $k = 0$ da funksiyalarning grafiglari kesishmaydi. Endi $k \neq 0$ holni qaraymiz. Bu holda $kx^2 - kx + 1 = 0$ kvadrat tenglama yechimga ega emas. Ma'lumki, kvadrat tenglamaning diskriminanti $D < 0$ bo'lsa, u yechimga ega bo'lmaydi. $D < 0$ shart $k^2 - 4k < 0 \iff k(k - 4) < 0$ shartga teng kuchli. Bu tengsizlik oraliqlar usuli yordamida oson yechiladi, uning yechimi $(0; 4)$ dan iborat. Bu intervalda 1, 2, 3 butun sonlari bor. Yuqorida ko'rsatildiki $k = 0$ da ham sistema yechimga ega emas edi. Shunday qilib k ning 4 ta butun qiymatida funksiyalarning grafiglari kesishmaydi. **Javob:** 4 (D).

7. (98-12-94) $y = (k - 2)x^2 - 3kx + 2$ va $y = kx^2 + kx + 4$ funksiyalarning grafiglari kesishmaydigan k ning barcha butun qiymatlari yig'indisini toping.
 A) 0 B) 1 C) -2 D) 3
8. (01-12-18) a ning qanday qiymatlarida $y = 2ax + 1$ va $y = (a - 6)x^2 - 2$ funksiyalarning grafiglari kesishmaydi?
 A) $(-3; 6)$ B) $(-\infty; 6) \cup (3; \infty)$
 C) \emptyset D) $(-6; 3)$
9. (99-3-11) a ning qanday qiymatlarida $y = 9x^2 - 12x + 35a$ parabola absissalar o'qi bilan ikkita umumiy nuqtaga ega bo'ladi?
 A) $a = \frac{4}{35}$ B) $a < \frac{4}{35}$ C) $a > \frac{4}{35}$ D) $a < \frac{18}{35}$
10. (98-8-17) Agar $f(x) = 2 - ax^2$ va $g(x) = 2b + x$ funksiyalarning qiymatlari $x = -1$ va $x = 0$ da teng bo'lsa, a va b ning qiymatini toping.
 A) $a = -1, b = 1$ B) $a = 1, b = 1$
 C) $a = 1, b = -1$ D) $a = 5, b = -1$
11. (98-10-59) $A(1; 1)$, $B(0; 3)$ va $C(2; 3)$ nuqtalardan o'tuvchi parabola qaysi funksiyaning grafigi hisoblanadi?
 A) $y = 2x^2 + 2x - 3$ B) $y = 2x^2 - 2x - 3$
 C) $y = 2x^2 - 4x + 3$ D) $y = 2x^2 - 3x + 2$

12. (98-11-79) m ning qanday qiymatida $y = 1$ to'g'ri chiziq, $y = x^2 - 2x + m$ parabola ga urinadi?
A) 4 B) 1 C) 3 D) 2
13. (00-6-11) a ning qanday qiymatlarida $y = ax^2 + 4x + c$ parabola koordinata o'qlarini $A(1; 0)$ va $B(0; 4)$ nuqtalarda kesib o'tadi?
A) -8 B) 4 C) -4 D) 1
14. (00-7-22) a ning qanday qiymatida $y = x^2 - 4x + 12 - a$ parabolaning uchi $M(2; 4)$ nuqtada yotadi?
A) 3 B) 2 C) 4 D) 5
- Yechish:** 3-xossaga ko'ra, parabola tenglamasi $y = (x - 2)^2 + 4$ ko'rinishda bo'ladi. Uni masala berilishidagi $y = x^2 - 4x + 12 - a$ bilan tenglashtirib, $4 + 4 = 12 - a$ ni hosil qilamiz. Bu yerdan $a = 4$. **Javob:** 4 (C).
15. (98-6-31) $y = 2x^2 + bx + c$ parabolaning uchi $(-3; -5)$ nuqtada joylashgan. Bu funksiya nol-larining o'rta arifmetigini toping.
A) -1 B) -2 C) -3 D) 1
16. (00-2-26) $A(1; 9)$ nuqta $y = -x^2 + ax + 4$ parabola ga tegishli. Parabola uchining ordinatasini toping.
A) 13 B) 6 C) 4 D) 2
17. (02-11-18) $y = -3x^2 + 12x - 16$ parabola uchining koordinatalari yig'indisini toping.
A) -1 B) 1 C) 0 D) -2
18. (02-11-19) a ning nechta butun qiymatida $y = (x - 4a)^2 + a^2 + 10a + 21$ parabola uchining absissasi musbat, ordinatasi esa manfiy bo'ladi?
A) 0 B) 1 C) 2 D) 3
19. (03-8-18) a ning nechta butun qiymatida $y = (x - 2a)^2 + a^2 - 9a + 14$ parabola uchining absissasi musbat, ordinatasi esa manfiy bo'ladi?
A) 1 B) 2 C) 4 D) 5
20. (97-12-21) Agar $a < 0$ va $b^2 - 4ac < 0$ bo'lsa, $y = ax^2 + bx + c$ funksiya grafigi koordinatalar tekisligining qaysi choraklarida joylashadi?
A) I, II B) III, IV C) II, III D) I, II va IV
- Yechish:** $a < 0$ shartdan parabola shoxlari pastga yo'nalgan ekanligi kelib chiqadi. $b^2 - 4ac < 0$ shartdan parabola Ox o'qini kesmasligi kelib chiqadi. Demak, parabola Ox o'qidan pastda, ya'ni III va IV choraklarda joylashgan. **Javob:** III, IV (B).
21. Agar $a > 0$ va $b^2 - 4ac < 0$ bo'lsa, $y = ax^2 + bx + c$ funksiya grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
A) I, IV B) I, II C) faqat IV D) III, IV
22. (98-11-13) Ushbu $y = -3x^2 + 8x - 8$ funksiyaning grafigi qaysi choraklarda joylashgan?
A) II, III, IV B) barcha choraklarda C) III, IV D) I, II, III
23. (00-8-11) Ushbu $f(x) = -4x^2 + 2x - 1$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
A) III; IV B) I; II; III C) I; III D) II; IV
24. (98-1-16) Chizmada qaysi funksiyaning grafigi tasvirlangan?
A) $y = 3x - x^2$ B) $y = 3x^2 - 3$
C) $y = 3(1 - x^2)$ D) $y = x^2 + 3x$
25. (01-9-38) $y = x^2 + px + q$ parabola $x = 5$ nuqtada Ox o'qiga urinadi. $\frac{q}{p}$ ni toping.
A) 1 B) -2 C) 2,5 D) $-2, 5$
- Yechish:** Masala shartidan parabolaning uchi $(5; 0)$ nuqtada ekanligi kelib chiqadi. 3-xossaga ko'ra $y = (x - 5)^2 = x^2 - 10x + 25$ ni olamiz. Bu yerdan $p = -10$, $q = 25$ ekanligi kelib chiqadi. Demak, $p/q = -2, 5$. **Javob:** $-2, 5$ (D).
26. (01-12-41) t ning qanday qiymatlarida $f(x) = 3x^2 + 2tx - (t - 1)^2$ funksiya $f(-1) = -2$ shartni qanoatlantiradi?
A) ± 3 B) ± 1 C) 3 D) ± 2
27. (01-2-25) Ushbu
$$y = 4x^2 + 4x + 1 \quad \text{va} \quad y = 2x + 1$$
funksiyalar grafiglari kesishish nuqtalari koordinatalarining yig'indisini toping.
A) $-0, 5$ B) 1 C) 0,5 D) 1,5
28. (02-5-12) m ning qanday qiymatlarida $y = (m + 4)x^2 - 2(m + 2)x + 1$ kvadrat funksiyaning grafigi absissalar o'qidan pastda joylashadi?
A) $(-\frac{1}{4}; 1)$ B) $(-2; 1)$ C) \emptyset D) $(-\infty; \infty)$
29. (03-5-34) $y = ax^2 + c$ funksiya grafigi $A(-1; -3)$ va $B(3; 0)$ nuqtalardan o'tishi ma'lum bo'lsa, $\frac{c}{a}$ ning qiymati nechaga teng.
A) -9 B) 9 C) -8 D) -10
30. (03-6-50) x ning qanday qiymatlarida $y = x^2$ funksiyaning qiymati 9 dan katta bo'ladi?
A) $-3 < x < 3$ B) $x < -3$
C) $x > 3$ D) $x < -3, x > 3$
31. (03-7-57) m ning qanday qiymatida $y = mx + 2$ to'g'ri chiziq va $y = -5x^2$ parabola absissasi $x = -1$ bo'lgan nuqtada kesishadi?
A) 3 B) -3 C) -7 D) 7

32. (03-8-46) $y = -2x^2 + 5x - 3$ funksiyaning eng katta qiymatini toping.

- A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) 5 D) -3

10.4 Teskari funksiya

Bizga X to'plamni Y to'plamga akslantiruvchi $y = f(x)$ funksiya berilgan bo'lsin. Faraz qilaylik, $D(f) = X$ va $E(f) = Y$ bo'lsin. Agar har bir $y \in Y$ uchun

$$f(x) = y \quad (10.1)$$

tenglama yagona $x \in D(f)$ yechimga ega bo'lsa, f funksiya teskarilanuvchan deyiladi. Agar f teskarilanuvchan funksiya bo'lsa, u holda har bir $y \in E(f)$ ga (10.1) tenglamaning yagona yechimi bo'lgan $x \in D(f)$ ni mos qo'yuvchi akslantirish f ga teskari funksiya deyiladi va u f^{-1} shaklda belgilanadi, ya'ni $x = f^{-1}(y)$. Teskari funksiya ta'rifidan quyidagilar kelib chiqadi.

1. $D(f) = E(f^{-1})$ va $D(f^{-1}) = E(f)$ tengliklar o'rinli.
2. Barcha $x \in D(f)$ uchun $f^{-1}(f(x)) = x$ o'rinli.
3. Barcha $x \in D(f^{-1})$ uchun $f(f^{-1}(x)) = x$ o'rinli.
4. Agar $(x_0; y_0)$ nuqta f funksiyaning grafigiga tegishli bo'lsa, u holda $(y_0; x_0)$ nuqta f^{-1} funksiyaning grafigiga tegishli bo'ladi.

Agar $f : D(f) \rightarrow R$ funksiya uchun, biror $y \in E(f)$ da (10.1) tenglama ikki yoki undan ko'p yechimga ega bo'lsa, u holda f funksiya teskarilanuvchan emas, ya'ni f ga teskari funksiya mavjud emas. Bu holda berilgan funksiyaning aniqlanish sohasi $D(f)$ ni "kichraytirish" hisobiga (10.1) tenglamani barcha $y \in E(f)$ larda yagona yechimga ega bo'ladigan qilish mumkin. Buni quyidagi misolda tushuntiramiz:

$$f : R \rightarrow R_+ = [0; \infty), \quad f(x) = x^2$$

funksiyani qaraymiz. $f(x) = 4 \iff x^2 = 4$ tenglama ikkita $x_1 = -2$, $x_2 = 2$ yechimlarga ega. Agar bu funksiyaning aniqlanish sohasini R_+ desak, u holda istalgan $y \in E(f)$ uchun $x^2 = y$ tenglama $D(f) = R_+$ da yagona $x = \sqrt{y}$ yechimga ega bo'ladi. Demak, $f : R_+ \rightarrow R_+$, $f(x) = x^2$ funksiyaga teskari funksiya $f^{-1}(y) = \sqrt{y}$ ekan.

1. (97-1-9) Quyidagilardan qaysi biri $y = \frac{3}{x+1} - 2$ funksiyaga teskari funksiya?

A) $y = \frac{3}{x-2}$ B) $y = \frac{x+1}{3} - 2$
 C) $y = \frac{x+1}{3} - \frac{1}{2}$ D) $y = \frac{3}{x+2} - 1$

Yechish: Berilgan funksiyaning aniqlanish sohasi $D(f) = (-\infty; -1) \cup (-1; \infty)$ dan qiymatlar sohasi esa $E(f) = (-\infty; -2) \cup (-2; \infty)$ dan iborat. Istalgan $y \in E(f)$ uchun $\frac{3}{x+1} - 2 =$

y tenglama yagona $x = \frac{3}{y+2} - 1$ yechimga ega. Demak, $y = f^{-1}(x) = \frac{3}{x+2} - 1$. **Javob:** (D).

2. (97-11-9) Quyidagilardan qaysi biri $y = \frac{3}{2-x} - 1$ funksiyaga teskari funksiya?

A) $y = x - 2$ B) $y = \frac{3}{x-2} + 1$
 C) $y = \frac{x-2}{3} + 1$ D) $y = 2 - \frac{3}{x+1}$
3. (00-3-61) Ushbu $y = x^2 - 4x + 7$ funksiyaga $(-\infty; 2]$ oraliqda teskari funksiyani toping.

A) $2 \pm \sqrt{x-3}$ B) $2 - \sqrt{x-3}$
 C) $2 + \sqrt{x-3}$ D) $2 + \sqrt{3-x}$
4. $y = \sqrt{x} + 1$, $x \geq 0$ funksiyaga teskari funksiyani toping.

A) $y = (x-1)^2$ B) $y = (x+1)^2$
 C) $y = x^2 + 1$ D) $y = x^2 - 1$
5. (01-8-19) Ushbu $y = \frac{4}{2-x} - 3$ funksiyaga teskari bo'lgan funksiyani ko'rsating.

A) $y = \frac{4}{x-3} - 2$ B) $y = \frac{4}{3-x} - 2$
 C) $y = \frac{4}{x+3} + 2$ D) $y = -\frac{4}{x+3} + 2$
6. (97-6-9) Quyidagilardan qaysi biri $y = \frac{2}{x-1} - 1$ funksiyaga teskari funksiya?

A) $y = 1 - \frac{2}{x+1}$ B) $y = 2 - \frac{3}{x}$
 C) $y = -\frac{2}{x+1}$ D) $y = \frac{2}{x+1} + 1$
7. (99-3-29) Funksiyaga teskari funksiyani toping.

$$y = \frac{x-1}{2-3x}$$

A) $y = \frac{2-3x}{x-1}$ B) $y = -\frac{2-3x}{x-1}$
 C) $y = \frac{2-3x}{1-x}$ D) $y = \frac{2x+1}{3x+1}$
8. (01-1-66) Ushbu $y = x^2 - 8$ ($x \geq 0$) funksiyaga teskari bo'lgan funksiyaning aniqlanish sohasini toping.

A) $(-8; \infty)$ B) $[-8; \infty)$ C) $(-8; 8)$ D) $[-8; 8]$

Yechish: Berilgan funksiyaning aniqlanish sohasi $D(f) = [0; \infty)$ dan qiymatlar sohasi esa $E(f) = [-8; \infty)$ dan iborat. 1-qoidaga ko'ra $D(f^{-1}) = [-8; \infty)$. **Javob:** $[-8; \infty)$ (B).
9. (98-11-15) Ushbu $y = 2x^2 - \frac{1}{2}$ ($x \geq 0$) funksiyaga teskari bo'lgan funksiyani toping.

A) $\sqrt{2x+1} \cdot 2^{-1}$ B) $\sqrt{2x+1} \cdot 4^{-1}$
 C) $\sqrt{2x+1} \cdot 2^{-1} - \frac{1}{2}$ D) $\sqrt{2x+1} \cdot 4^{-1} - \frac{1}{2}$
10. $D(f)$ qanday tanlansa $f(x) = x^2 - 2x + 3$ funksiyaga teskari funksiya mavjud bo'ladi.

A) $[0; \infty)$ B) $[1; \infty)$ C) $[-2; \infty)$ D) $[-2; 2)$

11. $D(f)$ qanday tanlansa $f(x) = |x - 3|$ funksiyaga teskari funksiya mavjud bo'ladi.
A) $[3; \infty)$ B) $[1; \infty)$ C) $[-2; \infty)$ D) $[-3; 5)$
12. $D(f)$ qanday tanlansa $f(x) = \{x\}$ funksiyaga teskari funksiya mavjud bo'ladi.
A) $[1; \infty)$ B) $[1; 5)$ C) $[-2; 0)$ D) $[0; 1)$
13. (98-6-14) Qaysi nuqta $y = x^3 + 5x - 2$ funksiyaga teskari funksiyaning grafigiga tegishli?
A) $(-2; 1)$ B) $(0; -2)$ C) $(4; 1)$ D) $(-8; 1)$
- Yechish:** 4-qoidaga ko'ra, agar $(x_0; y_0) \in Gr(f)$ bo'lsa, u holda $(y_0; x_0) \in Gr(f^{-1})$ bo'ladi. Shu sababli javoblarda keltirilgan nuqtalarning koordinatalari o'rinlarini almashtirib berilgan funksiya-ga qo'yib tekshiramiz. $y(1) = 4$ bo'lgani uchun $(1; 4)$ nuqta berilgan funksiyaning grafigiga tegishli bo'ladi. Demak, $(4; 1)$ nuqta unga teskari funksiyaning grafigiga tegishli. **Javob:** $(4; 1)$ (C).
5. (99-4-15) $y = \frac{k}{x+2}$, ($k > 0$) funksiyaning grafigi qaysi choraklar orqali o'tadi?
A) I va III B) II va IV
C) I, III, IV D) I, II va III
6. (03-6-40) $y = x|x|^{-1}$ funksiya grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?
A) III B) IV C) II, III D) I, III
7. (99-5-40) $y(x) = |x - 1| + |x - 3|$ funksiyaning qiymatlar sohasini toping.
A) $[0; \infty)$ B) $[1; \infty)$ C) $[2; \infty)$ D) $[3; \infty)$
- Yechish:** 3-qoidaga ko'ra, $E(y) = [2; \infty)$ ni olamiz.
Javob: $[2; \infty)$ (C).
8. (03-12-26) $y = |x - 1| + |x - 3|$ funksiyaning eng kichik qiymatini toping.
A) 3 B) 4 C) 2 D) 1
9. (00-9-45) $f(x) = |x + 2| + |x + 8|$ funksiyaning qiymatlar sohasini toping.
A) $[0; \infty)$ B) $[3; \infty)$ C) $[4; \infty)$ D) $[6; \infty)$

10.5 Aralash tipdagi masalalar

1. $y = f(x)$ va $y = g(x)$ funksiyalar grafiklari kesishgan nuqtalarning absissalari $f(x) = g(x)$ tenglamaning ildizlari bo'ladi.
2. Tekislikdagi $M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalar orasidagi masofa

$$|M_1M_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

3. $y = |x - a| + |x - b|$ ($a < b$) funksiyaning qiymatlar sohasi $E(y) = [b - a; \infty)$ dan iborat.

1. (00-3-59) Agar $f(x+1) = x^2 - 3x + 2$ bo'lsa, $f(x)$ ni toping.
A) $x^2 - 3x - 1$ B) $x^2 - 5x + 1$
C) $x^2 - 5x + 6$ D) $x^2 - 4$

Yechish: $x + 1 = t$ deb belgilash olamiz, u holda $x = t - 1$ bo'ladi. Uni $f(x + 1) = x^2 - 3x + 2$ tenglikka qo'yamiz:

$$f(t) = (t - 1)^2 - 3(t - 1) + 2 = t^2 - 5t + 6.$$

Javob: $f(x) = x^2 - 5x + 6$ (C).

2. (00-9-60) Agar $f(x-1) = x^2 + 3x - 2$ bo'lsa, $f(x)$ ni aniqlang.
A) $x^2 + 2x - 3$ B) $x^2 + 5x - 4$
C) $x^2 + 5x + 2$ D) $x^2 - x - 2$

3. (97-7-16) k ning qanday qiymatlarida $y = \frac{k}{x} - 1$ funksiyaning grafigi $C(-\frac{1}{2}; -3)$ nuqtadan o'tadi?
A) 1 B) -2 C) -1 D) $\frac{1}{2}$

4. (97-10-16) k ning qanday qiymatida $y = kx^3 + 2$ funksiyaning grafigi $B(-2; 10)$ nuqtadan o'tadi?
A) 2 B) 1 C) -0,5 D) -1

10. (02-7-13) $f(x) = \frac{3}{x-4}$ funksiyaning qiymatlar to'plamini toping.
A) $(-\infty; 0) \cup (0; \infty)$ B) $(-\infty; 4) \cup (4; \infty)$
C) $(-\infty; 3) \cup (3; \infty)$ D) $(-\infty; -1) \cup (-1; \infty)$
11. (00-3-60) Argumentning qanday qiymatida

$$y = \frac{5x}{2|x+1| - 5}$$

funksiya 2 ga teng?

- A) $-\frac{4}{3}$ B) $-\frac{5}{3}$ C) -2 D) $-\frac{14}{9}$

12. (00-7-17) Koordinata boshidan $y = x^2$ va $y = \frac{1}{x}$ funksiyalarning grafiklari kesishgan nuqttagacha bo'lgan masofani aniqlang.

- A) 2 B) 1,5 C) $\sqrt{2}$ D) $\frac{1}{2}\sqrt{2}$

Yechish: 1-qoidaga ko'ra, funksiyalar grafiklari kesishgan nuqtalar absissalari $x^2 = \frac{1}{x} \iff x^3 = 1$ tenglama yechimlari bo'ladi. Bu tenglamaning yechimi $x = 1$. Bundan $y = 1$ ni olamiz. Koordinata boshi $O(0; 0)$ dan $A(1; 1)$ nuqttagacha masofa (2-ga qarang)

$$|OA| = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}.$$

Javob: $\sqrt{2}$ (C).

13. (03-12-32) $3x + 4y + 7 = 0$ va $3x + y - 5 = 0$ to'g'ri chiziqlarning kesishish nuqtasi koordinata boshidan qanday masofada joylashgan?
A) 5 B) 6 C) 8 D) $8\sqrt{2}$
14. (01-4-12) $M(x, y)$ nuqtaning koordinatalari yig'indisi 3 ga teng. Bu nuqta va koordinata boshi orasidagi eng qisqa masofa qanchaga teng bo'ladi?
A) $3\sqrt{2}$ B) $2\sqrt{3}$ C) $1,5\sqrt{2}$ D) $4,5\sqrt{2}$

15. (00-10-8) $y = |x - 1| - 5$ va $y = 0$ funksiyalar grafiklari kesishgan nuqtalar absissalarining kvadratlarini yig'indisini toping.

A) 36 B) 48 C) 24 D) 52

Yechish: 1-qoidaga ko'ra, funksiyalar grafiklari kesishgan nuqtalar absissalari $|x - 1| - 5 = 0 \iff |x - 1| = 5$ tenglama yechimlari bo'ladi. Bu tenglamani geometrik usulda yechsak, sonlar o'qida koordinatasi 1 bo'lgan nuqtadan 5 birlik chapda $1 - 5 = -4$ ni, 5 birlik o'ngda $1 + 5 = 6$ ni topamiz. Ular kvadratlarini yig'indisi $(-4)^2 + 6^2 = 16 + 36 = 52$. **Javob:** 52 (D).

16. (98-11-12) $y = |x - 2| + 1$ va $y = 5$ funksiyalar grafiklari kesishgan nuqtalar absissalari kvadratlarining yig'indisini toping.

A) 52 B) 32 C) 40 D) 36

17. (01-11-14) Ushbu $y = x^4$ va $y = 2x^2 - 1$ funksiyalarning grafiklari nechta umumiy nuqtaga ega?

A) 4 B) 3 C) 1 D) 2

18. (02-1-53) Agar $y = x^3 + 1$ va $-1 < x < 2$ bo'lsa, y qanday oraliqda o'zgaradi?

A) $(-1; \infty)$ B) $(0; 9)$ C) $(1; 8)$ D) $(-1; 9)$

19. (02-12-47) $y = ax^3 + b$ kubik parabolaning grafigi $A(1; 18)$ va $B(-1; 14)$ nuqtalardan o'tadi. Qaysi nuqtada bu funksiyaning grafigi Ox o'qini kesib o'tadi?

A) $(0; 2)$ B) $(-3; 0)$ C) $(3; 0)$ D) $(-2; 0)$

20. (98-4-41) $f(x) = 1 - 2x$ funksiya berilgan. Agar $f(\varphi(x)) = x$ bo'lsa, $\varphi(x)$ funksiyani toping.

A) $\frac{1-x}{2}$ B) $\frac{x+1}{2}$ C) $\frac{x-1}{2}$ D) $\frac{2x-1}{4}$

Yechish: Masala shartiga ko'ra $f(\varphi(x)) = 1 - 2\varphi(x) = x$. Bu yerdan $1 - x = 2\varphi(x)$, ya'ni $\varphi(x) = \frac{1-x}{2}$. **Javob:** $\frac{1-x}{2}$ (A).

21. (00-9-43) $f(x) = 2x^2$ va $\varphi(x) = x + 1$ bo'lsa, x ning nechta qiymatida $f(\varphi(x)) = \varphi(f(x))$ bo'ladi?

A) 0 B) 1 C) 2 D) 3

22. Agar $f(x) = x^2 - 1$ va $\varphi(x) = x + 1$ bo'lsa, $f(\varphi(x))$ ning qiymatini toping.

A) x^2 B) $x^2 + 2x$ C) $2x$ D) $x^2 - 2x$

23. (01-7-46) Agar $f(x) = \frac{1-x}{1+x}$ bo'lsa, $f\left(\frac{1}{x}\right) + \frac{1}{f(x)}$ ning qiymatini toping.

A) $\frac{4x}{1-x^2}$ B) $\frac{4x}{x^2-1}$ C) $\frac{x^2+1}{x^2-1}$ D) $\frac{2(x^2+1)}{x^2-1}$

24. (01-6-18) p ning qanday qiymatida tenglamasi $x^2 + y^2 = 64$ bo'lgan aylana va $y = x^2 + p$ funksiyaning grafigi uchta umumiy nuqtaga ega bo'ladi.

A) 8 B) 6 C) -8 D) -6

25. (99-2-22) Ushbu $y = x^2$, $y = |x|$ funksiyalar grafiklarining Ox o'qida yotmaydigan kesishish nuqtalari orasidagi masofani toping.

A) 2 B) 2,5 C) 2,3 D) 1,5

11 -bob. Ko'rsatkichli tenglama va tengsizliklar

11.1 Ko'rsatkichli funksiya

$y = a^x$, $a > 0$, $a \neq 1$ ko'rinishdagi funksiyaga *ko'rsatkichli* funksiya deyiladi. Uning aniqlanish sohasi $D(y) = (-\infty; \infty)$ va qiymatlar sohasi $E(y) = (0; \infty)$. $y = a^x$ funksiya $a > 1$ da o'suvchi, $0 < a < 1$ da kamayuvchi. $y = a^x$ funksiyaning grafigi I va II choraklarda yotadi (11.1-chizmaga qarang).

Ixtiyoriy $a > 0$, $b > 0$ sonlar uchun quyidagi tengliklar o'rinli.

1. $a^{x+y} = a^x \cdot a^y$.

2. $(a^x)^y = a^{xy}$.

3. $\frac{a^x}{a^y} = a^{x-y}$.

4. $(a \cdot b)^x = a^x \cdot b^x$.

5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$.

6. $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$.

7. $a^{-x} = \frac{1}{a^x}$.

8. $a^{-m/n} = \frac{1}{a^{m/n}}$.

1. (98-7-23) Quyidagilardan qaysilari kamayuvchi funksiyalar?

1) $y = 0,37^x$; 2) $y = (\sqrt[3]{11})^x$; 3) $y = 3 \cdot \left(\frac{1}{2}\right)^x$;

4) $y = \left(\frac{3}{4}\right)^x$; 5) $y = \frac{1}{2} \cdot 3^x$

A) 1; 3; 5 B) 2; 3; 4 C) 1; 4 D) 1; 3; 4

Yechish: $y = a^x$ funksiya $0 < a < 1$ da kamayuvchi bo'lgani uchun 1; 3; 4 funksiyalar kamayuvchi bo'ladi. **Javob:** 1; 3; 4 (D).

2. Kamayuvchi funksiyalarni ajrating.

$y_1 = 2^x$; $y_2 = e^x$; $y_3 = \pi^{-x}$; $y_4 = \left(\frac{e}{\pi}\right)^x$

A) y_1 ; y_2 B) y_2 ; y_3 C) y_3 ; y_4 D) y_4 ; y_1

3. O'suvchi funksiyalarni ajrating.

$$y_1 = 3^x; \quad y_2 = e^x; \quad y_3 = \pi^x$$

A) y_1 B) y_2 C) y_3 D) barchasi

4. $f(x) = e^x - 1$ funksiyaning qiymatlar sohasini toping.

A) $(0; \infty)$ B) $(1; \infty)$ C) $(-1; \infty)$ D) $[0; \infty)$

5. $f(x) = 2^x - 2^{-x}$ funksiyaning qiymatlar sohasini toping.

A) $(0; \infty)$ B) $(1; \infty)$ C) $(-\infty; \infty)$ D) $[0; \infty)$

6. $f(x) = 2^x - 2^{-x}$ funksiyaning $x = 2$ nuqtadagi qiymatini toping.

A) 0 B) 2 C) 3,75 D) 3,25

Yechish: $f(2) = 2^2 - 2^{-2} = 4 - \frac{1}{2^2} = 4 - \frac{1}{4}$.

Demak, $f(2) = 3,75$ ekan. **Javob:** 3,75 (C).

7. $y = 2^x - 1$ funksiya grafigi qaysi choraklar orqali o'tadi?

A) I; III B) II; IV C) I; II D) II; III

8. Grafigi koordinata boshidan o'tuvchi funksiyani toping.

A) $y = 2^x - e^x$ B) $y = e^x + 1$
C) $y = 3^x - 3$ D) $y = 2^x - 2$

9. $y = 7^x - 1$ funksiya grafigi qaysi nuqta orqali o'tadi?

A) $(0; 1)$ B) $(1; 6)$ C) $(2; 13)$ D) $(-1; 0)$

10. (97-7-16) k ning qanday qiymatida $y = 2^{kx} - 1$ funksiyaning grafigi $C(2; 15)$ nuqtadan o'tadi?

A) 1 B) -2 C) 4 D) 2

Yechish: Masala shartidan $15 = 2^{2k} - 1$ tenglikni olamiz. Bu yerdan

$$16 = 2^{2k} \iff 2^4 = 2^{2k} \iff 4 = 2k$$

kelib chiqadi. Demak, $k = 2$. **Javob:** 2 (D).

11. (98-2-30) Quyidagi sonlardan qaysi biri 1 dan katta?

$$a = 0,7^{2,3} \cdot 0,3^{0,8}, \quad b = 3,2^{-4,2} \cdot 1,2^{-0,8},$$

$$c = 0,7^{-1,2} \cdot 0,6^{-0,4}, \quad d = 0,6^{0,4} \cdot 0,3^{0,6},$$

A) a, d B) b, c C) c D) d, c

12. (98-5-31) Ushbu $y = a^x$ funksiya uchun qaysi mulahaza noto'g'ri?

- A) aniqlanish sohasi barcha haqiqiy sonlar to'plami
B) qiymatlari to'plami barcha musbat haqiqiy sonlar to'plami
C) garifigi $(0;1)$ nuqtadan o'tadi
D) aniqlanish sohasida har doim o'suvchi

13. Toq funksiyalarni ajrating.

$$y_1 = a^x - a^{-x}; \quad y_2 = x(a^x + a^{-x}); \quad y_3 = x \cdot e^{|x|}.$$

A) y_1 B) y_2 C) $y_2; y_3$ D) barchasi

14. (99-3-27) Funksiyalardan qaysilari juft funksiya?

$$y_1 = \frac{a^x + a^{-x}}{2}; \quad y_2 = \frac{a^x + 1}{a^x - 1};$$

$$y_3 = \frac{x}{a^x - 1}; \quad y_4 = x \frac{a^x - 1}{a^x + 1};$$

A) y_1 B) y_2 C) $y_2; y_3$ D) $y_1; y_4$

11.2 Ko'rsatkichli tenglamalar

1. Bir xil asosga keltirib yechiladigan tenglamalar

Bu yerda quyidagi tengkuchlilikdan foydalaniladi:

$$1. a^{f(x)} = a^{g(x)} \iff f(x) = g(x), \quad a > 0, a \neq 1.$$

1. (97-9-94) Tenglamani yeching.

$$\left(\frac{25}{64}\right)^{7x^2-6} = \left(\frac{64}{25}\right)^{2+3x-6x^2}$$

A) -4; 1 B) -1; 4 C) 1; 4 D) -4; -1

Yechish: Tenglamani har ikkala qismini bir xil asosga keltiramiz (11.1-ning 6-xossasi)

$$\left(\frac{64}{25}\right)^{-(7x^2-6)} = \left(\frac{64}{25}\right)^{2+3x-6x^2}.$$

1-tengkuchlilikka ko'ra $-7x^2 + 6 = 2 + 3x - 6x^2$.

Bu yerdan $x^2 + 3x - 4 = 0$ kvadrat tenglamani hosil qilamiz va uni yechib $x_1 = -4, x_2 = 1$ ildizlarni olamiz. **Javob:** -4; 1 (A).

2. (96-1-34) Tenglamani yeching.

$$3^1 \cdot 3^2 \cdot 3^3 \cdot \dots \cdot 3^x = \frac{1}{9-33}$$

A) 12 va -11 B) 11 C) 12 D) 33

3. (96-6-51) Ushbu

$$\left(\frac{4}{3}\right)^x \cdot \left(\frac{3}{8}\right)^x = 2$$

tenglamani ildizi x_0 bo'lsa, quyidagi munosabatlardan qaysi biri o'rinli?

A) $x_0 > -1$ B) $x_0 < -1$
C) $x_0 = -1$ D) $\frac{x_0}{2} = -1$

4. (97-1-76) Tenglamani yeching.

$$(0,75)^{x-1} = \left(1\frac{1}{3}\right)^3$$

A) 1 B) -1 C) 2 D) -2

5. (97-6-57) Tenglamani yeching.

$$(0,8)^{3-2x} = (1,25)^3$$

A) 0 B) 1 C) 2 D) 3

6. (99-1-29) Tenglamani yeching.

$$4^{x-4} = 0,5$$

A) 3,5 B) 4,5 C) -4,5 D) -3,5

7. (99-6-8) Tenglamani yeching.

$$(3,5)^{x-5} = \left(\frac{4}{49}\right)^2$$

A) 3 B) 2 C) 1 D) 4

8. (99-6-27) Tenglamani yeching.

$$\frac{1}{27} \cdot \sqrt[4]{9^{3x-1}} = 27^{-\frac{2}{3}}$$

- A)
- -1
- B)
- 2
- C)
- 1
- D)
- -2

Yechish: Tenglamani har ikkala qismini 27 ga ko'paytirib, 27 va 9 larni 3^3 va 3^2 shaklda yozib, natijada

$$\sqrt[4]{(3^2)^{3x-1}} = 3^3 \cdot (3^3)^{-\frac{2}{3}} \iff 3^{(3x-1):2} = 3^1$$

ni olamiz. 1-ga ko'ra $(3x-1) : 2 = 1$ bo'ladi. Bu tenglamani yechimi $x = 1$ dir. **Javob:** 1 (C).

9. (99-6-58) Tenglamani yeching.

$$(0,1(6))^{3x-5} = 1296$$

- A)
- $\frac{1}{3}$
- B)
- 3
- C)
- -3
- D)
- $-\frac{1}{3}$

10. (99-10-39) Tenglamani ildizi 10 dan qancha kam?

$$3^{x+1} \cdot 27^{x-1} = 9^7$$

- A)
- 5
- B)
- 4
- C)
- 8
- D)
- 6

11. (00-3-32) Tenglamani yeching.

$$0,125 \cdot 4^{2x-3} = \left(\frac{\sqrt{2}}{8}\right)^{-x}$$

- A)
- 2
- B)
- -2
- C)
- 4
- D)
- 6

12. (96-10-37) Tenglamani yeching.

$$5^2 \cdot 5^4 \cdot 5^6 \cdot \dots \cdot 5^{2x} = 0,04^{-28}$$

- A)
- 5
- B)
- 10
- C)
- 14
- D)
- 7

13. (01-5-13) Tenglama ildizlarining ko'paytmasini toping.

$$2x^2 - 6x - 5/2 = 16\sqrt{2}$$

- A)
- -7
- B)
- -2
- C)
- 3
- D)
- 2

14. (01-6-35) Agar

$$\frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{24}{81}$$

bo'lsa, $x - y$ ning qiymatini toping.

- A)
- 6
- B)
- 5
- C)
- 4
- D)
- 3

15. (01-7-30) Tenglamani yeching.

$$(0,25)^{2-x} = \frac{1}{2^{x+3}}$$

- A)
- 2
- B)
- 3
- C)
- $\frac{1}{2}$
- D)
- $\frac{1}{3}$

16. (02-2-23) Tenglama ildizlarining ko'paytmasini toping.

$$\left(\frac{1}{4}\right)^{(4-x^2)/2} = 8^x$$

- A)
- -4
- B)
- 6
- C)
- 4
- D)
- -6

17. (02-3-17) Agar
- $3^{\alpha-3} = 11$
- bo'lsa,
- $3^{5-\alpha}$
- ning qiymatini toping.

- A)
- $\frac{9}{11}$
- B)
- 99
- C)
- $\frac{3}{16}$
- D)
- $\frac{11}{9}$

Yechish: $3^{5-\alpha}$ ni quyidagicha yozib olamiz:

$$3^{5-\alpha} = \frac{1}{3^{\alpha-5}} = \frac{3^2}{3^{\alpha-3}} = \frac{9}{11}$$

Javob: $\frac{9}{11}$ (A).

18. (02-7-53) Agar

$$\sqrt[4]{9^{\frac{n-3}{5}}} = 243$$

bo'lsa, n nechaga teng?

- A)
- 53
- B)
- 38
- C)
- 47
- D)
- 43

19. (03-3-31) Tenglama ildizlari yig'indisini toping.

$$\left(\frac{\sqrt{5}}{3}\right)^{2x^2-5x} = 1,8$$

- A)
- 5
- B)
- -5
- C)
- $2,5$
- D)
- $-2,5$

20. (03-4-29) Tenglamani ildizi 12 dan qancha kam?

$$\frac{2^{2x-1} \cdot 4^{x+1}}{8^{x-1}} = 64$$

- A)
- 8
- B)
- 9
- C)
- 6
- D)
- 10

21. (03-6-45) Tenglamani yeching.

$$\sqrt{5^2 - 4^2} = \sqrt[3]{81}$$

- A)
- 2
- B)
- 4
- C)
- 3
- D)
- 6

2. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish usuli

22. (98-2-31) Tenglamani kichik ildizini toping.

$$2^{-4x^2+2} - 3 \cdot 2^{-4x^2} = 2^{-16}$$

- A)
- 2
- B)
- -3
- C)
- -2
- D)
- -1

Yechish: Tenglamani chap qismida umumiy ko'paytuvchi 2^{-4x^2} ni qavsdan tashqariga chiqaramiz:

$$2^{-4x^2}(2^2 - 3) = 2^{-16} \iff 2^{-4x^2} = 2^{-16}$$

1-ga ko'ra $-4x^2 = -16 \iff x^2 = 4$. Bu tenglamani ildizlari $x_1 = -2$, $x_2 = 2$ lardir. Ularning kichigi $x_1 = -2$. **Javob:** -2 (C).

23. (98-8-34) Tenglamani yeching.

$$\left(\frac{1}{7}\right)^{-2x+3} + 49^{x-1} + 7^{2x-1} = 399$$

- A)
- 5
- B)
- 4
- C)
- 3
- D)
- 2

24. (98-9-31) 18 va
- $2^{x-4} + 2^{x+1} = 132$
- tenglama ildizi ayirmasini toping.

- A)
- 9
- B)
- 10
- C)
- 8
- D)
- 12

25. (99-3-18) Agar

$$3^{5x+1} + 3^{5x-1} = 30$$

bo'lsa, $\frac{x}{x+1}$ ning qiymatini hisoblang.

A) $\frac{2}{5}$ B) $\frac{1}{3}$ C) $\frac{2}{7}$ D) $\frac{4}{9}$

26. (01-7-31) Tenglamani yeching.

$$6 \cdot 9^{0,5x-2} + 2 \cdot 3^{x-6} = 56$$

A) 1 B) 2 C) 6 D) 3

27. (02-12-43) Agar

$$4^{x-1} - \frac{1}{2} \cdot 2^{2x} = -64$$

bo'lsa, $x + 13$ ning qiymatini toping.

A) 19 B) 15 C) 17 D) 13

3. Yangi noma'lum kiritish usuli

28. (99-6-49) Tenglamani yeching.

$$3^{\sqrt{x}} - 3^{1-\sqrt{x}} = \frac{26}{3}$$

A) \emptyset B) 9 C) 2 D) 4

Yechish: Tenglamada $3^{\sqrt{x}} = y > 0$ almashtirish olsak, u

$$y - \frac{3}{y} = \frac{26}{3} \iff 3y^2 - 26y - 9 = 0$$

ko'rinishni oladi. Bu kvadrat tenglamaning ildizlari $y_1 = -3^{-1}$, $y_2 = 9$ lardir. y_1 yechim $y > 0$ shartni qanoatlantirmaydi. $3^{\sqrt{x}} = y_2 \iff 3^{\sqrt{x}} = 9$ dan $x = 4$ ni olamiz. **Javob:** 4 (D).

29. (99-8-2) $5^x - 5^{3-x} = 20$ tenglamani yeching.

A) -5 B) 1 C) -5; 1 D) 2

30. (01-1-20) $5^x - 24 = 5^{2-x}$ tenglamani yeching.

A) -2 B) 0 C) -1 D) 2

31. (02-9-37) Tenglama ildizlari yig'indisini toping.

$$25^{x^2+0,5} - 5^{x^2} = 5^{x^2+3} - 25$$

A) 0 B) 1 C) $2\sqrt{2}$ D) 2

32. (02-11-28) Tenglama ildizlari ko'paytmasini toping.

$$8 \cdot 4^{|x|} - 33 \cdot 2^{|x|} + 4 = 0$$

A) 4 B) $\frac{1}{4}$ C) -4 D) $-\frac{1}{4}$

33. (03-7-19) Tenglamani yeching.

$$4^{x+1} - 2^{x+4} + 3 \cdot 2^{x+2} = 48$$

A) 1 B) 2 C) 3 D) 4

4. Guruhlash usuli

34. (97-6-26) Tenglamani yeching.

$$2^{3x+7} + 5^{3x+4} + 2^{3x+5} - 5^{3x+5} = 0$$

A) 1 B) 0 C) -1 D) 2

Yechish: 2 asosli darajalarni tenglamaning chap qismida qoldirib, 5 asosli darajalarni tenglamaning o'ng qismiga o'tkazib, umumiy ko'paytuvchini qavsdan tashqariga chiqarsak,

$$2^{3x+5}(2^2 + 1) = 5^{3x+4}(5 - 1) \iff 2^{3x+3} = 5^{3x+3}$$

ni olamiz. Tenglikning har ikkala qismini 5^{3x+3} ga bo'lib, $0,4^{3x+3} = 1 = 0,4^0$ ni hosil qilamiz. Bu yerdan $3x + 3 = 0$ yoki $x = -1$ ekanligi kelib chiqadi. **Javob:** -1 (C).

35. Tenglamani yeching.

$$5^{3x} - 7^x - 35 \cdot 5^{3x} + 35 \cdot 7^x = 0$$

A) 1 B) 0 C) -1 D) 2

36. $2^x = 5^x$ tenglamani yeching.

A) 1 B) 0 C) -1 D) 2

37. $3 \cdot 2^x = 2 \cdot 3^x$ tenglamani yeching.

A) 1 B) 0 C) -1 D) 2

38. Tenglamani yeching.

$$9 \cdot 16^x - 7 \cdot 12^x = 16 \cdot 9^x$$

A) 2 B) -2 C) 3 D) -1

Yechish: $m_1 \cdot a^x + m_2 \cdot b^x = m_3 \cdot c^x$ ko'rinishdagi tenglamalarni $ac = b^2$ ($a < b < c$) shartda kvadrat tenglamaga keltirib yechish mumkin. Bu misolda $16 \cdot 9 = 12^2$ tenglik bajariladi. Berilgan tenglamaning har ikkala qismini $16^x > 0$ ga bo'lib,

$$9 - 7 \cdot \left(\frac{12}{16}\right)^x = 16 \cdot \left(\frac{9}{16}\right)^x \iff 9 - 7 \cdot \left(\frac{3}{4}\right)^x = 16 \cdot \left(\frac{3}{4}\right)^{2x}$$

ni olamiz. Tenglamada $\left(\frac{3}{4}\right)^x = y > 0$ belgilash kiritib, uni $16y^2 + 7y - 9 = 0$ shaklga keltiramiz. Bu kvadrat tenglamaning ildizlari $y_1 = -1$, $y_2 = 9/16$. $y_1 = -1$ yechim $y > 0$ shartni qanoatlantirmaydi. $\left(\frac{3}{4}\right)^x = y_2 \iff \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^2$ dan $x = 2$ ni olamiz. **Javob:** 2 (A).

39. $4^x + 6^x = 2 \cdot 9^x$ tenglamani yeching.

A) 0 B) 0; -1 C) -1 D) 1

5. Tenglamalar sistemasi

40. (96-7-17) Agar $\begin{cases} 3^x = 9^{y+1} \\ 4y = 5 - x \end{cases}$ ekanligi ma'lum bo'lsa, $x + y$ ning qiymatini toping.

A) 3,5 B) 5 C) 2 D) -4

Yechish: Sistemaning 2-tenglamasidan $x = 5 - 4y$ ni olamiz. Uni sistemaning 1-tenglamasiga qo'yib $3^{5-4y} = 9^{y+1} \iff 3^{5-4y} = 3^{2y+2}$ ni olamiz. 1-tengkuchlilikka ko'ra, $5 - 4y = 2y + 2$. Bu yerdan $y = 0,5$ ni olamiz. Uni sistemaning 2-tenglamasi qo'yib $x = 3$ ni olamiz. Ularning yig'indisi $x + y = 3 + 0,5 = 3,5$. **Javob:** 3,5 (A).

41. (97-3-17) Agar $3^{x-1} = 9^y$ va $2x - y = 5$ bo'lsa, $x - y$ ni toping.
A) 2 B) 3 C) -1 D) -0,5
42. (97-7-17) Agar $2^{x+1} = 4^y$ va $x + y = -4$ bo'lsa, $y - x$ ni toping.
A) 4 B) -2 C) 2 D) -3
43. (00-3-30) Agar

$$\begin{cases} 9^{x+y} = 729 \\ 3^{x-y-1} = 1 \end{cases}$$

bo'lsa, $x^2 - y^2$ ni toping.

- A) 1 B) 4 C) 3 D) 2

44. (02-1-58) Agar

$$\begin{cases} 2^x + 2^y = 5 \\ 2^{x+y} = 4 \end{cases}$$

bo'lsa, $x \cdot y$ ni toping.

- A) 0 B) 1 C) 2 D) 3

45. (03-4-31) Agar $2^{x^2} \cdot 2^{y^2} = 64$ va $2^{xy} = \sqrt{8}$ bo'lsa, $|x + y|$ ning qiymatini toping.
A) 4,5 B) 3,5 C) 2,5 D) 3

11.3 Ko'rsatkichli tengsizliklar

$a^{f(x)} > a^{g(x)}$ yoki $a^{f(x)} < a^{g(x)}$ ko'rinishdagi tengsizliklar sodda ko'rsatkichli tengsizliklar deyiladi. Bunday tengsizliklarni yechish ko'rsatkichli $y = a^x$ funksiyaning $a > 1$ bo'lganda o'suvchi $0 < a < 1$ bo'lganda kamayuvchi ekanligiga asoslanadi, ya'ni:

1. Agar $0 < a < 1$ bo'lsa,

$$a^{f(x)} > a^{g(x)} \iff f(x) < g(x).$$

2. Agar $a > 1$ bo'lsa,

$$a^{f(x)} > a^{g(x)} \iff f(x) > g(x).$$

1. (98-2-32) Tengsizlikning eng katta butun yechimini toping.

$$\left(\frac{4}{9}\right)^x \cdot \left(\frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{-2x}$$

- A) 2 B) 3 C) 4 D) 1

Yechish: Tengsizlikning chap qismiga $a^x \cdot b^x = (ab)^x$ formulani, o'ng qismiga esa $a^x \cdot a^y = a^{x+y}$ formulani qo'llab

$$\left(\frac{4}{9} \cdot \frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^{6-2x} \iff \left(\frac{2}{3}\right)^x > \left(\frac{2}{3}\right)^{6-2x}$$

tengsizlikni hosil qilamiz. Asos $\frac{2}{3} < 1$ bo'lgani uchun 1-teng kuchlilikdan

$$x < 6 - 2x \iff 3x < 6 \iff x < 2$$

ni olamiz. Bu shartni qanoatlantiruvchi eng katta butun son 1 dir. **Javob:** 1 (D).

2. (96-6-54) Tengsizlikni yeching.

$$0,25^x \geq 0,5^{4x-8}$$

- A) $(-\infty; 4)$ B) $(-\infty; 2]$ C) $[2; \infty)$ D) $[4; \infty)$

3. (97-6-55) Tengsizlikni yeching.

$$2^{\sqrt{x}-1} \cdot (4x^2 - 4x + 1) > 0$$

- A) $(1; \infty)$ B) $[1; \infty)$
C) $[\frac{1}{2}; \infty)$ D) $[0; \frac{1}{2}) \cup (\frac{1}{2}; \infty)$

4. (97-9-76) x ning qanday qiymatlarida $y = 5^x - 5$ funksiya musbat qiymatlar qabul qiladi?

- A) $x < 1$ B) $x > 1$ C) $x \geq 1$ D) $x \leq 2$

5. (99-1-30) Tengsizlikni yeching.

$$(\sqrt{6})^x \leq \frac{1}{36}$$

- A) $(-\infty; -4]$ B) $[-4; \infty)$
C) $[-4; 4]$ D) $(-\infty; 6]$

Yechish: Tengsizlikning quyidagicha yozib olamiz:

$$6^{x/2} \leq 6^{-2} \iff \frac{x}{2} \leq -2 \iff x \leq -4$$

tengsizlikni hosil qilamiz. **Javob:** $(-\infty; -4]$ (A).

6. (99-2-35) Ushbu $\left(\frac{1}{2}\right)^{20-2x} > 1$ tengsizlikning eng kichik butun yechimini toping.

- A) 6 B) 11 C) 10 D) 9

7. (99-6-16) Tengsizlikning eng katta butun yechimini toping.

$$2^{3-6x} > 1$$

- A) 0 B) 1 C) -1 D) -2

8. (00-8-10) Tengsizlikni yeching.

$$\left(\frac{1}{2}\right)^{2x-1} > \frac{1}{16}$$

- A) $(-\infty; 2,5)$ B) $(2,5; \infty)$
C) $(-2,5; \infty)$ D) $(-\infty; 0) \cup (0; 2,5)$

9. (03-4-30) Tengsizlikning eng kichik butun yechimini toping.

$$\frac{1}{8} \cdot 2^{4x-2} > (\sqrt{2})^{10}$$

- A) 2 B) 1 C) 3 D) 4

10. (03-5-31) $f(x) = \sqrt{3^x - 4^x}$ funksiyaning aniqlanish sohasini toping.

- A) $(-\infty; 0]$ B) $(0; 1)$ C) $[0; 1)$ D) $[0; \infty)$

11. (00-6-31) Tengsizlikning butun yechimlari yig'indisini toping.

$$3^{8x} - 4 \cdot 3^{4x} \leq -3$$

- A) 8 B) 7 C) 4 D) 0

Yechish: Tengsizlikda $3^{4x} = y > 0$ belgilash olib, uni

$$y^2 - 4y + 3 \leq 0 \iff (y-1)(y-3) \leq 0$$

shaklda yozib olamiz. Bu tengsizlikni oraliqlar usuli bilan yechib $1 \leq y \leq 3$ ni olamiz. Belgilashga qaytib $1 \leq 3^{4x} \leq 3 \iff 3^0 \leq 3^{4x} \leq 3^1$ ni olamiz. Bu yerdan 2-ga ko'ra $0 \leq 4x \leq 1 \iff 0 \leq x \leq 0,25$. Tengsizlikning birgina butun yechimi 0 dir. **Javob:** 0 (D).

12. (02-5-20) Tengsizlikni yeching.

$$4^x - 5 \cdot 2^{x+1} + 16 \leq 0$$

- A) (1; 3) B) (0; 1) \cup (3; ∞)
C) [1; 3] D) [0; 1] \cup [3; ∞)

13. (01-4-30) Tengsizlikni yeching.

$$9^{-x} - 28 \cdot 3^{-x-1} + 3 < 0$$

- A) (-2; 1) B) $(-\infty; 2]$ C) [1; ∞) D) (-2; 0)

14. (01-1-21) Tengsizlikni yeching.

$$3^{\frac{1}{x+1}} > 9$$

- A) (-1; 1) B) $(-1; -\frac{1}{2})$
C) $(-\frac{1}{2}; 1)$ D) (0; 1)

15. (01-2-70) Nechta natural son

$$(0, 7)^{2+4+\dots+2n} > (0, 7)^{72}$$

tengsizlikni qanoatlantiradi?

- A) 7 B) 8 C) 9 D) 10

Yechish: Arifmetik progressiya dastlabki n ta hadi yig'indisi formulasidan foydalansak,

$$S_n = 2 + 4 + \dots + 2n = n(n+1).$$

Asos $0,7 < 1$ bo'lganligi uchun, 1-ga ko'ra

$$n(n+1) < 72 \iff (n-8)(n+9) < 0$$

tengsizlik o'rinli. Bu tengsizlikni oraliqlar usuli bilan yechib $-9 < n < 8$ ni olamiz. Bu shartni qanoatlantiruvchi natural sonlar 1, 2, 3, 4, 5, 6, 7 lardir. Ular 7 ta **Javob:** 7 (A).

16. (98-5-16) Ushbu $14 \leq 2^n < 64$ qo'sh tengsizlikni qanoatlantiruvchi natural sonlar nechta?

- A) 2 B) 3 C) 1 D) 4

17. (99-7-18) n ning nechta natural qiymati

$$9 \leq 3^n \leq 79$$

qo'sh tengsizlikni qanoatlantiradi?

- A) 1 B) 3 C) 4 D) 2

18. (01-8-32) Ushbu $3^{|x|+2} \leq 81$ tengsizlikning butun yechimlari yig'indisini toping.

- A) -1 B) 3 C) 4 D) 0

19. (01-9-18) Ushbu $0, 5^{x^2-4} > 0, 5^{3x}$ tengsizlikning butun yechimlari o'rta arifmetigini toping.

- A) 1, 5 B) 2 C) 1 D) 3

20. (02-2-25) Tengsizlikni yeching.

$$5^{\frac{1}{x}} + 5^{\frac{1}{x}+2} > 130$$

- A) (0; 1) B) (0; 3) C) $(0; \frac{3}{4})$ D) (1; 2)

Yechish: Umumiy ko'paytuvchi $5^{\frac{1}{x}}$ ni qavs oldiga chiqaramiz:

$$5^{\frac{1}{x}}(1 + 25) > 130 \iff 5^{\frac{1}{x}} > 5^1.$$

Asos $5 > 1$ bo'lganligi uchun, 2-ga ko'ra

$$\frac{1}{x} > 1 \iff \frac{1}{x} - 1 > 0 \iff \frac{1-x}{x} > 0.$$

Bu tengsizlikni oraliqlar usuli bilan yechib $0 < x < 1$ ni olamiz. **Javob:** (0; 1) (A).

21. (06-121-34) Tengsizlikni yeching.

$$3^{\frac{1}{x}} + 3^{\frac{1}{x}+3} > 84$$

- A) (0; 1) B) $(-\infty; 0)$
C) (1; ∞) D) (0; 1) \cup (1; ∞)

22. (03-6-58) Tengsizlikni yeching.

$$3^{3x-2} + 3^{3x+1} - 3^{3x} < 57$$

- A) $x > 1$ B) $x < 1\frac{1}{2}$ C) $x < 1$ D) $x > \frac{2}{3}$

23. (03-7-79) Tengsizlikning natural yechimlari yig'indisini toping.

$$3^{x+2} + 3^{x+3} \leq 972$$

- A) 1 B) 3 C) 6 D) 10

24. (02-5-22) Tengsizlik yechimlari orasida nechta tub son bor?

$$(1, 25)^{1-x} > (0, 64)^{2(1+\sqrt{x})}$$

- A) 5 B) 7 C) 9 D) 12

25. (97-2-54) Tengsizlikni yeching.

$$0, 2^{x^2+1} + 0, 2^{x^2-1} < 1, 04$$

- A) $(-\infty; -1)$ B) (1; ∞)
C) $(-\infty; -1] \cup [1; \infty)$ D) $(-\infty; -1) \cup (1; \infty)$

26. (03-10-32) Agar

$$\begin{cases} 5^x + 5^{-x} = 13 \\ 28^x < 17^x \end{cases}$$

bo'lsa, $5^x - 5^{-x} = ?$

- A) $\sqrt{135}$ B) $-\sqrt{145}$ C) $\sqrt{175}$ D) $-\sqrt{165}$

12 -bob. Logarifmik funksiya

Ko'rsatkichli $f(x) = a^x$, $a \neq 1$, $a > 0$ funksiyani qaraymiz. Har bir tayinlangan $y \in (0; \infty)$ uchun

$$a^x = y \quad (12.1)$$

tenglama yagona yechimga ega. Bu yechim $x = \log_a y$ shaklda yoziladi. y sonining a asosga ko'ra logarifmi deb y sonini hosil qilish uchun a ni ko'tarish kerak bo'lgan darajaga aytiladi. Demak, (12.1) tenglikda x soni y ning a asosga ko'ra logarifmi ekan. Yuqoridagilardan kelib chiqadiki $y = \log_a x$, $a > 0$, $a \neq 1$ funksiya, ko'rsatkichli $f(x) = a^x$ funksiyaga teskari funksiya ekan. $y = \log_a x$, $a > 0$, $a \neq 1$ funksiyaga *logarifmik* funksiya deyiladi. Uning aniqlanish sohasi $D(y) = (0; \infty) = E(f)$ va qiymatlar sohasi $E(y) = (-\infty; \infty) = D(f)$. Teskari funksiya ta'rifidan quyidagilar kelib chiqadi: barcha $x \in R$ lar uchun $\log_a a^x = x$ va

$$a^{\log_a x} = x, \quad x > 0. \quad (12.2)$$

Bu tenglik *asosiy logarifmik ayniyat* deyiladi. $y = \log_a x$ funksiya $a > 1$ da o'suvchi, $0 < a < 1$ da kamayuvchi. Agar logarifm asosida $e = 2,71 \dots$ bo'lsa, u $\ln x = \log_e x$ shaklda yoziladi. 10 asosga ko'ra logarifm $\lg x = \log_{10} x$ shaklda yoziladi. Logarifmik funksiyani grafik I va IV choraklarda yotadi (12.1-chizma).

Logarifmik funksiya quyidagi xossalarga ega.

Ixtiyoriy $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ uchun

1. $a^{\log_a x} = x$, $x > 0$.
2. $\log_a a = 1$, $\log_a 1 = 0$.
3. $\log_a(xy) = \log_a x + \log_a y$, $x, y > 0$.
4. $\log_a \frac{x}{y} = \log_a x - \log_a y$, $x, y > 0$.
5. $\log_a x^p = p \log_a x$, $x > 0$.
6. $\log_{a^p} x = \frac{1}{p} \log_a x$, $x > 0$.
7. $\log_a x = \frac{\log_b x}{\log_b a}$.
8. $\log_a b \cdot \log_b a = 1 \iff \log_a b = \frac{1}{\log_b a}$.

$$9. \log_a b \cdot \log_c d = \log_c b \cdot \log_a d.$$

$$10. \log_{a^x} b^y = \frac{y}{x} \log_a b.$$

$$11. a^{\log_b c} = c^{\log_b a}.$$

$$12. a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}.$$

12.1 Aniqlanish sohasi va xossalari

1. (96-6-52) Ushbu $y = \log_3(2 - x)$ funksiyani aniqlanish sohasini toping.

A) $(-\infty; 2)$ B) $(2; \infty)$ C) $(0; 2)$ D) $(0; 2]$

Yechish: Logarifmik funksiyani aniqlanish sohasi $(0; \infty)$ dan iborat. Shuning uchun $2 - x > 0 \iff x < 2$. Demak, funksiya $(-\infty; 2)$ to'plamda aniqlangan ekan. **Javob:** $(-\infty; 2)$ (A).

2. Ushbu $y = \log_7(4 - x^2)$ funksiyani aniqlanish sohasini toping.

A) $(0; 2)$ B) $(2; 4)$ C) $(-2; 2)$ D) $(0; 2]$

3. (98-12-42) $y = \log_3 x$ funksiyani grafik koordinalar tekisligining qaysi choraklarida yotadi?

A) I, IV B) I, II C) II, III D) III, IV

4. (99-2-36) $f(x) = \frac{\sqrt{8-x}}{\lg(x-1)}$ funksiyani aniqlanish sohasiga tegishli butun sonlar nechta?

A) 4 B) 8 C) 7 D) 6

5. (99-3-26) Funksiyani aniqlanish sohasini toping.

$$y = \frac{1}{\ln(1-x)} + \sqrt{x+2}$$

A) $[-2; \infty)$ B) $[-2; 1]$
C) $(-\infty; 1)$ D) $[-2; 0) \cup (0; 1)$

6. (99-5-39) Ushbu $f(x) = \log_2(64^{-x} - 8^{1-x})$ funksiyani aniqlanish sohasini toping.

A) $(-\infty; 0)$ B) $(-\infty; -1)$
C) $(-\infty; -2)$ D) $(1; \infty)$

7. (99-6-29) Ushbu $y = \log_3(x(x-3)) - \log_3 x$ funksiyani aniqlanish sohasini toping.

A) $(3; \infty)$ B) $(-\infty; 3)$ C) $[3; \infty)$ D) $(-\infty; 3]$

8. (97-2-52) Ushbu $y = \log_{x^2}(4-x)$ funksiyani aniqlanish sohasini toping.

A) $(-\infty; 4)$
B) $(-\infty; -1) \cup (-1; 0) \cup (0; 1) \cup (1; 4)$
C) $(-\infty; -1) \cup [-1; 1] \cup (1; 4)$
D) $(-\infty; 1) \cup (4; \infty)$

Yechish: $\log_a b$ son $b > 0$, $a > 0$, $a \neq 1$ da aniqlangan uchun

$$\begin{cases} 4-x > 0 \\ x^2 > 0 \\ x^2 \neq 1. \end{cases}$$

sistemani hosil qilamiz. Uni yechimi

$$\begin{cases} x < 4 \\ x \neq -1, 0, 1. \end{cases}$$

Demak, berilgan funksiya $(-\infty; -1) \cup (-1; 0) \cup (0; 1) \cup (1; 4)$ to'plamda aniqlangan ekan. **Javob:** (B).

9. (97-1-63) Ushbu $y = \log_x(3 - x)$ funksiyaning aniqlanish sohasini toping.
A) $(-\infty; 3)$ B) $(0; \infty)$
C) $(0; 1) \cup (1; 3)$ D) $(0; 3)$
10. (97-6-64) Ushbu $f(x) = \log_x(6 - x)$ funksiyaning aniqlanish sohasini toping.
A) $(-\infty; 6)$ B) $(1; 6)$
C) $(0; 1)$ D) $(0; 1) \cup (1; 6)$
11. (97-8-52) Ushbu $y = \log_{x-1}(x - \frac{1}{4})$ funksiyaning aniqlanish sohasini toping.
A) $(\frac{1}{4}; \infty)$ B) $(1; 2) \cup (2; \infty)$
C) $(-0, 25; 2) \cup (2; \infty)$ D) $[-0, 25; 2) \cup [2; \infty)$
12. (97-9-75) n ning qanday butun qiymatlarida $y = \lg(nx^2 - 5x + 1)$ funksiyaning aniqlanish sohasi $(-\infty; \frac{1}{4}) \cup (1; \infty)$ bo'ladi?
A) 1 B) 4 C) 3 D) 5
- Yechish:** Masalada $nx^2 - 5x + 1 > 0$ tengsizlikning yechimi $(-\infty; \frac{1}{4}) \cup (1; \infty)$ to'plam bo'ladigan $n \in Z$ sonini topish talab qilingan. Demak, parabolaning shoxlari yuqoriga yo'nalgan va Ox o'qini $\frac{1}{4}$ va 1 nuqtalarda kesib o'tadi. Bu yerdan $nx^2 - 5x + 1$ kvadrat uchhadning nollari $x_1 = \frac{1}{4}$ va $x_2 = 1$ ekanligi kelib chiqadi.

$$nx^2 - 5x + 1 = n(x - \frac{1}{4})(x - 1)$$

tenglikdan $n = 4$ ni olamiz. **Javob:** 4 (B).

13. (99-7-15) k ning qanday qiymatlarida $y = \lg(kx^2 - 2x + 1)$ funksiya faqat $x = 1$ nuqtada aniqlanmagan?
A) $k < 2$ B) $k < 3$ C) $k \leq 1$ D) $k = 1$
14. (99-8-34) Quyidagi funksiyalardan qaysi birining aniqlanish sohasi $(0; 1)$ oraliqdan iborat?
A) $y = \sqrt{1/(1-x)} + \log_2 x$ B) $y = 1/\sqrt{1-x^2}$
C) $y = \sqrt{1-x} - \sqrt{x}$ D) $y = \frac{1}{1-x}$
15. (99-8-36) Ushbu $f(x) = \log_3(x^2 - 6x + 36)$ funksiyaning eng kichik qiymatini toping.
A) 1 B) 9 C) 2 D) 3
16. (96-12-90) Agar $a = \log_{\frac{1}{6}} 4$, $b = \log_{\frac{1}{3}} 6$ va $c = \log_{\frac{1}{5}} 4$ bo'lsa, a , b va c sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?
A) $c < b < a$ B) $b < c < a$
C) $c < a < b$ D) $a < b < c$
- Yechish:** Asos $a \in (0; 1)$ da $y = \log_a x$ kamayuvchi, shunga ko'ra $b = \log_{\frac{1}{3}} 6 < \log_{\frac{1}{5}} 4 = c$. Endi a

bilan c ni taqqoslaymiz. Ularni 4 asosli logarifm shaklida tasvirlab olamiz:

$$a = \frac{\log_4 4}{\log_4 \frac{1}{6}} = -\frac{1}{\log_4 6}, \quad c = \frac{\log_4 4}{\log_4 \frac{1}{5}} = -\frac{1}{\log_4 5}.$$

Quyidagilardan $5 < 6 \implies \log_4 5 < \log_4 6$ yana

$$\frac{1}{\log_4 5} > \frac{1}{\log_4 6} \implies c = -\frac{1}{\log_4 5} < -\frac{1}{\log_4 6} = a.$$

Demak, $b < c < a$ ekan. **Javob:** $b < c < a$ (B).

17. (96-13-31) Agar $a = \log_{\frac{1}{5}} 4$, $b = \log_{\frac{1}{5}} 6$, $c = \log_{\frac{1}{6}} 4$ bo'lsa, a , b va c sonlar uchun quyidagi munosabatlarning qaysi biri o'rinli?
A) $b < c < a$ B) $c < a < b$
C) $a < c < b$ D) $b < a < c$
18. (03-5-63) Sonlarni o'sish tartibida joylashtiring.

$$a = 2 \log_2 5, \quad b = 3 \log_{\frac{1}{8}} \frac{1}{23}, \quad c = 4 \log_{\frac{1}{4}} \frac{5}{26}$$

- A) $b < a < c$ B) $a < b < c$
C) $b < c < a$ D) $c < b < a$

19. (02-2-20) Qaysi javobda manfiy son ko'rsatilgan?
A) $\log_{\frac{1}{2}} 2$ B) $\log_{\sqrt{2}} \sqrt{3}$
C) $\log_{\frac{1}{7}} \frac{1}{\sqrt{45}}$ D) $\log_2 1, 2$
20. (99-9-47) Agar $0 < p < 1$ va $1 < n < m$ bo'lsa, quyidagi ko'paytmalardan qaysi biri musbat?
A) $\log_p m \cdot \log_m 1$ B) $\log_p n \cdot \log_p m$
C) $\log_m p \cdot \log_n m$ D) $\log_p m \cdot \log_m 1$

21. (01-3-21) Ushbu $y = \log_{\sqrt{10}}(6 + x - x^2)$ funksiyaning aniqlanish sohasidagi butun sonlar yig'indisini toping.
A) 0 B) 3 C) 2 D) 5
22. (01-9-46) Ushbu

$$y = \log_{\pi} \frac{x^2 - 13x - 30}{25 - 9x^2}$$

funksiyaning aniqlanish sohasiga nechta natural son tegishli?

- A) 13 B) 15 C) 0 D) 8

23. (01-9-47) Ushbu

$$y = \log_{15} \frac{x^2 - 2x - 15}{2x + 3}$$

funksiyaning aniqlanish sohasiga tegishli eng katta manfiy butun sonni va funksiyaning shu nuqtadagi qiymatini toping.

- A) $y(-1) = \log_{15} 2$ B) $y(-5) = \log_{15} 20$
C) $y(-3) = 4$ D) $y(-2) = \log_{15} 7$

Yechish: Berilgan funksiyaning aniqlanish sohasi

$$\frac{x^2 - 2x - 15}{2x + 3} > 0 \iff \frac{(x + 3)(x - 5)}{2(x + 1,5)} > 0$$

tengsizlik yechimidan iborat. Bu tengsizlik oraliqlar isuli bilan yechiladi, uning yechimi $(-3; -1, 5) \cup (5; \infty)$ to'plamdir. Bu to'plamga tegishli eng katta manfiy butun son -2 dir. $y(-2) = \log_{15} 7$.

Javob: $y(-2) = \log_{15} 7$ (D).

24. (98-7-21) Tenglamada x ning qabul qilishi mumkin bo'lgan qiymatlar to'plamini ko'rsating.

$$\lg(x-3) - \lg(x+9) = \lg(x-2)$$

- A) (2; 3) B) (9; ∞) C) (-9; ∞) D) (3; ∞)

25. (02-7-20) $y = \lg\left(\frac{3x+1}{x+2} - 1\right)$ funksiyaning aniqlanish sohasini toping.

- A) $(-\infty; -2) \cup (\frac{1}{2}; \infty)$ B) $(-2; \frac{1}{2})$
C) $(-\infty; -2)$ D) $(\frac{1}{2}; \infty)$

26. (02-9-29) $y = \sqrt{2 + \log_{\frac{1}{2}}(3-x)}$ funksiyaning aniqlanish sohasini toping.

- A) (-1; 3) B) [-1; 3) C) $(-\infty; 3)$ D) $(-\infty; -1]$

27. (02-12-51) Funksiyaning aniqlanish sohasini toping.

$$f(x) = \sqrt{x+4} + \log_2(x^2 - 4)$$

- A) [-2; 2] B) (-4; 2)
C) (-2; 2) D) [-4; -2) \cup (2; ∞)

28. (03-6-43) Funksiyaning aniqlanish sohasini toping.

$$y = \sqrt{\frac{8}{|x|} - 1} + \lg(x^2 - 1)$$

- A) $-8 < x < -1$ B) $1 < x < 8$
C) $-1 < x < 1$ D) $-8 \leq x < -1, 1 < x \leq 8$

29. (03-10-38) $y = \frac{\ln(7-x^2)}{x+1}$ funksiyaning aniqlanish sohasiga tegishli butun sonlarning yig'indisini toping.

- A) 0 B) 1 C) -1 D) 2

12.1.1 Logarifmik ifodalarda shakl almashtirish

1. $\log_2 8$ ni hisoblang.

- A) 4 B) 3 C) 1 D) 2

Yechish: $8 = 2^3$ dan va 5-xossadan foydalanib $\log_2 2^3 = 3 \cdot \log_2 2 = 3$ ni olamiz. **Javob:** 3 (B).

2. $\log_4 8 + \log_4 32$ ni hisoblang.

- A) 4 B) 3 C) 5 D) 2

3. (02-4-38) Hisoblang.

$$\log_{\frac{1}{8}} 2 + \log_{\frac{1}{8}} 3$$

- A) -3 B) -1 C) 0 D) 1

4. $\log_2 18 - \log_2 9$ ni hisoblang.

- A) 4 B) 3 C) 1 D) 2

5. (08-121-28) $\log_2 \log_3 81$ ni hisoblang.

- A) 4 B) 3 C) 1 D) 2

6. (97-5-37) $\log_2 \lg 100$ ni hisoblang.

- A) 1 B) 4 C) 3 D) 2

7. (08-120-28) Hisoblang.

$$\frac{\log_9 12}{\log_{36} 3} - \frac{\log_9 4}{\log_{108} 3}$$

- A) 2 B) 3 C) 6 D) 1

8. (96-9-31) Hisoblang.

$$\frac{3}{(\sqrt[3]{7})^{\log_9 7}}$$

- A) 10 B) 9 C) 3 D) 7

Yechish: 8-xossadan foydalanib $\frac{1}{\log_9 7} = \log_7 9$ ni olamiz. Endi (12.2) asosiy ayniyatdan

$$\left(7^{\frac{1}{3}}\right)^{3 \cdot \log_7 9} = 7^{\log_7 9} = 9.$$

Javob: 9 (B).

9. (96-3-89) $\left(2^{\frac{1}{\log_3 16}}\right)^4$ ni hisoblang.

- A) $\sqrt{3}$ B) 4 C) 2 D) 3

10. (98-4-15) Hisoblang.

$$\frac{5^{\lg 20}}{20^{\lg 5+1}}$$

- A) 0,25 B) 0,1 C) 0,2 D) 0,05

11. (99-2-31) Hisoblang.

$$100^{\frac{1}{2} \lg 27 - \lg 3} \cdot 10$$

- A) 20 B) 40 C) 30 D) 10

12. (00-3-34) Hisoblang.

$$343^{\log_{49} 4}$$

- A) 8 B) 4 C) 7 D) 6

13. (00-10-42) Hisoblang.

$$\log_{2\sqrt{2}} 512$$

- A) 8 B) 6 C) 4 D) 10

14. (01-3-14) Hisoblang.

$$4^{\log_2 (\sqrt[3]{2\sqrt{2}})^2}$$

- A) 16 B) 2 C) 4 D) 64

15. (01-5-16) Ifodaning qiymatini toping.

$$49^{1-\log_7 2} + 5^{-\log_5 4}$$

- A) 12,5 B) 13 C) 14 D) 23

16. (96-9-84) Hisoblang.

$$\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9$$

- A) 1 B) 3 C) 6 D) 2

Yechish: 10 asosli logarifmga o'tamiz

$$\frac{\lg 4}{\lg 3} \cdot \frac{\lg 5}{\lg 4} \cdot \frac{\lg 6}{\lg 5} \cdot \frac{\lg 7}{\lg 6} \cdot \frac{\lg 8}{\lg 7} \cdot \frac{\lg 9}{\lg 8} = \frac{\lg 3^2}{\lg 3} = 2.$$

Javob: 2 (D).

17. (00-5-66) Hisoblang.

$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7$$

- A)
- $\frac{1}{2}$
- B)
- $\frac{1}{3}$
- C)
- $\frac{1}{4}$
- D)
- $\frac{1}{5}$

18. (99-6-13) Hisoblang.

$$\log_9 17 \cdot \log_{17} 7 \cdot \log_7 3$$

- A)
- $\frac{1}{2}$
- B)
- $\frac{1}{7}$
- C) 1 D) 2

19. (96-6-53) Sonlardan qaysi biri 2 dan kichik?

$$M = \log_5 100 - \log_5 4, \quad N = 4 \log_2 3 - \log_2 9$$

$$P = \log_6 72 - \log_6 2, \quad Q = \log_4 16 + \log_4 \frac{1}{8}$$

- A) N B) P C) M D) Q

Yechish: Yig'indini ko'paytmaga keltirish (12-bob 3-xossa) formulasi hamda 5-xossadan

$$Q = \log_4 16 + \log_4 \frac{1}{8} = \log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}.$$

Javob: Q (D).

20. (97-8-53) Sonlardan qaysi biri 2 dan kichik?

A) $\log_4 2 + \log_4 8$ B) $\log_2 36 - \log_2 3$

C) $2 \log_2 5 - \log_2 25$ D) $\log_2 6 + \frac{1}{2} \log_2 9$

21. (97-12-52) Sonlardan qaysi biri 1 ga teng emas?

A) $\log_3 12 - \log_3 4$ B) $\frac{1}{2} \log_4 36 + \log_4 \frac{2}{3}$

C) $\log_5 125 - \frac{1}{2} \log_5 625$ D) $2 \log_2 5 - \log_2 30$

22. (00-1-39) Eng katta sonni toping.

A) $\log_2 18 - \log_2 9$ B) $3^{\log_3 6}$

C) $\lg 25 + \lg 4$ D) $\log_{13} 169^2$

23. (03-1-20) Agar
- $x = \log_5 2 + \log_{11} 3$
- bo'lsa, quyidagi sonlarning qaysi biri eng katta bo'ladi?

A) x B) x^2 C) x^3 D) $\sqrt[3]{x}$

24. (98-1-33) Hisoblang.

$$\frac{\log_2^2 14 + \log_2 14 \log_2 7 - 2 \log_2^2 7}{\log_2 14 + 2 \log_2 7}$$

- A) 2 B)
- $\log_2 7$
- C)
- $-\log_2 7$
- D) 1

Yechish: $\log_2 7 = x$ belgilash olsak, $\log_2 14 = \log_2 2 + \log_2 7 = 1 + x$ bo'ladi. Endi kasr suratini hisoblaymiz: $(1+x)^2 + (1+x)x - 2x^2 = 1 + 2x + x^2 + x + x^2 - 2x^2 = 1 + 3x$. Kasr maxraji $1 + x + 2x = 1 + 3x$. Ularning nisbati 1 ga teng.**Javob:** 1 (D).

25. (98-8-33) Hisoblang.

$$\frac{2 \log_3^2 2 - \log_3^2 18 - \log_3 2 \cdot \log_3 18}{2 \log_3 2 + \log_3 18}$$

- A) 1 B)
- $\frac{1}{2}$
- C)
- -2
- D)
- $-\frac{1}{2}$

26. (01-6-36) Hisoblang.

$$2 \log_2 12 + \log_2 20 - \log_2 15 - \log_2 3$$

- A) 4 B) 5 C) 7 D) 6

27. (01-9-17) Soddashtiring

$$\frac{\lg^2(x^3)}{\lg^3(x^2)} \cdot \lg \sqrt{x}$$

- A)
- $\frac{9}{16}$
- B)
- $\frac{3}{4}$
- C)
- $1\frac{7}{9}$
- D)
- $\frac{3}{2}$

28. (01-11-25) Hisoblang.

$$\log_5 2 \cdot \log_2 5 \cdot \log_4 243 \cdot \log_3 4$$

- A) 4 B) 3 C) 5 D) 6

Yechish: Logarifmning 8-xossasiga ko'ra, $\log_5 2 \cdot \log_2 5 = 1$. Endi 5 va 8-xossalardan foydalansak

$$\log_4 243 \cdot \log_3 4 = \log_4 3^5 \cdot \log_3 4 = 5 \log_4 3 \cdot \log_3 4 = 5$$

bo'ladi. **Javob:** 5 (C).

29. (01-11-26) Hisoblang.

$$\frac{3 \lg 2 + 3 \lg 5}{\lg 1300 - \lg 13}$$

- A) 1,8 B) 1,6 C) 2,3 D) 1,5

30. (02-2-53) Hisoblang.

$$\frac{\log_5 30}{\log_{30} 5} - \frac{\log_5 150}{\log_6 5}$$

- A) 1 B)
- -1
- C)
- $\frac{1}{2}$
- D)
- $-\frac{1}{2}$

31. (02-3-32) Agar
- $a > 0$
- va
- $a \neq 1$
- bo'lsa,
- $\log_{\sqrt{a}} \sqrt[3]{a}$
- ifodaning qiymatini toping.

- A)
- $\frac{2}{3}$
- B)
- $\frac{3}{2}$
- C) 3 D) 6

32. (02-3-33) Hisoblang.

$$\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \frac{1}{\log_{16} 4} +$$

$$+ \frac{1}{\log_{32} 4} + \frac{1}{\log_{64} 4} + \frac{1}{\log_{128} 4}$$

- A) 14 B) 16 C) 7 D) 32

33. (02-5-24) Hisoblang.

$$\log_3^{-1} \sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}$$

- A) 27 B) -27 C) $\frac{1}{27}$ D) 3

Yechish: Logarifmning 5-xossasi va ildizning xossasidan foydalansak

$$\log_3^{-1} \sqrt[3]{\sqrt[3]{\sqrt[3]{3}}} = \frac{1}{\log_3 3^{\frac{1}{27}}} = \frac{1}{\frac{1}{27} \log_3 3} = 27$$

bo'ladi. **Javob:** 27 (A).

34. (03-5-39) Hisoblang.

$$y = \log_5 \log_5 \sqrt[5]{\sqrt[5]{\sqrt[5]{\sqrt[5]{5}}}}$$

- A) -4 B) $\frac{1}{5}$ C) $-\frac{1}{4}$ D) 4

35. (02-10-73) Hisoblang.

$$\left(\frac{\log_6 27 + 2 \log_6 2}{\log_6 \sqrt[3]{0,25} + \log_6 \frac{1}{3}} \right)^3$$

- A) -27 B) 27 C) -8 D) $8 \log_6 27$

36. (02-12-48) Hisoblang.

$$\frac{3 \lg 2 + 3 \lg 5}{\lg 1300 - \lg 0,13}$$

- A) 0,8 B) 0,6 C) 0,7 D) 0,75

37. (03-2-20) Hisoblang.

$$\frac{1 + 2 \log_3 2}{(1 + \log_3 2)^2} + \log_6^2 2$$

- A) 2 B) 0,5 C) 1 D) $\frac{1}{4}$

38. (03-3-33) Hisoblang.

$$\log_8 5^{2 \log_{25} 32}$$

- A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{5}{3}$ D) 2

39. (03-4-32) $\ln(3^{\log_3 0,64} + 8^{\log_8 0,36})$ ning qiymati -11 dan qancha ko'p?

- A) 10 B) 9 C) 11 D) 12

40. (03-4-33) Hisoblang.

$$2 \log_4 8 - 3 \log_8 4 + \log_2 32 + 18$$

- A) 22 B) 24 C) 26 D) 20

41. (98-5-29) Agar $\log_3 5 = a$, $\log_3 2 = b$ bo'lsa, $\log_6 45$ ni a va b orqali ifodalang.

- A) $\frac{b+2}{a+2}$ B) $\frac{2+a}{1+b}$ C) $\frac{a}{1+b}$ D) $\frac{b}{1+a}$

Yechish: $\log_3 5 = a$, $\log_3 2 = b$ berilgan. 7, 6, 3 hamda 2-xossalarga ko'ra

$$\log_6 45 = \frac{\log_3(9 \cdot 5)}{\log_3(2 \cdot 3)} = \frac{\log_3 3^2 + \log_3 5}{\log_3 2 + \log_3 3} = \frac{2+a}{b+1}$$

ekanini hosil qilamiz. **Javob:** $\frac{2+a}{b+1}$ (B).

42. (96-10-36) Agar $\log_4 125 = a$ bo'lsa, $\lg 64$ ni a orqali ifodalang.

- A) $\frac{3}{2}a + 4$ B) $\frac{2}{3}a + 6$ C) $\frac{18}{2a+3}$ D) $\frac{6}{3a+2}$

43. (96-9-28) Agar $a = \log_{50} 40$ bo'lsa, $\log_5 2$ ni a orqali ifodalang.

- A) $\frac{3a-1}{2-a}$ B) $\frac{a-3}{1-2a}$ C) $\frac{a-3}{2a-1}$ D) $\frac{1-2a}{a-3}$

44. (96-3-86) Agar $a = \log_{98} 56$ bo'lsa, $\log_7 2$ ni a orqali ifodalang.

- A) $\frac{3-a}{2a-1}$ B) $\frac{2a-1}{3-a}$ C) $\frac{a-3}{2a-1}$ D) $\frac{1-2a}{3-a}$

45. (00-1-38) Agar $a = \log_{12} 2$ bo'lsa, $\log_6 16$ ning qiymatini toping.

- A) $\frac{4a}{1+a}$ B) $\frac{2a}{1-a}$ C) $\frac{4a}{1-a}$ D) $\frac{3a}{1+a}$

46. (00-6-32) Agar $\log_{0,5} 27 = a$ bo'lsa, $\log_{\sqrt{3}} \sqrt[6]{1,5}$ ning qiymatini toping.

- A) $\frac{1}{3} + a^{-1}$ B) $a^2 - 1$ C) $3 + a^{-1}$ D) $1 + a^{-3}$

47. (00-10-34) Agar $a = \log_2 3$ bo'lsa, $\log_8 0,75$ ni a orqali ifodalang.

- A) $\frac{1}{3}(a-1)$ B) $\frac{1}{3}(a+1)$
C) $\frac{1}{3}(a-2)$ D) $\frac{1}{3}(a+2)$

48. (00-10-66) Agar $\log_a 27 = b$ bo'lsa, $\log_{\sqrt{3}} \sqrt[6]{a}$ ni toping.

- A) $\frac{1}{b}$ B) $\frac{2}{b}$ C) $-\frac{b}{2}$ D) $2b$

49. (99-10-35) Agar $\log_2 a = 2$ va $\log_3 b = 2$ bo'lsa, $\log_6 ab$ ning qiymatini toping.

- A) -2 B) 3 C) -3 D) 2

50. (02-8-12) Agar $7^{\log_5 b} = 4$ bo'lsa, $b^{\log_5 \sqrt{7}}$ ni hisoblang.

- A) 2 B) 3 C) 1 D) 4

51. (02-8-13) Agar $\lg 2 = a$ va $\lg 3 = b$ bo'lsa, $\log_9 20$ ni a va b orqali ifodalang.

- A) $\frac{1+a}{2b}$ B) $\frac{1-a}{2b}$ C) $\frac{b}{1+2a}$ D) $\frac{b}{1-2a}$

Yechish: Boshqa asosga o'tish formulasi 7-ga ko'ra

$$\log_9 20 = \frac{\lg 20}{\lg 9} = \frac{\lg 10 + \lg 2}{\lg 3^2} = \frac{1+a}{2b}$$

ekanini hosil qilamiz. **Javob:** $\frac{1+a}{2b}$ (A).

52. (02-9-38) Agar $\log_{\frac{1}{a}}\left(\frac{a^2}{b}\right) = -\frac{1}{2}$ bo'lsa, $\log_{a^2b}(ab)$ ni hisoblang.
A) $-\frac{1}{4}$ B) -1 C) 1 D) $0,8$
53. (02-10-27) $\lg 2 = a$ va $\log_2 7 = b$ bo'lsa, $\lg 56$ ni a va b orqali ifodalang.
A) $3a+ab$ B) $2a+3b$ C) $3a+2b$ D) $\frac{2a+5b}{3}$
54. (03-4-37) Agar $\log_a 8 = 3$ va $\log_b 243 = 5$ bo'lsa, ab ning qiymatini toping.
A) 4 B) 5 C) 6 D) 8
55. (03-7-67) Agar $\lg 5 = a$ va $\lg 3 = b$ bo'lsa, $\log_{30} 8$ ni a va b orqali ifodalang.
A) $\frac{a}{2a+3b}$ B) $\frac{b-3}{1-2a}$ C) $\frac{3a-3}{b+2}$ D) $\frac{3(1-a)}{1+b}$
56. (03-8-43) Agar $a = \log_5 4$ va $b = \log_5 3$ bo'lsa, $\log_{25} 12$ ni a va b orqali ifodalang.
A) $\frac{a+b}{2}$ B) $\frac{a-b}{4}$ C) $\frac{ab}{2}$ D) $\frac{a^2+b}{4}$

12.2 Logarifmik tenglamalar

O'zgaruvchisi logarifm belgisi ostida kelgan tenglamalar *logarifmik tenglamalar* deyiladi. Masalan,

$$\log_2 x = 3, \quad \log_x 2 = 1, \quad \log_3(x^2 - 5x + 3) = 0.$$

Eng sodda logarifmik tenglamaning ko'rinishi $\log_a x = b$ bo'lib, uning yechimi $x = a^b$. Agar

$$\log_a f(x) = \log_a g(x) \quad (12.3)$$

tenglamada $f(x) > 0$, $g(x) > 0$ shart bajarilganda, u

$$f(x) = g(x) \quad (12.4)$$

tenglamaga teng kuchli bo'ladi. (12.3) tenglamadan (12.4) tenglamaga o'tganda chet ildizlar paydo bo'lishi mumkin. Chet ildizlarni aniqlash uchun, ildizlarni dastlabki tenglamaga qo'yib tekshirib ko'riladi. Quyidagi tengkuchliliklarni keltiramiz.

- $\log_a f(x) = b \iff f(x) = a^b$.
- $\log_a f(x) = \log_a g(x) \iff \begin{cases} f(x) = g(x) \\ f(x) > 0 \\ g(x) > 0. \end{cases}$
- $\log_{f(x)} g(x) = b \iff \begin{cases} [f(x)]^b = g(x) \\ f(x) > 0, \quad f(x) \neq 1. \end{cases}$
 - $\log_a f(x) = b, \quad \log_a f(x) = \log_a g(x)$
ko'rinishdagi tenglamalar

- $\log_5 x = 2$ tenglamani yeching.
A) 10 B) 25 C) $\sqrt{5}$ D) 32
Yechish: 1-xossaga ko'ra $x = 5^2 = 25$.
Javob: 25 (B).

- $\lg x = -1$ tenglamani yeching.
A) 10 B) $0,1$ C) $\sqrt{10}$ D) -1
- $\ln x = \ln(8-x)$ tenglamani yeching.
A) e^2 B) $0,8$ C) $\sqrt{8}$ D) 4
- $\log_2 x^2 = 4$ tenglamani yeching.
A) 4 B) ± 2 C) ± 4 D) 16
- $\log_2 \log_3 x = 0$ tenglamani yeching.
A) 8 B) ± 3 C) 3 D) 9
- $\lg(x-4) = \lg(4-x)$ tenglama nechta ildizga ega.
A) 1 B) 2 C) 0 D) 4
- (00-7-33) a ning qanday qiymatlarida

$$\lg x + \lg(x-6) = \lg(-a)$$

tenglama bitta ildizga ega bo'ladi?

- A) 9 B) $a \in (-\infty; 0)$ C) 7 D) 6

Yechish: Berilgan tenglamaning aniqlanish sohasi $x > 6$ to'plamdan iborat. $\log_a b + \log_a c = \log_a bc$ formula yordamida tenglamaning chap qismini almashtiramiz. $\lg x(x-6) = \lg(-a)$. 2-tengkuchlilikdan $x(x-6) = -a$ ($x > 6$, $a < 0$). Hosil bo'lgan tenglamani yechamiz.

$$x^2 - 6x + a = 0; \quad D = 36 - 4a = 4(9 - a).$$

Bu tenglama $a \leq 9$ da yechimga ega va uning ildizlari quyidagilar

$$x_{1,2} = \frac{6 \pm 2\sqrt{9-a}}{2} = 3 \pm \sqrt{9-a}$$

$x_1 = 3 - \sqrt{9-a} \leq 3$ bo'lgani uchun u berilgan tenglamaning aniqlanish sohasiga kirmaydi. Demak, u chet ildiz. $x_2 = 3 + \sqrt{9-a}$ berilgan tenglamaning ildizi bo'lishi uchun $x_2 > 6$ bo'lishi kerak. Bu tengsizlikni yechamiz:

$$\sqrt{9-a} > 3 \iff 9-a > 9 \iff a < 0.$$

Shunday qilib, $a \in (-\infty; 0)$ da berilgan tenglama bitta ildizga ega. **Javob:** $a \in (-\infty; 0)$ (B).

- (98-9-34) Tenglamani yeching.
 $\lg(x^2 + 2x - 3) = \lg(x - 3)$
A) 0 B) -1 C) $0; -1$ D) \emptyset
- (99-6-26) Tenglamani yeching.
 $\log_{18} \log_2 \log_2 \left(-\frac{1}{x}\right) = 0$
A) $-\frac{1}{16}$ B) $-\frac{1}{8}$ C) $\frac{1}{8}$ D) $-\frac{1}{4}$
- (99-6-50) Tenglamani yeching.
 $\log_{\frac{1}{5}} \log_5 \sqrt{5x} = 0$
A) -5 B) 1 C) 0 D) 5

11. (00-2-22) Agar

$$\begin{cases} 3^x \cdot 2^y = 972, \\ \log_{\sqrt{3}}(x - y) = 2 \end{cases}$$

bo'lsa, xy ning qiymatini toping.

- A) 14 B) 12 C) 10 D) 8

12. (01-3-26) Tenglama ildizlari yig'indisini toping.

$$\lg\left(3\sqrt{\frac{x^2-4x}{x-3}} + 1\right) = 1$$

- A) 10 B) 2 C) 8 D) 25

13. (01-7-25) Tenglamani yeching

$$\lg(3 + 2\lg(1 + x)) = 0$$

- A) 0 B) 1 C) -15 D) -0,9

Yechish: Berilgan tenglamani

$$\lg(3 + 2\lg(1 + x)) = \lg 1$$

shaklda yozamiz. Bu yerdan $3 + 2\lg(1 + x) = 1$ ni, undan esa $\lg(1 + x) = -1$ ni olamiz. Ta'rifga ko'ra $1 + x = 10^{-1} \iff x = -0,9$. Bevosita tekshirish ko'rsatadiki, $x = -0,9$ tenglamani qanoatlantiradi. **Javob:** -0,9 (D).

14. (01-7-26) Tenglamani yeching

$$\log_2|x - 1| = 1$$

- A) 3 B) 2 C) -1 D) 3; -1

15. (01-9-41) Ushbu

$$\lg(5x - 2) = \lg(2 - 5x)$$

tenglamani aniqlanish sohasini toping.

- A)
- $(0, 4; \infty)$
- B)
- \emptyset
- C)
- $(-\infty; 0, 4)$
- D)
- $\{2, 5\}$

16. (02-3-35) Tenglama ildizlari ayirmasining moduli nechaga teng?

$$\log_3(4 \cdot 3^x - 1) = 2x + 1$$

- A) 1 B) 2 C) 3 D) 0

17. (02-10-69) Tenglamani yeching.

$$\log_2(2^{2x} + 16^x) = 2\log_4 12$$

- A)
- $\log_4 3$
- B)
- $\log_2 3$
- C) 2 D)
- $\log_4 6$

18. (02-10-71) Agar

$$\begin{cases} \log_2(x - y) = 1 \\ 2^x \cdot 3^{y+1} = 72 \end{cases}$$

bo'lsa, x va y ning o'rta proporsional qiymatini toping.

- A)
- $\sqrt{3}$
- B) 2 C)
- $\sqrt{2}$
- D)
- $2\sqrt{2}$

2. $a^{\log_a f(x)} = g(x)$ ko'rinishdagi tenglamalar

19. (96-6-55) Tenglamani ildizini toping.

$$3^{2\log_3 x} = 16$$

- A) 3 B) -4 C) 4 D)
- ± 4

Yechish: Tenglama $x > 0$ da aniqlangan. Logarifmning 5-xossasidan foydalanib, uni

$$3^{\log_3 x^2} = 16$$

shaklda yozamiz. Asosiy logarifmik ayniyatdan $x^2 = 16$ ni olamiz. Bu yerdan $x_1 = -4$, $x_2 = 4$ kelib chiqadi. $x_1 = -4$ tenglamani aniqlanish sohasida yotmaydi. $x = 4$ tenglamani qanoatlantiradi. **Javob:** 4 (C).

20. (97-2-55) Tenglamani ildizi 20 dan qancha katta?

$$4^{\log_4(x-5)} = 19$$

- A) 6 B) 2 C) 4 D) 3

21. (97-8-40) Tenglamani yeching.

$$4^{2\log_4 x} = 25$$

- A) 5 B)
- ± 5
- C) -5 D) 10

22. Tenglamani yeching.

$$2^{\log_4 x} = \frac{1}{4}$$

- A) 1 B) 2 C) 0,5 D) 0,25

23. (01-5-12) Tenglamani yeching

$$x^{\log_x(x^2-1)} = 3$$

- A) 2 B) 1 C) 3 D) 4

3. 3-4 va 5-xossalarga oid tenglamalar

24. (97-12-54) Tenglamani ildizi 8 dan qanchaga kam?

$$\log_2(x + 2) + \log_2(x + 3) = 1$$

- A) 7 B) 9 C) 10 D) 6

Yechish: Tenglamani aniqlanish sohasi $x > -2$ to'plam. 3-xossadan foydalansak, tenglamani $\log_2(x+2)(x+3) = \log_2 2$ shaklda yozish mumkin. Bu yerdan

$$(x + 2)(x + 3) = 2 \iff x^2 + 5x + 4 = 0$$

kelib chiqadi. Bu kvadrat tenglamani ildizlari $x_1 = -4$, $x_2 = -1$ dir. $x_1 = -4$ tenglamani aniqlanish sohasida yotmaydi. $x = -1$ tenglamani qanoatlantiradi. $8 - (-1) = 9$. **Javob:** 9 (B).

25. (00-3-38) Tenglamani yeching

$$\lg\left(\frac{1}{2} + x\right) = \lg \frac{1}{2} - \lg x$$

- A) 2 B)
- $\frac{1}{2}$
- C) 1 D) -1

26. (99-3-20) Tenglamani yeching

$$\lg \sqrt{x-5} + \lg \sqrt{2x-3} + 1 = \lg 30$$

- A) $\frac{1}{2}$ B) 6 C) $\frac{1}{2}; 6$ D) $\frac{1}{2}; 8$

27. (02-12-50) Agar $\lg(x^2 + y^2) = 2$, $\lg 2 + \lg xy = \lg 96$ va $x > 0$ bo'lsa, $x + y$ yig'indining qiymatini toping.

- A) 12 B) 14 C) 16 D) 18

28. (03-7-21) Tenglamani yeching.

$$\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$$

- A) $\frac{3}{2}$ B) $\frac{5}{4}$ C) 2 D) 2,5

29. (99-6-28) Tenglamani yeching.

$$\log_2(54 - x^3) = 3 \log_2 x$$

- A) -3 B) 2 C) 1 D) 3

Yechish: 5-xossadan foydalansak, tenglamani $\log_2(54 - x^3) = \log_2 x^3$ shaklda yozish mumkin. Bu yerdan

$$54 - x^3 = x^3 \iff 27 = x^3 \iff x = 3$$

kelib chiqadi. $x = 3$ tenglamani qanoatlantiradi.
Javob: 3 (D).

30. (00-2-24) $\log_5 x = 2 \log_5 3 + 4 \log_{25} 7$ bo'lsa, x ni toping.

- A) 441 B) 125 C) 256 D) 400

31. (00-3-28) Tenglamani yeching.

$$\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{\lg 4}{\lg 8}$$

- A) 3 B) 4 C) 2 D) 1

32. (00-8-15) Tenglamani yeching.

$$\log_2(9^{x-1} + 7) = 2 \log_2(3^{x-1} + 1)$$

- A) 2 B) 1 C) 3 D) 4

33. (01-5-11) Tenglamani yeching

$$\log_a x - \log_{a^2} x + \log_{a^4} x = \frac{3}{4}$$

- A) a B) a^2 C) a^4 D) 2

34. (03-4-34) Agar

$$\log_4 \frac{(2-x)^2}{(3-x)^3} = -3 \log_4 |3-x|$$

bo'lsa, $x - 27$ ni hisoblang.

- A) -25 B) -29 C) -26 D) -24

35. (03-11-13) Tenglamani yeching.

$$7^{(2x^2-5x-9)/2} = (\sqrt{2})^3 \log_2 7$$

- A) -1,5; 1 B) 1,5 C) -2,5; 4 D) -1,5; 4

4. 6-7 va 8-xossalarga oid tenglamalar

36. (99-6-55) Tenglamani yeching.

$$\log_{\sqrt{2}} x + \frac{2}{\log_x 2} = 4$$

- A) 2 B) 1 C) 3 D) 4

Yechish: 6-8-xossalardan foydalanib, tenglamani

$$\frac{1}{2} \log_2 x + 2 \log_2 x = 4 \iff 4 \log_2 x = 4$$

shaklda yozamiz. Bu yerdan $\log_2 x = 1$ kelib chiqadi. Logarifm ta'rifidan $x = 2^1$ ni olamiz.

Javob: 2 (A).

37. (98-11-45) Tenglama ildizlari ko'paytmasini toping.

$$\log_x 2 \log_{2x} 2 = \log_{4x} 2$$

- A) 1 B) $\frac{1}{\sqrt{2}}$ C) $-\frac{1}{\sqrt{2}}$ D) $\frac{1}{2}$

38. (99-3-21) Tenglamani yeching.

$$\log_4(x+12) \cdot \log_x 2 = 1$$

- A) 4 B) -3 C) 2 D) 4;2

39. (02-3-36) Tenglama ildizlari ko'paytmasini toping.

$$\log_x 2 + \log_{4x} 4 = 1$$

- A) 2 B) 4 C) 1 D) 8

5. 11-xossa yoki logarifmlash yordamida yechiladigan tenglamalar

40. (97-6-59) Tenglamani yeching.

$$x^{\lg 9} + 9^{\lg x} = 6$$

- A) 1 B) 10 C) $\sqrt{10}$ D) 2

Yechish: 11-xossadan foydalanib, tenglamani

$$9^{\lg x} + 9^{\lg x} = 6 \iff 2 \cdot 9^{\lg x} = 6 \iff 9^{\lg x} = 3$$

shaklda yozamiz. Bu tenglikning ikkala qismini 3 asosga ko'ra logarifmlaymiz

$$\log_3 9^{\lg x} = \log_3 3 \iff \lg x \cdot 2 = 1 \iff \lg x = \frac{1}{2}.$$

Logarifm ta'rifidan $x = 10^{1/2}$ ni olamiz. **Javob:** $\sqrt{10}$ (C).

41. (00-3-39) Tenglama ildizlari ko'paytmasini toping.

$$x^{\lg x-1} = 100$$

- A) 10 B) 20 C) 100 D) 1

42. (01-2-73) Tenglama ildizlari ko'paytmasini toping.

$$x^{(\lg x+5)/3} = 10^{5+\lg x}$$

- A) 100 B) 10 C) 1 D) 0,01

43. (01-9-9) Tenglama ildizlarining o'rta proporsional qiymatini toping.

$$x^{3-\log_3 x} = 9$$

- A) $3\sqrt{3}$ B) $\sqrt{2}$ C) $2\sqrt{3}$ D) $\sqrt{3}$

44. (02-6-34) Tenglama ildizlari ko'paytmasini toping.

$$x^{2\lg x} = 10x^2$$

- A) 1 B) 10 C) 100 D) 0,1

45. (02-7-9) Tenglamani yeching.

$$2 \cdot 3^{\log_7 x} + 3x^{\log_7 3} = 45$$

- A) 49 B) 4 C) 7 D) 8

46. (03-4-36) Agar

$$\begin{cases} x^{\lg y} = 1000, \\ \log_y x = 3 \end{cases}$$

bo'lsa, y ning qiymatini toping.

- A) 10 B) 0,01 C) 10 yoki 0,1 D) 30

6. Yangi o'zgaruvchi kiritish yordamida yechiladigan tenglamalar

47. (96-10-38) Tenglama ildizlari ko'paytmasini toping.

$$\log_2^2 x - 5 \cdot \log_2 x + 6 = 0$$

- A) 5 B) 6 C) 32 D) $\frac{3}{2}$

Yechish: Tenglamada $\log_2 x = y$ o'zgaruvchi kiritib, uni $y^2 - 5y + 6 = 0$ shaklda yozamiz. Bu kvadrat tenglamaning ildizlari $y_1 = 2$, $y_2 = 3$ lardir. Ularni belgilashga qo'yib, $\log_2 x = y_1 = 2$ va $\log_2 x = y_2 = 3$ larni olamiz. Bu yerdan $x_1 = 2^{y_1} = 4$, $x_2 = 2^{y_2} = 8$. Ularning ko'paytmasi $x_1 \cdot x_2 = 2^{y_1+y_2} = 2^5 = 32$. **Javob:** 32 (C).

Xulosa: Agar $\log_a^2 x + b \cdot \log_a x + c = 0$ ko'rinishdagi tenglamaning ikkita ildizi borligi ma'lum bo'lsa, u holda $x_1 \cdot x_2 = a^{-b}$ bo'ladi. Bu xulosadan foydalanib 48-51-misollarni og'zaki yechib ko'ring.

48. (96-1-35) Tenglama ildizlari ko'paytmasini toping.

$$\lg^2 x - \lg x - 2 = 0$$

- A) 1 B) -2 C) 10 D) 100

49. (96-9-86) Tenglama ildizlari ko'paytmasini toping.

$$\log_3^2 x - 4 \log_3 x + 3 = 0$$

- A) 4 B) 81 C) 24 D) $9\frac{1}{3}$

50. (98-3-33) Tenglama ildizlari ko'paytmasini toping.

$$\log_3^2 x - 3 \log_3 x + 2 = 0$$

- A) 6 B) 3 C) 27 D) 15

51. (98-6-24) Tenglama ildizlari ko'paytmasini toping.

$$\log_2^2 x - 4 \log_2 x - 1 = 0$$

- A) 8 B) 4 C) 16 D) $\frac{1}{8}$

52. (00-1-47) Tenglama ildizlari yig'indisini toping.

$$\log_2^2 x - 2 \log_2 x^2 + 3 = 0$$

- A) 4 B) -4 C) -10 D) 10

53. (02-11-33) Tenglama ildizlari ko'paytmasini toping.

$$\log_2^2 \frac{x}{2} - \log_2 4x = 3$$

- A) 2 B) 4 C) 6 D) 8

54. (03-3-34) Tenglama ildizlari ko'paytmasini toping.

$$\log_{0,2}^2 \frac{x}{25} + \log_{0,2} \frac{x}{5} = 1$$

- A) $\frac{1}{125}$ B) 125 C) 25 D) $\frac{1}{25}$

12.3 Logarifmik tengsizliklar

$\log_a f(x) > \log_a g(x)$ yoki $\log_a f(x) < \log_a g(x)$ ko'rinishdagi tengsizliklar sodda logarifmik tengsizliklar deyiladi. Bunday tengsizliklarni yechish $y = \log_a x$ logarifmik funksiyaning $a > 1$ bo'lganda o'suvchi $0 < a < 1$ bo'lganda kamayuvchi ekanligiga asoslanadi, ya'ni:

1. Agar $a > 1$ bo'lsa,

$$\log_a g(x) < \log_a f(x) \iff 0 < g(x) < f(x).$$

2. Agar $0 < a < 1$ bo'lsa,

$$\log_a g(x) < \log_a f(x) \iff 0 < f(x) < g(x).$$

1. (97-1-56) Tengsizlikni yeching.

$$\log_5(5 - 2x) \leq 1$$

- A) $(-\infty; 2, 5)$ B) $(0; 2, 5)$
C) $(-\infty; 2, 5]$ D) $[0; 2, 5)$

Yechish: Berilgan tengsizlikni quyidagicha $\log_5(5 - 2x) \leq \log_5 5$ yozib olamiz. 1-xossaga ko'ra $0 < 5 - 2x < 5$. Bu yerdan $-5 < -2x < 0$ ni, bundan esa $2,5 > x > 0$ ni hosil qilamiz. **Javob:** $(0; 2, 5)$ (B).

2. Tengsizlikni yeching.

$$\log_2(5 - x) \leq 1$$

- A) $(-\infty; 5)$ B) $[3; 5)$ C) $(-\infty; 3)$ D) $(3; 5]$

3. (97-3-33) Tengsizlikni yeching

$$\log_{\sqrt{3}} \left(\frac{3x}{3x-1,5} \right) > 0$$

- A) $(0, 5; \infty)$ B) $(0; 0, 5)$ C) $(-\infty; 0)$ D) $(0; \infty)$

4. (08-101-10) Tengsizlikni yeching:

$$3^{\log_3(4-x)} > 9$$

- A) $-5 < x < 4$ B) $x < 4$
 C) $x \leq -5$ D) $x < -5$

5. (08-104-10) Tengsizlikni yeching:

$$9^{\log_9(x-4)} > 3$$

- A) $4 < x < 7$ B) $x \geq 8$
 C) $x \geq 9$ D) $x > 7$

6. (08-110-10) Tengsizlikni yeching:

$$5^{\log_5(x-7)} \leq 4$$

- A) $x \geq 11$ B) $7 \leq x \leq 11$
 C) $x > 11$ D) $7 < x \leq 11$

7. (99-5-14) Tengsizlikni yeching.

$$\log_{0,5}(x+5)^4 > \log_{0,5}(3x-1)^4$$

- A) $(3; \infty)$ B) $(-\infty; 1)$ C) $(-\infty; 1) \cup (3; \infty)$
 D) $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$

Yechish: 2-ga ko'ra $0 < (x+5)^4 < (3x-1)^4$. Bu yerdan $0 < (x+5)^2 < (3x-1)^2$ ekanini hosil qilamiz. Bu qo'sh tengsizlik

$$(x+5)^2 - (3x-1)^2 < 0, \quad x \neq -5$$

ga teng kuchli. Tengsizlikning chap qismini ko'paytuvchiga ajratib $(x+5-3x+1)(x+5+3x-1) < 0$ ni, bundan $(6-2x)(4x+4) < 0$ ni, bu yerdan esa

$$2 \cdot 4(3-x)(x+1) < 0 \iff (3-x)(x+1) < 0$$

ni hosil qilamiz. Bu tengsizlik oraliqlar usuli bilan oson yechiladi, uning yechimi $(-\infty; -1) \cup (3; \infty)$ to'plamdan iborat. $x \neq -5$ bo'lganligi uchun -5 ni yechimdan chiqarib, $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$ yechimni hosil qilamiz.

Javob: $(-\infty; -5) \cup (-5; -1) \cup (3; \infty)$ (D).

8. (96-3-87) Ushbu $y = \log_2 \log_3 \sqrt{4x-x^2-2}$ funksiyaning aniqlanish sohasini toping.

- A) $(1, 5; 2, 5)$ B) $(1; 3)$
 C) $\{2\}$ D) $(-\infty; 1) \cup (3; \infty)$

9. (96-7-33) Tengsizlikni yeching.

$$\log_{\frac{1}{\sqrt{2}}} \frac{4x-1}{4x+8} < 0$$

- A) $(\frac{1}{4}; \infty)$ B) $(2; \infty)$ C) $(-2; \infty)$ D) $(-\infty; -2)$

10. (97-1-24) Tengsizlikni yeching.

$$\log_{\frac{1}{\sqrt{3}}}(x-5) + 2 \log_{\sqrt{3}}(x-5) < 4$$

- A) $(6; 15)$ B) $(5; 14)$ C) $(5; 81)$ D) $(10; 20)$

Yechish: $\frac{1}{\sqrt{3}} = (\sqrt{3})^{-1}$ tenglikdan hamda logarifmning 6-xossasidan foydalansak, berilgan tengsizlikni quyidagicha

$$-\log_{\sqrt{3}}(x-5) + 2 \log_{\sqrt{3}}(x-5) < 4$$

yoziq mumkin. Tengsizlikning chap qismini soddalashtirib, $4 = \log_{\sqrt{3}} 9$ dan va 1-xossadan

$$\log_{\sqrt{3}}(x-5) < \log_{\sqrt{3}} 9 \iff 0 < x-5 < 9$$

ni olamiz. Bu tengsizlikning barcha qismlariga 5 ni qo'shib $5 < x < 14$ ni olamiz. **Javob:** $(5; 14)$ (B).

11. (97-6-24) Tengsizlikni yeching.

$$\log_2(3-2x) - \log_{\frac{1}{8}}(3-2x) > \frac{4}{3}$$

- A) $(-\infty; 0; 5)$ B) $(-\infty; 1; 5)$
 C) $(-4; -1)$ D) $(0; 1)$

12. (97-11-24) Tengsizlikni yeching.

$$\log_{\frac{1}{3}}(x+2) - \log_9(x+2) > -\frac{3}{2}$$

- A) $(0; 1)$ B) $(1; \infty)$ C) $(2; 3)$ D) $(-2; 1)$

13. (98-2-37) Tengsizlikning barcha manfiy yechimlari to'plamini ko'rsating

$$\log_{0,2}(x^4 + 2x^2 + 1) > \log_{0,2}(6x^2 + 1)$$

- A) $(-2; 2)$ B) $(-2; 0)$
 C) $(-\infty; -2) \cup (0; 2)$ D) $(-\infty; -2)$

14. (97-4-16) x ning qanday qiymatlarida $y = 2 - \lg x$ funksiya manfiy qiymatlar qabul qiladi?

- A) $x > 100$ B) $x > 10$ C) $x \leq 100$ D) $x < 10$

15. (98-3-32) $\log_5(3-x) - \log_5 12 < 0$ tengsizlikni qanoatlantiradigan butun sonlar nechta?

- A) cheksiz ko'p B) 5 C) 10 D) 11

16. (99-2-33) $\log_{3x^2+5}(9x^4 + 27x^2 + 28) > 2$ tengsizlikning butun yechimini toping.

- A) 1 B) 2 C) -1 D) 0

17. (99-3-17) Tengsizlikni yeching.

$$\log_2 \log_{\frac{1}{3}} \log_5 x > 0$$

- A) $(0; \infty)$ B) $(-\infty; \sqrt[3]{5})$
 C) $(-\infty; 0) \cup (\sqrt[3]{5}; \infty)$ D) $(1; \sqrt[3]{5})$

Yechish: $0 = \log_2 1$ tenglikdan hamda 1-xossadan foydalanib $\log_{\frac{1}{3}} \log_5 x > 1 = \log_{\frac{1}{3}} \frac{1}{3}$ ni, 2-xossadan

$$0 < \log_5 x < \frac{1}{3} \iff \log_5 1 < \log_5 x < \log_5 5^{\frac{1}{3}}$$

ni olamiz. Bu tengsizlikning yechimi $1 < x < \sqrt[3]{5}$ dir. **Javob:** $(1; \sqrt[3]{5})$ (D).

18. (99-6-9) Tengsizlikning eng katta butun yechimini toping.

$$\log_2(2x - 1) < 3$$

- A) 2 B) 5 C) 1 D) 4

19. (00-9-22) Tengsizlikni yeching.

$$\log_{\frac{1}{5}}(x + 17)^8 \leq \log_{\frac{1}{5}}(x + 13)^8$$

- A) $(-15; -13) \cup (-13; \infty)$
 B) $[-15; -13) \cup (-13; \infty)$
 C) $(-13; \infty)$
 D) $(-\infty; -17) \cup (-17; -13) \cup (-13; \infty)$

20. (96-12-87) Funksiyaning aniqlanish sohasini toping.

$$y = \log_2 \log_{\frac{1}{2}} \sqrt{4x - 4x^2}$$

- A) $\{\frac{1}{2}\}$ B) $(0; \frac{1}{2})$ C) $(\frac{1}{2}; 1)$ D) $(0; \frac{1}{2}) \cup (\frac{1}{2}; 1)$

21. (96-13-28) Ushbu $y = \log_2(\log_3 \sqrt{4x - 4x^2})$ funksiyaning aniqlanish sohasini toping.

- A) $\{\frac{1}{2}\}$ B) \emptyset
 C) $(0; \frac{1}{2}) \cup (\frac{1}{2}; 1)$ D) $(-\infty; 0) \cup (1; \infty)$

22. (98-11-39) Tengsizlikni yeching.

$$\log_x 6 > \log_x 12$$

- A) $(0; \frac{1}{2})$ B) $(\frac{1}{2}; 1)$ C) $(0; 1)$ D) $(0; 2)$

23. (98-11-49) Tengsizlikni yeching.

$$x^{\log_2 x + 4} < 32$$

- A) $(2^{-1}; 2)$ B) $(2^{-2}; 2)$ C) $(2^{-3}; 2)$ D) $(2^{-5}; 2)$

24. (98-4-39) x ning nechta natural qiymatida

$$\frac{\sqrt{6-x}}{\log_{\frac{1}{3}}(x-3)} \geq 0$$

tengsizlik o'rinli bo'ladi?

- A) bunday qiymatlar yo'q
 B) 1 C) 2 D) 3

25. (01-1-24) Tengsizlikni yeching.

$$\log_2 x \leq \frac{2}{\log_2 x - 1}$$

- A) $(0; 1)$ B) $(0; 4]$ C) $(0; 2)$ D) $(0; \frac{1}{2}] \cup (2; 4]$

26. (01-2-28) Ushbu $\log_{x^2}(3 - 2x) > 1$ tengsizlikning butun yechimlari nechta?

- A) 4 B) 3 C) 2 D) 1

27. (00-4-41) Tengsizlikni yeching.

$$\log_{x^2}(x + 2) \leq 1$$

- A) $(-\infty; -1] \cup [2; \infty)$
 B) $(-\infty; -1) \cup [2; \infty)$
 C) $(-2; -1) \cup (-1; 0) \cup (0; 1) \cup [2; \infty)$
 D) $(-1; 2]$

28. (01-4-28) Tengsizlikni yeching.

$$\log_{1/3}(5 - 2x) > -2$$

- A) $(-2; -1)$ B) $(-2; 2; 5)$
 C) $(0; 2; 5)$ D) $(0; 2)$

29. (01-6-38) Tengsizlikning butun yechimini toping.

$$\log_{\frac{1}{2}}(2^x - 128) \geq -7$$

- A) 5 B) 6 C) 9 D) 8

Yechish: $\frac{1}{2} = 2^{-1}$ tenglikdan va logarifmning 6-xossasidan

$$-\log_2(2^x - 128) \geq -7 \log_2 2$$

ni olamiz. Tengsizlikning har ikkala qismini -1 ga ko'paytirib, logarifmning 5-xossasidan hamda logarifm tengsizlikning 1-xossasidan foydalanib

$$\log_2(2^x - 128) \leq \log_2 2^7 \iff 0 < 2^x - 128 \leq 2^7$$

ni hosil qilamiz. Bu tengsizlikning barcha qismlariga $128 = 2^7$ ni qo'shib

$$2^7 < 2^x \leq 2^8 \iff 7 < x \leq 8$$

ga kelamiz. Yechimlar ichida faqat 8 butun soni bor. **Javob:** 8 (D).

30. (01-6-39) Nechta butun son

$$\begin{cases} \log_2 x^2 \geq 2 \\ \log_5 x^2 \leq 2 \end{cases}$$

tengsizliklar sistemasini qanoatlantiradi?

- A) 6 B) 7 C) 9 D) 8

31. (01-7-28) Tengsizlikni yeching.

$$\log_{\frac{1}{3}}(x - 1) - 2 \log_{\frac{1}{9}}(2x - 3) < 0$$

- A) $(\frac{3}{2}; 2)$ B) $(-\infty; 2)$ C) $(2; \infty)$ D) $(\frac{3}{2}; \infty)$

32. (97-12-53) Tengsizlikning eng kichik butun musbat yechimini toping

$$\left(\frac{1}{2}\right)^{\log_{0,5} x(x-4)} > 0$$

- A) 4 B) 6 C) 5 D) 5,5

33. (01-7-35) Tengsizlikning eng katta butun yechimini toping.

$$0,5^{\log_3(x^2 + 6x - 7)} \geq \frac{1}{4}$$

- A) 1 B) 2 C) 4 D) 1,5

34. (02-4-42) Tengsizlikni qanoatlantiruvchi eng kichik butun sonni toping.

$$-\lg x < 1$$

- A) -2 B) -1 C) 10 D) 1

35. (02-4-43) Tengsizlikning eng kichik butun yechimini toping.

$$\log_{16}(3x+1) > \frac{1}{2}$$

- A) -2 B) -1 C) 0 D) 2

36. (01-9-45) Tengsizlikni yeching.

$$\sqrt{4x^2 - 5x - 9} < \ln \frac{1}{2}$$

- A) $(-5; 4)$ B) $(2; 3)$ C) $(-5; 2)$ D) \emptyset

Yechish: $\ln \frac{1}{2}$ logarifm asosi $e = 2, 71 \dots$ soniga teng. Logarifmning 4-xossasidan hamda $\ln 1 = 0$ tenglikdan

$$\sqrt{4x^2 - 5x - 9} < -\ln 2$$

ni olamiz. Tengsizlikning o'ng qismi $-\ln 2 < 0$, ya'ni manfiy, chap qismi esa manfiyemas. Shuning uchun berilgan tengsizlik yechimga ega emas.

Javob: \emptyset (D).

37. (01-11-32) Tengsizlikning butun yechimlari yig'indisini toping.

$$\frac{x-5}{\log_x^2 3} < 0$$

- A) 7 B) 8 C) 9 D) 10

38. (01-9-3) Tengsizlikni yeching.

$$\frac{2 \log_2(3-2x)}{\log_2 0,1} < 0$$

- A) $(-\infty; 1)$ B) $(-\infty; 1]$ C) $(1; \infty)$ D) $(-1; 2)$

39. (02-5-26) Tengsizlikni yeching.

$$2 \log_8(x-2) - \log_8(x-3) > \frac{2}{3}$$

- A) $(-\infty; 4)$ B) $\{2\} \cup (4; \infty)$
C) $(-\infty; 4) \cup (4; \infty)$ D) $(3; 4) \cup (4; \infty)$

40. (02-6-38) Tengsizlikni yeching.

$$(x^2 - 8x + 7) \cdot \sqrt{\log_5(x^2 - 3)} \leq 0$$

- A) $[-2; 1] \cup [2; 7]$ B) $[2; 7] \cup \{-2\}$
C) $[1; 7]$ D) $[3; 7]$

Yechish: Arifmetik ildiz manfiyemas, shuning uchun berilgan tengsizlik

$$\begin{cases} x^2 - 8x + 7 \leq 0 \\ \log_5(x^2 - 3) \geq 0 \end{cases} \iff \begin{cases} (x-1)(x-7) \leq 0 \\ x^2 - 3 \geq 1 \end{cases}$$

sistemaga teng kuchli. Bu tengsizliklarning ikkalasini ham oraliqlar usuli bilan yechish qulay. 1-tengsizlikning yechimi $[1; 7]$ dan iborat. 2-tengsizlik

$$x^2 - 3 \geq 1 \iff x^2 - 4 \geq 0 \iff (x-2)(x+2) \geq 0$$

ko'rinishda bo'lib, uning yechimi $(-\infty; -2] \cup [2; \infty)$ dan iborat. Ularning umumiy qismi (kesishmasi) $[2; 7]$ to'plamdir. $x = -2$ da tengsizlikning ikkala qismi ham nolga aylanadi. Shuning uchun uni yechimlar to'plamiga kiritib $[2; 7] \cup \{-2\}$ ni olamiz. **Javob:** $[2; 7] \cup \{-2\}$ (B).

41. (02-9-35) Tengsizlikning yechimlaridan nechtasi butun sondan iborat?

$$\lg(x-2) < 2 - \lg(27-x)$$

- A) 9 B) 8 C) 7 D) 6

42. (02-11-35) Tengsizlikning yechimlaridan iborat tub sonlarning yig'indisini toping.

$$\frac{2 \log_3 x}{2 + \log_3 x} \leq 1$$

- A) 5 B) 6 C) 16 D) 17

43. (03-1-29) Tengsizlikni yeching.

$$\log_x 3 < 2$$

- A) $(\sqrt{3}; \infty)$ B) $(3; \infty)$
C) $(0; 1) \cup (\sqrt{3}; \infty)$ D) $(0; 1)$

44. Tengsizlik nechta butun yechimga ega?

$$\log_4(2 - \sqrt{x+3}) < 1$$

- A) 6 B) 4 C) 5 D) 3

45. (07-121-32) $x^{\log_2 x+2} < 8$ tengsizlikni yeching.

- A) $(2^{-2}; 2)$ B) $(2^{-5}; 2)$ C) $(2^{-4}; 2)$ D) $(2^{-3}; 2)$

Yechish: Tengsizlikning aniqlanish sohasi $x > 0$. 2 asosli logarifmning o'suvchi ekanligidan

$$\log_2 x^{\log_2 x+2} < \log_2 8 \iff (\log_2 x + 2) \log_2 x < 3$$

ni olamiz. Bu tengsizlikda $t = \log_2 x$ deb, uni $(t+2)t < 3 \iff t^2 + 2t - 3 < 0$ yoki $(t-1)(t+3) < 0 \iff -3 < t < 1$ shaklda yozamiz. Belgilashga qaytib $-3 < \log_2 x < 1 \iff 2^{-3} < x < 2^1$ ni olamiz. **Javob:** $(2^{-3}; 2)$ (D).

46. (07-128-32) Tengsizlikni yeching.

$$\log_2 \log_{\frac{1}{3}} \log_8 x > 0$$

- A) $(-\infty; 0) \cup (0; 2)$ B) $(1; 2)$
C) $(-\infty; 2)$ D) $(0; 2)$

47. (06-111-34) $(x-2)^{\log_2(x^2-5x+5)} < (x-2)^{\log_2(x-3)}$ tengsizlik x ning qanday qiymatlarida o'rinli?

- A) $(2; 4)$ B) $(3; 8)$
C) $(-\infty; 2) \cup (4; \infty)$ D) $(\frac{5+\sqrt{5}}{2}; 4)$

48. (07-112-32) $4^{\log_2 x} + x^2 < 50$ tengsizlikning barcha butun yechimlari yig'indisini toping.

- A) 10 B) 6 C) 7 D) 15

49. (07-117-32) $\frac{2 \log_4 x}{2 + \log_4 x} \leq 1$ tengsizlikning tub sonlardan iborat yechimlari yig'indisini toping?

- A) 28 B) 17 C) 21 D) 41

13 - bob. Trigonometriya

13.1 Burchak va yoy, ularning o'lchovi

Tekislikda umumiy boshlang'ich nuqtaga ega bo'lgan ikki turli yarim to'g'ri chiziqdan iborat figura *burchak* deyiladi. Burchakning bu ta'rifi trigonometrik funksiyalarni o'rganish uchun qulay emas. Har qanday burchakni OA nurini o'zining boshlang'ich O nuqtasi atrofida burish (13.1-chizma) yordamida hosil qilish mumkin. To'la bir aylanish 360^0 deb qabul qilingan.

Agar OA nur O nuqta atrofida chorak aylanish qilsa, to'g'ri (90^0) burchak hosil bo'ladi (13.2a—chizma), agar OA nur O nuqta atrofida yarim aylanish qilsa yoyiq (180^0) burchak hosil bo'ladi (13.2b—chizma).

OA nur o'zining boshlang'ich O nuqtasi atrofida aylanib burchak yasaganda uning O nuqtadan boshqa istalgan nuqtasi aylana yoyini chizadi. Nurini boshlang'ich nuqtasi atrofida ikki yo'nalish bo'yicha aylantirish mumkin. Nurini soat strelkasiga qarama-qarshi yo'nalish bo'yicha aylantirishdan hosil bo'lgan burchak va yoyini musbat, soat strelkasi yo'nalishi bo'yicha aylantirishdan hosil bo'lgan burchak va yoyini manfiy deb atash qabul qilingan (13.3-chizma).

Burchak va yoyni o'lchov birligi qilib 1 gradusli burchak (1^0) va 1 gradusli yoy qabul qilingan. 1 gradusli burchak bu nurini o'z boshlang'ich nuqtasi atrofida soat strelkasiga qarama-qarshi yo'nalishda to'la aylanishni $1/360$ ga burishdan hosil bo'lgan burchakdir. 1 gradusli

yoy bu 1 gradusli burchakni hosil qilishda nurning nuqtasi chizadigan yoydir. Gradusning $1/60$ qismi bu minut ($1'$), minutning $1/60$ qismi bu sekund ($1''$) hisoblanadi. Uchi aylana markazida, tomonlari raduslardan iborat bo'lgan burchak markaziy burchak deyiladi. 13.4-chizmada AOB markaziy burchak, AB yoy markaziy burchakka mos yoydir. α markaziy burchakka mos AB yoy uzunligining radusga nisbati α burchakning *radian o'lchovi* deyiladi. Agar AB yoy uzunligi aylana radusiga teng bo'lsa, u holda AOB burchak 1 radianli burchak deyiladi. Agar α burchakning radian o'lchovi a , unga mos yoy uzunligi ℓ , radus r bo'lsa, u holda $a = \ell/r$ bo'ladi. Agar yoy uzunligi ikki radusga teng bo'lsa, u holda burchak 2 radianga, agar yoy uzunligi yarim radusga teng bo'lsa, u holda burchak $0,5$ radianga teng bo'ladi.

Ma'lumki, aylana uzunligining radusga nisbati 2π ga teng. Demak, 360^0 li burchakka 2π radian mos keladi. Xuddi shunday yoyiq burchakka π burchak radiani mos keladi. To'g'ri burchakka esa $\pi/2$ burchak radiani mos keladi. Endi radiandan gradusga, gradusdan radianga o'tish formulalarini beramiz.

$$1. \alpha \text{ radiandan gradusga o'tish: } \frac{180^0}{\pi} \cdot \alpha = \alpha^0.$$

$$2. n^0 \text{ gradusdan radianga o'tish: } \frac{\pi}{180} \cdot n^0 = n.$$

1. (97-2-31) $\frac{5\pi}{4}$ radian necha gradus bo'ladi?
A) 220^0 B) 230^0 C) 225^0 D) 240^0

Yechish: 1-formuladan foydalansak,

$$\frac{5\pi}{4} = \frac{180^0}{\pi} \cdot \frac{5\pi}{4} = \frac{5 \cdot 180^0}{4} = 225^0.$$

Javob: 225^0 (C).

2. $\frac{\pi}{4}$ radian necha gradus bo'ladi?
A) 20^0 B) 30^0 C) 45^0 D) 60^0
3. 1 radianga mos burchakni minut aniqligida toping.
A) $57^020'$ B) $57^018'$ C) $57^017'$ D) $57^019'$
4. (97-12-30) $\frac{4\pi}{3}$ radian necha gradusga teng?
A) 230^0 B) 220^0 C) 250^0 D) 240^0
5. $\frac{5\pi}{6}$ radian necha gradusga teng?
A) 130^0 B) 120^0 C) 150^0 D) 140^0

6. 240° ning radian o'lchovini toping.
 A) $\frac{5\pi}{4}$ B) $\frac{2\pi}{3}$ C) $\frac{4\pi}{3}$ D) $\frac{5\pi}{3}$

Yechish: 2-formuladan foydalansak,

$$240^{\circ} = \frac{\pi}{180} \cdot 240 = \frac{24 \cdot \pi}{18} = \frac{4\pi}{3}.$$

Javob: $\frac{4\pi}{3}$ (C).

7. 15° ning radian o'lchovini toping.
 A) $\frac{\pi}{15}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{9}$
8. (97-8-30) 216° ning radian o'lchovini toping.
 A) $\frac{4\pi}{3}$ B) $\frac{5\pi}{4}$ C) $\frac{3\pi}{2}$ D) $\frac{6\pi}{5}$
9. 30° ning radian o'lchovini toping.
 A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$
10. 45° ning radian o'lchovini toping.
 A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$
11. (00-8-58) 72° ning radian o'lchovini toping.
 A) 72 B) 1 C) 0,3 D) $\frac{2\pi}{5}$
12. Radiusi 3 ga teng bo'lgan aylananing 120° li yoyining uzunligini toping.
 A) π B) 2π C) $1,5\pi$ D) $1,6\pi$

13.2 Trigonometrik funksiyalar

Tekislikda to'g'ri burchakli Oxy koordinatalar sistemasini qaraymiz. Markazi koordinatalar boshida va radiusi birga teng bo'lgan aylana *birlik aylana* deyiladi. Uning tenglamasi $x^2 + y^2 = 1$ ko'rinishda bo'ladi. Koordinatalar boshi O ni burchakning uchi qilib, yarim musbat absissa o'qini OA nur deb qabul qilamiz. OA nur birlik aylanani koordinatalari $(1; 0)$ bo'lgan P_0 nuqtada kesib o'tadi. OA nurni α burchakka buramiz, natijada OB nurga ega bo'lamiz. OB nur birlik aylanani $P_{\alpha}(x; y)$ nuqtada kesib o'tsin (13.5-chizma).

aytiladi va $\cos \alpha = x$ shaklda yoziladi. α burchakning *tangensi* deb $P_{\alpha}(x; y)$ nuqta ordinatasining absissasiga nisbatiga aytiladi va quyidagicha yoziladi:

$$\operatorname{tg} \alpha = \frac{y}{x} = \frac{\sin \alpha}{\cos \alpha}.$$

α burchakning *kotangensi* deb $P_{\alpha}(x; y)$ nuqta absissasining ordinatasiga nisbatiga aytiladi, bu quyidagicha yoziladi:

$$\operatorname{ctg} \alpha = \frac{x}{y} = \frac{\cos \alpha}{\sin \alpha}.$$

Sinus, kosinus, tangens va kotangenslar α burchakning funksiyalaridir. Bu funksiyalar *trigonometrik funksiyalar* deyiladi.

Tangens va kotangenslarning quyidagi ta'rif qulay. $x = 1$ to'g'ri chiziq tangenslar o'qi, $y = 1$ to'g'ri chiziq esa kotangenslar o'qi deyiladi. OA nurni α ($\alpha \neq 90^{\circ} + 180^{\circ}n$, $n \in \mathbb{Z}$) burchakka buramiz, natijada bu nur yoki uning davomi tangenslar o'qini $P_{\alpha}(1; y)$ nuqtada kesib o'tadi (13.6-chizma). Bu nuqtaning ordinatasi α burchakning *tangensi* deyiladi, ya'ni $\operatorname{tg} \alpha = y$.

Xuddi shunday kotangens funksiyaga ta'rif berish mumkin. OA nurni α ($\alpha \neq n\pi$, $n \in \mathbb{Z}$) burchakka buramiz, natijada bu nur yoki uning davomi kotangenslar o'qini $P_{\alpha}(x; 1)$ nuqtada kesib o'tadi (13.7-chizma). Bu nuqtaning absissasi α burchakning *kotangensi* deyiladi, ya'ni $\operatorname{ctg} \alpha = x$.

α burchakning *sinusi* deb $P_{\alpha}(x; y)$ nuqtaning ordinatasiga aytiladi va $\sin \alpha = y$ shaklda yoziladi. α burchakning *kosinusi* deb $P_{\alpha}(x; y)$ nuqtaning absissasiga

Trigonometrik funksiyalarning ta'riflaridan foydalanib, to'g'ridan-to'g'ri ba'zi burchaklarda trigonometrik funksiyalarning son qiymatlarini hisoblash mumkin.

Masalan, $\alpha = 0^0$ bo'lsa, P_0 nuqtaning koordinatasi $(1; 0)$ bo'ladi. Shuning uchun $\sin 0^0 = 0$, $\cos 0^0 = 1$, $\operatorname{tg} 0^0 = 0$. $\operatorname{ctg} 0^0$ esa mavjud emas. Agar $\alpha = 90^0$ ($\alpha = \frac{\pi}{2}$) bo'lsa, P_α nuqtaning koordinatasi $(0; 1)$ bo'ladi. Shuning uchun $\sin 90^0 = 1$, $\cos 90^0 = 0$, $\operatorname{ctg} 0^0 = 0$. $\operatorname{tg} 0^0$ esa mavjud emas. Endi $\alpha = 45^0$ bo'lsin, u holda P_α nuqtaning koordinatasi (x, x) ko'rinishda bo'ladi. Bu nuqta birlik aylanada yotgani uchun uning tenglamasini qanoatlantiradi, ya'ni $x^2 + x^2 = 1$. Bu yerdan $x = \frac{\sqrt{2}}{2}$ ekanligi kelib chiqadi. Demak,

$$\sin 45^0 = \frac{\sqrt{2}}{2}, \quad \cos 45^0 = \frac{\sqrt{2}}{2}, \quad \operatorname{tg} 45^0 = 1, \quad \operatorname{ctg} 45^0 = 1.$$

Bundan tashqari 30^0 , 60^0 va 180^0 da ham trigonometrik funksiyalarning qiymatlarini hisoblash mumkin. Bu qiymatlarni quyidagi 13.1-jadval shaklida beramiz.

α	0^0	30^0	45^0	60^0	90^0	180^0	270^0
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\operatorname{tg} \alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	0	-
$\operatorname{ctg} \alpha$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-	0

13.1-jadval.

Ma'lumki, $M(x; y)$ nuqta tekislikning I choragida yotsa, uning koordinatalari $x > 0$, $y > 0$ shartni, II choragida yotsa, $x < 0$, $y > 0$ shartni, III choragida yotsa, $x < 0$, $y < 0$ shartni va IV chorakda yotsa, uning koordinatalari $x > 0$, $y < 0$ shartni qanoatlantiradi. Bulardan foydalanib trigonometrik funksiyalarning ishoralari uchun 13.2-jadvalni tuzish mumkin.

Funksiya	I chor ($0; \frac{\pi}{2}$)	II chor ($\frac{\pi}{2}; \pi$)	III chor ($\pi; \frac{3\pi}{2}$)	IV chor ($\frac{3\pi}{2}; 2\pi$)
$\sin \alpha$	+	+	-	-
$\cos \alpha$	+	-	-	+
$\operatorname{tg} \alpha$	+	-	+	-
$\operatorname{ctg} \alpha$	+	-	+	-

13.2-jadval.

Sinus va kosinus funksiyalar grafiklari 13.8-chizmada, tangens va kotangensning grafiklari 13.9-chizmada tasvirlangan.

Sinus, kosinus, tangens va kotangens funksiyalarning asosiy xossalarini keltitamiz.

1. Aniqlanish sohasi:

2. $D(\sin) = R = (-\infty; \infty)$.

3. $D(\cos) = R$.

4. $D(\operatorname{tg}) = R \setminus \{\frac{\pi}{2} + n\pi, \quad n \in Z\}$.

5. $D(\operatorname{ctg}) = R \setminus \{n\pi, \quad n \in Z\}$.

6. Qiymatlar sohasi:

7. $E(\sin) = [-1; 1]$.

8. $E(\cos) = [-1; 1]$.

9. $E(\operatorname{tg}) = R$.

10. $E(\operatorname{ctg}) = R$.

11. Davriyligi:

12. $\sin(x + 2\pi) = \sin x, \quad T = 2\pi$.

13. $\cos(x + 2\pi) = \cos x, \quad T = 2\pi$.

14. $\operatorname{tg}(x + \pi) = \operatorname{tg} x, \quad T = \pi$.

15. $\operatorname{ctg}(x + \pi) = \operatorname{ctg} x, \quad T = \pi$.

16. Juft-toqligi:

17. $\sin(-x) = -\sin x, \quad$ toq funksiya.

18. $\cos(-x) = \cos x, \quad$ juft funksiya.

19. $\operatorname{tg}(-x) = -\operatorname{tg} x, \quad$ toq funksiya.

20. $\operatorname{ctg}(-x) = -\operatorname{ctg} x, \quad$ toq funksiya.

21. Monotonligini jadval yordamida beramiz: ↗ o'suvchilik, ↘ kamayuvchilik belgisi.

Funksiya	I-chorak	II-chorak	III-chorak	IV-chorak
$\sin \alpha$	↗	↘	↘	↗
$\cos \alpha$	↘	↘	↗	↗
$\operatorname{tg} \alpha$	↗	↗	↗	↗
$\operatorname{ctg} \alpha$	↘	↘	↘	↘

13.3-jadval.

7. Hisoblang.

$$5 \sin 30^0 + 7 \cos 60^0 - 11 \sin 90^0 + 10 \cos 0^0$$

A) 3 B) 6 C) 5 D) 9

8. Hisoblang.

$$\sqrt{2} \sin 45^0 + 8\sqrt{3} \cos 30^0 - \sqrt{27} \operatorname{tg} 30^0 + \operatorname{ctg} 45^0$$

A) 10 B) 11 C) 15 D) 13

9. (96-11-58) Hisoblang.

$$\sin 180^0 + \sin 270^0 - \operatorname{ctg} 90^0 + \operatorname{tg} 180^0 - \cos 90^0$$

A) -1 B) 0 C) 1 D) -2

10. (96-12-11) Hisoblang.

$$3 \operatorname{tg} 0^0 + 2 \cos 90^0 + 3 \sin 270^0 - 3 \cos 180^0$$

A) 6 B) 0 C) -6 D) 9

1. (00-2-32) Quyidagilardan qaysi biri musbat?

A) $\cos 3$ B) $\sin 4$ C) $\sin 2$ D) $\operatorname{tg} 2$

Yechish: $\frac{\pi}{2} < 2 < \pi$ bo'lgani uchun 2 radianga mos burchak II chorakda yotadi. Shuning uchun $\sin 2$ musbat bo'ladi. **Javob:** $\sin 2$ (C).

2. (97-12-32) Quyidagi sonlardan qaysi biri manfiy?

A) $\sin 122^0 \cdot \cos 322^0$ B) $\cos 148^0 \cdot \cos 289^0$
C) $\operatorname{tg} 196^0 \cdot \operatorname{ctg} 189^0$ D) $\operatorname{tg} 220^0 \cdot \sin 100^0$

3. (96-6-33) Quyidagi sonlardan qaysi biri musbat?

$$M = \frac{\cos 320^0}{\sin 217^0}, \quad N = \frac{\operatorname{ctg} 187^0}{\operatorname{tg} 340^0}$$

$$P = \frac{\operatorname{tg} 185^0}{\sin 140^0}, \quad Q = \frac{\sin 135^0}{\operatorname{ctg} 140^0}$$

A) M B) N C) P D) Q

4. (97-2-33) Quyidagi sonlardan qaysi biri manfiy?

A) $\operatorname{tg} 247^0 \cdot \sin 125^0$ B) $\operatorname{ctg} 215^0 \cdot \cos 300^0$
C) $\operatorname{tg} 135^0 \cdot \operatorname{ctg} 340^0$ D) $\sin 247^0 \cdot \cos 276^0$

5. Quyidagi sonlardan qaysi biri manfiy?

1) $\cos 3, 78$; 2) $\operatorname{ctg} 2, 91$; 3) $\operatorname{tg} 4, 45$

A) 1; 2 B) 1 C) 2; 3 D) 1; 3

6. (96-3-56) Hisoblang.

$$5 \sin 90^0 + 2 \cos 0^0 - 2 \sin 270^0 + 10 \cos 180^0$$

A) -3 B) -6 C) -1 D) 9

Yechish: 13.1-jadvaldan ko'rish mumkinki,

$$\sin 90^0 = \cos 0^0 = 1, \quad \sin 270^0 = \cos 180^0 = -1.$$

Shu sababli berilgan ifoda

$$5 \cdot 1 + 2 \cdot 1 - 2 \cdot (-1) + 10 \cdot (-1) = 5 + 2 + 2 - 10 = -1$$

ga teng bo'ladi. **Javob:** -1 (C).

11. (01-2-58) Hisoblang.

$$\cos\left(\frac{12\pi}{5}(\log_2 0,25 + \log_{0,25} 2)\right)$$

A) 0 B) 1 C) -1 D) $\frac{1}{2}$

12. (96-12-58) Agar $\sin \alpha \cdot \cos \alpha < 0$ bo'lsa, α burchak qaysi chorakka tegishli?

A) I yoki II B) I yoki III
C) I yoki IV D) II yoki IV

13. (96-3-42) $P(3; 0)$ nuqtani koordinata boshi atrofida 90^0 ga burganda u qaysi nuqtaga o'tadi?

A) $(-3; 0)$ B) $(0; -3)$ C) $(3; 3)$ D) $(0; 3)$

Yechish: Berilgan $P(3; 0)$ nuqta absissa o'qida yotadi. OA nurni koordinata boshi atrofida 90^0 ga burganda u ordinata o'qining musbat yo'nalishi bilan ustma-ust tushadi. Demak, P nuqtaning koordinatasi $(0; 3)$ bo'ladi. **Javob:** $(0; 3)$ (D).

14. $P(3; 0)$ nuqtani koordinata boshi atrofida 180^0 ga burganda u qaysi nuqtaga o'tadi?

A) $(-3; 0)$ B) $(0; -3)$ C) $(3; 3)$ D) $(0; 3)$

15. (96-11-43) $P(-3; 0)$ nuqtani koordinata boshi atrofida 90^0 ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

A) $(3; 0)$ B) $(0; -3)$ C) $(3; 3)$ D) $(0; 3)$

16. $P(0; 2)$ nuqtani koordinata boshi atrofida 270^0 ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

A) $(-2; 0)$ B) $(0; -2)$ C) $(2; 2)$ D) $(2; 0)$

17. $P(1; 1)$ nuqtani koordinata boshi atrofida 135^0 ga burganda hosil bo'ladigan nuqtaning koordinatalarini toping.

A) $(\sqrt{2}; 0)$ B) $(0; \sqrt{2})$ C) $(-\sqrt{2}; 0)$ D) $(-1; 1)$

13.2.1 Trigonometriyaning asosiy ayniyatlari

1. $\sin^2 x + \cos^2 x = 1$ – asosiy trigonometrik ayniyat.

2. $1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$.

3. $1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}$.

4. $\operatorname{tg} x \cdot \operatorname{ctg} x = 1$.

1-4 ayniyatlardan yana bir nechta hosilaviy formulalar kelib chiqadi. Ularni 13.4-jadval ko'rishida beramiz:

Funks	$\sin \alpha$	$\cos \alpha$	$\operatorname{tg} \alpha$
$\sin \alpha$	$\sin \alpha$	$\pm \sqrt{1 - \cos^2 \alpha}$	$\frac{\pm \operatorname{tg} \alpha}{\sqrt{1 + \operatorname{tg}^2 \alpha}}$
$\cos \alpha$	$\pm \sqrt{1 - \sin^2 \alpha}$	$\cos \alpha$	$\frac{\pm 1}{\sqrt{1 + \operatorname{tg}^2 \alpha}}$
$\operatorname{tg} \alpha$	$\frac{\pm \sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$	$\frac{\sqrt{1 - \cos^2 \alpha}}{\pm \cos \alpha}$	$\operatorname{tg} \alpha$
$\operatorname{ctg} \alpha$	$\frac{\sqrt{1 - \sin^2 \alpha}}{\pm \sin \alpha}$	$\frac{\pm \cos \alpha}{\sqrt{1 - \cos^2 \alpha}}$	$\frac{1}{\operatorname{tg} \alpha}$

13.4-jadval.

Xususan $\operatorname{ctg} \alpha$ ning qiymati berilgan bo'lsa, $\sin \alpha$, $\cos \alpha$ va $\operatorname{tg} \alpha$ lar qiymatlari quyidagicha topiladi:

$$\sin \alpha = \frac{\pm 1}{\sqrt{1 + \operatorname{ctg}^2 \alpha}}; \cos \alpha = \frac{\pm \operatorname{ctg} \alpha}{\sqrt{1 + \operatorname{ctg}^2 \alpha}}; \operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}.$$

Asosiy trigonometrik ayniyatlar va ulardan kelib chiqadigan hosilaviy formulalar yordamida trigonometrik funksiyalardan birining qiymatiga ko'ra boshqalarining qiymatini topish mumkin. Yuqorida keltirilgan formulalarda + yoki – ishora α burchak qaysi chorakda bo'lsa, trigonometrik funksiyaning shu chorakdagi ishorasi olinadi.

1. (98-5-48) Agar $\frac{\pi}{2} < \alpha < \pi$ va $\sin \alpha = \frac{3}{5}$ bo'lsa, $\operatorname{tg} \alpha$ ni toping.

A) $-\frac{4}{5}$ B) $-\frac{3}{4}$ C) $\frac{3}{4}$ D) $-\frac{3}{5}$

Yechish: 13.4-jadvaldan

$$\operatorname{tg} \alpha = \frac{\pm \sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$$

ekanligini olamiz. Bu tenglikda $\sin \alpha = \frac{3}{5}$ desak

$$\operatorname{tg} \alpha = \pm \frac{3}{5} : \sqrt{1 - \frac{9}{25}} = \pm \frac{3}{5} : \frac{4}{5} = \pm \frac{3}{4}$$

bo'ladi. Berilgan α burchak II chorakda, tangens funksiyaning II chorakdagi qiymati manfiy.

Shuning uchun $\operatorname{tg} \alpha = -\frac{3}{4}$. **Javob:** $-\frac{3}{4}$ (B).

2. (99-7-47) Agar $\alpha \in (\frac{\pi}{2}; \pi)$ va $\sin \alpha = \frac{1}{4}$ bo'lsa, $\operatorname{ctg} \alpha$ ni hisoblang.

A) -4 B) $-\sqrt{17}$ C) $-\frac{1}{\sqrt{15}}$ D) $-\sqrt{15}$

3. (00-8-61) Agar $0 < \alpha < \frac{\pi}{2}$ va $\operatorname{tg} \alpha = 2$ bo'lsa, $\cos \alpha$ ni toping.

A) $\frac{5}{\sqrt{5}}$ B) $\frac{2}{\sqrt{5}}$ C) $\frac{\sqrt{5}}{5}$ D) $\sqrt{5}$

4. Agar $\alpha \in (\frac{3\pi}{2}; 2\pi)$ va $\cos \alpha = \frac{1}{4}$ bo'lsa, $\operatorname{tg} \alpha$ ni hisoblang.

A) $-\sqrt{15}$ B) $-\sqrt{17}$ C) $-\frac{1}{\sqrt{15}}$ D) $\sqrt{15}$

5. Agar $\pi < \alpha < \frac{3\pi}{2}$ va $\operatorname{ctg} \alpha = -\sqrt{15}$ bo'lsa, $\sin \alpha$ ni toping.

A) $-\frac{5}{\sqrt{5}}$ B) $-\frac{2}{5}$ C) $-\frac{1}{4}$ D) $-\sqrt{5}$

6. (01-7-37) Agar $\frac{\pi}{2} < \alpha < \pi$ va $\operatorname{tg} \alpha = -\frac{3}{4}$ bo'lsa, $\sin \alpha - \cos \alpha$ ning qiymatini toping.

A) $-\frac{1}{5}$ B) $\frac{1}{5}$ C) $\frac{7}{5}$ D) $-\frac{7}{5}$

7. Agar $\operatorname{tg} \alpha = 2$ bo'lsa,

$$\frac{3 \sin \alpha}{5 \sin^3 \alpha + 10 \cos^3 \alpha}$$

ning qiymati qanchaga teng bo'ladi?

A) $\frac{4}{5}$ B) $\frac{3}{5}$ C) $\frac{8}{15}$ D) $\frac{7}{15}$

Yechish: Qiymati izlanayotgan kasrning surat va maxrajini $\sin \alpha$ ga bo'lib

$$\frac{3}{5 \sin^2 \alpha + 10 \cos^2 \alpha \operatorname{ctg} \alpha}$$

olamiz. 4-ayniyatdan $\operatorname{ctg} \alpha = 2^{-1}$ kelib chiqadi. Bu qiymatni yuqoridagi tenglikka qo'yib

$$\frac{3}{5 \sin^2 \alpha + 10 \cos^2 \alpha \operatorname{ctg} \alpha} = \frac{3}{5 \sin^2 \alpha + 5 \cos^2 \alpha} = \frac{3}{5}$$

ni olamiz. **Javob:** $\frac{3}{5}$ (B).

8. (98-4-17) Agar $\operatorname{tg} \alpha = 3$ bo'lsa,

$$\frac{3 \sin \alpha}{5 \sin^3 \alpha + 10 \cos^3 \alpha}$$

ning qiymati qanchaga teng bo'ladi?

A) $\frac{16}{39}$ B) $\frac{4}{9}$ C) $\frac{8}{15}$ D) $\frac{18}{29}$

9. (02-8-41) Agar $\operatorname{ctg} \alpha = 2$ bo'lsa,

$$\frac{\sin^2 \alpha - 2 \cos^2 \alpha}{3 \sin \alpha \cdot \cos \alpha + \cos^2 \alpha}$$

ifodaning qiymatini hisoblang.

A) $-0,7$ B) $-0,5$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{3}}{2}$

10. (01-9-23) Agar $\cos \alpha = \frac{\sqrt{3}}{2}$ bo'lsa,

$$\frac{1 - \sin^2 \alpha + \cos^2 \alpha \cdot \sin \alpha}{1 + \sin \alpha}$$

ifodaning qiymatini toping.

- A) $\frac{3}{4}$ B) 1,5 C) $1\frac{1}{3}$ D) 1

11. (98-11-97) Agar $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = a$, $a > 0$ bo'lsa, $\sqrt{\operatorname{tg} \alpha} + \sqrt{\operatorname{ctg} \alpha}$ ning qiymati qanchaga teng bo'ladi?

- A) $\sqrt{a+2}$ B) $a-2$ C) $\sqrt{2} + \sqrt{a}$ D) $a+2$

Yechish: $\sqrt{\operatorname{tg} \alpha} + \sqrt{\operatorname{ctg} \alpha} = x \geq 0$ belgilash olib, bu tenglikni kvadratga ko'taramiz:

$$\operatorname{tg} \alpha + \operatorname{ctg} \alpha + 2 = x^2 \iff x^2 = a + 2.$$

Bu yerdan $x = \pm\sqrt{a+2}$. x ning nomanfiyligidan $x = \sqrt{a+2}$. **Javob:** $\sqrt{a+2}$ (A).

12. (98-8-62) Agar $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = p$ bo'lsa, $\operatorname{tg}^3 \alpha + \operatorname{ctg}^3 \alpha$ ni p orqali ifodalang.

- A) $-p^3 - 3p$ B) $p^3 - 3p$ C) $p^3 + 3p$ D) $3p - p^3$

13. (99-6-33) Agar

$$\frac{2 \sin x - \cos x}{2 \cos x + \sin x} = 3$$

bo'lsa, $\operatorname{tg} x$ ni hisoblang.

- A) 7 B) -3 C) 3 D) -7

14. (99-6-51) Soddashtiring.

$$\sin^6 \alpha + \cos^6 \alpha + 3 \sin^2 \alpha \cdot \cos^2 \alpha$$

- A) -1 B) 0 C) 1 D) 2

Yechish: Ikki son yig'indisining kubi formulasi-dan

$$\begin{aligned} \sin^6 \alpha + \cos^6 \alpha &= (\sin^2 \alpha)^3 + (\cos^2 \alpha)^3 = \\ &= (\sin^2 \alpha + \cos^2 \alpha)(\sin^4 \alpha - \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha) \end{aligned}$$

ni olamiz. Endi asosiy trigonometrik ayniyatga ko'ra, berilgan ifodani

$$\sin^4 \alpha + 2 \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha = (\sin^2 \alpha + \cos^2 \alpha)^2$$

shaklda yozish mumkin. Yana bir marta asosiy ayniyatdan foydalansak, berilgan ifodaning qiymati 1 ga tengligini olamiz. **Javob:** 1 (C).

15. (99-8-80) $\sin^2 x + \cos^2 x + \operatorname{tg}^2 x$ ni soddashtiring.

- A) $-\frac{1}{\cos^2 x}$ B) $-\frac{1}{\sin^2 x}$ C) $\frac{1}{\sin^2 x}$ D) $\frac{1}{\cos^2 x}$

16. (97-1-47) Soddashtiring.

$$(\operatorname{ctg} \alpha - \cos \alpha) \cdot \left(\frac{\sin^2 \alpha}{\cos \alpha} + \operatorname{tg} \alpha \right)$$

- A) $\cos^2 \alpha$ B) $\operatorname{tg} \alpha$ C) $\frac{1}{\cos \alpha}$ D) $\operatorname{ctg}^2 \alpha$

17. (98-1-55) Soddashtiring:

$$\frac{3 \sin^2 \alpha + \cos^4 \alpha}{1 + \sin^2 \alpha + \sin^4 \alpha}$$

- A) $2 \sin \alpha$ B) 2 C) $\operatorname{ctg}^2 \alpha$ D) 1

18. (98-8-55) Soddashtiring.

$$\frac{1 + \cos^2 \alpha + \cos^4 \alpha}{3 \cos^2 \alpha + \sin^4 \alpha}$$

- A) 3 B) 2 C) $1\frac{1}{2}$ D) 1

19. (02-4-30) Soddashtiring.

$$(\operatorname{tg} x + \operatorname{ctg} x)^2 - (\operatorname{tg} x - \operatorname{ctg} x)^2$$

- A) 0 B) -4 C) -2 D) 4

20. (02-7-39) Soddashtiring.

$$(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2 - 2$$

- A) 0 B) 4 C) $2 \sin 2\alpha$ D) 1

21. (01-1-69) Agar $\sin x + \cos x = 0,5$ bo'lsa, $16(\sin^3 x + \cos^3 x)$ ni toping.

- A) 8 B) 14 C) 11 D) 16

Yechish: Ikki son kublarining yig'indisi formulasi-dan va asosiy ayniyatdan foydalansak

$$\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x \cdot \cos x)$$

ni olamiz. Masala sharti $\sin x + \cos x = 0,5$ va

$$\sin x \cdot \cos x = \frac{1}{2}((\cos x + \sin x)^2 - 1)$$

tenglikdan foydalansak, berilgan ifoda ucun

$$16 \cdot 0,5(1 - \cos x \sin x) = 16 \cdot \frac{1}{2} \left(1 + \frac{1}{2} \cdot \frac{3}{4} \right) = 11$$

ni olamiz. **Javob:** 11 (C).

22. (00-2-45) Agar $\operatorname{ctg} \alpha = \frac{13}{4}$ bo'lsa,

$$\frac{2 \cos \alpha + \sin \alpha}{\cos \alpha - 2 \sin \alpha}$$

kasrning qiymatini toping.

- A) 6 B) 5 C) 6,2 D) 4,8

23. (03-8-55) Agar $\cos x = \frac{1}{\sqrt{10}}$ bo'lsa,

$$(1 + \operatorname{tg}^2 x)(1 - \sin^2 x) - \sin^2 x$$

ifodaning qiymatini toping.

- A) 0,1 B) 0,2 C) 0,3 D) $\frac{2}{\sqrt{10}}$

13.2.2 Trigonometrik funksiyalarning xossalari

1. $y = \sin x$ va $y = \cos x$ funksiyalarning eng kichik musbat davri 2π ga teng.

2. $y = \operatorname{tg} x$ va $y = \operatorname{ctg} x$ funksiyalarning eng kichik musbat davri π ga teng.

3. Agar $f(x)$ funksiyaning davri T bo'lsa, u holda $a f(x) + b$ funksiyaning davri T bo'ladi.

4. Agar $f(x)$ funksiyaning davri T bo'lsa, u holda $f(ax+b)$ funksiyaning davri T/a bo'ladi.

5. Agar $f(x)$ funksiyaning eng kichik musbat davri T_f , $g(x)$ funksiyaning eng kichik musbat davri T_g bo'lsa, $f(x) \pm g(x)$ funksiyaning eng kichik musbat davri $T = EKUK(T_f; T_g)$ bo'ladi.

6. Agar $g(x)$ davriy funksiya bo'lsa, u holda $f(g(x))$ funksiya ham davriy bo'ladi.

7. $y = \cos x$ funksiya juft.

8. $y = \sin x$, $y = \operatorname{tg} x$, $y = \operatorname{ctg} x$ funksiyalar toq.

9. Agar $f(x) + f(-x) \equiv 0$ bo'lsa, f toq, $f(x) - f(-x) \equiv 0$ bo'lsa, f juft funksiya bo'ladi.

Izoh. 5-xossada keltirilgan $EKUK(T_f; T_g)$ belgilash T_f va T_g sonlariga bo'lganda natural chiqadigan sonlarning eng kichigidir. Masalan, $f(x) = \operatorname{tg} 2x$, $g(x) = \sin 3x$ bo'lsin. U holda 1-2 va 4-xossalardan $T_f = \frac{\pi}{2}$, $T_g = \frac{2\pi}{3}$ ekanligi kelib chiqadi. T_f ga bo'lganda natural chiqadigan sonlar

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}, 4\pi, \frac{9\pi}{2}, 5\pi, \frac{11\pi}{2}, 6\pi, \dots$$

T_g ga bo'lganda natural chiqadigan sonlar

$$\frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3}, \frac{10\pi}{3}, 4\pi, \frac{14\pi}{3}, \frac{16\pi}{3}, 6\pi, \frac{20\pi}{3}, \dots$$

Bu ikki qatorda uchraydigan umumiy sonlarning eng kichigi 2π dir. Demak, $T = EKUK(T_f; T_g) = 2\pi$.

1. (03-1-17) $y = 2 + 3 \cos(8x - 7)$ funksiyaning eng kichik musbat davrini toping.

A) 2π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$

Yechish: 1 va 4-xossalarga ko'ra $y = \cos(8x - 7)$ funksiyaning eng kichik musbat davri

$$T = \frac{2\pi}{8} = \frac{\pi}{4}$$

ga teng. 3-xossaga ko'ra $y = 2 + 3 \cos(8x - 7)$ funksiyaning eng kichik musbat davri ham $\frac{\pi}{4}$ ga teng bo'ladi. **Javob:** $\frac{\pi}{4}$ (D).

2. $y = 3 + \sin 2x$ funksiyaning davrini toping.

A) $\frac{2\pi}{3}$ B) π C) $\frac{\pi}{3}$ D) 2π

3. (98-10-102) $y = \sin(3x + 1)$ funksiyaning davrini toping.

A) $\frac{2\pi}{3}$ B) π C) $\frac{\pi}{3}$ D) 2π

4. (98-12-56) $y = \cos(\frac{5x}{2} - \frac{5}{2})$ funksiyaning eng kichik musbat davrini aniqlang.

A) $\frac{4\pi}{5}$ B) 2π C) π D) $\frac{2\pi}{5}$

5. (01-11-35) Funksiyaning davrini toping.

$$f(x) = 2^{\sin x} + 3^{\operatorname{tg} x}$$

A) $\frac{\pi}{2}$ B) 2π C) 3π D) 4π

6. (96-12-105) Funksiyaning eng kichik musbat davrini toping.

$$y = \operatorname{tg} \frac{x}{3} - 2 \sin x + 3 \cos 2x$$

A) 6π B) 3π C) 4π D) 9π

Yechish: 4-xossaga ko'ra $y = \operatorname{tg} \frac{x}{3}$ funksiyaning eng kichik musbat davri $T_1 = \pi : \frac{1}{3} = 3\pi$ ga, $y = \sin x$ funksiyaning davri $T_2 = 2\pi$ ga, $y = \cos 2x$ funksiyaning davri $T_3 = \frac{2\pi}{2} = \pi$ ga teng. Ularning eng kichik umumiy karralisi $K(T_1; T_2; T_3) = 6\pi$ bo'lganligi uchun berilgan funksiyaning eng kichik musbat davri 6π ga teng. **Javob:** 6π (A).

7. (96-9-48) Funksiyaning eng kichik davrini toping.

$$y = \operatorname{tg} \frac{x}{3} - 2 \sin \frac{x}{2} + 3 \cos \frac{2}{3}x$$

A) 4π B) 6π C) 3π D) 12π

8. (96-13-14) Funksiyaning eng kichik davrini toping.

$$y = \operatorname{ctg} \frac{x}{3} + \operatorname{tg} \frac{x}{2}$$

A) 6π B) 2π C) 3π D) 12π

9. Qaysi funksiyaning davri eng kichik va u nechaga teng. $y_1 = \operatorname{tg} 3x$, $y_2 = \operatorname{ctg} 6x$, $y_3 = \cos(3x + 1)$, $y_4 = \sin(6x + 4)$

A) $y_1; \frac{2\pi}{3}$ B) $y_2; \frac{\pi}{6}$ C) $y_3; \frac{2\pi}{3}$ D) $y_4; \frac{\pi}{3}$

10. (02-3-41) Quyidagi funksiyalardan qaysi biri davriy emas?

A) $y = \sin \sqrt{x}$ B) $y = \sqrt{\sin x}$
C) $y = |\sin |x||$ D) $y = \sin^2 x$

11. (02-3-45) Quyidagi funksiyalardan qaysi birlari davriy emas?

1) $y = \sin \sqrt{x}$; 2) $y = \lg |\cos x|$
3) $y = x \cos x$; 4) $y = \sin^2 x + 1$

A) 1;3 B) 1;2 C) 2;3 D) 1;4

12. (03-4-38) Eng kichik musbat davrga ega bo'lgan funksiyaning ko'rsating.

A) $y = \sin \frac{4}{3}x$ B) $y = \cos \frac{5}{3}x$

C) $y = \operatorname{ctg} \frac{3}{2}x$ D) $y = \sin x \cos x$

13. (97-2-41) Quyidagi funksiyalardan qaysi biri juft?

A) $f(x) = \sin x + x^3$ B) $f(x) = \cos x \operatorname{tg} x$

C) $f(x) = x^2 \cdot \operatorname{ctg} x$ D) $f(x) = \frac{x^4 + x^2}{\cos x}$

Yechish: $f(x) = \sin x + x^3$ funksiya toq funksiya-
lar yig'indisi sifatida toq (10-bob, 7-xossa), $f(x) = \cos x \operatorname{tg} x$ va $f(x) = x^2 \cdot \operatorname{ctg} x$ funksiyalar juft va toq funksiyalar ko'paytmasi sifatida toq (10-bob, 4-xossa) funksiya bo'ladi. D javobdagi f funksiya juft funksiyalar nisbati sifatida juft (10-bob, 3-xossa) funksiya bo'lagi. **Javob:** (D).

14. (97-4-17) k ning qanday natural qiymatlarida $y = (\sin x)^{5k+4}$ juft funksiya bo'ladi?
A) toq qiymatlarida
B) juft qiymatlarida
C) 5 ga karrali qiymatlarida
D) barcha qiymatlarida
15. (00-10-72) Quyidagilardan qaysi biri toq funksiya?
A) $y = \lg \frac{1+x}{1-x}$ B) $y = \lg x^3$
C) $y = \cos(x-a)$ D) $y = \frac{a^x + a^{-x}}{2}$

16. (97-12-40) Quyidagi funksiyalardan qaysi biri toq?
A) $f(x) = \frac{\cos 5x + 1}{|x|}$ B) $f(x) = \frac{\sin^2 x}{x^2 - 1}$
C) $f(x) = \frac{\cos^2 x}{x(x^2 - 1)}$ D) $f(x) = \frac{\sin \frac{x}{2}}{x^3}$

17. (99-2-38) Quyidagi funksiyalardan qaysi biri toq funksiya?
A) $f(x) = \sin x \cdot \operatorname{tg} x$ B) $f(x) = \cos x \cdot \operatorname{ctg} x$
C) $f(x) = \sin |x|$ D) $f(x) = e^{|x|}$

18. (99-7-17) Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?
 $y_1 = 2^x + 2^{-x};$ $y_2 = 5^x - 5^{-x};$
 $y_3 = \sqrt{\sin x} + \sqrt{\cos x};$ $y_4 = \cos x + x^3.$
A) $y_1; y_2$ B) $y_1; y_3$ C) $y_3; y_4$ D) y_3

Yechish: 9-xossa yordamida ko'rsatish mumkinki y_1 juft, y_2 toq funksiya. y_4 funksiya juft va toq funksiyalar yig'indisi sifatida juft ham, toq ham bo'lmagan funksiya (10-bob, 11-xossa). y_3 funksiyaning juft yoki toqligini aniqlamasdan ham testning to'g'ri javobini topishimiz mumkin. Buning uchun quyidagicha yo'l tutamiz. Qaysi javoblarda y_1 yoki y_2 funksiyalar qatnashgan bo'lsa, ular to'g'ri javob bo'la olmaydi. Chunki y_1 va y_2 funksiyalar juft yoki toqlik xossasini namayon qiladi. To'g'ri javobda y_4 albatta qatnashishi kerak. Demak, to'g'ri javob C ekan. **Javob:** $y_3; y_4$ (C).

19. Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?
 $y_1 = \sin x + \cos(-x);$ $y_2 = \operatorname{tg} x - \operatorname{ctg}^2 x;$
 $y_3 = \sin^2 x + \cos^2 x;$ $y_4 = \cos^5 x + \sin^3 x.$
A) $y_1; y_2; y_4$ B) $y_1; y_3$ C) $y_3; y_4$ D) $y_2; y_3$
20. Funksiyalardan qaysi birlari juft ham, toq ham bo'lmagan funksiyalardir?
 $y_1 = \sin^3 x : \cos x;$ $y_2 = \cos(x-2);$
 $y_3 = \sin^7 x + 7;$ $y_4 = \cos^5 x \cdot \sin^3 x.$
A) $y_1; y_2; y_4$ B) $y_1; y_3$ C) $y_3; y_4$ D) $y_2; y_3$

21. Juft ham, toq ham bo'lmagan funksiyaning toping.
A) $1 + \sin x$ B) $1 - \cos^5 3x$
C) $1 + \operatorname{tg}^2 x$ D) $1 - \operatorname{ctg}^2 x$
22. (01-2-16) Quyidagi funksiyalardan qaysi biri toq funksiya?
A) $x^3 + x + 4$ B) $\cos x + \operatorname{tg} x$
C) $\sin x + \operatorname{tg} x - 1$ D) $\sin 2x \cdot \cos x / \operatorname{tg}^2 x$
23. (01-11-34) $y_1 = \sin(\frac{\pi}{2} - x);$ $y_2 = \operatorname{ctg}^2 x \sin^2 x$
va $y_3 = \lg(|x| + 1)$ funksiyalardan qaysi biri toq?
A) y_1 B) y_2 C) y_3
D) Berilgan funksiyalar ichida toq funksiya yo'q.

13.2.3 Qo'shish va keltirish formulalari

Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalari uchun formulalar keltiramiz. Quyida keltirilgan formulalar "qo'shish teoremlari" deb ham yuritiladi.

- $\sin(x+y) = \sin x \cos y + \cos x \sin y.$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y.$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y.$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y.$
- $\operatorname{tg}(x+y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}.$
- $\operatorname{tg}(x-y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y}.$

Argumentlari $\frac{n\pi}{2} \pm \alpha$, $n \in Z$ ko'rinishidagi trigonometrik funksiyalarni α burchakning trigonometrik funksiyalariga keltiruvchi formulalar keltirish formulalari deyiladi. Qo'shish teoremlaridan bevosita kelib chiqadigan va "keltirish formulalari" deb ataluvchi 13.5-jadvalni keltiramiz.

x	$\sin x$	$\cos x$	$\operatorname{tg} x$	$\operatorname{ctg} x$
$\frac{\pi}{2} - \alpha$	$\cos \alpha$	$\sin \alpha$	$\operatorname{ctg} \alpha$	$\operatorname{tg} \alpha$
$\frac{\pi}{2} + \alpha$	$\cos \alpha$	$-\sin \alpha$	$-\operatorname{ctg} \alpha$	$-\operatorname{tg} \alpha$
$\pi - \alpha$	$\sin \alpha$	$-\cos \alpha$	$-\operatorname{tg} \alpha$	$-\operatorname{ctg} \alpha$
$\pi + \alpha$	$-\sin \alpha$	$-\cos \alpha$	$\operatorname{tg} \alpha$	$\operatorname{ctg} \alpha$
$\frac{3\pi}{2} - \alpha$	$-\cos \alpha$	$-\sin \alpha$	$\operatorname{ctg} \alpha$	$\operatorname{tg} \alpha$
$\frac{3\pi}{2} + \alpha$	$-\cos \alpha$	$\sin \alpha$	$-\operatorname{ctg} \alpha$	$-\operatorname{tg} \alpha$
$2\pi - \alpha$	$-\sin \alpha$	$\cos \alpha$	$-\operatorname{tg} \alpha$	$-\operatorname{ctg} \alpha$

13.5-jadval.

Keltirish formulalaridan foydalanib, trigonometrik funksiyalarning ixtoyoriy burchakdagi qiymatini o'tkir burchakning trigonometrik funksiyalari orqali ifodalash mumkin. Bu yerdan kelib chiqadiki, trigonometrik funksiyalarning qiymatlarini hisoblash uchun, ularning faqat o'tkir burchakdagi qiymatlarini bilish kifoya ekan.

Quyidagi qoidalar yodda saqlansa, keltirish formulalarini eslab qolish oson bo'ladi.

1) $\pi \pm \alpha$, $2\pi \pm \alpha$ (butun π lar) burchak funksiyalaridan α burchak funksiyalariga o'tilayotganda funksiyaning nomi o'zgarmaydi.

2) $\frac{\pi}{2} \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$ (yarimtalik π lar) burchak funksiyalaridan α burchak funksiyalariga o'tilayotganda funksiyaning nomi o'zgaradi.

3) α ni o'tkir burchak deb hisoblab α burchak funksiyasi oldiga, keltirilayotgan funksiyaning $\beta = \frac{\pi}{2} \pm \alpha$ ($\pi \pm \alpha$, $\frac{3\pi}{2} \pm \alpha$ $2\pi \pm \alpha$) burchak yotgan chorakdagi ishorasi qo'yiladi.

Buni quyidagi misolda ko'rib chiqamiz.

1-misol. $\sin 300^\circ$ ni o'tkir burchakning trigonometrik funksiyasi orqali ifodalang.

Yechish: 1-usul. $\sin 300^\circ$ ni $\sin(360^\circ - 60^\circ)$ shaklda yozib olamiz. 360° — bu 2π radianga mos burchak, shuning uchun 60° o'tsak, sinus funksiyaning ismi o'zgar-maydi. Endi oldiga qo'yiladigan ishorani aniqlaymiz. $360^\circ - 60^\circ = 300^\circ$ li burchak, IV chorakka tegishli, IV chorakda sinus funksiya manfiy qiymatlar qabul qiladi, demak "—" ishorasi qo'yiladi. Shunday qilib, $\sin(360^\circ - 60^\circ) = -\sin 60^\circ$.

2-usul. $\sin 300^\circ = \sin \frac{10\pi}{6}$ ni $\sin(\frac{3\pi}{2} + \frac{\pi}{6})$ shaklda yozib olamiz. $\frac{3\pi}{2}$ (bir yarim π) bo'lganligi uchun $\frac{3\pi}{2}$ tashlanganda sinus funksiyaning ismi o'zgarib kosinusga almashadi. Endi oldiga qo'yiladigan ishorani aniqlaymiz. $\frac{3\pi}{2} < \frac{3\pi}{2} + \frac{\pi}{6} < 2\pi$ burchak IV chorakka tegishli, IV chorakda sinus funksiyaning qiymati manfiy, demak "—" ishorasi qo'yiladi. Shunday qilib, $\sin(\frac{3\pi}{2} + \frac{\pi}{6}) = -\cos \frac{\pi}{6}$. **Javob:** $\sin 300^\circ = -\sin 60^\circ = -\cos \frac{\pi}{6}$.

1. $\sin 15^\circ$ ni hisoblang.

- A) $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$ B) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$
 C) $\frac{\sqrt{2}}{2}(\sqrt{3} - 1)$ D) $\frac{\sqrt{2}}{4}(\sqrt{6} - \sqrt{2})$

Yechish: $\sin 15^\circ$ ni $\sin(45^\circ - 30^\circ)$ shaklda yozib, unga 2-formulani qo'llasak

$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

ni olamiz. 13.1-jadvaldan sinus va kosinuslar-ni 30° va 45° dagi qiymatlarini topib, ayirmaning hisoblaymiz:

$$\sin 15^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1).$$

Javob: $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$ (A).

2. $\cos 15^\circ$ ni hisoblang.

- A) $\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$ B) $\frac{1}{4}(\sqrt{6} - \sqrt{2})$
 C) $\frac{\sqrt{2}}{2}(\sqrt{3} + 1)$ D) $\frac{\sqrt{2}}{4}(\sqrt{3} + \sqrt{2})$

3. $\sin 75^\circ$ ni hisoblang.

- A) $\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$ B) $\frac{1}{4}(\sqrt{6} - \sqrt{2})$
 C) $\frac{\sqrt{2}}{2}(\sqrt{3} + 1)$ D) $\frac{\sqrt{2}}{4}(\sqrt{3} + \sqrt{2})$

4. $\cos 75^\circ$ ni hisoblang.

- A) $\frac{\sqrt{2}}{4}(\sqrt{3} - 1)$ B) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$
 C) $\frac{\sqrt{2}}{2}(\sqrt{3} - 1)$ D) $\frac{\sqrt{2}}{4}(\sqrt{6} - \sqrt{2})$

5. $\cos 75^\circ + \cos 105^\circ$ ni hisoblang.

- A) 0 B) $\sqrt{3}$ C) 1 D) $\sqrt{2}$

6. (98-6-54) Hisoblang.

$$\cos 45^\circ \cdot \cos 15^\circ + \sin 45^\circ \cdot \sin 15^\circ$$

- A) $\frac{1}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) 0

7. (01-6-27) Hisoblang.

$$\cos 15^\circ + \sqrt{3} \sin 15^\circ$$

- A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$

8. (00-5-31) $\sin 2010^\circ$ ni hisoblang.

- A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) 1

Yechish: $\sin 2010^\circ = \sin(6 \cdot 360^\circ - 150^\circ)$ tenglik-dan hamda sinusning 2π davriy va toq funksiya ekanligidan foydalansak

$$\sin 2010^\circ = -\sin 150^\circ = -\sin(180^\circ - 30^\circ)$$

ni olamiz. Bunga keltirish formulasini qo'llab, $\sin 30^\circ = 0,5$ ekanligini hisobga olsak, $\sin 2010^\circ = -0,5$ ni olamiz. **Javob:** $-\frac{1}{2}$ (A).

9. (01-2-17) $\cos 870^\circ$ ni hisoblang.

- A) $\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$

10. (02-4-29) $\sin^2 3570^\circ$ ni hisoblang.

- A) 0,2 B) 0,3 C) 0,25 D) 0,35

11. (03-2-41) $\operatorname{tg} 1395^\circ$ ni hisoblang.

- A) $\sqrt{3}$ B) $-\frac{1}{\sqrt{3}}$ C) -1 D) 1

12. $\operatorname{tg} 105^\circ$ ni hisoblang.

- A) $-2 - \sqrt{3}$ B) $2 - \sqrt{3}$
 C) $-2 + \sqrt{3}$ D) $2 + \sqrt{3}$

Yechish: $\operatorname{tg} 105^\circ$ ni $\operatorname{tg}(45^\circ + 60^\circ)$ shaklda yozib, unga 5-formulani qo'llasak

$$\operatorname{tg} 105^\circ = \frac{\operatorname{tg} 45^\circ + \operatorname{tg} 60^\circ}{1 - \operatorname{tg} 45^\circ \operatorname{tg} 60^\circ}$$

ni olamiz. 13.1-jadvaldan tangensning 45° va 60° dagi qiymatlarini topib, nisbatni hisoblaymiz:

$$\operatorname{tg} 105^\circ = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} = -2 - \sqrt{3}.$$

Javob: $-2 - \sqrt{3}$ (A).

13. $\operatorname{ctg} 105^0$ ni hisoblang.

- A) $\frac{-1}{2 + \sqrt{3}}$ B) $\frac{1}{2 - \sqrt{3}}$
 C) $\frac{1}{\sqrt{3} - 2}$ D) $\frac{1}{2 + \sqrt{3}}$

14. $\operatorname{ctg} 15^0$ ni hisoblang.

- A) $-2 - \sqrt{3}$ B) $2 - \sqrt{3}$
 C) $-2 + \sqrt{3}$ D) $2 + \sqrt{3}$

15. $\operatorname{tg} 75^0$ ni hisoblang.

- A) $-2 - \sqrt{3}$ B) $2 - \sqrt{3}$
 C) $-2 + \sqrt{3}$ D) $2 + \sqrt{3}$

16. $\operatorname{ctg} 75^0 + \operatorname{tg} 15^0$ ni hisoblang.

- A) $-4 - 2\sqrt{3}$ B) $4 - 2\sqrt{3}$
 C) $-4 + 2\sqrt{3}$ D) $4 + 2\sqrt{3}$

17. (03-4-23) Hisoblang.

$$(\operatorname{tg} 60^0 \cos 15^0 - \sin 15^0) \cdot 7\sqrt{2}$$

- A) 16 B) 12 C) 18 D) 14

18. (96-1-54) $2\operatorname{tg}(-765^0)$ ning qiymatini toping.

- A) $-\sqrt{2}$ B) $\frac{2}{\sqrt{3}}$ C) -2 D) 4

Yechish: $765^0 = 2 \cdot 360^0 + 45^0$ tenglikni va tangensning toq funksiya hamda π davriy ekanligini hisobga olsak

$$2\operatorname{tg}(-765^0) = -2\operatorname{tg}(2 \cdot 360^0 + 45^0) = -2\operatorname{tg} 45^0$$

ni olamiz. $\operatorname{tg} 45^0 = 1$ dan $2\operatorname{tg}(-765^0) = -2$ kelib chiqadi. **Javob:** -2 (C).

19. (97-11-43) Hisoblang.

$$\cos(-45^0) + \sin 315^0 + \operatorname{tg}(-855^0)$$

- A) 0 B) $\sqrt{2} - 1$ C) $1 + \sqrt{3}$ D) 1

20. (98-5-49) $\operatorname{tg} 1^0 \cdot \operatorname{tg} 2^0 \cdot \dots \cdot \operatorname{tg} 88^0 \cdot \operatorname{tg} 89^0$ ni hisoblang.

- A) 0 B) $\frac{1}{2}$ C) 1 D) $\sqrt{3}$

21. (98-10-36) Hisoblang.

$$\operatorname{tg} \frac{\pi}{6} \cdot \sin \frac{\pi}{3} \cdot \operatorname{ctg} \frac{5\pi}{4}$$

- A) 1,5 B) 0,5 C) $-\frac{1}{2}$ D) $\frac{\sqrt{3}}{4}$

22. (02-3-76) Hisoblang.

$$\sin \frac{\pi}{9} - \cos \frac{7\pi}{18}$$

- A) 0 B) $\frac{1}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$

23. (03-12-77) Hisoblang.

$$\left(\left(\operatorname{tg}^2 \frac{7\pi}{24} - \operatorname{tg}^2 \frac{\pi}{24} \right) : \left(1 - \operatorname{tg}^2 \frac{7\pi}{24} \cdot \operatorname{tg}^2 \frac{\pi}{24} \right) \right)^2$$

- A) $\frac{1}{9}$ B) 9 C) $\frac{1}{3}$ D) 3

24. (96-3-111) Agar $\operatorname{tg} \left(\frac{\pi}{4} - \alpha \right) = 2$ bo'lsa, $\operatorname{tg} \alpha$ ni toping.

- A) 3 B) -3 C) $\frac{1}{3}$ D) $-\frac{1}{3}$

Yechish: $\operatorname{tg} \alpha = \operatorname{tg} \left(\frac{\pi}{4} - \left(\frac{\pi}{4} - \alpha \right) \right)$ tenglikning o'ng tomoniga 6-formulani qo'llasak

$$\operatorname{tg} \alpha = \frac{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} \left(\frac{\pi}{4} - \alpha \right)}{1 + \operatorname{tg} \frac{\pi}{4} \operatorname{tg} \left(\frac{\pi}{4} - \alpha \right)} = \frac{1 - 2}{1 + 2} = -\frac{1}{3}.$$

Javob: $-\frac{1}{3}$ (D).

25. (96-9-46) Agar $\operatorname{tg} \left(\frac{\pi}{4} - \alpha \right) = 2$ bo'lsa, $\operatorname{ctg} \alpha$ ni toping.

- A) 3 B) $\frac{1}{3}$ C) $-\frac{1}{3}$ D) -3

26. (01-1-42) Agar

$$\operatorname{tg} \alpha = \frac{1}{2}, \quad \operatorname{tg} \beta = \frac{1}{3}, \quad \pi < \alpha + \beta < 2\pi$$

bo'lsa, $\alpha + \beta$ ning qiymatini toping.

- A) $\frac{7\pi}{3}$ B) $\frac{5\pi}{3}$ C) $\frac{5\pi}{4}$ D) $\frac{7\pi}{4}$

27. (97-1-60) Agar

$$\begin{cases} \operatorname{tg}(x + y) = 3 \\ \operatorname{tg}(x - y) = 2 \end{cases}$$

bo'lsa, $\operatorname{tg} 2x$ ni hisoblang.

- A) 5 B) 2,5 C) 1 D) -1

Yechish: Agar $\operatorname{tg} 2x = \operatorname{tg}((x + y) + (x - y))$ tenglikning o'ng tomoniga 5-formulani qo'llasak

$$\operatorname{tg} 2x = \frac{\operatorname{tg}(x + y) + \operatorname{tg}(x - y)}{1 - \operatorname{tg}(x + y) \cdot \operatorname{tg}(x - y)} = \frac{3 + 2}{1 - 3 \cdot 2} = -1.$$

Javob: -1 (D).

28. (97-6-60) Agar

$$\begin{cases} \operatorname{tg}(\alpha + \beta) = 5 \\ \operatorname{tg}(\alpha - \beta) = 3 \end{cases}$$

bo'lsa, $\operatorname{tg} 2\beta$ ni hisoblang.

- A) 15 B) 8 C) $\frac{1}{8}$ D) 1

29. (97-6-68) Agar

$$\begin{cases} 2\operatorname{tg} \alpha = 3 + \sqrt{x}, \\ 2\operatorname{tg} \beta = 3 - \sqrt{x} \\ 4(\alpha + \beta) = \pi \end{cases}$$

bo'lsa, x ni toping.

- A) $\frac{\pi}{3}$ B) -17 C) $-\frac{\pi}{6} + \pi k$ D) 17

30. (98-6-48) Agar $\operatorname{tg}(x + y) = 5$ va $\operatorname{tg} x = 3$ bo'lsa, $\operatorname{tg} y$ ni toping.

- A) 2 B) $\frac{1}{8}$ C) 8 D) $\frac{1}{2}$

Yechish: Agar $\operatorname{tg} y = \operatorname{tg}((x+y) - x)$ tenglikning o'ng tomoniga 6-formulani qo'llasak

$$\operatorname{tg} y = \frac{\operatorname{tg}(x+y) - \operatorname{tg} x}{1 + \operatorname{tg}(x+y) \cdot \operatorname{tg} x} = \frac{5-3}{1+5 \cdot 3} = \frac{1}{8}.$$

Javob: $\frac{1}{8}$ (B).

31. (00-1-29) Agar $\alpha = -45^\circ$ va $\beta = 15^\circ$ bo'lsa, $\cos(\alpha + \beta) + 2 \sin \alpha \cdot \sin \beta$ ning qiymatini toping.

A) $-\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$

32. (02-6-46) Agar

$$\begin{cases} \cos x \cdot \cos y = \frac{1}{6} \\ \operatorname{tg} x \cdot \operatorname{tg} y = 2 \end{cases}$$

bo'lsa, $\cos(x+y)$ ni toping.

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $-\frac{1}{2}$ D) $-\frac{1}{6}$

33. (01-1-49) Agar $\sin \alpha = -\frac{1}{3}$ va $\cos \beta = -\frac{1}{2}$ bo'lsa, $\sin(\alpha + \beta) \cdot \sin(\alpha - \beta)$ ning qiymatini toping.

A) $-\frac{23}{36}$ B) $\frac{23}{36}$ C) $\frac{3}{4}$ D) $-\frac{3}{4}$

34. (03-1-25) Agar

$$\begin{cases} 3 \sin x \cdot \cos y = -1 \\ 3 \cos x \cdot \sin y = 2 \end{cases}$$

bo'lsa, $\operatorname{ctg}(x-y)$ ni hisoblang.

A) 0 B) 1 C) $-\frac{1}{2}$ D) $\frac{1}{2}$

35. (00-1-26) Soddashtiring.

$$\frac{\sin(\frac{\pi}{2} - \alpha) \cdot \cos(\pi + \alpha)}{\operatorname{ctg}(\pi + \alpha) \cdot \operatorname{tg}(\frac{3\pi}{2} - \alpha)}$$

A) $-\sin^2 \alpha$ B) $-\sin^2 \alpha \cdot \operatorname{tg}^2 \alpha$
C) $-\cos^2 \alpha$ D) $\cos^2 \alpha \cdot \operatorname{ctg}^2 \alpha$

Yechish: Berilgan kasr suratiga keltirish formulasini qo'llab $\cos \alpha(-\cos \alpha) = -\cos^2 \alpha$ ni olamiz. Kasr maxrajiga ham keltirish formulasini qo'llab $\operatorname{ctg} \alpha \cdot \operatorname{ctg} \alpha = \operatorname{ctg}^2 \alpha$ ni olamiz. U holda berilgan kasr

$$-\cos^2 \alpha \cdot \operatorname{tg}^2 \alpha = -\cos^2 \alpha \cdot \frac{\sin^2 \alpha}{\cos^2 \alpha} = -\sin^2 \alpha$$

ga teng bo'ladi. **Javob:** $-\sin^2 \alpha$ (A).

36. (96-1-57) Soddashtiring.

$$\frac{\cos(\alpha + \beta) + 2 \sin \alpha \cdot \sin \beta}{\sin(\alpha + \beta) - 2 \cos \beta \cdot \sin \alpha}$$

A) $\operatorname{ctg}(\beta - \alpha)$ B) $\operatorname{tg}(\alpha - \beta)$
C) $2 \operatorname{tg}(\alpha + \beta)$ D) $2 \operatorname{ctg}(\alpha - \beta)$

37. (01-11-24) Soddashtiring.

$$\frac{\sin \alpha + \cos \alpha}{\sqrt{2} \cos(\frac{\pi}{4} - \alpha)}$$

A) 1,6 B) $\operatorname{ctg}(\frac{\pi}{4} + \alpha)$ C) 1,5 D) 1

38. (99-1-41) Soddashtiring.

$$\operatorname{tg} \alpha \cdot \operatorname{ctg}(\pi + \alpha) + \operatorname{ctg}^2 \alpha$$

A) $\frac{1}{\sin^2 \alpha}$ B) $\frac{1}{\cos^2 \alpha}$ C) $\operatorname{tg} \alpha$ D) $\operatorname{tg}^2 \alpha$

39. (00-8-60) Soddashtiring.

$$\frac{\operatorname{tg}(\pi - \alpha) \cdot \sin(\frac{3\pi}{2} + \alpha)}{\cos(\pi + \alpha) \cdot \operatorname{tg}(\frac{3\pi}{2} + \alpha)}$$

A) $\operatorname{tg}^2 \alpha$ B) $\operatorname{ctg}^2 \alpha$ C) $-\operatorname{tg}^2 \alpha$ D) $\frac{1}{\operatorname{tg} \alpha}$

13.2.4 Ikkilangan va yarim burchak formulari

Qo'shish teoremlaridan ikkilangan burchakning trigonometrik funksiyalari uchun quyidagilar kelib chiqadi.

1. $\sin 2x = 2 \sin x \cos x.$

2. $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x.$

3. $\sin 2x = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}.$

4. $\cos 2x = \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}.$

5. $\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}.$

Ikkilangan burchak formulalaridan yarim burchak uchun formulalar kelib chiqadi.

6. $\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}, \quad \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}.$

7. $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}, \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}.$

8. $\operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}.$

9. $\operatorname{ctg} \frac{x}{2} = \frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}.$

10. $\operatorname{tg}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}, \quad \operatorname{ctg}^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{1 - \cos \alpha}.$

1. (97-6-51) Hisoblang.

$$\sin \frac{\pi}{8} \cdot \cos^3 \frac{\pi}{8} - \sin^3 \frac{\pi}{8} \cdot \cos \frac{\pi}{8}$$

A) 0 B) 1 C) 2 D) $\frac{1}{4}$

Yechish: 1 va 2-dan, ya'ni $\sin \alpha \cdot \cos \alpha = \frac{1}{2} \sin 2\alpha$
va $\cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$ ekanligidan

$$\begin{aligned} \sin \frac{\pi}{8} \cdot \cos^3 \frac{\pi}{8} - \sin^3 \frac{\pi}{8} \cdot \cos \frac{\pi}{8} &= \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \cdot \\ &\cdot \left(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \right) = \frac{1}{2} \sin \frac{\pi}{4} \cos \frac{2\pi}{8} = \\ &= \frac{1}{2} \sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4} = \frac{1}{4} \sin \frac{\pi}{2} = \frac{1}{4} \end{aligned}$$

ni hosil qilamiz. **Javob:** $\frac{1}{4}$ (D).

2. (97-1-52) Hisoblang.

$$\begin{aligned} \sin \frac{\pi}{16} \cdot \cos^3 \frac{\pi}{16} - \sin^3 \frac{\pi}{16} \cdot \cos \frac{\pi}{16} \\ \text{A) } \frac{\sqrt{2}}{2} \quad \text{B) } \frac{\sqrt{2}}{3} \quad \text{C) } \frac{\sqrt{2}}{4} \quad \text{D) } \frac{\sqrt{2}}{8} \end{aligned}$$

3. (99-6-53) Hisoblang.

$$\begin{aligned} \cos \frac{\pi}{7} \cdot \cos \frac{4\pi}{7} \cdot \cos \frac{5\pi}{7} \\ \text{A) } -\frac{1}{8} \quad \text{B) } \frac{1}{4} \quad \text{C) } \frac{1}{2} \quad \text{D) } \frac{1}{8} \end{aligned}$$

Yechish: Keltirish formulasi $\cos \alpha = -\cos(\pi - \alpha)$ dan foydalanib berilgan ifodada quyidagicha shakl almashtirish qilamiz:

$$\begin{aligned} A &= \cos \frac{\pi}{7} \cdot \cos \frac{4\pi}{7} \cdot \cos \frac{5\pi}{7} = -\cos \frac{\pi}{7} \cdot \\ &\cdot \cos \frac{4\pi}{7} \cdot \cos(\pi - \frac{5\pi}{7}) = -\cos \frac{\pi}{7} \cdot \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}. \end{aligned}$$

Tenglikni 8 $\sin \frac{\pi}{7}$ ga ko'paytirib va 1-formulani bir necha marta qo'llab

$$\begin{aligned} 8A \sin \frac{\pi}{7} &= -8 \sin \frac{\pi}{7} \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \\ &= -4 \sin \frac{2\pi}{7} \cdot \cos \frac{2\pi}{7} \cdot \cos \frac{4\pi}{7} = -2 \sin \frac{4\pi}{7} \cos \frac{4\pi}{7} = \\ &= -\sin \frac{8\pi}{7} \text{ ni hosil qilamiz. Demak,} \end{aligned}$$

$$A = -\frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{\sin(\pi + \frac{\pi}{7})}{8 \sin \frac{\pi}{7}} = \frac{1}{8}.$$

Javob: $\frac{1}{8}$ (D).

4. (98-1-54) Agar $\operatorname{tg} \alpha = -\frac{1}{4}$ bo'lsa,

$$\frac{2 \cos^2 \alpha - \sin 2\alpha}{2 \sin^2 \alpha - \sin 2\alpha}$$

ni hisoblang.

$$\text{A) } -4 \quad \text{B) } 4 \quad \text{C) } \frac{1}{4} \quad \text{D) } -\frac{1}{2}$$

5. (98-10-101) Agar $\operatorname{tg} \alpha = \frac{1}{2}$ bo'lsa, $\operatorname{tg} 2\alpha$ ni toping.
A) $\frac{5}{3}$ B) $\frac{4}{3}$ C) $\frac{3}{4}$ D) $\frac{3}{5}$

6. (98-8-54) Agar $\operatorname{ctg} \alpha = \frac{1}{8}$ bo'lsa,

$$\frac{\sin 2\alpha + 2 \sin^2 \alpha}{\sin 2\alpha + 2 \cos^2 \alpha}$$

ni hisoblang.

$$\text{A) } \frac{1}{8} \quad \text{B) } 8 \quad \text{C) } \frac{1}{4} \quad \text{D) } 4$$

7. (96-10-35) Agar $\cos \alpha = \frac{1}{5}$ bo'lsa,

$$\frac{2 \sin \alpha + \sin 2\alpha}{2 \sin \alpha - \sin 2\alpha}$$

ni hisoblang.

$$\text{A) } 0,5 \quad \text{B) } 1,5 \quad \text{C) } 3 \quad \text{D) } \frac{3}{2}$$

8. (98-11-17) $\operatorname{tg} 22,5^\circ + \operatorname{tg}^{-1} 22,5^\circ$ ni hisoblang.

$$\text{A) } \sqrt{2} \quad \text{B) } \sqrt{2-1} \quad \text{C) } 4\sqrt{2} \quad \text{D) } 2\sqrt{2}$$

Yechish: 8-formuladan foydalansak

$$\operatorname{tg} 22,5^\circ + \operatorname{tg}^{-1} 22,5^\circ = \frac{1 - \cos 45^\circ}{\sin 45^\circ} + \frac{\sin 45^\circ}{1 - \cos 45^\circ}.$$

ni olamiz. Endi $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$ ni yuqori-dagi ifodaga qo'yib, uni soddalashtirsak $2\sqrt{2}$ ni olamiz. **Javob:** $2\sqrt{2}$ (D).

9. (98-10-32) $\operatorname{tg} 15^\circ - \operatorname{ctg} 15^\circ$ ni hisoblang.

$$\text{A) } 2\sqrt{3} \quad \text{B) } -2\sqrt{3} \quad \text{C) } -\frac{2\sqrt{3}}{3} \quad \text{D) } \frac{2\sqrt{3}}{3}$$

10. (98-4-29) Hisoblang.

$$\cos 92^\circ \cdot \cos 2^\circ + 0,5 \cdot \sin 4^\circ + 1$$

$$\text{A) } \frac{1}{2} \quad \text{B) } 1 \quad \text{C) } 0 \quad \text{D) } 2$$

11. (99-6-12) Hisoblang.

$$\frac{2 \operatorname{tg} 240^\circ}{1 - \operatorname{tg}^2 240^\circ}$$

$$\text{A) } -\sqrt{3} \quad \text{B) } \sqrt{3} \quad \text{C) } \frac{\sqrt{3}}{3} \quad \text{D) } \frac{2}{\sqrt{3}}$$

12. (00-10-13) Hisoblang.

$$\cos \frac{\pi}{5} \cdot \cos \frac{2\pi}{5}$$

$$\text{A) } \frac{1}{2} \quad \text{B) } \frac{1}{8} \quad \text{C) } \frac{1}{4} \quad \text{D) } \frac{1}{12}$$

13. (96-9-47) Soddalashtiring.

$$\frac{1 + \sin 2\alpha}{\sin \alpha + \cos \alpha} - \sin \alpha$$

A) $\cos \alpha$ B) $\sin \alpha$ C) $-\cos \alpha$ D) $-2 \sin \alpha$

Yechish: Asosiy trigonometrik ayniyat va 1-formuladan foydalanib berilgan kasr suratini quyidagicha yozib olamiz:

$$1 + \sin 2\alpha = 1 + 2 \sin \alpha \cos \alpha = (\sin \alpha + \cos \alpha)^2.$$

Endi soddalashtiramiz

$$\frac{(\sin \alpha + \cos \alpha)^2}{\sin \alpha + \cos \alpha} - \sin \alpha = \sin \alpha + \cos \alpha - \sin \alpha.$$

O'xshash hadlarni ixchamlab $\cos \alpha$ ga ega bo'lamiz.

Javob: $\cos \alpha$ (A).

14. (97-7-56) Soddalashtiring:

$$\frac{\sin(\pi - 2\alpha)}{1 - \sin(\frac{\pi}{2} - 2\alpha)}$$

A) $-\operatorname{tg} \alpha$ B) $2 \sin \alpha$ C) $\operatorname{ctg} \alpha$ D) $\operatorname{tg} \alpha$

15. (96-3-112) Soddalashtiring.

$$\frac{\sin 3\alpha}{\sin \alpha} - \frac{\cos 3\alpha}{\cos \alpha}$$

A) $2 \cos \alpha$ B) 2 C) $2 \sin \alpha$ D) 1

16. (96-12-85) Soddalashtiring.

$$\frac{2}{\operatorname{tg} \alpha + \operatorname{ctg} \alpha}$$

A) $\cos 2\alpha$ B) $\frac{1}{\cos 2\alpha}$ C) $\frac{1}{\sin 2\alpha}$ D) $\sin 2\alpha$

17. (96-13-38) Soddalashtiring.

$$\frac{2}{\operatorname{ctg} \alpha - \operatorname{tg} \alpha}$$

A) $\operatorname{ctg} 2\alpha$ B) $\sin 2\alpha$ C) $\operatorname{tg} 2\alpha$ D) $\cos 2\alpha$

18. (00-1-27) Soddalashtiring.

$$\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} + 1$$

A) $\cos^{-2} \alpha$ B) $\sin^{-2} \alpha$ C) $\sin^2 \alpha$ D) $\cos^2 \alpha$

19. (00-2-48) Soddalashtiring.

$$(\cos 3x + \cos x)^2 + (\sin 3x + \sin x)^2$$

A) $4 \cos^2 x$ B) $2 \cos^2 x$
C) $3 \sin^2 x$ D) $4 \sin^2 x$

20. (96-7-56) Soddalashtiring.

$$\frac{\sin 2\alpha + \cos(\pi - \alpha) \cdot \sin \alpha}{\sin(\frac{\pi}{2} - \alpha)}$$

A) $\cos \alpha$ B) $\sin \alpha$ C) $-2 \sin \alpha$ D) $-\cos \alpha$

21. (97-3-56) Soddalashtiring.

$$\frac{\cos 2\alpha + \cos(\frac{\pi}{2} - \alpha) \cdot \sin \alpha}{\sin(\frac{\pi}{2} + \alpha)}$$

A) $\cos \alpha$ B) $2 \sin \alpha$ C) $-\cos \alpha$ D) $\operatorname{tg} \alpha$

22. (97-10-56) Soddalashtiring.

$$\frac{\sin(2\alpha - \pi)}{1 - \sin(\frac{3\pi}{2} + 2\alpha)}$$

A) $\operatorname{tg} \alpha$ B) $-\operatorname{tg} \alpha$ C) $-2 \operatorname{ctg} \alpha$ D) $\sin \alpha$

23. (99-6-23) Soddalashtiring.

$$1 + \frac{\operatorname{tg}^2(-\alpha) - 1}{\sin(0,5\pi + 2\alpha)}$$

A) $-\operatorname{tg}^2 \alpha$ B) $\operatorname{tg}^2 \alpha$ C) $\operatorname{ctg}^2 \alpha$ D) $-\operatorname{ctg}^2 \alpha$

24. (98-8-57) Hisoblang.

$$\sin^4\left(\frac{23\pi}{12}\right) - \cos^4\left(\frac{13\pi}{12}\right)$$

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{\sqrt{2}}{2}$

Yechish: Argumentlarni $\frac{23\pi}{12} = 2\pi - \frac{\pi}{12}$ va $\frac{13\pi}{12} = \pi + \frac{\pi}{12}$ shaklda yozib, keyin keltirish formulalarini qo'llab, berilgan ifodani

$$\sin^4\left(\frac{\pi}{12}\right) - \cos^4\left(\frac{\pi}{12}\right) = \left(\sin^2\left(\frac{\pi}{12}\right)\right)^2 - \left(\cos^2\left(\frac{\pi}{12}\right)\right)^2$$

shaklda yozib olamiz. Bu ifodaga ikki son kvadratlari ayirmasi formulasini, keyin asosiy trigonometrik ayniyatni qo'llab

$$\sin^2 \frac{\pi}{12} - \cos^2 \frac{\pi}{12} = -(\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}) = -\cos \frac{2\pi}{12}$$

ni olamiz. $\cos \frac{2\pi}{12} = \cos \frac{\pi}{6}$ ning qiymatini 13.1-

jadvaldan qarab, berilgan ifodaning qiymati $-\frac{\sqrt{3}}{2}$

ekanligini olamiz. **Javob:** $-\frac{\sqrt{3}}{2}$ (C).

25. (98-12-90) Hisoblang.

$$\frac{\sqrt{3}}{\sin 100^\circ} + \frac{1}{\cos 260^\circ}$$

A) 2 B) -4 C) -3 D) -1

26. (01-11-18) Hisoblang.

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$$

A) $3,5$ B) $2,5$ C) 3 D) 4

27. (02-7-11) Hisoblang.

$$\sin^4 105^\circ \cdot \cos^4 75^\circ$$

A) $\frac{1}{256}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{1}{128}$ D) $\frac{\sqrt{6}}{4}$

28. (00-8-41) Hisoblang.

$$\log_2 \cos 20^\circ + \log_2 \cos 40^\circ + \log_2 \cos 60^\circ + \log_2 \cos 80^\circ$$

A) -4 B) -3 C) $\frac{1}{2}$ D) 1

29. (03-3-39) Hisoblang.

$$\operatorname{ctg} 35^{\circ} - \operatorname{tg} 35^{\circ} - 2\operatorname{tg} 20^{\circ}$$

- A) $\frac{1}{2}$ B) 0 C) 1 D) $\frac{\sqrt{3}}{2}$

30. (03-8-53) Hisoblang.

$$\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} \cdot \operatorname{tg} \frac{\pi}{8} \cdot \operatorname{ctg} \frac{9\pi}{8}$$

- A) $\frac{1}{2\sqrt{2}}$ B) $\sqrt{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

31. (00-8-46) Hisoblang.

$$\cos 50^{\circ} \cdot \cos 40^{\circ} - 2 \cos 20^{\circ} \cdot \sin 50^{\circ} \cdot \sin 20^{\circ}$$

- A) 0 B) 1 C) -1 D) $\cos 20^{\circ}$

32. (01-3-1) Ifodaning qiymatini hisoblang.

$$\sin 50^{\circ} + \sin 40^{\circ} \cdot \operatorname{tg} 20^{\circ}$$

- A) $\sin^2 20^{\circ}$ B) 0,5 C) 1 D) $\cos^2 20^{\circ}$

33. (01-1-43) Agar $\operatorname{tg} \alpha = -\frac{4}{3}$ bo'lsa, $\sin 2\alpha$ ning qiymatini toping.

- A) 0,96 B) -0,96 C) 0,25 D) -0,5

Yechish: 3-formuladan foydalansak

$$\sin 2\alpha = \frac{2\operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha} = \frac{2 \cdot (-\frac{3}{4})}{1 + (-\frac{3}{4})^2} = -\frac{3}{2} : \frac{25}{16}.$$

Uni soddalashtirib $\sin 2\alpha = -0,96$ ni olamiz. **Javob:** -0,96 (B).

34. (99-9-31) Agar $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = 4$ bo'lsa, $\sin 2\alpha$ ni hisoblang.

- A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$

35. (01-7-38) Agar $\operatorname{tg} \alpha = 2 - \sqrt{3}$ bo'lsa, α o'tkir burchakning qiymatini toping.

- A) $\frac{\pi}{8}$ B) $\frac{\pi}{12}$ C) $\frac{5}{12}\pi$ D) $\frac{3}{8}\pi$

36. (03-2-26) Agar $\operatorname{ctg} \alpha = \sqrt{2} - 1$ bo'lsa, $\cos 2\alpha$ ning qiymatini toping.

- A) $\sqrt{2}$ B) $\frac{\sqrt{2}+1}{2}$ C) $-\frac{1}{\sqrt{2}}$ D) $-\frac{1}{2}$

Yechish: 4-ayniyatdan va $\operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}$ dan foydalansak

$$\cos 2\alpha = \frac{1 - \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha} = \frac{\operatorname{ctg}^2 \alpha - 1}{\operatorname{ctg}^2 \alpha + 1}.$$

Endi $\operatorname{ctg} \alpha = \sqrt{2} - 1$ ning qiymatini oxirgi ifodaning o'ng tomoniga qo'ysak

$$\cos 2\alpha = \frac{(\sqrt{2}-1)^2 - 1}{(\sqrt{2}-1)^2 + 1} = \frac{1 - \sqrt{2}}{2 - \sqrt{2}} =$$

$$\frac{1 - \sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{-\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}.$$

Javob: $-\frac{1}{\sqrt{2}}$ (C).

37. (02-10-59) Agar $90^{\circ} < \alpha < 180^{\circ}$ va $\sin \alpha = \frac{1}{3}$ bo'lsa, $\operatorname{tg} 2\alpha$ ni hisoblang.

- A) $-\frac{4\sqrt{2}}{7}$ B) $-\frac{4\sqrt{3}}{7}$ C) $\frac{2}{3}$ D) $-\frac{\sqrt{2}}{4}$

38. (03-6-26) Agar $\sin 37^{\circ} = a$ bo'lsa, $\sin 16^{\circ}$ ni a orqali ifodalang.

- A) $a^2 - 1$ B) $a - 1$ C) $2a^2 - 1$ D) $1 - 2a^2$

39. (03-9-31) Agar $\operatorname{tg} \frac{\alpha}{2} = -2$ bo'lsa, $\sin \alpha + 2 \cos \alpha$ ning qiymatini hisoblang.

- A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) -2 D) $\frac{4}{5}$

40. (03-10-40) Agar $\operatorname{tg} \alpha = \frac{1}{2}$ bo'lsa

$$\sin\left(2\alpha + \frac{\pi}{4}\right)$$

ning qiymatini toping.

- A) $\frac{\sqrt{2}}{5}$ B) $\frac{2\sqrt{2}}{3}$ C) $\frac{2\sqrt{2}}{5}$ D) $\frac{7\sqrt{2}}{10}$

41. (03-11-22) α o'tkir burchak va

$$\sin^4 \alpha \cdot \cos^4 \alpha = \frac{1}{64}$$

bo'lsa, α quyidagilarning qaysi biriga teng?

- A) $\frac{\pi}{8}; \frac{3\pi}{8}$ B) $\frac{\pi}{8}; \frac{\pi}{4}$ C) $\frac{\pi}{16}$ D) $\frac{\pi}{6}; \frac{3\pi}{8}$

42. (98-10-100) $\sin 105^{\circ} + \sin 75^{\circ}$ ni hisoblang.

- A) $\frac{\sqrt{2} + \sqrt{3}}{2}$ B) $\frac{\sqrt{2} - \sqrt{3}}{2}$
C) $\sqrt{\sqrt{3} - \sqrt{2}}$ D) $\sqrt{2 + \sqrt{3}}$

Yechish: $105^{\circ} = 90^{\circ} + 15^{\circ}$, $75^{\circ} = 90^{\circ} - 15^{\circ}$ tengliklardan va keltirish formulasidan

$$\sin 105^{\circ} + \sin 75^{\circ} = \cos 15^{\circ} + \cos 15^{\circ} = 2 \cos 15^{\circ}.$$

Endi 7-ayniyat va 15° burchakni birinchi chorakda yotishini hisobga olsak

$$2 \cos 15^{\circ} = 2 \sqrt{\frac{1 + \cos 30^{\circ}}{2}} = \sqrt{2 + \sqrt{3}}$$

ni olamiz. **Javob:** $\sqrt{2 + \sqrt{3}}$ (D).

43. (96-7-55) $\sin \frac{\pi}{12}$ ni hisoblang.

- A) $\sqrt{2 - \sqrt{3}}$ B) $\frac{\sqrt{2 + \sqrt{3}}}{2}$
C) $\frac{\sqrt{2 - \sqrt{3}}}{2}$ D) $\frac{\sqrt{2 - \sqrt{2}}}{2}$

44. (97-7-55) $\cos \frac{5\pi}{12}$ ni hisoblang.

- A) $\frac{\sqrt{2 + \sqrt{3}}}{3}$ B) $\frac{\sqrt{3}}{4}$
C) $\frac{\sqrt{2 - \sqrt{2}}}{2}$ D) $\frac{\sqrt{2 - \sqrt{3}}}{2}$

45. (00-3-50) $\sin 112, 5^0$ ni hisoblang.

- A) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ B) $\frac{1}{2}\sqrt{1+\sqrt{2}}$
 C) $\frac{1}{2}\sqrt{2+\sqrt{2}}$ D) $\frac{1}{2}\sqrt{\sqrt{2}-1}$

46. (01-2-85) $\cos 2227^0 30'$ ni hisoblang.

- A) $\frac{\sqrt{2+\sqrt{2}}}{2}$ B) $\frac{\sqrt{2-\sqrt{2}}}{2}$
 C) $\frac{\sqrt{2-\sqrt{2}}}{2}$ D) $\frac{\sqrt{2+\sqrt{2}}}{4}$

47. (02-3-73) Hisoblang.

$$8 \sin^2 \frac{7\pi}{8} \cdot \cos^2 \frac{9\pi}{8}$$

- A) 0 B) $\frac{\sqrt{2}}{2}$ C) 1 D) $\frac{1}{2}$

Yechish: $\frac{7\pi}{8} = \pi - \frac{\pi}{8}$, $\frac{9\pi}{8} = \pi + \frac{\pi}{8}$ tengliklardan va keltirish formulasidan

$$8 \sin^2 \frac{7\pi}{8} \cdot \cos^2 \frac{9\pi}{8} = 8 \sin^2 \frac{\pi}{8} \cdot \cos^2 \frac{\pi}{8}$$

ni olamiz. Endi $2 \sin x \cos x = \sin 2x$ ayniyatdan

$$8 \sin^2 \frac{\pi}{8} \cdot \cos^2 \frac{\pi}{8} = 2 \left(\sin \frac{\pi}{4} \right)^2 = 2 \cdot \frac{2}{4} = 1$$

ni olamiz. **Javob:** 1 (C).

48. (98-1-57) Hisoblang.

$$8 \sin^2 \frac{15\pi}{16} \cdot \cos^2 \frac{17\pi}{16} - 1$$

- A) $-\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$

49. (00-3-53) Qaysi α o'tkir burchak uchun

$$\cos \alpha = \frac{1}{2}\sqrt{2+\sqrt{3}}$$

tenglik to'g'ri?

- A) $7, 5^0$ B) $22, 5^0$ C) 75^0 D) 15^0

Yechish: Kvadrat ildiz xossalaridan

$$\cos \alpha = \sqrt{\frac{2+\sqrt{3}}{4}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{1+\cos 30^0}{2}}$$

dan $\alpha = 15^0$ ni olamiz. **Javob:** 15^0 (D).

50. (97-5-28) $8 \cos 30^0 + \operatorname{tg}^2 15^0$ ni hisoblang.

- A) 5 B) 6 C) 7 D) 8

51. (01-3-3) Hisoblang.

$$\sin^4 15^0 + \cos^4 15^0$$

- A) $\frac{5}{6}$ B) $\frac{2}{3}$ C) $\frac{7}{8}$ D) $\frac{5}{7}$

52. (02-9-39) Hisoblang.

$$\frac{2 \sin^2 70^0 - 1}{2 \operatorname{ctg} 115^0 \cdot \cos^2 155^0}$$

- A) -1 B) 1 C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

53. (97-9-28) $4 \operatorname{ctg} 30^0 + \operatorname{tg}^2 15^0$ ni hisoblang.

- A) 5 B) 7 C) 9 D) 8

54. (97-6-44) Agar

$$\cos \alpha = \frac{1}{2}, \quad \frac{3\pi}{2} < \alpha < 2\pi$$

bo'lsa, $\sin(\pi - \frac{\alpha}{2})$ ni toping.

- A) $-\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$

Yechish: Keltirish formulasiga ko'ra $\sin(\pi - \frac{\alpha}{2}) = \sin \frac{\alpha}{2}$. 6-ayniyat va $\frac{\alpha}{2}$ burchak ikkinchi chorakda yotganligi sababli

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{\frac{1 - \frac{1}{2}}{2}} = \frac{1}{2}.$$

Javob: $\frac{1}{2}$ (D).

55. (96-1-55) Agar $\cos 2\alpha = \frac{1}{2}$ bo'lsa, $\cos^2 \alpha$ ni hisoblang.

- A) $\frac{1}{4}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{3}{4}$ D) $\frac{3}{8}$

56. (97-1-45) Agar

$$\cos \alpha = -\frac{1}{2}, \quad \pi < \alpha < \frac{3\pi}{2}$$

bo'lsa, $\sin(\frac{\pi}{2} + \frac{\alpha}{2})$ ni toping.

- A) $\frac{1}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$

57. (98-11-20) Agar

$$\cos \alpha = \frac{7}{18}, \quad 0 < \alpha < \frac{\pi}{2}$$

bo'lsa, $6 \cos \frac{\alpha}{2}$ ni toping.

- A) 3 B) 5 C) 6 D) 4

Yechish: 4-ayniyatga asosan va $\frac{\alpha}{2}$ birinchi chorakda joylashganligi sababli

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}}.$$

Bundan

$$6 \cos \frac{\alpha}{2} = 6 \sqrt{\frac{1 + \frac{7}{18}}{2}} = 6 \sqrt{\frac{25}{36}} = 5.$$

Javob: 5 (B).

58. (01-1-68) Agar

$$\sin \alpha = -0,8, \quad \alpha \in (\pi; \frac{3\pi}{2})$$

bo'lsa, $\operatorname{tg} \frac{\alpha}{2}$ ni hisoblang.

- A) 1 B) -1 C) 2 D) -2

59. (02-3-74) Agar $\cos(\pi - 4\alpha) = -\frac{1}{3}$ bo'lsa,

$\cos^4\left(\frac{3\pi}{2} - 2\alpha\right)$ ni hisoblang.

- A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) $\frac{8}{9}$

60. (02-7-16) Soddashtiring.

$$\frac{2 \cos^2(45^\circ - \frac{\alpha}{2})}{\cos \alpha}$$

- A) $\operatorname{ctg}(45^\circ - \frac{\alpha}{2})$ B) $\sin \frac{\alpha}{2}$
 C) $2 \sin(45^\circ - \frac{\alpha}{2})$ D) $\cos \frac{\alpha}{2}$

61. (02-11-42) Agar

$$\operatorname{ctg} \alpha = \frac{5}{12}, \quad \alpha \in (540^\circ; 630^\circ)$$

bo'lsa, $\sin \frac{\alpha}{2}$ ning qiymatini hisoblang.

- A) $\frac{3}{4}$ B) $-\frac{3}{4}$ C) $-\frac{1}{2}$ D) $-\frac{3}{\sqrt{13}}$

62. (03-5-40) Agar $\sin \alpha \left(1 - 2 \sin^2 \frac{\alpha}{2}\right) = \frac{1}{3}$ bo'lsa,

$\cos\left(\frac{\pi}{4} - \alpha\right) \cdot \sin\left(\frac{3\pi}{4} - \alpha\right)$ ni hisoblang.

- A) $\frac{5}{6}$ B) $\frac{3}{4}$ C) $\frac{4}{5}$ D) $\frac{\sqrt{3}}{4}$

63. (03-7-35) Agar

$$\cos 15^\circ + \sin 15^\circ = \frac{a}{4 \cos 15^\circ}$$

bo'lsa a ning qiymatini toping.

- A) $\sqrt{3}$ B) $\sqrt{3} + 1$ C) $\sqrt{3} + 2$ D) $\sqrt{3} + 3$

64. (02-8-40) Soddashtiring.

$$\cos(\pi + 2\alpha) + \sin(\pi + 2\alpha) \cdot \operatorname{tg}\left(\frac{\pi}{2} + \alpha\right)$$

- A) 1 B) 2 C) $\sin \alpha$ D) $\cos \alpha$

Yechish: Keltirish formulalaridan, berilgan ifodaning $-\cos 2\alpha + \sin 2\alpha \cdot \operatorname{ctg} \alpha$ ga tengligini olamiz. Bu ifodani $\sin 2x = 2 \sin x \cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ ayniyatlardan foydalanib soddashtiramiz:

$$\begin{aligned} & -\cos^2 \alpha + \sin^2 \alpha + 2 \sin \alpha \cos \alpha \cdot \frac{\cos \alpha}{\sin \alpha} = \\ & = -\cos^2 \alpha + \sin^2 \alpha + 2 \cos^2 \alpha = \sin^2 \alpha + \cos^2 \alpha = 1. \end{aligned}$$

Javob: 1 (A).

65. (01-9-21) Soddashtiring.

$$\sin^2 \alpha \operatorname{tg} \alpha + \cos^2 \alpha \operatorname{ctg} \alpha + \sin 2\alpha$$

- A) $\frac{2}{\sin 2\alpha}$ B) $\frac{2}{\sin \alpha \cos \alpha}$ C) 1 D) $\sin^2 \alpha$

66. (03-4-24) Soddashtiring.

$$\frac{1 - \cos 2\alpha}{1 + \operatorname{tg}^2 \alpha}$$

- A) $\sin^2 2\alpha$ B) $\frac{1}{2} \sin^2 2\alpha$ C) $\cos^2 2\alpha$ D) $\frac{1}{2} \cos^2 2\alpha$

67. (02-11-41) Soddashtiring.

$$\frac{1 + \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}{1 - \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}}$$

- A) $\operatorname{tg} \frac{\alpha}{4}$ B) $\cos \frac{\alpha}{2}$ C) $-\operatorname{ctg} \frac{\alpha}{4}$ D) $\sin \frac{\alpha}{4}$

68. (02-12-38) Soddashtiring.

$$\frac{\operatorname{tg} \alpha + \sin \alpha}{2 \cos^2 \frac{\alpha}{2}}$$

- A) $\operatorname{ctg} \alpha$ B) $\operatorname{tg} \alpha$ C) $\operatorname{tg} \frac{\alpha}{2}$ D) $\operatorname{ctg} \frac{\alpha}{2}$

69. (99-8-76) Soddashtiring.

$$\frac{\sin^2 2,5\alpha - \sin^2 1,5\alpha}{\sin 4\alpha \cdot \sin \alpha + \cos 3\alpha \cdot \cos 2\alpha}$$

- A) $2 \operatorname{tg} 2\alpha$ B) $\operatorname{tg} 2\alpha \cdot \operatorname{tg} \alpha$ C) $2 \sin 2\alpha$ D) $4 \sin^2 \alpha$

13.2.5 Yig'indi va ayirma uchun formulalar

1. $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$.

2. $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$.

3. $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$.

4. $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$.

1, 2 va 3-formulalarda x, y o'zgaruvchilardan $\alpha = \frac{x+y}{2}$, $\beta = \frac{x-y}{2}$ o'zgaruvchilarga o'tib ko'paytma uchun 5-7 formulalarga ega bo'lamiz:

5. $\sin \alpha \cdot \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$.

6. $\cos \alpha \cdot \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$.

7. $\sin \alpha \cdot \cos \beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(\alpha + \beta))$.

1. (98-11-103) $\sin 75^\circ - \sin 15^\circ$ ni hisoblang.

- A) $\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{2}$ D) $-\sqrt{2}$

Yechish: 4-ayniyatga asosan

$$\sin 75^\circ - \sin 15^\circ = 2 \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}.$$

Javob: $\frac{\sqrt{2}}{2}$ (A).

2. (00-1-28) Hisoblang.

$$\frac{\sin 35^\circ + \cos 65^\circ}{2 \cos 5^\circ}$$

- A) 0,25 B) 0,75 C) 0,5 D) 0,6

3. (00-8-59) Hisoblang.

$$\sin 10^0 + \sin 50^0 - \cos 20^0$$

- A) 0 B) -1 C) 1 D)
- $\cos 20^0$

4. (99-5-54) Hisoblang.

$$\sqrt[3]{8 + \left(\cos \frac{\pi}{5} + \cos \frac{2\pi}{5} + \cos \frac{3\pi}{5} + \cos \frac{4\pi}{5}\right)^3}$$

- A) 1 B) 2 C) 3 D) 4

5. (96-6-35) Soddalashtiring.

$$\frac{\cos \alpha - \cos 3\alpha}{\sin \alpha}$$

- A)
- $-2 \cos 2\alpha$
- B)
- $2 \cos 2\alpha$
- C)
- $\sin 2\alpha$
- D)
- $2 \sin 2\alpha$

Yechish: 1-ayniyat va sinusning toq funksiya ekanligidan foydalanib ifodani quyidagicha yozamiz

$$\frac{-2 \sin \frac{\alpha + 3\alpha}{2} \sin \frac{\alpha - 3\alpha}{2}}{\sin \alpha} = \frac{-2 \sin 2\alpha (-\sin \alpha)}{\sin \alpha}.$$

Bu yerdan esa ifodaning qiymati $2 \sin 2\alpha$ ekanligiga kelamiz. **Javob:** $2 \sin 2\alpha$ (D).

6. (97-12-34) Soddalashtiring.

$$\frac{\cos 6\alpha - \cos 4\alpha}{\sin 5\alpha}$$

- A)
- $2 \sin \alpha$
- B)
- $2 \cos \alpha$
- C)
- $-2 \cos \alpha$
- D)
- $-2 \sin \alpha$

7. (98-10-35) Soddalashtiring.

$$\frac{\sin 4\alpha - \sin 6\alpha}{\cos 5\alpha}$$

- A)
- $\sin 2\alpha$
- B)
- $2 \sin \alpha$
- C)
- $-2 \cos \alpha$
- D)
- $-2 \sin \alpha$

8. (98-8-58) Soddalashtiring.

$$\frac{1 - \sin \alpha - \cos 2\alpha + \sin 3\alpha}{\sin 2\alpha + 2 \cos \alpha \cdot \cos 2\alpha}$$

- A)
- $2 \operatorname{ctg} \alpha$
- B)
- $\operatorname{tg} \alpha$
- C)
- $2 \sin \alpha$
- D)
- $\operatorname{ctg} \alpha$

9. (01-7-40) Soddalashtiring.

$$\frac{\sin \alpha + \sin 2\alpha - \sin(\pi + 3\alpha)}{2 \cos \alpha + 1}$$

- A)
- $\sin \alpha$
- B)
- $\cos \alpha$
- C)
- $\sin 2\alpha$
- D)
- $\cos 2\alpha$

10. (00-8-48) Hisoblang.

$$\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}$$

- A)
- $-\frac{1}{2}$
- B)
- $\frac{1}{4}$
- C)
- $\frac{1}{3}$
- D)
- $\frac{\sqrt{2}}{3}$

Yechish: Berilgan ifodani A bilan belgilaymiz

$$A = \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7}.$$

Bu tenglikni $2 \sin \frac{\pi}{7}$ ga ko'paytirib, har bir qo'shiluvchiga 7-formulani qo'llab

$$\begin{aligned} 2A \sin \frac{\pi}{7} &= 2 \sin \frac{\pi}{7} \cos \frac{2\pi}{7} + 2 \sin \frac{\pi}{7} \cos \frac{4\pi}{7} + \\ &+ 2 \sin \frac{\pi}{7} \cos \frac{6\pi}{7} = -\sin \frac{\pi}{7} + \sin \frac{3\pi}{7} - \sin \frac{3\pi}{7} + \\ &+ \sin \frac{5\pi}{7} - \sin \frac{5\pi}{7} + \sin \frac{7\pi}{7} = -\sin \frac{\pi}{7} \end{aligned}$$

ni olamiz. U holda $A = -\frac{1}{2}$. **Javob:** $-\frac{1}{2}$ (A).

11. (96-3-57) Hisoblang.

$$\sin 20^0 \cdot \sin 40^0 \cdot \sin 80^0$$

- A)
- $\frac{1}{2}$
- B)
- $\frac{1}{3}$
- C)
- $\frac{1}{4}$
- D)
- $\frac{\sqrt{3}}{8}$

12. (01-1-45)
- $5^0, 10^0, 15^0, \dots$
- burchaklarning qiymatlari arifmetik progressiya tashkil qiladi. Shu progressiyaning birinchi hadidan boshlab eng kamida nechtasini olganda, ularning kosinuslari yig'indisi nolga teng bo'ladi?

- A) 18 B) 17 C) 19 D) 35

13. (03-9-30) Hisoblang.

$$\cos 55^0 \cdot \cos 65^0 \cdot \cos 175^0$$

- A)
- $-\frac{1}{8}$
- B)
- $-\frac{\sqrt{3}}{8}$
- C)
- $\frac{\sqrt{3}}{8}$
- D)
- $-\frac{1}{8}\sqrt{2 + \sqrt{3}}$

13.2.6 Qiymatlar sohasi va monotonligi

1. $y = \sin x$ va $y = \cos x$ funksiyalarning qiymatlar sohasi $[-1; 1]$ kesmadan iborat.

2. $y = \operatorname{tg} x$ va $y = \operatorname{ctg} x$ funksiyalarning qiymatlar sohasi $(-\infty; \infty)$ oraliqdan iborat.

3. $y = a \sin x + c$ va $y = a \cos x + c$ funksiyalarning qiymatlar sohasi $[c - |a|; c + |a|]$ kesmadan iborat.

4. $y = a \sin x + b \cos x + c$ funksiyaning qiymatlar sohasi $[c - \sqrt{a^2 + b^2}; c + \sqrt{a^2 + b^2}]$ kesmadan iborat.

5. $y = a \sin^2 x + b \cos^2 x$ ($a < b$) funksiyaning qiymatlar sohasi $[a; b]$ kesmadan iborat.

6. $y = \sin x$ funksiya $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ kesmada o'suvchi.

7. $y = \cos x$ funksiya $[0; \pi]$ kesmada kamayuvchi.

8. $y = \operatorname{tg} x$ funksiya $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ oraliqda o'suvchi.

9. $y = \operatorname{ctg} x$ funksiya $(0; \pi)$ oraliqda kamayuvchi.

1. $y = 3 \sin x$ funksiyaning qiymatlar sohasini toping.

A) $[0; 3]$ B) $(-3; 3)$ C) $[-3; 0]$ D) $[-3; 3]$

Yechish: 3-xossaga ko'ra $y = 3 \sin x$ ($a = 3$, $c = 0$) funksiyaning qiymatlar to'plami $[-3; 3]$ kesmadan iborat. **Javob:** $[-3; 3]$ (D).

2. $y = 2 \cos x$ funksiyaning qiymatlar sohasini toping.

A) $[0; 2]$ B) $(-2; 2)$ C) $[-2; 2]$ D) $[-2; 0]$

3. $y = 2 + 5 \operatorname{tg} 3x$ funksiyaning qiymatlar sohasini toping.

A) $(-\infty; 2]$ B) $(-\infty; \infty)$
C) $[2; \infty)$ D) $[-2; 2]$

4. $y = 5 - 7 \operatorname{ctg} (3x + 2)$ funksiyaning qiymatlar sohasini toping.

A) $(-\infty; 5]$ B) $(-\infty; \infty)$
C) $[2; \infty)$ D) $[12; \infty)$

5. $y = 2 \cos x - 3$ funksiyaning qiymatlar sohasini toping.

A) $[-5; 2]$ B) $[-5; 1)$ C) $[-5; -1]$ D) $[-3; 2]$

6. (01-10-51) Funksiyaning qiymatlar sohasini toping.

$$y = (\sin x + \cos x)^2 - \frac{1 - \cos 4x}{2 \sin 2x} - \cos x$$

A) $[0; 2]$ B) $(0; 2)$
C) $(0; 1) \cup (1; 2)$ D) $[0; 1) \cup (1; 2]$.

Yechish: Trigonometriyaning asosiy ayniyati $\sin^2 x + \cos^2 x = 1$ hamda $1 - \cos 4x = 2 \sin^2 2x$ ayniyatdan foydalanib berilgan funksiyani quyidagicha yozamiz:

$$y = (\sin x + \cos x)^2 - \frac{1 - \cos 4x}{2 \sin 2x} - \cos x = 1 - \cos x.$$

3-xossadan $y = 1 - \cos x$ ($a = -1$, $c = 1$) funksiyaning qiymatlar to'plami $[0; 2]$ kesmadan iborat ekanligi kelib chiqadi. Ammo $\sin 2x \neq 0 \iff x \neq \frac{\pi}{2}k$, $k \in Z$ shartni e'tiborga olsak, berilgan funksiyaning qiymatlar sohasi $(0; 1) \cup (1; 2)$ to'plamdan iborat ekanligiga kelamiz.

Javob: $(0; 1) \cup (1; 2)$ (C).

7. (01-11-23) Ushbu $f(x) = 2 \cos \frac{x}{2} + 3$ funksiyaning qiymatlar sohasini toping.

A) $[3; 5]$ B) $[4; 5]$ C) $[2; 5]$ D) $[1; 5]$

8. (02-2-60) $y = \cos^4 x - 2 \sin^2 x + 7$ funksiyaning eng kichik qiymatini toping.

A) 5 B) 3 C) 2 D) 1

9. (02-12-53) Funksiyaning qiymatlar sohasini toping.

$$f(x) = 3^{\log_2(3 \sin^2 x + 1)}$$

A) $[1; 9]$ B) $[0; 9]$ C) $[0; 9)$ D) $(1; 9)$

10. (00-1-25) Sonlarning eng kattasini toping.

A) $\sin 1$ B) $\cos(\frac{\pi}{2} - \frac{1}{2})$ C) $\sin 4$ D) 1

Yechish: $\cos(\frac{\pi}{2} - \frac{1}{2}) = \sin \frac{1}{2}$ va $\sin 4 = \sin(\pi - (\pi - 4)) = \sin(\pi - 4)$. Endi $-\frac{\pi}{2} < \pi - 4 < \frac{1}{2} < 1 < \frac{\pi}{2}$ va sinus funksiyaning $[-\frac{\pi}{2}, \frac{\pi}{2}]$ kesmada o'suvchiligini hisobga olsak $\sin(\pi - 4) < \sin \frac{1}{2} < \sin 1 < \sin \frac{\pi}{2} = 1$. Demak, berilgan sonlar ichida eng kattasi 1 ekan. **Javob:** 1 (D).

11. (03-4-22) $\operatorname{tg} 240^0$, $\sin 120^0$, $\cos 150^0$ va $\operatorname{ctg} 225^0$ sonlardan eng kattasining eng kichigiga ko'paytmasini toping.

A) $-1, 4$ B) $-1, 5$ C) $-\frac{\sqrt{6}}{2}$ D) $1, 5$

12. (03-2-11) $y = \cos^2 x - \frac{\sqrt{3}}{2} \sin 2x$ funksiyaning eng katta va eng kichik qiymatlari yig'indisini toping.

A) 1,5 B) 0,5 C) 1 D) 2

13. (03-2-29) Funksiyaning qiymatlar sohasini toping.

$$y = (1 + \operatorname{tg}^2 x) \cos^2 x - \frac{\sin 2x}{2 \cos x}$$

A) $[0; 2]$ B) $(0; 2)$ C) $[-1; 1]$ D) $(-2; 0)$

14. (01-3-15) Ushbu

$$f(x) = \sin \frac{x}{2} \cdot \cos^3 \frac{x}{2} - \sin^3 \frac{x}{2} \cdot \cos \frac{x}{2}$$

funksiyaning eng katta qiymatini toping.

A) 1 B) $\frac{1}{2}$ C) 2 D) $\frac{1}{4}$

Yechish: Bu ifodani ko'paytuvchilarga ajratib

$$f(x) = \sin \frac{x}{2} \cdot \cos \frac{x}{2} (\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2})$$

ga kelamiz. 13.2.4-ning 1 va 2-formularidan foydalanib

$$f(x) = \frac{1}{2} \sin x \cdot \cos x = \frac{1}{4} \sin 2x$$

tenglikka kelamiz. $g(x) = \sin 2x$ funksiyaning eng katta qiymati 1 bo'lganligi sababli, $f(x) = \frac{1}{4} \sin 2x$ ning eng katta qiymati $\frac{1}{4}$ ga teng bo'ladi.

Javob: $\frac{1}{4}$ (D).

15. (03-7-36) $y = (\cos x + 5) \cdot (3 - \cos x)$ funksiyaning eng katta qiymatini toping.

A) 8 B) 12 C) 15 D) 16

16. (03-7-49) $y = \frac{3}{4} \cdot \cos^2(x - \frac{\pi}{4}) - 1$ funksiyaning qiymatlar sohasini toping.

A) $[-\frac{3}{4}; \frac{3}{4}]$ B) $[-1; 0]$
C) $[-1; -0, 25]$ D) $[-0, 25; 0]$

17. (96-10-15) $y = 2\sin x + \cos x$ funksiyaning eng katta qiymatini toping.

A) 3 B) $\sqrt{5}$ C) 2 D) -1

Yechish: 4-qoidaga ko'ra $y = 2\sin x + \cos x$ funksiyaning qiymatlar sohasi $[-\sqrt{5}; \sqrt{5}]$ kesmadan iborat bo'ladi. Shuning uchun uning eng katta qiymati $\sqrt{5}$ dir. **Javob:** $\sqrt{5}$ (B).

18. (00-1-30) Agar α - o'zgaruvchi miqdor bo'lsa; $4(\sqrt{3}\cos\alpha + \sin\alpha)$ ning eng katta qiymati qanchaga teng bo'ladi?

A) 9,5 B) 7 C) 8 D) 6,5

19. (00-3-57) $y = 6\sin 2x + 8\cos 2x$ funksiyaning qiymatlar to'plamini toping.

A) $[-10; 10]$ B) $[-14; 14]$
C) $(-\infty; \infty)$ D) $[0; 6]$

20. (01-6-43) Ushbu $f(x) = \sin x + \cos x$ funksiyaning eng katta qiymatini toping.

A) 1,4 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 1,6

21. (01-7-45) Ushbu $f(x) = (\sin x + \cos x)^2$ funksiyaning qiymatlar sohasini toping.

A) $[-1; 1]$ B) $[-2; 2]$ C) $[0; 2]$ D) $[-\frac{1}{2}; \frac{1}{2}]$

22. (02-1-18) $y = (2, (1)+1, (8)) \sin x + (1, (2)+1, (7)) \cos x$ funksiyaning qiymatlar to'plamini toping.

A) $[-5; 5]$ B) $[-4; 4]$ C) $[-3; 3]$ D) $(-4; 4)$

23. (02-4-34) $y = 3\sin x - 4\cos x$ funksiyaning eng katta qiymatini toping.

A) 3 B) 4 C) 5 D) 6

24. (03-12-23) $y = 1 - 6\sin 2x + 8\cos 2x$ funksiyaning eng katta qiymatini toping.

A) 15 B) 14 C) 13 D) 11

25. (03-12-27) Funksiyaning eng katta qiymatini toping.

$$y = \frac{8\sin x - 15\cos x + 3}{4}$$

A) 6,5 B) 7,5 C) 5 D) 6

26. (96-7-57) Ushbu

$$x = \cos \frac{11\pi}{12}, \quad y = \cos\left(-\frac{\pi}{3}\right), \quad z = \sin \frac{11\pi}{12}$$

sonlar uchun qo'yidagi munosabatlardan qaysi biri o'rinli?

A) $x < y < z$ B) $x < z < y$
C) $y < z < x$ D) $z < y < x$

Yechish: Kosinusning juft funksiya ekanligidan

$$y = \cos\left(-\frac{\pi}{3}\right) = \cos \frac{\pi}{3}$$

tenglikni, $\sin \alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$ ekanligidan esa

$$z = \sin \frac{11\pi}{12} = \cos\left(\frac{\pi}{2} - \frac{11\pi}{12}\right) = \cos\left(-\frac{5\pi}{12}\right) = \cos \frac{5\pi}{12}$$

ni hosil qilamiz. Endi $x = \cos \frac{11\pi}{12}$, $y = \cos \frac{\pi}{3}$, $z = \cos \frac{5\pi}{12}$ sonlarni taqqoslaymiz.

$$\frac{\pi}{3} < \frac{5\pi}{12} < \frac{11\pi}{12}$$

tengsizlikdan va $y = \cos x$ funksiyaning $[0; \pi]$ kesmada kamayuvchiligidan

$$\cos \frac{\pi}{3} > \cos \frac{5\pi}{12} > \cos \frac{11\pi}{12}$$

munosabatni olamiz. **Javob:** $x < z < y$ (B).

27. (97-3-57) Ushbu $x = \operatorname{tg} \frac{5\pi}{7}$; $y = \sin \frac{\pi}{6}$; $z = \operatorname{tg} \frac{3\pi}{7}$ sonlar uchun quyidagi munosabatlardan qaysi biri o'rinli?

A) $z > y > x$ B) $x > z > y$
C) $y > x > z$ D) $x > y > z$

28. (97-7-57) Ushbu $x = \operatorname{tg}\left(\frac{5\pi}{6}\right)$, $y = \cos\left(\frac{2\pi}{5}\right)$,

$z = \operatorname{tg}\left(-\frac{\pi}{8}\right)$ sonlarni kamayish tartibida yozing.

A) $x > y > z$ B) $y > x > z$
C) $x > z > y$ D) $y > z > x$

29. (98-9-21) Quyidagi ayirmalardan qaysi birining qiymati manfiy?

A) $\sin 140^\circ - \sin 150^\circ$ B) $\cos 10^\circ - \cos 50^\circ$
C) $\operatorname{tg} 87^\circ - \operatorname{tg} 85^\circ$ D) $\operatorname{ctg} 45^\circ - \operatorname{ctg} 40^\circ$

30. (98-11-98) Eng katta sonni toping.

A) $\sin 170^\circ$ B) $\sin 20^\circ$
C) $\sin(-30^\circ)$ D) $\sin 100^\circ$

31. (99-1-50) $x = \sin 60^\circ$, $y = \cos(-600^\circ)$,

$z = \operatorname{ctg} \frac{31\pi}{6}$ sonlarni kamayish tartibida yozing.

A) $z > x > y$ B) $x > y > z$
C) $y > z > x$ D) $z > y > x$

32. (99-6-32) Sonlarni o'sish tartibida joylashtiring.

$a = \cos(-13^\circ)$, $b = -\sin(-75^\circ)$, $c = \sin 100^\circ$

A) $b < a < c$ B) $a < b < c$
C) $a < c < b$ D) $b < c < a$

33. (99-9-27) $M = \sin 72^\circ$, $N = \cos 220^\circ$ va

$Q = \operatorname{ctg} 184^\circ \cdot \sin 4^\circ$ sonlarni kamayish tartibida yozing.

A) $N > Q > M$ B) $N > M > Q$
C) $Q > M > N$ D) $Q > N > M$

34. (99-10-26) $k = \operatorname{tg} 248^\circ$, $t = \cos 32^\circ$ va $q = \sin 112^\circ$ sonlarni o'sish tartibida joylashtiring.

A) $q < t < k$ B) $k < t < q$
C) $t < k < q$ D) $t < q < k$

35. (96-6-32) $y = 2\sin^2 x + \cos^2 x$ funksiyaning eng katta qiymatini toping.

A) 1 B) 1,5 C) 2,6 D) 2

Yechish: 5-qoidadan $y = 2\sin^2 x + \cos^2 x$ funksiyaning qiymatlar sohasi $[1; 2]$ kesmadan iborat ekanligi kelib chiqadi. Demak, funksiyaning eng katta qiymati 2 ekan. **Javob:** 2 (D).

36. (96-1-56) $y = 2 \sin 3x + \cos 3x$ funksiyaning eng katta qiymatini toping.
A) 3 B) 2 C) $\sqrt{5}$ D) 4

37. (96-7-30) $y = 5^{1-\sin x} - e^{\ln 2}$ funksiyaning eng kichik qiymatini toping.
A) $1 - e^2$ B) 3 C) -1 D) $-2, 29$

Yechish: Asosiy logarifmik ayniyatga ko'ra $e^{\ln 2} = 2$ va u o'zgarmas. 3-qoidaga ko'ra $g(x) = 1 - \sin x$ funksiyaning qiymatlar sohasi $[0; 2]$ kesmadan iborat. $f(t) = 5^t$ o'suvchi funksiya ekanligidan $y = 5^{1-\sin x} - e^{\ln 2}$ funksiyaning eng kichik qiymati $y_0 = 5^0 - 2 = 1 - 2 = -1$ ekanligi kelib chiqadi.
Javob: -1 (C).

38. (97-1-21) $y = 1 + \cos x$ funksiyaning $[\frac{\pi}{3}; \frac{\pi}{2}]$ kesmadagi eng kichik qiymatini toping.

A) 0 B) 1 C) $1 + \frac{1}{2}$ D) $1 + \frac{\sqrt{3}}{2}$

39. (97-8-31) $\sin^2 \alpha + 2 \cos^2 \alpha$ ning eng katta qiymatini toping.

A) 1,2 B) 1,4 C) 1,6 D) 2

40. (97-10-30) Quyidagilardan qaysi biri

$$y = \frac{10}{5^{|\cos x|}} + 2 \ln e^3$$

funksiyaning eng katta qiymati?

A) 8 B) 16 C) $2 + 2e^3$ D) 18

41. (97-11-21) $y = 2 - 2 \sin x$ funksiyaning $[0; \frac{\pi}{6}]$ kesmadagi eng kichik qiymatini toping.

A) 0 B) $\frac{1}{2}$ C) $2 - \sqrt{3}$ D) 1

Yechish: Agar $y = f(x)$ o'suvchi funksiya bo'lsa, u holda ixtiyoriy $c < 0$ uchun $y = c f(x) + b$ funksiya kamayuvchi bo'ladi. Bu yerdan va sinus funksiyaning $[0; \frac{\pi}{6}]$ kesmada o'suvchiligidan $y = 2 - 2 \sin x$ funksiyaning $[0; \frac{\pi}{6}]$ kesmada kamayuvchi ekanligi kelib chiqadi. Demak, $y_0 = 2 - 2 \sin \frac{\pi}{6} = 2 - 1 = 1$ berilgan funksiyaning $[0; \frac{\pi}{6}]$ kesmadagi eng kichik qiymati bo'ladi. **Javob:** 1 (D).

42. (97-3-30) Funksiyaning eng katta qiymatini toping.

$$y = \frac{1}{2^{\cos x}} + \ln e^2$$

A) 2,5 B) 3 C) $1 + e^2$ D) 4

43. (98-8-30) Ushbu $y = 2 - \sin x$ funksiyaning $[0; \frac{7\pi}{6}]$ oraliqdagi eng katta qiymatini toping.

A) 3 B) 2 C) 2,5 D) 1

44. (98-5-14) $f(x) = 5 \sin x + 6$ funksiyaning eng katta qiymatini toping.

A) -1 B) 11 C) 1 D) 6

45. (98-1-30) $y = 0,5 \cos x$ funksiyaning $[-\frac{\pi}{4}; \frac{3\pi}{4}]$ kesmadagi eng kichik qiymatini toping.

A) $-\frac{1}{2}$ B) -1 C) 0 D) $-\frac{\sqrt{2}}{4}$

46. (00-7-24) Ifodaning eng kichik qiymatini toping.

$$2 \sin^2 x + \sqrt{3} \cos 2x$$

A) -1 B) 1 C) $2 - \sqrt{3}$ D) $3 - 2\sqrt{2}$

47. (00-2-30) Qaysi funksiya $x \in (-\frac{\pi}{6}; \frac{5\pi}{6})$ oraliqda faqat musbat qiymatlarni qabul qiladi?

A) $y = \sin(x + \frac{\pi}{6})$ B) $y = \sin(x + \frac{5\pi}{6})$

C) $y = \sin(x - \frac{5\pi}{6})$ D) $y = \sin(x - \frac{\pi}{6})$

48. (01-2-18) Ushbu $y = \frac{x}{2} + \sin^2 x$ funksiyaning

$[-\frac{\pi}{2}; \frac{\pi}{2}]$ kesmadagi eng katta qiymatini toping.

A) $-\frac{\pi}{2} + 1$ B) $-\frac{\pi}{4} + 1$ C) $\frac{\pi}{6} + 1$ D) $\frac{\pi}{4} + 1$

13.3 Teskari trigonometrik funksiyalar

Ma'lumki, tayinlangan $y \in [-1; 1]$ da $\sin x = y$ tenglama yechimi yagona emas. Masalan, $x_0 = 0$ va $x_1 = \pi$ sonlari $\sin x = 0$ tenglamaning yechimlari bo'ladi. Bu esa $y = \sin x$ funksiyaga teskari funksiya mavjud emasligini bildiradi. Xuddi shunday mulohaza yuritib $y = \cos x$ ($y = \operatorname{tg} x$, $y = \operatorname{ctg} x$) funksiya uchun ham teskari funksiya mavjud emasligiga kelamiz.

Agar $y = \sin x$ funksiyaning aniqlanish sohasini qisqartirib, uni $D(y) = [-\frac{\pi}{2}; \frac{\pi}{2}]$ deb olsak, u holda har bir tayinlangan $y \in [-1; 1]$ da $\sin x = y$ tenglama yagona yechimga ega bo'ladi, ya'ni teskari funksiya mavjud. Demak, aniqlanish sohasi $D(\sin) = [-\frac{\pi}{2}; \frac{\pi}{2}]$ bo'lgan $y = \sin x$ funksiyaga teskari funksiya mavjud bo'lib, u *arksinus* deyiladi va $x = \arcsin y$ shaklda yoziladi.

Xuddi shunday mulohaza yuritib $y = \cos x$, $D(\cos) = [0; \pi]$; $y = \operatorname{tg} x$, $D(\operatorname{tg}) = (-\frac{\pi}{2}; \frac{\pi}{2})$, $y = \operatorname{ctg} x$, $D(\operatorname{ctg}) = (0; \pi)$ funksiyalar uchun ham teskari funksiyalar mavjudligiga kelamiz. Ularga teskari bo'lgan funksiyalar mos ravishda *arkkosinus*, *arktangens* va *arkkotangens* deyiladi va $y = \arccos x$, $y = \operatorname{arctg} x$ va $y = \operatorname{arccotg} x$ shaklda yoziladi. Teskari funksiya ta'rifidan foydalanib, teskari trigonometrik funksiyalar qiymatlarini quyidagi 13.6a – 13.6b jadvallar shaklida berish mumkin:

a	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arcsin a$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\arccos a$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0

13.6a jadval.

b	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\arctg b$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\text{arctg} b$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

13.6b jadval.

Teskari trigonometrik funksiyalarning asosiy xossalari keltiramiz.

1. Aniqlanish sohasi:

$$D(\arcsin) = D(\arccos) = [-1; 1],$$

$$D(\arctg) = D(\text{arctg}) = R.$$

2. Arksinus va arktangenslar toq funksiya:

$$\arcsin(-x) = -\arcsin x, \quad \arctg(-x) = -\arctg x$$

3. Arkkosinus va arkkotangenslar uchun

$$\arccos(-x) = \pi - \arccos x,$$

$$\text{arctg}(-x) = \pi - \text{arctg} x$$

tengliklar o'rinli.

To'g'ri va teskari funksiya ta'rifidan quyidagi 4-11 xossalar kelib chiqadi:

4. $\arcsin(\sin x) = x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}.$

5. $\sin(\arcsin x) = x, \quad -1 \leq x \leq 1.$

6. $\arccos(\cos x) = x, \quad 0 \leq x \leq \pi.$

7. $\cos(\arccos x) = x, \quad -1 \leq x \leq 1.$

8. $\arctg(\text{tg} x) = x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$

9. $\text{tg}(\arctg x) = x, \quad x \in R.$

10. $\text{arctg}(\text{ctg} x) = x, \quad 0 < x < \pi.$

11. $\text{ctg}(\text{arctg} x) = x, \quad x \in R.$

Quyidagi 12-20 tengliklar o'rinli:

12. $\sin(\arccos x) = \cos(\arcsin x) = \sqrt{1-x^2}.$

13. $\arcsin a + \arccos a = \frac{\pi}{2}.$

14. $\arctg x + \text{arctg} x = \frac{\pi}{2}.$

15. $\arcsin x + \arcsin y = \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}).$

16. $\arccos x + \arccos y = \arccos(x\sqrt{1-y^2} - y\sqrt{1-x^2}).$

17. $\arccos x - \arccos y = \arccos(x\sqrt{1-y^2} + y\sqrt{1-x^2}).$

18. $\arcsin x - \arcsin y = \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}).$

19. $\arctg x + \arctg y = \arctg \frac{x+y}{1-xy}.$

20. $\arctg x - \arctg y = \arctg \frac{x-y}{1+xy}.$

1. (98-9-20) Hisoblang.

$$\arccos\left(-\frac{1}{2}\right) - \arcsin\left(-\frac{\sqrt{2}}{2}\right)$$

A) $\frac{11\pi}{12}$ B) $\frac{7\pi}{4}$ C) $\frac{\pi}{12}$ D) $\frac{5\pi}{6}$

Yechish: 2 va 3-xossalardan foydalansak, berilgan ifodani $\pi - \arccos \frac{1}{2} + \arcsin \frac{\sqrt{2}}{2}$ shaklda yozish mumkin. 13.6a - jadvaldan $\arccos \frac{1}{2} = \frac{\pi}{3}$ va

$\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$ qiymatlarni topib, ularni bu ifodaga qo'yib $\frac{11\pi}{12}$ ni olamiz. **Javob:** $\frac{11\pi}{12}$ (A).

2. (98-2-22) Hisoblang.

$$\arccos\left(-\frac{\sqrt{2}}{2}\right) - \arctg \frac{1}{\sqrt{3}}$$

A) -75^0 B) 75^0 C) -105^0 D) 105^0

3. (98-9-23) Hisoblang.

$$\arccos\left(-\frac{\sqrt{2}}{2}\right) - \arcsin\left(-\frac{\sqrt{3}}{2}\right)$$

A) $\frac{7\pi}{12}$ B) $\frac{\pi}{12}$ C) $\frac{13}{12}\pi$ D) $\frac{5\pi}{12}$

4. (99-8-68) Hisoblang.

$$2 \arcsin\left(-\frac{1}{2}\right) + \frac{1}{2} \arccos \frac{\sqrt{3}}{2}$$

A) $-\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) 0 D) $\frac{\pi}{3}$

5. (01-6-31) Hisoblang.

$$\sin\left(2 \arccos \frac{\sqrt{3}}{2}\right)$$

A) $\frac{\sqrt{2}}{2}$ B) $\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{6}}{2}$

6. (98-3-57) Hisoblang.

$$\arcsin\left(\sin \frac{5\pi}{8}\right) + \arccos\left(\cos \frac{8\pi}{7}\right)$$

A) $\frac{99\pi}{56}$ B) $\frac{83\pi}{56}$ C) $\frac{85\pi}{56}$ D) $\frac{69\pi}{56}$

Yechish: $\sin \alpha = \sin(\pi - \alpha)$ formuladan

$$1) \quad \sin \frac{5\pi}{8} = \sin\left(\pi - \frac{5\pi}{8}\right) = \sin \frac{3\pi}{8}$$

ni hosil qilamiz. $\arcsin(\sin \alpha) = \alpha, \quad -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ tenglikdan foydalanamiz.

$$-\frac{\pi}{2} < \frac{3\pi}{8} < \frac{\pi}{2}$$

munosabatlar o'rinli. Shuning uchun

$$\arcsin(\sin \frac{5\pi}{8}) = \arcsin(\sin \frac{3\pi}{8}) = \frac{3\pi}{8}.$$

$\cos \alpha = \cos(2\pi - \alpha)$ ekanligidan

$$2) \quad \cos \frac{8\pi}{7} = \cos(2\pi - \frac{8\pi}{7}) = \cos \frac{6\pi}{7}.$$

$\arccos(\cos \alpha) = \alpha$, $0 \leq \alpha \leq \pi$ dan

$$\arccos(\cos \frac{8\pi}{7}) = \arccos(\cos \frac{6\pi}{7}) = \frac{6\pi}{7}$$

bo'ladi. Shuning uchun berilgan ifoda $\frac{3\pi}{8} + \frac{6\pi}{7} = \frac{69\pi}{56}$ ga teng. **Javob:** $\frac{69\pi}{56}$ (D).

7. (98-5-47) Hisoblang.

$$\sin\left(\arcsin \frac{\sqrt{2}}{2} - \arccos \frac{\sqrt{2}}{2}\right)$$

A) 0 B) 1 C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{3}}{2}$

8. (99-7-46) Hisoblang.

$$\operatorname{tg}\left(\arcsin \frac{\sqrt{3}}{2} + \operatorname{arctg} \sqrt{3}\right)$$

A) $\sqrt{3}$ B) $-\frac{\sqrt{3}}{3}$ C) $-\frac{\sqrt{2}}{2}$ D) $-\sqrt{3}$

9. (98-4-16) Hisoblang.

$$\arccos\left(\sin \frac{\pi}{8}\right)$$

A) $1 - \left(\frac{\pi}{8}\right)^2$ B) $\frac{5\pi}{8}$ C) $\frac{7\pi}{8}$ D) $\frac{3\pi}{8}$

10. (98-10-104) Hisoblang.

$$\operatorname{arctg}\left(\operatorname{tg}\left(-\frac{3\pi}{5}\right)\right) + \operatorname{arcctg}\left(\operatorname{ctg}\left(-\frac{3\pi}{5}\right)\right)$$

A) $-\frac{6\pi}{5}$ B) $-\frac{7\pi}{10}$ C) $\frac{4\pi}{5}$ D) $-\frac{4\pi}{5}$

11. (00-4-46) $\sin(2\operatorname{arctg} 3)$ ning qiymatini toping.

A) 0,6 B) 0,8 C) 0,75 D) 0,36

Yechish: $\alpha = \operatorname{arctg} 3$ bo'lsin. U holda $\operatorname{tg} \alpha = 3$ va

$$\sin 2\alpha = \frac{2\operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha} = \frac{2 \cdot 3}{1 + 9} = \frac{6}{10} = 0,6$$

Javob: 0,6 (A).

12. (99-2-25) $m = \arcsin \frac{\sqrt{3}}{2}$, $n = \arccos\left(-\frac{1}{2}\right)$

va $p = \operatorname{arctg} 1$ sonlarni kamayish tartibida joylashtiring.

A) $m > p > n$ B) $m > n > p$
C) $n > m > p$ D) $p > n > m$

13. (01-12-31) Qaysi ifoda ma'noga ega emas?

1) $\sqrt{\lg \frac{11\pi}{8}}$; 2) $\sqrt{\sin \frac{19\pi}{12}}$; 3) $\log_{\sqrt{\frac{\pi}{3}}} \sqrt[3]{\frac{3\pi}{8}}$.

A) 1; 3 B) 3 C) 2 D) 1; 2

Yechish: 1) ifodada $1 < \frac{11\pi}{8}$ bo'lgani sababli $\lg \frac{11\pi}{8}$ musbat. Shu sababli $\sqrt{\lg \frac{11\pi}{8}}$ ma'noga ega. $\sin \frac{19\pi}{12} = \sin\left(2\pi - \frac{5\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right) < 0$ bo'lgani uchun 2) dagi ildiz ma'noga ega emas. 3) ifodada $\sqrt[3]{\frac{3\pi}{8}} > 0$ bo'lgani uchun, $\log_{\sqrt{\frac{\pi}{3}}} \sqrt[3]{\frac{3\pi}{8}}$ ma'noga ega. **Javob:** 2 (C).

14. (97-4-37) Ma'noga ega ifodalarni ko'rsating.

1) $\arcsin(\log_2 5)$; 2) $\arccos \frac{\pi}{\sqrt{17}}$
3) $\arccos \frac{a^2 + b^2}{a^2 + b^2 + c^2}$ 4) $\arcsin \frac{a^2 + b^2 + \sqrt{2}}{a^2 + b^2 + 1}$
A) 1); 2) B) 1); 3) C) 2); 3) D) 3); 4)

15. (97-9-97) Ma'noga ega ifodalarni ko'rsating.

1) $\lg(\arccos 1)$; 2) $\arcsin(\lg \frac{1}{2})$
3) $\arccos\left(\frac{a^4 + 1}{(a^2 + 1)^2}\right)$ 4) $\arcsin(\sqrt[10]{2})$
A) 1); 2) B) 2); 4) C) 3); 4) D) 2); 3)

16. (01-9-19) Hisoblang.

$$\frac{\sin(\pi + \arcsin \frac{\sqrt{3}}{2})}{\cos(0,5\pi + \arcsin \frac{1}{2})}$$

A) $\sqrt{3}$ B) $-\frac{\sqrt{3}}{2}$ C) $-\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

17. (97-9-30) Hisoblang.

$$\operatorname{arcctg}(\operatorname{ctg}(-3))$$

A) $\pi + 3$ B) $2\pi - 3$ C) $\frac{2\pi}{3} - 3$ D) $\pi - 3$

Yechish: $\operatorname{ctg}(-3) = -\operatorname{ctg} 3$ tenglikdan hamda 3-xossadan $\operatorname{arcctg}(\operatorname{ctg}(-3)) = \pi - \operatorname{arcctg}(\operatorname{ctg} 3)$ kelib chiqadi. Endi $0 < 3 < \pi$ bo'lganligi uchun 10-xossadan foydalansak

$$\operatorname{arcctg}(\operatorname{ctg}(-3)) = \pi - \operatorname{arcctg}(\operatorname{ctg} 3) = \pi - 3$$

ni olamiz. **Javob:** $\pi - 3$ (D).

18. (96-7-60) Hisoblang.

$$\sin\left(2\arcsin \frac{1}{3}\right)$$

A) $\frac{2}{3}$ B) $\frac{2\sqrt{2}}{3}$ C) $\frac{4\sqrt{2}}{9}$ D) $\frac{2\sqrt{2}}{9}$

19. (97-4-63) Hisoblang.

$$\sin\left(2\arccos \frac{1}{3}\right)$$

A) $\frac{2}{3}$ B) $\frac{2}{9}$ C) $\frac{4\sqrt{2}}{9}$ D) $\frac{4\sqrt{2}}{3}$

20. (00-6-54) Ifodaning qiymatini toping.

$$\cos\left(2 \arcsin \frac{2}{5}\right)$$

- A) $\frac{9}{25}$ B) $\frac{1}{5}$ C) $\frac{4}{5}$ D) $\frac{17}{25}$

21. (98-12-76) Hisoblang.

$$\sin\left(\frac{1}{2} \arccos \frac{1}{9}\right)$$

- A) $\frac{2}{3}$ B) $\frac{4}{9}$ C) $\frac{8}{9}$ D) $\frac{3}{4}$

22. (01-1-47) Ifodaning qiymatini toping:

$$\operatorname{arctg} 3 - \arcsin \frac{\sqrt{5}}{5}$$

- A) 0 B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$

23. (02-1-54) Hisoblang.

$$\cos(\operatorname{arctg} \sqrt{3} + \arccos \frac{\sqrt{3}}{2})$$

- A) 1 B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) 0

24. (02-2-50) Tenglamani yeching.

$$\frac{\pi}{24}(8x + 1) = \arccos\left(-\frac{1}{2}\right) + \arcsin \frac{1}{2} - \frac{1}{2} \operatorname{arctg} 1$$

- A) 4 B) 6 C) 5 D) 2

25. (02-4-31) Hisoblang.

$$12 \arcsin\left(-\frac{1}{2}\right) : \pi$$

- A) 0 B) -2 C) 2 D) 1

26. (02-4-32) $\operatorname{tg}(\operatorname{arctg} 3 + \operatorname{arctg} 7)$ ni hisoblang.

- A) 0 B) 0,5 C) -0,5 D) 0,25

Yechish: Agar $\operatorname{arctg} 3 = \alpha$, $\operatorname{arctg} 7 = \beta$ desak, 9-xossadan $\operatorname{tg} \alpha = 3$, $\operatorname{tg} \beta = 7$ ni olamiz. U holda qo'shish formulalarining 5-dan

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{3 + 7}{1 - 3 \cdot 7} = -\frac{1}{2}$$

kelib chiqadi. **Javob:** -0,5 (C).

27. (02-7-19) Hisoblang.

$$\operatorname{arcctg} 3 - \operatorname{arctg} 2$$

- A) $-\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

28. (02-7-34) $\operatorname{tg}\left(\pi - \arcsin \frac{3}{5}\right)$ ni hisoblang.

- A) $-\frac{3}{4}$ B) $\frac{3}{4}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

29. (02-12-39) Hisoblang.

$$\cos\left(2 \arcsin \frac{4}{5}\right)$$

- A) -0,28 B) 0,28 C) -0,26 D) 0,26

2.4 Qisqa ko'paytirish formulalari

	0	1	2	3	4	5	6	7	8	9
0		B	C	A	C	D	A	C	D	D
1	B	C	A	B	A	B	A	D	D	D
2	D	C	A	B	A	A	C	D	A	D
3	D	D	C	D	A	A	D	C	B	A

2.5 Ko'phadni ko'paytuvchilarga ajratish

	0	1	2	3	4	5	6	7	8	9
0		B	D	D	B	B	B	A	C	C
1	D	B	D	C	A	B	B	C	C	C
2	C	C	C	C	A	D	C	D	B	D
3	D									

2.6 Algebraik kasrlar

	0	1	2	3	4	5	6	7	8	9
0		D	C	D	B	D	C	B	A	D
1	D	B								

2.7 Ratsional ifodalar

	0	1	2	3	4	5	6	7	8	9
0		D	C	D	D	D	C	B	B	B
1	D	D	B	A	D	A	B	D	C	D

2.8 Aralash tipdagi masalalar

	0	1	2	3	4	5	6	7	8	9
0		A	D	B	D	D	A	C	B	C
1	D	C	A	D	B	B	C	B	C	C
2	B	D	A	D	D	C	A	D	D	A

3.1 Arifmetik ildiz. Kvadrat ildiz

	0	1	2	3	4	5	6	7	8	9
0		C	A	A	C	D	B	A	C	D
1	C	B	D	A	A	D	C	D	D	B
2	D	D	C	B	D	D	D	C	C	C
3	B	B	C	D	A	A				

3.1.1 Hisoblashga oid misollar

	0	1	2	3	4	5	6	7	8	9
0		D	D	C	A	D	C	C	C	A
1	A	D	C	D	D	D	A	A	B	B
2	D	D	C	B	D					

3.1.2 Ildizli ifodalarni soddalashtirish

	0	1	2	3	4	5	6	7	8	9
0		D	B	D	D	D	B	A	A	C
1	D	D	C	A	A	A	B			

3.2 n - darajali ildiz

	0	1	2	3	4	5	6	7	8	9
0		A	C	B	B	D	D	D	D	A
1	A	C	A	C	B	D	D	C	C	

3.2.1 Hisoblashga oid misollar

	0	1	2	3	4	5	6	7	8	9
0		A	C	B	A	A	B	D	D	C
1	C	A	A	D	A	D	D	A	D	B
2	A	A	A	C	C	D	C	B	C	C
3	C	D	D	A	A	B	C	B	A	C

3.3 O'rta qiymatlar

	0	1	2	3	4	5	6	7	8	9
0		A	B	B	B	B	C	A	D	C
1	D	A	B	B	C	D	B	B	C	D
2	B	A	D	B	A	B	A	B	A	C
3	D	A	C	C	D	D	B	D	A	B
4	A	A	A	C	B					

4.1 Ayniyat va tenglama

	0	1	2	3	4	5	6	7	8	9
0		C	C	D	B	C	B	B	A	D
1	A	B	D	C	B	C				

4.2 Chiziqli tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		D	C	D	B	C	A	A	D	C
1	B	B	B	B	A	B	A	D	B	B
2	C	A	B	C	A	D	D	D	C	B
3	A	B	A							

4.2.1 Proporsiya

	0	1	2	3	4	5	6	7	8	9
0		A	A	A	C	A	A	C	B	A

4.3 Kvadrat tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		A	C	B	D	A	A	A	C	B
1	D	B	B	A	D	D	D	C	D	D
2	C	D	D	A	D	C	C	D	B	D
3	D	A	D	D	D	D	D	A		

4.3.1 Parametrlil kvadrat tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		A	D	B	B	D	A	A	C	D
1	A	A	A	D	D	C	D	C	B	C
2	C	C	A	B	C	A	D	A	A	A
3	C	C	D	B	D	C	A	D	B	B
4	A	C	C	D	B	B	A	A	A	A
5	A	A								

4.4 Ratsional tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		D	C	D	C	D	D	A	C	B
1	D	A	C	D	A	B	A	D	D	B
2	C	D	A	B	D					

4.5.1 Chiziqli tenglamalar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		D	C	C	C	A	D	A	B	D
1	C	C	B	C	D	C	A	D	B	C
2	C	B	A	B						

4.5.2 Parametrlı tenglamalar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		A	D	D	D	B	C	D	C	A
1	B	D	D	A	A	B	D			

4.5.3 Ikkinchi va undan yuqori darajali tenglamalar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		D	D	C	B	C	D	D	C	B
1	C	C	A	A	B	D	C	A	B	D
2	A	C	C	D	D	D	A	B	C	A
3	C	A	B	D	D	A	A	B	B	D
4	D	C	D	A	A	D	D	A	A	D
5	A									

5.1 Chiziqli tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		C	A	C	A	D	A	D	D	B
1	A	A	A	C						

5.2 Chiziqli tengsizliklar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		C	D	C	D	B	A	C	C	A
1	B	B	C	D	A	A	D	D	D	A
2	D	B	C	D	D	C	C			

5.3 Oraliqlar usuli

	0	1	2	3	4	5	6	7	8	9
0		B	C	D	D	A	C	B	B	C
1	C	D	A	A	D	A	C	D	A	C
2	C	D	D	B	D	C	D	D	A	C
3	D	C	A	A	D	C	D	C	B	B
4	A	A	A	B	A	B	C	A	A	D
5	B	D	A	D	A	A	A	D	D	B
6	B	B	D	D	C	B	B	D	A	D

5.3.1 Parametrlı tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		D	D	D	B	A	C	B	D	B
1	C									

5.3.2 Shartli tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		B	D	B	D	C	D	A	C	

6.1 Modulli tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		A	C	B	A	D	A	D	B	C
1	D	D	D	B	D	D	B	D	C	C
2	C	A	D	C	D	C	D	D	D	D
3	C	A	A	A						

6.2 Modulli tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		B	B	A	D	C	A	A	A	C
1	A	C	C	A	B	C	A	B	C	C
2	B	D	D	D	D	B				

6.3 Modulli tenglamalar va tengsizliklar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		D	D	C	D	C	A	D	D	D
1	A	B								

7.1 Irratsional tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		B	A	D	A	A	C	A	D	A
1	C	D	B	A	B	D	A	D	D	A
2	A	A	D	D	D	B	A	C	A	B
3	D	C	C	A						

7.2 Irratsional tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		C	C	B	D	C	C	D	D	D
1	A	C	C	D	D	D	A	C	D	A
2	B	A	B	D	D	C	C	A	C	B

8.1 Arifmetik progressiya

	0	1	2	3	4	5	6	7	8	9
0		C	B	B	A	B	C	D	D	C
1	A	A	B	C	C	C	C	C	C	C
2	D	C	D	C	D	A	C	C	C	D
3	C	B	D	A	D	C	A	D	A	B
4	B	B	C	A	D	A	C	B	A	D
5	D	B	D	D	B	B	B	D	D	C
6	D	A	D	B	C	A				

8.2 Geometrik progressiya

	0	1	2	3	4	5	6	7	8	9
0		A	C	A	B	C	B	A	B	D
1	A	D	D	B	D	B	D	C	D	C
2	B	D	B	A	B	B	B	C	D	C
3	C	C	C	B	D	C	C	D	C	D
4	D	C	D	A	C	A	A	A	D	A

9.1 Sonlarga oid masalalar

	0	1	2	3	4	5	6	7	8	9
0		D	B	C	C	C	D	D	D	C
1	C	A	B	D	C	C	D	B	D	A
2	D	A	D	C	C	D	B	D	C	D
3	C	A	C	D	A	B	A	D	A	C
4	D	B	C	D	D	C	B	C	B	A
5	B	B	C	A	A	C	C	C	A	C
6	D	A	D	D	A	A	C	D	C	A
7	D	A	D	A	A	D	A	A	B	C
8	C	B	C	D	C	C	D	A	A	B
9	D	B	A							

9.2 Foizga oid masalalar

	0	1	2	3	4	5	6	7	8	9
0		B	A	D	D	D	A	D	D	D
1	B	A	B	C	C	B	A	C	D	C
2	B	C	A	C	D	D	D	A	A	C
3	D	B	D	D	C	D	C	D	A	A
4	B	B	B							

9.3 Harakatga oid masalalar

	0	1	2	3	4	5	6	7	8	9
0		B	C	B	D	D	D	D	C	D
1	C	A	D	A	D	A	C	C	B	

9.4 Ishga oid masalalar

	0	1	2	3	4	5	6	7	8	9
0		D	A	A	C	A	C	A	A	C
1	B	A								

10 Funksiyalar

	0	1	2	3	4	5	6	7	8	9
0		A	D	B	A	B	D	A	D	C
1	C	B	C	B	C	D	C	D	D	A
2	B	B	A	C	B	A	A	D	A	D
3	A	D	D	A						

10.1 Tekislikda koordinatalar sistemasi

	0	1	2	3	4	5	6	7	8	9
0		A	B	C	D	A	B	C	A	C
1	A	D	D	B	D	D	B	A	D	

10.2 Chiziqli funksiya

	0	1	2	3	4	5	6	7	8	9
0		A	A	C	B	B	D	C	A	D
1	D	B	B	C	D	D	D	A	C	A
2	D	B	B	D	C	A	C	D	A	A
3	D	A	C	B	B	A				

10.3 Kvadrat funksiya

	0	1	2	3	4	5	6	7	8	9
0		D	A	D	D	C	D	A	D	B
1	B	C	D	A	C	C	A	D	A	C
2	B	B	C	A	C	D	D	C	C	A
3	D	D	A							

10.4 Teskari funksiya

	0	1	2	3	4	5	6	7	8	9
0		D	D	B	A	D	D	D	B	A
1	B	A	D	C						

10.5 Aralash tipdagi masalalar

	0	1	2	3	4	5	6	7	8	9
0		C	C	A	D	D	D	C	C	D
1	A	D	C	A	C	D	C	D	B	D
2	A	B	B	A	C	A				

11.1 Ko'rsatkichli funksiya

	0	1	2	3	4	5	6	7	8	9
0		D	C	D	C	C	C	A	A	B
1	D	C	D	D	D					

11.2 Ko'rsatkichli tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		A	B	C	D	D	A	C	C	A
1	D	D	D	A	D	D	A	A	A	C
2	D	B	C	D	D	C	C	C	D	D
3	D	A	C	B	C	B	B	A	A	A
4	A	A	C	C	A	D				

11.3 Ko'rsatkichli tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		D	D	D	B	A	B	A	A	C
1	A	D	C	A	B	A	A	D	D	A
2	A	A	C	C	C	D	D			

12.1 Aniqlanish sohasi va xossalari

	0	1	2	3	4	5	6	7	8	9
0		A	C	A	D	D	B	A	B	C
1	D	B	B	D	A	D	B	D	A	A
2	B	C	A	D	D	A	B	D	D	B

12.1.1 Logarifmik ifodalarda shakl almashtirish

	0	1	2	3	4	5	6	7	8	9
0		B	A	B	C	D	A	D	B	D
1	D	C	A	B	C	A	D	B	A	D
2	C	D	B	D	D	C	D	A	C	D
3	A	A	A	A	A	A	D	C	C	C
4	B	B	C	D	B	C	A	C	A	D
5	A	A	D	A	C	D	A			

12.2 Logarifmik tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		B	B	D	C	C	C	B	D	D
1	D	C	C	D	D	B	A	A	A	C
2	C	A	C	A	B	B	B	B	C	D
3	A	C	A	A	C	D	A	A	A	A
4	C	A	D	A	B	A	C	C	C	B
5	C	C	D	D	B					

12.3 Logarifmik tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		B	B	A	D	D	D	D	B	D
1	B	A	D	B	A	D	D	D	D	B
2	D	B	C	D	B	D	D	D	B	D
3	D	A	C	B	D	D	D	C	A	D
4	B	B	D	C	B	D	B	D	A	A

13.1 Burchak va yoy, ularning o'lchovi

	0	1	2	3	4	5	6	7	8	9
0		C	C	B	D	C	C	C	D	D
1	B	D	B							

13.2 Trigonometrik funksiylar

	0	1	2	3	4	5	6	7	8	9
0		C	B	C	D	A	C	C	B	A
1	B	B	D	D	A	B	D	C		

13.2.1 Trigonometriyaning asosiy ayniyatlari

	0	1	2	3	4	5	6	7	8	9
0		B	D	C	A	C	C	B	D	A
1	A	A	B	D	C	D	A	D	D	D
2	A	C	A	A						

13.2.2 Trigonometrik funksiyalarning xossalari

	0	1	2	3	4	5	6	7	8	9
0		D	B	A	A	B	A	D	A	B
1	A	A	C	D	B	A	C	B	C	A
2	D	A	D	D						

13.2.3 Qo'shish va keltirish formulalari

	0	1	2	3	4	5	6	7	8	9
0		A	A	A	A	A	C	B	A	C
1	C	C	A	A	D	D	B	D	C	D
2	C	B	A	D	D	D	C	D	C	D
3	B	D	D	A	A	A	A	D	A	A

13.2.4 Ikkilangan va yarim burchak formulalar

	0	1	2	3	4	5	6	7	8	9
0		D	D	D	B	B	B	B	D	B
1	B	A	C	A	C	B	D	C	A	A
2	B	A	B	A	C	B	D	A	A	B
3	A	A	C	B	A	B	C	A	D	C
4	D	A	D	C	D	C	C	C	A	D
5	C	C	A	B	D	C	D	B	D	A
6	A	A	A	D	A	A	B	C	B	D

13.2.5 Yig'indi va ayirma uchun formulalar

	0	1	2	3	4	5	6	7	8	9
0		A	C	A	B	D	D	D	B	C
1	A	D	D	D						

13.2.6 Qiymatlar sohasi va monotonligi

	0	1	2	3	4	5	6	7	8	9
0		D	C	B	B	C	C	D	A	A
1	D	B	C	B	D	D	C	B	C	A
2	B	C	A	C	D	C	B	A	D	D
3	D	A	A	C	D	D	C	C	B	D
4	B	D	D	C	B	D	C	A	D	A
5	D									

13.3 Teskari trigonometrik funksiyalar

	0	1	2	3	4	5	6	7	8	9
0		A	D	C	A	C	D	A	D	D
1	C	A	C	C	C	D	A	D	C	C
2	D	A	D	D	D	B	C	A	A	A
3	A									

13.4 Trigonometrik tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		D	D	B	A	C	A	A	C	B
1	B	B	A	A	B	A	A	A	A	A
2	C	A	B	C	D	D	C	C	B	C
3	C	C	C	B	B	A	D	D	D	D
4	A	D	A	A	B	B	C	D	A	B
5	D	B	C	C	D	A	B	B	A	A
6	B	C	D	D	D	A	B	B	A	A
7	D	A	C	C	C	C	C	A	B	A
8	B	B	D	B	A	D				

13.5 Trigonometrik tengsizliklar

	0	1	2	3	4	5	6	7	8	9
0		B	B	A	C	C	D	C	C	A
1	C	B	C	A	B	A	D	D	D	D
2	D	D	D	B	D	A	A	A	B	

13.6 Aralash tipdagi masalalar

	0	1	2	3	4	5	6	7	8	9
0		A	D	B	B	B	C	A	A	A
1	D	C	B	A	D	D	C	D	A	D
2	D	A	D	C	C	B	C			

14.1 Elementar funksiyalarning hosilasi

	0	1	2	3	4	5	6	7	8	9
0		B	B	D	A	A	A	A	B	B
1	A	B	D	A	B	D	B	C	C	D
2	D	D	A	A	A	A	D	D		

14.1.1 Murakkab funksiyaning hosilasi

	0	1	2	3	4	5	6	7	8	9
0		B	B	A	C	A	D	B	C	A
1	B	B	D	D	D	C	D	A	C	C
2	B	D	A	D	A	C	D	A	A	C
3	D	A	B	B	A	C	B	B	D	A
4	A	A	D	D	C	D				

14.2 Funksiyani hosila yordamida tekshirish

	0	1	2	3	4	5	6	7	8	9
0		B	D	B	B	C	A	D	C	B
1	A	C	D	D	D	D	C	D	A	D
2	B	A	B	C	B	B	B	D	A	B
3	B	D	D	B	D	C	D	A	B	D
4	C	D	A	D	A	D	B	A	B	A
5	C	C	A	B	D	D	C	A	B	C
6	A	C	D	A	D	D	D	A		

14.3 Hosilaning geometrik va mexanik ma'nosi

	0	1	2	3	4	5	6	7	8	9
0		C	B	A	A	B	C	C	B	A
1	C	B	A	B	B	A	D	C	D	A
2	A	A	D	A	D	A	C	A	D	A
3	B	B	D	D	D	D	D	B	D	B
4	A	C	C							

14.4 Boshlang'ich funksiya va integral

	0	1	2	3	4	5	6	7	8	9
0		B	A	B	D	A	A	A	A	D
1	A	A	C	D	C	A	B	B	A	A
2	B	D	B	C	D	D	C	A	C	D
3	D	C	B	A	A	C	A	D	D	B
4	D	A	D	D	A					

14.4.1 Aniq integral

	0	1	2	3	4	5	6	7	8	9
0		A	B	B	A	B	B	C	B	D
1	B	A	B	C	A	B	D	A	C	C
2	A	A	D	D	D	A	B	A	D	C
3	D	A	A	D	A	B	A	B	A	A
4	D	A	B	D	A	A	D	D	D	A

14.5 Maxsus yo'l bilan yechiladigan tenglamalar

	0	1	2	3	4	5	6	7	8	9
0		C	D	D	C	D	C	D	C	B
1	B	A	B	D	C	B	B	B	B	A
2	D	C	B	D	A	C	B	D	B	A
3	B	C	A	A	D	B	D	C	B	D
4	D	A	D	D	C	A	A	A	B	A
5	D	D	D	D	D	A	D			

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