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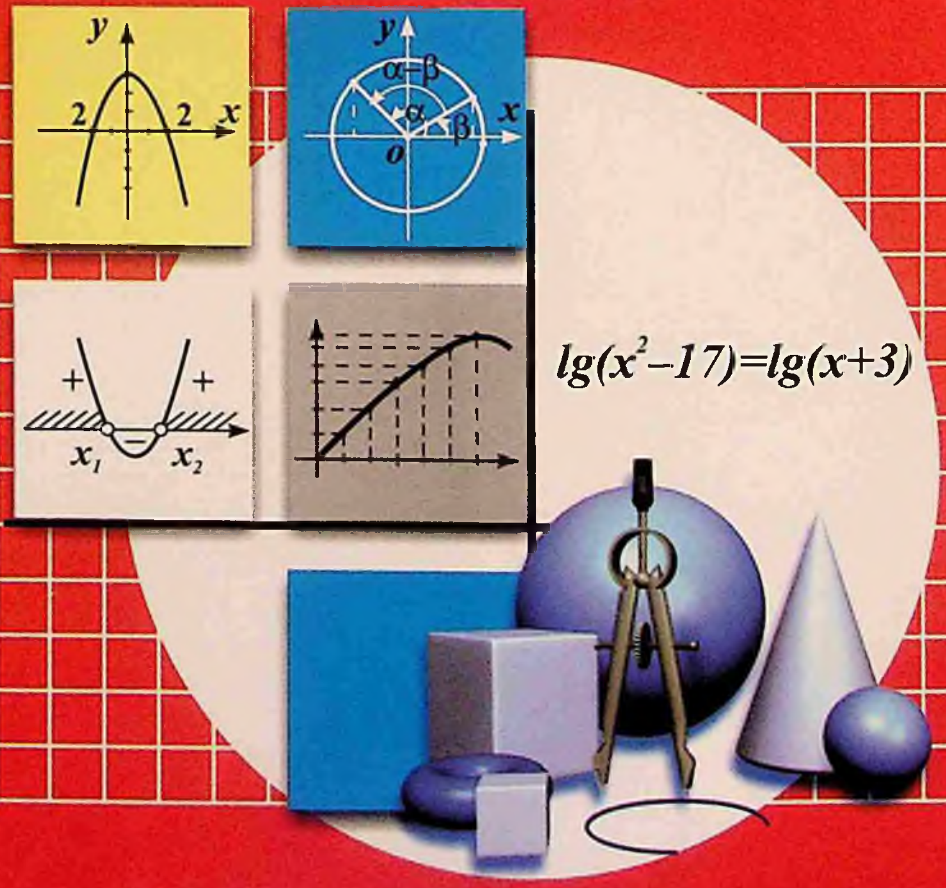
ALGEBRA VA ANALIZ ASOSLARI

(qo'llanma)

I qism



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Mazkur qo‘llanmadan, o‘qituvchilar, o‘rta umumta’lim maktab yuqori sinf o‘quvchilari, litsey va kollej talabalari va barcha qiziqqanlar foydalanishlari mumkin.

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SO'ZBOSHI

Bizlar hozirgi vaqtda axborot texnologiyalari asrida yashayapmiz. Bunda biz, albatta, zamonaviy taraqqiyotda fan va texnikaga suyanib olg'a boramiz. Bu taraqqiyotga ko'tarilishda dunyo olimlari va ajdodlarimiz yaratgan fan yangiliklari orqali muvaffaqiyatlarga yerishib kelmoqdamiz. Bunday yangiliklarni yaratishda matematika fani asosiy o'rin tutadi.

O'rta bilim yurtlarini tamomlagan o'quvchilar zamonaviy texnologiyani o'zlashtirishda, ba'zi amaliy masalalarni yechishda va oliy o'quv yurtlariga kirishdagi sinov imtihonlari vaqtida algebradan olgan bilimlari yetarli emasligini his qilmoqdalar.

Bu qo'llanma o'quvchilarning yuqoridagi kamchiliklarga barham berish va mustahkam algebraik bilimga ega bo'lishini istaganlar uchun yaratildi.

Bu qo'llanmani o'rta maktabda ishlagan pedagogik davrida orttirgan pedagogik malakalarimiz va o'rta maktablar uchun chiqarilgan algebraik qo'llanmalar asosida yaratdik.

Bu qo'llanma II qism, XXIII bob va 141-§ dan iborat. Mazkur I qismga XVI bob, 94-§ kiritilgan. Bu qo'llanmada har bir mavzuga oid nazariy qism, masalalarning yechimlari, mavzuga oid savollar va mustaqil yechish uchun misol va masalalar berilgan.

Qo'llanmadan maktab o'quvchilari kollej, litsey o'quvchilari, oliy o'quv yurti qoshidagi tayyorlov kurslarining tinglovchilari to'liq foydalanishlari mumkin.

Muallif qo'llanmani yaratishda Toshkent viloyati, Bo'stonliq tumani matematiklarining uslubiyat seksiyasi rahbari, oliy toifali matematika o'qituvchisi M. Normetovga, Bo'stonliq tumanidagi 20-umumta'lim maktabining 1-toifali matematika o'qituvchisi A. Umirzoqovga, o'zining qimmatli maslahatlarini bergan barcha o'qituvchilarga minnatdorchilik bildiradi.

Hurmatli kitobxonlarning qo'llanma haqidagi tanqidiy fikr va mulohazalarini chuqur mamnuniyat bilan qabul qilamiz.

Mualliflar

ALGEBRANI VUJUDGA KELISHIDAGI TARIXIY MA'LUMOTLAR

Insoniyat taraqqiyotining dastlabki bosqichlarida predmetlarni sanash arifmetikaning eng sodda tushunchalarini vujudga keltirdi. Og'zaki sanoq sistemasi asosida yozma sanoq sistemasi paydo bo'ldi va asta-sekin natural sonlar ustida to'rt arifmetik amalni bajarish takomillasha boshladi. Ayniqsa, qadimgi Misr va Bobilda arifmetika, geometriya fan sifatida shakllana boshlagan, savdo-sotiq va astronomiyaning rivojlanishi asosida algebra va trigonometriyaga oid ma'lumotlar to'plana boshlagan edi.

Qadimgi Misr va Bobilda matematik faktlar tarqoq holda bo'lgan bo'lsa, qadimgi Yunonistonda mantiqiy sistemaga solingan matematik bilimlar vujudga kela boshladi.

Qadimgi Yunonistonda matematikaning taraqqiyoti Fales va Pifagor nomi bilan bog'liq. Pifagor falsafiy maktabining vakillari matematik bilimlarni to'plab, sistemaga solishda, matematikaning ilk rivojlanishida katta o'rin tutdi.

Matematika qadimgi Yunon va ellinizm davri bilan bir vaqtda Xitoy va Hindistonda ham rivojlangan. Masalan, Chjan San va Szin Chou-channing «To'qqiz bobli arifmetika» asarida natural sonlardan kvadrat va kubildiz chiqarish qoidalari keltirilgan. Qadimgi Xitoy matematiklari asarlarida chiziqli tenglamalar sistemasini yechishda noma'lumni chiqarish usuli, Pifagor teoremasining arifmetik varianti, chegirmalarga oid masalalar uchraydi. Hind matematikasining rivojlanishi, asosan, V–XII asrlarda, yuqori bosqichga ko'tarilgan.

O'nli sanoq sistemasi, nol raqami, kvadrat irratsionalliklar va manfiy sonlar Hindistonda keng qo'llanilgan.

IX asrga kelib arab tilida ijod qilgan matematiklar matematik-astronomik jadvallar tuzishda, algebra, geometriya, trigonometriya sohasida qator yutuqlarga erishdilar.

IX asrda bizning vatandoshimiz, buyuk o‘zbek olimlaridan biri **Abu Abdulloh Muhammad ibn Muso Al-Xorazmiy** arab tilida algebra fanini mustaqil fan sifatida vujudga kelishi va rivojlanishida buyuk asos bo‘lgan «Al-kitob al-muxtasar fi hisob Al-jabr val-muqobala» (**«Al-jabr val-muqobala hisobi haqida qisqacha kitob»**) nomli risolasini yaratdi. Bu risola dunyoda birinchi marta algebrani sistemali ravishda bayon qilgan asar edi. Bu asar keyinchalik «Al-jabr val-muqobala» nomi bilan butun dunyoga mashhur bo‘ldi.

Kitob nomidagi «al-jabr» so‘zidan «algebra» so‘zi kelib chiqqan. «Al-jabr» so‘zi Yevropaga g‘arbiy arablar orqali o‘tgan. G‘arbiy arablar «j»ni «g» deb talaffuz qiladilar. Shu yo‘sinda «Al-jabr» Yevropada va keyinchalik, yer yuzining barcha joyida **«algebra»** deb yuritiladigan bo‘ldi.

Avvalo Muhammad Xorazmiy asaridagi «Al-jabr», «val-muqobala» terminlarining ma’nosini bilib olaylik. Bu arabcha terminlar algebradagi ikki amalni bildiradi:

«Al-jabr» – «to‘ldirish» – tenglamaning biror qismidagi ayriluvchi hadni uning ikkinchi tomoniga qo‘shiluvchi qilib o‘tkazish demakdir. Masalan, $3x^2-4x+2=2x^2+7$ tenglamaga «al-jabr» amali ishlatilsa, tenglama quyidagi ko‘rinishga keladi: $3x^2+2=2x^2+4x+7$.

«Al-muqobala» – «qarama-qarshi qo‘yish» – tenglamaning har ikkala qismidagi teng qo‘shiluvchilarni o‘zaro yeyishtirish (tashlab yuborish) demakdir. Oldingi tenglamaga «al-muqobala» amalini tatbiq etsak, u quyidagi ko‘rinishda bo‘ladi: $\mathbf{x^2=4x+5}$.

Bu asar juda ko‘p Yevropa tillariga tarjima qilingan. Ulardan eng qimmatlisi 1145-yili ingliz **Rober Chester** va 1160-yili italiyalik **Gerardo** qilgan lotincha tarjimalardir.

Bu asar bizgacha 1342-yili ko‘chirilgan arabcha nusxada yetib kelgan. U Oksford universiteti kutubxonasida saqlanadi. Muhammad Xorazmiyning «Al-jabr val-muqobala hisobidan qisqacha kitob» nomli asari rus tiliga matematika tarixchisi professor **B.A. Rozenfeld** tomonidan tarjima qilingan va 1964-yili O‘zSSR «Fan» nashriyotida chop etilgan.

Arifmetikada konkret sonlar ustida birinchi to'rt amal o'rganiladi. Algebrada esa bu amallarning har qanday son va son bo'lmagan boshqa matematik obyektlar uchun o'rinli umumiy xossalari tekshiriladi.

Bunda hosil qilinadigan natijalarning umumiy bo'lishiga erishish uchun miqdorlarning qiymatlari harflar bilan belgilanib, harfiy ifodalar ustida bajariladigan amallarning qoida va qonunlari ko'rsatiladi, ifodalar shaklini o'zgartirish va tenglamalarni yechish qoidalari o'rganiladi.

«Mashhur shoir va matematik **Umar Xayyom** algebrani tenglamalar yechish haqidagi fan deb ta'riflagan edi. Uning bu ta'rifi o'tgan asrgacha ham o'z kuchini yo'qotmay keldi.

1074-yilda Umar Xayyom «Al-jabr val-muqobala» masalalariga oid mashhur risolasini yozdi. Bu asar fransuz tiliga tarjima qilinishi bilan butun Yevropaga tarqaldi. Uning «Al-jabr» degan boshqa bir kitobida chiziqli va kvadrat tenglamalarni yechish, uchinchi darajali tenglamalarning ildizlarini geometrik usul bilan izlash va boshqa juda ko'p masalalarni yechish yo'llari ko'rsatilgan.

O'nli sanoq sistemasining Hindistondan butun dunyoga tarqalishi va takomillashuvida Xorazmiy, Beruniy, Koshiy va boshqalarning xizmati katta.

Tabobat ilmining buyuk arbobi **Abu Ali Ibn Sino** asarlarida ham o'sha zamon uchun alohida ahamiyatga ega bo'lgan arifmetika va algebra masalalarining yechimlari berilgan. Uning algebra va arifmetikaga oid ishlarida sonlarni kvadrat va kubga ko'tarish amallari tekshirilgan.

Shunisi diqqatga sazovorki, qadimgi dunyo tarixidan to Al-Xorazmiy davriga qadar matematika algebra va arifmetika kabi bo'limlarga ajralmagan edi. Faqat Al-Xorazmiy davridan boshlab, algebra matematikaning alohida bo'limi bo'lib ajraldi.

XV asr oxirida «+» (plyus) va «–» (minus) ishoralari algebraga kiritildi. Bundan keyingi davrda masalada qatnashadigan miqdorlar, shuningdek noma'lumlar harflar bilan belgilanadigan bo'ldi.

XVI asrda hozirgi zamon algebrasi uchun xarakterli bo'lgan harfiy formulalar birinchi marta paydo bo'ldi.

XVII asrda noma'lum sonlar uchun $x, y, z, \ldots$ harflarni ishlatish **Dekartdan** boshlangan bo'lib, hozir ham shunday qilinmoqda.

Algebrada kvadrat tenglamalarni yechish qadimgi dunyodan ma'lum, ammo uchinchi va to'rtinchi darajali tenglamalarni yechish formulalarini esa faqat XVI asrda Italyan matematiklari **Kardano, Tartalya** va **Ferrarilar** yaratib beradi. XIX asrda **Abel** hamda **Galualar** darajasi 4 dan yuqori algebralik tenglamalar ildizlarining koeffitsiyentlari orqali ratsional amallar bilan radikal ko'rinishda ifoda etish mumkin emasligini isbot qildilar (Galua nazariyasi).

Algebra fanining rivojlanishiga bir qator o'zbek olimlari, chunonchi: **T.A. Sarimsoqov, S.H. Sirojiddinov, R.I. Iskandarov, M.M. Sultonova** va boshqalar o'z hissalarini qo'shdilar.

I qism. ALGEBRA

I bob. ALGEBRAIK IFODALAR VA ULARDA SHAKL ALMASHTIRISH

1-§. O'zgaruvchili ifodalar (algebraik ifodalar)

Bizga ma'lumki, sonli ifodalar sonlardan amal ishoralari va qavslar yordamida tuziladi. Masalan, $12{,}4+3\cdot 1{,}5$; $5\cdot(3{,}8-0{,}84)$ va boshqalar sonli ifoda bo'ladi.

Sonli ifodadagi amallar bajarilgandan keyin hosil bo'lgan son **ifodaning qiymati** deyiladi.

$12{,}4+3\cdot 1{,}5=16{,}9$ da ifodaning qiymati 16,9 ga teng.

1-masala. Akasi singlisidan 3-yosh katta.

Agar: 1) Singlisi 5-yoshda bo'lsa, akasi necha yoshda bo'ladi?

2) Singlisi 11-yoshda bo'lsa, akasi necha yoshda bo'ladi?

3) Singlisi 25,5-yoshda bo'lsa, akasi necha yoshda bo'ladi?

Akasi: 1) $5+3=8$; 2) $11+3=14$; 3) $25{,}5+3=28{,}5$-yoshda bo'ladi.

Umuman, akasining yoshini topish uchun singlisining yoshiga 3-yosh qo'shib topiladi. Masalani umumiy holda quyidagicha ifodalaymiz:

Akasi singlisidan 3-yosh katta. Agar singlisi m yoshda bo'lsa, akasi necha yoshda bo'ladi? Bunda m singlisining yoshi bo'lgani uchun akasining yoshi $m+3$-yosh bo'ladi.

Bundagi $m+3$-yozuvdagi m o'rniga turli sonlarni qo'yib, g'oyat ko'p o'xshash masalalar tuzish mumkin (m ning o'rniga turli sonlar qo'yib).

$\boldsymbol{m+3}$-yozuv bir-biriga o'xshash masalalar yechimining ***umumiy formulasi*** bo'ladi.

2-masala. O'quvchi donasi 200 so'mdan n dona daftar va 800 so'mga bitta darslik oldi. O'quvchi bularning hammasiga qancha pul to'ladi?

Buning uchun bir daftarning bahosini daftarlar soniga ko'paytirib, $200\cdot n$ so'mni hosil qilamiz. Natijaga darslikka to'langan 800 so'mni qo'shib, to'langan pul $200\cdot n+800$ so'mni topamiz.

Bundan buyon harf bilan son ko‘paytuvchi orasidagi ko‘paytirish belgisi yozilmaydi. $200 \cdot n+800$-yozuvni $200n+800$ ko‘rinishda yozamiz. Avvalgi masaladagi $m+3$ va $200n+800$-yozuvlarda sonlardan boshqa m va n harflar ham ishlatildi.

***Ta’rif.* Raqamlar yoki harflar bilan belgilangan sonlarni amal ishoralari yordamida birlashtirishdan hosil bo‘lgan yozuvni algebraik ifoda (o‘zgaruvchili ifoda) deyiladi.**

Ba’zan, «algebraik ifoda» («o‘zgaruvchili ifoda»)ni qisqacha «ifoda» deb ataladi. Algebraik ifodaga misollar keltiramiz: $3mn+7$; $\frac{a+3}{b-2}$; $9(2p-5{,}2\,q)$ va hokazo.

Bu misollardan algebraik ifodaning faqat birgina harfdan iborat bo‘lishi va bir necha harflardan iborat bo‘lishi mumkin. $3mn+7$ ifodadagi m va n larning o‘rniga son qo‘yib, natijani hisoblaymiz.

Masalan, $m=6$; $n=10$ bo‘lsin, $3mn+7=3\cdot 6\cdot 10+7=187$. Bu 187 son $3mn+7$ ifodaning $m=6$, $n=10$ lardagi qiymati bo‘ladi.

Algebraik ifodaning qiymati deb, shu ifodadagi harflar o‘rniga berilgan son qiymatlarni qo‘yib, shu sonlar ustida ko‘rsatilgan amallar bajarilgandan keyin hosil bo‘lgan songa aytiladi.

1) $x(x+8)$ ifodani ko‘rib chiqamiz. Bu ifodadagi x o‘zgaruvchining har qanday qiymatiga mos qiymatni hisoblab topish mumkin. Bunday hollarda ifoda o‘zgaruvchining barcha qiymatlarida ma’noga ega deyiladi. Masalan, $x=-7$ da, $x(x+8)=-7\cdot(-7+8)=-7\cdot 1=-7$.

2) Ba’zi ifodalar o‘zgaruvchining har qanday qiymatida ma’noga ega bo‘lmaydi. Masalan, $\frac{12b}{b-7}$ kasr $b=7$ bo‘lganda ma’noga ega bo‘lmaydi. Bu holda $b-7=7-7=0$, nolga esa bo‘lish mumkin emas. b ning qolgan barcha qiymatlarida bu ifoda ma’noga ega bo‘ladi.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday ifodalarni sonli ifodalar deyiladi?
2. Qanday ifodalarni algebraik ifoda (o‘zgaruvchili ifoda) deyiladi?
3. Algebraik ifodalarga misollar keltiring.

4. Algebraik ifodaning qiymati deb nimaga aytiladi?
5. O'zgaruvchining barcha qiymatlarida ma'noga ega bo'ladigan ifodalarga misol keltiring.
6. O'zgaruvchining ba'zi qiymatlarida ma'noga ega bo'lmaydigan ifodalarga misol keltiring.

MASALALARNI YECHING

1. x ning jadvalda ko'rsatilgan qiymatlarida $3x-1$ va $-3x+1$ ifodalarning qiymatlarini hisoblab, jadvalni to'ldiring:

x	-2	-1	0	1	2	3	4	5	6
$3x-1$									
$-3x+1$									

2. x o'zgaruvchining qiymati 5; 3; 0; -3; 10 ga teng:

a) $(2x-5)x$; b) $2x-(x+5)$ ifodaning o'zgaruvchining shu qiymatlariga mos ifodaning qiymatlarini toping.

3. m va n o'zgaruvchilarning qiymatlari quyidagicha bo'lganda $5m-3n$ ifodaning qiymati nimaga teng bo'ladi?

a) $m=-\frac{2}{5}$, $n=\frac{2}{3}$; b) $m=0{,}2$, $n=1{,}4$.

4. $4a-3b$ ifodaning qiymatini topib, jadvalni to'ldiring:

a	5	-2	4	0,5	-6	4,25
b	-3	4	0	-1	2,5	-5
$4a-3b$						

5. Quyidagi jumlani ifoda shaklida yozing:

a) b va c sonlarning yig'indisi;

b) a va m sonlarning ayirmasi;

d) x sonning kvadrati;

e) y sonning kubi;

f) m bilan x va y sonlari bo'linmasining ayirmasi;

g) x bilan a va b sonlari ko'paytmasining yig'indisi;

h) a bilan x va y sonlari yig'indisining ko'paytmasi.

6. Quyidagi ifoda o'zgaruvchining qanday qiymatlarida ma'noga ega bo'ladi?

a) $5y+2$; b) $\frac{18}{y}$; d) $\frac{1}{x-11}$; e) $\frac{m-1}{4}$; f) $\frac{7a}{3+a}$; g) $\frac{2b}{10-b}$.

2-§. Formulalar

Ko'pgina masalalarni umumiy shaklda yechish uchun o'zgaruvchilar orasidagi bog'liqlikni ifodalovchi formulalar tuziladi.

Masalan:

1-misol. Kvadratning tomoni a sm bo'lsin. Unda kvadratning yuzi a^2 sm^2 bo'ladi. Kvadratning yuzini S harfi bilan belgilab, $\boldsymbol{S=a^2}$ (kvadrat birlik) formulani hosil qilamiz.

2-misol. Har qanday juft m soni biror butun n sonini 2 ga ko'paytirilganiga teng, ya'ni $\boldsymbol{m=2n}$. Bunda: m – juft son, n – butun son. Bu formula juft son formulasi deyiladi.

3-misol. Agar jism bir xil v m/s tezlik bilan harakat qilsa, unda jism t soat ichida vt m yo'l bosadi. Jism bosib o'tgan yo'lni (metr) S harfi bilan belgilab, yo'l, tezlik va harakat vaqti orasidagi bog'liqlikni ifoda qiluvchi formula chiqaramiz:

$\boldsymbol{S=vt}$ (uzunlik birligi).

Odatda masalaning ma'nosiga ko'ra, formulalardagi o'zgaruvchilarning qanday qiymatlar qabul qila olishi aniq bo'ladi. Masalan, kvadratning yuzi $S=a^2$ formulasidagi a o'zgaruvchi har qanday musbat qiymat oladi, biroq nolga teng bo'la olmaydi va manfiy qiymat ola olmaydi. $m=2n$ juft son formulasida n o'zgaruvchi har qanday butun qiymat olishi mumkin, ammo kasr qiymat olmaydi.

TAKRORLASH UCHUN SAVOLLAR

1. Ko'pchilik masalalarni umumiy shaklda yechish uchun nimadan foydalaniladi?
2. Kvadratning yuzi qanday formula yordamida topiladi?
3. Juft sonlar qanday formula yordamida topiladi?
4. Jism bosib o'tgan yo'lni topish formulasini ayting (yozing).

MASALALARNI YECHING

7. To‘g‘ri to‘rtburchakning perimetrini hisoblaydigan formula tuzing.

8. To‘g‘ri to‘rtburchakning perimetri formulasidan foydalanib, uning perimetrini toping:

   a) 4,5 m va 3 m; b) 30 sm va 18 sm;

   d) 8,4 sm va 1,6 sm; e) 42 dm va 82 sm.

9. a soni b sonining $p\%$ iga teng. a ni p va b lar orqali ifodalovchi formulani yozing. Topilgan formula orqali sonli masala tuzib, uni yeching.

10. Har qanday toq sonni ifodalovchi formula tuzing. Uni misollarda ko‘rsating.

11. Quyidagi songa karrali sonning formulasini tuzing:

    a) 3 ga karrali; b) 5 ga karrali; d) 10 ga karrali.

12. 7 ga karrali sonning formulasini yozing. 7 ga karrali ikkita uch xonali sonni yozing.

3-§. Ifodalarni aynan almashtirish. Ayniyat

$x(y+7)$ va $xy+7x$ ifodalarni ko‘rib chiqamiz. Ularning $x=9$ va $y=-2$ bo‘lgandagi qiymatlarini hisoblaymiz:

$$x(y+7)=9\cdot(-2+7)=45$$

$$xy+7x=9\cdot(-2)+7\cdot 9=45.$$

Ko‘rinib turibdiki, $x=9$ va $y=-2$ bo‘lganda $x(y+7)$ va $xy+7x$ ifodalarning mos qiymatlari bir-biriga teng.

O‘zgaruvchilarning har qanday qiymatlarida bu ifodalarning mos qiymatlari bir-biriga teng. Bunday ifodalar **aynan teng ifodalar** bo‘ladi.

Ta’rif. **Agar o‘zgaruvchilarning qiymati har qanday bo‘lganda ham ikki ifodaning mos qiymatlari bir-biriga teng bo‘lsa, bunday ikki ifoda aynan teng ifodalar deyiladi.**

Tenglamalar yechishda, ifodalarning qiymatlarini hisoblashda va boshqa hollarda bir ifodaga aynan teng bo'lgan boshqa ifodaga almashtiriladi. Bir ifodani unga aynan teng bo'lgan boshqa ifodaga almashtirish **aynan almashtirish** yoki soddagina qilib **ifodani almashtirish** deyiladi. O'zgaruvchili ifodalar sonlar ustida bajariladigan amallarning xossalariga asosan almashtiriladi.

Ifodalarni aynan almashtirishda o'xshash qo'shiluvchilarni ixchamlash, qavslarni ochish va o'rin almashtirish qonunlaridan foydalaniladi.

1-misol. $5x+2x-3x$ yig'indida o'xshash qo'shiluvchilarni ixchamlaymiz.

Ma'lumki, o'xshash qo'shiluvchilarni ixchamlash uchun ularning koeffitsiyentlarini qo'shish va natijani ularning umumiy harfiy qismiga ko'paytirish kerak. Masalan, $5x+2x-3x=(5+2-3)x=4x$. Bu almashtirish ko'paytirishning taqsimot xossasiga asoslangan.

2-misol. $2a-(b-3a)$ ifodada qavsni ochib, o'xshash qo'shiluvchilar ixchamlanadi. Oldida «minus» ishorasi turgan qavslarni ochish qoidasidan foydalanib, unda qavsni tashlab, qavs ichidagi hadlar qarama-qarshi ishora bilan yoziladi va o'xshash qo'shiluvchilar ixchamlanadi: $2a-(b-3a)=2a-b+3a=2a+3a-b=(2+3)a-b=5a-b$.

Bundan keyingi ixchamlashlarda $2a+3a$ ifodani bir yo'la $5a$ kabi yozamiz.

Agar $5(a-b)-3b$ ifodada qavslar ochilsa, so'ngra o'xshash qo'shiluvchilar ixchamlansa, unda bu ifodaga aynan teng bo'lgan $5a-8b$ ifoda hosil bo'ladi, ya'ni $5(a-b)-3b=5a-5b-3b=5a-8b$.

Demak, $5(a-b)-3b=5a-8b$ aynan teng bo'lgan ikki ifoda.

Bu aynan teng ifodalar o'zgaruvchilarning har qanday qiymatlarida to'g'ri bo'ladi. Bunday tengliklar **ayniyat** deyiladi.

Ta'rif. **O'zgaruvchilarning qiymati har qanday bo'lganda ham to'g'ri bo'lgan tenglik ayniyat deyiladi.**

Sonli to'g'ri tengliklar ham ayniyat bo'ladi. Ayniyatga misollar keltiramiz:

$a+b=b+a$; $a+(-a)=0$; $a(-b)=-ab$;

$a(bc)=(ab)c=(ac)b$; $(-a)(-b)=ab$; $a\cdot 1=a$.

Ayniyatni isbotlash uchun ifodalarni aynan almashtirishdan foydalaniladi.

Masalan, $7(2+b)-(14-b)=8b$ ayniyatni isbotlaymiz.

$7(2+b)-(14-b)=14+7b-14+b=8b$. Demak bu tenglik ayniyat.

Ayniyatni isbotlashda ba'zan tenglikning ikkala qismida ham shakl almashtiriladi. Masalan, $d(c-a)+ab=a(b-d)+cd$ ayniyatni isbotlaymiz:

$d(c-a)+ab=cd-ad+ab$;

$a(b-d)+cd=ab-ad+cd=cd-ad+ab$.

Berilgan tenglikning chap qismi ham, o'ng qismi ham ayni bir ifodaga aynan teng. Demak, berilgan tenglik ayniyat ekan.

Ba'zida tenglikning bir qismidan ikkinchi qismi ayrilib, ayirmani nolga tengligi ko'rsatiladi (teng ifodalar ayirmasi nol bo'lgani uchun).

Masalan, $11(2-3a)+6a=22-27a$ tenglikni ayniyat ekanligini isbot qilamiz.

Isbotlash uchun chap tomonidan o'ng tomonini ayiramiz:

$$11(2-3a)+6a-(22-27a)=22-33a+6a-22+27a=(22-22)+$$
$$+(-33a+6a+27a)=0+(-33a+33a)=0+0=0.$$

Demak, berilgan tenglik ayniyat bo'ladi.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday ifodalarni aynan teng ifodalar deyiladi?
2. Aynan shakl almashtirish qachon amalga oshiriladi?
3. O'xshash qo'shiluvchilarni ixchamlash uchun nima qilinadi?
4. Qanday tenglikni ayniyat deyiladi?
5. Ayniyatga misollar keltiring.

MASALALARNI YECHING

13. Quyidagi ifodalar aynan teng ifodalarmi?

a) $2c\cdot 3$ va $6c$; e) $2x=14$ va $2(x+7)$;
b) $7+(a+b)$ va $(7+a)+b$; f) $(a+b)\cdot 0$ va $a+b$;
d) $x-y$ va $y-x$; g) $(a+b)\cdot 1$ va $a+b$.

14. Ko'paytirishning o'rin almashtirish va guruhlash xossalaridan foydalanib, ifodani soddalashtiring:

a) $6{,}2a\cdot 5$; d) $0{,}3x\cdot(-12y)$;
b) $4c\cdot(-1{,}25)$; e) $-0{,}1b\cdot(-2{,}3c)$.

15. Ko'paytirishning taqsimot xossasidan foydalanib, ifodani aynan teng ifodaga almashtiring:

a) $7(x-y)$;
b) $1{,}2(a-4b)$;
d) $-23\cdot(2a-3b+1)$;
e) $-0{,}1(100a+10b-c)$.

16. O'xshash qo'shiluvchilarni ixchamlang:

a) $5a+27-a$;
b) $12b-17b-b$;
d) $6x-14-13x+26$;
e) $-8-y+17-10y$.

17. Qavslarni oching:

a) $x+(b+c+d-m)$;
b) $a-(b-c-d)$;
d) $x+y-(b+c-m)$;
e) $x+(a-b)-(c+d)$.

18. Qavslarni ochib, o'xshash qo'shiluvchilarni ixchamlang:

a) $x+(2x+0{,}5)$;
b) $13x-(5x-7)$;
d) $4a-(a+6)$;
e) $6b+(10-4{,}5b)$.

19. Ifodani soddalashtiring:

a) $(x-1)+(12-7{,}5x)$;
b) $(2p+1{,}9)-(7-p)$;
d) $(3-0{,}4a)-(10-0{,}8a)$;
e) $b-(4-2b)+(3b-1)$;
f) $y-(y+4)+(y-4)$;
g) $4x-(1-2x)+(2x-7)$.

20. Qavslarni ochib, o'xshash qo'shiluvchilarni ixchamlang:

a) $3(6-5x)+17x-10$;
b) $8(3y+4)+29y+14$;
d) $2(7{,}3-1{,}6a)+3{,}2a-9{,}6$;
e) $-4(3{,}3-8c)+4{,}8c+5{,}2$.

21. Ifodani soddalashtiring va uning qiymatini toping:

a) $0{,}6(p-3)+p-2$, bunda $p=0{,}5$;

b) $4(0{,}5q-6)-14q+21$, bunda $q=\frac{1}{3}$;

d) $-0{,}5(3a+4)+1{,}9a-1$, bunda $a=-\frac{1}{4}$;

e) $10(0{,}7-3b)+14b+13$, bunda $b=-16$.

22. Quyidagi tenglik ayniyatmi?

a) $6(x-y)=6x-6y$;
b) $25(a-a)=25$;
d) $3a-4=(2a-4)+a$;
e) $a\cdot 5b=5ab$.

23. Tenglikning ayniyat ekanligini isbotlang:

a) $7x-42=7(x-6)$; d) $7(2b-3)+18=14b-3$;

b) $3(2m-1)=2(3m-1,5)$; e) $3a=6b+3(a-2b)$.

24. Ayniyatni isbotlang:

a) $x(y-2)-3y=y(x-3)-2x$;

b) $2a(b-5)+b=b(2a+1)-10a$.

25. $4p(1-q)+5q$ va $q(5-4p)+4p$ ifodalarning aynan tengligini isbot qiling.

26. Quyidagi tenglikni bir tomonidan ikkinchi tomonini ayirish bilan isbotlang:

a) $3(2m-4)=2(3m-6)$;

b) $2(2,5a+10b)=5(4b+a)$;

d) $12x-1=4(15+3x)-61$;

e) $8(2x+5)+7=47+16x$.

27. Ifodaning qiymatini aynan shu songa tengligini ko‘rsating:

a) $\frac{3}{8}\cdot 2,4+\frac{2}{3}\cdot 0,15=1$;

b) $2,08:\frac{2}{3}-0,15\cdot\frac{4}{5}=3$;

d) $(1,25\cdot 1,7\cdot 0,8-1,7)\cdot 3,45=0$;

e) $3,947:(3,6-2,6\cdot 4\cdot 0,25)=3,947$;

f) $19,6\cdot 2\frac{1}{5}+\left(5,25\cdot 1\frac{1}{5}-4,5\cdot\frac{4}{5}\right)=45,82$.

4-§. Natural ko'rsatkichli darajaning ta'rifi

Bir nechta bir xil ko'paytuvchilarning ko'paytmasi $5\cdot5\cdot5\cdot5\cdot5\cdot5\cdot5=5^7$ ko'rinishda yoziladi. Bundagi 5^7 ifoda: «Beshning yettinchi darajasi» deb o'qiladi.

Ta'rif. **a sonining natural n ko'rsatkichli darajasi deb har biri a ga teng bo'lgan n ta ko'paytuvchilarning ko'paytmasiga aytiladi.**

a sonining n ko'rsatkichli darajasi a^n ko'rinishida yoziladi. Bunda: a^n **– daraja, a – darajaning asosi, n – daraja ko'rsatkichi** deb aytiladi.

Masalan, $a^1=a$; $a^2=a\cdot a$; $a^3=a\cdot a\cdot a$; ...

Umuman, $a^n=\underbrace{a\cdot a\cdot a...a}_{n\ \text{ta}}$

Misollar: $3^4=3\cdot3\cdot3\cdot3=81$; $(-6)^3=(-6)\cdot(-6)^{-6}=-216$.

Bundagi 81 va – 216 – darajaning qiymatlari.

1) Ravshanki, musbat sonni darajaga ko'targanda musbat son hosil bo'ladi. Masalan, $4^3=4\cdot4\cdot4=64$.

2) Manfiy sonni darajaga ko'targanda musbat son ham, manfiy son ham hosil bo'ladi. Masalan,

$(-2)'=-2$

$(-2)^2=(-2)\cdot(-2)=4$.

$(-2)^3=(-2)\cdot(-2)\cdot(-2)=-8$.

$(-2)^4=(-2)\cdot(-2)\cdot(-2)\cdot(-2)=16$.

a) Ko'rsatkichi juft bo'lgan manfiy sonning darajasi musbat bo'ladi, chunki juft dona manfiy ko'paytuvchilarning ko'paytmasi musbat bo'ladi ($(-2)^2=4$; $(-2)^6=64$).

b) Ko'rsatkichi toq bo'lgan manfiy sonning darajasi manfiy son bo'ladi, chunki toq dona manfiy ko'paytuvchilarning ko'paytmasi manfiy bo'ladi ($(-2)^3=-8$; $(-2)^7=-128$).

Har qanday sonning kvadrati musbat son yoki nol, ya'ni a har qanday bo'lganda ham $a^2 \geq 0$ bo'ladi.

1-misol. $4 \cdot 10^3$ ifodaning qiymatini topamiz:

1) $10^3 = 10 \cdot 10 \cdot 10 = 1000$; 2) $4 \cdot 1000 = 4000$.

2-misol. $-2^6 + (-3)^4$ ifodaning qiymatini topamiz:

1) $2^6 = 64$; 3) $(-3)^4 = 81$;

2) $-2^6 = -64$; 4) $-64 + 81 = 17$. Demak, $-2^6 + (-3)^4 = 17$.

TAKRORLASH UCHUN SAVOLLAR

1. a sonining natural n ko'rsatkichli darajasi deb nimaga aytiladi?
2. Manfiy sonning juft darajasi qanday son bo'ladi?
3. Manfiy sonning toq darajasi qanday son bo'ladi?
4. Og'zaki hisoblang:

a) 3^3; b) $(-3)^2$; d) $(-3)^3$; e) $(-4)^3$; f) $(-4)^4$.

MASALALARNI YECHING

28. Ko'paytmani daraja ko'rinishida yozing:

a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$; d) $b \cdot b \cdot b \cdot b \cdot b \cdot b$; f) $\left(-\frac{2}{5}\right) \cdot \left(-\frac{2}{5}\right) \cdot \left(-\frac{2}{5}\right)$;

b) $(-5) \cdot (-5) \cdot (-5) \cdot (-5)$; e) $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$; g) $(xy) \cdot (xy) \cdot (xy) \cdot (xy)$.

29. Darajaning asosini va ko'rsatkichini ayting:

a) $5{,}2^3$; d) $(-100)^5$; f) $(-a)^5$;

b) $(-0{,}1)^4$; e) $(-0{,}1)^7$; g) $\left(\frac{2}{3}y\right)^6$.

30. Darajani hisoblang:

a) 2^4; d) 5^3; f) $(7{,}1)^2$; h) $\left(\frac{3}{4}\right)^4$;

b) 4^3; e) $(-3)^5$; g) $(-1{,}5)^3$; i) $\left(-\frac{2}{5}\right)^5$.

31. Jadvalni to'ldiring:

n	1	2	3	4	5	6	7	8	9	10
2^n										
3^n										

32. a) 0,25; 0,64; 121; 225; $\frac{16}{49}$; $1\frac{24}{25}$; 0,0004 sonni kvadrat ko‘rinishida ifoda qiling:

b) 27; 125; 0,008; –216; $-\frac{1}{64}$; $4\frac{17}{27}$ sonni kub ko‘rinishida yozing.

33. Amallarni bajaring:

a) $7\cdot 4^2$; d) $(-0,4)^3$; f) $-3\cdot 2^5$; h) $(-0,1)^4$;

b) $(7\cdot 5)^2$; e) $-0,4^3$; g) $-6^2\cdot(-12)$; i) $-0,1^4$.

34. Hisoblang:

a) 7^2+3^3; e) $10-5\cdot 2^4$; h) $5\frac{1}{16}\cdot\left(\frac{4}{3}\right)^4+9$;

b) 8^2-4^3; f) $2^3\cdot 5-5^3\cdot 2$; i) $3^4-\frac{2}{5}\cdot\left(2\frac{1}{2}\right)^2$;

d) $(8+3)^3$; g) $-6^2-(-3)^3$; j) $8\cdot 0,5^3+25\cdot 0,2^3$.

35. $2x^4-5x^3+x^2+3x+2$ ifodaning $x=5$; -5 bo‘lgandagi qiymatini toping.

5-§. Darajalarni ko‘paytirish va bo‘lish

a^2a^3 ifodani bir xil asosli daraja ko‘rinishida yozamiz:

$a^2a^3=(a\cdot a)\cdot(a\cdot a\cdot a)=a\cdot a\cdot a\cdot a\cdot a=a^5$.

Demak, $a^2\cdot a^3=a^{2+3}=a^5$.

a^2a^3 ko‘paytma xuddi shunday asosli darajaga teng bo‘lib, uning ko‘rsatkichi ko‘paytuvchilar daraja ko‘rsatkichlarining yig‘indisiga teng.

Teorema. ***Har qanday a soni va ixtiyoriy natural m va n sonlar uchun $a^ma^n=a^{m+n}$ bo‘ladi.***

Isbot. Darajaning ta’rifi va ko‘paytirish xossasidan foydalanib, a^ma^n ifodani dastlab har qaysisi a ga teng ko‘paytuvchilarning ko‘paytmasi ko‘rinishida, so‘ngra daraja ko‘rinishida ifoda qilamiz:

$$a^ma^n=\underbrace{aaa...a}_{m\text{ ta}}\cdot\underbrace{aaa...a}_{n\text{ ta}}=\underbrace{aaa...a}_{(m+n)\text{ ta}}=a^{m+n}.$$

Shunday qilib, $a^m a^n = a^{m+n}$ (1).

Isbotlangan bu tenglik ***darajaning asosiy xossasi*** deyiladi.

Bu xossa uchta va undan ham ko'p darajalar ko'paytmasi uchun to'g'ri bo'ladi, ya'ni $a^m a^n a^k = a^{m+n} a^k = a^{(m+n)+k} = a^{m+n+k}$.

Masalan, $m^3 m^4 m^5 = m^{3+4+5} = m^{12}$.

Demak, ***bir xil asosli darajalarni ko'paytirishda asosi avvalgicha qoldirilib, darajalarning ko'rsatkichlari bir-biriga qo'shiladi.***

Masalan: $x^7 x^6 = x^{7+6} = x^{13}$; $yy^{16} = y^1 y^{16} = y^{1+16} = y^{17}$; $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{3+2} = \left(\frac{2}{3}\right)^5 =$

$= \frac{32}{243}$; $0{,}2^3 \cdot 0{,}04 = 0{,}2^3 \cdot 0{,}2^2 = 0{,}2^{3+2} = 0{,}2^5 = 0{,}00032$.

Bir xil asosli ikki darajaning bo'linmasini ko'rib chiqamiz:

$$\frac{a^7}{a^4} = \frac{aaaaaaa}{aaaa} = \frac{aaa}{1} = a^3.$$

Demak, $a^7 : a^4 = a^3 = a^{7-4}$; yoki $a^7 : a^4 = a^{7-4} = a^3$.

Biz bildikki, $a^7 : a^4$ bo'linma xuddi shunday asosli darajaga teng bo'lib, uning ko'rsatkichi bo'linuvchi bilan bo'luvchi ko'rsatkichlarining ayirmasiga teng.

Teorema. ***Har qanday $a \neq 0$ soni va $m > n$ kabi ixtiyoriy natural m va n sonlari uchun $a^m : a^n = a^{m-n}$ bo'ladi.***

Isbot. $a^m : a^n = a^{m-n}$ tenglikni isbotlash uchun bo'lish amalining ta'rifiga asosan $a^{m-n} \cdot a^n = a^m$ ekanligini ko'rsatish kifoya. Haqiqatan ham, darajaning asosiy xossasiga asosan, $a^{m-n} \cdot a^n = a^{(m-n)+n} = a^{m-n+n} = a^m$.

Demak, $a^m : a^n = a^{m-n}$. (2).

Shunday qilib, ***bir xil asosli darajalarni bo'lishida asosni avvalgicha qoldirib, bo'linuvchining daraja ko'rsatkichidan bo'luvchining daraja ko'rsatkichi ayriladi.***

Masalan: $c^{10} : c^8 = c^{10-8} = c^2$; $p^8 : p = p^8 : p^1 = p^{8-1} = p^7$.

$0{,}3^5 : 0{,}3^2 = 0{,}3^{5-2} = 0{,}3^3 = 0{,}027$; $c^n : c^{11} = c^{n-11}$; $c^7 : c^m = c^{7-m}$.

Agar $m=n$ bo‘lsa, formula $a^n : a^n = a^{n-n} = a^0 = 1$ bo‘lib, bo‘linma 1 ga teng bo‘ladi.

Ta'rif. **Noldan boshqa har qanday sonning nolinchi darajasi 1 ga teng, ya'ni $a^0 = 1$.**

Masalan: $7^0 = 1$; $(-3{,}5)^0 = 1$; $\left(-3\frac{2}{11}\right)^0 = 1$; $(-2a^2b)^0 = 1$.

TAKRORLASH UCHUN SAVOLLAR

1. Bir xil asosli natural ko‘rsatkichli darajalarning ko‘paytmasi nimaga teng?
2. Bir xil asosli natural ko‘rsatkichli darajalarning bo‘linmasi nimaga teng?
3. Har qanday (noldan boshqa) sonning nolinchi darajasi nimaga teng?
4. Og‘zaki hisoblang: a) $3^7 : 3^5$; b) $5^8 - 5^7$; d) $11^4 - 11^3$.

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36. Ko‘paytmani daraja ko‘rinishida yozing:

a) x^5x^8; d) y^4y^9; f) x^9x; h) $2^6 \cdot 2^4$;
b) $x^4 \cdot x^3$; e) b^8b^{15}; g) yy^{11}; i) $7^0 \cdot 7^{12}$.

37. Ko‘paytmani daraja ko‘rinishida yozing:

a) $x^3x^5x^2$; b) y^3yy^4; d) $a^4a^5a^6$; e) $5 \cdot 5^3 \cdot 5^9$.
f) $7^3 \cdot 7^4 \cdot 7$; g) $n^5n^4n^2$; h) $0{,}4^3 \cdot 0{,}4^2 \cdot 0{,}4^6$; i) $\left(\frac{2}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^5$

38. Ifodani daraja ko‘rinishida yozing:

a) $5^8 \cdot 25$; d) $6^{15} \cdot 36$; f) $0{,}4^5 \cdot 0{,}16$;
b) $3^{12} \cdot 27$; e) $2^9 \cdot 32$; g) $0{,}001 \cdot 0{,}1^7$.

39. Ifodani 2 asosli daraja ko‘rinishida yozib, qiymatini toping:

a) $2^4 \cdot 2$; b) $2^6 \cdot 4$; d) $8 \cdot 2^7$; e) $16 \cdot 32 \cdot 2^2$.

40. Bo‘linmani daraja ko‘rinishida ifodalang:

a) $x^5 : x^3$; d) $a^{21} : a$; f) $c^{12} : c^3$; h) $3^8 : 3^5$;
b) $y^{10} : y^7$; e) $b^{19} : b^{18}$; g) $p^{20} : p^{10}$; i) $0{,}7^9 : 0{,}7^4$.

41. Darajani xuddi shu asosli darajalar bo‘linmasi ko‘rinishida biror usul bilan yozing:

a) c^2; b) x^4; d) 2^5; e) $0{,}5^4$; f) $(-1{,}2)^3$ g) $\left(\frac{2}{3}\right)^6$.

42. Ifodaning qiymatini toping:

a) $5^6:5^4$; d) $0{,}5^{10}:0{,}5^7$; f) $2{,}73^{13}:2{,}73^{12}$;

b) $10^{15}:10^{13}$; e) $\left(1\frac{1}{3}\right)^8:\left(1\frac{1}{3}\right)^6$; g) $\left(-\frac{2}{3}\right)^7:\left(-\frac{2}{3}\right)^4$.

43. Hisoblang:

a) $\frac{7^9\cdot7^5}{7^{12}}$; b) $\frac{3^{15}}{7^5\cdot3^6}$; d) $\frac{5^{16}\cdot5^4}{5^{18}}$; e) $\frac{0{,}6^8\cdot0{,}6^4}{0{,}6^6\cdot0{,}6^4}$.

44. Hisoblang:

a) $7^0+3\cdot2^5$; b) $(4\cdot2^{10})^0$; d) $(8^0-2\cdot3)^3$; e) $-4^2-12\cdot6^0$.

6-§. Ko‘paytmani, bo‘linma (kasr)ni va darajani darajaga ko‘tarish

$(ab)^4$ ifoda a va b ko‘paytuvchilar ko‘paytmasining to‘rtinchi darajasi. Bu ifodani $(ab)^4=(ab)\cdot(ab)\cdot(ab)=(aaaa)\cdot(bbbb)+a^4b^4$ ko‘rinishda yozish mumkin.

Demak, $(ab)^4=a^4b^4$ ab ko‘paytmaning to‘rtinchi darajasi to‘rtinchi darajali a va b ko‘paytuvchilarning ko‘paytmasiga teng.

Teorema. ***Har qanday a va b, hamda ixtiyoriy natural n soni uchun $(ab)^n=a^nb^n$ bo‘ladi.***

Isbot. Darajaning ta’rifiga muvofiq

$$(ab)^n=\underbrace{(ab)\cdot(ab)\cdot\ldots\cdot(ab)}_{n\text{ ta}}=\underbrace{(aa\ldots a)}_{n\text{ ta}}\cdot\underbrace{(bb\ldots b)}_{n\text{ ta}}=a^n\cdot b^n.$$

Demak, $(ab)^n=a^nb^n$.

Ko‘paytma darajasining $(ab)^n=a^nb^n$ xossasi uchta va undan ortiq ko‘paytuvchilar ko‘paytmasining darajasi uchun ham to‘g‘ridir. Masalan, $(2yz)^5=2^5y^5z^5=32y^5z^5$.

Shunday qilib, ***ko‘paytmani darajaga ko‘tarishda har bir ko‘paytuvchi shu darajaga ko‘tarilib, darajalar bir-biriga ko‘paytiriladi.***

Masalan: a) $(3ab)^3=3^3a^3b^3=27a^3b^3$;

b) $(-2xyz)^8=(-2)^8x^8y^8z^8=256x^8y^8z^8$.

$(a:b)^4$ – bo'linmaning darajasini kasrning darajasi $\left(\frac{a}{b}\right)^4$ bilan almashtiramiz.

$$(a:b)^4=\left(\frac{a}{b}\right)^4=\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}=\frac{aaaa}{bbbb}=\frac{a^4}{b^4}$$

Demak, $\frac{a}{b}$ bo'linmaning (kasrning) to'rtinchi darajasi suratining to'rtinchi darajasini maxrajining to'rtinchi darajasiga bo'linganiga teng.

Teorema. ***Har qanday a va b, hamda ixtiyoriy natural n soni uchun*** $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$ ***bo'ladi.***

Isbot. Darajaning ta'rifiga muvofiq

$$\left(\frac{a}{b}\right)^n=\frac{a}{b}\cdot\frac{a}{b}\cdot\ldots\cdot\frac{a}{b}=\frac{aa\ldots a}{bb\ldots b}=\frac{a^n}{b^n}.$$

Demak, $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$.

Masalan: 1) $\left(\frac{x}{y}\right)^3=\frac{x^3}{y^3}$; 2) $\left(\frac{2y}{3c}\right)^4=\frac{(2y)^4}{(3c)^4}=\frac{2^4\cdot y^4}{3^4c^4}=\frac{16y^4}{81c^4}$

Shunday qilib, ***kasrni darajaga ko'tarishda uning surat va maxrajlari shu darajaga ko'tariladi.***

3. $(a^3)^4$ ifoda asosining o'zi daraja bo'lgan darajadir. Bu ifodani asosi a bo'lgan daraja ko'rinishida yozamiz:

$(a^3)^4=a^3a^3a^3a^3=a^{3+3+3+3}=a^{3\cdot4}=a^{12}$.

a^3 darajani to'rtinchi darajaga ko'tarishda a asosli daraja hosil qildik, uning ko'rsatkichi 3 va 4 ko'rsatkichlar ko'paytmasiga teng bo'ladi.

Teorema. ***Har qanday a soni va ixtiyoriy natural m va n sonlar uchun*** $(a^m)^n=a^{mn}$ ***bo'ladi.***

Isbot. Darajaning ta'rifiga asosan,

$$(a^m)^n=a^ma^m\ldots a^m=a^{\overbrace{m+m+\ldots+m}^{n\text{ ta}}}=a^{mn}.$$

Demak, $(a^m)^n = a^{mn}$.

Masalan: 1) $(x^5)^3 = x^{5\cdot 3} = x^{15}$; 2) $((-2)^3)^2 = (-2)^{3\cdot 2} = (-2)^6 = 64$; 3) $(y^2y^5)^3 = (y^2)^3(y^5)^3 = y^6y^{15} = y^{6+15} = y^{21}$.

Shunday qilib, darajani darajaga koʻtarishda asosi avvalgicha qoldirilib, koʻrsatkichlari bir-biriga koʻpaytiriladi.

TAKRORLASH UCHUN SAVOLLAR

1. Koʻpaytmaning darajasi haqidagi teoremani ayting.
2. Kasrning darajasi haqidagi teoremani ayting.
3. Darajaning darajasi haqidagi teoremani ayting.
4. Ogʻzaki darajaga koʻtaring:

a) $(xy)^3$; b) $(3c)^4$; d) $\left(\frac{3}{4}d\right)^2$; e) $(-3x^2)^3$; d) $(-2^2)^3$.

MASALALARNI YECHING

45. Darajaga koʻtaring:

a) $(2x)^3$; d) $\left(\frac{2x}{y}\right)^3$; f) $(-5\,xy)^3$; h) $(2\cdot 10)^3$;

b) $(-3y)^4$; e) $\left(-\frac{3c}{4d}\right)^4$; g) $(-0{,}5\,bd)^4$; i) $(-3\cdot 0{,}1)^4$.

46. Koʻpaytmani daraja koʻrinishida ifoda qiling:

a) b^3x^3; d) $x^2y^2z^2$; f) $32\,a^5$; h) $\frac{27x^3}{y^3}$;

b) $a^7\cdot y^7$; e) $-a^3b^3$; g) $0{,}027\,m^3$; i) $\frac{c^5}{32d^5}$.

47. Ifodaning qiymatini toping:

a) $2^4\cdot 5^4$; d) $0{,}25^{15}\cdot 4^{15}$; f) $\left(\frac{5}{7}\right)^{10}\cdot 1{,}4^9$;

b) $4^3\cdot 25^3$; e) $\left(\frac{2}{3}\right)^7\cdot 1{,}5^7$; g) $0{,}2^6\cdot 50^7$.

48. Darajaga ko'taring:

a) $(x^3)^2$; d) $(a^5)^4$; f) $(y^7)^3$; h) $\left(\frac{c^2}{d^5}\right)^5$;

b) $(x^2)^3$; e) $\left(\left(\frac{2}{3}\right)^3\right)^4$; g) $(-2^3)^4$; i) $\left(-\frac{b^2}{c^4}\right)^3$.

49. Ifodani asosi a bo'lgan daraja ko'rinishida yozing.

a) a^3a^n; d) $a^m a^3$; f) $\frac{a^3}{a^m}$; h) $(a^2)^m$;

b) aa^m; e) $a^{-3}an$; g) $\frac{a^n}{a^5}$; i) $(a^n)^3$.

50. 2^{20} ni asosi: a) 2^2; b) 2^4; d) 2^5; e) 2^{10} bo'lgan daraja ko'rinishida yozing.

51. 2^{60} ni asosi: a) 4; b) 8; d) 16; e) 32 bo'lgan daraja ko'rinishida yozing.

52. a^{12} ifodani bir necha usulda daraja ko'rinishida yozing.

53. Ifodani soddalashtiring:

a) $x^3 \cdot (x^2)^5$; d) $(a^2)^3 \cdot (a^4)^2$; f) $\left(\frac{a^5}{a^3}\right)^4$;

b) $(a^3)^2 \cdot a^5$; e) $(m^3m^4)^5$; g) $\left(\frac{2b^7}{b^4}\right)^5$.

54. Ifodani soddalashtiring:

a) $x^5 \cdot (x^2)^3$; b) $(x^3)^4 \cdot x^8$; d) $(x^4)^3 \cdot (x^5)^2$; e) $\left(\frac{y^5}{y^2}\right)^7$; f) $\frac{(y^6)^5}{(y^3)^4}$.

55. Ifodaning qiymatini toping:

a) $\frac{2^5 \cdot (2^3)^4}{2^{13}}$; b) $\frac{(5^8)^2 \cdot 5^7}{5^{22}}$; d) $\frac{25^4 \cdot 5^6}{125^2 \cdot 5^5}$; e) $\frac{9^5 \cdot 27}{(3^4)^3}$.

56. Agar kvadratning tomoni 2 marta orttirilsa, uning yuzi qanday o'zgaradi? 3 marta; 10 marta; n marta orttirilsa-chi?

7-§. Birhad va uning standart shakli

24, 5 a, $2b^2c$, $-3xy^3$, $3\frac{5}{7}\,a^2bc$ va boshqa ifodalar bilan tanishmiz.

Sonlar, o'zgaruvchilar (harflar) ularning darajalari hamda ko'paytmalari birhad deyiladi.

$2b^2(-3)bc$ birhadda ko'paytuvchilarning joylarini o'zgartiramiz va 2 bilan (–3) ning ko'paytmasini – 6 bilan, b^2 va b ko'paytuvchilarni b^3 bilan almashtiramiz. Natijada $2b^2(-3)bc=2\cdot(-3)b^2bc=-6b^3c$ ifoda hosil bo'ladi.

Birhadni birinchi ko'paytuvchisi son va har xil o'zgaruvchilarning darajalari ko'paytmasi ko'rinishida ifodalanganiga birhadning ***standart shakli*** deyiladi.

Masalan $-6b^3c$; $\frac{3}{8}x^5y$; $-0{,}13\,cd^2e^3$ va hokazo.

$7a^2b^3c$ birhadda hamma o'zgaruvchilar daraja ko'rsatkichlarining yig'indisi 6(1+2+3) ga teng. 6 soni $7a^2b^3c$ birhadning darajasi deyiladi.

Demak, ***birhadning darajasi deb birhad tarkibidagi barcha o'zgaruvchilar daraja ko'rsatkichlarining yig'indisiga aytiladi.***

Agar birhadda o'zgaruvchi bo'lmasa (birhad faqat son bo'lsa), unda birhadning darajasi nolga teng bo'ladi, masalan 17 birhadning darajasi nol bo'ladi.

Standart shaklida yozilgan birhadning son ko'paytuvchisi birhadning ***koeffitsiyenti*** deyiladi. Masalan, $-7{,}3\,b^4c^2$ *birhadning* ***koeffitsiyenti*** – 7,3 ga teng; – ab birhadning koeffitsiyenti – 1 ga teng.

Ta'rif. **Bir necha birhadning algebraik yig'indisi ko'phad deyiladi. Masalan, $2a^2$; $-4bc^3$; 4,7 ab^3 birhadlarning yig'indisi $2a^2+(-4bc^3)+4{,}7\,ab^3c$ ko'phad bo'ladi.**

TAKRORLASH UCHUN SAVOLLAR

1. Birhad deb nimaga aytiladi?
2. Birhadning standart shakli deb nimaga aytiladi? Misollar keltiring.
3. Birhadning darajasi deb nimaga aytiladi? Misollar keltiring.
4. Birhadning koeffitsiyenti deb nimaga aytiladi? Misollar keltiring.
5. $-7{,}4\, b^2c^3$ birhadning: a) koeffitsiyentini ayting; b) darajasini ayting.

MASALALARNI YECHING

57. Quyidagi ifoda birhadmi?

a) $3{,}4\, x^2y$; e) x^2+y; h) $-m$;

b) $-0{,}7\, xy^3$; f) x^3x; i) $2(x+y)^2$;

d) $a(-8)b^2$; g) $-\frac{3}{4}\, m^3nm^2$; j) $a-c$.

58. Birhad standart shaklida yozilganmi?

a) $6\, xy$; d) $0{,}5\, m2n$; f) $-x^2y^3$;

b) $-2aba$; e) $-bca$; g) $5\, p^3p$.

59. Birhadni standart shaklida yozing va uning koeffitsiyentini ayting:

a) $8x^2x$; d) $3xy(-1{,}7)y$; f) $\frac{2}{3}m^2n\cdot 4{,}5\, n^3$;

b) $1{,}2abc5a$; e) $6c^2(-0{,}8c)$; g) $2\frac{1}{3}a^2x\left(-\frac{3}{7}\right)a^3x^2$.

60. Birhadning qiymatini toping:

a) $5x^3$, bunda $x=-3$; d) $12x^2y$, bunda $x=0{,}3$; $y=\frac{1}{6}$;

b) $-0{,}125y^4$, bunda $y=-2$; e) $-9x^5y^2$, bunda $x=-1$; $y=\frac{1}{3}$.

61. Birhadning darajasini ayting:

a) $-7x^5y^6$; d) $0{,}8\, mn^3n^2$; f) $-6m^7$;

b) $\frac{1}{3}\, abc$; e) ab^2c^3; g) 27.

62. $5x^3$; $-11x^2$; $1{,}2x$; -18 birhadlardan uchta ko‘phad tuzing.

8-§. O'xshash hadlarni ixchamlash

Masala. Bitta daftar a so'm. Karim 3 ta, Vali 7 ta, Lola esa 5 ta daftar oldi. Olingan hamma daftar qancha pul turadi?

Yechish. Ularning har biri $3a$; $7a$; $5a$ so'mdan to'ladi.

Olingan hamma daftar uchun $3a+7a+5a$ to'landi. Oxirgi ifodani tarqatish qonuniga asosan soddalashtiramiz: $3a+7a+5a=(3+7+5)a=$ $=15a$. Bundagi $5a$ ifoda $3a+7a+5a$ ifodaning soddalashgan ko'rinishi. Bu $3a$; $7a$; $5a$ ifodalarni o'xshash birhadlar deb ataymiz. Masalan, $5ax^2y^3$ va $10ax^2y^3$ birhadlar o'xshash, ammo $5ax^2y^3$ bilan $5a^2x^2y^3$ birhadlar o'xshash emas, chunki ular a ning daraja ko'rsatkichi bilan farq qiladi. O'xshash birhadlar faqat koeffitsiyentlari bilan farq qiladi.

***Ta'rif.* O'xshash hadlarning yig'indisini shu yig'indiga aynan teng bo'lgan birhadga almashtirish – o'xshash hadlarni ixchamlash deyiladi.**

Demak, o'xshash hadlarni ixchamlash – aynan shakl almashtirish ($3a+7a+5a=15a$ kabi).

1-misol. $3a^2b-a^2b+7{,}4a^2b=(3-1+7{,}4)a^2b=9{,}4a^2b.$

2-misol. $2x^2-x+4x^3+2x+5x^2+7$ bunga qo'shishning o'rin almashtirish va guruhlash qonunlarini qo'llaymiz; birinchi o'ringa darajasi yuqori had $4x^3$, so'ngra x^2 li ikkita o'xshash hadni yozamiz; undan keyin x li o'xshash hadlarni, nihoyat, ozodni yozamiz:

$$4x^3+(2x^2+5x^2)+(-x+2x)+7=4x^3+7x^2+x+7.$$

Bunda biz o'xshash hadlarni ixchamladik va x ning darajasi pasayib borishi bo'yicha yozdik.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday birhadlar o'xshash birhadlar deyiladi?
2. O'xshash hadlarni ixchamlash deb nimaga aytiladi?
3. O'xshash hadlarni ixchamlang (og'zaki):
   a) $4x+11x$; b) $7ab-15ab$; d) $8y^2c-10y^2c+30y^2c$.

MASALALARNI YECHING

63. Quyidagi ko'phadlarning o'xshash hadlarini ixchamlang:

a) $5a-2a$; d) $8m-5m$; f) $15ab+4ab-10ab$;

b) $8x-10x$; e) $-2q+2q$; g) $-25p^4-32p^4+48p^4$.

64. a) $2d^2-1,5d^2-3,5d^2$; d) $\frac{3}{4}a^5-\frac{1}{2}a^5-\frac{5}{8}a^5$;

b) $5q^4-1\frac{1}{2}q^4+6\frac{1}{2}q^4$; e) $\frac{2}{3}x^3+\frac{5}{6}x^3-\frac{1}{2}x^3$.

65. a) $11x^2+4x-x^2-4x$; d) $2y^2-3y+2y-y^2$;

b) $-a-5-2a+3$; e) $7m^2-n^2+2m^2-11n^2$.

66. a) $\frac{1}{3}x^2-\frac{1}{3}y+\frac{2}{3}x^2+\frac{1}{3}y$;

b) $0,3c^3-0,1c^2-0,5c^3$;

d) $5ab-4a^2b^2-8ab+3ab-ab^2-4a^2b^2$;

e) $-1\frac{2}{3}ab^3+2a^3b-4\frac{1}{2}a^2b-ab^3-\frac{1}{2}a^2b-a^3b$.

9-§. Birhadlarni va ko'phadlarni qo'shish

1. Quyidagi birhadlarni qo'shishni bajaramiz. $13x^3$; $10x$; $-5x^3$; -5; $6x$. Bulardan quyidagi yig'indini hosil qilamiz.

$13x^3+10x+(-5x^3)+(-5)+(6x)$. Bu ifodadagi qo'shish ishoralarini tashlab, qisqacha bunday yozamiz:

$13x^3+10x-5x^3-5+6x$. Bu ifodada $13x^2$ va $-5x^3$ o'xshash hadlarni ixchamlab, $8x^3+16x-5$-yig'indini hosil qilamiz va ko'phadni x ga nisbatan darajasi kamayib boradigan qilib joylashtiramiz.

Demak, birhadlarni qo'shish uchun, ularni birin-ketin algebraik yig'indi shaklida yozib chiqish kifoya. Hosil bo'lgan ifodada o'xshash hadlar bo'lsa, ular ixchamlanadi.

1-misol. $(3p^2q)+(-11p^2)+(0,3p^2)+(-8p^2q)+(-20)$ birhadlar yig'indisidagi o'xshash hadlarni ixchamlang: Buni algebraik yig'indi shaklida yozib, so'ngra o'xshash hadlarni ixchamlaymiz:

$3p^2q-11p^2+0,3p^2-8p^2q-20=(3p^2q-8p^2q)+(-11p^2+0,3p^2)-20=$
$=-5p^2q-10,7p^2-20.$

2. Quyidagi masalani ko'rib chiqamiz.

Birinchi savatda x olma bor.

Ikkinchi savatda $x+y$ olma bor.

Uchinchi savatda $x+y-27$ olma bor.

Uchala savatda $x+(x+y)+(x+y-27)$ olma bo'ladi.

Olingan javob bitta birhad bilan ikkita ko'phadning yig'indisidan iborat. Bu javobni soddalashtirishda ifodalarning har biri algebraik yig'indi ekanligidan ularni qo'shish qoidasiga muvofiq quyidagicha yozamiz:

$x+(x+y)+(x+y-27)=x+x+y+x+y-27=3x+2y-27$ hosil bo'ladi.

Demak, **ko'phadlarni qo'shish uchun, ularning hamma hadlarini ketma-ket algebraik yig'indi shaklida o'z ishoralari bilan yozib chiqiladi. Hosil bo'lgan ifodada o'xshash hadlar bo'lsa, ular ixchamlanadi.**

Izoh. Agar ifoda qavs bilan boshlanib, uning oldida hech qanday ishora bo'lmasa, plyus ishora bor deb tushuniladi, masalan:

$(a^2-3a+2)+(3a-7)=a^2-3a+2+3a-7=a^2-5.$

2-misol. $3xy+x^2+0{,}4y^2$ va $6xy-2x^2-2{,}6y^2$ ko'phadlarning yig'indisini topamiz: $(3xy+x^2+0{,}4y^2)+(6xy-2x^2-2{,}6y^2)$.

Qavslarni ochib, o'xshash hadlarni ixchamlaymiz:

$$(3xy+x^2+0{,}4y^2)+(6xy-2x^2-2{,}6y^2)=$$
$$=3xy+x^2+0{,}4y^2+6xy-2x^2-2{,}6y^2=$$
$$=3xy+6xy+x^2-2x^2+0{,}4y^2-2{,}6y^2=$$
$$=9xy-x^2-2{,}2y^2.$$

TAKRORLASH UCHUN SAVOLLAR

1. Birhadlarga misollar keltiring.
2. Birhadlar qanday qo'shiladi?
3. Ko'phadlar deb nimaga aytiladi?
4. Ko'phadlarga misollar keltiring.
5. Ko'phadlar qanday qo'shiladi?
6. Og'zaki ixchamlang.

a) $2a+7a$; b) $2a+(-7a)$; b) $8x^2y+(-10{,}3x^2y)$;

d) $(2x+y)+(2x-y)$; e) $(-1{,}8\,xy^3)+(-8{,}2\,xy^3)$.

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67. Quyidagi birhadlarni qo'shing:

a) $4a^2+(-6a^2)+8a^2$; b) $-7ab^2+(-5ab^2)$;

d) $-0{,}3x^2+(-6{,}7x^2)+11x^2$; e) $15c^3+(-23c^3)+20c^3$.

68. a) $7{,}5a^2+(-2a^2)+(-14{,}7a^2)$;

b) $(-8xy)+10xy+(-3xy)$;

d) $3a^2b+(-a^2b)+2a^2b+(-6a^2b)$;

e) $(-0{,}8q+(-1{,}6q)+(-14{,}2q)+20{,}6q$.

69. Quyidagi yig'indini hisoblang:

a) $\left(-\frac{3}{4}ab\right)+\frac{2}{3}a^2b+7\frac{3}{4}ab+\left(-\frac{5}{6}a^2b\right)$;

b) $\left(-\frac{1}{2}xy^2\right)+\left(-\frac{3}{8}x^2y\right)+\frac{3}{4}x^2y+\frac{1}{2}xy^2$.

70. Quyidagi birhadlarni qo'shing:

a) $3pq+(-4{,}2p^2)+0{,}3p^2+2q+(-5pq)+(-3q)$;

b) $(-0{,}3\ ab)+(-0{,}2\ a^2)+1{,}4b+(-5a^2)+(-2{,}3\ ab)$.

71. Ko'phadlarni qo'shing:

a) $5a$ va $3a+7$; d) $2m-3n$ va $m-n$;

b) $1-6x$ va $15x$; e) $1{,}5a^2+2b^2$ va $2a^2-b^2$.

72. Qo'shishni bajaring:

a) $8a+(3b+5a)$; d) $\left(\frac{1}{2}x+\frac{3}{4}\right)+\left(2\frac{1}{2}-x\right)$;

b) $(4x+2)+(-x+1)$; e) $0{,}4y+(1{,}2y-0{,}1)$.

73. a) $(4a^2b-3ab^2)+(-a^2b+2ab^2)$;

b) $(a^2+2ab+b^2)+(6a^2+9b^2)$;

d) $(x^2+4x-5)+(x^2-3x+2)$;

e) $(5m^2-5m+3)+(-4m^2+5m+3)$.

74. a) $(5x^2+ax+a^2)+(3x^2+2ax-3a^2)+(-8x^2+2a^2)$;

b) $(2a^4+5a^3b+3a^2b^2-ab^3)+(-15a^3b-3a^2b^2+11ab^3)$.

10-§. Birhadlarni va ko‘phadlarni ayirish

1. ***Birhadlarni ayirish.***

$16x^2y$ birhaddan $3xy^2$ birhadni ayirishni $16x^2y-(3xy^2)$ ko‘rinishda yozamiz.

Bundagi $3xy^2$ bilan $-3xy^2$ birhadlar qarama-qarshi, chunki ularning yig‘indisi nol bo‘ladi.

Ayirish amali kamayuvchiga ayriluvchining qarama-qarshisini qo‘shish orqali bajariladi, ya’ni $16x^2y-3xy^2=16x^2y+(-3xy^2)$ kabi bajariladi.

Xulosamizning to‘g‘riligini ayirmaga ayriluvchini qo‘shib tekshiriladi. Haqiqatan, $16x^2y+(-3xy^2)+3xy^2=16x^2y$ kamayuvchini hosil qildik.

Shunday qilib, **birhadni ayirish uchun uni qarama-qarshi ishora bilan kamayuvchining yoniga yozish kerak.**

Masalan: 1) $-12abc^2-(-5abc^2)=-12abc^2+5abc^2=-7abc^2$ (o‘xshash hadlar ixchamlandi).

2) $0{,}2\ m^2n^2-4{,}2\ m^2n^2=0{,}2\ m^2n^2+(-4{,}2\ m^2n^2)=-4\ m^2n^2$;

3) $12\ a^3b-8\ a^3b-10\ a^3b=12\ a^3b+(-8\ a^3b)+(-10\ a^3b)=-6\ a^3b$.

2. ***Ko‘phadlarni ayirish.***

Qarama-qarshi ikkita sonning absolyut qiymatlari teng va ishoralari qarama-qarshi ekanini bilamiz. Masalan, 5 bilan -5 va a bilan $-a$ sonlar, ularda $5+(-5)=0$ va $a+(-a)=0$ bo‘ladi.

Absolyut qiymatlari bir xil, lekin ishoralari qarama-qarshi bo‘lgan hadlardan tuzilgan ikkita ko‘phadlar ham qarama-qarshi bo‘ladi.

Masalan, $x^2+2ax-3a^2$ va $-x^2-2ax+3a^2$ larni qo‘shamiz:

$$\begin{array}{r} x_2+2ax-3a^2 \\ +\ \underline{-x^2-2ax+3a^2} \\ 0 \end{array}$$

Demak, bu ko‘phadlar qarama-qarshi ekan, ularni $x^2+2ax-3a^2=$ $=-(-x^2-2ax+3a^2)$ yoki $-(-x^2-2ax+3a^2)=x^2+2ax-3a^2$ ko‘rinishda yozamiz.

Shunday qilib, ***oldida minus ishora turgan qavsni ochish uchun, qavsni tashlab, qavs ichida turgan hadlarning barchasini qarama-***

qarshi ishoralar bilan yozib chiqiladi. $5x^2-3xy+y^2$ ko‘phaddan $6x^2+8xy+y^2$ ko‘phadni ayirish kerak bo‘lsin, ya’ni $5x^2-3xy+y^2-$ $-(6x^2+8xy+y^2)$.

Ayirish ta’rifiga asosan ayirish amali kamayuvchiga ayriluvchining qarama-qarshisini qo‘shish orqali bajariladi, ya’ni $(5x^2-3xy+y^2)-$ $-(-(6x^2+8xy+y^2))=(5x^2-3xy+y^2)+(-6x^2-8xy-y^2)=5x^2-3xy+$ $+y^2-6x^2-8xy-y^2=(5x^2-6x^2)+(-3xy-8xy)+(y^2-y^2)=-x^2-11xy.$

Ayirishni to‘g‘ri bajarilganligini tekshiramiz: buning uchun ayirma ko‘phadni ayriluvchi ko‘phadga qo‘shamiz: $(-x^2-11xy)+(6x^2+8xy-y^2)=$ $=-x^2-11xy+6x^2+8xy+y^2=5x^2-3xy+y^2$. Demak, kamayuvchi hosil bo‘ldi. Shunday qilib, **ko‘phadni ayirish uchun, kamayuvchining yoniga ayriluvchining hamma hadlarini qarama-qarshi ishoralar bilan yoziladi. Hosil bo‘lgan ifodada o‘xshash hadlar bo‘lsa, ular ixchamlanadi.**

Masalan: 1) $5ab^2-(5a^2-11ab^2)=5ab^2-5a^2+11ab^2=16ab^2-5a^2$;

2) $\left(\frac{1}{5}ab+\frac{1}{7}ab-\frac{2}{3}ac\right)-\left(-\frac{4}{5}ab+\frac{3}{14}bc-\frac{1}{5}ac\right)=$

$$=\frac{1}{5}ab+\frac{1}{7}bc-\frac{2}{3}ac+\frac{4}{5}ab-\frac{3}{14}bc+\frac{1}{5}ac=ab-\frac{1}{14}bc-\frac{7}{15}ac.$$

TAKRORLASH UCHUN SAVOLLAR

1. Ayirish amali deb nimaga aytiladi?
2. Birhadlar qanday ayriladi?
3. Ayirishni og‘zaki bajaring:
   a) $2a-9a$; b) $7{,}5x^2y-(10{,}5x^2y)$; d) $-12a^3-(-10a^3)$.
4. Ko‘phadlar qanday ayriladi?
5. Ayirishni og‘zaki bajaring:
   a) $8a-(a+7)$; b) $(2xy^2-c)-(-6c)$; d) $(bc+a)-(a-bc)$.

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75. Ayirishni bajaring:

a) $15x^2-10x^2$;
d) $2{,}8\ a^2-(-4{,}2\ a^2)$;
b) $3xy-(-7xy)$;
e) $\frac{2}{3}x^3y-2\frac{2}{3}x^3y$.

76. a) $0{,}2\,m^2n-(-1{,}8\,m^2n)$;   d) $6a^3-15a^3-4a^3$;

b) $1\frac{3}{5}x-\left(-\frac{2}{3}x\right)$;   e) $12{,}4\,bc-6{,}4\,bc-(-5\,bc)$.

77. $a-(b+c)=a-b-c$ tenglikning to'g'riligini harflarning quyidagi qiymatlarida tekshiring:

a	7	10	–5	0,8	$\frac{3}{4}$
b	4	8	–4	1,3	$\frac{1}{2}$
c	3	2	–3	2,5	$\frac{5}{8}$

78. Ayirishni bajaring va mumkin bo'lgan o'rinlarda o'xshash hadlarni ixchamlang:

a) $5x-(3x+2y)$;   d) $(6a^2-5a)-(a^2-7a)$;

b) $(2m-3n)-(5m+6n)$;   e) $15x-(7x-2y-10)$.

79. a) $(3a^2b-13b^2)-(-8a^2b-10b^2)$;

b) $(4x^2y-8xy^2)-(3x^2y-5xy^2)$;

d) $(13x-11y+10z)-(-15x+10y-15z)$;

e) $(7m^2-4mn-n^2)-(2m^2-mn+2n^2)$.

80. Ifodani soddalashtiring:

a) $(y^2-1{,}75y-3{,}2)-(0{,}3y^2+4-2y)$;

b) $6xy-2x^2-(3xy+4x^2-7)$;

d) $-(2ab^2-ab+b)-3ab^2-4b-(5ab-ab^2)$;

e) $\left(\frac{1}{2}x^2y^2-\frac{2}{3}ab-\frac{5}{6}a^2b^2-1\right)-a^2b^2+\frac{1}{3}x^2y^2-\left(-\frac{1}{12}ab+\frac{1}{4}\right)$.

81. a) $1\frac{3}{4}a^2-\frac{5}{8}ab+\left(3\frac{1}{2}ac-\frac{2}{25}a^2\right)-\left(5bc-\frac{23}{200}ab\right)$;

b) $-(-1{,}4x^2-2{,}24xy)-1{,}5y^2+10\frac{3}{4}x^2-\left(-\frac{5}{8}xy-1\frac{1}{2}y^2\right)$.

82. $3x^3-2x^2-x+4$ ifodani qandaydir ikki hadlar:

a) yig'indisi ko'rinishida yozilsin;

b) ayirmasi ko'rinishida yozilsin.

83. Ketma-ket kelgan uchta natural sonning yig‘indisi 3 ga bo‘linishini isbotlang.

11-§. Birhadni birhadga va birhadni ko‘phadga ko‘paytirish

1. Birhadni birhadga ko‘paytirish

$13a^2b^3c$ va $5a^3bc^3d^3$ birhadlarni ko‘paytiramiz.

Ko‘paytmaga ko‘paytirish qoidasiga asosan $13a^2b^3c\cdot 5\cdot a^3\cdot b\cdot c^3\cdot d^3$ bunday yozamiz va o‘rin almashtirish va guruhlash qonunlaridan foydalanib quyidagicha yozamiz:

$13a^2b^4c\cdot 5a^3bc^3d^3=13a^2b^3c\cdot 5\cdot a^3\cdot b\cdot c^3\cdot d^3=(13\cdot 5)(a^2\cdot a^3)(b^3\cdot b)$ $(c\cdot c^3)\cdot d^3=65a^5b^4c^4d^3.$

Ikkitadan ortiq birhadlarni ko‘paytirish ham xuddi shu yo‘l bilan bajariladi, masalan, $2x^2y^3\cdot 0{,}5x^3y^2\cdot 4yz^3=(2\cdot 0{,}5\cdot 4)(x^2\cdot x^3)$ $(y^3\cdot y^2)\cdot z^3=4x^5y^5z^3$ kabi bajariladi.

Shunday qilib, **birhadlarni ko‘paytirishda ularning koeffitsiyentlari bir-biriga ko‘paytiriladi, ko‘paytuvchilarda bo‘lgan bir xil harflarning daraja ko‘rsatkichlari qo‘shiladi, faqat bitta ko‘paytuvchida bo‘lgan harflar o‘z ko‘rsatkichlari bilan ko‘paytmaga olinadi.**

Masalan, $(-2{,}1\,m^2n^3p)\cdot(-3{,}5\,m^3np^4)\cdot\left(-\frac{2}{5}m^2n^4p^2\right)=\left((-2{,}1\cdot(-3{,}5)\times\right.$ $\left.\times\left(-\frac{2}{5}\right)\right)(m^2\cdot m^3\cdot m^2)(n^3\cdot n\cdot n^4)\cdot(p\cdot p^4\cdot p^2)=-2{,}94m^7n^8\cdot p^7.$

2. Ko‘phadni birhadga ko‘paytirish $3a^3-5a^2b+6a^2c$ ko‘phadni $4ab^2$ birhadga ko‘paytiramiz. Ko‘phad – algebraik yig‘indidan iborat bo‘lgani uchun tarqatish qonuniga asosan ko‘paytmani quyidagicha yozamiz: $(3a^3-5a^2b+6a^2c)\cdot 4ab^2=3a^3\cdot 4ab^2-5a^2b\cdot 4ab^2+6a^2c\cdot 4ab^2=$ $=12a^4b^2-20a^3b^3+24a^3b^2c.$ Bundan quyidagi qoidani chiqaramiz.

Ko‘phadni birhadga ko‘paytirish uchun, ko‘phadning har bir hadini shu birhadga ko‘paytirib, hosil bo‘lgan ko‘paytmalar qo‘shiladi.

Masalan, $(4x^2yz-7xy^2z^2-3x^2yz^2)\cdot(-5x^2yi)$ (birhadni birhadga ko‘paytmasi og‘zaki bajariladi)$=-20x^4y^2z^2+35x^3y^3z^3-15x^4y^2z^3.$

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1. Birhadni birhadga qanday ko'paytiriladi?
2. Og'zaki bajaring:
   a) $a^2 \cdot (-a)$; b) $-x \cdot y \cdot (-2z)$; d) $2b \cdot (-3c)$; e) $(-4a) \cdot (-5ac)$.
3. Ko'phadni birhadga qanday ko'paytiriladi?
4. Og'zaki bajaring:
   a) $a \cdot (a^2 + b)$; b) $-2a(-4 + 5a^2)$; d) $(-2{,}4c(b - 5c)$.

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84. Quyidagi amallarni bajaring:

a) $a \cdot (-b) \cdot c$; d) $-4a \cdot (-4b) \cdot (-5)$;

b) $-x \cdot (-2y) \cdot 5z$; e) $8c \cdot (-0{,}5c) \cdot 10\,c^2$.

85. a) $-3x^2 \cdot (-4x^3) \cdot (-5x)$; d) $\left(-1\frac{1}{2}x^2y^3z\right) \cdot \left(-1\frac{1}{3}xy^2z\right)$;

b) $(-m^2n^2) \cdot \frac{5}{6}m^3n \cdot \left(-1\frac{1}{5}mn^3\right)$; e) $(-2{,}5\,m^3n^2p) \cdot (-3{,}4\,m^2n^3p)$.

86. Amallarni bajaring:

a) $(3-y)^2$; d) $(4\,m^2)^3$; f) $\left(-\frac{1}{2}b^2\right)^3$;

b) $(-4c^2)^2$; e) $(-3c^2)^4$; g) $\left(-2\frac{1}{2}ab^2\right)^3$.

87. a) $(2a^3)^2 \cdot (-3a)^3$; d) $(x^n)^2 \cdot (2x)^3$;

b) $(-7m^4)^2 \cdot (2m)^3$; e) $(-2a^m)^2 \cdot (3a^3)^3$.

88. Ko'paytmani ko'phadga aylantiring:

a) $1{,}5y \cdot (y^2 - 2{,}4y + 6)$; e) $(-2x^2 - 3x + 4) \cdot (-0{,}5x^2)$;

b) $-4b^2(5b - 3b - 2)$; f) $(-3y^2 + 0{,}6y - 8) \cdot (-1{,}5y^3)$;

d) $(3a^3 - a^2 - a) \cdot (-5a^2)$; g) $4ab(-3a^2 + 2b^3 - 5ab)$.

89. a) $(-2ax^2 + 3ax - a^2)(-a^2x^2)$;

b) $2{,}5a^2b(4a^2 - 2ab + 0{,}2b^2)$;

d) $(6{,}3x^3y - 3y^2 - 0{,}7x) \cdot 10x^2y^2$;

e) $-\frac{2}{5}a^2y^5\left(5ay^2 - \frac{1}{2}a^2y - \frac{5}{6}a^3\right)$.

90. a) $(2x^3-3x^2+3x-1)\cdot 4x^2y^2$;

b) $(-2a^2b)(8a^8-4a^2b^2-3ab^2+5b^3)$;

d) $(-2a^3x+5a^2x^2-5ax^3+3x^4)\cdot(-3ax^2)$;

e) $(-5xyz)(4xy^2z-7x^2yz^2+3x^2yz-2x^2y^2z^2)$.

91. Amallarni bajaring va soddalashtiring:

a) $a(a+b)-b(a-b)$; d) $7(2m-3n)+3(m+n)-17m$;

b) $-2(a-3b)+3(a-2b)$; e) $6(3p+4q)-8(5p-q)+22p$.

92. a) $2a^2-a(2a-5b)-b(2a-b)$;

b) $6m^2-5m(-m+2n)+4m\left(-3m-2\frac{1}{2}n\right)$.

d) $10x(5x^2-7y)-6x(5y+7x^2)-3xy$;

e) $4a(5b-2a)-4(7a^2-3ab)-2a(3a-3b)$.

12-§. Ko'phadni ko'phadga ko'paytirish

$a+b$ va $(c+d+e)$ ko'phadlarning ko'paytmasini $(a+b)\cdot(c+d+e)$ ko'rinishda yozamiz.

Buning uchun $a+b$ ko'phadni x harfi bilan belgilaymiz va birhadni ko'phadga ko'paytirish qoidasi bo'yicha ko'paytiramiz. $a+b=x$ bo'lsin, y holda $(a+b)(c+d+e)=x(c+d+e)=xc+xd+xe$.

Bundagi x ning o'rniga $a+b$ ni qo'yib, ko'phadni birhadga ko'paytirish qoidasi bo'yicha ko'paytiramiz. $xc+xd+xe=(a+b)c+(a+b)d+(a+b)e=ac+bc+ad+bd+ae+be$. Umuman, har qanday ikkita ko'phadning ko'paytmasini ko'phad ko'rinishida ifodalash mumkin.

Shunday qilib, ko'phadni ko'phadga ko'paytirish uchun bir ko'phadning har bir hadini ikkinchi ko'phadning har bir hadiga ko'paytirib, chiqqan ko'paytmalarni qo'shish kerak.

Masalan, $(5-2a+a^2)(4a^2-3a-1)$ ko'paytmani ko'phad ko'rinishida yozing. $(-5-2a+a^2)(4a^2-3a-1)=-20a^2+15a+5-8a^3+6a^2+2a+4a^4-3a^3-a^2=4a^4-11a^3-15a^2+17a+5$.

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1. Birhadni ko'phadga qanday ko'paytiriladi?
2. Ko'phadni ko'phadga qanday ko'paytiriladi?
3. Og'zaki ko'paytiring:

a) $2x\cdot(x+y)$; b) $-3b(2b-c)$; d) $-a(3a-6b)$; e) $(x+y)(x-y)$.

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93. Ko'phadlarni ko'paytiring:

a) $(x+m)(x+5)$; d) $(2y-1)\cdot(3y+2)$;
b) $(2-a)(a-8)$; e) $(5x-3)(4-3x)$.

94. a) $(m^2-n)(m^2+2n^2)$; d) $(a-2)\cdot(4a^3-3a^2)$;
b) $(4a^2+b^2)\cdot(3a^2-b^2)$; e) $(5a+3a^3)\cdot(4a+1)$.

95. Ifodani ko'phad ko'rinishida yozing:

a) $(x^2+xy-y^2)\cdot(x+y)$; d) $(x^2-x+1)(5x-2)$;
b) $(a+x)\cdot(a^2-ax-x^2)$; e) $(25x^2+10xy+4y^2)\cdot(5x-2y)$.

96. a) $(1+0{,}6m+0{,}12n^2)\cdot(m-0{,}5n^2)$;

b) $(1{,}44\,p^2+0{,}6\,pq+0{,}25\,q^2)\cdot(1{,}2p-0{,}5q)$;

d) $\left(\frac{2}{3}x+1\frac{1}{2}y\right)\cdot\left(\frac{4}{9}x^2-xy+2\frac{1}{4}y^2\right)$.

97. Ifodani soddalashtiring va berilgan sondagi qiymatini toping:

a) $(a-7)(a+5)+(a+8)(a-6)$, bunda $a=7$;

b) $(b+9)(b+4)+(b-12)(b-1)$, bunda $b=5$.

98. x o'zgaruvchining qiymati har qanday bo'lganda:

a) $(x-3)(x+7)+(x+5)(x-1)$ ifodaning qiymati -16 ga teng ekanligini isbotlang;

b) $x^4+(x^2+7)(x^2+7)$ ifodaning qiymati 49 ga teng ekanligini isbotlang.

99. a) n ning qiymati har qanday natural son bo'lganda $n(n+5)+$ $+(n+3)(n+2)$ ifodaning qiymati 6 ga karrali ekanligini isbotlang;

b) n ning qiymati 2 dan katta har qanday natural son bo'lganda $(n-1)(n+1)-(n-7)(n-5)$ ifodaning qiymati 12 ga karrali ekanligini isbotlang.

13-§. Qisqa ko'paytirish formulalari

Ko'pincha ko'paytirishning ba'zi bir xususiy hollari uchraydiki, bunda hosil bo'ladigan ko'paytmani esda tutish foydali, chunki bunday hollar uchraganda, har gal hadma-had ko'paytirib o'tirmasdan, birdaniga chiqadigan ko'paytmani yozish mumkin bo'ladi. Shuning uchun bu ko'paytmalar **qisqa ko'paytirish formulalari** deb ataladi.

1. Yig'indining kvadrati.

a va b sonlar yig'indisini kvadratga ko'taramiz $(a+b)^2=(a+b)(a+b)=$ $=a^2+ab+ab+b^2=a^2+2ab+b^2$.

$\boldsymbol{(a+b)^2=a^2+2ab+b^2}$ formula hosil bo'ldi. Bu formulani aytilishini esda saqlaylik:

Ikki son yig'indisining kvadrati – birinchi son kvadrati, plyus shu ikki sonning ikkilangan ko'paytmasi, plyus ikkinchi son kvadrati.

Masalan, $(3a+2b)^2=(3a)^2+2\cdot 3a\cdot 2b+(2b)^2=9a^2+12ab+4b^2$. Bu misolda ko'rsatilgandek, oraliqdagi yozuvlarni bajarmasdan, birdaniga $(3a+2b)^2=9a^2+12ab+4b^2$ kabi oxirgi natijani yozishga o'rganish kerak.

2. Ayirmaning kvadrati.

$(a-b)^2=(a-b)(a-b)=a^2-ab-ab+b^2=a^2-2ab+b^2$.

$\boldsymbol{(a-b)^2=a^2-2ab+b^2}$.

Ikki son ayirmasining kvadrati – birinchi son kvadrati, minus shu ikki sonning ikkilangan ko'paytmasi, plyus ikkinchi son kvadrati.

Masalan, $(4a^2b-3ab)^2=(4a^2b)^2-2\cdot 4a^2b\cdot 3ab+(3ab)^2=16a^4b^2-24a^3b^2+$ $+9a^2b^2$.

Bunda ham, oraliqdagi hisoblashni dilda bajarib, $(4a^2b-3ab)^2=$ $=16a^4b^2-24a^3b^2+9a^2b^2$ kabi birdaniga yozish kerak.

3. Ikki son yig'indisi bilan u sonlar ayirmasining ko'paytmasi. $(a+b)(a-b)=a^2+ab-ab-b^2=a^2-b^2$.

Ikki son yig'indisi bilan shu sonlar ayirmasining ko'paytmasi shu sonlar kvadratining ayirmasiga teng.

Masalan: 1) $(5a^2b+2b)(5a^2b-2b)=(5a^2b)^2-(2b)^2=25a^4b^2-4b^2$.

2) $\left(\frac{2}{3}x^3y+7xy^2\right)\left(\frac{2}{3}x^3y-7xy^2\right)=\left(\frac{2}{3}x^3y\right)^2-(7xy^2)^2=$

$=\frac{4}{9}x^6y^2-49x^2y^4=x^2y^2\left(\frac{4}{9}x^4-49y^2\right)$.

3) $102^2-101^2=(102+101)(102-101)=203\cdot 1=203$.

4. Yig'indining kubi.

$(a+b)^3=(a+b)^2\cdot(a+b)=(a^2+2ab+b^2)\cdot(a+b)=a^3+2a^2b+ab^2+a^2b+$
$+2ab^2+b^3=a^3+3a^2b+3ab^2+b^3$. $\mathbf{(a+b)^3=a^3+3a^2b+3ab^2+b^3}$.

Ikki son yig'indisining kubi – birinchi son kubi, plyus birinchi son kvadrati bilan ikkinchi sonning uchlangan ko'paytmasi, plyus ikkinchi son kvadrati bilan birinchi sonning uchlangan ko'paytmasi, plyus ikkinchi son kubi.

Masalan:

1) $(2a+3b)^3=(2a)^3+3\cdot(2a)^2\cdot 3b+3\cdot 2a\cdot(3b)^2+(3b)^3=8a^3+36a^2b+$
$+54ab^2+27b^3$.

2) $\left(\frac{1}{3}a^2+\frac{3}{4}b^3\right)^3=\left(\frac{1}{3}a^2\right)^3+3\cdot\left(\frac{1}{3}a^2\right)^2\cdot\frac{3}{4}b^3+3\cdot\frac{1}{3}a^2\cdot\left(\frac{3}{4}b^3\right)^2+\left(\frac{3}{4}b^3\right)^3=$

$=\frac{1}{27}a^6+\frac{1}{4}a^4b^3+\frac{9}{16}a^2b^6+\frac{27}{64}b^9$

3) $11^3=(10+1)^3=1000+300+30+1=1331$.

5. Ayirmaning kubi.

$(a-b)^3=(a-b)^2\cdot(a-b)=(a^2-2ab+b^2)(a-b)=a^3-2a^2b+ab^2-a^2b+2ab^2-b^3=$
$=a^3-3a^2b+3ab^2-b^3$. $\mathbf{(a-b)^3=a^3-3a^2b+3ab^2-b^3}$.

Ikki son ayirmasining kubi – birinchi son kubi, minus birinchi son kvadrati bilan ikkinchi sonning uchlangan ko'paytmasi, plyus ikkinchi son kvadrati bilan birinchi sonning uchlangan ko'paytmasi, minus ikkinchi son kubi.

Masalan:

1) $(2x-5)^3=(2x)^3-3\cdot(2x)^2\cdot 5+3\cdot 2x\cdot 5^2-5^3=8x^3-60x^2+150x-125$.

2) $(3a^2-4b^3)^3=(3a^2)^3-3\cdot(3a^2)^2\cdot 4b^3+3\cdot 3a^2\cdot(4b^3)^2-(4b^3)^3=27a^6-108a^4b^3+$
$+144\,a^2b^6-64\,b^9$.

3) $18^3=(20-2)^3=20^3-3\cdot 20^2\cdot 2+3\cdot 20\cdot 2^2-2^3=8000-2400+240-8=5832$.

6. Ikki son yig'indisi bilan shu sonlar ayirmasining to'liqsiz kvadratining ko'paytmasi.

$(a+b)(a^2-ab+b^2)=a^3-a^2b+ab^2+a^2b-ab^2+b^3=a^3+b^3$.

$\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}$.

a^2+ab+b^2 – ikki son yig'indisining to'liqsiz kvadrati.

a^2-ab+b^2 – ikki son ayirmasining to'liqsiz kvadrati.

Ikki son yig'indisi bilan shu sonlar ayirmasi to'liqsiz kvadratining ko'paytmasi shu sonlar kublarining yig'indisiga teng.

Masalan: 1) $(3x+y)\,(9x^2-3xy+y^2)=(3x)^3+y^3=27x^3+y^3$.

2) $(3ab+2c)(9a^2b^2+6abc+4c^2)=(3ab)^3+(2c)^3=27a^3b^3+8c^3$.

Ba'zida bu formulaning chap va o'ng qismlari almashtirib yoziladi, ya'ni $\boldsymbol{a^3+b^3=(a+b)(a^2-ab+b^2)}$.

Ikki son kublarining yig'indisi shu sonlar yig'indisi bilan ular ayirmasi to'liqsiz kvadratining ko'paytmasiga teng.

Masalan: 1) $64a^3+8c^3=(4a)^3+(2c)^3=(4a+2c)(16a^2-8ac+4c^2)$.

2) $\frac{1}{27}b^3+0{,}125c^3=\left(\frac{1}{3}b\right)^3+(0{,}5c)^3=\left(\frac{1}{3}b+0{,}5c\right)\left(\frac{1}{9}b^2-\frac{1}{6}bc+0{,}25c^2\right)$.

7. Ikki son ayirmasi bilan shu sonlar yig'indisining to'liqsiz kvadratining ko'paytmasi.

$(a-b)(a^2+ab+b^2)=a^3+a^2b+ab^2-a^2b-ab^2-b^3=a^3-b^3$.

$\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}$.

Ikki son ayirmasi bilan shu sonlar yig'indisi to'liqsiz kvadratining ko'paytmasi shu sonlar kublarining ayirmasiga teng.

Masalan: 1) $(0{,}3x-2y)(0{,}09x^2+0{,}6xy+4y^2)=(0{,}3x)^3-(2y)^3=0{,}027x^3-8y^3$.

2) $(2x^4-5y^3)(4x^8+10x^4y^3+25y^6)=(2x^4)^3-(5y^3)^3=8x^{12}-125y^9$.

Bu formulani ham chap va o'ng tomonlari almashtirilib, $a^3-b^3=(a-b)\times$ $\times(a^2+ab+b^2)$ kabi yoziladi.

Bu formula quyidagicha o‘qiladi:

Ikki son kublarining ayirmasi shu sonlar ayirmasi bilan ular yig‘indisi to‘liqsiz kvadratining ko‘paytmasiga teng.

Masalan: 1) $1-64x^3=1^3-(4x)^3=(1-4x)(1+4x+16x^2)$.

2) $0,027b^3c^9-\frac{1}{27}d^{12}=(0,3bc^3)^3-\left(\frac{1}{3}d^4\right)^3=$

$=\left(0,3bc^3-\frac{1}{3}d^4\right)\left(0,09b^2c^6+0,1bc^3d^4+\frac{1}{9}d^8\right)$.

3) 731^3-631^3 ayirmani 100 ga bo‘linishini isbotlang.

$731^3-631^3=(731-631)(731^2+731\cdot631+631^2)=100\cdot(731^2+731\cdot631+$ $+631^2)$ ko‘paytma 100 ga bo‘linadi.

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100. Quyidagi formulalarni yozing:

a) $(a+b)^2=$ $(a-b)^2=$ $(a+b)^3=$ $(a-b)^3=$

b) $a^2-b^2=$ $a^3+b^3=$ $a^3-b^3=$

d) $(a+b)(a-b)=$ $(a+b)(a^2-ab+b^2)=$ $(a-b)(a^2+ab+b^2)=$

101. Ifodani ko‘phad ko‘rinishida yozing:

a) $(x-5)^2$; b) $(3+y)^2$; d) $(5x-12)^2$; e) $(6x+y)^2$;

f) $\left(\frac{1}{3}x+5\right)^2$; g) $(5xy-0,2y^2)^2$; h) $(3a^2+5\,ab)^2$; i) $(8xy+3\text{-}y^2)^2$;

j) $(a^3b^3-1)^2$; k) $(2+x^4y^2)^2$; l) $(x^6-3\text{-}y^4)^2$; m) $(y^8-2\,x^4y)^2$.

102. Ifodani ko‘phadga aylantiring:

a) $(0,7\,x^3y-2xy^3)^2$; b) $\left(\frac{3}{4}a^3b-\frac{2}{3}ab^3\right)^2$;

d) $(0,2\,p^3q+0,3pq^3)^2$; e) $\left(\frac{1}{8}bc^4+\frac{8}{9}b^2c^3\right)^2$.

103. Ifodani ko'phadga aylantiring:

a) $(x+5)^3$; e) $(3b-2c)^3$; h) $(0{,}2x^2-5y^2)^3$;

b) $(y-3)^3$; f) $(3x^2+y)^3$; i) $\left(\frac{2}{3}b^2-\frac{3}{4}c\right)^3$;

d) $(2x+y)^3$; g) $(3c^3+2d)^3$; j) $(1{,}4b^3c+5b^2c)^3$.

104. Ko'paytuvchilarga ajrating:

a) $27y^3+1$; d) $0{,}027x^3-100$; f) $d^3+0{,}027c^3$;

b) $0{,}008-x^3$; e) $y^6-0{,}001\,x^3$; g) $125-0{,}064\,p^9$.

105. Ifodani ko'paytmaga aylantiring:

Namuna. $16a^5+54a^2=2a^2(8a^3+27)=2a^2((2a)^3+3^3)=$
$=2a^2\cdot(2a+3)\cdot((2a)^2-2a\cdot3+3^2)=2a^2(2a+3)(4a^2-6a+9)$.

a) $12a^2-12b^2$; d) $1{,}3a^3+1{,}3b^3$; f) $5a^6-125a^4$;

b) $4x^2-16y^2$; e) $2a^4-8b^2$; g) $a^5+0{,}64a^2$.

106. Ko'paytuvchilarga ajrating:

Namuna. $x^3-y^3-5xy(x^2+xy+y^2)=(x-y)(x^2+xy+y^2)-5xy(x^2+xy+y^2)=$
$=(x^2+xy+y^2)\cdot(x-y-5xy)$.

a) $a^2-b^2+2(a+b)$; e) $5a^2-5-4(a+1)^2$;

b) $b^2-c^2-10(b+c)^2$; f) $x^3+y^3+2xy(x+y)$;

d) $2(x-y)^2+3x^2-3y^2$; g) $a^3-b^3+5a^2b-5ab^2$.

107*. Ko'paytma ko'rinishida yozing:

a) $x^3+y^3+2x^2-2xy+2y^2$; d) $a^4+ab^3-a^3b-b^4$;

b) $a^3-b^3+3a^2+3ab+3b^2$; e) $x^4+x^3y-xy^3-y^4$.

14-§. Birhadlarni bo'lish

$10a^5b^3c$ ifodani $4a^3b$ ga bo'lamiz. Bo'lishni $10a^5b^3c:4a^3b$ kabi yozamiz.

Bo'lish amalining xossalaridan foydalanib, bo'linuvchini 4 ga bo'lamiz. Buning uchun 10 koeffitsiyentni 4 ga bo'lamiz, ya'ni $2{,}5\,a^5b^3c$. Bu bo'linmani a^3 ga bo'lamiz. Buning uchun a^5 ni a^3 ga bo'lamiz, ya'ni $2{,}5\,a^2b^3c$. Chiqqan natijani, b ga bo'lamiz. Buning uchun b^3 ni b ga bo'lamiz, nihoyat, $2{,}5\,a^2b^2c$ bo'linma hosil bo'ladi.

Demak, $10a^5b^3c:4a^3b=2{,}5\,a^2b^2c$.

Tekshirish: $4a^3b\cdot 2{,}5a^2b^2c=10a^5b^3c$ bo'linuvchi hosil bo'ldi, bo'lish to'g'ri bajarilgan.

Bundan keyin bo'lishlarning hammasini ketma-ket bajarmay, natijani birdaniga yozamiz.

Masalan: 1) $18x^6y^2z^3:6x^2y^2z=3x^4z^2$.

Bunda: 18 ni 6 ga bo'lib, 3 ni yozamiz, x^6 ni x^2 ga bo'lib x^4 ni;

y^2 ni y^2 ga bo'lib 1 ni va

z^3 ni z ga bo'lib, z^2 ni yozamiz.

Natijada $3x^4z^2$ hosil bo'ladi.

2) $6ax^3:3ax^2=4$ tenglamani yechamiz.

Buning uchun bo'lishni bajarib, x ni topamiz:

$2x=4$ bo'lib, $x=2$ bo'ladi.

Birhadni birhadga bo'lish uchun:

1) bo'linuvchining koeffitsiyentini bo'luvchining koeffitsiyentiga bo'linadi;

2) chiqqan bo'linma yoniga bo'linuvchining har bir harfini bo'linuvchi va bo'luvchidagi shu harflar ko'rsatkichlarining ayirmasiga teng ko'rsatkich bilan olib, ko'paytuvchi qilib yoziladi.

Masalan: 1) $-12pqr:6p=-2qr$ $(p:p=1)$.

2) $s^7:c^4=c^{7-4}=c^3$. 3) $a^n:a^5=a^{n-5}$($a^m:a^n=a^{m-n}$ ga asosan).

4) $0{,}5\ a^mb^nc^3:\left(-\frac{2}{3}a^2b^3c\right)=\left(-0{,}5:\frac{2}{3}\right)a^{m-2}b^{n-3}c^2=-\frac{3}{4}a^{m-2}b^{n-3}c^2$.

TAKRORLASH UCHUN SAVOLLAR

1. $a:b=c$ bo'lsa, bo'linuvchi a nimaga teng.
2. Bo'lish amalining to'g'ri bajarilganligi qanday tekshiriladi?
3. Birhadni birhadga bo'lish qanday bajariladi?
4. Bo'lishni og'zaki bajaring:

a) $10a:5$; b) $(-12m):(-4)$; d) $15x:10x$; e) $6xy:(-4x)$.

MASALALARNI YECHING

108. Bo'lishni bajaring:

a) $24a:(-8)$; e) $15mn:4m$; h) $a^5:a^3$;

b) $-6x:(-x)$; f) $-13b^2:5b$; i) $-y^7:y^4$;

d) $-12bc:(-4c)$; g) $-12z^2:\frac{3}{4}z$; j) $8x^6:2x^4$.

109. a) $-18y^8:(-6y^4)$; e) $x^m:x^k$;
b) $16x^7y:(-12x^2y)$; f) $y^{n+1}:y^5$;
d) $20m^7n^5:(-4m^4n)$; g) $x^{m+n}:x^n$.

110. a) $-c^{n+3}:c^{10}$; e) $\left(-\frac{2}{3}a^4x^6\right):\left(-\frac{1}{2}a^2x^5\right)$;
b) $y^{2n}:y^n$; f) $\left(-\frac{3}{4}a^5b^3c\right):1\frac{1}{2}a^3b^2c$;
d) $d^{2n+1}:d^{11}$; g) $(-1{,}2a^8b^3c^4):(-0{,}3a^5bc^4)$.

111. Bo‘lish amalini algebraik kasr shaklida yozing:
a) $a:x$; d) $2m:3n$; f) $5a^2b:4pq$;
b) $b:7$; e) $-7d:2c$; g) $1{,}8a^{n+3}b^{n+2}:0{,}9a^mb^3$.

15-§. Ko‘phadni birhadga bo‘lish

Quyidagi bo‘lishni bajaramiz: $(6a^4b^2-7a^3b+3{,}6a^2b^3):2a^2b$ bunga yig‘indini bo‘lish qoidasiga asosan ko‘phadning har bir hadini $2a^2b$ ga bo‘lib, quyidagi bo‘linmani hosil qilamiz: $(6a^4b^2-7a^3b+3{,}6a^2b^3):$ $:2a^2b=6a^4b^2:2a^2b-7a^3b:2a^2b+3{,}6a^2b^3:2a^2b=3a^2b-3{,}5a+1{,}8b^2$. Bo‘lish amalini to‘g‘ri bajarilganligini bo‘luvchiga bo‘linmani ko‘paytirish bilan tekshiriladi. $(3a^2b-3{,}5a+1{,}8b^2)\cdot 2a^2b=6a^4b^2-7a^3b+3{,}6a^2b^3$. Bo‘linuvchi hosil bo‘ldi. Demak, bo‘lish to‘g‘ri bajarilgan. Bulardan quyidagi qoida kelib chiqadi:

Ko‘phadni birhadga bo‘lish uchun, ko‘phadning har bir hadini shu birhadga bo‘lish va chiqqan bo‘linmalarni qo‘shish kerak.

Bu qoidadan, ko‘phadning har bir hadi birhadga bo‘linsagina ko‘phad birhadga qoldiqsiz bo‘linadi, deyish mumkin.

Agar bo‘luvchidagi biror harfning ko‘rsatkichi bo‘linuvchi ko‘phadning birorta hadida shu harfning ko‘rsatkichidan katta bo‘lsa, bo‘lish amali bajarilmaydi. Masalan, $(30a^4b^3:12a^2b^5):6a^3b^2$ da $30a^4b^3:6a^3b^2=5ab$, ammo – $12a^2b^5:6a^3b^2$ butun bo‘linmagani uchun ko‘phad birhadga bo‘linmaydi.

Masalan: 1) $(-24x^2+12x^3y^2-16x^4y^3):(-4x^2)=-24x^2:(-4x^2)+12x^3y^2:$ $:(-4x^2)-16x^4y^3:(-4x^2)=6-3xy^2+4x^2y^3.$

Birhadni birhadga bo'lishni bundan keyin dilda bajaramiz.

2) $\left(\frac{3}{4}a^6x^3+\frac{6}{5}a^3x^4-\frac{9}{10}ax^5\right):\frac{3}{5}ax^3=1\frac{1}{4}a^5+2a^2x-1\frac{1}{2}x^2.$

TAKRORLASH UCHUN SAVOLLAR

1. $8a:4=2a$ da bo'lish amali to'g'ri bajarilganmi?
2. Ko'phadni birhadga bo'lish qanday bajariladi?
3. Qanday holda ko'phadni birhadga bo'lish amali bajarilmaydi?
4. Bo'lishni og'zaki bajaring:

a) $(15x+6):3$; d) $(10x-25):5$;

b) $(-12y+8z):4$; e) $(-21y+9):(-3)$.

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112. Bo'lishni bajaring:

a) $(6a+18b):3$; e) $(3ab+4ac):a$;

b) $(36m-48n):(-12)$; f) $(15xy:10xz):5x$;

d) $(21k-14l):(-7)$; g) $(8a^3:32a):4a$.

113. a) $(10\,m^3n^5-20\,m^2n^3):5\,m^2n^3$;

b) $(18\,p^4q^3-27p^3q^2):(-9p^2q)$;

d) $(-10x^3+5x^2-20x):5x$;

e) $(18a^4x^3-24\,a^3x^4+36\,a^2x^5):6\,a^2x^3$.

114. a) $(-a^3+3a^2-4a):\left(-\frac{1}{3}\right)$;

b) $(6a^2x^5-9a^3x^4+15a^4x^3):\frac{3}{2}a^2x^3$;

d) $(p^4+4p^3-6p^2-8p):(-2p)$;

e) $(0{,}01\,a^4+0{,}02\,a^3+0{,}04\,a^2+0{,}002\,a):0{,}01\,a$.

115. Amallarni bajaring:

a) $9a^2 \cdot (a^2-2ab)+(9ab^3+12a^4b^2):3ab$;

b) $(9a^2b^3-12a^4b^4):3a^2b-(2+3a^2b)\cdot b^2$.

116. a) $(m^2-mn):m-(n^2-mn):n-(m-n)$;

b) $-4\left(\frac{1}{4}x-\frac{5}{8}\right)+(16x^2-8x):(-4x)-(x+2)$.

117. a) $4(4t^2-4t+1)\left(0,5t+\frac{1}{4}\right)+(2t^6-t^5):\left(-\frac{1}{4}t^3\right)$;

b) $\left((a-3b)(a+3b)-\frac{1}{6}(2a-1,5b)(3a+6b)\right):\frac{1}{4}b$.

16-§. Birinchi darajali bir noma'lumli tenglama va uning xossalari

Ko'pchilik masalalarni yechish uchun tenglamalardan foydalanamiz. Masalan, birorta songa 17 ni qo'shib, 46 hosil qilindi. Shu sonni toping. Bu masalani yechish uchun noma'lum sonni biror harf bilan, masalan, x harfi bilan belgilaymiz. Unga 17 ni qo'shib, $x+17$ ifodani hosil qilamiz. Masala shartiga ko'ra, $x+17$ ifoda 46 ga teng bo'lishi kerak. Demak, $x+17=46$. Bunda x va 17 qo'shiluvchilar, 46-yig'indi. x qo'shiluvchi ayirish natijasiga teng. Demak, $x=46-17$-yoki $x=29$.

x ning o'rniga 29 ni qo'yib tenglamani tekshiramiz: $x+17=46$ da $26+17=46$, $46=46$. Masala to'g'ri yechilgan. Noma'lum sonni o'z ichiga olgan tengliklar alohida nom bilan ataladi.

1-ta'rif. **Harf bilan belgilangan noma'lum sonni o'z ichiga olgan tenglik tenglama deyiladi.**

Masalan: $2x+8=20$; $42-3x^2=30$; $4x^3-20=12$ va hokazo.

Tenglamadagi noma'lumning yuqori darajasi tenglamaning darajasi deyiladi. $2x+8=20$ ning darajasi 1 ga teng, chunki x ning darajasi 1, $42-3x^2=30$ ning darajasi 2 ga teng, chunki x^2 ning darajasi 2, $4x^3-20=12$ ning darajasi 3 ga teng, chunki x^3 ning darajasi 3.

Tenglamada tenglik belgisidan chapda turgan ifoda tenglamaning chap qismi, o'ngdagi ifoda tenglamaning o'ng qismi deyiladi. $2x+8=20$ tenglamada $2x+8$ – tenglamaning chap qismi, 20 – tenglamaning o'ng qismi. $x=6$ da $2x+8=20$ tenglamaning ikkala qismi teng, ya'ni $12+8=20$-yoki $20=20$.

2-ta'rif. **Tenglamani yechish – noma'lumning tenglamani ikkala qismini bir xil songa teng qiladigan qiymatlarini topish demakdir.** $x=6$ qiymat $2x+8=20$ ning ikkala qismini teng qiladi.

3-ta'rif. **Noma'lumning tenglamani qanoatlantiradigan qiymatlari tenglamaning ildizlari deyiladi.** $x=6$ qiymat $2x+8=20$ tenglamaning ildizi.

Tenglamadagi noma'lum sonlar ko'pincha lotin alfavitining oxirgi harflari x, y, z, ...lar bilan belgilanadi.

4-ta'rif. **Birinchi darajali bir noma'lumli tenglama deb $ax+b=0$ ko'rinishdagi tenglamaga aytiladi.**

Bunda: x – noma'lum son, a (koeffitsiyent); $a\neq0$ har qanday son, b (ozod had) – har qanday son.

Masalan: $2x-12=0$; $-0{,}5x+8=0$; $\frac{5}{7}x-\frac{3}{4}=0$; va hokazo.

Ko'pgina tenglamalar ba'zi bir shakl almashtirishlardan keyin $ax+b=0$ ko'rinishiga keltiriladi (tenglamaning xossalariga asoslanib).

Umumiy holda $ax+b=0$ tenglama quyidagicha yechiladi:

$ax=0-b$

$ax=-b$

$x=-\frac{b}{a}$ ildizga ega.

Masalan, $0{,}5x+8=0$.

$0{,}5x=0-8$

$0{,}5x=-8$;

$x=\frac{-8}{0{,}5}$

$x=-16$. Tekshirish: $x=-16$ da

$0{,}5\cdot(-16)+8=0$.

$-8+8=0$.

$0=0$.

Tenglamaning xossalarini ko'rib chiqamiz: $x+10=16$ tenglamani noma'lum qo'shiluvchini topish qoidasi bo'yicha yechamiz: $x=16-10$; $x=6$. Ildizi: 6.

1) Bu tenglamaning ikkala qismiga bir xil son 20 ni qo'shib, tenglamani yechamiz.

$x+10+20=16+20$.

$x+30=36$.

$x=6$. Ildizi: 6.

2) $x+10=16$ ning ikkala qismiga $2x$ ni qo‘shamiz: $x+10+2x=16+2x$ bundan $3x+10=16+2x$ bo‘lib, bunda: 16 va $2x$ – qo‘shiluvchilar, $3x+10$ – yig‘indi. Bundagi 16 qo‘shiluvchi bo‘lgani uchun $3x+10-2x=16$ ni yozamiz. O‘xshash hadlarni ixchamlab,

$x+10=16$

$x=16-10$

$x=6$. Ildizi: 6.

Demak:

1-xossa. **Tenglamaning ikkala qismiga bir xil son yoki noma'lumli bir xil ifoda qo‘shilsa, yoki ayrilsa, tenglamaning ildizi o‘zgarmaydi.**

1-misol. $5x-6=18-3x$ tenglamani yechamiz.

Yechish. 1-xossaga asosan berilgan tenglamaning ikkala qismiga $3x$ ni qo‘shamiz.

$5x-6+3x=18-3x+3x$. Buni soddalashtirib, $5x-6+3x=18$.

$8x-6=18$ ni hosil qilamiz. Buning ikki tomoniga 6 ni qo‘shamiz;

$8x-6+6=18+6$ yoki $8x=18+6$($8x-6=18$ edi).

$8x=24$

$x=3$.

$x=3$ ni berilgan tenglamaga qo‘yib, tenglikning to‘g‘riligini tekshiramiz.

$5\cdot3-6=18-3\cdot3$

$9=9$. Ildizi: $x=3$.

3) a) $5x+3=38$ tenglamani $5x$ qo‘shiluvchini topish orqali yechamiz:

$5x=38-3$; $5x=35$; $x=7$.

b) $5x+3=38$ tenglamaning hamma hadlarini 4 ga ko‘paytirib, so‘ngra tenglamani yechamiz:

$5x\cdot4+3\cdot4=38\cdot4$

$20x+12=152$

$20x=152-12$

$20x=140$

$x=7$ Ildizi: 7.

Tenglamaning barcha hadlarini bir xil songa bo‘lsak ham, uning ildizi o‘zgarmaydi.

Demak:

2-xossa. **Tenglamaning barcha hadlarini bir xil songa ko‘paytirilsa yoki bo‘linsa, tenglamaning ildizi o‘zgarmaydi.**

2-misol. $\frac{x-1}{2}+\frac{x-4}{3}=\frac{x+1}{2}$ tenglamani yechamiz.

Yechish. 2-xossaga asosan berilgan tenglamaning barcha hadlarini 6 ga ko'paytiramiz: $3(x-1)+2\cdot(x-4)=3(x+1)$. Qavslarni ochib, ifodani soddalashtiramiz:

$3x-3+2x-8=3x+3.$

$5x-11=3x+3.$ Ikkala qismiga $11-3x$ ni qo'shamiz:

$5x-11+11-3x=3x+3+11-3x;$

$5x-3x=3+11;$

$2x=14;$

$x=7.$

Tekshirish: $x=7$ da $\frac{7-1}{2}+\frac{7-4}{3}=\frac{7+1}{2}$

$3+1=4$

$4=4.$ Ildizi: 7.

Yuqorida yechilgan ikkita tenglamadan quyidagicha xulosa qilamiz:

Tenglamaning istalgan hadini uning ishorasini qarama-qarshisiga o'zgartirib, tenglamaning bir qismidan ikkinchi qismiga o'tkazish mumkin.

3-misol. $9x-42+3x=4x-18$ tenglamani yechamiz.

Yechish. Tenglamadagi $4x$ ni chap tomonga $-4x$ qilib, -42 ni o'ng tomonga 42 qilib o'tkazamiz.

$9x-3x-4x=-18+42$. Ularni ixchamlab,

$2x=24$ kabi yozamiz.

$x=12.$

Tekshirish: $9\cdot12-42-3\cdot12=4\cdot12-18$

$30=30.$

Ildizi: $x=12$.

4-misol. $3(x+1)^2+(x-4)^3=101+(x-3)^3$ tenglamani yechamiz.

Yechish. $3(x^2+2x+1)+x^3-12x^2+48x-64=101+x^3-9x^2+27x-27;$

$3x^2+6x+3+x^3-12x^2+48x-64-x^3+9x^2-27x=101-27;$

$27x=135;$

$x=135:27$

$x=5.$

Tekshirish: $x=5$ da $3\cdot6^2+1^3=101+2^3$.

$109=109.$

5-misol.

$$9(x-1)+2(x-4)=72-3(x+1)$$
$$9x-9+2x-8=72-3x-3$$
$$9x+2x+3x=72-3+9+8$$
$$14x=86$$
$$x=\frac{86}{14}=\frac{43}{7}=6\frac{1}{7}.$$
$$x=6\frac{1}{7}.$$

Tekshirish. $9\cdot\left(6\frac{1}{7}-1\right)+2\left(6\frac{1}{7}-4\right)=72-3\left(6\frac{1}{7}+1\right)$ tenglikni soddalashtirsak $50\frac{4}{7}=50\frac{4}{7}$.

Demak, $x=6\frac{1}{7}$ tenglamaning ildizi.

Haqiqatan ham umumiy holda birinchi darajali bir noma'lumli tenglamaning birgina ildizi bor.

$ax+b=0$ tenglamani umumiy holda yechamiz. Buning uchun b ni tenglikning o'ng qismiga o'tkazamiz, $ax=-b$ hosil bo'ladi. Tenglamaning ikkala qismini $a\neq 0$ ga bo'lib, birgina ildizni hosil qilamiz, ya'ni $x=-\frac{b}{a}$.

TAKRORLASH UCHUN SAVOLLAR

1. Tenglama deb nimaga aytiladi? (16-§ da)
2. Tenglamani yechish deb nimaga aytiladi? (16-§ da)
3. Tenglamaning ildizi deb nimaga aytiladi? (16-§ da)
4. Tenglamaning birinchi xossasini ayting? (16-§ da)
5. Tenglamaning ikkinchi xossasini ayting (16-§ da)
6. Tenglamaning biror hadi uning ikkinchi tomoniga qanday ishora bilan o'tkaziladi?
7. Tenglamani og'zaki yeching:
   a) $x-5=0$; b) $2x-30=0$; d) $3x+18=6$; e) $0,5x-7=9$.

MASALALARNI YECHING

118. Tenglamalarni yeching:

a) $3x+2=14$; d) $20+5x+30$; f) $\frac{2}{3}x+4=5\frac{1}{3}$;

b) $7x-8=27$; e) $112-4x=48$; g) $0,4x+3,2=8,5$.

119. x ning qanday qiymatlarida ikki hadning qiymatlari teng bo'ladi?

a) $2x+1$ va $x-5$; b) $3x-5$ va $2x+4$.

120. Ildizlari 1; 5; –3 va 0,5 sonlari bo'ladigan birinchi darajali tenglamalar tuzing.

121. Tenglamalarni yeching:

a) $8x-3=5x+6$; d) $10x-3=x+3$;
b) $2x-19=7x+31$; e) $5y-9=7y-13$.

122. a) $5-6z=9z-5$; d) $x-7+8x=9x-3-4x$;
b) $19-x=100-10x$; e) $11x+42-2x=100-9x-22$.

123. a) $10x+7+13x=x+5+24x$;

b) $2x-\frac{3}{5}x=\frac{3}{2}x-\frac{1}{2}-\frac{2}{5}x+2$;

d) $2\frac{1}{3}x-3\frac{1}{2}x+1=x-5\frac{1}{3}x+3\frac{1}{5}x$;

e) $x+1\frac{1}{2}x+9=\frac{2}{3}x+4+\frac{5}{6}x-\frac{6}{5}x+\frac{1}{5}$.

124. a) $3+2{,}25x+2{,}6=2x+5+0{,}4x$;

b) $0{,}75x-2x=9+0{,}6x-0{,}5x$;

d) $5{,}76+4{,}8x-0{,}05x=6{,}99x-1{,}995x+5{,}13$;

e) $5x+3{,}48-2{,}35x=5{,}381-2{,}9x+10{,}42$.

125. a) x ning qanday qiymatida $10x+7+13$ va $x+5+24$ ko'phadlarning qiymatlari bir xil bo'ladi?

b) x ning qanday qiymatida $3x-20+6x-2$ va $8x-10+2x$ ko'phadlarning qiymatlari bir xil bo'ladi?

Tenglamalarni yeching:

126. a) $10y+2(7y-2)=5(4y+3)+3y$;
b) $26-4x=12x-7(x+4)$;
d) $8(3z-2)-13z=5(12-3z)+7z$;
e) $4y-3(20-y)=6y-7(11-y)$.

127. a) $17(2-3x)-5(x-12)=8(1-7x)$;

b) $(x-3)(x+4)-2(3x-2)=(x-4)^2$;

d) $12-2(x-1)^2=4(x-2)-(x-3)(2x-5)$;

e) $5(x-1)^2-2(x+3)^2=3(x+2)^2-7(6x-1)$.

128. a) $2x^2+(x+5)^2-2(x+7)^2=2(3x-72)^5+(x-6)^2$;

b) $(x+1)^3-(x-1)^3=6(x^2+x+1)$.

129. Tenglamalarni yeching:

a) $\frac{5x-4}{2}=\frac{16x+1}{7}$; d) $\frac{1-9y}{5}=\frac{19+3y}{8}$;

b) $\frac{5-z}{8}=\frac{18-5z}{12}$; e) $\frac{4t+33}{21}=\frac{17+t}{14}$.

17-§. Tenglamalar yordamida masalalar yechish

Masalalarni tenglamalar yordamida yechishda asosan quyidagi tartibga rioya qilish foydali:

1. Masalada topish kerak bo'lgan soni biror harf bilan (ko'pincha x, y, z...)lar bilan belgilab olinadi.
2. Shu harf va masalada berilgan boshqa sonlar orqali biror ifoda tuziladi.
3. Bu ifodani masalada berilgan songa yoki boshqa ifodaga tenglashtirib tenglama tuziladi.
4. Tenglamani yechib, noma'lum sonni (x, y, z...) topiladi.
5. Topilgan son orqali qolgan noma'lum sonlar ham topiladi.

Masala yechish namunasi:

1-masala: Ikki sonning yig'indisi 84 ga teng. Ularning biri ikkinchisidan 32 ta ko'p bo'lsa, shu sonlarni toping.

Yechish: Sonlardan biri x bo'lsin.

Ikkinchi son $x+32$ bo'ladi.

Ularning yig'indisi: $x+x+32=84$

Tenglamani yechamiz: $x+x+32=84$

$2x+32=84$

$2x=84-32$ Ikkinchi son: $x+32=26+32=58$

$2x=52$ Tekshirish: $26+58=84$.

$x=52:2$

$x=26$ *Javob:* 26 va 58.

2-masala. 158 ta kitobni uchta tokchaga: birinchi tokchada ikkinchi tokchadagiga qaraganda 8 ta kam kitob va uchinchi tokchadagiga qaraganda 6 ta ortiq kitob joylashgan. Har qaysi tokchada nechtadan kitob bo'lgan?

Yechish: 1-tokchada x ta kitob bo'lsin.

2-tokchada $x+8$ ta kitob bo'ladi.

3-tokchada $x-6$ ta kitob bo'ladi.

$x+x+8+x-6=158$

$3x=158-8+6$ 2-tokchada $x+8=52+8=60$

$3x=156$ 3-tokchada $x-6=52-6=46$

$x=156:3$

$x=52$ (ta kitob) *Javob:* 52 ta, 60 ta, 46 ta.

3-masala. Maktablarning matematika olimpiadasida yechish uchun 10 ta masala berildi. To'g'ri yechilgan har bir masala uchun 5 ball beriladi, noto'g'ri yechilgan har bir masala uchun 3 ball olib tashlanadi. 34 ball olgan o'quvchi nechta masalani to'g'ri yechgan?

Yechish. To'g'ri yechilgan masala x ta bo'lsin.

Noto'g'ri yechilgan masalalar $10-x$ ta bo'ladi.

To'g'ri yechilgan masalalar $5x$ ball bo'ladi.

Noto'g'ri yechilgan masalalar $3\cdot(10-x)$ ball bo'ladi.

To'plangan barcha ballar: $5x-3(10-x)$ ta bo'ladi.

Tenglama tuzamiz:

$5x-3(10-x)=34$

$5x-30+3x=34$

$8x=34+30$

$8x=64$

$x=8$ (ta). *Javob:* 8 ta masala.

MASALALARNI YECHING

130. Savatdagi olmalar yashikdagiga qaraganda ikki marta kam bo‘lgan. Yashikdagi olmadan 12 ta olib savatga solinsa, ulardagi olmalar teng bo‘lib qoladi. Savatda qancha olma bo‘lgan?

131. Zavodning uchta sexida 1274 kishi ishlaydi. Ikkinchi sexda birinchi sexga qaraganda 70 kishi ortiq ishlaydi, uchinchi sexda esa, ikkinchi sexdagiga qaraganda 84 kishi ortiq. Har qaysi sexda qanchadan kishi ishlaydi?

132. Sviter, shapka va sharf to‘qish uchun 555 g jun sarf qilingan. Bunda shapka uchun sviterga qaraganda 320 g kam, sharfga qaraganda esa 5 g ortiq jun ketgan. Bularning har biriga qanchadan jun sarf qilingan?

133. Doskaga biror son yozib qo‘yilgan. Bu sonni o‘quvchilardan biri 23 ta orttirdi, ikkinchisi 1 ta kamaytirdi. Birinchi bolaning yozgan soni ikkinchi bolaning yozgan sonidan 7 marta katta bo‘lgan. Doskada qanday son yozilgan?

134. Bir tarvuz ikkinchi tarvuzdan 2 kg yengil, uchinchi tarvuzdan 5 marta yengil. Birinchi va uchinchi tarvuzlar birgalikda ikkinchi tarvuzdan 3 marta og‘ir. Har qaysi tarvuzning og‘irligini toping.

135. Ikki qopning har birida 50 kg dan shakar bor edi. Birinchi qopdan ikkinchisidan olinganiga qaraganda 3 marta ko‘p shakar olinganidan so‘ng, birinchi qopda ikkinchisiga qaraganda 2 marta kam shakar qolgan. Har qaysi qopda qanchadan shakar qolgan?

136. Teploxod daryo oqimi bo‘ylab 9 soatda oqimga qarshi 11 soatda o‘tgancha yo‘l yurdi. Agar daryo oqimining tezligi 2 km/soat bo‘lsa, teploxodning tezligini toping.

137. Kassada 200 so‘mlik va 500 so‘mlik pullardan 400 dona bo‘lgan. Ularning qiymati (jami pul) 155000 so‘mni tashkil qilgan. Kassada 200 so‘mlik va 500 so‘mlik pullardan qancha dona bo‘lgan?

138. Ikki kishining omonat kassaga qo‘ygan puli 2490000 so‘m bo‘lgan. Ulardan birining 6,5% puli, ikkinchisining 8,5% puliga teng. Ularning har biri omonatga qanchadan pul qo‘ygan?

139. Ikki ishchining oylik maoshi birgalikda 870000 so'm bo'lgan. Birinchi ishchining maoshi ikkinchisining maoshidan 50000 so'm ortiq bo'lsa, har birining oylik maoshi qanchadan bo'lgan?

140. Sirdaryo uzunligining Norin daryosi uzunligiga nisbati 20 : 11 kabi. Sirdaryo Norin daryosidan 900 km ortiq bo'lsa, har qaysi daryoning uzunligini toping.

141. Poyezd *A* shahridan *B* shahriga 10 soatu 40 minutda bordi. Agar poyezdning tezligi soatiga 10 km kam bo'lsa, u *B* shahriga 2 soat 8 minut kechikib boradi. Shaharlar orasidagi masofani va poyezdning tezligini toping.

142. Vertolyot ikki shahar orasidagi masofani shamol yo'nalishi tomoniga 5 soatu 30 minutda, shamolga qarshi 6 soatda uchib o'tadi. Agar shamolning tezligi 10 km bo'lsa, shaharlar orasidagi masofani va vertolyotning o'z tezligini toping.

143*. Zavodda birinchi sex ishchilari sonining ikkinchi sex ishchilarining soniga nisbati 3:2 kabi. Agar birinchi sexdan 18 ishchi ikkinchiga olinsa, u vaqtda birinchi sex ishchilari sonining ikkinchi sex ishchilarining soniga nisbati 5:4 ga teng bo'ladi. Har qaysi sex ishchilarining sonini toping.

18-§. Ikki o'zgaruvchili chiziqli tenglama

Ikki o'zgaruvchili $5x+2y=10$, $-3x+7y=5$, $0{,}4x-1\frac{1}{2}y=-7$ va hokazo tenglamalarning har biri $ax+by=c$ ko'rinishga ega, bunda a, b va c – biror sonlar. Bunday tenglamalar **ikki o'zgaruvchili chiziqli tenglamalar** deyiladi.

Ta'rif. **Ikki o'zgaruvchili chiziqli tenglama deb $ax+by=c$ ko'rinishdagi tenglamaga aytiladi, bunda x va y – o'zgaruvchilar, a, b va c sonlar.** a va b sonlar o'zgaruvchilar oldidagi **koeffitsiyentlar,** c soni **ozod had.**

Agar chiziqli tenglamada y oldidagi koeffitsiyent $b\neq0$ bo'lsa, bu tenglamada y ni x orqali ifodalash mumkin. Masalan, $3x+2y=6$ tenglamadan:

$2y=-3x+6$; $y=-1,5x+3$. Bu $y=-1,5x+3$ formula grafigi to‘g‘ri chiziq bo‘lgan chiziqli funksiyani ifodalaydi. O‘sha to‘g‘ri chiziqning o‘zi $3x+2y=6$ tenglamaning grafigi hamdir, chunki bu tenglama $y=1,5x+3$ tenglamaga teng kuchli.

Agar $b=0$, $a=2$ bo‘lsa, tenglama $2x+0y=6$ bo‘lib, bunda $x=3$ bo‘ladi. y istalgan son bo‘lganda ham $2x+0y=6$ tenglamani qanoatlantiradi. Bu sonlar jufti $(3;\ y)$ dan iborat. Bu juftlar ordinatalar $0y$ o‘qiga parallel bo‘lgan to‘g‘ri chiziq bo‘ladi (1-*a* chizma).

Agar $a=0$, $b=4$ bo‘lsa, tenglama $0\cdot x+4y=6$ bo‘lib, bunda $y=1,5$ bo‘ladi. x istalgan son bo‘lganda ham $0\cdot x+4y=6$ tenglamani qanoatlantiradi. Bu sonlar jufti $(x;\ 1,5)$ dan iborat bo‘lib, OX o‘qiga parallel to‘g‘ri chiziq bo‘ladi (1-*a* chizma).

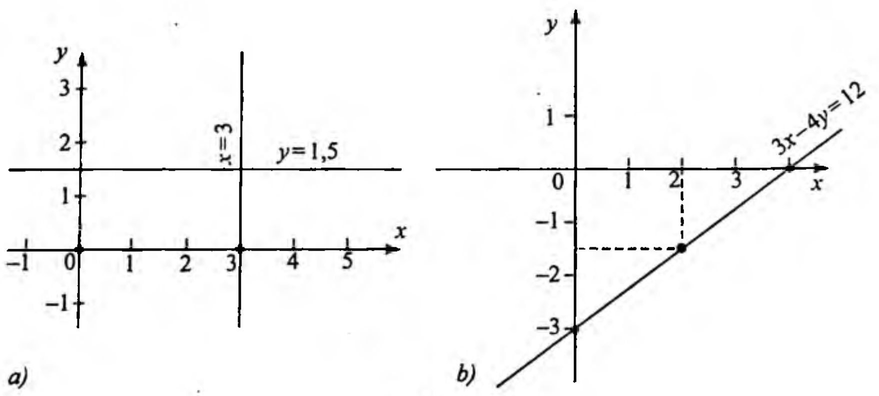
Shunday qilib, **o‘zgaruvchilar oldidagi koeffitsiyentlardan aqalli bittasi nolga teng bo‘lmagan ikki o‘zgaruvchili chiziqli tenglamaning grafigi to‘g‘ri chiziq bo‘ladi.**

1-misol. $3x-4y=12$ tenglamaning grafigini chizamiz. Bu tenglamaning koeffitsiyentlari noldan farqli. Shuning uchun grafigi to‘g‘ri chiziq bo‘ladi. To‘g‘ri chiziq ikki nuqtasi bilan chizilganligi sababli uning istalgan ikki nuqtasining koordinatalarini topamiz:

agar $x=0$ bo‘lsa, $y=-3$ bo‘lib, $(0;\ -3)$ bo‘ladi;

agar $x=2$ bo‘lsa, $y=-1,5$ bo‘lib, $(2;\ -1,5)$ bo‘ladi.

Bu nuqtalar orqali to‘g‘ri chiziq o‘tkazamiz (1-*b* chizma).



1-chizma.

U to‘g‘ri chiziq $3x-4y=12$ tenglamaning grafigi bo‘ladi.

2-misol. $-2(3x+y)+5y=3$ tenglamaning grafigini chizamiz.

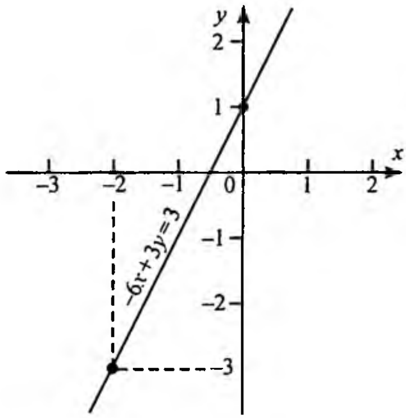
Avval tenglamani soddalashtiramiz:

$-6x-2y+5y=3$; $-6x+3y=3$; $3y=6x+3$; $y=2x+1$ chiziqli tenglama hosil bo‘ldi.

Bunda: $x=0$ da, $y=1$ bo‘lib, (0; 1)

$x=-2$ da, $y=-3$ bo‘lib, (–2; –3) hosil bo‘ldi.

Natijada $A(0;\ 1)$, $B(-2;\ -3)$ nuqtalarni yasab, ulardan $-6x+3y=3$ to‘g‘ri chiziqni o‘tkazamiz (2-chizma).



2-chizma.

TAKRORLASH UCHUN SAVOLLAR

1. Ikki o‘zgaruvchili chiziqli tenglamalarga misollar keltiring.
2. Ikki o‘zgaruvchili chiziqli tenglamaning umumiy ko‘rinishini yozing.
3. Chiziqli tenglamaning grafigi qanday chiziq bo‘ladi?
4. $x=-3{,}2$ va $y=2\frac{1}{3}$ larning grafiklari qanday to‘g‘ri chiziq bo‘ladi?

MASALALARNI YECHING

144. Ikki o‘zgaruvchili quyidagi tenglamalardan chiziqli tenglamasi yozing:

a) $3x-y=17$; b) $x^2-2y=5$; d) $13x+6y-0$; e) $xy-2x=9$.

145. $4x-3y=12$ chiziqli tenglamadan: a) y ni x orqali; b) x ni y orqali ifodalang.

146. Quyidagi chiziqli tenglamaning ixtiyoriy beshta yechimini toping:
a) $x-3y=12$; b) $u+2v=4{,}2$.

147. $21x-45y=100$ tenglamaning yechimi ikki son bo'lib, ulardan biri $x=5$. y ning unga mos qiymatini toping.

148. Tenglamaning grafigini chizing:
a) $2x-y=6$; b) $1{,}5x+2y=3$; d) $2x=7$; e) $-5y=21$.

149. Tenglamaning grafigini chizing:
a) $x-y=0$; d) $2(x-y)+3y=4$;
b) $3x=y+4$; e) $-2(x+y)-(x-y)=4$.

19-§. Chiziqli tenglamalar sistemasi

Masala. Ikki savatda 12 kg olma bor, bunda birinchi savatda ikkinchisidagidan 2 kg ortiq. Har qaysi savatda qancha olma bor?

Yechish: Birinchi savatda x kg, ikkinchi savatda y kg olma bo'lsin. Masalaning shartiga asosan, ikkala savatda 12 kg olma bo'lgani uchun $x+y=12$ tenglamani tuzamiz. Birinchi savatdagi olmalar ikkinchisidagidan 2 kg ortiq bo'lgani uchun $x-y=2$ tenglamani tuzamiz.

Masalaning savoliga javob berish uchun, ham birinchi tenglamaning, ham ikkinchi tenglamaning yechimi bo'ladigan ikki qiymatni topishimiz kerak. Bunday ishni **tenglamalar sistemasini yechish** deyiladi. Tenglamalar sistemasini katta qavs yordamida yozishga kelishilgan, ya'ni

$$\begin{cases} x+y=12 \\ x-y=2 \end{cases}$$ kabi yoziladi.

O'zgaruvchilarning $x=7$ va $y=5$ qiymatlari jufti sistemadagi har qaysi tenglamaning yechimi bo'ladi, chunki $7+5=12$ va $7-5=2$ tengliklar to'g'ri.

Bu sonlar jufti (7; 5) ni ***sistemaning yechimi*** deyiladi. Sistemaning boshqa yechimlari yo'q.

Javob: 1 – savatda 7 kg, 2 – savatda 5 kg olma bor.

Ta'rif. **Chiziqli tenglamalar sistemasining yechimi deb o'zgaruvchilarning sistemadagi har qaysi tenglamani to'g'ri tenglikka aylantiruvchi qiymatlari juftiga aytiladi.**

Tenglamalar sistemasini yechish – uning hamma yechimlarini topishni yoki yechimi yo'q ekanligini isbotlash tushuniladi.

1-misol. Chiziqli $\begin{cases} 2x+3y=5 \\ 3x-y=-9 \end{cases}$ tenglamalar sistemasini grafik usulda yechishni o'rganamiz. Buning uchun ayni bir koordinata tekisligida sistemadagi tenglamalarning grafigini chizamiz. Bu ikkala tenglamaning grafigi to'g'ri chiziq bo'ladi.

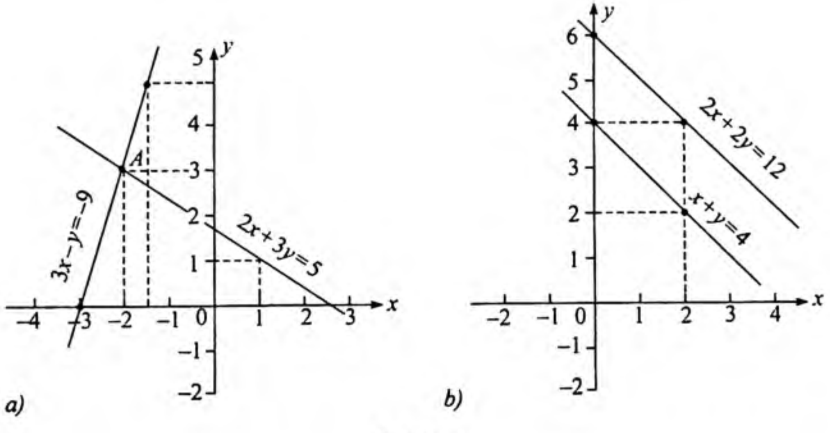
Grafiklar (to'g'ri chiziqlar) kesishgan nuqtaning koordinatalari ikkala tenglamani qanoatlantiradi, ya'ni sistemaning yechimi bo'ladi. Tenglamalarning grafigi bitta $A(-2; 3)$ nuqtada kesishadi.

$2x+3y=5$ ning grafigi $x=1$ da $3y=5-2=3$, ya'ni (1; 1). $x=2{,}5$ da $3y=5-5=0$, ya'ni (2,5; 0).

$3x-y=-9$ ning grafigi $x=-3$ da $-y=-9+9=0$; $y=0$; (–3; 0).

$x=-1{,}5$ da $-y=-9+4{,}5=-4{,}5$; $y=4{,}5$; (–1,5; 4,5).

Demak, sistema yagona (–2; 3) yechimga ega (3-*a* chizma).



3-chizma.

2-misol. $\begin{cases} x+y=4 \\ 2x+2y=12 \end{cases}$ sistemani yechamiz.

Sistemadagi tenglamalarning grafiklarini chizamiz:

$x+y=4$ ning grafigi	$x=0$ da $y=4$; (0; 4)
	$x=2$ da $y=4-2=2$; (2; 2)
$2x+2y=12$ ning grafigi	$x=0$ da $2y=12$; $y=6$; (0; 6)
	$x=2$ da $2y=12-4=8$; $y=4$; (2; 4)

Bu sistemadagi chiziqli tenglamalarning grafiklari parallel to‘g‘ri chiziqlar bo‘lgani uchun ular umumiy nuqtaga ega bo‘lmaydi, shuning uchun sistema yechimga ega emas (3-*b* chizma).

Agar sistemadagi to‘g‘ri chiziqlar ustma-ust tushsa, sistema cheksiz ko‘p yechimga ega bo‘ladi. Masalan, $\begin{cases} x+y=4 \\ 4x+4y=16 \end{cases}$ sistemadagi tenglamalarning grafiklari ustma-ust tushadi, ya’ni sistema cheksiz ko‘p yechimga ega.

Shuni eslatamizki, grafik usulda yechim taqriban topiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Tenglamalar sistemasini yechish deganda nimani tushunamiz?
2. Chiziqli tenglamalar sistemasining yechimi deb nimaga aytiladi?
3. Chiziqli tenglamalar sistemasini grafik usulda yechilganda qanday holda faqat bitta yechimga ega bo‘ladi?
4. Chiziqli tenglamalar sistemasini grafik usulda yechganda qanday holda cheksiz ko‘p yechimga ega bo‘ladi?
5. Chiziqli tenglamalar sistemasini grafik usulda yechilganda qanday holda sistema yechimga ega bo‘lmaydi?

MASALALARNI YECHING

150. Quyidagi sonlar jufti $\begin{cases} x+y=3 \\ 3x-y=1 \end{cases}$ sistemaning yechimi bo‘ladimi?

a) $x=3, y=0$; b) $x=1, y=2$; d) $x=3; y=-5$.

151. (3; –1) sonlar jufti sistemaning yechimi bo'ladimi (bu yerda birinchi o'rinda y o'zgaruvchining qiymati, ikkinchi o'rinda z o'zgaruvchining qiymati)?

a) $\begin{cases} 3y+z=8 \\ 7y-2z=23 \end{cases}$; b) $\begin{cases} 2y+z=5 \\ 5y-2z=17 \end{cases}$.

152. Yechimi o'zgaruvchilarning: a) $x=2$ va $y=5$;

b) $x=0$ va $y=3$ qiymatlari jufti bo'lgan tenglamalar sistemasini tuzing.

153. Sistemani grafik usulda yeching:

a) $\begin{cases} y-x=0 \\ 3x+y=8 \end{cases}$; b) $\begin{cases} 2x-y=3 \\ x+y=3 \end{cases}$.

154. Quyida berilgan sistemalar yechimga egami, agar bo'lsa, ular nechta:

a) $\begin{cases} 4y-x=12 \\ 3y+x=-5 \end{cases}$; b) $\begin{cases} x+2y=3 \\ 0,5x+y=0 \end{cases}$; d) $\begin{cases} 2x+3y=11 \\ 6y=22-4x \end{cases}$.

20-§. Chiziqli tenglamalar sistemasini yechishning o'rniga qo'yish usuli

Ushbu $\begin{cases} 3x+y=7 \\ -5x+2y=3 \end{cases}$ chiziqli tenglamalar sistemasini yechamiz. Birinchi tenglamadan y ni x orqali ifodalaymiz: $y=7-3x$.

Ikkinchi tenglamadagi y ning o'rniga $7-3x$ ifodani qo'yib, $\begin{cases} 3x+y=7 \\ -5x+2(7-3x)=3 \end{cases}$ sistema hosil qilamiz. Bu sistema berilgan sistema bilan bir xil yechimga ega, chunki y ning o'rniga unga teng bo'lgan $7-3x$ qo'yildi.

Shunday qilib, berilgan sistema bilan hosil bo'lgan sistemalar bir xil yechimga ega bo'ladi. Bunday sistemalarni **teng kuchli sistemalar** deyiladi.

Keyingi sistemaning ikkinchi tenglamasi bitta o'zgaruvchili tenglama bo'ldi. Shu tenglamani yechamiz:

$-5x+2(7-3x)=3$

$-5x+14-6x=3$

$-11x=-11$

$x=1.$

Berilgan sistemaning $3x+y=7$ tenglamasidagi x ning o'rniga 1 sonini qo'yib, y ning mos qiymatini topamiz:

$y=7-3x=7-3\cdot1=4.$

Topilgan (1; 4) sonlar jufti berilgan sistemaning yechimi bo'ladi.

Chiziqli tenglamalar sistemasini yechishning bu usulini **o'rniga qo'yish usuli** deyiladi.

O'rniga qo'yish usulini qo'llab yana bitta sistemani yechamiz.

2-misol.

$\begin{cases} 7x+6y=6 \\ 3x+4y=9 \end{cases}$ ikkinchi tenglamadan x ni y orqali ifodalaymiz.

$3x=9-4y$

$x=\frac{9-4y}{3}$ buni birinchi tenglamadagi x ning o'rniga $\frac{9-4y}{3}$ ifodani qo'yamiz.

$7\cdot\frac{9-4y}{3}+6y=6$ bu bir o'zgaruvchili tenglamani yechamiz.

$7(9-4y)+3\cdot6y=3\cdot6$

$63-28y+18y=18$

$-10y=-45$

$y=4,5.$

Bu $y=4{,}5$ sonni $x=\frac{9-4y}{3}$ ga qo'yib, ya'ni $x=\frac{9-4\cdot4{,}5}{3}=\frac{-9}{3}=-3$.

Javob: $x=-3$; $y=4{,}5$-yoki $(-3;\ 4{,}5)$.

3-misol. $\begin{cases}5y+8(x-3y)=7x-12\\9x+3(x-9y)=11y+46\end{cases}$ sistemani yechamiz.

Yechish: $\begin{cases}5y+8x-24y-7x=-12\\9x+3x-27y-11y=46\end{cases}$; $\begin{cases}x-19y=-12\\12x-38y=46\end{cases}$

$\begin{cases}x=19y-12\\12x-38y=46\end{cases}$; $12\cdot(19y-12)-38y=46$.

$228y-144-38y=46$

$190y=190$

$y=1$. bu $y=1$ sonni $x=19y-12$ ga qo'yib,

$x=19\cdot1-12=7$. *Javob:* $(7;\ 1)$.

TAKRORLASH UCHUN SAVOLLAR

1. Chiziqli tenglamalar sistemasining yechimi deganda nimani tushunamiz?
2. Teng kuchli tenglamalar sistemasi deb qanday sistemalarga aytiladi?
3. Tenglamalar sistemasini o'rniga qo'yib yechish usuli nimadan iborat?

MASALALARNI YECHING

155. Tenglamalar sistemasini yeching:

a) $\begin{cases}y=2x+1\\7x-y=a\end{cases}$; b) $\begin{cases}7x-3y=13\\x-2y=5\end{cases}$; d) $\begin{cases}x+y=6\\3x-5y=2\end{cases}$.

e) $\begin{cases}4x-y=11\\6x-2y=13\end{cases}$; f) $\begin{cases}y-x=20\\2x-15y=-1\end{cases}$; g) $\begin{cases}25-x=-4y\\3x-2y=30\end{cases}$.

156. Tenglamalar sistemasini yeching:

a) $\begin{cases} 2u+5v=0 \\ -8u+15v=7 \end{cases}$; d) $\begin{cases} 4u+3v=14 \\ 5u-3v=25 \end{cases}$;

b) $\begin{cases} 5p-3q=0 \\ 3p+4q=29 \end{cases}$; e) $\begin{cases} 10p+7q=-2 \\ 2p-22=5q \end{cases}$.

157. a) $\begin{cases} 2(3x-2y)+1=7x \\ 12(x+y)-15=7x+12y \end{cases}$; d) $\begin{cases} 5(x+2y)-3=3x+5 \\ 4(x-3y)-50=-33y \end{cases}$;

b) $\begin{cases} 3(x+y)-7=12+y \\ 6(y-2x)-1=-45x \end{cases}$; e) $\begin{cases} 4x+1=5(x-3y)-6 \\ 3(x+6y)+4=9y+19 \end{cases}$.

158. a) $\begin{cases} \frac{x}{3}-\frac{y}{2}=-4 \\ \frac{x}{2}+\frac{y}{4}=-2 \end{cases}$; d) $\begin{cases} \frac{2m}{5}+\frac{n}{3}=1 \\ \frac{m}{10}-\frac{7n}{6}=4 \end{cases}$;

b) $\begin{cases} \frac{a}{6}-2b=6 \\ -3a+\frac{b}{2}=-37 \end{cases}$; e) $\begin{cases} 7x-\frac{3y}{5}=-4 \\ x+\frac{2y}{5}=-3 \end{cases}$.

21-§. Chiziqli tenglamalar sistemasini yechishning qo‘shish usuli

Tenglamalar sistemasini yechishning **qo‘shish usuli** deb ataladigan yana bir usulini ko‘rib chiqamiz. Sistemalarni bu usul bilan yechishda sistemadan unga teng kuchli bo‘lgan boshqa sistemaga o‘tamiz, unda tenglamalardan biri faqat, bir o‘zgaruvchili bo‘ladi.

1-misol. $\begin{cases} 2x+3y=-5 \\ x-3y=38 \end{cases}$ sistemani yechamiz.

Sistemadagi tenglamalarda y lar oldidagi koeffitsiyentlar qarama-qarshi sonlar. Tenglamaning chap va o'ng qismlarini hadma-had qo'shib, bir o'zgaruvchili tenglama hosil qilamiz, ya'ni $+\begin{cases}2x+3y=-5\\x-3y=38\end{cases}$

$$3x=33$$

Berilgan sistemadagi tenglamalardan birini, masalan, birinchisini $3x=33$ tenglama bilan almashtirib, $\begin{cases}3x=33\\x-3y=38\end{cases}$ sistema hosil qilamiz.

Bu sistemadagi tenglamadan $3x=33$ ni olib, undan $x=11$ ekanini topamiz. $x=11$ ni $x-3y=38$ tenglamaga qo'yib, y o'zgaruvchini topamiz:

$$11-3y=38$$
$$-3y=-11+38$$
$$-3y=27$$
$$y=-9.$$

Demak, (11; –9) sonlar jufti berilgan sistemani qanoatlantiradi.

Javob: (11; –9).

2-misol. $\begin{cases}2x-7y=2\\6x-11y=26\end{cases}$ sistemani yechamiz:

Ikkinchi tenglamada x ning oldidagi koeffitsiyent birinchidagiga qaraganda uch marta katta. Shuning uchun birinchi tenglamaning ikkala qismini –3 ga ko'paytirib, berilgan tenglamaga teng kuchli sistema hosil qilamiz, ya'ni $\begin{cases}2x-7y=2\\6x-11y=26\end{cases}\Bigg|\,-3 \quad +\begin{cases}-6x+21y=-6\\6x-11y=26\end{cases}.$

$$10y=20$$
$$y=2.$$

Bularni qo'shib, yig'indini yozamiz. Berilgan sistemaning birinchi tenglamasidagi y ning o'rniga 2 ni qo'yib, x o'zgaruvchini topamiz:

$$2x-7\cdot 2=2$$
$$2x=16$$
$$x=8.$$

Javob: (8; 2).

3-misol. $\begin{cases} \frac{5x}{6} - y = -\frac{5}{6} \\ \frac{2x}{3} + 3y = -\frac{2}{3} \end{cases}$ sistemani yechamiz.

$$\begin{cases} \frac{5x}{6} - y = -\frac{5}{6} \\ \frac{2x}{3} + 3y = -\frac{2}{3} \end{cases} \begin{array}{|l} 6 \\ 3 \end{array} \begin{cases} 5x - 6y = -5 \\ 2x + 9y = -2 \end{cases} \begin{array}{|l} 3 \\ 2 \end{array} + \begin{cases} 15x - 18y = -15 \\ 4x + 18y = -4 \end{cases}$$

$$19x = -19;$$

$$x = -1$$

$x=-1$. Sistemaning $2x+9y=-2$ tenglamasidagi x ning o‘rniga -1 qo‘yib, y o‘zgaruvchini topamiz:

$2 \cdot (-1) + 9y = -2$

$9y = 0$

$y = 0.$ *Javob:* $(-1; 0)$.

TAKRORLASH UCHUN SAVOLLAR

1. Tenglamalar sistemasini yechishning qo‘shish usuli qanday bajariladi?
2. Tenglamalar sistemasini yechishning grafik usuli qanday bajariladi?
3. Tenglamalar sistemasini yechishning o‘rniga qo‘yish usuli qanday bajariladi?
4. Sistemaning og‘zaki yechimini toping:

$$\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}.$$

MASALALARNI YECHING

159. Sistemani qo‘shish usuli bilan yeching:

a) $\begin{cases} 2x + 11y = 15 \\ 10x - 11y = 9 \end{cases}$; d) $\begin{cases} 4x - 7y = 30 \\ 4x - 5y = 90 \end{cases}$;

b) $\begin{cases} 9x - 17y = -4 \\ -9x + 15y = 12 \end{cases}$; e) $\begin{cases} 13x - 8y = 28 \\ 11x - 8y = 24 \end{cases}$.

160. Sistemani qo'shish usuli bilan yeching:

a) $\begin{cases} 40x+3y=10 \\ 20x-7y=5 \end{cases}$; d) $\begin{cases} 13x-12y=14 \\ 11x-18y=4 \end{cases}$;

b) $\begin{cases} 5x-2y=1 \\ 15x-3y=-3 \end{cases}$; e) $\begin{cases} 10x-9y=8 \\ 15x+21y=0,5 \end{cases}$.

161. Sistemaning yechimini toping:

a) $\begin{cases} 0,75x+20y=95 \\ 0,32x-25y=7 \end{cases}$; d) $\begin{cases} 10x=4,6+3y \\ 4y+3,2=6x \end{cases}$;

b) $\begin{cases} 0,5u-0,6v=0 \\ 0,4u+1,7v=10,9 \end{cases}$; e) $\begin{cases} -3b+10a-0,1=0 \\ 15a+4b-2,7=0 \end{cases}$.

162. Sistemani yeching:

a) $\begin{cases} 5(x+2y)-3=x+5 \\ y+4(x-3y)=50 \end{cases}$; b) $\begin{cases} 2,5(x-3y)-3=-3x+0,5 \\ 3(x+6y)+4=9y+19 \end{cases}$.

163. Sistemaning yechimini toping:

a) $\begin{cases} \frac{x}{3}+\frac{y}{4}-5=0 \\ 2x-y=10 \end{cases}$; d) $\begin{cases} \frac{1}{3}x-\frac{1}{12}y=4 \\ 6x+5y=150 \end{cases}$;

b) $\begin{cases} 2x-7y=4 \\ \frac{x}{6}-\frac{y}{6}=0 \end{cases}$; e) $\begin{cases} \frac{1}{3}v-\frac{1}{8}u=3 \\ 7u+9v=-2 \end{cases}$.

164. Sistema yechimga egami va nechta yechimga ega?

a) $\begin{cases} -5x+2y=7 \\ 15x-6y=-21 \end{cases}$; b) $\begin{cases} 2x-y=1 \\ -6x+3y=2 \end{cases}$.

22-§. Masalalarni tenglamalar sistemasi yordamida yechish

Ko‘p masalalarning yechilishi ikki noma’lumli tenglama sistemasini yechishga keltiriladi. Buning uchun avval noma’lum sonlar harflar bilan belgilanadi. So‘ngra tenglamalar sistemasi tuziladi, yechiladi va nihoyat, chiqqan natija masala shartini qanoatlantirishi tekshiriladi.

1-masala. Maktab uchun 55000 so‘mga 5 komplekt shaxmat va 8 komplekt shashka sotib olindi. Agar 3 komplekt shaxmat 4 komplekt shashkadan 2200 so‘m qimmat bo‘lsa, bir komplekt shaxmat va bir komplekt shashka qancha turadi?

Yechish. Bir komplekt shaxmat x so‘m, bir komplekt shashka y so‘m tursin. U holda 5 komplekt shaxmat $5x$ so‘m, 8 komplekt shashka $8y$ so‘m turadi. To‘langan pul 55000 so‘m bo‘lgani uchun $5x+8y=550000$ tenglama tuzamiz.

3 komplekt shaxmat $3x$ so‘m va 4 komplekt shashka $4y$ so‘m bo‘lgani uchun va ular orasidagi farq 2200 so‘m bo‘lganidan $3x-4y=2200$ tenglama tuzamiz. Masalaning javobini topish uchun x va y ning shunday qiymatlarini topish kerakki, ular birinchi tenglamani ham, ikkinchi tenglamani ham qanoatlantirsin, ya’ni $\begin{cases} 5x+8y=55000 \\ 3x-4y=2200 \end{cases}$ sistemani qanoatlantirsin.

Sistemani yechamiz.

$$\begin{cases} 5x+8y=55000 \\ 3x-4y=2200 \end{cases} \Big| \, 2 \quad + \begin{cases} 5x+8y=55000 \\ 6x-8y=4400 \end{cases}$$

$$11x=59400$$

$$x=5400 \text{ (so‘m)}.$$

$3x-4y=2200$ dan

$x=5400$ bo‘lganda

$4y=3x-2200=3\cdot 5400-2200=16200-2200=14000.$

$4y=1400$

$y=3500$ (so‘m). Demak, $x=5400$ so‘m, $y=3500$ so‘m.

Javob: 1 komplekt shaxmat 5400 so‘m,

1 komplekt shashka 3500 so‘m.

2-masala. Ikki bolada 32 ta yong'oq bor. Agar bir bola ikkinchisiga 6 ta yong'oq bersa, o'zida o'rtog'ining qo'lidagidan 3 marta kam yong'oq qoladi. Har qaysi bolada nechta yong'oq bor?

Yechish. Birinchi bolada x ta, ikkinchi bolada y ta yong'oq bo'lsin. Ikkalasida $x+y=32$ ta yong'oq bo'ladi. 6 ta yong'oqni ikkinchisiga bergandan keyin birinchi bolada $x-6$ ta, ikkinchi bolada $y+6$ ta yong'oq bo'ladi. Natijada birinchi bolada ikkinchisiga qaraganda 3 marta kam bo'lganidan quyidagicha tenglik to'g'ri bo'ladi $3\cdot(x-6)=y+6$.

Ikkala tenglamadan sistema tuzib, uni yechamiz.

$$+\begin{cases} x+y=32 \\ 3x-y=24 \end{cases}$$
$$4x=56$$

$x=14$ (yong'oq).

$x+y=32$ dan $y=32-x=32-14=18$ (yong'oq).

Javob: 1-bolada 14 ta, 2-bolada 18 ta yong'oq.

MASALALARNI TENGLAMALAR SISTEMASI YORDAMIDA YECHING

165. Ikki sonning yig'indisi 13, ayirmasi 2. Shu sonlarni toping.

166. To'g'ri to'rtburchakning perimetri 30 m, uning eni bo'yidan 1 m qisqa. To'g'ri to'rtburchakning tomonlari uzunligini toping.

167. Fermer bug'doy va suli ekish uchun 700 ga yer ajratdi, bunda bug'doy uchun ajratilgan maydon suli uchun ajratilgan maydondan 60 ga ortiq. Bug'doy uchun va suli uchun necha gektar yer ajratilgan?

168. 8 ta ot bilan 15 ta sigirni boqish uchun kuniga 162 kg pichan beriladi. Agar 5 ta otga 7 ta sigirga berilganiga qaraganda 3 kg ortiq pichan berilsa, kuniga har bir otga qancha va har bir sigirga qancha pichan beriladi?

169. Ikki usta birga ishlab 117000 so'm olishdi. Birinchi usta 15 kun, ikkinchisi 14 kun ishlagan. Agar birinchi ustaning 4 kunga olgani ikkinchi ustaning 3 kunga olganidan 11000 so'm ortiq bo'lsa, har qaysi usta kuniga qancha pul olgan?

170. Bir paravoz va 15 ta vagondan tuzilgan passajir poyezdi tarkibining og'irligi 370,5 t. Paravozning og'irligi 4 ta vagonning og'irligidan 13,3 t ortiq. Bitta vagonning va paravozning og'irligini toping.

171. Turistlar 4 soat avtomashinada va 7 soat poyezdda yurib, 640 km masofani o'tishdi. Agar poyezdning tezligi avtomashinaning tezligidan 5 km/s ortiq bo'lsa, poyezdning tezligini toping.

172. Oralaridagi masofa 280 km bo'lgan *A* va *B* shaharlardan bir vaqtda ikki avtomobil yo'lga chiqdi. Agar avtomobillar bir-biriga qarab yursa, ular 2 soatdan keyin uchrashadi. Agar ular bir yo'nalishda yursa, *A* shahardan chiqqan avtomobil B shahardan chiqqan avtomobilni 14 soatda quvib yetadi. Har qaysi avtomobilning tezligini toping.

173. Motorli qayiq oqim bo'yicha bir portdan ikkinchisiga 4 soatda boradi. Qaytishda 5 soat yuradi. Agar qayiq 70 km ni oqim bo'yicha 3,5 soatda o'tsa, uning turg'un suvdagi tezligini va oqim tezligini toping.

174. Daraxt kesuvchilarning ikki brigadasi yanvarda 900 m^3-yog'och tayyorladi. Fevralda birinchi brigada yanvardagidan 15% ortiq, ikkinchi brigada 12% ortiq tayyorlab, ikkala brigada birgalikda 1020 m^3-yog'och tayyorladi. Fevralda har qaysi brigada necha kub metr yog'och tayyorlagan?

175. 10 ot bilan 14 sigirni boqish uchun kuniga 180 kg pichan beriladi. Otlarga beriladigan pichan normasi 25% va sigirlarniki $33\frac{1}{3}$ % ko'paytirilgandan keyin kuniga 232 kg pichan beriladigan bo'ldi. Boshda kuniga bir otga qancha kilogramm va bir sigirga qancha kilogramm pichan berilgan?

176. Bak ikki quvur orqali to'ladi. Agar birinchi quvurdan 20 minut, ikkinchi quvurdan 10 minut suv oqib tushsa, bakka 120 m^3 suv yig'iladi. Agar birinchi quvur 15 minut, ikkinchi quvur 7 minut ochiq tursa, bakka 88,5 m^3 suv tushadi. Bakka har qaysi quvur orqali minutiga necha kub metr suv oqib tushadi?

177. Ishchilarning ikki brigadasi plan bo'yicha bir oyda 680 ta detal tayyorlashi kerak edi. Birinchi brigada oylik planni 20% oshirib, ikkinchi brigada 15% oshirib bajardi, shuning uchun ikkala brigada plandagidan 118 ta ortiq detal tayyorladi. Har qaysi brigada bir oyda plan bo'yicha nechta detal tayyorlashi kerak edi?

23-§. Algebraik kasr haqida tushuncha

Agar ko‘phadlarni bo‘lishda butun bo‘linma hosil bo‘lmasa, bo‘linma kasr ifoda shaklida yoziladi. Bunda bo‘linuvchi surat, bo‘luvchi maxraj bo‘ladi.

Kasr ifodalarga misollar:

$\frac{a}{3b}$; $\frac{a+b}{2b-c}$; $\frac{3(x+y)+5}{8x^3}$; $\frac{(a+c)^3-(a-2c)^3}{x^2+xy+y^2}$ va hokazo.

$\frac{x^2-10x}{8}=\frac{1}{8}(x^2-10x)$ butun ifoda bo‘ladi.

Kasr ifodaning surat va maxrajining o‘zlari ham kasr ifodalar bo‘lishi mumkin, masalan:

$\frac{\frac{a+b}{c}}{4a^2+c^3}$; $\frac{2x+\frac{b}{c}}{\frac{a}{b}+2c^2}$ va hokazo.

***Ta'rif.* Surat va maxraji ko‘phadlardan iborat bo‘lgan kasr shaklidagi ifoda – algebraik kasr deyiladi.**

Algebraik kasrning surat va maxraji kasrning hadlari deyiladi. Butun ifodani ham kasr ifoda shaklida yozish mumkin. Buning uchun suratiga berilgan ifodani, maxrajiga esa 1 ni yozish kifoya. Masalan, $x+2y=\frac{x+2y}{1}$.

Butun va kasr ifodalar ***ratsional ifodalar*** deyiladi.

Algebraik kasrning suratiga kiruvchi harflarga (agar biron qo‘shimcha cheklashlar bo‘lmasa) har qanday qiymatlar berish mumkin.

Maxrajiga kiruvchi harflarga esa maxrajni nolga aylantirmaydigan qiymatlarni berish mumkin.

Masalan: $\frac{2a+1}{b}$ 1) kasrning suratidagi a ga barcha sonlarni berish mumkin, ammo maxraji b ga esa noldan boshqa barcha sonlarni berish mumkin.

2) $\frac{2x^2-41}{a+12}$ kasrning suratidagi x ga barcha sonlarni berish mumkin, ammo maxrajidagi a ga esa, -12 dan boshqa barcha sonlarni berish mumkin, chunki $-12+12=0$ bo'ladi.

Algebraik kasrning qiymati butun yoki kasr son, musbat yoki manfiy son, yoki nol bo'lish mumkin. Masalan:

1) $a=12$; $b=8$ bo'lganda $\frac{a+b}{a-8}=\frac{12+8}{12-8}=\frac{20}{4}=5.$

2) $x=10$; $y=-1$ bo'lganda $\frac{x^2+20y}{2x-5y}=\frac{10^2+20\cdot(-1)}{2\cdot10-5\cdot(-1)}=\frac{80}{25}=\frac{16}{5}=3{,}2$

O'zgaruvchilarning ifoda ma'noga ega bo'ladigan qiymatlari **o'zgaruvchilarning qabul qiladigan qiymatlari** deyiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Kasr ifoda qanday hosil bo'ladi?
2. Kasr ifodaga misollar keltiring.
3. Qanday ifodani algebraik kasr deyiladi?
4. Qanday ifodani ratsional ifodalar deyiladi?
5. Algebraik kasrning surati va maxrajida qatnashgan harflarga qanday qiymatlar berish mumkin?
6. Algebraik kasr o'zgaruvchining (harfning) qanday qiymatlarida ma'noga ega bo'lmaydi?
7. 3 ta butun ifoda va 3 ta kasr ifoda yozing.
8. $\frac{x+5}{x}$ ifoda x ning qanday qiymatida ma'noga ega emas.

MASALALARNI YECHING

178. Ushbu $2x^2y$; $4a^2-b(a-3b)$; $\frac{x^2}{8}-12$; $9x-\frac{1}{2}$; $\frac{x+y}{x}$ ifodalarning qaysilari butun ifoda, qaysilari kasr ifodalar?

179. $\frac{x-1}{x}$ kasr ifodaning qiymatini $x=3$; -2; -15; $\frac{1}{3}$; $1{,}6$; 100 bo'lganda toping.

180. Ifodaning qiymatini toping:

a) $x=\frac{1}{2}$ da $x+\frac{8}{x-1}$; b) $y=1,5$ da $\frac{y+3}{y}-\frac{2y}{y-3}$.

181. Ifodani soddalashtiring:

a) $x(6-x)-2(1+3x)$;

b) $5a(a+2)-a(9+4a)$;

d) $(x^2-x+1)(x-1)-x(x^2-4)$;

e) $y^3(y-4)-(y^2+y-2)(y^2+2)$.

182. Ko'phadga aylantiring:

a) $(a-5b)(a+5b)$;

b) $(2y+3)(2y-3)$

d) $(3-2x)(2x+3)$;

e) $(8x+y)(y-8x)$;

f) $(x+7)^2$;

g) $(a-2x)^2$;

h) $(y+3)(y^2-3y+9)$;

i) $(m+5n)(m^2-5mn+25n^2)$.

183. Ko'phadni ko'paytuvchilarga ajrating:

a) $6m-18$;

b) $15ax+20ay$;

d) x^2-xy;

e) c^2-16;

f) $(b+7)^2$;

g) $(ab+1)^2$;

h) $(p-q)^3$;

i) $(2y-8)^3$.

184. O'zgaruvchining qanday qiymatlarida quyidagi ratsional ifodalar ma'noga ega?

a) $\frac{x}{x-2}$; b) $\frac{b^2+4}{b+7}$; d) $\frac{y^2-1}{y}+\frac{y}{y-3}$; e) $\frac{c+10}{c^2+5}$.

185. Ifodadagi o'zgaruvchilarning qabul qiladigan qiymatlarini ko'rsating.

a) x^2-8x+9;

b) $\frac{4}{6x-30}$;

d) $\frac{3x-15}{1,7}$;

e) $\frac{x^2-8}{3x-7}$;

f) $\frac{x^2-9}{x(x+1)}$;

g) $\frac{x-5}{x^2+16}-\frac{3}{x}$.

24-§. Kasrning asosiy xossasi. Kasrlarni qisqartirish

$\frac{a}{b}$ ko‘rinishdagi ratsional ifoda ratsional kasr deyiladi.

$\frac{2x}{x^2-5}$; $\frac{3x+2}{5y}$; $\frac{2ab-3}{\frac{a}{b}+12}$ va hokazo ratsional kasrlar.

Oddiy kasrda quyidagi xossa bor. Agar kasrning suratini va maxrajini ayni bir songa ko‘paytirilsa, kasrning qiymati o‘zgarmaydi.

Ratsional kasrlarda ham, a, b va c ning istalgan qiymatlarida $\frac{a}{b}=\frac{ac}{bc}$ tenglik to‘g‘ri bo‘lishini ko‘rsatamiz (bunda $b\neq 0$ va $c\neq 0$).

Isbot $\frac{a}{b}=m$ bo‘lsin. Bo‘linmaning ta’rifiga ko‘ra $a=bm$. Bu tenglikning ikkala qismini c ga ko‘paytiramiz: $ac=(bm)\cdot c$.

Ko‘paytirishning o‘rin almashtirish qonuniga ko‘ra $ac=(bc)\cdot m$ o‘rinli. Bunda $bc\neq 0$ edi. Bo‘linmaning ta’rifiga ko‘ra $\frac{ac}{bc}=m$ bo‘lib, bundan $\frac{a}{b}=\frac{ac}{bc}$ kelib chiqadi.

O‘zgaruvchining hamma qiymatlarida to‘g‘ri bo‘ladigan tengliklarni ayniyat deb aytgan edik.

Ta’rif. **Tenglikka kiruvchi o‘zgaruvchilarning barcha qabul qiladigan qiymatlarida to‘g‘ri bo‘ladigan tenglik ayniyat deyiladi.**

Demak, $\frac{a}{b}=\frac{ac}{bc}$ tenglik ayniyat bo‘ladi. $(a+b)(a-b)=a^2-b^2$ ham ayniyat bo‘ladi. Yuqoridagi ayniyatning chap va o‘ng qismlarini almashtirib, $\frac{ac}{bc}=\frac{a}{b}$ ni hosil qilamiz. Bu ayniyat $\frac{ac}{bc}$ kasrni surat va maxrajning umumiy ko‘paytuvchisi c ga qisqartirish degan ma’noni beradi. Demak, $\frac{ac}{bc}=\frac{a}{b}$.

1-misol. $\frac{21y}{7y^2}=\frac{3\cdot 7y}{y\cdot 7y}=\frac{3}{y}$ (kasrni $7y$ ga qisqartirdik).

2-misol. $\frac{a^2-9}{ab+3b}$ ni qisqartirish uchun uning surat va maxrajini ko'paytuvchilarga ajratamiz. $\frac{a^2-9}{ab+3b}=\frac{(a+3)(a-3)}{b(a+3)}=\frac{a-3}{b}$ ($a+3$ ga qisqartirdik).

3-misol. $\frac{4x}{5y}$ kasrning maxrajini $75y^4$ ga keltiramiz. $75y^4=5y\cdot 15y^3$ kabi yozamiz. $\frac{4x}{5y}$ kasrning surat va maxrajini $15y^3$ ga ko'paytiramiz $\frac{4x}{5y}=\frac{4x\cdot 15y^3}{5y\cdot 15y^3}=\frac{60xy^3}{75y^4}$ hosil bo'ldi.

Bundagi $15y^3$ ko'paytuvchi $\frac{4x}{5y}$ kasrning qo'shimcha ko'paytuvchisi deyiladi.

Agar kasr suratining (yoki maxrajining) ishorasi o'zgartirilsa, shu kasrning ishorasi ham o'zgaradi, ya'ni $\frac{a}{b}$ kasrda:

$$\frac{-a}{b}=-\frac{a}{b} \text{ yoki } \frac{a}{-b}=-\frac{a}{b};\quad \frac{a}{b}=-\frac{-a}{b}=-\frac{a}{-b}.$$

Masalan: 1) $\frac{16}{2x-y}=-\frac{16}{-(2x-y)}=-\frac{16}{(y-2x)}$.

2) Kasrlarni qisqartiramiz:

a) $\frac{\overset{1}{\cancel{10}}xy^2}{\underset{3}{\cancel{30}}xy}=\frac{y}{3}$;

b) $\frac{\overset{4}{\cancel{24}}a^2c^2}{\underset{3}{\cancel{18}}ac^3}=\frac{4a}{3c}$;

d) $\frac{a^{\cancel{16}\,4}}{a^{\cancel{12}\,1}}=\frac{a^4}{1}=a^4$;

e) $\frac{15b+20c}{10b}=\frac{\cancel{5}^1(3b+4c)}{{}_2\cancel{10}b}=\frac{3b+4c}{2b}$;

f) $\frac{5x^2+15xy}{x+3y}=\frac{5x\cancel{(x+3y)}}{\cancel{x+3y}}=\frac{5x}{1}=5x$;

g) $\frac{25-a^2}{3a-15}=\frac{5^2-a^2}{3(a-5)}=\frac{(5-a)(5+a)}{3(a-5)}=\frac{-(a-5)(a+5)}{3(a-5)}=-\frac{a+5}{3}$;

h) $\frac{a^2-2a+4}{8-a^3}=\frac{a^2-2a+4}{-(a^3-2^3)}=\frac{(a-2)^{\cancel{2}\,1}}{-\cancel{(a-2)}(a^2+2a+4)}=-\frac{a-2}{a^2+2a+4}=\frac{2-a}{a^2+2a+4}$;

i) $y=-0{,}2$ da $\frac{y^{12}-y^{10}}{y^8-y^6}$ ifodaning qiymatini topamiz.

$$\frac{y^{12}-y^{10}}{y^8-y^6}=\frac{y^{10}(y^2-1)}{y^6(y^2-1)}=\frac{y^4}{1}=y^4=(-0{,}2)^4=0{,}0016.$$

j) $\frac{2x}{x^2+xy+y^2}$ kasrni x^3-y^3 maxrajga keltiramiz.

$x^3-y^3=(x-y)\cdot(x^2+xy+y^2)$ bo'lgani uchun $\frac{2x}{x^2+xy+y^2}$ kasrning surat va maxrajini $x-y$ ga ko'paytiramiz.

$$\frac{2x}{x^2+xy+y^2}=\frac{2x\cdot(x-y)}{(x^2+xy+y^2)\cdot(x-y)}=\frac{2x(x-y)}{x^3-y^3}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Ratsional kasrning asosiy xossasini ayting.
2. Qanday tenglikni ayniyat deyiladi?
3. Kasrning biror hadining ishorasi o'zgartirilsa, kasr ishorasi qanday o'zgaradi?
4. $\frac{3x}{15y}$; $\frac{2a}{18a}$; $\frac{4a^2}{12a}$; $\frac{2x^7}{x^5}$ kasrlarni qisqartiring.

MASALALARNI YECHING

186. Surat va maxrajning umumiy ko'paytuvchisini ko'rsating va kasrni qisqartiring:

a) $\frac{6a}{24}$; b) $\frac{3x^2}{7x}$; d) $\frac{7ab}{21bc}$; e) $\frac{10xy}{15yz}$;

f) $\frac{8x^2y^2}{24xy}$; g) $\frac{12ay^3}{4a^2}$; h) $\frac{24a^2c^2}{36a^2c}$; i) $\frac{63x^2y^3}{42x^6y^4}$.

187. Bo'linmani kasr ko'rinishida yozing va kasrni qisqartiring:

a) $4a^2b^3:(2a^4b^2)$; d) $24\,p^3q^4:79(48\,p^2q^2)$;

b) $3xy^2:(6x^3y^2)$; e) $36\,m^2n:(18\,mn)$; f) $-32b^5c^4:(8b^4c^3)$.

188. Kasrni qisqartiring:

a) $\frac{56m^3n^2}{35m^4n}$; d) $\frac{a(b-2)}{5(b-2)}$; f) $\frac{ab(y+3)}{a^2b(y+3)}$;

b) $-\frac{25p^4q^2}{100p^5q}$; e) $\frac{9(x+4)}{6(x+4)}$; g) $\frac{15a^3(a-b)}{20a^2b(a-b)}$.

189. Kasrning surat va maxrajini ko'paytuvchilarga ajrating va uni qisqartiring:

a) $\frac{3a+12b}{6ab}$; d) $\frac{6a-4}{8(a-2)}$; f) $\frac{a^2-3ab}{a-3b}$;

b) $\frac{15b-20c}{10b}$; e) $\frac{9x(y+2)}{6y+12}$; g) $\frac{3x^2+15xy}{6x+30y}$.

190. Kasrni qisqartiring:

a) $\frac{y^2-16}{3y+12}$; d) $\frac{(c+2)^2}{7c^2+14c}$; f) $\frac{a^2+10a+25}{a^2-25}$;

b) $\frac{5x-15y}{x^2-9y^2}$; e) $\frac{6cd-18c}{(d-3)^2}$; g) $\frac{y^2-9}{y^2-6y+9}$.

191. Bo'linmani kasr ko'rinishida yozing va kasrni qisqartiring.

a) $(9x^2-y^2):(3x+y)$; d $(x^2+2x+4):(x^3-8)$;

b) $(2ab-a):(4b^2-4b+1)$; e) $(1+a^3):(1+a)$.

192. Ifodani soddalashtiring:

a) $\frac{x^6+x^4}{x^4+x^2}$; b) $\frac{y^6-y^8}{y^4-y^2}$; d) $\frac{b^7-b^{10}}{b^5-b^2}$; e) $\frac{c^6-c^4}{c^3+c^2}$.

193. a) $\frac{x}{a-b}$ kasrni $(a-b)^2$ maxrajga keltiring;

b) $\frac{y}{x-a}$ kasrni x^2-a^2 maxrajga keltiring;

d) $\frac{2y}{x-1}$ kasrni x^3-1 maxrajga keltiring;

e) $\frac{a}{a-2}$ kasrni a^2-2a maxrajga keltiring;

f) $\frac{1}{x+1}$ kasrni x^3+1 maxrajga keltiring.

25-§. Bir xil maxrajli kasrlarni qo'shish va ayirish

Bir xil maxrajli kasrlarni qo'shish (ayirishda) ularning suratlari qo'shiladi (ayriladi), maxrajlari esa o'zgarishsiz qoldiriladi. Masalan: $\frac{5}{7}+\frac{3}{7}=\frac{5+3}{7}=\frac{8}{7}=1\frac{1}{7}$; $\frac{5}{7}-\frac{3}{7}=\frac{5-3}{7}=\frac{2}{7}$.

Bir xil maxrajli ratsional kasrlar ham xuddi shunday qo'shiladi (ayriladi): $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ tenglikni a, b va $c\neq 0$ ning istalgan qabul qiladigan qiymatlarida to'g'ri bo'lishini isbotlaymiz.

$\frac{a}{c}=m$ va $\frac{b}{c}=n$ bo'lsin. Bo'linmaning ta'rifiga asosan $a=cm$ va $b=bn$. Bundan $a+b=cm+cn=c(m+n)$, ya'ni $a+b=c(m+n)$. Bunda $c\neq 0$ bo'lgani uchun bo'linmaning ta'rifiga asosan $m+n=\frac{a+b}{c}$ bo'lib, $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ kelib chiqadi. Ratsional kasrlarni ayirish ham shuningdek isbotlanadi, ya'ni $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$.

Demak, **bir xil maxrajli kasrlarni qo'shish (ayirish) uchun ularning suratlarini qo'shish (ayirish), maxrajini esa o'zgarishsiz qoldirish kerak.**

Masalan: $\frac{3a-7b}{15ab}$ va $\frac{2a+2b}{15ab}$ kasrlarni qo'shing.

1-misol. $\frac{3a-7b}{15ab}+\frac{2a+2b}{15ab}=\frac{3a-7b+2a+2b}{15ab}=\frac{5a-5b}{15ab}=\frac{5(a-b)}{15ab}=\frac{a-b}{3ab}$.

2-misol. $\frac{a^2+9}{5a-15}$ kasrdan $\frac{6a}{5a-15}$ kasrni ayiring.

$$\frac{a^2+9}{5a-15}-\frac{6a}{5a-15}=\frac{a^2+9-6a}{5a-15}=\frac{(a-3)^2}{5(a-3)}=\frac{a-3}{5}.$$

3-misol. $\frac{x^2-3}{x^2+2x}+\frac{2}{x^2+2x}-\frac{2x-1}{x^2+2x}$ ifodani soddalashtiring.

$$\frac{x^2-3}{x^2+2x}+\frac{2}{x^2+2x}-\frac{2x-1}{x^2+2x}=\frac{x^2-3+2-(2x-1)}{x^2+2x}=\frac{x^2-3+2-2x+1}{x^2+2x}=\frac{x^2-2x}{x^2+2x}=\frac{x(x-2)}{x(x+2)}=$$

$$=\frac{x-2}{x+2}.$$

Qarama-qarshi maxrajli kasrlarni qo‘shish va ayirish bir xil maxrajli kasrlarni qo‘shish va ayirishga keltirib hisoblanadi.

Masalan, $\frac{3a}{2x-a}+\frac{6x}{a-2x}=\frac{3a}{2x-a}+\frac{6x}{-(2x-a)}=\frac{3a}{2x-a}-\frac{6x}{2x-a}=\frac{3a-6x}{2x-a}=$

$=\frac{3(a-2x)}{2x-a}=\frac{-3(2x-a)}{2x-a}=-3.$

TAKRORLASH UCHUN SAVOLLAR

1. Bir xil maxrajli kasrlar qanday qo‘shiladi?
2. Bir xil maxrajli ratsional kasrlar qanday qo‘shiladi va qanday ayriladi?
3. Kasrlarni og‘zaki qo‘shing:

a) $\frac{a}{4}+\frac{c}{4}$; b) $\frac{b}{y}+\frac{4}{y}$; d) $\frac{2x}{y}+\frac{3x}{y}$; e) $\frac{c+d}{a}+\frac{c}{a}$.

4. Kasrlarni og‘zaki ayiring:

a) $\frac{x}{7}-\frac{y}{7}$; b) $\frac{12}{9}-\frac{8}{9}$; d) $\frac{a}{c}-\frac{5}{c}$; e) $\frac{12x}{d}-\frac{5x}{d}$.

MASALALARNI YECHING

194. Qo‘shishni yoki ayirishni bajaring:

a) $\frac{a}{x}+\frac{5a}{x}$; e) $\frac{5b^2}{c}+\frac{13b^2}{c}$; h) $\frac{a+b}{6}+\frac{a-2b}{6}$;

b) $\frac{b}{7}-\frac{c}{7}$; f) $\frac{x+y}{9}-\frac{x}{9}$; i) $\frac{x+5}{9}-\frac{x+2}{9}$;

d) $\frac{a}{y}+\frac{5}{y}$; g) $\frac{m}{p}-\frac{m-n}{p}$; j) $\frac{11x-5}{14x}+\frac{3x-2}{14x}$.

195. Ifodani kasrga almashtiring:

a) $\frac{2x-3y}{4xy}+\frac{11x-2y}{4xy}$; d) $\frac{3x-y^4}{4y^5}-\frac{y^4+3x}{4y^5}$;

b) $\frac{5a+b^2}{8b}-\frac{5a-7b^2}{8b}$; e) $\frac{a-2}{8a}+\frac{2a+5}{8a}-\frac{3-a}{8a}$.

196. Ifodani soddalashtiring:

a) $\frac{17-12x}{x}+\frac{10-x}{x}$; e) $\frac{3p-q}{5p}-\frac{2p+6q}{5p}+\frac{p-4q}{5p}$;

b) $\frac{12p-1}{3p^2}-\frac{1-3p}{3p^2}$; f) $\frac{5c-2d}{4c}-\frac{3d}{4c}+\frac{d-5c}{4c}$;

d) $\frac{6y-3}{5y}-\frac{y+2}{5y}$; g) $\frac{5c+2d}{4d}+\frac{3d-8c}{4d}-\frac{d-3c}{4d}$.

197. Ifodani soddalashtiring:

a) $\frac{16}{x-4}-\frac{x^2}{x-4}$; e) $\frac{x-3}{x^2-64}+\frac{11}{x^2-64}$;

b) $\frac{25}{a+5}-\frac{a^2}{a+5}$; f) $\frac{2a+b}{(a-b)^2}+\frac{2b-5a}{(a-b)^2}$;

d) $\frac{3a-1}{a^2-b^2}-\frac{3b-1}{a^2-b^2}$; g) $\frac{13x+6y}{(x+y)^2}-\frac{11x+4y}{(x+y)^2}$.

198. Ifodani soddalashtiring:

a) $\frac{a}{c-3}+\frac{6}{3-c}$; e) $\frac{a^2+16}{a-4}+\frac{8a}{4-a}$;

b) $\frac{x}{y-1}-\frac{5}{1-y}$; f) $\frac{x^2}{x^2-16}-\frac{8(x+2)}{16-x^2}$;

d) $\frac{5p}{2q-p}+\frac{10q}{p-2q}$; g) $\frac{x^2}{(x-5)^2}-\frac{25}{(5-x)^2}$.

26-§. Har xil maxrajli kasrlarni qo‘shish va ayirish

Har xil maxrajli kasrlarni qo‘shish bir xil maxrajli kasrlarni qo‘shishga keltiriladi.

$\frac{a}{b}$ va $\frac{c}{d}$ kasrlarni qo‘shish talab qilingan bo‘lsin. Bu kasrlarni bir xil maxrajga keltiramiz yoki kasrlarni umumiy maxrajga keltiramiz. Buning uchun $\frac{a}{b}$ kasrning surat va maxrajini d ga ko‘paytirib $\frac{ad}{bd}$ kasrni, $\frac{c}{d}$ kasrning surat va maxrajini b ga ko‘paytirib $\frac{bc}{bd}$ kasrni hosil qilamiz. Natijada $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$ ni hosil qilamiz. Demak, $\boldsymbol{\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}}$.

Har xil maxrajli kasrlarni ayirishda ham xuddi shunga o‘xshash ish bajariladi, ya’ni $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$; $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$.

Har xil maxrajli kasrlarni qo‘shish (ayirish) uchun oldin ularni umumiy maxrajga keltiramiz, so‘ngra teng maxrajli kasrlarni qo‘shish (ayirish) qoidasi bo‘yicha bajariladi.

Har xil maxrajli kasrlarni qo‘shish va ayirishda soddaroq bo‘lgan umumiy ko‘paytuvchini topishga keltiriladi.

1-misol. $\frac{x}{4a^3b}$ va $\frac{5}{6ab^4}$ kasrlarni qo‘shamiz.

Kasrlarning maxrajlari birhadlardir. Bu birhadlarning eng sodda umumiy maxraji $12a^3b^4$ bo‘lgan birhaddir. Bu birhadning koeffitsiyenti kasr maxrajlari koeffitsiyentlarining eng kichik umumiy karralisiga teng, har bir o‘zgaruvchi esa kasrlarning maxrajiga kirgan eng katta ko‘rsatkichi bilan olingan. Bu kasrlarning suratlari va maxrajlariga qo‘shimcha ko‘paytuvchilar $3b^2$ va $2a^2$ ga teng.

Natijada: $\frac{^{3b^3/}x}{4a^3b}+\frac{^{2a^2/}5}{6ab^4}=\frac{x\cdot 3b^3+5\cdot 2a^2}{12a^3b^4}=\frac{3b^3x+10a^2}{12a^3b^4}$.

2-misol. $\frac{a+3}{a^2+ab}-\frac{b+3}{ab+b^2}$ ayirmani hisoblaymiz.

Kasrni umumiy maxrajga keltirish uchun har bir kasrning maxrajini ko‘paytuvchilarga ajratamiz:

$\frac{a+3}{a^2+ab}-\frac{b+3}{ab+b^2}=\frac{a+3}{a(a+b)}-\frac{b+3}{b(a+b)}$. Eng sodda umumiy maxraj $ab(a+b)$ ifoda bo‘ladi. Bu kasrlarning qo‘shimcha ko‘paytuvchilari a va b ga teng. Natijada $\frac{a+3}{a^2+ab}-\frac{b+3}{ab+b^2}=\frac{^{b/}a+3}{a(a+b)}-\frac{^{a/}b+3}{b(a+b)}=\frac{b\cdot(a+3)-a\cdot(b+3)}{ab(a+b)}=$

$$=\frac{ab+3b-ab-3a}{ab(a+b)}=\frac{3b-3a}{ab(a+b)}=\frac{3(b-a)}{ab(a+b)}.$$

3-misol. $x+y-\frac{x^2+y^2}{x+1}$ ifodani soddalashtiramiz. $x+y$ ifodani maxraji 1 ga teng bo‘lgan kasrga almashtiramiz va kasrlarni ayiramiz:

$$x+y-\frac{x^2+y^2}{x+y}=\frac{^{x+y/}x+y}{1}-\frac{^{1/}x^2+y^2}{x+y}=\frac{(x+y)^2-x^2-y^2}{x+y}=\frac{x^2+2xy+y^2-x^2-y^2}{x+y}=\frac{2xy}{x+y}.$$

4-misol. $\frac{b}{a^2-2ab+b^2}-\frac{a+b}{b^2-ab}$ ayirmani kasr ko'rinishida ifodalaymiz:

$$\frac{b}{a^2-2ab+b^2}-\frac{a+b}{b^2-ab}=\frac{b}{(a-b)^2}-\frac{a+b}{b(b-a)}=\frac{b}{(a-b)^2}-\frac{a+b}{-b(a-b)}=\frac{^{b/}b}{(a-b)^2}+$$

$$+\frac{^{a-b/}a+b}{b(a-b)}=\frac{b^2+(a-b)(a+b)}{b(a-b)^2}=\frac{b^2+a^2-b^2}{b(a-b)^2}=\frac{a^2}{b(a-b)^2}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Har xil maxrajli oddiy kasrlar qanday qo'shiladi?
2. $\frac{a}{b}$ va $\frac{c}{d}$ kasrlar bir xil maxrajga qanday keltiriladi?
3. a) $\frac{5}{6}+\frac{3}{8}$ yig'indini hisoblang; b) $\frac{14}{15}-\frac{11}{20}$ ayirmani hisoblang.
4. Kasrlarni og'zaki qo'shing yoki ayiring:

a) $\frac{a}{3}+\frac{a}{2}$; b) $\frac{b}{5}+\frac{3b}{15}$; d) $\frac{5c}{8}-\frac{7c}{12}$;

e) $\frac{3}{2x}-\frac{2}{3x}$; f) $\frac{a}{c}+\frac{b}{d}$; g) $\frac{x}{y}-\frac{y}{x}$.

MASALALARNI YECHING

199. Kasr ko'rinishida yozing:

a) $\frac{a}{6}+\frac{a}{18}$; d) $\frac{3c}{8}+\frac{c}{6}$; f) $\frac{5x}{8y}+\frac{x}{4y}$;

b) $\frac{c}{4}-\frac{c}{6}$; e) $\frac{a}{5c}+\frac{3a}{4c}$; g) $\frac{17y}{24c}-\frac{25y}{36c}$.

200. Ifodani kasrga aylantiring:

a) $\frac{7a}{12b}+\frac{3a}{15b}$; d) $\frac{15a-b}{12a}-\frac{a-4b}{9a}$; f) $\frac{a+b}{a^2}+\frac{a-b}{ab}$;

b) $\frac{9p}{10}-\frac{7p}{12}$; e) $\frac{7x+4}{8y}-\frac{3x-1}{6y}$; g) $\frac{2a-3b}{a^2b}+\frac{4a-5b}{ab^2}$.

201. Kasr ko‘rinishida yozing:

a) $\frac{3b+2c}{9b^2c}-\frac{2c-5b}{6bc^2}$; d) $\frac{b-a}{a^2}-\frac{b-a}{b^2}+\frac{a+b}{ab}$;

b) $\frac{2x-7y}{2x^2y}+\frac{8x-5y}{5xy^2}$; e) $\frac{2a^2-4b^2}{2ab}+\frac{a+2b}{a}-\frac{a-5b}{b}$.

202. Ifodani kasrga almashtiring:

a) $2p-\frac{4p^2+1}{2p}$; e) $\frac{a+b}{4}-a+b$;

b) $5y^2+7-\frac{15y^2-9}{3}$; f) $\frac{a+b}{4}-\frac{a+b}{6}$;

d) $2-\frac{b}{a}-\frac{a}{b}$; g) $5+\frac{2b^2-1}{b}-b$.

203. Kasr ko‘rinishida ifodalang:

a) $\frac{c-a}{a}+\frac{b}{b+c}$; e) $\frac{2a}{2a-1}-\frac{1}{2a+1}$; h) $\frac{3}{ax-ay}+\frac{2}{by-bx}$;

b) $\frac{x+1}{x-2}-\frac{x+y}{y}$; f) $\frac{p}{3p-1}-\frac{p}{3p+1}$; i) $\frac{13c}{bm-bn}-\frac{12b}{cn-cm}$;

d) $\frac{m}{m-n}-\frac{n}{m+n}$; g) $\frac{3x}{5(x+y)}-\frac{2y}{3(x+y)}$; j) $\frac{p}{2x+1}-\frac{p}{3x-2}$.

204. Ifodani soddalashtiring:

a) $\frac{a^2}{ax-x^2}+\frac{x}{x-a}$; d) $\frac{b}{2a^2-ab}-\frac{4a}{2ab-b^2}$;

b) $\frac{b^2-4by}{2y^2-by}-\frac{4y}{b-2y}$; e) $\frac{4y}{3x^2+2xy}-\frac{9x}{3xy+2y^2}$.

205. Amallarni bajaring:

a) $\frac{x^2-3xy}{(x+y)(x-y)}+\frac{y}{x-y}$; d) $\frac{a+3}{a^2-1}-\frac{1}{a^2+a}$;

b) $\frac{c}{b-c}+\frac{b^2-3bc}{b^2-c^2}$; e) $\frac{b}{ab-5a^2}-\frac{15b-25a}{b^2-25a^2}$.

206. Amallarni bajaring:

a) $\frac{a^2+b^2}{a^3+b^3}-\frac{1}{a+b}$; d) $\frac{1-a}{a^2-a+1}+\frac{a^2}{a^3+1}$;

b) $\frac{1}{p-q}-\frac{3pq}{p^3-q^3}$; e) $\frac{6a^3+48a}{a^3+64}-\frac{3a^2}{a^2-4a+16}$;

207. Kasr ko'rinishida tasvirlang:

a) $\frac{b}{a^2-2ab+b^2}-\frac{a+b}{b^2-ab}$; d) $\frac{x-2}{x^2+2x+4}-\frac{6x}{x^3-8}+\frac{1}{x-2}$;

b) $\frac{1}{(a-3)^2}-\frac{2}{a^2-9}+\frac{1}{(a+3)^2}$; e) $\frac{2a^2+7a+3}{a^3-1}-\frac{1-2a}{a^2+a+1}-\frac{3}{a-1}$.

208. Ifodalarni aynan tengligini isbotlang:

a) $\frac{3}{a^2-3a}+\frac{a^2}{a-3}$ va $a+3+\frac{9a+3}{a^2-3a}$;

b) $\frac{a^3}{a^2-4}+\frac{a}{a-2}-\frac{2}{a+2}$ va $a-1$.

27-§. Kasrlarni ko'paytirish. Kasrni darajaga ko'tarish

Oddiy kasrlarni ko'paytirishda ularning suratlari ko'paytirilib suratiga, maxrajlari ko'paytirib maxrajiga yoziladi.

Masalan, $\frac{2}{7}\cdot\frac{9}{11}=\frac{2\cdot 9}{7\cdot 11}=\frac{18}{77}$.

Har qanday ratsional kasrlar ham xuddi shunga o'xshash ko'paytiriladi: $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ **bo'ladi ($b\neq 0$; $d\neq 0$).**

Isbot. $\frac{a}{b}=m$ va $\frac{c}{d}=n$ bo'lsin. U holda $m\cdot n=\frac{a}{b}\cdot\frac{c}{d}$ bo'ladi.

Bo'linmaning ta'rifiga asosan $a=bm$ va $c=dn$. Bundan $ac=(bm)\cdot(dn)=(bd)\cdot mn$, ya'ni $ac=(bd)\cdot(mn)$. $bd\neq 0$ bo'lganidan bo'linmaning ta'rifga ko'ra $mn=\frac{ac}{bd}$ va $mn=\frac{a}{b}\cdot\frac{c}{d}$ ekanidan $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ kelib chiqadi.

Demak, **kasrni kasrga ko‘paytirish uchun ularning suratlarini ko‘paytirish va ularning maxrajlarini ko‘paytirish hamda birinchi ko‘paytmani kasrning surati qilib yozish, ikkinchi ko‘paytmani maxraji qilib yozish kerak.**

1-misol. $\frac{a^3}{4b^2}$ kasrni $\frac{6b}{a^2}$ kasrga ko‘paytiramiz: $\frac{a^3}{4b^2}\cdot\frac{6b}{a^2}=\frac{a^3\cdot\cancel{6}^3 b}{{}_2\cancel{4}b^2\cdot a^2}=\frac{3a}{2b}$.

2-misol. $\frac{pm+2p}{m}$ kasrni $\frac{pm^2}{m^2-4}$ kasrga ko‘paytiramiz:

$$\frac{pm+2p}{m}\cdot\frac{pm^2}{m^2-4}=\frac{p\cancel{(m+2)}\cdot pm^2}{m\cdot(m-2)\cancel{(m+2)}}=\frac{p^2m}{m-2}.$$

3-misol. $\frac{x-1}{x+n}\cdot\frac{x+1}{x}$ ko‘paytmani ratsional kasr ko‘rinishida ifodalaymiz: $\frac{x-1}{x+2}\cdot\frac{x+1}{x}=\frac{(x-1)\cdot(x+1)}{(x+2)\cdot x}=\frac{x^2-1}{x^2+2x}$.

4-misol. $\frac{x+a}{x-a}$ kasrni x^2-a^2 ko‘phadga ko‘paytiramiz.

$$\frac{x+a}{x-a}\cdot(x^2-a^2)=\frac{x+a}{\cancel{x-a}_1}\cdot\frac{(\cancel{x-a}^1)\cdot(x+a)}{1}=\frac{(x+a)^2}{1}=(x+a)^2.$$

Kasrlarni ko‘paytirish qoidasi uchta va undan ortiq ko‘paytuvchilar uchun ham tegishli.

Masalan, $\frac{a}{b}\cdot\frac{c}{d}\cdot\frac{m}{n}=\frac{ac}{bd}\cdot\frac{m}{n}=\frac{acm}{bdn}$.

$\frac{a}{b}$ kasrning n – darajasi bo‘lgan $\left(\frac{a}{b}\right)^n$ ifoda berilgan bo‘lsin.

Darajaning ta’rifiga ko‘ra: $\left(\frac{a}{b}\right)^n=\underbrace{\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}\cdot\ldots\cdot\frac{a}{b}}_{n}=\frac{\overbrace{a\cdot a\cdot a\ldots a}^{n}}{\underbrace{b\cdot b\cdot b\ldots b}_{n}}=\frac{a^n}{b^n}$.

Demak, $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$.

Kasrni biron *n* darajaga ko‘tarish uchun berilgan kasrning suratini va maxrajini shu *n* darajaga ko‘tarish kifoya.

5-misol. $\frac{2a^2}{b^4}$ kasrni uchinchi darajaga ko‘taramiz:

$$\left(\frac{2a^2}{b^4}\right)^3=\frac{(2a^2)^3}{(b^4)^3}=\frac{8a^6}{b^{12}}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Oddiy kasrlar qanday ko‘paytiriladi?
2. Kasrni kasrga ko‘paytirish uchun nima qilinadi?
3. Kasrni darajaga ko‘tarish uchun nima qilinadi?
4. Ko‘paytirishni og‘zaki bajaring:

a) $\frac{5}{6}\cdot\frac{7}{15}$; b) $1\frac{2}{3}\cdot\frac{9}{10}$; d) $\frac{5a}{8}\cdot\frac{3}{10b}$; e) $\frac{c^2}{10}\cdot\frac{15}{2c}$;

f) $\frac{9a}{4}\cdot\frac{8b}{3}$; g) $\frac{18}{a^3}\cdot\frac{3a^2}{24}$; h) $\frac{15x^2}{18}\cdot\frac{9}{20x}$.

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209. Ko‘paytirishni bajaring:

a) $\frac{5}{9x^2}\cdot\frac{3x}{20}$; d) $\frac{12x^5}{25}\cdot\frac{15}{8x^2}$; f) $\frac{m^2}{16n}\cdot\frac{24}{5m}$;

b) $\frac{18}{c^4}\cdot\frac{c^3}{24}$; e) $\frac{15}{4a^3}\cdot\frac{16a^4}{9}$; g) $\frac{8y^2}{9x^3}\cdot\frac{27x^2}{16y}$.

210. Ifodani soddalashtiring:

a) $\frac{48x^2}{49y^4}\cdot\frac{7y^2}{16x^3}$; d) $-\frac{15p^4}{8q^6}\cdot\frac{16q^5}{25p^3}$; f) $4m^2\cdot n\cdot\frac{13x}{12mn^2}$;

b) $\frac{18m^3}{11n^3}\cdot\frac{22n^4}{9m^2}$; e) $-\frac{72x^4}{25y^5}\cdot\frac{5y^4}{27x^5}$; g) $-ab\cdot\left(-\frac{11x^2}{3a^2b^2}\right)$.

211. Ifodani soddalashtiring:

a) $\frac{2a^2b}{3xy}\cdot\frac{3x^2y}{4ab^2}\cdot\frac{6ax}{15b^2}$; b) $\frac{6m^3n^2}{35p^3}\cdot\frac{49n^4}{m^5p^3}\cdot\frac{5m^4p^2}{42n^6}$.

212. Darajaga ko‘taring:

a) $\left(\frac{n^2}{10m}\right)^3$; d) $\left(-\frac{2a^2b}{3mn^3}\right)^2$; f) $\left(-\frac{10m^2}{n^2p}\right)^3$;

b) $\left(\frac{2a}{p^2q^3}\right)^4$; e) $\left(-\frac{3x^2}{2y^3x}\right)^3$; g) $\left(-\frac{b^3c^2}{8a^3}\right)^4$.

213. Kasr ko'rinishida yozing:

a) $\frac{kx+k^2}{x^2}\cdot\frac{x}{x+k}$; d) $\frac{xy}{a^2+a^3}\cdot\frac{a+a^2}{x^2y^2}$;

b) $\frac{ax+ay}{xy^2}\cdot\frac{x^2y}{3x+3y}$; e) $\frac{6a}{x^2-x}\cdot\frac{2x-2}{3ax}$.

214. Ifodaning qiymatini toping:

a) $\frac{5mn-m}{4m+n}\cdot\frac{16m^2-n^2}{5n-1}$, bunda $m=\frac{1}{4}$; $n=-3$;

b) $\frac{(x+2)^2}{3x+9}\cdot\frac{2x+6}{x^2-4}$, bunda $x=0{,}5$ va $x=-1{,}5$.

215. Ifodani soddalashtiring:

a) $\frac{x^2-10x+25}{3x+12}\cdot\frac{x^2-16}{2x-10}$; b) $\frac{1-a^2}{4a+8b}\cdot\frac{a^2+4ab+4b^2}{3-3a}$.

28-§. Kasrlarni bo'lish

Oddiy kasrlarni bo'lishda birinchi kasr (bo'linuvchi) ikkinchisiga (bo'luvchiga) teskari kasrga ko'paytiriladi. Masalan, $\frac{3}{8}:\frac{2}{5}=\frac{3}{8}\cdot\frac{5}{2}=\frac{15}{16}$.

Istalgan ratsional kasrni bo'lishda ham xuddi shunday qilinadi.

$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$. Bunda $b\neq 0$, $c\neq 0$ va $d\neq 0$.

Isbot. $\frac{a}{b}=m$ va $\frac{c}{d}=n$ bo'lsin. Bundan $a=mb$ va $d=\frac{c}{n}$ bo'lib,

$a\cdot d=mb\cdot\frac{c}{n}=(bc)\cdot\frac{m}{n}$ yoki $ad=(bc)\cdot\frac{m}{n}$ dan $\frac{m}{n}=\frac{ad}{bc}$; ya'ni $m:n=\frac{ad}{bc}$

yoki $\frac{a}{b}:\frac{c}{d}=\frac{a\cdot d}{b\cdot c}$; ya'ni $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$.

Demak, **kasrni kasrga bo'lish uchun bo'linuvchini bo'luvchiga teskari kasrga ko'paytirish kerak.**

1-misol. $\frac{7a^2}{b^3}$ kasrni $\frac{14a}{b}$ kasrga bo‘lamiz: $\frac{7a^2}{b^3}:\frac{14a}{b}=\frac{{}^{1}\cancel{7}a^2}{b^3}\cdot\frac{b}{{}_{2}\cancel{14}a}=\frac{a}{2b^2}$.

2-misol. $\frac{x-2}{x}$ kasrni $\frac{x+1}{x+2}$ kasrga bo‘lamiz:

$$\frac{x-2}{x}:\frac{x+1}{x+2}=\frac{x-2}{x}\cdot\frac{x+2}{x+1}=\frac{x^2-4}{x(x+1)}.$$

3-misol. $\frac{a^2-9}{3ay-9y}:\frac{a+3}{6y^2}=\frac{a^2-9}{3ay-9y}\cdot\frac{6y^2}{a+3}=\frac{(a-3)(a+3)}{{}_{1}\cancel{3}y(a-3)}\cdot\frac{{}^{2}\cancel{6}y^2}{a+3}=\frac{2y}{1}=2y.$

TAKRORLASH UCHUN SAVOLLAR

1. Oddiy kasrlarni bo‘lish qanday bajariladi?
2. Oddiy kasrlarni bo‘lishga misollar keltiring.
3. Ratsional kasrlarni bo‘lish formulasini yozing.
4. Ratsional kasrlar qanday bo‘linadi?
5. Bo‘lishni og‘zaki bajaring.

a) $\frac{5}{7}:\frac{3}{4}$; b) $\frac{5}{8}:\frac{15}{16}$; d) $\frac{x}{y}:\frac{c}{d}$; e) $\frac{y^2}{x}:\frac{y}{x^2}$.

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216. Bo‘lishni bajaring:

a) $\frac{5m}{6n}:\frac{15m^2}{8}$; d) $\frac{a^2}{12b}:\frac{ab}{36}$; f) $\frac{11x}{4y^2}:22x^2$;

b) $\frac{14}{9x^3}:\frac{7x}{2y^2}$; e) $\frac{3x}{10^3}:\frac{1}{5a^2}$; g) $27a^3:\frac{18a4}{7b^2}$.

217. Ifodani soddalashtiring:

a) $\frac{6x^2}{5y}:\frac{3x}{10y^3}$; d) $\frac{a^2b^2}{11mn^2}:\left(-\frac{4ab^3}{33mn}\right)$; f) $\frac{8mx^2}{3y^3}:4m^2x$;

b) $\frac{8c}{21d^2}:\frac{6c^2}{7d}$; e) $-\frac{6xy^2}{5ab}:\frac{9x^2y^2}{10ab}$; g) $15a^2bx:\frac{a^3b^2}{30x^2}$.

218. Kasr ko‘rinishida yozing:

a) $\frac{3x^2}{5y^3}:\frac{9x^2}{2y^2}\cdot\frac{5y}{3x}$; d) $\frac{2ab}{3c^2d}\cdot\frac{2cd^2}{9ab}:\frac{a^2b}{c^3d}$;

b) $\frac{7p^4}{10q^3}\cdot\frac{5q}{14p^2}:\frac{3p}{4q^4}$;   e) $\frac{8x^2y}{7ab^2}:\frac{4xy^2}{7a^2b}:\frac{2x^3y}{ab}$.

219. Ifodani soddalashtiring:

a) $\frac{x^2-4y^2}{xy}:\frac{x^2-2xy}{3y}$;   d) $\frac{a^2-3a}{a^2-25}:\frac{a^2-9}{a^2+5a}$;

b) $\frac{ab^2}{a^2-1}:\frac{5b}{a-a^2}$;   e) $\frac{3m^2-3n^2}{m^2+mp}:\frac{6m-6n}{p+m}$.

220. Ifodaning qiymatini toping:

a) $\frac{4x^2-4x}{x+3}:(2x-2)$, bunda $x=2{,}5$; $x=-1$;

b) $(3a+6b):\frac{2a^2-8b^2}{a+b}$, bunda $a=26$; $b=-12$.

221. Ifodani soddalashtiring:

a) $\frac{m^2+6m+9}{2x^2y}:\frac{am+3a}{4xy}$;   d) $\frac{a^2+ax+x^2}{x-1}:\frac{a^3-x^3}{x^2-1}$;   f) $\frac{\frac{1}{x}-\frac{1}{2x}}{\frac{1}{x^2}-\frac{1}{2x^2}}$;

b) $\frac{ab^3}{7-7p}:\frac{a^2b^2}{1-2p+p^2}$;   e) $\frac{ap^2-9a}{p^3-8}:\frac{p+3}{2p-4}$.   g) $\frac{\frac{x}{x-1}-\frac{x+1}{x}}{\frac{x}{x+1}-\frac{x-1}{x}}$.

29-§. Ratsional ifodalarni shakl almashtirish

Istalgan ratsional ifodani shakl almashtirish kasrlarni qo'shish, ayirish, ko'paytirish va bo'lishga keltiriladi. Demak, har qanday ratsional ifodani surat va maxraji ko'phad bo'lgan kasr ko'rinishida tasvirlash mumkin ekan. Misollar keltiramiz:

1-misol. $x+1-\frac{1}{x+2}\cdot\frac{x^2-4}{x}$ ratsional ifodani kasrga almashtiring:

Avval kasrlarni ko'paytiramiz. Keyin olingan natijani $x+1$ ko'phaddan ayiramiz:

1) $\frac{1}{x+2}\cdot\frac{x^2-4}{x}=\frac{1}{x+2}\cdot\frac{(x-2)(x+2)}{x}=\frac{x-2}{x}$;

2) ${}^{x/}x+1-\frac{x-2}{x}=\frac{x(x+1)-(x-2)}{x}=\frac{x^2+x-x+2}{x}=\frac{x^2+2}{x}$.

2-misol. Ushbu $\left(\frac{a-b}{ab}\cdot\frac{ab}{a+b}-\frac{a+b}{a-b}\right):ab$ ifodani ratsional kasr ko‘rinishida tasvirlaymiz.

1) $\frac{a-b}{ab}\cdot\frac{ab}{a+b}=\frac{a-b}{a+b};$

2) $\frac{^{a-b/}a-b}{a+b}-\frac{^{a+b/}a+b}{a-b}=\frac{(a-b)^2-(a+b)^2}{(a+b)(a-b)}=\frac{a^2-2ab+b^2-a^2-2ab-b^2}{a^2-b^2}=-\frac{4ab}{a^2-b^2};$

3) $-\frac{4ab}{a^2-b^2}:ab=\frac{-4ab}{a^2-b^2}\cdot\frac{1}{ab}=\frac{-4}{a^2-b^2}=\frac{4}{b^2-a^2}.$

3-misol. $\left(\frac{x^2+y^2}{x^2-y^2}\right)^2:\left(\left(\frac{x+y}{x-y}+\frac{x}{y}\right)\cdot\left(\frac{x+y}{x-y}-\frac{y}{x}\right)\right)$ ifodani soddalashtiramiz.

1) $\frac{^{y/}x+y}{x-y}+\frac{^{x-y/}x}{y}=\frac{xy+y^2+x^2-xy}{y(x-y)}=\frac{x^2+y^2}{y(x-y)};$

2) $\frac{^{x/}x+y}{x-y}-\frac{^{x-y/}y}{x}=\frac{x^2+xy-xy+y^2}{x(x-y)}=\frac{x^2+y^2}{x(x-y)};$

3) $\frac{x^2+y^2}{y(x-y)}\cdot\frac{x^2+y^2}{x\cdot(x-y)}=\frac{(x^2+y^2)^2}{xy(x-y)^2};$

4) $\left(\frac{x^2+y^2}{x^2-y^2}\right):\frac{(x^2+y^2)^2}{xy(x-y)^2}=\frac{\cancel{(x^2+y^2)}^2}{(x^2-y^2)^2}\cdot\frac{xy(x-y)^2}{\cancel{(x^2+y^2)}^2}=\frac{xy(x-y)^2}{(x-y)^2(x+y)^2}=$

$=\frac{xy\,\cancel{(x-y)}^2}{\cancel{(x-y)}^2(x+y)^2}=\frac{xy}{(x+y)^2}.$

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222. Amallarni bajaring:

a) $\left(\frac{x}{y^2}-\frac{1}{x}\right):\left(\frac{1}{y}+\frac{1}{x}\right);$

b) $\left(\frac{a}{m^2}+\frac{a^2}{m^3}\right):\left(\frac{m^2}{a^2}+\frac{m}{a}\right);$

d) $\frac{ab+b^2}{3}:\frac{b^3}{3a}+\frac{a+b}{b};$

e) $\frac{x-y}{x}-\frac{5y}{x^2}\cdot\frac{x^2-xy}{5y}.$

223. Ifodani soddalashtiring:

a) $\left(\frac{4a}{2-a}-a\right):\frac{a+2}{a-2}$;   d) $\left(\frac{2m+1}{2m-1}-\frac{2m-1}{2m+1}\right):\frac{4m}{10m-5}$;

b) $\frac{x-2}{x-3}\cdot\left(x+\frac{x}{2-x}\right)$;   e) $\frac{x+3}{x^2+9}\cdot\left(\frac{x+3}{x-3}+\frac{x-3}{x+3}\right)$.

224. Ifodani soddalashtiring:

a) $\left(2x+1-\frac{1}{1-2x}\right):\left(2x-\frac{4x^2}{2x-1}\right)$;

b) $\left(\frac{pq}{p^2-q^2}+\frac{q}{q-p}\right):\left(p-q+\frac{4q^2-p^2}{p+q}\right)$;

d) $x-\frac{ax}{a-x}:\frac{a+x}{a}+\frac{ax^2}{a^2-x^2}$;

e) $\frac{ax-2a}{x+1}\cdot\frac{ax-x}{2x^2-8}\cdot\frac{x+2}{a-1}-\frac{a}{2}$.

225. Amallarni bajaring:

a) $\frac{4xy}{y^2-x^2}:\left(\frac{1}{y^2-x^2}+\frac{1}{x^2+2xy+y^2}\right)$;

b) $\left(\frac{x-2y}{x^2+2xy}-\frac{1}{x^2-4y^2}:\frac{x+2y}{(2y-x)^2}\right)\cdot\frac{(x+2y)^2}{4y^2}$;

d) $\left(\frac{a^2}{a+n}-\frac{a^3}{a^2+n^2+2an}\right):\left(\frac{a}{a+n}-\frac{a^2}{a^2-n^2}\right)$;

e) $\left(\frac{2a}{2a+b}-\frac{4a^2}{4a^2+4ab+b^2}\right):\left(\frac{2a}{4a^2-b^2}+\frac{1}{b-2a}\right)$.

226. Ifodani soddalashtiring:

a) $\left(\frac{a-1}{3a+(a-1)^2}-\frac{1-3a+a^2}{a^3-1}-\frac{1}{a-1}\right):\frac{a^2+1}{1-a}$;

b) $\left(\frac{1}{x+1}-\frac{3}{x^3+1}+\frac{3}{x^2-x+1}\right)\left(x-\frac{2x-1}{x+1}\right)$;

d) $\left(\frac{2x^2+x}{x^3-1}-\frac{x+1}{x^2+x+1}\right)\left(1+\frac{x+1}{x}-\frac{x+5}{x+1}\right)$;

e) $\frac{x^2-x}{x^2+ax+a^2}:\frac{x^2-1}{x^3-a^3}+\frac{x^3+a^3}{x^2-1}:\frac{x^2-ax+a^2}{ax-a}$.

227. Ifodaning qiymatini toping:

a) $\frac{a-2}{4a^2+16a+16}:\left(\frac{a}{2a-4}-\frac{a^2+4}{2a^2-8}-\frac{2}{a^2+2a}\right)$, bunda $a=-3$;

b) $\left(\frac{a-x}{a^2+ax+x^2}-\frac{1}{a-x}\right)\left(\frac{2x+a}{a}-\frac{2a+x}{x}\right)$, bunda $a=-2, x=-4$.

228. Ifodani soddalashtiring:

a) $\dfrac{\frac{2a-b}{b}+1}{\frac{2a+b}{b}-1}$; b) $\dfrac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}}$; d) $\dfrac{\frac{a-b}{c}+3}{\frac{a+b}{c}-1}$;

229. O'rniga qo'ying va hosil bo'lgan ifodani soddalashtiring:

a) $\frac{x-a}{x-b}$, bunda $x=\frac{ab}{a+b}$; b) $\frac{ax}{a+x}-\frac{bx}{b-x}$, bunda $x=\frac{ab}{a-b}$;

d) $\left(\frac{x}{y}-\frac{y}{x}\right):\left(\frac{x}{y}+\frac{y}{x}-2\right):\left(1+\frac{y}{x}\right)=\frac{x}{x-y}$ ayniyatni isbotlang.

30-§. Arifmetik kvadrat ildiz

Masala. Kvadratning yuzi 64 sm^2. Bu kvadrat tomonining uzunligi qancha?

Kvadratning tomoni x sm bo'lsin. U holda kvadratning yuzi x^2 sm^2 bo'ladi. Demak, $x^2=64$. Bu tenglamaning ikkita ildizi bor: 8 va –8. Haqiqatan ham, $8^2=64$ va $(-8)^2=64$. Uzunlik musbat son bilan ifodalangani uchun masala shartini ildizlardan faqat bittasi, ya'ni 8 soni qanoatlantiradi. Shunday qilib, kvadrat tomonining uzunligi 8 sm ga teng.

Kvadrati 64 ga teng sonlar: 8 va –8 lar 64 sonining kvadrat ildizlari deyiladi.

***1-ta'rif.* *a* sonining kvadrat ildizi deb kvadrati *a* soniga teng bo'lgan songa aytiladi.**

Masalan, $\frac{1}{4}$ va $-\frac{1}{4}$ sonlar $\frac{1}{16}$ ning kvadrat ildizlaridir, chunki $\left(\frac{1}{4}\right)^2=\frac{1}{16}$ va $\left(-\frac{1}{4}\right)^2=\frac{1}{16}$.

0 sonining kvadrat ildizi 0 ga teng, chunki $0^2=0$; 0 ning kvadrat, ildizi faqat bitta.

–49 ning kvadrat ildizi yo'q, chunki kvadrati –49 ga teng bo'ladigan son mavjud emas.

$x^2=64$ tenglamaning musbat kvadrat ildizi bo'lgan 8 soni 64 ning **arifmetik kvadrat ildizi** deyiladi.

***2-ta'rif.* *a* sonning arifmetik kvadrat ildizi deb kvadrati *a* ga teng bo'lgan musbat songa aytiladi.**

$\sqrt{\ }$ belgini arifmetik kvadrat ildizi belgisi deyiladi. $\sqrt{a}$ yozuv «*a* ning arifmetik kvadrat ildizi» deb o'qiladi. Arifmetik kvadrat ildizlarni hisoblashga misollar keltiramiz:

$\sqrt{4}=2$, chunki 2 – musbat son va $2^2=4$;

$\sqrt{1,21}=1,1$, chunki 1,1 – musbat son va $1,1^2=1,21$;

$\sqrt{0}=0$, chunki 0 – nomanfiy son va $0^2=0$.

Istalgan sonning kvadrati musbat son bo'lgani uchun $a<0$ bo'lgan $\sqrt{a}$ ifodaning ma'nosi yo'q. Masalan, $\sqrt{-16}$; $\sqrt{-5,4}$ kabi ifodalarning ma'nosi yo'q.

$x^2=64$ tenglamaning kvadrat ildizlari. $x_1=\sqrt{64}$ va $x_2=-\sqrt{64}$ yoki $x_{1/2}=\pm 8$ kabi yoziladi.

TAKRORLASH UCHUN SAVOLLAR

1. $x^2=25$ tenglamaning ikkita ildizini ayting.
2. a sonining kvadrat ildizi deb nimaga aytiladi?
3. $x^2=25$ tenglamaning arifmetik kvadrat ildizini ayting.
4. a sonining arifmetik kvadrat ildizi deb nimaga aytiladi?
5. Arifmetik kvadrat ildiz qanday belgi bilan yoziladi?
6. $\sqrt{-36}$ kvadrat ildizning nima uchun ma'nosi yo'q?

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230. Sonlarning kvadrat ildizlarini toping va ularning arifmetik kvadrat ildizini ayting: a) 16; b) 25; d) 100; e) 900; f) 0,04; g) 0,64.

231. Tenglikning to'g'ri ekanini isbotlang:

a) $\sqrt{121}=11$; b) $\sqrt{169}=13$; d) $\sqrt{2,25}=1,5$; e) $\sqrt{0,09}=0,3$.

232. Ildizning qiymatini toping:

a) $\sqrt{81}$; e) $\sqrt{1600}$; h) $\sqrt{0,04}$; k) $\sqrt{\frac{4}{81}}$;

b) $\sqrt{64}$; f) $\sqrt{2500}$; i) $\sqrt{0,25}$; l) $\sqrt{2\frac{1}{4}}$;

d) $\sqrt{36}$; g) $\sqrt{10000}$; j) $\sqrt{0,09}$; m) $\sqrt{1\frac{24}{25}}$.

233. Ifodaning qiymatini toping (arifmetik ildizi bo'yicha):

a) $\sqrt{36}\cdot\sqrt{16}$; d) $3\sqrt{9}-16$;

b) $\sqrt{81}:\sqrt{100}$; e) $-7\sqrt{0,36}+5,4$;

f) $\sqrt{0,09}+\sqrt{0,25}$;

g) $\sqrt{0,04}-\sqrt{0,01}$;

h) $0,1\sqrt{400}+0,2\sqrt{1600}$;

i) $\frac{1}{3}\sqrt{0,36}+\frac{1}{5}\sqrt{900}$.

234. 11 dan 20 gacha natural sonlar kvadratlari jadvalidan foydalanib toping:

a) $\sqrt{289}$;

b) $\sqrt{225}$;

d) $\sqrt{169}$;

e) $\sqrt{361}$;

f) $\sqrt{144}+\sqrt{256}$;

g) $\sqrt{400}-\sqrt{324}$;

h) $\sqrt{196}-\sqrt{900}$;

i) $\sqrt{1600}-\sqrt{2500}$;

j) $\sqrt{2,56}$;

k) $\sqrt{2,25}$;

l) $\sqrt{2,89}+\sqrt{1,69}$;

m) $\sqrt{1,44}-\sqrt{2,56}$.

235. Ifodaning ma'nosi bormi?

a) $\sqrt{100}$;

b) $\sqrt{-100}$;

d) $-\sqrt{100}$;

e) $\sqrt{(-10)^2}$;

f) $\sqrt{(-25)\cdot(-4)}$;

g) $\sqrt{-25\cdot 4}$.

236. Tenglamani yeching:

a) $x^2=81$;

b) $x^2-0,16=0$;

d) $x^2-36=0$;

e) $x^2-100=0$;

f) $5a^2=1,8$;

g) $2x^2-128=0$.

31-§. Irratsional sonlar

1-masala. Kvadratning yuzi 20 m^2 bo'lsa, uning tomonini toping.

Kvadratning tomoni x m bo'lsin, u holda uning yuzi $x^2=20$ m^2 tenglamani hosil qiladi. Bu tenglamaning arifmetik ildizi $x=\sqrt{20}$ bo'ladi. Bundan butun kvadrat ildiz chiqmaydi. Bu ildiz butungacha aniqlikda 4 bilan 5 oralig'ida bo'ladi, ya'ni $4<\sqrt{20}<5$-yoki $16<20<25$.

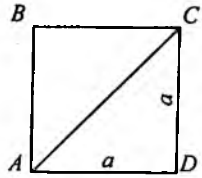
0,1 gacha aniqlikda $4,4<\sqrt{20}<5$-yoki $19,36<20<20,25$.

0,01 gacha aniqlikda $4,47<\sqrt{20}<4,48$-yoki $19,6809<20<20,0704$.

0,001 gacha aniqlikda $4,472<\sqrt{20}<4,473$-yoki $19,988784<20,007729$.

Bu usulda kvadrat ildiz chiqarishni tugatib bo'lmaydi.

2-masala. Tomoni a ga teng bo'lgan kvadratning diagonalini uning tomoniga nisbatini toping.



$ABCD$ – kvadrat. $AB=BC=CD=AD=a$.

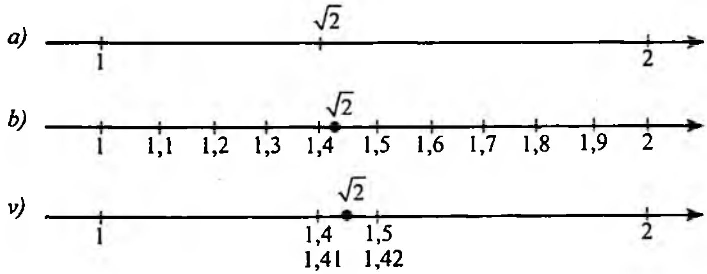
Pifagor teoremasiga asosan $AC^2=a^2+a^2=2a^2$;

$AC=\sqrt{2a^2}=\sqrt{2}a$. $\frac{AC}{AB}=\frac{\sqrt{2}a}{a}=\sqrt{2}$; Demak, $\frac{AC}{AB}=\sqrt{2}$.

$\sqrt{2}$ dan butun kvadrat ildiz chiqmaydi. $\sqrt{2}$ dan butungacha, 0,1; 0,01; 0,001 va hokazo aniqlikda kvadrat ildiz chiqariladi. Butungacha aniqlikda $1<\sqrt{2}<2$-yoki $1<2<4$. 0,1 gacha aniqlikda $1{,}4<\sqrt{2}<1{,}5$-yoki $1{,}96<2<2{,}25$. 0,01 gacha aniqlikda $1{,}41<\sqrt{2}<1{,}42$-yoki $1{,}9881<2<2{,}0164$.

0,001 gacha aniqlikda $1{,}414<\sqrt{2}<415$-yoki $1{,}999396<2<2{,}002225$.

Bu hisoblashni davom ettirib, biz $\sqrt{2}$ sonning o‘nli yozuvida borgan sari yangi raqamlarini topamiz. Ushbu $\sqrt{2}=1{,}414213...$ songa ega bo‘lamiz. Bu sonni son o‘qidagi o‘rni 4-chizmada tasvirlangan.



4-chizma.

3-misola. Aylana uzunligining diametriga nisbatini toping.

s – aylana uzunligi, d – aylana diametri bo‘lsin. nisbatni π bilan belgilanadi. Bu nisbat $\frac{c}{d}=\pi=3{,}14159...$ ga teng doimiy son. Bu son ham cheksiz va davr siz o‘nli kasr.

4-masala. Matematikada muhim ahamiyatga ega bo‘lgan $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n=2{,}71828047...$ cheksiz, davrsiz son mavjud.

Biz bilgan ratsional sonlar $\frac{m}{n}(n\neq 0)$ ko‘rinishda chekli yoki davrli o‘nli kasrdan iborat bo‘ladi. Masalan: $\frac{13}{8}=1{,}625$; $\frac{125}{32}=3{,}90625$; $\frac{37}{21}=1{,}(761904)$; $\frac{11}{12}=0{,}91(6)$ va hokazo.

Biz o‘rganib o‘tgan $\sqrt{20}$; $\sqrt{2}$, π va l sonlarni $\frac{m}{n}(n \neq 0)$ kasr ko‘rinishida tasvirlab bo‘lmaydi. Shuning uchun bunday sonlarni ***noratsional yoki irratsional*** sonlar deyiladi.

1-ta’rif. **$\frac{m}{n}$ kasr ko‘rinishida (bunda m – butun son, n – natural son) tasvirlab bo‘lmaydigan sonlar irratsional son deyiladi.**

3; 5; 6; 5; 7; ... kabi sonlardan chiqarilgan kvadrat ildiz ham irratsional sonlar bo‘ladi. $\sqrt{3}$ va $-\sqrt{3}$; $\sqrt{5}$ va $-\sqrt{5}$ va hokazolar ham irratsional son.

2-ta’rif. **Ratsional va irratsional sonlarni haqiqiy sonlar deyiladi.**

Biz shu vaqtgacha o‘rgangan barcha sonlarimiz **haqiqiy sonlarni** tashkil qiladi. Bizlar raqamlar, natural sonlar, butun sonlar, ratsional sonlar, irratsional sonlar bilan tanishib chiqdik.

1-misol. $\frac{1}{3}$ va $\sqrt{3}$ sonlarning yig‘indisini topamiz.

0,1 gacha aniqlikda $\frac{1}{3} \approx 0,3$ va $\sqrt{3} \approx 1,7$.

$\frac{1}{3}$ va $\sqrt{3}$ sonlarning yig‘indisini 0,1 gacha aniqlikda taqribiy topamiz.

$\frac{1}{3} + \sqrt{3} \approx 0,3 + 1,7 = 2,0$ hosil qildik.

0,01 ga aniqlikda olsak, ya’ni $\frac{1}{3} \approx 0,33$ va $\sqrt{3} \approx 1,73$. $\frac{1}{3} + \sqrt{3} \approx 0,33 +$ $+ 1,73 = 2,06$ hosil qildik.

2-misol. r radiusi 5 m bo‘lgan aylananing uzunligini topamiz.

Aylana uzunligi $c = 2\pi r$ formula bilan hisoblanadi. Bunda $\pi \approx 3,14$ deb olib, quyidagini hisoblaymiz:

$$c \approx 2 \cdot 3,14 \cdot 5 = 31,4 \text{ (m)}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Kvadrat diagonalini uning tomoniga nisbati qanday son bilan ifodalaniladi?
2. Qanday sonlarni ratsional sonlar deyiladi?
3. Qanday sonlarni irratsional sonlar deyiladi?

4. Haqiqiy sonlar deb nimaga aytiladi?

5. $-20{,}3$; $\frac{7}{11}$ -4; $-12{,}5$; 8; $\frac{5}{\sqrt{3}}$; $-\frac{\sqrt{13}}{6}$ sonlardan raqamlarni, natural sonlarni, butun sonlarni, ratsional sonlarni, irratsional sonlarni va haqiqiy sonlarni ayting.

MASALALARNI YECHING

237. $\sqrt{9}$; $\sqrt{12}$; $\sqrt{121}$; $\sqrt{\frac{9}{16}}$; $\sqrt{\frac{5}{49}}$; $\sqrt{0{,}36}$ sonlardan:

a) Ratsional sonlarni ajratib yozing;

b) Irratsional sonlarni ajratib yozing.

238. $\sqrt{10}$ sonning o'nli yozuvidagi birlar va o'ndan birlar xonasidagi raqamlarni toping.

239. $\frac{1}{7}$; 0; 0,(25); –2,3; $\sqrt{121}$; 3,14; $\sqrt{5}$; 0,8181181118... sonlardan ratsional sonlarni, irratsional sonlarni ko'rsating.

240. Sonlarni o'sib borishi tartibida yozing: π; 3,141; 3,142; 3,14...; $3\frac{1}{7}$; 3,(14).

241. Tomonlari $\sqrt{2}$ m va $\sqrt{3}$ m ga teng bo'lgan to'g'ri to'rtburchak perimetrining taqribiy qiymatini toping ($\sqrt{2}$ va $\sqrt{3}$ sonlarining taqribiy qiymatlarini 0,01 gacha aniqlikda oling).

32-§. $y=\sqrt{x}$ funksiya va uning grafigi

Kvadrat tomonining uzunligi a sm, kvadratning yuzi S sm² bo'lsin. Kvadrat tomonining har bir a qiymatiga yuzining yagona S qiymati mos keladi. Kvadrat yuzining a tomonga bog'liqligi $S=a^2$ formula bilan ifodalanadi, bunda $a \geq 0$.

Kvadratning a tomoni uchun S yuziga bog'liqligi $a=\sqrt{S}$ formula bilan ifodalanadi.

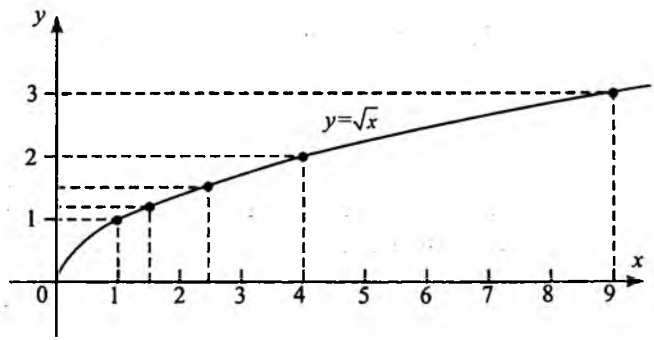
$S=a^2$ formulada kvadratning tomoni yerkli o'zgaruvchi bo'lsa, $a=\sqrt{S}$ formulada kvadratning yuzi yerkli o'zgaruvchi bo'ladi.

Agar har bir holda yerkli o'zgaruvchini x harfi bilan, yerksiz o'zgaruvchini y harfi bilan belgilasak, quyidagi formulalarga ega bo'lamiz: $y=x^2$ (bunda $x \geq 0$) va $y=\sqrt{x}$.

$y=x^2$ (bunda $x \geq 0$) funksiyaning grafigi parabolaning bir qismi – uning o‘ng tarmog‘i ekanligini bilamiz.

Yendi $y=\sqrt{x}$ funksiyaning grafigini yasaymiz $y=\sqrt{x}$ funksiya $x \geq 0$ bo‘lganda ma’noga ega. Agar $x=0$ bo‘lsa, $y=0$ bo‘ladi, ya’ni grafik koordinatalar boshidan o‘tadi.

x	0	0,5	1	2	3	4	5	6	7	8	9
$y=\sqrt{x}$	0	0,7	1	1,4	1,7	2	2,2	2,4	2,6	2,8	3



4-chizma.

$x>0$ da $\sqrt{x}$ ifoda musbat qiymatlar qabul qiladi. Demak, grafik birinchi koordinata choragida joylashadigan x argumentga mos y ning qiymatlari parabolaning qismi bo‘ladi. x argumentga mos y ning qiymatlari 0,1 ga aniqlikda olingan (4-chizma).

$y=\sqrt{x}$ funksiyaning grafigi yordamida $\sqrt{1,5}$ va $\sqrt{2,5}$ ifodalarning qiymatini taqqoslaymiz. Biz grafikdan $\sqrt{1,5} < \sqrt{2,5}$ ekanligini ko‘ramiz.

Demak, katta songa arifmetik kvadrat ildizning katta qiymati mos kelishini ko‘rish mumkin.

TAKRORLASH UCHUN SAVOLLAR

1. Kvadratning a tomoniga ko‘ra uning yuzini ifodalovchi funksiyani yozing.
2. Kvadratning S yuziga ko‘ra uning tomonini ifodalovchi formulani yozing.
3. $y=x^2$ ($x \geq 0$) funksiyaning grafigi qanday chiziq bo‘ladi?

4. $y=\sqrt{x}(x \geq 0)$ funksiyaning grafigi qanday chiziq bo'ladi?

5. $\sqrt{6}$ bilan $\sqrt{8}$ larni grafikdan foydalanib taqqoslang.

MASALALARNI YECHING

242. Doiraning yuzi $S=\pi r^2$ (bunda r – doira radiusi) yoki $S=\frac{\pi d^2}{4}$ (bunda d – doira diametri) formula berilgan:

a) r ning S ga bog'liqligini;

b) d ning S ga bog'liqligini formula bilan ifodalang.

243. $y=\sqrt{x}$ funksiyaning grafigidan foydalanib:

a) $\sqrt{x}$ ning $x=2{,}5$; $5{,}5$; $8{,}4$ dagi qiymatini toping;

b) x ning $\sqrt{x}=1{,}2$; $1{,}7$; $2{,}5$ ga mos kelgan qiymatini toping.

244. Quyidagi nuqtalar $y=\sqrt{x}$ funksiyaning grafigiga tegishlimi?

$M(64; 8)$; $N(10000; 100)$; $E(-81; 9)$; $F(25; -5)$.

245. x o'zgaruvchining qiymatini toping:

a) $\sqrt{x}=4$; d) $2\sqrt{x}=0$; f) $\sqrt{x}-8=0$;

b) $\sqrt{x}=0{,}5$; e) $4\sqrt{x}=1$; g) $3\sqrt{x}-2=0$.

246. $y=\sqrt{x}$ funksiyaning grafigi yordamida sonlarni taqqoslang:

a) $\sqrt{0{,}5}$ va $\sqrt{0{,}8}$; b) $\sqrt{4{,}2}$ va $\sqrt{5{,}7}$;

d) $\sqrt{7}$ va $\sqrt{8}$; e) $2\sqrt{5}$ va $2\sqrt{4}$.

33-§. Ko'paytmadan va kasrdan chiqarilgan kvadrat ildiz

$\sqrt{81\cdot 4}$ va $\sqrt{81}\cdot\sqrt{4}$ ifodalarning qiymatlarini taqqoslaymiz: Demak, $\sqrt{81\cdot 4}=\sqrt{324}=18$ va $\sqrt{81}\cdot\sqrt{4}=9\cdot 2=18$. Demak, $\sqrt{81\cdot 4}=$ $=\sqrt{81}\cdot\sqrt{4}$.

1-teorema. ***Agar*** $a\geq 0$ ***va*** $b\geq 0$ ***bo'lsa,*** **u** ***holda*** $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ ***bo'ladi.***

Isbot. $a\geq 0$ va $b\geq 0$ da $\sqrt{ab}$ va $\sqrt{a}\cdot\sqrt{b}$ larning har biri ma'noga ega. Kvadrat ildiz ta'rifiga asosan $(\sqrt{ab})^2=ab$.

1) $\sqrt{a}\cdot\sqrt{b}$ ko'paytma musbat, chunki $\sqrt{a}\geq 0$ va $\sqrt{b}\geq 0$ edi. Ularning ko'paytmasi ham musbat, ya'ni $\sqrt{a}\cdot\sqrt{b}\geq 0$.

2) Ko'paytmaning darajasi xossasiga asosan $\left(\sqrt{a}\cdot\sqrt{b}\right)^2=\left(\sqrt{a}\right)^2\cdot$ $\cdot(\sqrt{b})^2=ab$, ya'ni $\left(\sqrt{a}\cdot\sqrt{b}\right)^2=ab$. Demak, arifmetik kvadrat ildizning ta'rifiga ko'ra, $a\geq 0$ va $b\geq 0$ bo'lganda $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ tenglik kelib chiqadi.

Isbotlangan teorema ildiz ishorasi ostidagi ko'paytuvchilar ikkitadan ortiq bo'lganda ham o'rinli.

Masalan, agar $a\geq 0$, $b\geq 0$, $c\geq 0$ bo'lsa, $\sqrt{abc}=\sqrt{a}\cdot\sqrt{b}\cdot\sqrt{c}$ bo'ladi.

Shunday qilib:

Manfiy bo'lmagan ko'paytuvchilarning ko'paytmasidan chiqarilgan kvadrat ildiz bu ko'paytuvchilardan chiqarilgan kvadrat ildizlarning ko'paytmasiga teng.

2-teorema. ***Agar $a\geq 0$ va $b\geq 0$ bo'lsa, u holda $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ bo'ladi.***

Isbot. $a\geq 0$ va $b\geq 0$ bo'lganidan $\sqrt{\frac{a}{b}}$ va $\frac{\sqrt{a}}{\sqrt{b}}$ ifodalarning har biri ma'noga ega. Kvadrat ildiz ta'rifiga asosan $\left(\sqrt{\frac{a}{b}}\right)^2=\frac{a}{b}$.

1) $\frac{\sqrt{a}}{\sqrt{b}}\geq 0$, chunki musbat sonlarning bo'linmasi.

2) Kasrning darajasi xossasiga asosan

$\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2=\frac{\left(\sqrt{a}\right)^2}{\left(\sqrt{b}\right)^2}=\frac{a}{b}$, ya'ni $\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2=\frac{a}{b}$.

Demak, arifmetik kvadrat ildizning ta'rifiga ko'ra $\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2=\frac{a}{b}$ tenglikdan $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ tenglik kelib chiqadi.

Shunday qilib: **surat nomanfiy, maxraji musbat bo'lgan kasrning ildizi suratning ildizini maxrajining ildiziga bo'linganiga teng.**

$\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ va $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ formulalarning chap qismini o‘ng qismi bilan almashtirib, **ko‘paytirish va bo‘lish qoidalarini** ifodalovchi ayniyatlarni hosil qilamiz:

$\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$, bunda $a\geq 0$ va $b\geq 0$;

$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$, bunda $a\geq 0$ va $b>0$.

1-misol. $\sqrt{64\cdot 0{,}04}$ ifodaning qiymatini topamiz. 1-teoremaga asosan $\sqrt{64\cdot 0{,}04}=\sqrt{64}\cdot\sqrt{0{,}04}=8\cdot 0{,}2=1{,}6$.

2-misol. $\sqrt{32\cdot 98}$ ifodaning qiymatini hisoblaymiz.

$\sqrt{32\cdot 98}=\sqrt{(16\cdot 2)\cdot(49\cdot 2)}=\sqrt{16\cdot 49\cdot 4}=4\cdot 7\cdot 2=56.$

3-misol. $\sqrt{\frac{36}{169}}$ ifodaning qiymatini topamiz. 2-teoremaga asosan $\sqrt{\frac{36}{169}}=\frac{\sqrt{36}}{\sqrt{169}}=\frac{6}{13}$.

4-misol. $\sqrt{20}\cdot\sqrt{5}$ ko‘paytmaning qiymatini topamiz.

$\sqrt{20}\cdot\sqrt{5}=\sqrt{20\cdot 5}=\sqrt{100}=10.$

5-misol. $\frac{\sqrt{80}}{\sqrt{5}}=\sqrt{\frac{80}{5}}=\sqrt{16}=4.$

TAKRORLASH UCHUN SAVOLLAR

1. Manfiy bo‘lmagan ko‘paytuvchilardan chiqarilgan ildiz haqidagi teoremani ayting.
2. Kasrdan chiqarilgan ildiz haqidagi teoremani ayting.
3. Kvadrat ildizlarning ko‘paytmasi va bo‘linmasi haqidagi formulalarni yozing.
4. $\sqrt{9\cdot 16}$; $\sqrt{\frac{25}{4}}$ ildizni hisoblang.

MASALALARNI YECHING

247. Ildizning qiymatini hisoblang:

a) $\sqrt{100\cdot 9}$; d) $\sqrt{144\cdot 0{,}25}$; f) $\sqrt{49\cdot 16\cdot 25}$;

b) $\sqrt{81\cdot 400}$; e) $\sqrt{2{,}25\cdot 0{,}04}$; g) $\sqrt{1{,}21\cdot 0{,}09\cdot 0{,}0001}$.

248. Ildizning qiymatini hisoblang:

a) $\sqrt{\frac{9}{64}}$; d) $\sqrt{\frac{121}{49}}$; f) $\sqrt{1\frac{9}{16}}$;

b) $\sqrt{\frac{36}{25}}$; e) $\sqrt{\frac{144}{169}}$; g) $\sqrt{2\frac{7}{81}}$.

249. Ifodaning qiymatini toping:

a) $\sqrt{49\cdot 0,36\cdot 0,16}$; d) $\sqrt{1\frac{5}{9}\cdot 1\frac{9}{16}\cdot 0,64}$; f) $\sqrt{12\cdot 75}$;

b) $\sqrt{0,09\cdot 49\cdot 0,25}$; e) $\sqrt{3\cdot 12}$; g) $\sqrt{45\cdot 80}$.

250. Ifodaning qiymatini hisoblang:

a) $\sqrt{32\cdot 72}$; d) $\sqrt{90\cdot 6,4}$; f) $\sqrt{4,9\cdot 40}$;

b) $\sqrt{2,5\cdot 14,4}$; e) $\sqrt{160\cdot 2,5}$; g) $\sqrt{3,6\cdot 6,4}$.

251. Ildizdan chiqaring:

a) $\sqrt{13^2-12^2}$; d) $\sqrt{45,8^2-44,2^2}$; f) $\sqrt{6,8^2-3,2^2}$;

b) $\sqrt{313^2-312^2}$; e) $\sqrt{21,8^2-18,2^2}$; g) $\sqrt{\left(1\frac{1}{16}\right)^2-\left(\frac{1}{2}\right)^2}$.

252. Ko'paytmaning qiymatini toping:

a) $\sqrt{2}\cdot\sqrt{8}$; d) $\sqrt{28}\cdot\sqrt{7}$; f) $\sqrt{5}\cdot\sqrt{45}$;

b) $\sqrt{27}\cdot\sqrt{3}$; e) $\sqrt{13}\cdot\sqrt{52}$; g) $\sqrt{1,2}\cdot\sqrt{3\frac{1}{3}}$.

253. Bo'linmaning qiymatini toping:

a) $\frac{\sqrt{2}}{\sqrt{18}}$; d) $\frac{\sqrt{52}}{\sqrt{117}}$; f) $\sqrt{\frac{2}{3}}:\sqrt{\frac{8}{27}}$;

b) $\frac{\sqrt{23}}{\sqrt{2300}}$; e) $\frac{\sqrt{999}}{\sqrt{111}}$; g) $\sqrt{12,96}:\sqrt{2,25}$.

254. $\sqrt{2}\approx 1,41$ va $\sqrt{5}\approx 2,24$ ekanidan foydalanib, ifodaning taqribiy qiymatini toping:

a) $\sqrt{10}$; b) $\sqrt{0,5}$; d) $\sqrt{2,5}$; e) $\sqrt{0,4}$.

34-§. Darajaning kvadrat ildizi

$\sqrt{x^2}$ ifodaning qiymatini $x=5$ va $x=-6$ da topamiz:

$$\sqrt{5^2}=\sqrt{25}=5;\quad \sqrt{(-6)^2}=\sqrt{36}=6.$$

Ko‘rib chiqilgan misollarning har birida sonning kvadratidan chiqarilgan ildiz shu sonning moduliga teng, ya’ni $\sqrt{5^2}=|5|$; $\sqrt{(-6)^2}=|-6|$.

Teorema. ***x ning istalgan qiymatida*** $\sqrt{x^2}=|x|$ ***bo‘ladi.***

Isbot. Sonning moduli ta’rifidan x ning istalgan qiymatida $|x|\geq 0$ bo‘ladi. $|x|^2=x^2$ ni to‘g‘ri ekanligini ko‘rsatamiz.

Haqiqatan: 1) agar $x\geq 0$ bo‘lsa, $|x|=x$ dan $|x|^2=x^2$ bo‘ladi, 2) agar $x\leq 0$ bo‘lsa, $|x|=-x$ dan $|x|^2=(-x)^2=x^2$ kelib chiqadi.

$$\sqrt{x^2}=|x| \text{ yoki } \sqrt{x^2}=\begin{cases} x & \text{agar } x\geq 0; \\ -x & \text{agar } x<0. \end{cases}$$

Isbotlangan teorema juft ko‘rsatkichli darajadan kvadrat ildiz chiqarilganda qo‘llaniladi.

1-misol. $\sqrt{a^{16}}$ ifodani soddalashtiramiz. a^{16} darajani biror ifodaning kvadrati ko‘rinishida ifodalaymiz va teoremaga asosan: $\sqrt{a^{16}}=\sqrt{(a^8)^2}=|a^8|$. Bundagi a ning istalgan qiymatida $a^8\geq 0$ bo‘lgani uchun $|a^8|=a^8$ bo‘ladi. Demak, $\sqrt{a^{16}}=a^8$.

2-misol. $\sqrt{x^{10}}$ ifodani (bunda $x<0$) shakl almashtiramiz.

x^{10} ni biror ifodaning kvadrati ko‘rinishida tasvirlaymiz va teoremaga asosan $\sqrt{x^{10}}=\sqrt{(x^5)^2}=|x^5|$ ni hosil qilamiz. $x<0$ bo‘lgani uchun $x^5<0$ bo‘ladi.

Shuning uchun $|x^5|=-x^5$. Demak, $\sqrt{x^{10}}=-x^5$.

3-misol. $\sqrt{a^{14}b^6}$ ifodani (bunda $a>0$, $b<0$) soddalashtiramiz.

a^{14} va b^6 ifodalarning har birini kvadrat ko‘rinishida ifodalab, teoremaga asosan $\sqrt{x^2}=|x|$ ayniyatni qo‘llab, $\sqrt{a^{14}b^6}=\sqrt{(a^7)^2\cdot(b^3)^2}=$ $=\sqrt{(a^7)^2}\cdot\sqrt{(b^3)^2}=|a^7|\cdot|b^3|$.

Bunda $a>0$ bo‘lgani sababli $a^7>0$, ya’ni $|a^7|=a^7$; $b<0$ bo‘lganidan $b^3<0$ bo‘ladi va $|b^3|=-b^3$.

Demak, $\sqrt{a^{14}b^6}=|a^7|\cdot|b^3|=a^7\cdot(-b^3)=-a^7b^3$.

4-misol. $\sqrt{2025}$ ifodaning qiymatini topamiz.

$$\sqrt{2025}=\sqrt{3^4\cdot5^2}=\sqrt{3^4}\cdot\sqrt{5^2}=\sqrt{(3^2)^2}\cdot\sqrt{5^2}=3^2\cdot5=45.$$

TAKRORLASH UCHUN SAVOLLAR

1. a sonining arifmetik kvadrat ildizi deb nimaga aytiladi?
2. x ning istalgan qiymatlaridan $\sqrt{x^2}$ nimaga teng.
3. $x\geq$ da $\sqrt{x^4}$ va $\sqrt{x^{10}}$ nimaga teng?
4. $x<0$ bo‘lsa, nimaga teng?

MASALALARNI YECHING

255. Hisoblang:

a) $\sqrt{(0,1)^2}$; e) $\sqrt{(1,7)^2}$; h) $2\sqrt{(-23)^2}$;

b) $\sqrt{(-0,4)^2}$; f) $\sqrt{(-19)^2}$; i) $5\sqrt{52^2}$;

d) $\sqrt{(-0,8)^2}$; g) $\sqrt{24^2}$; j) $0,2\sqrt{(-61)^2}$.

256. Ifodaning qiymatini toping:

a) $\sqrt{x^2}$, bunda $x=22$; -35;

b) $2\sqrt{a^2}$, bunda $a=-7$; 12;

d) $0,1\sqrt{y^2}$, bunda $y=-15$; 27.

257. Ifodani shakl almashtiring:

a) $\sqrt{n^2}$, bunda $n<0$; b) $3\sqrt{c^2}$, bunda $c>0$;

d) $5r\sqrt{r^2}$, bunda $r\geq 0$; f) $2b^{11}\sqrt{b^2}$, bunda $b<0$;

e) $2c^3\sqrt{c^2}$, bunda $c>0$; g) $-1{,}5x\sqrt{x^2}$, bunda $x<0$.

258. Ildizning qiymatini toping:

a) $\sqrt{2^4}$; d) $\sqrt{(-5)^4}$; f) $3\sqrt{(-2)^6}$; h) $0{,}1\sqrt{(-3)^8}$;

b) $\sqrt{3^4}$; e) $\sqrt{(-2)^8}$; g) $-2\sqrt{10^4}$; i) $100\sqrt{0{,}1^{10}}$.

259. Ifodani soddalashtiring:

a) $\sqrt{y^8}$, bunda $y\geq 0$; e) $5\sqrt{a^8}$;

b) $\sqrt{m^{10}}$, bunda $m\geq 0$; f) $\frac{1}{3}c\sqrt{c^{12}}$,

d) $\sqrt{x^{10}}$, bunda $x<0$; g) $1{,}5t^2\sqrt{t^{14}}$, bunda $t<0$.

260. Birhad ko'rinishida yozing:

a) $\sqrt{b^6c^8}$, e) $ab\sqrt{a^4b^4}$;

b) $\sqrt{16x^4y^{12}}$; f) $x^6\sqrt{x^2y^8}$, bunda $x>0$;

d) $\sqrt{0{,}25p^2y^4}$, bunda $p>0$; g) $36^5\sqrt{a^{12}b^6}$, bunda $b<0$.

261. Ifodani almashtiring:

a) $\sqrt{\frac{p^4}{a^8}}$; d) $2y^2\sqrt{\frac{4x^2}{y^6}}$, bunda $x>0$; $y>0$;

b) $\sqrt{\frac{16a^{12}}{b^{10}}}$, bunda $b>0$; e) $3c^2\sqrt{\frac{c^6}{9d^2}}$, bunda $c<0$; $d>0$.

262. Hisoblang:

a) $\sqrt{576}$; b) $\sqrt{2304}$; d) $\sqrt{1764}$; e) $\sqrt{4225}$.

35-§. Ko'paytuvchini ildiz ishorasi ostidan chiqarish. Ko'paytuvchini ildiz ishorasi ostiga kiritish

$3\sqrt{7}$ va $\sqrt{54}$ ifodalarning qiymatlarini taqqoslaymiz.

Bu masalani $\sqrt{54}$ ni shakl almashtirib hal qilish mumkin. 54 sonini biri biror sonning kvadratiga teng bo'ladigan ikki sonning ko'paytmasi ko'rinishida tasvirlaymiz. $\sqrt{54}=\sqrt{9\cdot 6}$ yozib, bunga ko'paytmaning ildizi haqidagi teoremani qo'llaymiz: $\sqrt{9\cdot 6}=$ $=\sqrt{9}\cdot\sqrt{6}=3\sqrt{6}$. $3\sqrt{7}>3\sqrt{6}$ bo'lgani uchun $3\sqrt{7}>\sqrt{54}$.

Bunday $\sqrt{54}=\sqrt{9\cdot 6}=3\sqrt{6}$ almashtirish, **ko'paytuvchini ildiz belgisi ostidan chiqarish** deyiladi.

Endi $\sqrt{13}$ va $2\sqrt{3}$ ifodalarning qiymatlarini taqqoslaymiz. Bulardan ko'paytuvchini ildiz belgisi ostidan chiqarib taqqoslab bo'lmaydi.

$2\sqrt{3}$ ifodani arifmetik kvadrat ildiz ko'rinishida yozamiz. Buning uchun 2 ni 4 ning arifmetik kvadrat ildizi bilan almashtiramiz, ya'ni $2\sqrt{3}=\sqrt{4}\cdot\sqrt{3}=\sqrt{12}$. $13>12$ bo'lgani uchun $\sqrt{13}>\sqrt{12}$ bo'ladi. Demak, $\sqrt{13}>2\sqrt{3}$.

Bunday almashtirish **ko'paytuvchini ildiz belgisi ostiga kiritish deyiladi.**

1-misol. $\sqrt{a^7}$ ifodada ko'paytuvchini ildiz belgisi ostidan chiqaramiz.

$\sqrt{a^7}$ ifodada faqat $a\geq 0$ da ma'noga ega (agar $a<0$ bo'lsa, $a^7<0$ bo'ladi).

a^7 ifodani ko'paytuvchilardan biri birhadning kvadratiga teng bo'ladigan ko'paytma ko'rinishida tasvirlaymiz, ya'ni $a^7=a^6\cdot a=(a^3)^2\cdot a$. Ko'paytmaning ildizi haqidagi teoremaga asosan $\sqrt{a^7}=\sqrt{(a^3)^2\cdot a}=\sqrt{(a^3)^2}\cdot\sqrt{a}=|a^3|\sqrt{a}=a^3\sqrt{a}$.

2-misol. $-4\sqrt{x}$ ifodada ko'paytuvchini ildiz belgisi ostiga kiritamiz.

-4 manfiy ko‘paytuvchini kvadrat ildiz belgisi ostiga kiritib bo‘lmaydi, shuning uchun $-4=-1\cdot 4$ kabi yozib, ifodani quyidagicha almashtiramiz: $-4\sqrt{x}=-1\cdot 4\sqrt{x}=-1\cdot\sqrt{16}\cdot\sqrt{x}=-\sqrt{16x}$.

3-misol. $a\sqrt{2}$ ifodada ko‘paytuvchini kvadrat ildiz belgisi ostiga kiritamiz.

$a\sqrt{2}$ ifodadagi a ko‘paytuvchi nomanfiy bo‘lishi ham, manfiy bo‘lishi ham mumkin. Shunga bog‘liq holda almashtirish turlicha bajariladi.

$a\geq 0$ bo‘lsa, u holda $a\sqrt{2}=\sqrt{a^2}\cdot\sqrt{2}=\sqrt{2a^2}$; agar $a<0$ bo‘lsa, u holda $a\sqrt{2}=-(-a)\sqrt{2}=-\sqrt{(-a)^2\cdot 2}=-\sqrt{2a^2}$.

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263. Kvadrat ildizdan chiqaring:

a) $\sqrt{12}$; e) $\sqrt{48}$; h) $\sqrt{363}$;

b) $\sqrt{18}$; f) $\sqrt{125}$; i) $\sqrt{432}$;

d) $\sqrt{80}$; g) $\sqrt{108}$; j) $\sqrt{845}$.

264. Ko‘paytuvchini ildiz belgisi ostidan chiqaring va soddalashtiring:

a) $\frac{1}{2}\sqrt{24}$; d) $-\frac{1}{7}\sqrt{147}$; f) $0{,}1\sqrt{20000}$;

b) $\frac{2}{3}\sqrt{45}$; e) $-\frac{1}{5}\sqrt{275}$; g) $-0{,}05\sqrt{28800}$.

265. Ko‘paytuvchini ildiz belgisi ostiga kiriting:

a) $7\sqrt{10}$; d) $6\sqrt{x}$; f) $3\sqrt{2a}$;

b) $5\sqrt{3}$ e) $10\sqrt{x}$; g) $5\sqrt{3b}$.

266. Ifodani arifmetik kvadrat ildiz ko‘rinishida tasvirlang:

a) $3\sqrt{\frac{1}{3}}$; d) $\frac{1}{3}\sqrt{9}$; f) $5\sqrt{\frac{a}{5}}$;

b) $2\sqrt{\frac{3}{4}}$; e) $-10\sqrt{0{,}02}$; g) $-\frac{1}{2}\sqrt{12x}$.

267. Ifodaning qiymatini taqqoslang:

a) $5\sqrt{4}$ va $4\sqrt{5}$;
e) $\frac{1}{3}\sqrt{54}$ va $\frac{1}{5}\sqrt{150}$;

b) $\sqrt{40}$ va $3\sqrt{5}$;
f) $\frac{2}{3}\sqrt{72}$ va $7\sqrt{\frac{2}{3}}$;

d) 24 va $\frac{1}{3}\sqrt{216}$;
g) $3\sqrt{120}$ va $2\sqrt{270}$.

268. Ko'paytuvchini ildiz belgisi ostidan chiqaring:

a) $\sqrt{6x^2}$, bunda $x \geq 0$;
d) $\sqrt{9a^3}$;
f) $\sqrt{0{,}01b^5}$, bunda $b>0$;

b) $\sqrt{3y^2}$, bunda $y \geq 0$;
e) $\sqrt{50b^4}$;
g) $\sqrt{27c^6}$, bunda $c<0$.

269. Ko'paytuvchini ildiz belgisi ostiga kiriting:

a) $a\sqrt{3}$, bunda $a \geq 0$;
e) $x\sqrt{\frac{2}{x}}$; $x<0$;

b) $a\sqrt{3}$, bunda $a < 0$;
f) $x\sqrt{-\frac{2}{x}}$;

d) $x\sqrt{\frac{2}{x}}$, bunda $x > 0$;
g) $-2b\sqrt{7}$, bunda $b<0$.

36-§. Kvadrat ildizlar qatnashgan ifodalarni shakl almashtirish

Ifodalarni shakl almashtirishga ko'paytmadan, kasrdan va darajadan ildiz chiqarish, ko'paytuvchini ildiz ostidan chiqarish, ko'paytuvchini ildiz belgisi ostiga kiritishga doir masalalarning yanada murakkabroqlarini ko'rib chiqamiz.

1-m i s o l. $3\sqrt{5a}-\sqrt{20a}+4\sqrt{45a}$ ifodani soddalashtiramiz. $\sqrt{20a}=\sqrt{4\cdot 5a}=2\sqrt{5a}$; $\sqrt{45a}=\sqrt{9\cdot 5a}=3\sqrt{5a}$ kabi yozamiz.

Natijada $3\sqrt{5a}-\sqrt{20a}+4\sqrt{45a}=3\sqrt{5a}-2\sqrt{5a}+12\sqrt{5a}=$
$=\sqrt{5a}(3-2+12)=13\sqrt{5a}$.

$3\sqrt{5a}-2\sqrt{5a}+12\sqrt{5a}$ yig'indini ixchamlashda oraliq natijani yozmasdan $\left(\sqrt{5a}(3-2+12)\text{ ni}\right)$ $13\sqrt{5a}$ ifodani yozganimiz qulay bo'ladi.

2-misol. $(3\sqrt{5}-6\sqrt{2})(\sqrt{5}+2\sqrt{2})$ ifodani shakl almashtiring.

Birinchi yig'indining har bir hadini ikkinchisining har birhadiga ko'paytirib, quyidagini hosil qilamiz: $(3\sqrt{5}-6\sqrt{2})(\sqrt{5}+2\sqrt{2})=$ $=3(\sqrt{5})^2-6\sqrt{2}\cdot\sqrt{5}+6\sqrt{5}\cdot\sqrt{2}-12\cdot(\sqrt{2})^2=3\cdot5-12\cdot2=15-24=-9.$

Bu ifodaning shakl almashtirishni ikki son kvadratlarining ayirmasi formulasi yordamida ham bajarish mumkin, ya'ni

$$(3\sqrt{5}-6\sqrt{2})(\sqrt{5}+2\sqrt{2})=3(\sqrt{5}-2\sqrt{2})\cdot(\sqrt{5}+2\sqrt{2})=3\left((\sqrt{5})^2-(2\sqrt{2})^2\right)=$$

$$=3(5-8)=3\cdot(-3)=-9.$$

3-misol. $\frac{x^2-3}{x+\sqrt{3}}$ kasrni qisqartiramiz. $a\geq 0$ bo'lganda $(\sqrt{a})^2=a$ tenglikdan foydalanib, $3=(\sqrt{3})^2$ tenglikni yozamiz. Natijada quyidagini hosil qilamiz.

$$\frac{x^2-3}{x+\sqrt{3}}=\frac{x^2-(\sqrt{3})^2}{x+\sqrt{3}}=\frac{(x-\sqrt{3})\cancel{(x+\sqrt{3})}}{\cancel{x+\sqrt{3}}}=x-\sqrt{3}.$$

4-misol. $\frac{c}{\sqrt{2}}$ kasrning maxrajini ildiz belgisidan qutqaramiz.

Buning uchun $\frac{c}{\sqrt{2}}$ kasrning surat va maxrajini $\sqrt{2}$ ga ko'paytiramiz:

$$\frac{c}{\sqrt{2}}=\frac{c\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{\sqrt{2}c}{(\sqrt{2})^2}=\frac{\sqrt{2}c}{2}.$$

5-misol. $\frac{7}{\sqrt{3}+1}$ kasrning maxrajini ildiz belgisidan qutqaramiz.

Buning uchun surat va maxrajini $\sqrt{3}-1$ ifodaga ko'paytiramiz, ya'ni $\frac{7}{\sqrt{3}+1}=\frac{7\cdot(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{7(\sqrt{3}-1)}{(\sqrt{3})^2-1}=\frac{7(\sqrt{3}-1)}{3-1}=\frac{7(\sqrt{3}-1)}{2}=3,5(\sqrt{3}-1).$

6-misol. Kasr $\frac{1-2\sqrt{x}+4x}{1-2\sqrt{x}}$ ning maxrajini ildiz belgisidan qutqaramiz.

$$\frac{1-2\sqrt{x}+4x}{1-2\sqrt{x}}=\frac{(1-2\sqrt{x}+4x)(1+2\sqrt{x})}{(1-2\sqrt{x})(1+2\sqrt{x})}=\frac{1+\cancel{2\sqrt{x}}-\cancel{2\sqrt{x}}-\cancel{4x}+\cancel{4x}+8x\sqrt{x}}{1^2-(2\sqrt{x})^2}=\frac{1+8x\sqrt{x}}{1-4x}.$$

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270. Ifodani soddalashtiring:

a) $\sqrt{9a}+\sqrt{25a}-\sqrt{36a}$;

b) $\sqrt{5a}-2\sqrt{20a}-3\sqrt{80a}$;

d) $\sqrt{75}+\sqrt{48}-\sqrt{300}$;

e) $\sqrt{75}-0,1\sqrt{300}-\sqrt{27}$;

f) $\sqrt{3}\cdot(\sqrt{12}+\sqrt{15})$;

g) $(4\sqrt{3}-2\sqrt{6})\cdot 2\sqrt{3}$;

h) $\sqrt{3}\cdot(\sqrt{12}-2\sqrt{27})$;

i) $\sqrt{8}-\sqrt{5}\cdot(\sqrt{10}-\sqrt{5})$.

271. Ko'paytiring:

a) $(1+3\sqrt{2})(1-2\sqrt{2})$;

b) $(3+\sqrt{3})(2+\sqrt{3})$;

d) $(\sqrt{5}-\sqrt{8})(\sqrt{5}-3\sqrt{2})$;

e) $(2\sqrt{2}-\sqrt{3})(3\sqrt{2}-2\sqrt{3})$;

f) $(a+\sqrt{b})(a-\sqrt{b})$;

g) $(x-\sqrt{y})(x+\sqrt{y})$.

272. Kvadratlar ayirmasi formulasidan foydalanib, ko'paytiring:

a) $(2+\sqrt{5})(2-\sqrt{5})$;

b) $(3\sqrt{5}+1)(3\sqrt{5}-1)$;

d) $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$;

e) $(\sqrt{10}+\sqrt{7})(\sqrt{7}-\sqrt{10})$;

f) $(2\sqrt{5}+1)(2\sqrt{5}-1)$;

g) $(5\sqrt{7}-\sqrt{13})(\sqrt{13}+5\sqrt{7})$;

h) $(3\sqrt{2}-2\sqrt{3})(2\sqrt{3}+3\sqrt{2})$;

i) $(0,5\sqrt{14}+\sqrt{3})(\sqrt{3}-0,5\sqrt{14})$.

273. Amallarni bajaring:

a) $(\sqrt{a}+\sqrt{b})^2$;

b) $(\sqrt{m}-2\sqrt{n})^2$;

d) $(\sqrt{6}+\sqrt{5})^2-\sqrt{120}$;

e) $\sqrt{60}+(\sqrt{3}-\sqrt{5})^2$;

f) $(\sqrt{14}-3\sqrt{2})^2+6\sqrt{28}$;

g) $(3\sqrt{5}+\sqrt{15})^2-10\sqrt{27}$;

h) $(5\sqrt{7}-13)(5\sqrt{7}+13)$;

i) $(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})$.

274. Ifodani ko'paytuvchilarga ajrating:

a) a^2-5;

b) $6-c^2$;

d) $y-3$;

e) $x-y$;

f) $3+\sqrt{3}$;

g) $10-2\sqrt{10}$;

h) $\sqrt{x}+x$;

i) $a-5\sqrt{a}$;

j) $\sqrt{a}-\sqrt{2a}$;

k) $\sqrt{3m}+\sqrt{5m}$;

l) $\sqrt{14}-\sqrt{7}$;

m) $\sqrt{33}-\sqrt{22}$.

275. Kasrni qisqartiring:

a) $\frac{b^2-3}{b+\sqrt{3}}$; d) $\frac{a-b}{\sqrt{b}+\sqrt{a}}$; f) $\frac{\sqrt{7}-7}{\sqrt{7}-1}$; h) $\frac{\sqrt{x}+1}{x+\sqrt{x}}$;

b) $\frac{m+\sqrt{6}}{6-m^2}$; e) $\frac{2\sqrt{x}-3\sqrt{y}}{4x-9y}$; g) $\frac{3+\sqrt{x}}{3\sqrt{x}+x}$; i) $\frac{\sqrt{2a}-\sqrt{12}}{3\sqrt{a}-3\sqrt{6}}$.

276. Kasrning maxrajini ildiz belgisidan qutqaring:

a) $\frac{3}{\sqrt{2}}$; e) $\frac{a}{2\sqrt{3}}$; h) $\frac{2}{1+\sqrt{b}}$; k) $\frac{9}{3-2\sqrt{2}}$;

b) $\frac{1}{\sqrt{p}}$; f) $\frac{3}{\sqrt{1+x}}$; i) $\frac{1}{\sqrt{x+\sqrt{y}}}$; l) $\frac{14}{1+5\sqrt{2}}$;

d) $\frac{5}{3\sqrt{c}}$; g) $\frac{5}{4\sqrt{15}}$; j) $\frac{15}{2\sqrt{5}+5}$; m) $\frac{12}{\sqrt{3}+\sqrt{6}}$.

277. Kasrni qisqartiring:

a) $\frac{\sqrt{70}-\sqrt{30}}{\sqrt{35}-\sqrt{15}}$; d) $\frac{2\sqrt{10}-5}{4-\sqrt{10}}$; f) $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{6}}{2+\sqrt{6}-\sqrt{2}}$;

b) $\frac{\sqrt{15}-5}{\sqrt{6}-\sqrt{10}}$; e) $\frac{9-2\sqrt{3}}{3\sqrt{6}-2\sqrt{2}}$; g) $\frac{(\sqrt{10}-1)^2-3}{\sqrt{10}+\sqrt{3}-1}$.

278. Kasrning maxrajini ildiz belgisidan qutqaring:

a) $\frac{1}{\sqrt{2}+\sqrt{3}+1}$; b) $\frac{1}{\sqrt{5}-\sqrt{3}+2}$.

37-§. Kvadrat tenglamaning ta'rifi. Chala kvadrat tenglamalar

$15x^2-3x+1,2=0$ tenglamaning chap qismi ikkinchi darajali ko'phad, o'ng qismi esa nol sonidan iborat. Bunday ko'rinishdagi tenglama **ikkinchi darajali tenglama yoki kvadrat tenglama** deyiladi.

Ta'rif. **$ax^2+bx+c=0$ ko'rinishdagi tenglama kvadrat tenglama deyiladi, bunda x – o'zgaruvchi, $a \neq 0$, b va c – istalgan sonlar.**

Masalan, $x^2-4x+7=0$; $-2x^2+17x-32=0$; $-0,2x^2-1\frac{1}{3}x-9,6=0$ kvadrat tenglamalardir. Agar x^2 oldidagi koeffitsiyent manfiy bo'lsa, tenglamaning ikkala qismini -1 ga ko'paytirib, uni musbat qila olamiz.

Masalan, $-3x^2-5x-7=0$ ni -1 ko'paytirib, $3x^2+5x+7=0$ holda keltiramiz. Shuning uchun bundan buyon doim $a>0$ deb yuritamiz. Xususiy hollarda b va c yoki ikkalasi ham nolga teng bo'lishi mumkin. U vaqtda tenglamani **chala kvadrat** tenglama deb ataymiz.

1) $c=0$ bo'lsa, $ax^2+bx=0$
2) $b=0$ bo'lsa, $ax^2+s=0$
3) $b=c=0$ bo'lsa, $ax^2=0$

} chala kvadrat tenglamalar.

Agar normal shakldagi tenglamada $a=1$ bo'lsa, tenglama x^2+px+ $+q=0$ shaklga keltirib, uni **keltirilgan kvadrat tenglama** deyiladi.

Bunda p va q – ixtiyoriy sonlar.

Masalan, $x^2-7x+0,3=0$, $x^2-1\frac{1}{3}x-0,07=0$

$ax^2+bx+c=0$, bunda: a – birinchi koeffitsiyent, b – ikkinchi koeffitsiyent, c – ozod had deyiladi.

$13x^2-1,4x-3\frac{3}{4}=0$ da, $a=13$; $b=-1,4$; $c=-3\frac{3}{4}$.

I. $ax^2+bx=0$ tenglamani yechamiz.

Masalan, $5x^2-12x=0$ ni yeching.

x ni qavsdan chiqaramiz:

$x(5x-12)=0$ bu ko‘paytma 0 ga teng bo‘lishi uchun ko‘paytuvchilardan kamida bittasi nol bo‘lishi mumkin.

1) $x=0$ bo‘lsa, 2) $5x-12=0$ bo‘lsa, $5x=12$; $x=12:5$; $x=2{,}4$.

Tenglamaning ildizlari: $x_1=0$ va $x_2=2{,}4$.

Endi $ax^2+bx=0$ tenglamani umumiy shaklda yechamiz. Bundan x ni qavsdan tashqariga chiqaramiz:

$x(ax+b)=0$

1) $x_1=0$; 2) $ax+b=0$, bundan $ax=-b$; $x_2=-\frac{b}{a}$;

Demak, $ax^2+bx=0$ ning ildizlari: $x_1=0$; $x_2=-\frac{b}{a}$.

II. $ax^2+c=0$ tenglamani yechamiz:

Masalan, $4x^2-144=0$, buni yechish uchun tenglamaning ikkala qismini 4 ga bo‘lamiz.

$x^2-36=0$

$x^2=36$, bundan $x=\pm\sqrt{36}=\pm6$

Endi $ax^2+c=0$ tenglamani umumiy holda yechamiz.

Ozod had c ni o‘ng tomonga o‘tkazamiz.

$ax^2=-c$, ikkala qismini a ga bo‘lamiz,

$x^2=-\frac{c}{a}$ dan $x_{1,2}=\pm\sqrt{-\frac{c}{a}}$, $-\frac{c}{a}\geq0$ bo‘lishi kerak.

$x_1=-\sqrt{-\frac{c}{a}}$, $x_2=\sqrt{-\frac{c}{a}}$

Masalan, 1) $9x^2-4=0$ 2) $2x^2+98=0$

$9x^2=4$ $2x^2=-98$

$x^2=\frac{4}{9}$ $x^2=-\frac{98}{2}=-49<0$

$x=\pm\sqrt{\frac{4}{9}}=\pm\frac{2}{3}$ $x=\pm\sqrt{-49}$ bu ildiz mavjud emas.

$x_1=-\frac{2}{3}$; $x_2=\frac{2}{3}$. Tenglama ildizga ega emas.

Javob: $\pm\frac{2}{3}$.

III. $ax^2=0$ bu tenglama faqat $x=0$ ildizga ega.

Masalan, $7x^2=0$ $\quad -3\frac{5}{8}x^2=0;\quad x=\frac{0}{-3\frac{5}{8}}=0,\quad x=0.$

$x=0$

TAKRORLASH UCHUN SAVOLLAR

1. Qanday tenglamani kvadrat tenglama deyiladi?
2. Normal shakldagi kvadrat tenglamani ayting va yozing?
3. Undagi birinchi, ikkinchi va ozod hadlarni ayting va misol keltiring.
4. Keltirilgan kvadrat tenglamani yozing.
5. Chala kvadrat tenglamalarni umumiy ko'rinishlarini yozing.

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279. Chala kvadrat tenglamalarni yeching:

a) $x^2=16;\quad x^2=0{,}25;\quad x^2=\frac{9}{16};\quad x^2=3\frac{1}{16};$

b) $x^2-4=0;\quad x^2-1=8;\quad 2x^2=32;\quad 5x^2-20=0;$

d) $x^2-0{,}4x=0;\quad x^2+1\frac{2}{3}x=0;\quad x^2-2\frac{7}{9}x=0;$

280. a) $2x^2=32;$ b) $3x^2-48=0;$

d) $7x^2+224x=0;$ e) $11x^2-100x=0.$

Namuna: 1-misol. $x(3x+4)=2(17-5x)-34$ tenglamani yechamiz. Qavslarni ochib, barcha hadlarni chap tomonga o'tkazib, soddalashtiramiz.

$3x^2+4x=34-10x-34$

$3x^2+4x-34+10x+34=0$

$3x^2+14x=0$

$x(3x+14)=0$

$x_1=0;\qquad 3x+14=0$

$3x=-14$

$x=-\frac{-14}{3}=-4\frac{2}{3};$

$x_2=-4\frac{2}{3};$ Ildizlari: $0;\ -4\frac{2}{3};$

2-misol. $\frac{3x^2-11}{8}+\frac{74-2x^2}{12}=10$ tenglamani yechamiz.

Bu tenglamani yechish uchun barcha hadlarini 24 (umumiy maxraj) ga ko‘paytiramiz. $\frac{^{3/}3x^2-11}{8}+\frac{^{2/}74-2x^2}{12}={}^{24/}10$

$3(3x^2-11)+2(74-2x^2)-240=0$

$9x^2-33+148-4x^2-240=0$

$5x^2-125=0$

$\mathbf{x^2-25=0;\ x^2=25;\ x_{1,2}=\pm 5.}$ **Ildizlari: ±5.**

281. Tenglamalarni yeching:

1) $4x^2+5x=9x^2-15x$; 2) $13x+7x^2=5x^2+8x$;

3) $12x^2-5x=9x^2+7x$; 4) $8{,}5x-3x^2=3{,}5x+2x^2$.

282. 1) $x(x-15)=3(108-5x)$; 3) $10(x-2)+19=(5x-1)(5x+1)$;

2) $(x-7)(x+3)+(x-1)(x+5)=102$; 4) $(x-8)^2+(4x+2)^2=85$.

283. 1) $\frac{5x^2-9}{6}-\frac{4x^2-9}{5}=3$; 2) $\frac{13x^2-4}{12}-\frac{20-3x^2}{18}=3\frac{5}{9}$;

3) $\frac{x}{x+1}+\frac{x}{x-1}=2\frac{2}{3}$; 4) $\frac{x+3}{x-3}+\frac{x-3}{x+3}=3\frac{1}{3}$

38-§. Keltirilgan kvadrat tenglama

Masala. To‘g‘ri to‘rtburchakning bir tomoni ikkinchi tomonidan 7 sm qisqa. Uning yuzi 60 sm² ga teng. Uning tomonlarini toping.

Yechish: Katta tomoni x sm bo‘lsin, ikkinchi tomoni $x-7$ sm, to‘g‘ri to‘rtburchakning yuzi $x(x-7)$ sm² bo‘ladi. Masala shartiga ko‘ra, $x(x-7)=60$ tenglama hosil bo‘ldi. Bu tenglamani normal holga keltirsak, $x^2-7x-60=0$ hosil qildik.

Buni yechish uchun undan ikki had kvadratini ajratamiz.

$x^2-7x-60=x^2-2\cdot 3{,}5x+3{,}5^2-3{,}5^2-60=(x-3{,}5)^2-72{,}25.$

Tenglama $(x-3,5)^2-72,25=0$ ko'rinishga keldi. $(x-3,5)^2=72,25$; $x-3,5=\pm\sqrt{72,25}=\pm 8,5$.

Bu tenglamadan $x-3,5=\pm 8,5$-yoki $x=3,5\pm 8,5$ bo'lib, $x_1=3,5-8,5=-5$; $x_2=3,5+8,5=12$.

Bu ikkala –5 va 12 ildizlar $x^2-7x-60=0$ tenglamani qanoatlantiradi, lekin masala shartiga ko'ra musbat ildiz $(x=12)$ olinadi.

Demak, to'g'ri to'rtburchakning katta tomoni 12 sm, kichik tomoni $12-7=5$ (sm) ga teng.

$x^2+px+q=0$ tenglamani umumiy holda yechamiz.

Bundagi $p=2\cdot\frac{p}{2}$ ko'rinishda yoziladi. $\left(\frac{p}{2}\right)^2=\frac{p^2}{4}$ bo'ladi.

$$x^2+px+q=\underbrace{x^2+2\cdot\frac{p}{2}x+\left(\frac{p}{2}\right)^2}_{\left(x+\frac{p}{2}\right)^2}-\left(\frac{p}{2}\right)^2+q=\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+q=0$$

$\left(x+\frac{p}{2}\right)^2-\left(\frac{p^2}{4}-q\right)=0$ bundan $\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}-q$, bo'lib:

1-hol. $\frac{p^2}{4}-q>0$ bo'lsa, $x+\frac{p}{2}=\pm\sqrt{\frac{p^2}{4}-q}$ ga teng.

$x_{1,2}=-\frac{p}{2}=\pm\sqrt{\frac{p^2}{4}-q}$ ikkita ildizga ega bo'ladi.

2-hol. $\frac{p^2}{4}-q<0$ bo'lsa, $\left(x+\frac{p}{2}\right)^2\neq\frac{p^2}{4}-q$ (tenglik to'g'ri emas), chunki $\left(x+\frac{p}{2}\right)^2>0$ (musbat).

Bu holda tenglama ildizga ega emas.

3-hol. $\frac{p^2}{4}-q=0$ bo'lsa, $\left(x+\frac{p}{2}\right)^2=0$ bo'lib, $x=-\frac{p}{2}$ bo'ladi.

Demak, **$x^2+px+q=0$ ildizi: $x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}$**

1-misol. $x^2-4x-32=0$ tenglamani yechamiz.

Bunda $p=-4$; $q=-32$.

$$x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=-\frac{-4}{2}\pm\sqrt{\frac{16}{4}-(-32)}=2\pm\sqrt{36}=2\pm6;$$

$x_1=2-6=-4$; $x_2=2+6=8$. **Ildizlari:** -4 va 8.

2-misol. $x^2-7x+12=0$ da $p=-7$; $q=12$.

$$x_{1,2}=-\frac{p}{2}=\pm\sqrt{\frac{p^2}{4}-q}=-\frac{-7}{2}\pm\sqrt{\frac{49}{4}-12}=3,5\pm\sqrt{\frac{1}{4}}=3,5\pm0,5;$$

$x_1=3,5-0,5=3$; $x_2=3,5+0,5=4$. **Ildizlari:** 3 va 4.

3-misol. $2(x-3)^2-(x-5)^2=2x+14$.
$2(x^2-6x+9)-(x^2-10x+25)-2x-14=0$.
$2x^2-12x+18-x^2+10x-25-2x-14=0$.
$x^2-4x-21=0$ da, $p=-4$; $q=-21$.

$$x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=-\frac{-4}{2}\pm\sqrt{\frac{16}{4}-(-21)}=2\pm\sqrt{4+21}=2\pm5;$$

$x_1=2-5=-3$; $x_2=2+5=7$. **Ildizlari:** -3 va 7.

TAKRORLASH UCHUN SAVOLLAR

1. Keltirilgan kvadrat tenglamaning umumiy ko'rinishini yozing.
2. Keltirilgan kvadrat tenglama ildizlari.

MASALALARNI YECHING

284. Tenglamalarni yeching:

a) $x^2+12x-64=0$; d) $x^2+14x=-24$;

b) $x^2-4x=45$; e) $x^2=11x+60$.

285. a) $x^2=x+12$; d) $x^2-4\frac{1}{2}x+4\frac{1}{2}=0$;

b) $x^2+12=7x$; e) $x^2+3\frac{5}{12}x+2=0$;

286. a) $x^2+2,4x-13+0$; d) $x^2-5,6x+6,4=0$;

b) $-3x^2-2x+8+0$; e) $x^2=x+2,64$.

287. x ning qanday qiymatlarida $y=x^2+7x+6$ uchhad va $y=x+1$ ikkihadning qiymatlari teng bo'ladi va y qiymatlar qaysilar?

288. Biri ikkinchisidan 12 ta kam bo'lgan sonlarning ko'paytmasi 133 bo'lsa, shu sonlarni toping.

289. Ikkita ketma-ket natural sonning ko'paytmasi ularning yig'indisidan 109 ta ortiq. Shu sonlarni toping.

39-§. Umumiy shakldagi kvadrat tenglama

Ushbu $4x^2-5x-21=0$ tenglamani yechamiz. Uning hadlarini 4 ga bo'lamiz.

$x^2-\frac{5}{4}x-\frac{21}{4}=0$ keltirilgan tenglama hosil bo'ladi. Bunda $p=-\frac{5}{4}$;

$q=-\frac{21}{4}$ dan $x=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=\frac{5}{8}\pm\sqrt{\frac{25}{64}+\frac{21}{4}}=\frac{5}{8}\pm\sqrt{\frac{25+336}{64}}=\frac{5}{8}\pm\frac{19}{8}$;

$x_1=\frac{5}{8}-\frac{19}{8}=-\frac{7}{4}$; $x_2=\frac{5}{8}+\frac{19}{8}=\frac{24}{8}=3$. **Ildizi:** $-\frac{7}{4}$ va 3.

Umumiy shakldagi $ax^2+bx+c=0$ tenglamani ham yuqoridagi usul bilan yechamiz.

Tenglamani a ga bo'lamiz:

$x^2+\frac{b}{a}x+\frac{c}{a}=0$ bo'lib, unda $p=\frac{b}{a}$ va $q=\frac{c}{a}$.

$$x=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Tenglamaning ildizi: $x_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Bundagi $\boldsymbol{b^2-4ac=D}$ kabi belgilab, uni tenglamaning ***diskriminanti*** deyiladi.

1-hol. $D=b^2-4ac>0$ bo'lsa, tenglama

$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$ va $x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$ ikki ildizga ega bo'ladi.

2-hol. $D=b^2-4ac=0$ bo'lsa, tenglama bitta $x=\frac{-b}{2a}$ ildizga ega bo'ladi.

3-hol. $D=b^2-4ac<0$ bo'lsa, tenglama ildizga ega bo'lmaydi.

1-misol. $3x^2+11x=6=0$ tenglamani yeching. $a=3$; $b=11$; $c=6$.

$$x_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-11\pm\sqrt{121-4\cdot3\cdot6}}{2\cdot3}=\frac{-11\pm\sqrt{49}}{6}=\frac{-11\pm7}{6};$$

$$x_1=\frac{-11-7}{6}=-3;\quad x_2=\frac{-11+7}{6}=-\frac{4}{6}=-\frac{2}{3};\ \textbf{Ildizi: } -3 \text{ va } -\frac{2}{3}.$$

Agar tenglamada x oldidagi koeffitsiyent juft bo'lsa ($b=2k$), $ax^2+2kx+c=0$ tenglamaning ildizi boshqacha ko'rinishda bo'ladi.

$$x_{1,2}=\frac{-2k\pm\sqrt{4k^2-4ac}}{2a}=\frac{-2k\pm2\sqrt{k^2-ac}}{2a}=\frac{-k\pm\sqrt{k^2-ac}}{a}$$

Ya'ni $x_{1,2}=\frac{-k\pm\sqrt{k^2-ac}}{a}$ kabi bo'ladi, bunda $k=\frac{b}{2}$ (yarmi).

2-misol. $5x^2-8x+3=0$ tenglamani yeching $2k=-8$; $k=-4$.

$$x_{1,2}=\frac{-k\pm\sqrt{k^2-ac}}{a}=\frac{4\pm\sqrt{16-5\cdot3}}{5}=\frac{4\pm1}{5};$$

$$x_1=\frac{4-1}{5}=\frac{3}{5};\quad x_2=\frac{4+1}{5}=1;\quad \textbf{Ildizi: } \frac{3}{5} \text{ va } 1.$$

3-misol. $2y^2-9y+10=0$ da $k=-4{,}5$.

$$y_{1,2}=\frac{-k\pm\sqrt{k^2-ac}}{a}=\frac{4{,}5\pm\sqrt{20{,}25-2\cdot10}}{2}=\frac{4{,}5\pm\sqrt{0{,}25}}{2}=\frac{4{,}5\pm0{,}5}{2};$$

$$y_1=\frac{4{,}5-0{,}5}{2}=\frac{4}{2}=2;\quad y_2=\frac{4{,}5+0{,}5}{2}=2{,}5;\quad \textbf{Ildizi: } 2 \text{ va } 2{,}5.$$

3-masala. Futbol o'yinida birinchilikni olish uchun musobaqada 55 ta o'yin o'ynaldi. Bunda har bir komanda qolgan komandalar bilan bir martadan o'ynadi. O'yinga nechta komanda qatnashgan?

Yechish: O'yinda x ta komanda qatnashgan deylik. Har bir komanda boshqa komandalar bilan $x-1$ tadan o'yin o'ynagan. Barcha komandalar

$x(x-1)$ ta o'yinda 2 tadan komanda o'ynagani uchun barcha o'yinlar soni $\frac{x(x-1)}{2}$ ta bo'ladi.

Masala shartiga ko'ra $\frac{x(x-1)}{2}=55$ tenglamani tuzamiz. Tenglamani 2 ga ko'paytirsak, $x(x-1)=110$ bo'ladi.

$x^2-x-110=0$

$x_{1,2}=0,5\pm\sqrt{0,25+110}=0,5\pm 10,5;$

$x_1=0,5-10,5=-10$; $x_2=0,5+10,5=11$. Masala javobi: 11 ta komanda.

4-misol. $5x+6=\frac{7}{2x+9}$ tenglamani yechamiz: Avval tenglamaning maxrajlarini tenglashtiramiz. $\frac{(5x+6)(2x+9)}{(2x+9)}=\frac{7}{2x+9}$ (bunda $x\neq-4,5$ bo'ladi).

Maxrajlari teng bo'lgan kasrning suratlari ham teng bo'ladi, ya'ni $(5x+6)(2x+9)=7$. Bu tenglamani yechamiz.

$10x^2+45x+12x+54-7=0.$

$10x^2+57x+47=0.$

$x_{1,2}=\frac{-28,5\pm\sqrt{812,25-470}}{10}=\frac{-28,5\pm 18,5}{10};\quad x_1=-4,7;\ x_2=-1.$

Ildizi: $-4,7$ va -1.

TAKRORLASH UCHUN SAVOLLAR

1. Umumiy shakldagi kvadrat tenglamani yozing.
2. Umumiy shakldagi kvadrat tenglamaning ildizlari formulasini yozing.
3. Tenglama qanday bo'lganda ikkita har xil ildizga ega bo'ladi?
4. Tenglama qanday bo'lganda bitta ildizga ega bo'ladi?
5. Tenglama qanday bo'lganda ildizga ega bo'lmaydi?
6. Tenglamada $b=2k$ bo'lganda uning ildizi qanday bo'ladi?

TENGLAMALARNI YECHING

290. a) $3x^2-7x+4=0$; e) $2y^2-9y+10=0$;

b) $y^2-10y-24=0$; f) $5y^2-6y+1=0$;

d) $3x^2-13x+14=0$; g) $4x^2+x-33=0$.

291. a) $7y^2=4y+6y^2+96$;   d) $x^2-20x=20x+100$;

b) $13p^2-10p=10p^2-3$;   e) $25x^2+13x=10x^2-7$.

292. a) $(x+4)^2=3x+40$;   d) $(x+1)^2=7918-2x$;

b) $(2x-3)^2+11x+19$;   e) $(x+2)^2=3131-2x$.

293. Tenglamalarni yeching:

a) $\frac{3x-7}{x+5}=\frac{x-3}{x+2}$;   d) $\frac{5(x-1)}{4}=\frac{x}{6}+\frac{6}{x}$;

b) $\frac{5+2x}{4x-3}=\frac{3x+3}{7-x}$;   e) $\frac{7}{x}-\frac{21+65x}{7}+8x+11=0$.

294. a) $\frac{6}{5x-1}=3x+8$;   d) $4-\frac{x-1}{x+1}=\frac{3(x+7)}{x+1}-\frac{x+11}{x^2-1}$;

b) $\frac{5x-1}{9}+\frac{3x-1}{5}=\frac{2}{x}+x-1$;   e) $\frac{14}{x^2-9}+\frac{4-x}{x+3}=\frac{7}{3+x}-\frac{1}{3-x}$.

295. a) $1-\frac{3-2x}{5-x}=\frac{3}{3-x}-\frac{x+3}{x+1}$;   d) $\frac{30}{x^2-1}-\frac{13}{x^2+x+1}=\frac{7+18x}{x^3-1}$;

b) $\frac{20+x}{2x-2}-\frac{9x^2+x+2}{6x^2-6}=\frac{5-3x}{x+1}-\frac{10-4x}{3x+3}$;   e) $\frac{2}{x^2-x+1}=\frac{1}{x+1}+\frac{2x-1}{x^3+1}$.

296*. a) $\frac{5}{x^2-4}-\frac{8}{x^2-1}=\frac{2}{x^2-3x+2}-\frac{20}{x^2+3x+2}$;   d) $\sqrt{2z^2}+4\sqrt{3z}-2\sqrt{2}=0$;

b) $z^2+2(\sqrt{3}+1)z+2\sqrt{3}=0$;   e) $\frac{1}{x-6}+\frac{1}{x-4}=\frac{1}{x+2}+\frac{1}{x-7}$.

40-§. Kvadrat tenglamalar yordamida masalalar yechish

297. Bo‘yi enidan 4 sm ortiq bo‘lib, yuzi 60 sm^2 ga teng bo‘lgan to‘g‘ri to‘rtburchakning perimetrini toping.

298. Tomonlaridan biri ikkinchisidan 10 marta katta bo‘lgan to‘g‘ri to‘rtburchak shaklidagi poliz maydonini devor bilan o‘rash kerak. Agar maydonning yuzi 12000 m^2 bo‘lsa, uning perimetrini toping.

299. Kvadrat shaklidagi karton varag‘idan eni 3 sm bo‘lgan to‘g‘ri to‘rtburchak qirqib olindi, shundan so‘ng qolgan to‘g‘ri to‘rtburchak shaklidagi varaqning yuzi 70 sm^2 ga teng bo'ldi. Varaqning dastlabki o'lchamini toping.

300. Shaxmat turniriga qatnashuvchilarning har biri qolganlari bilan bir marta o'ynasa, hammasi bo'lib 231 o'yin o'ynaladi. Turnirga necha kishi qatnashgan?

301. Maktabni tamom qiladigan sinf o'qituvchilari bir-birlari bilan rasmlarini almashtirdilar. Agar 870 ta rasm almashtirilgan bo'lsa, sinfda nechta o'quvchi bo'lgan?

302. Kinoteatrda qatordagi o'rinlar soni qatorlar sonidan 8 ta ortiq. Agar kinoteatrda hammasi bo'lib, 884 ta o'rin bor bo'lsa, unda nechta qator bor?

303. Kvadratlarining yig'indisi 869 ga teng bo'lgan uchta ketma-ket butun sonni toping.

304. Ikki avtomobil bir shahardan ikkinchi shaharga qarab bir vaqtda yo'lga chiqdi. Birinchi avtomobil tezligi ikkinchinikiga qaraganda soatiga 10 km ortiq, shuning uchun birinchi avtomobil belgilangan joyga ikkinchidan 1 soat oldin keldi. Agar shaharlarning orasi 560 km bo'lsa, har qaysi avtomobilning tezligini toping.

Suv ikki quvurdan kelganda suv haydash baki 2 soatu 55 minutda to'ladi. Birinchi quvur suv haydash bakini ikkinchiga qaraganda 2 soat oldin to'ldiradi. Har qaysi quvurning o'zi suv haydash bakini qancha vaqtda to'ldiradi.

Yechish namunasi:

Birinchi quvur bakni x soatda to'ldirgan.

Ikkinchi quvur bakni $x+2$ soatda to'ldiradi.

Birinchi quvur bir soatda bakning $\frac{1}{x}$ qismini to'ldiradi.

Ikkinchi quvur bir soatda bakning $\frac{1}{x+2}$ qismini to'ldiradi.

Ikkalasi bir soatda bakning $\frac{1}{x}+\frac{1}{x+2}=\frac{2x+2}{x(x+2)}$ qismini to'ldiradi.

Ikkalasi 2 soatu 55 minut $=2\frac{55}{60}$ soat $=2\frac{11}{12}$ soatda bakni to'liq to'ldirib bo'ladi.

Masala shartiga ko'ra $\frac{2x+2}{x(x+2)}\cdot 2\frac{11}{12}=1$ (to'la bakni 1 butun deb olamiz). Tenglamani tuzamiz: $\frac{2x+2}{x(x+2)}\cdot\frac{35}{12}=1;$

$$\frac{35(2x+2)}{70x+70}=\frac{12x(x+2)}{12x^2+24x}$$

$$6x^2-23x-35=0.$$

$$x_{1/2}=\frac{11{,}5\pm\sqrt{132{,}25+210}}{6}=\frac{11{,}5\pm18{,}5}{6};\quad x_1=-\frac{7}{6};\ x_2=5$$

$x_1=-7$ ildiz masala shartini qanoatlantirmaydi. Demak, birinchi quvur 5 soatda, ikkinchi quvur $5+2=7$ (soat)da bakni to'ldiradi.

305. Aerodromdan 1600 km uzoqlikdagi joyga ikki samolyot bir vaqtda uchib ketdi. Samolyotlardan birining tezligi ikkinchisining tezligidan soatiga 80 km ortiq, shuning uchun birinchi samolyot tayinlangan joyga ikkinchi samolyotdan 1 soat oldin keldi. Har qaysi samolyotning tezligini toping.

306. Klubning tomosha zalida 320 ta joy bor edi. Har bir qatordagi joylar sonini 4 ta orttirib, yana bir qator qo'shilgandan keyin zalda 420 ta joy bo'ldi. Klubda necha qator joy bo'ldi.

307. Elektr poyezd 4 minut to'xtab qoldi va 20 km yo'lda tezligini soatiga 10 km oshirib, kechikishni yo'qotdi. Poyezd shu yo'lda jadvalga muvofiq qanday tezlik bilan yurishi kerak edi.

308. *A* va *B* stansiyalari orasidagi yo'lning o'rtasida poyezd 10 minut to'xtab qoldi. *B* stansiyaga kechikmasdan borish uchun, mashinist poyezdning dastlabki tezligini soatiga 12 km oshirdi. Agar stansiyalarning orasi 120 km bo'lsa, poyezdning dastlabki tezligini toping.

309. Narxlar ketma-ket ikki marta bir xil protsentdan tushirilgandan keyin, fotoapparatning narxi 30 so'mdan 19 so'm 20 tiyinga tushdi. Fotoapparatning narxi har safar necha protsent tushirilgan?

310. Shahar aholisi yaxlit raqamlar bilan olganda ikki yilda 20 000 kishidan 22050 kishiga yetdi. Shu shahar aholisining o‘rtacha yillik ko‘payish protsentini toping.

41-§. Viyet teoremasi

$x^2-7x+10=0$ tenglamaning ildizlari 2 va 5. Bu ildizlarning yig‘indisi 7 ga teng, ko‘paytmasi 10 ga teng. Ildizlarning yig‘indisi $2+5=7$ qarama-qarshi ishora bilan olingan ikkinchi koeffitsiyentga, ildizlarning ko‘paytmasi $2\cdot5=10$ esa ozod hadga teng ekanligini ko‘ramiz. Ildizga ega bo‘lgan kvadrat tenglama quyidagi xossaga ega ekanini isbotlaymiz.

Teorema (*Viyet teoremasi*). **Keltirilgan kvadrat tenglama ildizlarining yig‘indisi qarama-qarshi ishora bilan olingan ikkinchi koeffitsiyentga teng, ildizlarining ko‘paytmasi esa ozod hadga teng.**

Isbot. Ushbu $x^2+px+q=0$ tenglamada $D=b^2-4ac>0$ bo‘lsin. U holda berilgan tenglama $x_1=\frac{-p-\sqrt{D}}{2}$ va $x_2=\frac{-p+\sqrt{D}}{2}$ ildizlarga ega bo‘ladi. Bu ildizlarning yig‘indisi va ko‘paytmasini topamiz:

$$x_1+x_2=\frac{-p-\sqrt{D}}{2}+\frac{-p+\sqrt{D}}{2}=\frac{-2p}{2}=-p,$$

$$x_1\cdot x_2=\frac{-p-\sqrt{D}}{2}\cdot\frac{-p+\sqrt{D}}{2}=\frac{(-p)^2-(\sqrt{D})^2}{4}=\frac{p^2-D}{4}=\frac{p^2-(p^2-4q)}{4}=$$

$$=\frac{p^2-p^2+4q}{4}=\frac{4q}{4}=q.$$

Demak, $\boldsymbol{x_1+x_2=-p}$ va $\boldsymbol{x_1\cdot x_2=q}$.

$D=0$ bo‘lganda $x^2+px+q=0$ kvadrat tenglama ikkita teng ildizga ega bo‘lib, teoremani bu holga ham tatbiq qilish mumkin. Bu teorema mashhur fransuz matematigi *Fransua Viyet* (1540–1603-y.) nomi bilan ***Viyet teoremasi*** deyiladi.

Viyet teoremasiga teskari teorema ham to‘g‘ri. Teorema (teskari teorema). **Agar *m* va *n* sonlarning yig‘indisi $-p$ ga teng, ko‘paytmasi q ga teng bo‘lsa, bu sonlar $x^2+px+q=0$ tenglamaning ildizlari bo‘ladi.**

Isbot. Shartga ko'ra $m+n=-p$, $m \cdot n=q$ bo'lib, bularni $x^2+px+q+0$ ga qo'yamiz.

$x^2-(m+n)x+mn=0$. Buni aynan shakl almashtiramiz.

$x^2-mx-nx+mn=0$

$x(x-m)+n(x-m)=0$

$(x-m)(x-n)=0$. Bu tenglamani yechib, $x_1=m$ va $x_2=n$ ildizlar topiladi. $(x-m)(x-n)=0$ tenglama $x^2+px+q=0$ ga teng kuchli bo'lgani uchun m va n sonlar $x^2+px+q=0$ tenglamaning ham ildizi bo'ladi.

1-misol. $3x^2-5x+2=0$ tenglama ildizlarining yig'indisini va ko'paytmasini topamiz $D=25-4 \cdot 3 \cdot 2=1>0$. Tenglamaning ikkala qismini 3 ga bo'lib, $x^2-\frac{5}{3}x+\frac{2}{3}=0$ keltirilgan kvadrat tenglama hosil qilamiz. Viyet teoremasiga ko'ra ildizlari yig'indisi $\frac{5}{3}$ ga teng, ko'paytmasi esa $\frac{2}{3}$ ga teng, ya'ni $x_1+x_2=-p=\frac{5}{3}$ va $x_1 \cdot x_2=q=\frac{2}{3}$.

2-misol. $x^2+3x-40=0$ tenglamani yechamiz va **Viyet teoremasiga** teskari teoremaga ko'ra tekshiramiz. $D=3^2+4 \cdot 40=169>0$.

$x_{1/2}=-1,5 \pm \sqrt{2,25+40}=-1,5 \pm 6,5$; $x_1=-8$; $x_2=5$.

Haqiqatan ham, $x_1+x_2=-8+5=-3=-p$. $x_1 \cdot x_2=-8 \cdot 5=-40=q$.

3-misol. $10x^2-33x+c=0$ tenglamaning ildizlaridan biri 5,3 ga teng. Ikkinchi ildizini va c ozod hadni toping.

Berilgan tenglamaning barcha hadlarini 10 ga bo'lib, keltirilgan $x^2-3,3x+0,1c=0$ tenglamani hosil qilamiz. Bu tenglamaning ildizlari x_1 va x_2 bo'lsin. Berilganga ko'ra: $x_1=5,3$; $x_1+x_2=3,3$; $x_1 \cdot x_2=0,1c$. Bularda, $5,3+x_2=3,3$; $x_2=-2$; $x_1 \cdot x_2=0,1c$ da $5,3 \cdot (-2)=0,1c$, $c=\frac{-10,6}{0,1}=-106$. *Javob:* $x_2=-2$; $c=-106$.

TAKRORLASH UCHUN SAVOLLAR

1. Keltirilgan kvadrat tenglamaning umumiy ko'rinishini ayting.
2. Viyet teoremasini ayting.
3. Ildizlari yig'indisini va ko'paytmasini yozing.
4. Viyet teoremasiga teskari teoremani ayting.
5. Viyet kim bo'lgan?

MASALALARNI YECHING

311. Tenglama ildizlarining yig'indisi va ko'paytmasini toping:

a) $x^2-37x+27=0$; e) $-z^2+2z+15=0$;

b) $y^2-41y-371=0$; f) $2x^2-9x-10=0$;

d) $x^2-19=0$; g) $3x^2+5x=0$.

312. Tenglamani yeching va Viyet teoremasiga teskari teorema bo'yicha tekshiring:

a) $x^2-2x-9=0$; d) $2x^2+7x-6=0$;

b) $3x^2-4x-4=0$; e) $2x^2+9x+8=0$.

313. $x^2+px-35=0$ tenglamaning ildizlaridan biri 7 ga teng. Ikkinchi ildizni va p koeffitsiyentni toping.

314. $x^2-13x+q=0$ tenglamaning ildizlaridan biri 12,5 ga teng. Ikkinchi ildizni va q ni toping.

315. $5x^2+bx+24=0$ tenglamaning ildizlaridan biri 8 ga teng. Ikkinchi ildizini va b koeffitsiyentini toping.

316. $x^2-12x+q=0$ kvadrat tenglama ildizlarining ayirmasi 2 ga teng. q ni toping.

317. Tenglamaning ildizlari bir xil ishorali bo'lmasligini isbotlang:

a) $3x^2+113x-7=0$; b) $5x^2-219x-16=0$.

318. Tenglamani yechmasdan ildizlarining ishoralarini aniqlang (agar ildizlari bo'lsa):

a) $x^2+7x-1=0$; d) $19x^2-23x+5=0$;

b) $5x^2+17x+16=0$; e) $-2x^2-5x+11=0$.

42-§. Kvadrat uchhadni ko'paytuvchilarga ajratish

$3x^2-21x+30$ kvadrat uchhadni ko'paytuvchilarga ajratish ataylab qilingan bo'lsin. Avval 3 sonini qavsdan tashqariga chiqaramiz.

Natijada $3x^2-21x+30=3(x^2-7x+10)$.

Bundagi $-7x$ ni $-2x$ va $-5x$ birhadlarning yig'indisi ko'rinishida yozamiz va guruhlash usulini qo'llab, ko'paytuvchilarga ajratamiz. $3(x^2-7x+10)=3(x^2-2x-5x+10)=3(x(x-2)-5(x-2))=3(x-2)(x-5)$.

Demak, $3x^2-21x+30=3(x-2)(x-5)$.

Shunday qilib, biz $3x^2-21x+30$ kvadrat uchhadni uchta ko'paytuvchilarga ajratdik. Bunda birinchi ko'paytuvchi x^2 oldidagi koeffitsiyentga, ikkinchi ko'paytuvchi x o'zgaruvchi bilan uchhad ildizlaridan birining ayirmasiga, uchinchi ko'paytuvchi esa x o'zgaruvchi bilan ikkinchi ildizning ayirmasiga teng.

Bunday ko'paytmani ildizga ega bo'lgan istalgan kvadrat uchhad uchun hosil qilish mumkin.

Teorema. **Agar x_1 va x_2 sonlar ax^2+bx+c kvadrat uchhadning ildizlari bo'lsa, u holda $ax^2+bx+c=a(x-x_1)(x-x_2)$ bo'ladi.**

Isbot. ax^2+bx+c ko'phaddagi a ko'paytuvchini qavsdan tashqariga chiqaramiz. Bundan: $ax^2+bx+c=a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$. x_1 va x_2 sonlar ax^2+ $+bx+c$ kvadrat uchhadning ildizlari bo'lgani uchun Viyet teoremasiga ko'ra: $x_1+x_2=-\frac{b}{a}$; $x_1\cdot x_2=\frac{c}{a}$. Bundan:

$\frac{b}{a}=-(x_1+x_2)$, $\frac{c}{a}=x_1\cdot x_2$ bo'lgani uchun

$$x^2+\frac{b}{a}x+\frac{c}{a}=x^2-(x_1+x_2)x+x_1x_2=x^2-x_1x-x_2x+x_1x_2=x(x-x_1)-x_2(x-x_1)=$$
$$=(x-x_1)(x-x_2).$$

Shunday qilib, $ax^2+bx+c=a(x-x_1)(x-x_2)$.

Agar kvadrat uchhad ildizga ega bo'lmasa, uni birinchi darajali ko'phaddan iborat bo'lgan ko'paytuvchilarga ajratib bo'lmaydi.

1-misol. $2x^2-5x-3$ kvadrat uchhadni ko'paytuvchilarga ajratamiz. Uning ildizlarini topamiz, ya'ni $2x^2-5x-3=0$ tenglamani yechamiz. $D=25-4\cdot2\cdot(-3)=49$.

$$x=\frac{5\pm7}{4};\quad x_1=-\frac{1}{2};\quad x_2=3.$$

Kvadrat uchhadni ko'paytuvchilarga ajratish formulasiga asosan

$2x^2-5x-3=2\left(x+\frac{1}{2}\right)(x-3)$ hosil bo'ladi.

2-misol. $-25x^2+10x-1$ kvadrat uchhadni ko‘paytuvchilarga ajratamiz.

Bunda $D=10^2-4\cdot(-25)\cdot(-1)=0,\ x_1=x_2=\frac{1}{5}$.

Demak, $-2x^2+10x-1=-25\left(x-\frac{1}{5}\right)\left(x-\frac{1}{5}\right)=-25\left(x-\frac{1}{5}\right)^2=-(5x-1)^2$.

Bundan: $-25x^2+10x-1=-(5x-1)^2$.

3-misol. Kasr $\frac{3x+2}{3x^2-13x-10}$ ni qisqartiramiz.

Maxraj $3x^2-13x-10$ ni ko‘paytuvchilarga ajratamiz: $3x^2-13x-10=0$.

$D=169+120=289,\ x=\frac{6{,}5\pm\sqrt{42{,}25+30}}{3}=\frac{6{,}5\pm8{,}5}{3}$ ning davomi.

$x_1=-\frac{2}{3}$ va $x_2=5$. Bundan:

$$3x^2-13x-10=3\left(x+\frac{2}{3}\right)(x-5)=(3x+2)(x-5).$$

Demak, $\frac{3x+2}{3x^2-13x-10}=\frac{3x+2}{(3x+2)(x-5)}=\frac{1}{x-5}$.

TAKRORLASH UCHUN SAVOLLAR

1. Kvadrat uchhad ildizga ega bo‘lsa, u qanday ko‘paytuvchilarning ko‘paytmasiga teng.
2. Kvadrat uchhadni ko‘paytuvchilarga ajratish haqidagi teoremani ayting.
3. Ildizga ega bo‘lgan kvadrat uchhadni ko‘paytuvchilarga ajratish formulasini yozing.
4. Ildizga ega bo‘lmagan uchhad ko‘paytuvchilarga ajraladimi?

MASALALARNI YECHING

319. Kvadrat uchhadni ko‘paytuvchilarga ajrating:

a) $3x^2-24x+21$; e) $x^2-11x+30$;

b) $5x^2+10x-15$; f) $-y^2+6y-5$;

d) $\frac{1}{6}x^2+\frac{1}{2}x+\frac{1}{3}$; g) $-2x^2+5x+7$.

320. Kvadrat uchhadni ko'paytuvchilarga ajrating:

a) $2x^2-2x+\frac{1}{2}$; d) $16a^2+24a+9$;

b) $-9x^2+12x-4$; e) $0{,}25m^2-2m+4$.

321. Ayniyatni isbotlang:

a) $10x^2+19x-2=10(x-0{,}1)(x+2)$;

b) $0{,}5x^2-5{,}5x+15=0{,}5(x-6)(x-5)$.

322. Kvadrat uchhadni birinchi darajali ko'phadlar ko'paytmasi ko'rinishida ifodalash mumkinmi?

a) $-3y^2+3y+11$; d) $y^2-7y+11$;

b) $4b^2-9b+7$; e) $3y^2-12y+12$.

323. Kasrni qisqartiring:

a) $\frac{7x^2+x-8}{7x-7}$; b) $\frac{5a+110}{2a^2+13a+18}$; d) $\frac{b^2-8b+15}{b^2-25}$;

e) $\frac{y^2-5y-36}{81-y^2}$; f) $\frac{c^2-c-10}{22+9c-c^2}$; g) $\frac{2m^2-5m-3}{2m^2-3m-2}$.

324. Kasrlarni qisqartiring:

a) $\frac{3x-12}{x^2+x-20}$; b) $\frac{2x^2+7x+3}{x^2+3x}$; d) $\frac{5a^2+8a+3}{14+3a-11a^2}$.

325. $y=\frac{x^2+x-2}{x-1}$ funksiyaning grafigi $y=x+2$ funksiyaning grafigidan nima bilan farq qiladi?

43-§. Kasr ratsional tenglamalarni yechish

Ushbu $2x+6=7(2-x)$, $x-\frac{5}{x}=-4x+19$, $\frac{x-4}{2x+1}=\frac{x-9}{5x}$ tenglamalarda chap va o'ng qismlari ratsional ifodalar. Bunday tenglamalarni ***ratsional tenglamalar*** deyiladi. Ham chap, ham o'ng qismi butun ifodalar bo'lgan ratsional tenglama ***butun ratsional tenglama*** deyiladi.

$2x+6=7(2-x)$ – butun ratsional tenglama.

Ta'rif. **Chap yoki o'ng qismi kasr ifodalar bo'lgan ratsional tenglama kasr ratsional tenglama deyiladi.** Masalan, $x-\frac{5}{x}=-4x+19$ va $\frac{x-4}{2x+1}=\frac{x-9}{5x}$ kasr ratsional tenglamalar.

Kasr ratsional tenglamalarni yechish usulini ko'rib chiqamiz. Masalan, $2-\frac{x-7}{x-5}=\frac{x+5}{x^2-5x}-\frac{1}{x}$ tenglamani yechamiz.

Bu tenglamaning chap va o'ng qismlarini umumiy maxrajli kasr ko'rinishida tasvirlaymiz: Umumiy maxraj $x^2-5x=x(x-5)$ bo'ladi $\frac{2x(x-5)-x(x-7)}{x(x-5)}=\frac{x+5-(x-5)}{x(x-5)}$.

Bu kasrlar teng, maxrajlari noldan farqli bo'lgan qiymatlarida va faqat shu qiymatlarda o'zaro teng bo'ladi. Demak, $2x(x-5)-x(x-7)=$ $=x+5-(x-5)$ tenglamani yechish kerak. Yoki berilgan tenglamaning ikkala qismini umumiy maxraj $x(x-5)$ ga ko'paytirib $2x(x-5)-x(x-7)=$ $=x+5-(x-5)$ tenglama hosil qilinadi. Bundagi qavslar ochilib, hamma hadlari ixchamlansa, $x^2-3x-10=0$ hosil bo'ladi.

Bu tenglamani yechib:

$x = 1{,}5 \pm \sqrt{2{,}25 + 10} = 1{,}5 \pm 3{,}5$, bundan $x_1 = -2$ va $x_2 = 5$ topiladi.

Bu ildizlarda $x(x-5) \neq 0$ tekshiriladi:

Agar $x=-2$ da $x(x-5) \neq 0$ bo‘ladi. Demak, 2 ildiz, $x=5$ da esa $x(x-5)=0$ bo‘ladi. Demak, 5 soni tenglamaning ildizi bo‘lmaydi.

Javob: – 2 soni tenglamaning ildizi.

2-misol. Ushbu $\frac{30}{x^2-1} - \frac{13}{x^2+x+1} = \frac{18x+7}{x^3-1}$ tenglamani yechamiz. Tenglamadagi kasrlarning umumiy maxrajini topamiz. Buning uchun kasr maxrajlarini ko‘paytuvchilarga ajratamiz: $\frac{x^2+x+1/_{30}}{(x-1)(x+1)} - \frac{x^2-1/_{13}}{x^2+x+1} = \frac{x+1/_{18x+7}}{(x-1)(x^2+x+1)}$.

Umumiy maxraj $(x-1)(x-1)(x^2-x-1)$ bo‘ladi.

Berilgan tenglamaning ikkala qismini umumiy maxrajga ko‘paytirib, qo‘shimcha ko‘paytuvchini kasr suratiga ko‘paytirib, butun tenglama hosil qilamiz: $30(x^2+x+1)-13(x^2-1)=(x+1)(18x+7)$. Bundagi qavslarni ochib, o‘xshash hadlar ixchamlansa, $x^2-5x-36=0$ kvadrat tenglama hosil bo‘ladi.

Tenglamani yechib: $x = 2{,}5 \pm \sqrt{6{,}25 + 36} = 2{,}5 \pm 6{,}5$,

$x_1=-4$ va $x_2=9$ ildizlarni topamiz.

$x_1=-4$ va $x_2=9$ larda umumiy maxraj – $(x^2-1)(x^2+x+1) \neq 0$ bo‘ladi.

Demak, –4 va 9 sonlar berilgan tenglamaning ildizlari. Odatda, kasr tenglamalarni yechishda quyidagi tartibga rioya qilinadi:

1) tenglamadagi kasrlarning umumiy maxraji topiladi.

2) berilgan tenglamaning ikkala qismini umumiy maxrajga ko‘paytirib, uni butun tenglamaga aylantiriladi.

3) hosil qilingan butun tenglama yechiladi.

4) uning ildizlari orasidan umumiy maxrajni nolga aylantiradiganlari tashlab yuboriladi.

TAKRORLASH UCHUN SAVOLLAR

1. Butun ratsional tenglamalarga misollar keltiring.
2. Ratsional tenglamalarga misollar keltiring.
3. Kasr ratsional tenglamalar deb qanday tenglamalarga aytiladi?
4. Kasr ratsional tenglamalarni yechish tartibini aytib bering.

MASALALARNI YECHING

326. Tenglamaning ildizlarini toping:

a) $\frac{y^2}{y+3}=\frac{y}{y+3}$; b) $\frac{x^2}{x^2-4}=\frac{5x-6}{x^2-4}$; d) $\frac{2x^2}{x-2}=\frac{-7x+6}{2-x}$;

e) $\frac{2x-1}{x+7}=\frac{3x+4}{x-1}$; f) $\frac{2y+3}{2y-1}=\frac{y-5}{y+3}$; g) $\frac{1+3x}{1-3x}=\frac{5-2x}{1+2x}$.

327. Tenglamani yeching:

a) $\frac{2x-5}{x+5}-4=0$; b) $\frac{x^2-4}{x}=\frac{3+2x}{2}$; d) $\frac{10}{2x-3}=x-1$;

e) $\frac{x^2+4x}{x+2}=\frac{2x}{3}$; f) $x+2=\frac{15}{4x+1}$; g) $\frac{y^2}{y^2-6y}=\frac{4(3-2y)}{y(6-y)}$.

328. Tenglamani yeching:

a) $\frac{3x+1}{x+2}-\frac{x-1}{x-2}=1$; b) $\frac{4}{9y^2-1}-\frac{4}{3y+1}=\frac{5}{1-3y}$;

d) $\frac{2}{x^2-x+1}=\frac{1}{x+1}+\frac{2x-1}{x^3+1}$; e) $\frac{3y-2}{y}-\frac{1}{y-2}=\frac{3y+4}{y^2-2y}$;

f) $\frac{4}{x+3}-\frac{5}{3-x}=\frac{1}{x-3}-1$; g) $\frac{3}{y+2}-\frac{4}{y^2-2y+4}=\frac{2}{y^3+8}$.

329. Tenglamani yeching:

a) $\frac{5}{y-2}-\frac{4}{y-3}=\frac{1}{y}$; b) $\frac{1}{2(x+1)}+\frac{1}{x+2}=\frac{3}{x+3}$;

d) $\frac{10}{y^3-y}+\frac{1}{y-y^2}=\frac{1}{1+y}$; e) $1+\frac{45}{x^2-8x+16}=\frac{14}{x-4}$;

f) $\frac{5}{x-1}-\frac{4}{3-6x+3x^2}=3$; g) $\frac{10}{(x-5)(x+1)}+\frac{x}{x+1}=\frac{3}{x-5}$.

330. Tenglamaning ildizlarini toping.

a) $\frac{4}{(x+1)^2}-\frac{1}{(x-1)^2}+\frac{1}{x^2-1}=0$; d) $\frac{4}{9x^2-1}+\frac{1}{3x^2-x}=\frac{4}{9x^2-6x+1}$;

b) $\frac{17}{(x-3)(x+4)}-\frac{1}{x-3}=\frac{x}{x+4}$; e) $\frac{18}{4x^2+4x+1}-\frac{1}{2x^2-x}=\frac{6}{4x^2-1}$.

331.* Tenglamani yeching:

a) $\frac{32}{x^3-2x^2-x+2}+\frac{1}{(x-1)(x-2)}=\frac{1}{x+1}$;

b) $\frac{1}{3(x-4)}+\frac{1}{2(x^2+3)}+\frac{1}{x^3-4x^2+3x-12}=0$;

d) $\frac{x\sqrt{3}+\sqrt{2}}{x\sqrt{3}-\sqrt{2}}+\frac{x\sqrt{3}-\sqrt{2}}{x\sqrt{3}+\sqrt{2}}=\frac{10x}{3x^2-2}$;

e) $\frac{1-y\sqrt{5}}{1+y\sqrt{5}}+\frac{1+y\sqrt{5}}{1-y\sqrt{5}}=\frac{9y}{1-5y^2}$.

44-§. Ratsional tenglamalar yordamida masalalar yechish

Ko'pgina masalalar kasr ratsional tenglamalar yordamida yechiladi.

1-masala. Motorli qayiq daryo oqimi bo'yicha 25 km, oqimga qarshi 3 km o'tdi va butun yo'lga 2 soat sarfladi. Agar oqimning tezligi 3 km/soat bo'lsa, qayiqning turg'un suvdagi tezligi qanday?

Yechish. Qayiqning turg'un suvdagi tezligi x km/soat bo'lsin. U holda qayiqning oqim bo'yicha tezligi $x+3$ km/soat, oqimga qarshi tezligi $x-3$ km/soat bo'ladi.

Qayiq oqim bo'yicha 25 km yo'lni $\frac{25}{x+3}$ soat, oqimga qarshi 3 km yo'lni soatda $\frac{3}{x-3}$ o'tadi. Butun yo'lga sarflangan vaqt $\frac{25}{x+3}+\frac{3}{x-3}$ soat bo'ladi. Masala shartiga ko'ra butun yo'lga sarflagan vaqt 2 soat bo'lgani uchun quyidagi tenglamani tuzamiz.

$\frac{25}{x+3}+\frac{3}{x-3}=2$. Bu tenglamaning ikkala qismini $(x+3)(x-3)$ umumiy maxrajga ko'paytiramiz. Natijada $25(x-3)+3(x+3)=2(x-3)(x+3)$ tenglamani hosil qilib, uni soddalashtiramiz va $-2x^2+28x-48=0$ tenglamani hosil qilamiz. Bu tenglama $x^2-14x+24=0$ tenglamaga teng kuchli bo'ladi. Bu tenglamani yechib, $x_1=2$ va $x_2=12$ ildizlarni topamiz. Bu $x=2$ va $x=12$ ildizlar umumiy maxraj $(x-3)(x-12)$ ni nolga aylantirmaydi. Bu 2 va 12 sonlar dastlabki tenglamaning ildizi bo'ladi. $x=2$ da $\frac{3}{x-3}<0$ bo'lgani uchun. *Javob:* 12 km/soat.

2-masala. Elektr poyezdi 840 km yo‘l bosishi kerak edi. Yo‘lning yarmida 20 minut tutilib qoldi, shu sababli u kechikmaslik uchun tezligini soatiga 6 km oshirdi. Elektr poyezdi butun yo‘lga qancha vaqt sarf qilgan?

Yechish. Poyezdning dastlabki tezligi x km/soat. Poyezdning keyingi tezligi $x+6$ km/soat bo‘ladi.

Poyezd dastlabki tezlik bilan 420 km ni $\frac{420}{x}$ soat yuradi, keyingi tezligi bilan $\frac{420}{x+6}$/soat yuradi. 20 minut $\frac{20}{60}=\frac{1}{3}$ soat.

Masala shartiga ko‘ra $\frac{^{3(x+6)/}420}{x}-\frac{^{3x/}420}{x+6}=\frac{^{x(x+6)/}1}{3}$ tenglamani tuzamiz. Bu tenglamaning umumiy maxraji $3x(x+6)$ bo‘lib, tenglamani shu umumiy maxrajga ko‘paytirib, $1260(x+6)-1260x=x(x+6)$ butun ratsional tenglama hosil qilamiz. Bu tenglamani soddalashtirib, $x^2+6x-7560=0$ kvadrat tenglama hosil qilamiz.

Bu tenglamani yechib, $x_1=-90$ va $x_2=84$ ildizlarni topamiz. Bu ildizlarning 84 (km/soat)i masala shartini qanoatlantiradi.

Elektr poyezdi butun yo‘lni $\frac{840}{84}=10$ (soat) da bosib o‘tadi.

Javob: 10 soat.

MASALALARNI YECHING

332. Oddiy kasrning surati maxrajidan bitta ortiq. Agar kasrning suratiga 3, maxrajiga esa 18 qo‘shilsa, u holda dastlabki kasrdan 1 ta kam kasr hosil bo‘ladi. Dastlabki kasrni toping.

333. Oddiy kasrning maxraji suratidan 4 ta ortiq. Agar kasrning surati 3 ta kamaytirilsa, maxraji esa 5 ta orttirilsa, hosil qilingan kasr dastlabki kasrdan $\frac{1}{3}$ ta kam bo‘ladi. Dastlabki kasrni toping.

334. Velosipedchi 27 km yo‘lni 1 soatu 40 minutda bosib o‘tdi, bunda oxirgi 17 km yo‘lni avvalgi qismini o‘tganidan 2 km/soat ortiq tezlik bilan o‘tdi. Velosipedchi dastlab qanday tezlik bilan yurgan?

335. Velosipedchi shahardan qishloqqa uzunligi 24 km bo'lgan yo'ldan, qaytishda esa 30 km uzunlikdagi yo'ldan yurdi. Qaytishda u tezligini 2 km/soat orttirgan bo'lsa ham, qaytishga shahardan qishloqqa borishdagidan 6 minut ortiq vaqt sarfladi. Velosipedchi qanday tezlik bilan qaytgan?

336. Teploxod 1 soatda ko'lda 9 km va daryo oqimi bo'yicha 20 km o'tdi. Agar daryo oqimining tezligi 3 km/soat bo'lsa, teploxodning ko'ldagi tezligini toping.

337. Poyezd 1 soat kechikishini yo'qotish uchun 720 km yo'lda tezligini jadvaldagidan 10 km/soat orttirdi. Poyezdning jadval bo'yicha tezligi qanday?

338. Ikki brigada birgalikda ishlab, bir ishni 6 kunda tugalladi. Agar bu ishni bajarish uchun bir brigadaga ikkinchisiga qaraganda 5 kun ortiq vaqt kerak bo'lsa, har bir brigada bu ishni necha kunda bajarar edi?

339. Quvvatlari har xil bo'lgan ikkita traktor 4 kunda birga ishlab, yerning $\frac{2}{3}$ qismini haydadi. Agar butun yerni birinchi traktor ikkinchisiga qaraganda 5 kun tezroq hayday olsa, butun yerni har qaysi traktor yolg'iz o'zi necha kunda hayday oladi?

340. Turg'un suvdagi tezligi 15 km/soat bo'lgan motorli qayiq daryo oqimi bo'yicha 35 km va daryo oqimiga qarshi 25 km suzib o'tdi. U daryo oqimi bo'yicha yo'lga daryo oqimiga qarshi yo'lga sarflagancha vaqt ketkazdi. Daryo oqimining tezligi qanday?

341. Poyezd 450 km masofani jadval bo'yicha ma'lum vaqtda bosib o'tishi kerak edi. Biroq yo'lning 40% qismi o'tilganda u 18 daqiqa to'xtab turishga majbur bo'ldi, stansiyaga jadvaldagi muddatda yetib kelishi uchun tezlikni 10 km/soat orttirdi. Yo'lni poyezd jadval bo'yicha qanday tezlikda o'tishi kerak?

45-§. Sonli tengsizliklar

Biz ikki natural sonni; ikki oddiy kasrni; ikki o'nli kasrni ikki manfiy sonni va hokazolarni qanday taqqoslashni bilamiz.

Masalan: 1) $\frac{5}{8}$ va $\frac{4}{7}$ larni taqqoslash uchun avval ularni umumiy maxrajga keltiramiz. $\frac{5}{8}=\frac{35}{56}$; $\frac{4}{7}=\frac{32}{56}$ bularda $35>32$ bo'lgani uchun $\frac{35}{56}>\frac{32}{56}$ bo'ladi, ya'ni $\frac{5}{8}>\frac{4}{7}$.

2) 25,6783 va 25,6779 o'nli kasrlarni taqqoslaymiz. Bularda butunlar, o'ndan birlar va yuzdan birlar xonasidagi raqamlar bir xil, mingdan birlar xonasida esa birinchi kasrda 8 raqami, ikkinchi kasrda 7 raqami yozilgan, $8>7$ bo'lgani uchun $25{,}6783>25{,}6779$ bo'ladi.

3) –35 va –24 manfiy sonlarni taqqoslaymiz.

Bularda $|-35|=35$; $|-24|=24$; $24<35$ bo'lib, ikkinchi sonning moduli birinchi sonning modulidan kichik. Manfiy sonlarda moduli kichik son moduli kattasidan katta bo'ladi, ya'ni $-35<-24$. Biz sonlarning ko'rinishiga qarab, ularni taqqoslashning turli usullaridan foydalandik. Biroq sonlarni taqqoslashning hamma usullarini ham o'z ichiga oluvchi taqqoslash usuliga egamiz. Bu usul shundan iboratki, bunda sonlarning ayirmasi hisoblanadi va hosil bo'lgan son musbat ekanligi, manfiy ekanligi yoki nolga teng bo'lishi aniqlanadi.

Ta'rif. **Agar $a-b$ ayirma musbat bo'lsa, a son b sondan katta bo'ladi; agar $a-b$ ayirma manfiy bo'lsa, a son b sondan kichik bo'ladi; agar $a-b$ ayirma nolga teng bo'lsa, a son b songa teng bo'ladi.**

Ya'ni, 1) $a-b>0$ bo'lsa, $a>b$ bo'ladi;

2) $a-b<0$ bo'lsa, $a<b$ bo'ladi;

3) $a-b=0$ bo'lsa, $a=b$ bo'ladi.

Istalgan ikkita a va b sonlar uchun $a>b$, $a<b$, $a=b$ munosabatlardan bittasi va faqat bittasi bajariladi. Ta'rifdan quyidagilar kelib chiqadi:

agar $a>b$ bo'lsa, $b<a$ bo'ladi;

agar $a<b$ bo'lsa, $b>a$ bo'ladi.

Agar a son b dan katta, yoki a son b songa teng bo'lsa, ular $a \geq b$ kabi yoziladi. U «a son b sondan katta yoki teng» deb o'qiladi. $a \leq b$ yozuv: «a son b sondan kichik yoki teng» deb o'qiladi.

$>$ va $<$ tengsizliklarni qat'iy tengsizliklar, $\geq$ va $\leq$ tengsizliklarni ***noqat'iy tengsizliklar*** deyiladi.

Ta'rifga oid misollar ko'rib chiqamiz.

1-misol. $(a-3)(a-5)$ ifoda a ning istalgan qiymatida $(a-4)^2$ dan kichik ekanligini ko'rsating.

Yechish. Ta'rifga asosan kattasidan kichik ifodaning ayirmasini musbat ekanligini ko'rsatish kifoya.

$(a-4)^2-(a-3)(a-5)=a^2-8a+16-a^2+5a+3a-15=-8a+16+8a-$
$-15=1>0.$

Demak, $(a-4)^2>(a-3)(a-5)$.

2-misol. Istalgan ikki son kvadratlarining yig'indisi ular ko'paytmasining ikkilangandan kichik emasligini isbotlang.

Yechish. a va b ixtiyoriy sonlar bo'lsin. $a^2+b^2>2ab$ ekanligini ko'rsatamiz.

$a^2+b^2-2ab=(a-b)^2>0$. Demak, $a^2+b^2>2ab$.

TAKRORLASH UCHUN SAVOLLAR

1. Ikki sonni taqqoslashning qanday usullarini bilasiz?
2. Sonlarni taqqoslashning hamma usullarini o'z ichiga olgan usul nimadan iborat?
3. Sonlarni taqqoslashning ta'rifini ayting.
4. Qanday tengsizliklarni qat'iy tengsizliklar, qanday tengsizliklarni noqat'iy tengsizliklar deyiladi?
5. Agar $c-d>0$ bo'lsa, c bilan d o'zaro qanday bo'ladi? $c-d<0$ bo'lsa-chi?

MASALALARNI YECHING

342. Agar $a-b$ ayirma -10 ga; 3 ga; 0 ga teng bo'lsa, a va b sonlarni taqqoslang.

a) $x-y=-0{,}83$; b) $x-y=0$; d) $x-y=3{,}2$ bo'lsa, x va y sonlarni taqqoslang.

343. $a<b$ ekani ma'lum. $a-b$ ayirma 4,5; –7,2; 0 sonlardan qaysi biriga teng bo'lishi mumkin?

344. $2b(3b+5{,}5)$ va $(3b+1)(2b+3)$ ifodalar b ning istalgan qiymatida birinchi ifodani ikkinchi ifodadan katta yoki kichikligini aniqlang.

345. O'zgaruvchining istalgan qiymatida tengsizlik to'g'ri ekanligini isbotlang:

a) $3(a+1)+a<4(2+a)$; d) $(a+2)^2>a(a+4)$;

b) $(7p-1)(7p+1)<49p^2$; e) $(2a+3)(2a+1)>4a(a+2)$.

346. Tengsizlikni isbotlang:

a) $a(a+b)\geq ab$; d) $2bc\leq b^2+c^2$;

b) $m^2-mn+n^2\geq mn$; e) $a(a-b)\geq b(a-b)$.

347. Musbat c son bilan unga teskari sonning yig'indisi 2 dan kichik emasligini isbotlang.

348. $a\geq 0$, $b\geq 0$ bo'lsa, $\frac{a+b}{2}\geq\sqrt{ab}$ ekanligini isbotlang.

46-§. Sonli tengsizliklarning xossalari

Sonli tengsizliklarning xossalarini ifodalovchi teoremalarni ko'rib chiqamiz.

1-teorema. Agar $a<b$ va $b<c$ bo'lsa, u holda $a<c$ bo'ladi.

Isbot. Bunda biz $a<c$ yoki $a-c<0$ ekanligini ko'rsatamiz.

$a<b$ da $a>b<0$ va $b<c$ da $b-c<0$ bo'ladi.

$a-c$ ayirmaga b va $-b$ larni qo'shamiz.

$a-c+b-b=(a-b)+(b-c)$ bunda $a-b$ va $b-c$ lar manfiy sonlar. Manfiy sonlarning yig'indisi ham manfiy son bo'ladi. Demak, $a-c<0$, ya'ni $a<c$ (5-a, chizma).

Shuningdek $a>b$ va $b>c$ bo‘lsa, u holda $a>c$ bo‘ladi (5-*b*, chizma).

5-chizma.

2-teorema. Agar $a<b$ va c istalgan son bo‘lsa, u holda $a+c<b+c$ bo‘ladi.

Isbot. Buning uchun $(a+c)-(b+c)<0$ ekanligini ko‘rsatamiz. $a<b$ da, $a-b<0$ bo‘ladi. $(a+c)-(b+c)$ ayirmani ko‘rib chiqamiz. $(a+c)-(b+c)=a+c-b-c=a-b<0$ (shartga ko‘ra).

Demak, $(a+c)-(b+c)$ ayirma manfiy son.

Bundan $a+c<b+c$.

Shunday qilib, to‘g‘ri tengsizlikning ikkala qismiga ayni bir son qo‘shilsa, to‘g‘ri tengsizlik hosil bo‘ladi.

3-teorema. Agar $a<b$ va c musbat son bo‘lsa, u holda $ac<bc$ bo‘ladi. Agar $a<b$ va c manfiy son bo‘lsa, u holda $ac>bc$ bo‘ladi.

Isbot. 1) $a<b$ va $c>0$ bo‘lsa, $ac<bc$ ekanligini;

2) $a<b$ va $c<0$ bo‘lsa, $ac>bc$ ekanligini ko‘rsatamiz.

$a<b$ da $a-b<0$ bo‘ladi.

$ac-bc$ ayirmani ko‘paytma ko‘rinishida tasvirlaymiz.

1) $ac-bc=c(a-b)$, bunda $c>0$; $a-b<0$ ekanligidan $c(a-b)$ ko‘paytma manfiy bo‘lib, $ac-bc<0$ bo‘ladi.

Demak $ac<bc$ bo‘ladi.

2) $ac-bc=c(a-b)$, bunda $c<0$ va $a-b<0$ ekanligidan ko‘paytma $c(a-b)$ musbat, demak $ac>bc$ bo‘ladi.

Bo‘lishni bo‘luvchiga teskari songa ko‘paytirish bilan almashtirish mumkin bo‘lgani sababli tengsizlikni songa bo‘lish ham yuqoridagidek isbotlanadi.

Shunday qilib, **agar to‘g‘ri tengsizlikning ikkala qismini ayni bir musbat songa ko‘paytirilsa yoki bo‘linsa, to‘g‘ri tengsizlik hosil bo‘ladi;**

agar to‘g‘ri tengsizlikning ikkala qismi ayni bir manfiy songa ko‘paytirilsa yoki bo‘linsa va tengsizlikning ishorasi teskarisiga almashtirilib, to‘g‘ri tengsizlik hosil bo‘ladi.

Natija. Agar *a* va *b* musbat sonlar va *a*>*b* bo'lsa, u holda $\frac{1}{a}<\frac{1}{b}$ bo'ladi.

Isbot. $a>b$ tengsizlikning ikkala qismini musbat ab songa bo'lamiz: $\frac{a}{ab}>\frac{b}{ab}$, so'ngra bu kasrni qisqartiramiz.

$\frac{1}{b}>\frac{1}{a}$ tengsizlikni $\frac{1}{a}<\frac{1}{b}$ ko'rinishda yozamiz.

1-misol. a) $18>-11$ tengsizlikning ikkala qismiga 7 soni; 3,2 soni; –20 soni qo'shilganda va ayrilganda hosil bo'lgan to'g'ri tengsizliklarni yozing.

b) $-15<6$ tengsizlikning ikkala qismini 2 ga; –3 ga; $-\frac{1}{5}$ ga ko'paytirganda va bo'lganda hosil bo'lgan to'g'ri tengsizliklarni yozing.

Yechish. a) $18+7>-11+7;$ $\quad 18+3{,}2>-11+3{,}2;$ $\quad 18+(-20)>-11+(-20)$

$25>-4$ $\quad 21{,}2>-7{,}8$ $\quad -2>-3{,}1;$

$18-7>-11-7$ $\quad 18-3{,}2>-11-3{,}2$ $\quad 18-(-20)>-11-(-20)$

$11>-18$ $\quad 14{,}8>-14{,}2$ $\quad 38>9$

b) $-15\cdot 2<6\cdot 2$ $\quad -15\cdot(-3)>6\cdot(-3)$ $\quad -15\cdot\left(-\frac{1}{5}\right)>6\cdot\left(-\frac{1}{5}\right)$

$-30<12$ $\quad 45>-18$ $\quad 3>-1{,}2$

$-15:2<6:2$ $\quad -15:(-3)>6:(-3)$ $\quad -15:\left(-\frac{1}{5}\right)>6:\left(-\frac{1}{5}\right)$

$-7{,}5<3$ $\quad 5>-2$ $\quad 75>-30$

2-misol. Tomoni a bo'lgan muntazam oltiburchakning perimetrini $5{,}4<a<5{,}5$ bo'lganda baholang.

Yechish. $5{,}4<a<5{,}5$ tengsizlikni $5{,}4<a$ va $a<5{,}5$ larga ajratib, ularning har birini 6 ga ko'paytiramiz, $5{,}4\cdot 6<6\cdot a$ va $6\cdot a<5{,}5\cdot 6$-yoki $32{,}4<6a$ va $6a<33$ tengsizliklarni hosil qilamiz $p=6a$ bo'lgani uchun bu tengsizliklarni birlashtirib $32{,}4<p<33$ (sm) ni hosil qilamiz. *Javob:* $32{,}4<p<33$ (sm).

TAKRORLASH UCHUN SAVOLLAR

1. Tengsizlikni xossalarini ifodalovchi birinchi teoremani ayting.
2. 2-teoremani ayting.
3. 2-teoremaning natijasini ayting.

4. 3-teoremani ayting.
5. 3-teoremaning natijasini ayting.
6. Koordinata to‘g‘ri chizig‘ida $a<b$ va $b<c$ bo‘lsa, a bilan c ni taqqoslang.

MASALALARNI YECHING

349. $a<b$ ekani ma’lum. Quyidagi sonlarni taqqoslang.
a va $b+1$; $a-3$ va b; $a-5$ va $b+2$; $a-8$ va $b-7$.

350. Tengsizlikning xossalaridan foydalanib:

a) $18>-11$ tengsizlikning ikkala qismiga 9 soni; 4,7 soni – 6 soni qo‘shilganda hosil bo‘ladigan to‘g‘ri tengsizlikni yozing;

b) $12>-8$ tengsizlikning ikkala qismidan 7 soni; 4,3 soni; –12 soni ayrilganda hosil bo‘ladigan to‘g‘ri tengsizlikni yozing;

d) $-10<23$ tengsizlikning ikkala qismini 7 ga; –4 ga va $-\frac{1}{5}$ ga ko‘paytirganda hosil bo‘ladigan to‘g‘ri tengsizlikni yozing.

e) $15>-9$ tengsizlikning ikkala qismini 3 ga; –1 ga; $-\frac{2}{3}$ ga bo‘lganda hosil bo‘ladigan to‘g‘ri tengsizlikni yozing.

351. a ning ishorasi qanday?

a) $5a>2a$; b) $7a<3a$; d) $-4a<4a$; e) $-12a>-2a$.

352. $a<b$ ekani ma’lum. * belgisi o‘rniga < yoki > belgilarni to‘g‘ri tengsizlik hosil bo‘ladigan qilib qo‘ying:

a) $-12{,}7a * -12{,}7\,b$; d) $\frac{a}{3} * \frac{b}{3}$;

b) $0{,}17a * 0{,}17\,b$; e) $-\frac{a}{2} * -\frac{b}{2}$.

353. $5<x<8$ ekanini bilgan holda quyidagi ifodaning qiymatini baholang:
a) $6x$; b) $-10x$; d) $x-5$; e) $3x+2$.

354. a) Tomoni a (sm) ga teng kvadratning perimetrini baholang, bunda $5{,}1<a<5{,}2$.

b) Kvadratning perimetri p (sm) ga tengligini bilgan holda kvadrat tomonining uzunligini baholang, bunda $15{,}2 \le p \le 15{,}6$.

355. $\frac{1}{y}$ ifodaning qiymatini baholang, bunda:

a) $5<y<8$; b) $0{,}125<y<0{,}25$.

47-§. Sonli tengsizliklar ustida amallar

Bir xil ishorali $5<8$ va $3<6$ to‘g‘ri tengsizliklarni hadma-had qo‘shamiz va ko‘paytiramiz. Natijada $5+3<8+6$, ya’ni $8<14$ va $5\cdot3<8\cdot6$, ya’ni $15<18$ tengsizliklarni hosil qilamiz, bularning har biri to‘g‘ri tengsizlik.

Qanday holda bir xil ishorali to‘g‘ri tengsizliklarni hadma-had qo‘shish va ko‘paytirish mumkinligiga quyidagi teoremalar javob beradi.

4-teorema. Agar $a<b$ va $c<d$ bo‘lsa, u holda $a+c<b+d$ bo‘ladi.

Isbot. $a<b$ tengsizlikning ikkala qismiga c ni qo‘shib, $a+c<b+c$ ni hosil qilamiz. $c<d$ tengsizlikning ikkala qismiga b ni qo‘shib, $b+c<b+d$ ni hosil qilamiz.

$a+c<b+c$ va $b+c<b+d$ tengsizliklardan 1-teoremaga asosan $a+c<b+d$ kelib chiqadi.

Qaralayotgan tengsizliklar soni ikkitadan ortiq bo‘lgan holda ham teorema to‘g‘ri bo‘ladi.

Shunday qilib, **agar bir xil ishorali to‘g‘ri tengsizliklar hadma-had qo‘shilsa, to‘g‘ri tengsizlik hosil bo‘ladi.**

1-misol. $3<a<4$ va $4<b<5$ bo‘lsin. Quyidagilarni baholang:

a) $a+b$; b) $a-b$.

Yechish. a) $3<a$ va $4<b$ larni, so‘ngra $a<4$ va $b<5$ tengsizliklarni qo‘shib, $3+4<a+b$ va $a+b<4+5$ larni hosil qildik.

Natijani qo‘sh tengsizlik ko‘rinishida yozamiz, ya’ni $7<a+b<9$. Odatda yozuv qisqaroq qilinib bunday yoziladi.

$$+\begin{array}{r}3<\ \ a\ \ <4\\ 4<\ \ b\ \ <5\\ \hline 7<a+b<9\end{array}$$

b) $a-b$ ayirmani baholaymiz. $a-b$ ayirmani $a+(-b)$ yig‘indi ko‘rinishida yozamiz. Avval $-b$ ning qiymatini baholaymiz. $4<b<5$ bo‘lgani uchun $-4>-b>-5$, ya’ni $-5<-b<-4$ bo‘ladi.

Endi qo‘sh tengsizliklarni hadma-had qo‘shamiz.

$$\begin{array}{r} + \begin{array}{rcl} 3< & a & <4 \\ -5< & -b & <-4 \end{array} \\ \hline -2<a-b<0 \end{array}$$

5-teorema. Agar *a<b* va *c<d* (bunda *a, b, c* va *d* – musbat sonlar) bo'lsa, u holda *ac<bd* bo'ladi.

Isbot. $a<b$ tengsizlikning ikkala qismini musbat c songa ko'paytirib, $ac<bc$ ni hosil qilamiz. $c<d$ tengsizlikning ikkala qismini musbat b songa ko'paytirib, $bc<bd$ ni hosil qilamiz. $ac<bc$ va $bc<bd$ tengsizliklardan $ac<bd$ ekani kelib chiqadi.

Teorema qaralayotgan tengsizliklar soni ikkitadan ortiq bo'lgan holda ham to'g'ri bo'ladi.

Shunday qilib, **agar o'ng va chap qismlari musbat sonlar bo'lgan bir xil ishorali to'g'ri tengsizliklar hadma-had ko'paytirilsa, to'g'ri tengsizlik hosil bo'ladi.**

Natija. Agar *a* va *b* sonlar musbat, hamda *a<b* bo'lsa, u holda *an<bn* bo'ladi (*n* – natural son).

a va b musbat sonlar bo'lgan n ta $a<b$ to'g'ri tengsizlikni hadma-had ko'paytirib, $a^n<b^n$ to'g'ri tengsizlikni hosil qilamiz.

2-misol. $10<x<11$ va $7<y<8$ ekani ma'lum bo'lsin, xy va $\frac{x}{y}$ ifodalarni baholaymiz.

Yechish. a) xy ko'paytmani baholaymiz. x va y sonlarning har biri musbat sonlar orasida joylashgani uchun ular ham musbat sonlar bo'ladi.

Tengsizliklarni hadma-had ko'paytirish haqidagi teoremani (5-teoremani) qo'llaymiz.

$$\begin{array}{r} \times \begin{array}{rcl} 10< & x & <11 \\ 7< & y & <8 \end{array} \\ \hline 70<xy<88 \end{array}$$

b) $\frac{x}{y}$ bo'linmani baholaymiz. Buning uchun $\frac{x}{y}$ bo'linmani $x\cdot\frac{1}{y}$ ko'paytma ko'rinishga keltiramiz.

Avval $\frac{1}{y}$ ifodani baholaymiz. $7<y<8$ bo'lgani uchun $\frac{1}{7}>\frac{1}{y}>\frac{1}{8}$, ya'ni $\frac{1}{8}<\frac{1}{y}<\frac{1}{7}$ bo'ladi. Tengsizliklarni hadma-had ko'paytiramiz.

$$\times\begin{array}{l} 10 < x < 11 \\ \frac{1}{8} < \frac{1}{y} < \frac{1}{7} \\ \hline 1\frac{1}{4} < \frac{x}{y} < 1\frac{4}{7} \end{array}$$

3-masala. To‘g‘ri to‘rtburchak shaklidagi xonaning a bo‘yini va b enini o‘lchab (sm bilan), $5{,}8<a<5{,}9$ va $4{,}2<b<4{,}3$ ekani topildi.

a) To‘g‘ri to‘rtburchakning perimetrini baholang;

b) To‘g‘ri to‘rtburchakning yuzini baholang.

Yechish. a) To‘g‘ri to‘rtburchakning perimetri $P=2a+2b$ formula bilan topiladi.

$2\cdot 5{,}8<2a<2\cdot 5{,}9$; $11{,}6<2a<11{,}8$;

$2\cdot 4{,}2<2b<2\cdot 4{,}3$; $8{,}4<2b<8{,}6$. Bu qo‘sh tengsizliklarni hadma-had qo‘shamiz.

$$\times\begin{array}{l} 11{,}6 < \quad 2a \quad < 11{,}8 \\ 8{,}4 < \quad 2b \quad < 8{,}6 \\ \hline 20 < 2a+2b < 20{,}4 \end{array};\ 20<p<20{,}4 \text{ sm.}$$

b) To‘g‘ri to‘rtburchakning yuzi $S=ab$ formula bilan topiladi. $5{,}8<a<5{,}9$ va $4{,}2<b<4{,}3$ qo‘sh tengsizliklarni hadma-had ko‘paytiramiz.

$$\times\begin{array}{l} 5{,}8 < a < 5{,}9 \\ 4{,}2 < b < 4{,}3 \\ \hline 24{,}36 < ab < 25{,}37 \end{array};\ 24{,}36<S<25{,}37 \text{ sm.}$$

TAKRORLASH UCHUN SAVOLLAR

1. Bir xil ishorali to‘g‘ri tengsizliklarni qo‘shish haqidagi teoremani ayting.
2. Bir xil ishorali to‘g‘ri tengsizliklarni (hadlari musbat bo‘lgan) ko‘paytirish haqidagi teoremani ayting.
3. Bir xil ishorali to‘g‘ri tengsizliklar (hadlari musbat bo‘lgan) qanday bo‘linadi?
4. $15<16$ va $4<5$ tengsizliklarni hadma-had:
   a) qo‘shing; b) ayiring; d) ko‘paytiring; e) bo‘ling.

MASALALARNI YECHING

356. Tengsizliklarni hadma-had qo'shing:

a) $15>12$ va $5>-4$;   b) $-3,4<-0,7$ va $-7,3<-1,7$.

357. Tengsizliklarni hadma-had ayiring.

a) $3,2<4,8$ va $-4<-1,5$;   b) $-\frac{2}{3}<\frac{1}{2}$ va $-1\frac{1}{5}<0,8$.

358. $12<a<15$ va $-4<b<-2$ bo'lsin:

a) $a+b$;   b) $a-b$ larni baholang.

359. $6\le x\le 6,5$ va $12\le y\le 12,5$ bo'lsa,

a) xy;   b) $\frac{y}{x}$ ifodalarni baholang.

360. $1,4<\sqrt{2}<1,5$ va $1,7<\sqrt{3}<1,8$ bo'lsa,

a) $\sqrt{2}+\sqrt{3}$;   b) $\sqrt{3}-\sqrt{2}$;   d) $\sqrt{6}$;   e) $\sqrt{1,5}$ ni toping.

361. Teng yonli uchburchakning asosi va yon tomonining millimetr bilan ifodalangan a va b uzunliklari ma'lum: $26\le a\le 28$ va $41\le b\le 43$. Shu uchburchakning perimetrini baholang.

362. To'g'ri to'rtburchak shaklidagi xonaning a uzunligi va b enining chegaralari (m bilan) Ma'lum: $7,5\le a\le 7,6$ va $5,4\le b\le 5,5$.

Yuzi kamida 40 m^2 bo'lishi kerak bo'lgan kutubxona uchun bu xona yaroqlimi?

363. Isbotlang: a) $9a+\frac{1}{a}\ge 6$, bunda $a>0$;   b) $25b+\frac{1}{b}\le -10$, bunda $b<0$.

48-§. Bir o'zgaruvchili tengsizliklar. Sonli oraliqlar

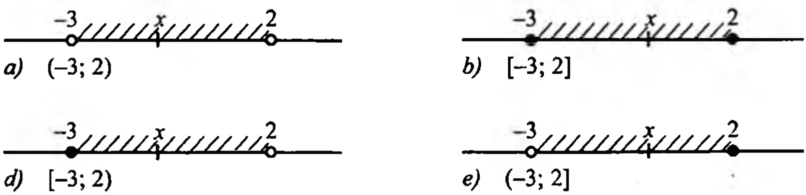
Tengsizlik o'zgaruvchi ba'zi qiymatlarida to'g'ri tengsizlikka aylanadi, ba'zi qiymatlarida esa to'g'ri tengsizlikka aylanmaydi. Masalan, $4x-15>2$ tengsizlik x o'zgaruvchining 5 ga teng qiymatida to'g'ri sonli tengsizlikka aylanadi, ya'ni $4\cdot 5-15>2$-yoki $5>2$. Agar x ning o'rniga 3 qo'ysak $4\cdot 3-15>2$-yoki $-3>2$ (noto'g'ri tengsizlik) hosil bo'ladi.

Bunday hollarda 5 soni $4x-15>2$ **tengsizlikning yechimi** yoki tengsizlikni qanoatlantiradi deyiladi.

Ta'rif. **Bir o'zgaruvchili tengsizlikning yechimi deb o'zgaruvchining tengsizlikni to'g'ri sonli tengsizlikka aylantiradigan qiymatiga aytiladi.**

$4x-15>2$ tengsizlikning boshqa bir qancha yechimlarini ko'rsatish mumkin, masalan: 8; 25; 5,2; 200; 2002 va hokazo. Ammo 2; 4,4; –10; –100 va hokazo sonlar tengsizlikning yechimi bo'lmaydi.

Agar x son $x>-3$ tengsizlikning ham, $x<2$ tengsizlikning ham yechimi bo'lsa, y $-3<x<2$ qo'sh tengsizlikni qanoatlantiradi. $-3<x<2$ qo'sh tengsizlikni qanoatlantiruvchi barcha sonlar to'plami sonlar oralig'i deyiladi va bunday (–3; 2) belgilanadi (6-*a*, chizma).



6-chizma.

$-3 \le x \le 2$ qo'sh tengsizlikni qanoatlantiruvchi x son nuqta bilan tasvirlanib, bu nuqta –3 va 2 sonlar, ular orasidagi barcha sonlarni o'z ichiga oladi va bunday [–3; 2] belgilanadi (6-*b*, chizma) $-3 \le x<2$ tengsizlik [–3; 2) kabi va $-3<x \le 2$ tengsizlik (–3; 2] kabi belgilanadi (6-*d, e,* chizma).

$x>6$ bo'lgan tengsizlikni qanoatlantiruvchi hamma x sonlar to'plami koordinatasi 6 bo'lgan nuqtadan o'ngda joylashgan yarim to'g'ri chiziq bilan tasvirlanadi (7-*a,* chizma), ya'ni (6; +∞) kabi. Bu oraliq 6 dan «plyus cheksizlikkacha» deb o'qiladi.

$x \ge 6$ tengsizlikni qanoatlantiruvchi oraliq [6; +∞) kabi belgilanadi (6 ham shu oraliqqa tegishli) (7-*b*, chizma).

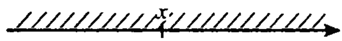
7-chizma.

$x<6$ tengsizlikning qanoatlantiruvchi oraliq $(-\infty;\ 6)$ (8-*a*, chizma) va $x\leq 6$ tengsizlikni qanoatlantiruvchi oraliq $(-\infty;\ 6]$ kabi (unga 6 ham tegishli) belgilanadi (8-*b*, chizma).

a) $(-\infty;\ 6)$ *b)* $(-\infty;\ 6]$

8-chizma.

Hamma haqiqiy sonlar to‘plami butun koordinata to‘g‘ri chizig‘i bilan tasvirlanadi. U bunday $(-\infty;\ +\infty)$ belgilanadi (9-chizma).



$(-\infty;\ +\infty)$

9-chizma.

Masalan, a) $[-10;\ 3)$; b) $(-4{,}5;\ 8{,}2]$ oraliqlarga tegishli eng kichik va eng katta butun sonni ayting.

Yechish. a) $[-10;\ 3)$ ga tegishli eng kichik butun son -10, shu oraliqqa tegishli eng katta butun son 2 bo‘ladi.

b) $(-4{,}5;\ 8{,}2]$ ga tegishli eng kichik butun son -4, shu oraliqqa tegishli eng katta butun son 8 bo‘ladi.

TAKRORLASH UCHUN SAVOLLAR

1. $4x>12$ tengsizlikning yechimi deb nimaga aytiladi?
2. Bir o‘zgaruvchili tengsizlikning yechimi deb nimaga aytiladi?
3. $-10<x<12$ tengsizlikni sonlar oralig‘i orqali qanday yoziladi va koordinata to‘g‘ri chizig‘ida qanday tasvirlanadi?
4. $1{,}5\leq x\leq 10$ tengsizlikni sonlar oralig‘i orqali qanday yoziladi va koordinata to‘g‘ri chizig‘ida qanday tasvirlanadi?
5. $x>8$ va $x\geq 8$ tengsizliklarni sonli oraliq ko‘rinishida yozing va koordinata to‘g‘ri chizig‘ida tasvirlang.
6. $x<4$ va $x\leq 4$ tengsizliklarni sonli oraliq ko‘rinishida yozing va koordinata to‘g‘ri chizig‘ida tasvirlang.
7. Koordinata to‘g‘ri chizig‘idagi sonlarni sonli oraliq yordamida yozing?

364. x ning: a) 4; b) 2; d) –5; e) 4,5 ga teng qiymati $10x>2(x+8)$ tengsizlikning yechimi bo'ladimi?

365. $2x<x+7$ tengsizlikning ixtiyoriy ikkita yechimini ko'rsating.

366. Oraliqlarni koordinata to'g'ri chizig'ida tasvirlang:

a) (3; 7); b) (4; 6]; d) (–8; 5,5]; e) (–∞; –7].

367. Quyidagi tengsizlikni qanoatlantiruvchi sonlar to'plamini koordinata to'g'ri chizig'ida tasvirlang:

a) $x\geq -4$; b) $x<8$; d) $x\leq -2$;
e) $-2<x<1,3$; f) $-5\leq x<3,5$; g) $-6\leq x\leq 7,5$.

368. Butun sonlardan qaysilari quyidagi oraliqqa tegishli bo'ladi?

a) [–2; 3); b) (–3; 4); d) (–5; 2,2]; e) [–4,5; 5,4].

369. Quyidagi oraliqqa tegishli bo'lgan eng katta butun sonni va eng kichik butun sonni ayting:

a) [0; 8); b) (–12; –3]; d) [–2,4; 5,2]; e) (– ∞; 5).

370. $\frac{a^4+1}{2}\geq a^2$ tengsizlikni isbotlang.

49-§. Bir o'zgaruvchili tengsizliklarni yechish

1-misol. To'g'ri to'rtburchak bir tomonining uzunligi 6 sm ga teng. To'g'ri to'rtburchakning perimetri 50 sm dan katta bo'lishi uchun uning ikkinchi tomonining uzunligi qanday bo'lishi kerak?

To'g'ri to'rtburchak ikkinchi tomonining uzunligi x sm ga teng bo'lsin. U holda to'g'ri to'rtburchakning perimetri $(2x+12)$ sm ga teng bo'ladi. Masalaning shartiga ko'ra, to'g'ri to'rtburchakning perimetri 50 sm dan katta bo'lishi kerak, ya'ni $2x+12>50$. Masala shartiga javob berish uchun $\mathbf{12x+12>50}$ tengsizlikni yechish kerak. Demak, **tengsizlikni yechish – uning hamma yechimlarini topish yoki yechimlari yo'qligini isbotlash demakdir.**

Ayni bir xil yechimga ega bo'lgan tengsizliklar **teng kuchli tengsizliklar** deyiladi. Yechimi bo'lmagan tengsizliklar ham teng kuchli hisoblanadi.

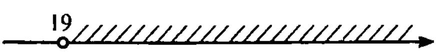
Tengsizliklarni yechishda quyidagi xossalardan foydalaniladi.

1) Agar tengsizlikning bir qismidan ikkinchisiga qo'shiluvchini qarama-qarshi ishora bilan o'tkazilsa, unga teng kuchli tengsizlik hosil bo'ladi.

2) Agar tengsizlikning ikkala qismi ayni bir musbat songa ko'paytirilsa, yoki bo'linsa, unga teng kuchli tengsizlik hosil bo'ladi;

3) Agar tengsizlikning ikkala qismi ayni bir manfiy songa ko'paytirilsa yoki bo'linsa va bunda tengsizlikning ishorasini qarama-qarshisiga o'zgartirilsa, unga teng kuchli tengsizlik hosil bo'ladi.

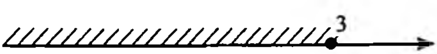
Yuqoridagi xossalarga asosan $2x+12>50$ tengsizlikdagi 12 qo'shiluvchini qarama-qarshi ishora bilan tengsizlikning chap qismidan o'ng qismiga o'tkazamiz. Bunda $2x>50-12$ tengsizlik hosil bo'ladi. Uni soddalashtirsak $2x>38$ hosil bo'ladi. 2-xossaga asosan $2x>38$ tengsizlikning ikkala qismini 2 ga bo'lamiz, ya'ni $x>19$ tengsizlikni hosil qilamiz. Demak, to'g'ri to'rtburchakning ikkinchi tomonining uzunligi 19 sm dan katta bo'lishi kerak. $x>19$ tengsizlikni koordinata o'qida tasvirlab (10-chizma):



$(19; +\infty)$ oraliqdagi javobni topamiz.

10-chizma.

2-misol. $8(2x-4)\leq 2x+10$ tengsizlikni yechamiz. Qavsni ochib, $16x-32\leq 2x+10$ ni hosil qilamiz. Bundagi $2x$ ni cham tomonga, -32 ni o'ng tomonga o'tkazib, ularni ixchamlaymiz. $16x-2x\leq 10+32$, $14x\leq 42$. Tengsizlikning ikkala qismini 14 ga bo'lib, $x\leq 3$ tengsizlikni hosil qilamiz. Natijada



$(-\infty; 3]$ oraliq topiladi.

11-chizma.

3-misol. $\frac{x}{2}+\frac{x}{5}<14$ tengsizlikni yechamiz. Tengsizlikdagi kasrlarning maxrajini yo'qotish uchun ikkala qismini 10 ga ko'paytiramiz, ya'ni $5x+2x<140$;

$7x<140$;

$x<20$; *Javob:* $(-\infty; 20)$.

4-misol. $2(x+8)-5x<4-3x$ tengsizlikni yechamiz.

$2x+16-5x<4-3x$;

$2x-5x+3x<4-16$

$0\cdot x<-12$ – bu tengsizlik yechimga ega emas, chunki x ning istalgan qiymatida uning chap qismi nolga teng, 0 soni -12 dan kichik bo'lishi mumkin emas.

Javob: tengsizlik yechimga ega emas.

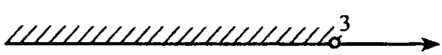
Ko'rib chiqilgan misollarda qavslar ochilib, kasr hadlarni, ifodalar ixchamlangandan keyin tengsizliklar $ax>b$ yoki $ax<b$ ko'rinishdagi teng kuchli tengsizliklar bilan almashtiriladi. Bunday ko'rinishdagi tengsizliklar **bir o'zgaruvchili chiziqli tengsizliklar** deyiladi.

Bu tengsizlikning ikkala qismini a ga bo'lib, $x>\frac{b}{a}$ yoki $x<\frac{b}{a}$ yechim topiladi.

5-misol. a ning qanday qiymatlarida $\frac{3a-1}{2}$ va $\frac{1+5a}{4}$ kasrlarning ayirmasi manfiy bo'ladi? Yechish $\frac{3a-1}{2}-\frac{1+5a}{4}<0$ tengsizlikni yechamiz. Tengsizlikning ikkala qismini 4 ga ko'paytiramiz. $2(3a-1)-(1+5a)<0$.

$6a-2-1-5a<0$.

$a<3$, ya'ni



Javob: $(-\infty; 3)$

TAKRORLASH UCHUN SAVOLLAR

1. Tengsizlikni yechish deganda nimani tushunamiz?
2. Tengsizlikning hadlari uning bir qismidan ikkinchi qismiga qanday o'tkaziladi?
3. Tengsizlikning ikkala qismi biror musbat songa ko'paytirilganda yoki bo'linganda ishorasi qanday o'zgaradi?

4. Tengsizlikning ikkala qismi biror manfiy songa ko'paytirilganda yoki bo'linganda ishorasi qanday o'zgaradi?
5. Tengsizliklarni og'zaki yeching:
   a) $x-4>0$; b) $x+7<0$; d) $2x>-9$; e) $3x-12>0$.

MASALALARNI YECHING

371. Tengsizlikni yeching:

a) $x+0,4\le 0$; b) $3x>-15$; d) $-5x<-35$; e) $x-\le -1$;

f) $12y<-6$; g) $27b\ge 12$; h) $0,5y>-4$; i) $\frac{1}{7}y<\frac{5}{14}$.

372. Tengsizlikni yeching va uning yechimlarining to'plamini koordinata to'g'ri chizig'ida tasvirlang:

a) $2x<17$; b) $5x\ge -3$; d) $-12x<-48$; e) $-2x<-7,5$;

f) $-15x<-27$; g) $\frac{1}{6}x<2$; h) $-\frac{1}{3}x<0$; i) $0,2x -0,6$.

373. $3x-2<6$ tengsizlikni yeching. 4; 2,8; $2\frac{4}{7}$ soni shu tengsizlikning yechimi bo'ladimi?

374. Tengsizlikni yeching:

a) $7x-2,4<0,4$; b) $2x-17\ge -27$; d) $2-3a\le 8$;

e) $17-x>10-6x$; f) $64-6y\ge 1-y$; g) $8+5y\le 21+6y$.

375. a ning qanday qiymatlarida $2a-1$ ifodaning qiymatlari $7-0,5a$ ifodaning qiymatlaridan kichik bo'ladi?

376. Tengsizlikni yeching:

a) $5(x-1)+7\le 1-3(x+2)$; d) $4(b-1,5)-1,2\ge 6b-1$;

b) $4(a+8)-7(a-1)<12$; e) $a+2<5(2a+8)+13(4-a)$.

377. Tengsizlikni yeching:

a) $4(2-3x)-(5-x)>11-x$;

b) $2(3-z)-3(2+z)\le z$;

d) $1>1,5(4-2a)+0,5(2-6a)$;

e) $3,2(a-6)-1,2a<3(a-8)$.

378. Tengsizlikni yeching va koordinata to'g'ri chizig'ida uning yechimlari to'plamini ko'rsating:

a) $a(a-4)-a^2>12-6a$;

b) $(2x-1)2x-5x<4x^2-x$;

d) $5y^2-5y(y+4)\leq 100$;

e) $6a(a-1)-2a(3a-2)<6$.

379. Tengsizlikni yeching:

a) $\frac{2x}{5}>1$; b) $\frac{x}{3}<2$; d) $\frac{6x}{7}\geq 0$; e) $\frac{5x-1}{4}>2$;

f) $\frac{3-2x}{2}\leq 1$; g) $\frac{12-7x}{42}\geq 0$; h) $\frac{2+3x}{18}<0$.

380. Tengsizlikni yeching:

a) $\frac{2a-1}{2}-\frac{3a-3}{5}>a$; d) $\frac{5x-1}{5}+\frac{x+1}{2}>a$;

b) $x-\frac{2x+3}{2}\leq\frac{x-1}{4}$; e) $\frac{y-1}{2}-\frac{2y+3}{8}-y>2$.

381. n ning qanday natural qiymatlarida:

a) $(2-2n)-(5n-27)$ ayirma musbat bo'ladi;

b) $(-27{,}1+3n)+(7{,}1+5n)$ yig'indi manfiy bo'ladi?

50-§. Bir o'zgaruvchili tengsizliklar sistemalarini yechish

1-masala. Agar turist o'z tezligini 1 km/soat orttirsa, u 4 soatda 20 km dan ortiq masofani bosib o'tadi. Agar tezligini 1 km/soat kamaytirsa, 5 soatda 20 km dan kam masofani bosib o'tadi. Turistning tezligini toping.

Yechish. Turistning tezligi x km/soat bo'lsin. Agar turist $(x+1)$ km/soat tezlik bilan yursa, 4 soatda $4(x+1)$ km masofani bosib o'tadi. Masalaning shartiga ko'ra $4(x+1)>20$ tengsizlikni hosil qilamiz.

Agar turist $(x-1)$ km/soat tezlik bilan yursa, 5 soatda $5(x-1)$ km masofani bosib o'tadi. Masalaning shartiga ko'ra $5(x-1)<20$ tengsizlikni hosil qilamiz.

Masalani yechishda x ning ham $4(x+1)>20$ tengsizlik, ham $5(x-1)<20$ tengsizlik to'g'ri bo'ladigan qiymatlarini topish talab qilinadi. Buning uchun $4(x+1)>20$ va $5(x-1)<20$ tengsizliklardan tuzilgan tengsizliklar sistemasi yechish talab qilinadi.

U tengsizliklar sistemasi $\begin{cases} 4(x+1)>20 \\ 5(x-1)<20 \end{cases}$ ko'rinishda yoziladi.

Bu sistemadagi tengsizliklarni soddalashtirib unga teng kuchli bo'lgan $\begin{cases} x>4 \\ x<5 \end{cases}$ sistemani hosil qilamiz.

Demak, x ning qiymati $4<x<5$ shartni qanoatlantirishi kerak.

Javob: turistning tezligi 4 km/soatdan ortiq, lekin 5 km/soat-dan kam.

Ta'rif. **Bir o'zgaruvchili tengsizliklar sistemasining yechimi deb, o'zgaruvchining sistemadagi har bir tengsizlikni to'g'ri tengsizlikka aylantiradigan qiymatiga aytiladi.**

Sistemani yechish – uning hamma yechimlarini topish yoki uning yechimi yo'qligini isbotlash demakdir.

2-misol. Tengsizliklar sistemasini yeching.

$$\begin{cases} 2x-1>6 \\ 5-3x>-13 \end{cases}$$

Yechish. Bu sistemani ixchamlab,

$\begin{cases} 2x>7 \\ -3x>-18 \end{cases}$ ga ega bo'lamiz. Bundan:

$\begin{cases} x>3,5 \\ x<6 \end{cases}$ bu yechimni koordinata to'g'ri chizig'ida tasvirlab, sistemaning yechimlari x ning $3,5<x<6$ qo'sh tengsizlikni qanoatlantiruvchi qiymatlari ekanligini ko'ramiz (12-chizma).

Javob: Sistemaning yechimlari to'plami (3,5; 6) oraliq.

12-chizma.

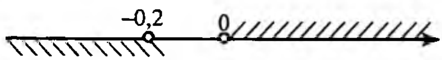
3-misol. $\begin{cases} 3x-2>25 \\ 1-x<0 \end{cases}$ tengsizliklar sistemasini yeching.

Yechish. $\begin{cases} 3x>27 \\ -x<-1 \end{cases}$; $\begin{cases} x>9 \\ x>1 \end{cases}$

Yechim (9; +∞).

4-misol. $\begin{cases} 1-12y<3y+1 \\ 2-6y>4+4y \end{cases}$ tengsizliklar sistemasini yeching.

Yechish. $\begin{cases} -12y-3y<1-1 \\ -6y-4y>4-2 \end{cases}$; $\begin{cases} -15y<0 \\ -10y>2 \end{cases}$; $\begin{cases} y>0 \\ y<-0,2 \end{cases}$.

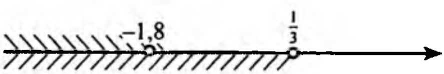


13-chizma.

Koordinata to‘g‘ri chizig‘ida $\begin{cases} y>0 \\ y<-0,2 \end{cases}$ tengsizliklar sistemasini qanoatlantiruvchi sonlar to‘plami mavjud emas, ya’ni sistema yechimga ega emas (13-chizma).

5-misol. $\begin{cases} x(x-1)-(x^2-10)<1-6x \\ 3,5-(x-1,5)<6-4x \end{cases}$ tengsizliklar sistemasini yeching.

Yechish. $\begin{cases} x^2-x-x^2+10<1-6x \\ 3,5-x+1,5<6-4x \end{cases}$; $\begin{cases} -x+6x<1-10 \\ -x+4x<6-5 \end{cases}$; $\begin{cases} 5x<-9 \\ 3x<1 \end{cases}$; $\begin{cases} x<-1,8 \\ x<\frac{1}{3} \end{cases}$.

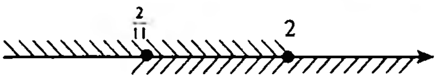


Javob: (–∞; –1,8).

6-misol. $\begin{cases} \frac{5a+8}{3}-a\geq 2a \\ 1-\frac{6-15a}{4}\geq a \end{cases}$ tengsizliklar sistemasini yeching.

Yechish. Birinchi tengsizlikni 3 ga, ikkinchi tengsizlikni 4 ga ko‘paytiramiz.

$$\begin{cases} 5a+8-3a \geq 6a \\ 4-6+15a \geq 4a \end{cases}; \quad \begin{cases} 5a-3a-6a \geq -8 \\ 15a-4a \geq 6-4 \end{cases}; \quad \begin{cases} -4a \geq -8 \\ 11a \geq 2 \end{cases}; \quad \begin{cases} a \leq 2 \\ a \geq \frac{2}{11} \end{cases};$$



Javob: $\left[\frac{2}{11}; 2\right]$.

TAKRORLASH UCHUN SAVOLLAR

1. Tengsizliklar sistemasini yechish deganda biz nimani tushunamiz?
2. Tengsizliklar sistemasining yechimi yo‘q deganda biz nimani tushunamiz?
3. Quyidagi tengsizliklar sistemasini og‘zaki yeching:

a) $\begin{cases} x>3 \\ x>8 \end{cases}$; b) $\begin{cases} x<4 \\ x>-3 \end{cases}$; d) $\begin{cases} x>-2 \\ x<15 \end{cases}$; e) $\begin{cases} x \geq 4 \\ -x \geq 2 \end{cases}$.

MASALALARNI YECHING

382. –2; 0; 5; 10 sonlardan qaysi biri?

$$\begin{cases} 3x-22>0 \\ 2x-1>3 \end{cases}$$

383. Tengsizliklar sistemasini yeching:

a) $\begin{cases} x>17 \\ x>12 \end{cases}$; b) $\begin{cases} x<1 \\ x<5 \end{cases}$; d) $\begin{cases} x>-2 \\ x<5 \end{cases}$; e) $\begin{cases} x \geq 8 \\ x<-3 \end{cases}$; f) $\begin{cases} x>8 \\ x \leq 20 \end{cases}$.

384. Tengsizliklar sistemasini yeching:

a) $\begin{cases} 2x-12>0 \\ 3x>9 \end{cases}$; b) $\begin{cases} 4y<-4 \\ 5-y>0 \end{cases}$; d) $\begin{cases} 3x-10<0 \\ 2x>0 \end{cases}$; e) $\begin{cases} 6y \geq 12 \\ 4y+12 \leq 0 \end{cases}$.

385. Tengsizliklar sistemasini yeching va uning yechimlari bo‘lgan uchta sonni ko‘rsating:

a) $\begin{cases} x-0{,}8>0 \\ 5x-1>9 \end{cases}$; d) $\begin{cases} 5x+6 \leq x \\ 3x+12 \leq x+17 \end{cases}$;

b) $\begin{cases} 10x > 2x + 16 \\ 2 + x \geq 7 \end{cases}$; e) $\begin{cases} 2x - 1 < 1,4 - x \\ 3x - 2 > x - 4 \end{cases}$.

386. Tengsizliklar sistemasini yeching:

a) $\begin{cases} 57 - 7x > 3x - 2 \\ 22x - 1 < 2x + 47 \end{cases}$ d) $\begin{cases} 102 - 73y > 2y + 2 \\ 81 + 11y \geq 1 + y \end{cases}$;

b) $\begin{cases} 1 - 12y < 3y + 1 \\ 2 - 6y > 4 + 4y \end{cases}$; e) $\begin{cases} 6 + 5,2x \geq 12 - 1,8x \\ 2 - x \geq 3,5 - 2x \end{cases}$.

387. Tengsizliklar sistemasini yeching:

a) $\begin{cases} 2y - (y - 4) < 6 \\ y > 3(2y - 1) + 18 \end{cases}$; d) $\begin{cases} 3,3 - 3(1,2 - 5x) > 0,6(10x + 1) \\ 1,6 - 4,5(4x - 1) < 2x + 26,1 \end{cases}$;

b) $\begin{cases} 3(2 - 3p) - 2(3 - 2p) > p \\ 6 < p^2 - p(p - 8) \end{cases}$; e) $\begin{cases} 5,8(1 - a) - 1,8(6 - a) < 5 \\ 8 - 4(2 - 5a) > -(5a + 6) \end{cases}$.

388. Tengsizliklar sistemasini yeching:

a) $\begin{cases} \frac{3x-1}{2} - x \leq 2 \\ 2x - \frac{x}{3} \geq 1 \end{cases}$; d) $\begin{cases} \frac{x-1}{2} - \frac{x-3}{3} < 2 \\ \frac{13x-1}{2} > 0 \end{cases}$;

b) $\begin{cases} 2y - \frac{y-2}{5} > 4 \\ \frac{y}{2} - \frac{y}{8} \leq 6 \end{cases}$; e) $\begin{cases} 4 - \frac{y-1}{3} \geq y \\ \frac{7y-1}{8} \geq 6 \end{cases}$.

389. Tengsizliklar sistemasini yeching:

a) $\begin{cases} x > 6 \\ x > 10 \\ x < 21 \end{cases}$; b) $\begin{cases} 2x - 1 < 3 \\ 5x - 4 > 0 \\ x + 0,5 > 0 \end{cases}$; d) $\begin{cases} 2x - 1 < x + 3 \\ 5x - 1 > 6 - 2x \\ x - 3 < 0 \end{cases}$.

51-§. Butun manfiy ko'rsatkichli darajaning ta'rifi

Ba'zi adabiyotlarda Quyoshning massasi $1{,}985 \cdot 10^{33}$ g ga, vodorod atomining massasi $1{,}674 \cdot 10^{-24}$ g ga teng ekanligi keltirilgan. Bulardagi 10^{33}-yozuvni har biri 10 ga teng bo'lgan 33 ta ko'paytuvchilarning ko'paytmasini bildiradi. 10^{-24}-yozuvning ma'nosi qanday?

10 sonining 0, 1, 2, 3, ... va hokazo ko'rsatkichli darajalarining ketma-ketligini yozamiz:

$$10^0, 10^1, 10^2, 10^3, \ldots \qquad (1)$$

Bu satrdagi har bir son undan keyingi sondan 10 marta kichik. (1) satrni shu qonun bo'yicha chapga davom ettirib, 10^0 sonidan oldin $\frac{1}{10}=\frac{1}{10^1}$ ni, $\frac{1}{10^1}$ sonidan oldin $\frac{1}{100}=\frac{1}{10^2}$ ni, $\frac{1}{10^2}$ sonidan oldin $\frac{1}{10^3}$ sonini yozish kerak va hokazo. Natijada ..., $\frac{1}{10^3}$, $\frac{1}{10^2}$, $\frac{1}{10^1}$, 10^0, 10^1, 10^2, 10^3, ... (2) hosil bo'ladi. (2) satrda 10^0 sonidan o'ngda har bir darajaning ko'rsatkichi undan keyingi darajaning ko'rsatkichidan 1 ta kamayib boradi. Bu qonunni 10^0 sonidan chapdagi sonlarga qo'llab, bu sonlarni 10 sonining manfiy ko'rsatkichli darajasi ko'rinishida yoziladi. $\frac{1}{10^1}$ o'rniga 10^{-1}; $\frac{1}{10^2}$ o'rniga 10^{-2}, $\frac{1}{10^3}$ o'rniga 10^{-3}-yoziladi va hokazo.

Natijada quyidagicha ketma-ketlik hosil bo'ladi:

$$\ldots, 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2, 10^3, \ldots$$

Shunday qilib, $10^{-1}=\frac{1}{10^1}$ ga, $10^{-2}=\frac{1}{10^2}$ ga $10^{-3}=\frac{1}{10^3}$ ga va hokazo tenglik hosil bo'ladi.

Bunday kelishuv noldan farqli istalgan asosli darajalar uchun to'g'ri bo'ladi.

Ta'rif. **Agar $a \neq 0$ va n–butun manfiy son bo'lsa, u holda $a^n = \frac{1}{a^{-n}}$ bo'ladi.**

Masalan: 1) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$; 2) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$; 3) $\left(-\frac{2}{3}\right)^{-3} = \frac{1}{\left(-\frac{2}{3}\right)^3} = \frac{1}{-\frac{8}{27}} = -\frac{27}{8} = -3\frac{3}{8}$.

n butun manfiy son bo'lganda 0^n ifoda ma'noga ega bo'lmaydi. $\frac{a^2}{5b^3}$ ifoda $\frac{a^2}{5b^3} = 5^{-1}a^2b^{-3}$ butun ifodaga aylanadi.

a^3 bilan $\frac{1}{a^3}$ o'zaro teskari bo'lgani uchun ta'rifga asosan a^3 bilan a^{-3} ifodalar o'zaro teskari bo'ladi, ya'ni $a^3 \cdot a^{-3} = a^3 \cdot \frac{1}{a^3} = 1$. Endi vodorod atomi massasini ifodalovchi $1{,}674 \cdot 10^{-24} = 1{,}674 \times \times \frac{1}{10^{24}} = \frac{1{,}674}{\underbrace{1000...0}_{24 \text{ ta nol}}} = 0{,}\underbrace{000...01674}_{27 \text{ ta raqam}}$.

TAKRORLASH UCHUN SAVOLLAR

1. ..., 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2, 10^3, ... qatordagi sonning darajasi o'zidan oldin yoki keyin turgan sonning darajasida qancha ortiq yoki kam bo'ladi?
2. 10^{-4}; 5^{-3} sonlar qanday sonni anglatadi?
3. Butun manfiy ko'rsatkichli darajaning ta'rifini ayting.
4. Hisoblang: a) 5^{-2}; b) 3^{-4}; d) 2^{-6}.

MASALALARNI YECHING

390. Ifodani kasrga almashtiring:

a) 10^{-4}; b) 9^{-2}; d) a^{-1}; e) x^{-12}; f) $3y^{-5}$; g) $-0{,}3\ s^{-6}$.

391. a) 8; 4; 2; 1, $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$ sonlarni 2 asosli daraja ko'rinishida yozing;

b) $\frac{1}{125}$; $\frac{1}{25}$; $\frac{1}{5}$; 1; 5; 25; 125 sonlarni 5 asosli daraja ko'rinishida yozing.

392. Hisoblang:

a) 4^{-3}; d) $\left(\frac{1}{7}\right)^{-2}$; f) -10^{-4}; h) $(-1)^{-9}$;

b) $(-3)^{-4}$; e) $(0,3)^{-3}$; g) $-(-2)^{-3}$; i) $(-1)^{-20}$.

393. Ifodaning qiymatini toping:

a) $8\cdot 4^{-3}$; e) $10\cdot\left(\frac{1}{5}\right)^{-2}$; h) $0,5^{-2}+\left(\frac{1}{3}\right)^{-1}$;

b) $-2\cdot 10^{-5}$; f) $3^{-2}+4^{-1}$; i) $0,3^{0}+0,1^{-4}$;

d) $18\cdot(-9)^{-1}$; g) $2^{-3}-(-2)^{-4}$; j) $(-2,1)^{0}-(-0,02)^{-3}$.

394. Kasrni butun ifoda ko‘rinishida yozing:

a) $\frac{3}{b^2}$; d) $\frac{2a^8}{c^5}$; f) $\frac{1}{x^2y^3}$; h) $\frac{2a}{(a-2)^2}$;

b) $\frac{x}{y}$; e) $\frac{a^5}{7b^3}$; g) $\frac{d}{b^4c^4}$; i) $\frac{(a+b)^2}{2(a-b)^4}$.

395. Ifodani kasr ko‘rinishida tasvirlang:

a) $a^{-2}+b^{-2}$; d) $(a+b^{-1})(a^{-1}-b)$; f) $(a^{-1}+b^{-1})(a+b)^{-1}$;

b) $xy^{-1}+xy^{-2}$; e) $(x-2y^{-1})(x^{-1}+2y)$; g) $(a-b)-2(a^{-2}-b^{2})$.

52-§. Butun ko‘rsatkichli darajaning xossalari

Natural ko‘rsatkichli darajalar uchun aniqlangan xossalar istalgan butun ko‘rsatkichli darajalar uchun ham to‘g‘ridir.

Ya’ni: istalgan $a\neq 0$ va istalgan butun m va n sonlar uchun:

1. $a^m\cdot a^n=a^{m+n}$;
2. $a^m : a^n=a^{m-n}$;
3. $(a^m)^n=a^{mn}$;

Istalgan $a\neq 0$, $b\neq 0$ va istalgan n butun son uchun:

4. $(ab)^n=a^nb^n$;
5. $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$.

Bu xossalarni butun manfiy ko‘rsatkichli darajaning ta’rifiga va natural ko‘rsatkichli darajaning xossalariga tayangan holda isbotlash mumkin.

Masalan, $a^m\cdot a^n=a^{m+n}$ xossaning to‘g‘riligini darajalarning ko‘rsatkichlari manfiy sonlar bo‘lgan hol uchun isbotlaymiz. Ya’ni,

$a^{-m} \cdot a^{-n} = a^{-m-n}$ xossani isbotlaymiz, bunda $a \neq 0$, m va n – natural sonlar,

$$a^{-m} \cdot a^{-n} = \frac{1}{a^m} \cdot \frac{1}{a^n} = \frac{1}{a^m \cdot a^n} = \frac{1}{a^{m+n}} = a^{-m(+n)} = a^{-m-n}.$$

Demak, $a^{-m} \cdot a^{-n} = a^{-m-n}$. Bunda biz butun manfiy ko'rsatkichli darajaning ta'rifidan va natural ko'rsatkichli darajaning ko'paytmasi xossasidan foydalandik.

1-misol. $a^{-17} \cdot a^{21}$ ko'paytmani topamiz: $a^{-17} \cdot a^{21} = a^{-17+21} = a^4$.

Shunday qilib, **bir xil asosli darajalarni ko'paytirishda asosi o'zgarishsiz qoldiriladi, daraja ko'rsatkichlari esa qo'shiladi.**

Butun ko'rsatkichli darajaning qolgan barcha xossalari ham yuqoridagi kabi, butun manfiy ko'rsatkichli, darajaning ta'rifidan va natural ko'rsatkichli darajaning xossalaridan foydalanib isbotlanadi.

2-misol. $b^3 : b^5$ bo'linmani topamiz.

Bir xil asosli darajalarni bo'lishda asosi o'zgarishsiz qoldiriladi, bo'linuvchining daraja ko'rsatkichidan bo'luvchining daraja ko'rsatkichi ayriladi. Demak, $b^3 : b^5 = b^{3-5} = b^{-2}$.

3-misol. $(2a^{-3}b^{-5})^{-2}$ ifodani soddalashtiramiz. Avvalo, ko'paytma darajaga ko'paytiriladi, so'ngra darajalarni darajaga ko'tariladi nihoyat, ko'paytuvchilar o'zaro ko'paytiriladi.

Demak, $(2a^{-3}b^{-5})^{-2} = 2^{-2} \cdot (a^{-3})^{-2} \cdot (b^{-5})^{-2} = \frac{1}{4} a^6 b^{10}$.

TAKRORLASH UCHUN SAVOLLAR

1. Natural ko'rsatkichli darajaning xossalarini yozing.
2. Butun manfiy ko'rsatkichli darajaning xossalarini nimalarga asoslanib isbotlanadi?
3. Og'zaki hisoblang:
   a) $2^{-2} \cdot 2^{-3}$; b) $3^{-3} \cdot 3^{-1}$; d) $4^{-3} \cdot 4^2$; e) $4^{-8} \cdot 4^{-6}$.

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396. Ifodaning qiymatini toping:

a) $3^{-4} \cdot 3^6$; e) $2^{10} : 2^{12}$; h) $(2^{-4})^{-1}$;

b) $2^4 \cdot 2^{-3}$; f) $5^{-3} : 5^{-5}$; i) $(5^2)^{-2} \cdot 5^3$;

d) $10^8 \cdot 10^{-5} \cdot 10^{-6}$; g) $3^{-4} : 3^{-8}$; j) $3^{-4} \cdot (3^{-2})^{-4}$.

397. Ifodani 5 asosli daraja ko'rinishida yozing, unda m – butun son:

a) $5^m \cdot 5^{m+1} \cdot 5^{1-m}$; b) $(5^m)^2 \cdot (5^{-3})^m$; d) $625 : 5^{4m-2}$.

398. Hisoblang:

a) $8^{-2} \cdot 4^3$; e) $125^{-4} : 25^{-5}$; h) $\frac{4^{-2} \cdot 8^{-6}}{2^{-22}}$;

b) $9^{-6} \cdot 27^5$; f) $16^{-3} \cdot 8^4$; i) $\frac{3^{-10} \cdot 9^8}{(-3)^2}$;

d) $10^0 : 10^{-3}$; g) $\frac{2^{-21}}{4^{-5} \cdot 4^{-6}}$; j) $\frac{5^{-5} \cdot 25^{10}}{125^3}$.

399. x asosan daraja ko'rinishida yozing (bunda n – butun son):

a) $x^{10} : x^{12}$; d) $x^{n-1} : x^{-8}$;

b) $x^0 : x^{-5}$; e) $x^6 : x^{n+2}$.

400. Ifodani soddalashtiring:

a) $1{,}5ab^{-3} \cdot 6a^{-2}b$; e) $3{,}2x^{-1}y^{-5} \cdot \frac{5}{8}xy$;

b) $\frac{3}{4}m^{-2}n^4 \cdot 8m^3n^{-2}$; f) $\frac{1}{2}p^{-1}q^{-3} \cdot \frac{1}{6}p^2q^{-5}$;

d) $0{,}6c^2d^4 \cdot \frac{1}{3}c^{-1}d^{-4}$; g) $3\frac{1}{3}a^5b^{-18} \cdot 0{,}6a^{-1}b^{20}$.

401. Ifodaning qiymatini toping:

a) $0{,}2a^{-2}b^4 \cdot 5a^3b^{-3}$, bunda $a=-0{,}125$; $b=8$;

b) $\frac{1}{27}a^{-1}b^{-5} \cdot 81a^2b^4$, bunda $a=\frac{1}{7}$; $b=\frac{1}{14}$.

402. Ifodani soddalashtiring:

a) $\frac{12x^{-5}}{y^{-6}} \cdot \frac{y}{36x^{-9}}$; d) $\frac{5x^{-1}y^3}{3} \cdot \frac{9x^6}{y^{-2}}$;

b) $\frac{63a^2}{2b^{-5}} \cdot \frac{18b^2}{7a}$; e) $\frac{16p^{-1}q^2}{5} \cdot \frac{25p^6}{64q^{-8}}$.

Namuna. $\left(\frac{2^{-1}x^{-2}}{3y^{-2}\cdot z^{3}}\right)^{-3}:\left(\frac{x^{2}y^{3}}{9z^{5}}\right)^{-2}$ ifodani soddalashtiramiz.

1) $\left(\frac{2^{-1}x^{-2}}{3y^{-2}z^{3}}\right)^{-3}=\frac{2^{3}x^{6}}{3^{-3}y^{6}z^{-9}}=\frac{8x^{6}}{\frac{1}{27}y^{6}z^{-9}}=216x^{6}y^{-6}z^{9};$

2) $\left(\frac{x^{2}y^{3}}{9z^{5}}\right)^{-2}=\frac{x^{-4}y^{-6}}{9^{-2}z^{-10}}=81x^{-4}y^{-6}z^{10};$

3) $216x^{6}y^{-6}z^{9}:81x^{-4}y^{-6}z^{10}=2\frac{2}{3}x^{10}z^{-1}.$

403. Ifodani soddalashtiring:

a) $\left(0,25x^{-4}y^{-3}\right)^{2}\cdot\left(\frac{x^{-3}}{4y^{2}}\right)^{-3};$

b) $\left(\frac{a^{-3}b^{4}}{9}\right)^{-2}\cdot\left(\frac{3}{a^{-2}b^{3}}\right)^{-3};$

d) $\left(\frac{c^{-4}}{10a^{5}b^{2}}\right)^{-2}\cdot(5a^{3}bc^{2})^{-2};$

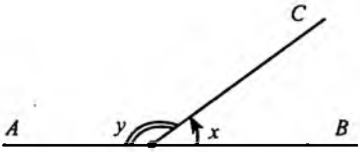
e) $\left(\frac{x^{2}y^{-3}}{6z^{4}}\right)^{-3}\cdot\left(\frac{x^{2}y^{-2}}{9z^{5}}\right)^{2}.$

53-§. O'zgaruvchi va o'zgarmas miqdorlar. Funksiyaning ta'rifi

O'zgaruvchi va o'zgarmas miqdorlarga doir bir nechta misollarni ko'rib chiqamiz.

1-misol. Biz biror koptokga dam bersak, uning diametri, sirti va hajmi kattalashib boradi, ammo koptok vazni o'zgarmaydi.

2-misol. Agar poyezdning bir bekatdan ikkinchi bekatgacha bo'lgan harakatida poyezdning bekatlar orasidagi masofalari, yoqilg'isi, yurgan vaqti o'zgarib boradi, ammo vagonlar soni, undagi g'ildiraklar soni va boshqalar soni o'zgarmaydi.



14-chizma.

3-misol. Ikki qo'shni burchakning umumiy tomoni (OC) o'ngdan chapga qarab harakatlansa, x va y qo'shni burchaklar o'zgarib boradi. Ularning yig'indisi $x+y=180°$ bo'lib, o'zgarmay qoladi (14-chizma).

Bu misollardan ma'lum bo'ldiki, turmush va texnikada uchraydigan turli hodisalarda ayrim miqdorlar doim o'z qiymatini saqlashlari; ayrimlari esa o'zgarib turishlarini ko'ramiz.

1-ta'rif. ***Berilgan masala shartida faqat birgina son qiymatiga ega bo'lgan miqdorlar o'zgarmas miqdorlar deyiladi.***

2-ta'rif. ***Berilgan masala shartida har xil qiymatlarga ega bo'la oladigan miqdorlar o'zgaruvchi miqdorlar deyiladi.***

Yuqoridagi misollarda uchraydigan o'zgaruvchi miqdorlar orasida ma'lum bog'lanish mavjud.

1-misoldagi koptokga dam berishni x bilan, ikkinchi o'zgaruvchi koptok sirtini y bilan belgilasak, unda x ning ortishi bilan y ham ortishini bilamiz.

Uchinchi misoldagi qo'shni burchaklardan biri x ortganda, ikkinchi burchak y kamayib boradi. x va y miqdorlar orasidagi bog'lanishni $x+y=180°$ yoki $y=180°-x$ ko'rinishda yozamiz. Bu bog'lanishdan x miqdorning har bir qiymatiga y miqdorning ma'lum bir qiymati mos keladi.

3-ta'rif. ***O'zgaruvchi ikki miqdor birining har bir qiymatiga ikkinchisining ma'lum bir qiymati mos kelsa, bunday o'zgaruvchi miqdorlar orasida funksional bog'lanish bor deyiladi.***

Masalan: 1) olingan mahsulot miqdori (x) bilan unga to'langan pul (y) miqdori. Bu o'zgaruvchilar $y=kx$ (k – o'zgarmas miqdor) funksional bog'lanishda bo'ladi. Masalan, $y=8x$, $y=-\frac{3}{5}x$.

4-ta'rif. ***O'zgaruvchi x miqdor qabul qilishi mumkin bo'lgan har bir qiymatiga u miqdorning aniq bir qiymati mos kelsa, u miqdor x miqdorning funksiyasi deyiladi.*** Funksiyalar y, $y(x)$, $f(x)$, $g(x)$, $h(x)$ va hokazo kabi belgilanadi. Bundagi x – ***yerkli o'zgaruvchi*** yoki ***argument*** deyiladi. Funksiyalar ko'pincha $y=kx$; $y=180-x$ va hokazo formula shaklida beriladi.

Ikkita x va y sonlarning ko'paytmasi biror k songa teng bo'lsa, o'zgaruvchilar orasidagi funksional bog'lanish $x \cdot y=k$ formula bilan beriladi. Bundan $y=\frac{k}{x}$ tenglikni yozish mumkin. Bu funksiyadagi x ning qabul qilishi mumkin qiymatlar to'plami $x \neq 0$ bo'lgan barcha sonlardan iborat. Shuningdek, ba'zi funksiyalarda x ning (argumentning) qiymatlari cheklangan bo'ladi.

Masalan, $y=\sqrt{x-3}$ funksiyada $x \geq 3$ bo'lishi kerak. $x<3$ da funksiya ma'noga ega emas, chunki manfiy sonlarning kvadrat ildizi mavjud emas.

Argumentning qabul qilishi mumkin bo'lgan barcha qiymatlarini funksiyaning aniqlanish sohasi deb ataladi. $y=\frac{k}{x}$ ning aniqlash sohasi $x \neq 0$, $y=\sqrt{x-3}$ ning aniqlanish sohasi $x \geq 3$ dan iborat.

Funksiyaning aniqlanish sohasi bo'yicha topilgan qiymati funksiyaning qiymatlari to'plami deyiladi. $y=\frac{k}{x}$ (k – o'zgarmas son) funksiyaning qiymatlar to'plami $y \neq 0$ barcha sonlar.

$y=\sqrt{x-3}$ funksiyaning qiymatlar to'plami barcha haqiqiy sonlar to'plamidan iborat.

Berilgan funksiyaning aniqlanish sohasini topishda quyidagilarga amal qilish lozim:

1. Funksiya ratsional ifoda shaklda berilgan bo'lsa, uning maxraji nolda farqli bo'lishi, kerak.

Masalan, $y=\frac{3x}{x^2-4}$ ni aniqlanish sohasini toping.

Yechish. Kasrning maxrajini nolga aylantirmaydigan x ning qiymatlarini topamiz. Buning uchun $x^2-4\neq 0$; $(x-2)(x+2)=0$. Bu tenglik $x_1=2$ va $x_2=-2$ larda nolga aylanadi.

Demak, $y=\frac{3x}{x^2-4}$ kasrning aniqlanish sohasi $x\neq\pm 2$.

2. Funksiya juft ko'rsatkichli ildiz ostida berilgan bo'lsa, ildiz ostidagi ifoda manfiy bo'lmasligi kerak, ya'ni $y=\sqrt[2n]{f(x)}$ bo'lib, bunda n – natural son, $f(x)\geq 0$ bo'lishi kerak.

Masalan, $y=\sqrt{\frac{84}{3-5x}}$ funksiyaning aniqlanish sohasini toping.

Yechish. 1) $3-5x\neq 0$ bo'lishi kerak, ya'ni $x\neq 0{,}6$.

2) $\frac{84}{3-5x}>0$ bo'lishi kerak, bunda $84>0$ dan $3-5x>0$ bo'lishi kerak yoki $x<0{,}6$. Bundagi $x\neq 0{,}6$ va $x<0{,}6$ shartlardan $x<0{,}6$.

Javob: Aniqlanish sohasi $x<0{,}6$.

TAKRORLASH UCHUN SAVOLLAR

1. O'zgaruvchi va o'zgarmas miqdorlarga misollar keltiring.
2. Masala shartidagi qanday miqdorlarni o'zgarmas miqdorlar va o'zgaruvchi miqdorlar deyiladi?
3. O'zgaruvchi ikki miqdorlar orasidagi bog'lanishni qanday bog'lanish deyiladi?
4. u miqdor qanday holda x miqdorning funksiyasi deyiladi?
5. Funksiyalar qanday belgilanadi?
6. Funksiyaning aniqlanish sohasi deb nimaga aytiladi? Qiymatlar sohasi deb nimaga aytiladi?

MASALALARNI YECHING

Funksiyaning aniqlanish sohasini toping.

404. a) $y=2x$; b) $y=-2x+7$; d) $y=\frac{3}{x}$; e) $y=\sqrt{3x}$.

405. a) $y=\frac{8x-13}{35}$; b) $y=\frac{18}{x+1}$; d) $y=\frac{60x}{4x-5}$; e) $y=\frac{-13x}{0,2x-7}$.

406. a) $y=\sqrt{5x}$; b) $y=\sqrt{x-7}$; d) $y=\sqrt{\frac{15}{x+4}}$; e) $y=\sqrt{\frac{-8}{10-2x}}$.

407. a) $y=\frac{x+5}{x(2x-8)}$; d) $y=\sqrt{x}+\sqrt{3x+9}$;

b) $y=\frac{1}{5x}+\sqrt{10x}$; e) $y=\sqrt{\frac{6x}{2x-4}}$.

408. To‘g‘ri burchakli uchburchakning bir o‘tkir burchagi y ni ikkinchi o‘tkir burchagi x ning funksiyasi orqali ifoda qiling va bu funksiyaning aniqlanish sohasini toping.

409. Teng yonli uchburchakning uchidagi burchagi y ni uning asosiga yopishgan burchagi x ning funksiyasi orqali ifoda qiling va bu funksiyaning aniqlanish sohasini toping.

54-§. Funksiyalarning berilish usullari

O‘zgaruvchi ikki miqdor orasidagi funksional bog‘lanishni o‘rganishda biz ulardan birining son qiymatlarining o‘zgarishi bilan ikkinchisi qanday son qiymatlariga ega bo‘lishini aniqlash usullarini o‘rganamiz.

Bu moslik turli usullar bilan aniqlanishi mumkin.

1. Jadval usuli

Ikki o‘zgaruvchining qiymatlari orasidagi moslik jadval yordamida berilgan bo‘lsa, bu funksional moslikni funksiyani jadval usulda berilishi deyiladi.

Quyidagi jadvalda 1-o‘zgaruvchi x (ming so‘m) – konfet narxi, 2 – o‘zgaruvchi y (kg) – konfet kilogrami berilgan. Berilgan 100 000 so‘m pulga olingan konfetni jadvalda ko‘rsatamiz.

x (ming so'm)	2	4	5	8	10	16	20	25	40	50
y (kg)	50	25	20	12,5	10	6,25	5	4	2,5	2

Bu jadvalda ko'rsatilgan x (so'm) ning har bir qiymatiga y (kg) ning aniq bir qiymati mos keladi. Bu x va y o'zgaruvchilar orasidagi moslik jadval yordamida berilgan bo'lib, bu moslikni funksiya deb ataymiz.

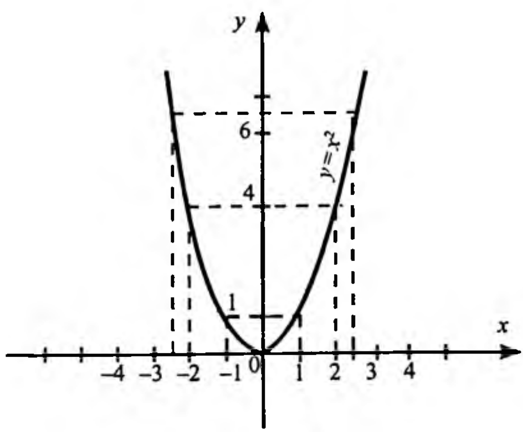
2. Grafik usuli

Funksiyaning jadval tarzida berilishida funksiyaning qiymatlari argumentning faqat jadvalda keltirilgan qiymatlariga mosini topa olamiz, xolos.

Funksiya grafik usulda berilganda argumentning har bir qiymati uchun tekislikda ayrim nuqta yasaladiki, bu nuqtaning absissasi argumentning berilgan qiymatiga, ordinatasi esa funksiyaning shunga mos qiymatiga teng bo'ladi.

Abssissaning (argumentning) mumkin bo'lgan hamma o'zgarishlarida unga mos nuqtalar biror chiziq hosil qiladi. Bu chiziq berilgan funksiyaning grafigi bo'ladi.

Bu grafikda absissa o'qidagi har qanday son argumentning qiymati, unga mos ordinata esa shu argument funksiyasining qiymatiga teng (15-chizma).



15-chizma.

Masalan, $x=2$ da $y=4$; va $x=2{,}5$ da $y=6{,}3$ kabi. Amalda ko'pincha funksiya tayyor chizilgan grafik bilan beriladi. Bundan argumentning istalgan qiymatiga mos funksiya qiymatini ma'lum aniqlikda topish mumkin.

Hozirgi zamon ishlab chiqarishida o'ziyurar asboblar keng qo'llaniladi, ular avtomat tarzda ba'zi miqdorlarning (temperatura, bosim va boshqalarning) o'zgarish grafiklarini chizib beradi. Shu grafikdan kerakli ma'lumotlar topib (hisoblab) olinadi. Berilgan diagrammalar orqali ham funksiya hosil qilish mumkin.

3. Analitik usul

Funksiya, argumentning berilgan qiymati bo'yicha funksiyaning mos qiymatini qanday hisoblashni ko'rsatuvchi formula vositasida ham berilishi mumkin.

Funksiyani bu usulda berish – analitik usul deyiladi. Ko'pincha argument x harfi bilan, funksiya esa y harfi bilan belgilanadi.

1-misol. Jismning biron tekis harakati $s=v\cdot x$ formula yordamida beriladi. Bundagi v (tezlik) – o'zgarmas miqdor, x (vaqt) – argument (erkli o'zgaruvchi), y (yo'l) – funksiya (vaqtning funksiyasi).

Bu formula yordamida istalgan vaqtga mos keluvchi o'tilgan yo'lni topish mumkin.

Masalan, $x=3$ (soat)da, $S=3v$ (km) yo'l bosadi.

Agar $v=30$ km/soat bo'lsa, $S=3\cdot30=90$ (km) $x=4{,}5$ da, $S=30\cdot4{,}5=$ $=135$ (km) yo'l bosadi va hokazo.

2-misol. $y=\frac{2x+3}{7x-28}$ formula bo'yicha $x(x\neq4)$ning berilgan qiymatlariga mos funksiyaning qiymatlarini topamiz.

Masalan, $x=3$ da $y=\frac{2\cdot3+3}{7\cdot3-28}=\frac{9}{-7}=-1\frac{2}{7}$;

$$x=-2 \text{ da } y=\frac{2\cdot(-2)+3}{7\cdot(-2)-28}=\frac{-1}{-42}=\frac{1}{42} \text{ va hokazo.}$$

Ba'zan, o'zgaruvchi miqdorlar faqat birgina o'zgaruvchi miqdorlarga emas, balki ikki, uch va undan ortiq o'zgaruvchi miqdorga bog'liq bo'ladi. U holda funksiyani ikki, uch va undan ortiq o'zgaruvchi miqdorlarning funksiyasi deb qaraladi.

Masalan, to'g'ri burchakli parallelepipedning (masalan, xonaning) hajmi parallelepiped asosining bo'yi, eni va balandliklari kabi o'zgaruvchilarning ko'paytmasiga teng, ya'ni $V=a\cdot b\cdot c$ bo'lib, bunda a; b; c – o'zgaruvchilar. V (hajm) – funksiya.

TAKRORLASH UCHUN SAVOLLAR

1. Jadval orqali funksiya qanday hosil qilinadi?
2. Grafik yordamida funksiya qanday hosil qilinadi?
3. Funksiya formula yordamida qanday hosil qilinadi?
4. $y=2{,}4\,x$ formula orqali funksiya qanday topiladi?

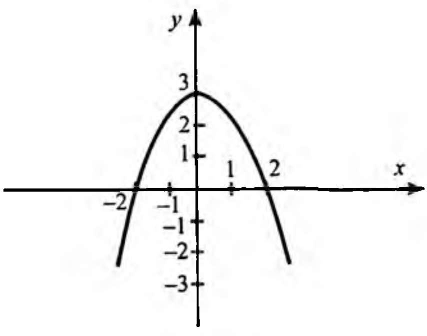
MASALALARNI YECHING

410.

x	–3	–2	–1	0	1	2	3	4	5
y	11	9	7	5	3	1	–1	–3	–5

jadval orqali funksiyani toping.

411. Berilgan grafikdan $f(-2)$; $f(-1)$; $f(0)$ va $f(2)$ funksiya qiymatlarini toping (16-chizma).



16-chizma.

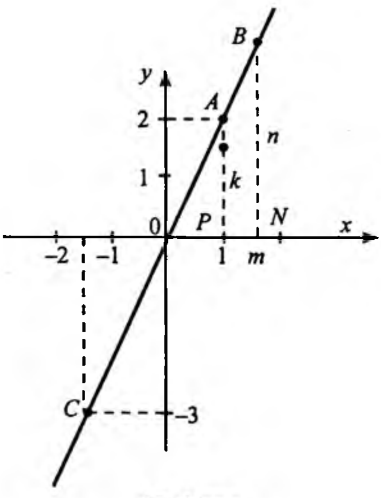
412. $y=\dfrac{x^2-7x}{x+6}$ formuladan $f(-4)$; $f(-1{,}2)$; $f\left(1\frac{2}{3}\right)$ va $f(\sqrt{2})$ funksiyaning qiymatlarini toping.

55-§. To'g'ri proporsionallikning grafigi

Biz quyi sinf darsliklarida to'g'ri proporsionallik haqida boshlang'ich ma'lumotlarga ega bo'lganmiz.

O'zaro to'g'ri proporsional bo'lgan ikki miqdorlar orasidagi moslik $y=kx$ formula bilan berilgan. Bundagi $k\neq0$ – o'zgarmas son, x va y – o'zgaruvchi miqdorlar, ya'ni x – argument, y esa x ning funksiyasi bo'ladi. Ikki o'zgaruvchi miqdorlar orasidagi to'g'ri proporsional moslikni $y=kx$ funksiya deb yuritamiz.

Teorema. ***$y=kx$ funksiyaning grafigi koordinatalar boshidan o'tuvchi to'g'ri chiziqdir.***



17-chizma.

Grafikning ikkita nuqtasini yasaymiz: $x=0$ da $y=k\cdot0=0$; $O(0;\ 0)$, $x=1$ da $y=k\cdot1=k$; $A(1;\ k)$. (17-chizma)

Demak, O va A nuqtalar $y=kx$ funksiya grafigida yotadi, ya'ni OA to'g'ri chiziqda yotadi. OA to'g'ri chiziq $y=kx$ funksiyaning grafigi ekanligini isbotlaymiz. Bu isbotni ikki qismga ajratamiz.

1) OA to'g'ri chiziqning barcha nuqtalari $y=kx$ funksiyaning grafigiga tegishli ekanligini;

2) OA to'g'ri chiziqda yotmagan nuqta $y=kx$ funksiya grafigiga tegishli emasligini.

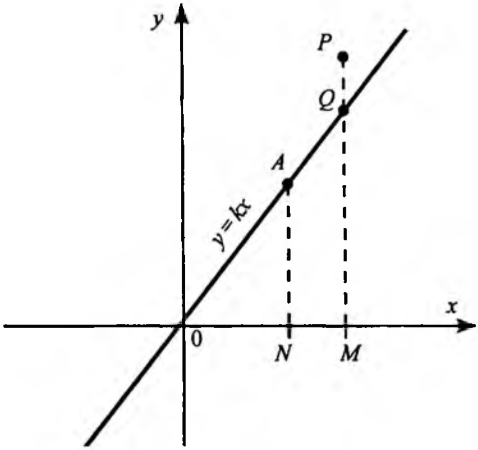
1) Isbotni avval $k>0$ bo'lgan hol uchun bajaramiz. OA to'g'ri chiziqda (17-chizma) ixtiyoriy $B(m;\ n)$ nuqtani olamiz. $\Delta OPA \backsim \Delta ONB$, chunki ular, bitta to'g'ri burchak va $\angle AOP$ larga ega.

Bu o'xshashlikdan $\frac{BN}{AP}=\frac{ON}{OP}$ bunda $BN=n$, $ON=m$; $AP=k$; $OP=1$ bo'lganidan $\frac{n}{k}=\frac{m}{1}$; $n=km$.

Bu $n=mk$ tenglik $y=kx$ funksiyani qanoatlantirgani uchun $B(m;\ n)$ nuqta $y=kx$ funksiya grafigiga tegishli.

B nuqta *OA* to‘g‘ri chiziqning ixtiyoriy olingan nuqtasi bo‘lgani uchun to‘g‘ri chiziqning barcha nuqtalari $y=kx$ funksiya grafigiga tegishli. Demak, $y=kx$ funksiyaning grafigi koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq bo‘ladi.

2) *OA* to‘g‘ri chiziqda yotmagan nuqta $y=kx$ funksiya grafigiga tegishli emasligini ko‘rsatamiz.



18-chizma.

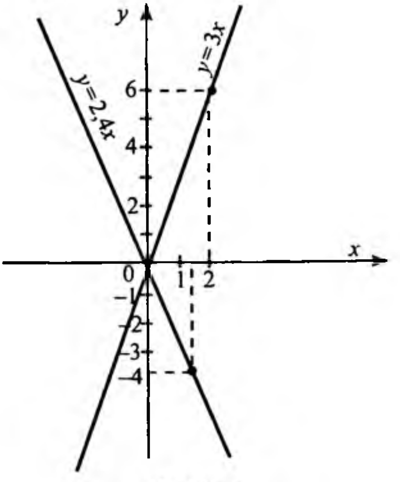
OA to‘g‘ri chiziqdan yuqorida yotgan ixtiyoriy *P* nuqtani olaylik (18-chizma). Bu nuqtadan absissa o‘qiga o‘tkazilgan perpendikulyar *OA* to‘g‘ri chiziqni *Q* nuqtada kesib o‘tadi. Ilgari isbotlanganga ko‘ra, *Q* nuqta $y=kx$ funksiyaning grafigiga tegishli. Demak, $QM=k\cdot OM$. Lekin $PM>QM$ ekanidan $PM>k\cdot OM$ bo‘ladi. Bu tengsizlik *P* nuqtaning koordinatalari $y=kx$ tenglikni qanoatlantirmaydi. Shunday ekan, *P* nuqta $y=kx$ funksiyaning grafigiga tegishli emasligini anglatadi.

Yana shu yo‘l bilan *OA* to‘g‘ri chiziqdan pastda yotgan har qanday nuqta $y=kx$ funksiyaning grafigiga tegishli emasligini isbotlash mumkin.

Demak, *OA* to‘g‘ri chiziqning barcha nuqtalari $y=kx$ funksiyaning grafigiga tegishli ekanligi isbotlandi. Agar $k<0$ bo‘lsa, $y=kx$ funksiyaning grafigi II, IV choraklar va koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq bo‘ladi.

1-misol. $y=3x$ funksiyaning grafigini chizing. Bu funksiyaning grafigi koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq bo‘ladi.

Grafikning koordinatalar boshidan boshqa biror nuqtasining koordinatalarini masalan, $x=2$ da $y=3\cdot2=6$ bo‘lgan (2; 6) nuqtani belgilaymiz. Bu nuqta va koordinatalar boshi orqali o‘tuvchi to‘g‘ri chiziq $y=3x$ funksiyaning grafigi bo‘ladi (19-chizma).



19-chizma.

2-misol. $y=-2{,}4x$ funksiyaning grafigini chizing.

Bu funksiyaning grafigi koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq bo‘ladi. Bu to‘g‘ri chiziq o‘tadigan ikkinchi nuqta, $x=1{,}5$ da $y=-2{,}4\cdot1{,}5=-3{,}6$ bo‘lgan (1,5; –3,6) nuqta bo‘ladi (19-chizma).

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani to‘g‘ri proporsionallik funksiyasi deyiladi?
2. $y=kx$ funksiyaning grafigi haqidagi teoremani ayting.
3. $y=kx$ funksiyaning grafigi asosan qanday ikki nuqta orqali chiziladi?
4. $y=kx$ funksiyaning grafigi $k>0$ bo‘lganda va $k<0$ bo‘lganda qaysi koordinatalar choraklaridan o‘tadi?

MASALALARNI YECHING

413. To‘g‘ri proporsionallik: a) $y=5x$; b) $y=-\frac{1}{3}x$ funksiya bilan berilgan. $A(1;\ 1)$, $B(6;\ -2)$, $C(-2;\ -10)$, $D\left(-\frac{1}{3};\ 1\frac{2}{3}\right)$, $E(0;\ 0)$, $F(-3;\ -1)$ lar to‘g‘ri proporsionallik grafigiga tegishli bo‘ladimi?

414. $y=4x$ funksiyaning grafigini chizing.

Grafikdan foydalanib:

a) x ning 1; 1,5; 2,5; 3; –2; –0,5 ga teng qiymatiga mos y ning qiymatini toping;

b) x ning qanday qiymatida y ning qiymati –3; 0; 3; 5 ga teng.

415. Quyidagi funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida joylashgan?

a) $y=2{,}3\,x$; b) $y=-4{,}5x$; d) $y=-0{,}7x$; e) $y=1\frac{2}{3}x$.

416. Teng tomonli uchburchakning tomoni x sm ga teng. Shu uchburchakning y perimetrini x orqali ifoda qiling. y funksiyaning grafigini chizing. Uchburchakning perimetri 6,8 (sm) ga teng bo'lgandagi uchburchakning tomonini grafikdan foydalanib toping.

56-§. Chiziqli funksiya va uning grafigi

1-masala. Sayohatchi shahardan 7 km uzoqlikda joylashgan. Sayohatchi soatiga 4 km tezlik bilan yuradi. Sayohatchi x soatdan keyin shahardan qancha masofada bo'ladi?

Yechish. Sayohatchi x soatda $4x$ km yo'l bosadi. Yana y shahardan 7 km uzoqda joylashgan. Demak, x soatdan keyin sayohatchining shahardan uzoqlashgan masofasi $4x+7$ km bo'ladi. Bu masofani y bilan belgilab, ushbu $y=4x+7$ bog'lanishni hosil qilamiz. Bundagi x ni argument, y ni funksiya desak, $y=4x+7$ funksiyani hosil qilamiz.

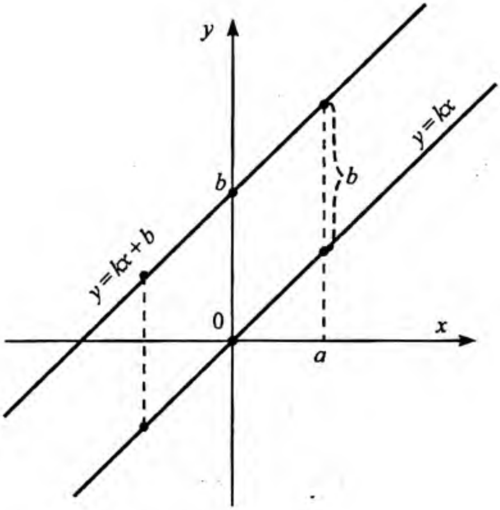
Ta'rif. **Argument x ga nisbatan $y=kx+b$ ko'rinishdagi funksiyani chiziqli funksiya deyiladi.**

Bunda: k va b – ixtiyoriy sonlar. Agar $b=0$ bo'lsa, funksiya $y=kx$ bo'lib, bu oldingi paragrafda qarab chiqilgan to'g'ri proporsionallik funksiyasi bo'ladi.

Teorema. **Chiziqli funksiyaning grafigi to'g'ri chiziqdir.**

Isbot. Avvalo $y=kx+b$ chiziqli funksiyada $b=0$ bo'lgan hol uchun $y=kx$ funksiyaning grafigini yasaymiz (20-chizma).

x absissaga ixtiyoriy $x=a$ qiymat beramiz. U holda $y=kx$ to'g'ri chiziq nuqtasining ordinatasi $y=ka$ ga teng ($k>0$ bo'lsin). U holda $y=kx+b$ funksiyaning ordinatasi, $y=ka+b$ bo'ladi.



20-chizma.

Biz x absissani ixtiyoriy olganimiz uchun, $y=kx+b$ funksiya grafigining istalgan nuqtasining ordinatasi, $y=kx$ to‘g‘ri chiziqning o‘sha absissali nuqtasining ordinatasi bilan b ning yig‘indisiga tengdir. Bu xulosaga asoslanib $y=kx+b$ funksiyaning grafigi $y=kx$ funksiya grafigiga ($y=kx$ to‘g‘ri chiziqqa) parallel bo‘lgan to‘g‘ri chiziq bo‘ladi. $y=kx+b$ funksiya grafigining bir nuqtasini $x=0$ da $y=k\cdot 0+b=b$ bo‘lgan (0; b) nuqtani yasab olib, bu nuqtadan, $y=kx$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziq o‘tkazamiz (20-chizma).

2-masala. $y=-0{,}4x+6$ funksiyaning grafigini chizamiz.

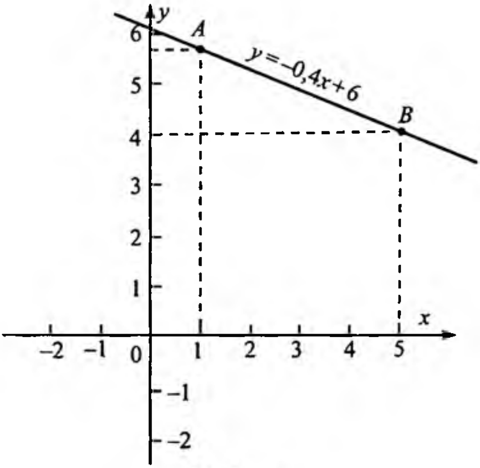
Yechish. Chiziqli funksiyaning grafigi to‘g‘ri chiziq bo‘ladi. To‘g‘ri chiziqi chizish uchun chiziqli funksiyaning ikki nuqtasi koordinatalari topilib chiziladi.

Ya’ni $x=1$, $y=-0{,}4\cdot 1+6=5{,}6$; $A(1;\ 5{,}6)$.

$x=5$, $y=-0{,}4\cdot 5+6=4$. $B(5;\ 4)$.

$A(1;\ 5{,}6)$ va $B(5;\ 4)$ nuqtalarni belgilaymiz. Bu nuqtalar orqali to‘g‘ri chiziq o‘tkazamiz (21-chizma).

Chiziqli funksiyaning grafigini chizishda, odatda to‘g‘ri chiziqning koordinata o‘qlari bilan kesishadigan ikki nuqtasi olinadi.



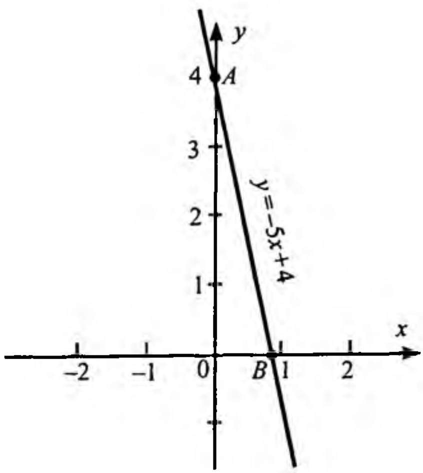
21-chizma.

Masalan, $y=-5x+4$ funksiya grafigini chizamiz.

$x=0$ da $y=-5\cdot0+4=4$; $A(0;\ 4)$

$y=0$ da $0=-5x+4$; $5x=4$; $x=0{,}8$; $B(0{,}8;\ 0)$.

Bu nuqtalarni koordinata o'qlarida belgilab ulardan AB to'g'ri chiziqni o'tkazamiz. U to'g'ri chiziq $y=-5x+4$ chiziqli funksiyaning grafigi bo'ladi (22-chizma).



22-chizma.

Umumiy holda ikki nuqta:

$x=0$ da, $y=k\cdot 0+b=b$; $A(0;\ b)$

$y=0$ da, $0=kx+b$; $kx=-b$; $x=\frac{-b}{k}$; $B\left(-\frac{b}{k};\ 0\right)$; kabi topilib, AB to‘g‘ri chiziq chiziladi.

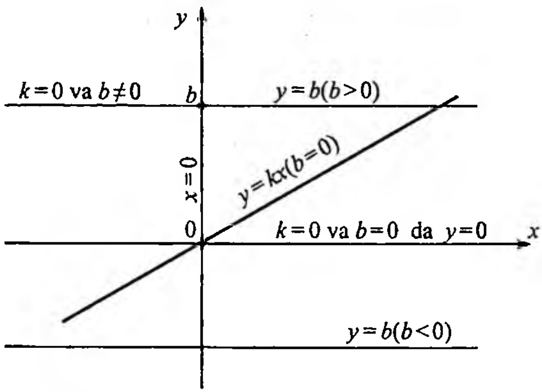
$y=kx+b$ funksiya grafigining xususiy hollari:

1) $b=0$ da, $y=kx$ bo‘lib, to‘g‘ri chiziq (0; 0) nuqtadan o‘tuvchi to‘g‘ri chiziq;

2) $k=0$ va $b\neq 0$ da, $y=b$ bo‘lib, to‘g‘ri chiziq $(0;\ b)$ nuqtadan OX o‘qiga parallel bo‘ladi.

3) $y=0$ to‘g‘ri chiziq OX o‘qidan iborat bo‘ladi.

4) $x=0$ to‘g‘ri chiziq OY o‘qidan iborat bo‘ladi (23-chizma).



23-chizma.

3-masala. $y=1{,}2x-7$ funksiyaning grafigini chizmasdan shu grafikning: a) $A(-10;\ 5)$, b) $B(100;\ 113)$; d) $C(-15;\ -25)$, $D(300;\ 358)$ nuqtadan o‘tish va o‘tmasligini aniqlang.

Yechish. a) $x=-10$ da, $y=1{,}2\cdot(-10)-7=-12-7=-19\neq 5$. $A(-10;\ 5)$ nuqta grafikka tegishli emas.

b) $x=100$ da, $y=1{,}2\cdot 100-7=113$. $B(100;\ 113)$ nuqta grafikka tegishli.

d) $x=-15$ da, $y=1{,}2\cdot(-15)-7=-18-7=-25$. $C(-15;\ -25)$ nuqta grafikka tegishli.

e) $x=300$ da, $y=1{,}2\cdot 300-7=360-7=353\neq 358$. $D(300;\ 358)$ nuqta grafikka tegishli emas.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani chiziqli funksiya deyiladi?
2. Chiziqli funksiyaning grafigi qanday chiziq bo'ladi?
3. $y=kx+b$ funksiyaning grafigi $y=kx$ funksiya grafigidan qanday farq qiladi?
4. $y=kx+b$ funksiyaning grafigi $k>0$ bo'lsa, koordinatalar tekisligining qaysi choragida joylashadi? $k<0$ bo'lganda koordinatalar tekisligining qaysi choragida joylashadi?
5. A(–2; 3) va B(3,5; 6) nuqtalardan qaysi biri $y=2x-1$ chiziqli funksiyaning grafigiga tegishli?

MASALALARNI YECHING

417. Funksiyaning grafigini chizing:

a) $y=-1\frac{3}{4}x$; b) $y=0{,}2x+5$; d) $y=-3x+4$;

e) $y=-0{,}6x+1{,}5$; f) $y=-0{,}3x=1\frac{1}{3}$; g) $y=-0{,}4x-2$.

418. Quyidagi funksiyalarning grafiklarini ayni bir koordinatalar sistemasida chizing:

a) $y=1{,}5x$ va $y=1{,}5x-3$; b) $y=2x$ va $y=-2x+3$.

419. $y=1{,}5x+4$ chiziqli funksiyaning grafigini chizing. Grafikdan foydalanib: a) y ning $x=3{,}5$; $-1{,}5$ va $2\frac{1}{3}$ larga mos qiymatini toping;

b) x ning $y=2{,}5$; $-0{,}5$ va $-1{,}4$ larga mos qiymatlarini toping.

420. Bitta koordinatalar tekisligida $y=6$; $y=0{,}8$; $y=-3$ va $y=0$ funksiyalarning grafiklarini chizing.

421. Bitta koordinatalar tekisligida $x=-3$; $x=-0,4$; $x=2,5$; $x=0$ funksiyalarning grafiklarini chizing.

422. $y=0,4x+4$ funksiyaning grafigini chizmasidan, shu grafikning a) $A(-5;\ 3)$; b) $B(-1,5;\ 4,4)$; d) $C(-2,5;\ 3)$ nuqtadan o'tish-o'tmasligini aniqlang.

57-§. Teskari proporsional funksiyaning grafigi

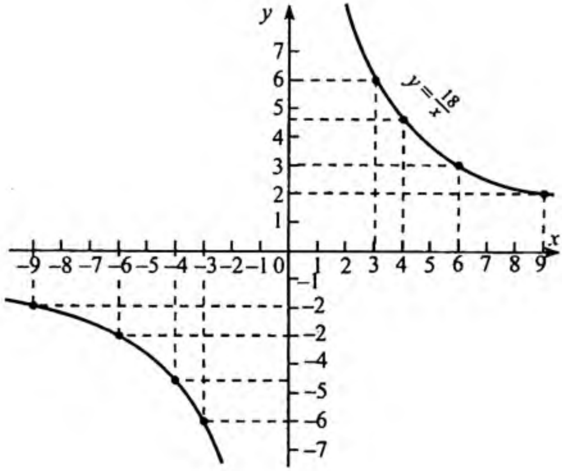
Quyi sinfda teskari proporsional miqdorlar haqida boshlang'ich ma'lumot berilgan. O'zaro teskari proporsional bo'lgan ikki miqdorlar orasidagi moslik $y=\frac{k}{x}$ formula bilan berilgan. Bundagi k – o'zgarmas son, $x\neq0$ va y – o'zgaruvchi miqdorlar orasidagi $y=\frac{k}{x}$ moslikni teskari proporsionallik funksiyasi deyiladi. Masalan, $y=\frac{18}{x}$ funksiya grafigini quyidagi jadval asosida chizamiz. Bunda $k=18>0$.

x	–18	–9	–6	–4	–3	–2	–1	0	1	2	3	4	6	9	18
$y=\frac{18}{x}$	–1	–2	–3	–4,5	–6	–9	–18	–	18	9	6	4,5	3	2	1

Jadvalda ko'rsatilgan qiymatlarini chizmaga ko'chirib, grafikda olingan barcha nuqtalarni egri chiziq bilan, tutashtirib chiqsak (qo'l bilan yoki lekalo bilan), $y=\frac{18}{x}$ teskari proporsional funksiya grafigini hosil qilamiz (24-chizma).

Yana ham aniqroq grafik hosil qilish uchun imkoni boricha ko'proq nuqtalar yasash kerak.

Bu grafikda absissa x cheksiz ravishda ortganda egri chiziq ordinatasi tobora kamayib nolga yaqinlashib boradi, egri chiziq o'ng tomonga qancha uzaytirilsa, OX o'qiga shuncha yaqinlashib boradi, lekin unga hech qachon yetisha olmaydi ($\frac{18}{x}$ kasr hech qachon nolga tenglashmaydi).



24-chizma.

Xuddi shuningdek, absissa x nolga yaqinlashib borganda kasr qiymati borgan sari kattalashib boradi, grafik esa cheksiz darajada yuqoriga ko‘tariladi.

Absissa x manfiy qiymatlar qabul qilganda $y=\frac{18}{x}$ funksiya grafigi 3-chorakda joylashib, y ham 1-chorakdagi kabi OX va OY o‘qlarga yaqinlasha boradi.

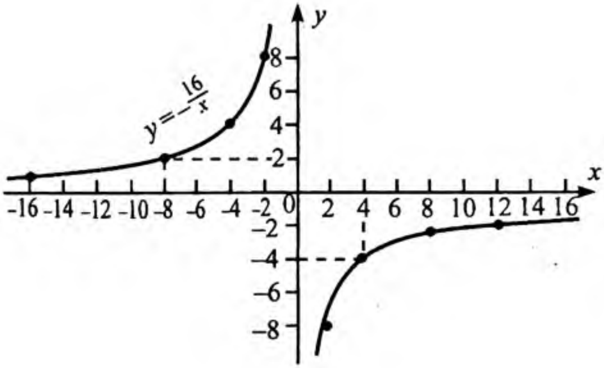
2-misol. $y=-\frac{16}{x}$ funksiya grafigini chizing.

Bunda $k=-16<0$ bo‘lgan jadvalni tuzamiz.

x	–16	–8	–4	–2	–1	0	1	2	4	8	16
$y=-\frac{16}{x}$	1	2	4	8	16	–	–16	–8	–4	–2	–1

$y=\frac{k}{x}$ funksiyaning grafigi **giperbola** deb ataladi.

$y=\frac{k}{x}$ funksiyada $k>0$ bo‘lsa, ***giperbola*** 1 va 3 – koordinat choraklarida, $k<0$ bo‘lsa, giperbola 2 va 4 – koordinat choraklarida joylashadi.



25-chizma.

TAKRORLASH UCHUN SAVOLLAR

1. Teskari proporsional bog'lanishda bo'lgan ikki o'zgaruvchiga misollar keltiring.
2. Qanday funksiyani teskari proporsional funksiya deyiladi?
3. $y=\frac{k}{x}$ funksiyaning grafigi qanday chiziladi?
4. Teskari proporsional funksiyaning grafigi qanday nomlanadi?
5. $y=\frac{k}{x}$ funksiyaning grafigi $k>0$ bo'lganda va $k<0$ bo'lganda qaysi koordinat choraklarida joylashadi?

MASALALARNI YECHING

423. a) $y=x$; b) $y=\frac{1}{x}$ funksiyalarning grafiklarini chizing.

424. a) $y=\frac{8}{x}$; b) $y=-\frac{12}{x}$ funksiyaning grafiklarini chizing.

425. $A(5;\ -4)$, $B(-4;\ -5)$, $C(10;\ -2)$ va $D(-2;\ -10)$ nuqtalar $y=\frac{20}{x}$ funksiya grafigiga tegishlimi?

426. $y=\frac{2,4}{x}$ funksiyaning grafigini chizing.

Grafikdan:

a) x ning 2,2; 3,4; –1,5 va –4,2 qiymatlariga mos y ning qiymatini toping;

b) y ning 1,2; 4; 8; –6; –4; –2,8 qiymatlariga mos x ning qiymatini toping.

58-§. Kasr chiziqli funksiyaning grafigi

$y=\frac{ax+b}{cx+d}$ (1) funksiya kasr chiziqli funksiya deb ataladi. Bunda $x\neq-\frac{d}{c}$ va a, b, c, d – parametrlar.

Kasr chiziqli funksiyaga misollar keltiramiz:

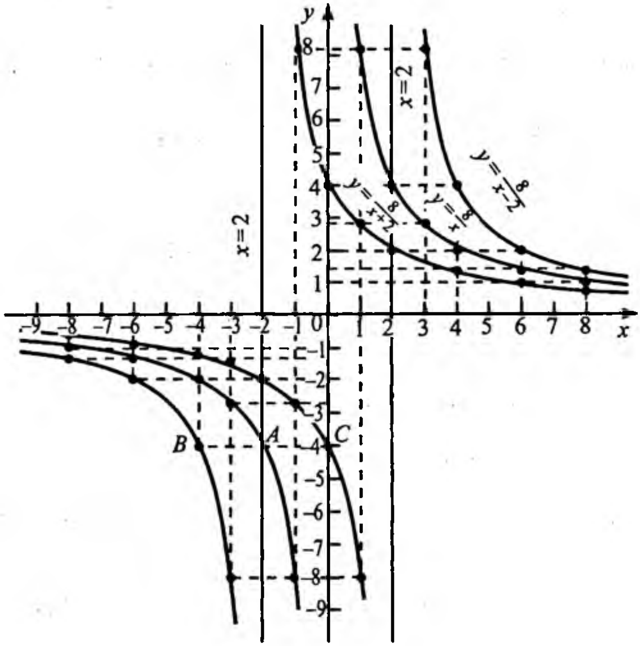
I. (1) funksiyada $a=d=0$, $b=8$, $c=1$ bo'lsa, funksiya $y=\frac{8}{x}$ ko'rinishda;

(1) funksiyada $a=0$, $b=8$, $c=1$, $d=2$ bo'lsa, funksiya $y=\frac{8}{x+2}$ ko'rinishda;

(1) funksiyada $a=0$; $b=8$; $c=1$, $d=2$ bo'lsa, funksiya $y=\frac{8}{x-2}$ ko'rinishda bo'ladi. $y=\frac{8}{x}$, $y=\frac{8}{x+2}$ va $y=\frac{8}{x-2}$ funksiyalar grafiklarini quyidagi jadval yordamida yasaymiz.

x	–8	–6	–4	–3	–2	–1	0	1	2	3	4	5	6
$y=\frac{8}{x}$	–1	$-1\frac{1}{3}$	–2	$-2\frac{2}{3}$	–4	–8	–	8	4	2	2	$1\frac{1}{3}$	1
$y=\frac{8}{x+2}$	$-1\frac{1}{3}$	–2	–4	–8	–	8	4	$2\frac{2}{3}$	2	1,6	$1\frac{1}{3}$	1	0,8
$y=\frac{8}{x-2}$	–0,8	–1	$-1\frac{1}{3}$	–1,6	–2	$-2\frac{2}{3}$	–4	–8	–	8	4	2	$1\frac{1}{3}$

26-chizmada $y=\frac{8}{x}$, $y=\frac{8}{x+2}$, $y=\frac{8}{x-2}$ funksiyalar grafiklaridan quyidagilarni bilish mumkin:



26-chizma.

1) Grafikda ordinatalari teng bo'lgan nuqtalar (*A, B, C*) bir-biridan 2 birlik farq bilan chapga yoki o'ngga suriladi.

2) $y=\frac{8}{x+2}$ funksiyaning grafigi $y=\frac{8}{x}$ funksiya grafigini absissa o'qi bo'ylab 2 birlik chapga surishdan hosil bo'ladi.

3) $y=\frac{8}{x-2}$ funksiyaning grafigi $y=\frac{8}{x}$ funksiya grafigini absissa o'qi bo'ylab 2 birlik o'ngga surishdan hosil bo'ladi.

Umuman, $y=\frac{k}{x+m}$ funksiyaning grafigi $y=\frac{k}{x}$ funksiya grafigini $m>0$ bo'lganda m birlik chapga, $m<0$ bo'lganda m birlik o'ngga, absissa o'qi bo'ylab suriladi.

$y=\frac{k}{x+m}$ funksiyaning grafigi $k>0$ bo'lsa, I va III koordinat choraklarida, $k<0$ bo'lsa II va IV koordinat choraklarida joylashadi.

II (1) funksiyada $a\neq0$, $b\neq0$, $d=0$ bo‘lsa, $y=\frac{ax+b}{cx}=\frac{a}{c}+\frac{b}{cx}=\frac{a}{c}+\frac{\frac{b}{c}}{x}$ bo‘lib, bundagi $\frac{a}{c}=n$, $\frac{b}{c}=k$ desak, $y=\frac{k}{x}+n$ ko‘rinishga keladi.

x ga bir xil qiymatlar berilganda $y=\frac{k}{x}+n$ funksiyaning qiymati $y=\frac{k}{x}$ funksiyaning qiymatidan $n>0$ da n birlik ortiq bo‘ladi. $y=\frac{k}{x}+n$ funksiyaning qiymati $y=\frac{k}{x}$ funksiyaning qiymatidan $n<0$ da n birlik kam bo‘ladi.

Shuning uchun $y=\frac{k}{x}+n$ funksiyaning grafigi $y=\frac{k}{x}$ funksiyaning grafigi kabi giperbola bo‘lib, $n>0$ bo‘lganda n birlik yuqoriga, $n<0$ bo‘lganda esa n birlik pastga surilgan bo‘ladi.

Agar (1) funksiyada $a\neq0$, $d\neq0$, $b=0$ bo‘lsa, $y=\frac{ax}{cx}=\frac{a}{c}$ ko‘rinishda bo‘lib, $y=\frac{a}{c}$ ning grafigi OX o‘qidan $\frac{a}{c}$ birlik yuqori (past)dan unga parallel to‘g‘ri chiziq bo‘ladi.

Masalan, $y=\frac{5}{x}+3$ funksiyaning grafigi $y=\frac{5}{x}$ funksiyaning grafigini OY o‘qi bo‘ylab 3 birlik yuqoriga ko‘tarish orqali hosil qilingan giperbola bo‘ladi.

III. (1) funksiyada $a\neq0$ va $d\neq0$ bo‘lsa, funksiya $y=\frac{ax+b}{cx+d}$ bo‘lib, uni boshqa ko‘rinishga keltiramiz.

$$y=\frac{ax+b}{cx+d}=\frac{a\left(x+\frac{b}{a}\right)}{c\left(x+\frac{d}{c}\right)}=\frac{a\left(x+\frac{d}{c}+\frac{b}{a}-\frac{d}{c}\right)}{c\left(x+\frac{d}{c}\right)}=\frac{a\left(x+\frac{d}{c}+\frac{bc-ad}{ac}\right)}{c\left(x+\frac{d}{c}\right)}=\frac{a}{c}+\frac{\frac{bc-ad}{c}}{c\left(x+\frac{d}{c}\right)}=$$

$$=\frac{a}{c}+\frac{\frac{bc-ad}{c^2}}{x+\frac{d}{c}}$$ bundagi $\frac{a}{c}=n$, $\frac{d}{c}=m$; $\frac{bc-ad}{c^2}=k$ deb olsak, u holda

$$y=\frac{ax+b}{cx+d}=\frac{k}{x+m}+n$$ ko‘rinishga keladi.

$y=\dfrac{k}{x+m}+n$ funksiyaning grafigini chizishda quyidagicha ketma-ketlikka rioya qilinadi.

$y=\dfrac{k}{x+m}+n$ funksiyaning grafigi $y=\dfrac{k}{x}+n$ funksiyaning grafigi

$y=\dfrac{k}{x}$ funksiyaning grafigi giperboladan iborat bo'lib, koordinatalar o'qiga nisbatan parallel ravishda ketma-ket surilib hosil qilinadi.

1) Avval, $y=\dfrac{k}{x}$ funksiyaning grafigi OX o'qi bo'ylab m birlik chapga yoki o'ngga (m ning musbat yoki manfiyligiga qarab) suriladi. Natijada $y=\dfrac{k}{x+m}$ funksiyaning grafigi hosil bo'ladi.

2) So'ngra, $y=\dfrac{k}{x+m}$ funksiyaning grafigini OY o'qi bo'ylab, n birlik yuqoriga yoki pastga (n ning musbat yoki manfiyligiga qarab) surish bilan $y=\dfrac{k}{x+m}+n$ funksiyaning grafigi hosil qilinadi.

Masalan, $y=\dfrac{3x-1}{2x+1}$ funksiyaning grafigini chizamiz.

$$y=\frac{3x-1}{2x+1}=\frac{3\left(x-\frac{1}{3}\right)}{2\left(x+\frac{1}{2}\right)}=\frac{3\left(x+0{,}5-0{,}5-\frac{1}{3}\right)}{2(x+0{,}5)}=1{,}5-\frac{3\cdot\frac{5}{6}}{2(x+0{,}5)}=1{,}5-\frac{1{,}25}{x+0{,}5}=$$

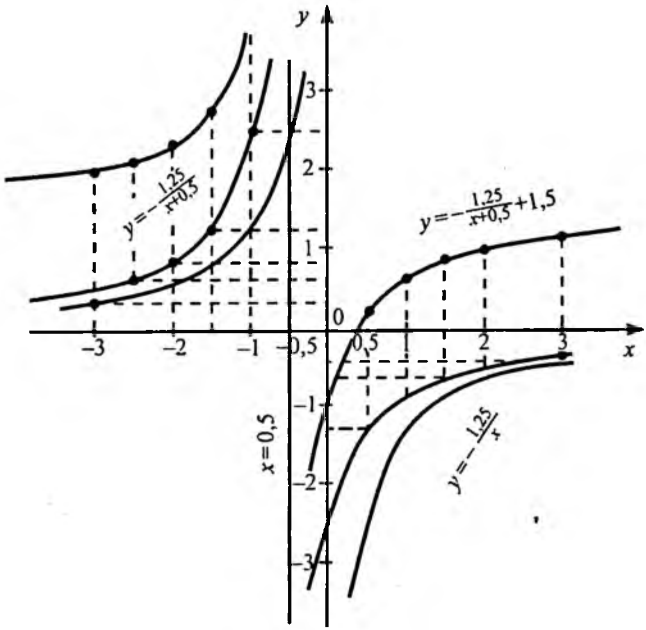
$=-\dfrac{1{,}25}{x+0{,}5}+1{,}5;$ $\quad y=-\dfrac{1{,}25}{x+0{,}5}+1{,}5$ funksiyaning grafigini chizamiz.

1. Buning uchun avval $y=-\dfrac{1{,}25}{x}$ funksiyaning grafigini jadval yordamida chizamiz.

x	–3	–2	–1,5	–1	–0,5	0,5	1	1,5	2	3
$y=-\dfrac{1{,}25}{x}$	≈0,4	≈0,6	0,8	1,25	2,5	≈–2,5	–1,25	≈–0,8	≈–0,6	≈–0,4

2. $y=-\frac{1,25}{x}$ funksiya grafigining har bir nuqtasini OX o'qiga parallel ravishda 0,5 birlik chapga surib, $y=-\frac{1,25}{x+0,5}$ funksiyaning grafigini hosil qilamiz (27-chizma).

3. Nihoyat, $y=-\frac{1,25}{x+0,5}$ funksiya grafigining har bir nuqtasini OY o'qiga parallel ravishda 1,5 birlik yuqoriga suramiz. Natijada, $y=-\frac{1,25}{x+0,5}+1,5$ funksiyaning grafigi hosil bo'ladi.



27-chizma.

TAKRORLASH UCHUN SAVOLLAR

1. Chiziqli funksiya qanday ko'rinishda bo'ladi? Chiziqli funksiyaga misollar keltiring.
2. Qanday funksiyani kasr chiziqli funksiya deyiladi? Misollar keltiring.
3. $y=\frac{k}{x}$ funksiyaning grafigi qanday chiziq bo'ladi?

4. $y=\frac{k}{x+m}$ funksiyaning grafigi qanday chiziladi?

5. $y=\frac{k}{x}+n$ funksiyaning grafigi qanday chiziladi?

6. $y=\frac{k}{x+m}+n$ funksiyaning grafigi qanday chiziladi?

7. $y=\frac{4}{x}$ funksiyaning grafigidan:

a) $y=\frac{4}{x}+2$; b) $y=\frac{4}{x}-2$ funksiya grafigi qanday yasaladi?

8. $y=\frac{6}{x}$ funksiyaning grafigidan:

a) $y=\frac{6}{x+3}$; b) $y=\frac{6}{x-3}$ funksiya grafigi qanday yasaladi?

9. a) $y=\frac{8}{x+1}+2,5$; b) $y=\frac{8}{x+1}-2,5$ funksiya grafigi qanday yasaladi?

MASALALARNI YECHING

427. $y=\frac{6}{x}$ funksiyaning grafigini chizing.
Undan foydalanib: a) $y=\frac{6}{x}+2$; b) $y=\frac{6}{x}-2$ funksiya grafiklarini shu chizmada chizing.

428. a) $y=-\frac{4}{x}+3$; b) $y=-\frac{4}{x}-3$ funksiyaning grafigini chizing.

429. $y=-\frac{8}{x}$ funksiyaning grafigini chizing.
Undan foydalanib shu chizmada:
a) $y=-\frac{8}{x+2}$; b) $y=-\frac{8}{x-2}$ funksiyaning grafigini chizing.

430. $y=\frac{10}{x}$ funksiyaning grafigini chizing. Undan foydalanib shu chizmada:
a) $y=\frac{10}{x+2}$; b) $y=\frac{10}{x+2}-3$ funksiyaning grafigini chizing.

431. Grafigi $y=\frac{3x+2}{x}$ funksiya grafigining OY o‘q bo‘yicha yuqoriga 3 birlik surishdan hosil bo‘ladigan funksiyaning analitik ko‘rinishi qanday bo‘ladi?

432. Quyidagi kasr chiziqli funksiyalarning grafigini chizing:

a) $y=\frac{x-6}{x}$; b) $y=\frac{8}{2x+5}$; d) $y=\frac{2x-4}{x-3}$.

59-§. Kvadrat uchhadning grafigi

Kvadrat uchhadning umumiy shakli ax^2+bx+c ekanligini bilamiz, bunda $a\neq 0$, b, c – istalgan sonlar, x – o‘zgaruvchi miqdor bo‘lib unga istalgancha qiymatlar berib, uchhadning mos qiymatlarini hosil qilamiz.

Demak, uchhad x argumentning funksiyasidir. Bu funksiyani y bilan belgilab $y=ax^2+bx+c$ kvadrat funksiyani hosil qilamiz.

Kvadrat uch hadni nolga aylantiradigan argumentning qiymatlari uchhadning ildizlari deyiladi.

Masalan, $y=x^2-4x-5$ uchhadning ildizlari –1 va 5 ga teng.

$y=ax^2+bx+c$ uchhadda:

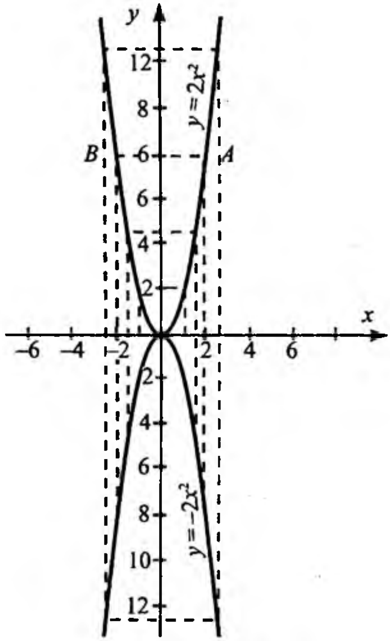
I. $b=c=0$ bo‘lsa, uchhad $y=ax^2$ shaklga keladi. Bunda $a=1$ bo‘lsa, funksiya $y=x^2$ bo‘ladi.

Masalan, $y=2x^2$ va $y=-2x$ funksiyalarning grafigini bitta koordinata tekisligida chizamiz.

Buning uchun x ning qiymatlari va y ning ularga mos qiymatlaridan quyidagi jadvalni tuzamiz.

x	–2,5	–2	–1,5	–1	–0,5	0	0,5	1	1,5	2	2,5
$y=2x^2$	12,5	8	4,5	2	0,5	0	0,5	2	4,5	8	12,5
$y=-2x^2$	–12,5	–8	–4,5	–2	–0,5	0	–0,5	–2	–4,5	–8	–12,5

Bu jadval bo‘yicha koordinatalar tekisligida nuqtalarni yasaymiz (28-chizma).



28-chizma.

Agar x ga jadvalda olingan nuqtalar orasidagi qiymatlarni ham bersak, nuqtalar tekislikda zichroq joylashadi. Bu nuqtalarning hammasi biror (egri) chiziqqa joylashgan bo'ladi. Bu egri chiziq **parabola** deb ataladi.

Demak, grafikning hamma nuqtalarining har bir jufti ordinata o'qiga nisbatan simmetrik ravishda joylashganligini ko'ramiz. Masalan, $A(2; 8)$ nuqta bilan $B(-2; 8)$ nuqta OY o'qiga nisbatan simmetrik bo'ladi.

Umumiy holda grafikning $A(x; y)$ nuqtasiga shu grafikning ordinatalar o'qining ikkinchi tomonida undan o'shancha masofada joylashgan $B(-x; y)$ nuqta to'g'ri keladi. Parabolaning simmetriya o'qi bilan kesishish nuqtasi parabolaning uchi deyiladi. $y=0,5x^2$ va $y=-0,5x^2$ funksiyalarning grafiklari ham $y=2x^2$ va $y=-2x^2$ funksiyalarning grafiklari kabi $y=0,5x^2$ ning ham tarmoqlari yuqoriga qaragan parabola, $y=-0,5x^2$ ning ham tarmoqlari pastga qaragan parabola bo'ladi. Qarab chiqilgan misollardan ma'lum bo'ldiki, $y=ax^2$ parabolaning tarmoqlari $a>0$ da yuqoriga yo'nalgan bo'ladi, chunki $a>0$, $x^2>0$ lardan $y=ax^2>0$. $a<0$ da $y=ax^2$ parabolaning tarmoqlari $a>0$ da yuqoriga yo'nalgan bo'ladi, $a<0$ da $y=ax^2$ parabolaning tarmoqlari pastga yo'nalgan bo'ladi, chunki $a<0$, $x^2>0$ lardan $y=ax^2<0$.

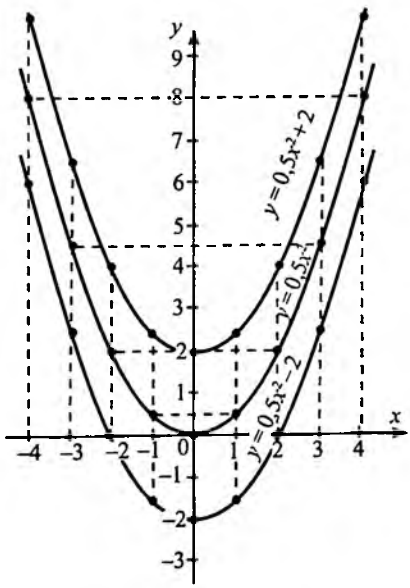
$y=ax^2$ parabolada istalgan a da, $x=0$ bo'lsa, $y=0$ bo'lgani uchun parabola doim koordinatalar boshidan o'tadi.

II. $y=ax^2+bx+c$ kvadrat uch hadda $\boldsymbol{b=0}$ bo'lsa, $\boldsymbol{y=ax^2+c}$ ko'rinishga keladi.

Masalan, 1) $y=0,5x^2$; 2) $y=0,5x^2+2$; 3) $y=0,5x^2-2$ funksiyalarning grafiklarini jadval yordamida chizamiz.

x	–4	–3	–2	–1	0	1	2	3	4
$y=0,5x^2$	8	4,5	2	0,5	0	0,5	2	4,5	8
$y=0,5x^2+2$	10	6,5	4	2,5	2	2,5	4	6,5	10
$y=0,5x^2-2$	6	2,5	0	–1,5	–2	–1,5	0	2,5	6

Ma'lumki, x argumentning bir xil qiymatida $y=0,5x^2+2$ funksiyaning ordinatasi, $y=0,5x^2$ funksiyaning mos kelgan ordinatasidan 2 birlik ortiq, $y=0,5x^2-2$ funksiyaning ordinatasi esa $y=0,5x^2$ funksiyaning mos kelgan ordinatasidan 2 birlik kam. Shunga ko'ra $y=0,5x^2+2$ va $y=0,5x^2-2$ funksiyalar chizmada $y=0,5x^2$ funksiyadek o'sha parabola bilan tasvirlanadi, faqat parabola 2 birlik yuqoriga ko'tarilgan ($y=0,5x^2-2$ funksiyada 2 birlik pastga tushirilgan) bo'ladi (29-chizma).



29-chizma.

Umuman $y=ax^2+b$ funksiyaning grafigi, $y=ax^2$ ni tasvirlaydigan paraboladan iborat bo'lib, faqat, agar $b>0$ bo'lsa, parabola b birlik qadar yuqoriga ko'tarilgan bo'ladi. Agar $b<0$ bo'lsa, b birlik qadar pastga tushirilgan bo'ladi.

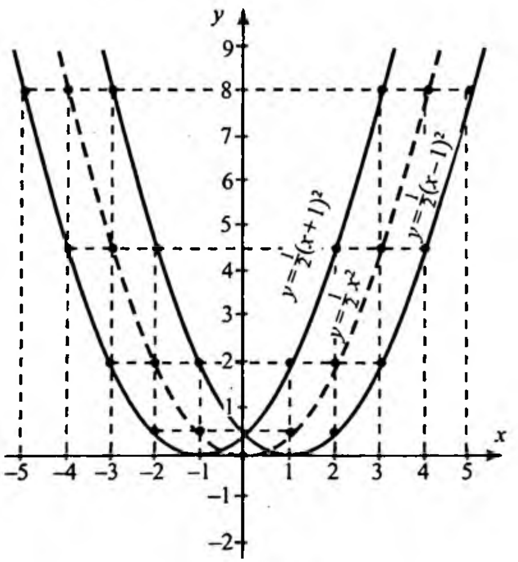
Yuqoridagi hollar kabi $y=ax^2+b$ funksiyada $a<0$ bo'lsa, parabolaning tarmoqlari pastga qaragan bo'ladi.

III. $y=a(x+m)^2$ ko'rinishdagi kvadrat funksiyaning grafigini chizamiz. Masalan, 1) $y=\frac{1}{2}x^2$; 2) $y=\frac{1}{2}(x+1)^2$; $y=\frac{1}{2}(x-1)^2$.

Shu uch funksiyaning qiymatlari jadvalini tuzib olamiz.

	x	–4	–3	–2	–1	0	1	2	3	4
1)	$y=\frac{1}{2}x^2$	8	4,5	2	0,5	0	0,5	2	4,5	8
2)	$y=\frac{1}{2}(x+1)^2$	4,5	2	0,5	0	0,5	2	4,5	8	12,5
3)	$y=\frac{1}{2}(x-1)^2$	12,5	8	4,5	2	0,5	0	0,5	2	4,5

Jadvaldagi qiymatlarning hammasini 30-chizmaga ko'chirib tasvirlangan uchta grafikni hosil qilamiz.



30-chizma.

Chizmani ko'ramiz: $y=\frac{1}{2}(x+2)^2$ parabola $y=\frac{1}{2}x^2$ parabolaning o'zi, faqat 1 birlik chapga surilgan, $y=\frac{1}{2}(x-2)^2$ parabola ham parabola $y=\frac{1}{2}$ x^2 parabolaning o'zginasi; faqat 1 birlik o'ngga surilgan.

Bu natijalarni umumlashtirib, $y=a(x+m)^2$ funksiyaning grafigi $y=ax^2$ funksiyani ifoda qiluvchi paraboladan iborat, faqat bu parabola, m ning absolyut miqdori qancha birlik bo'lsa, shuncha birlik chapga (agar $m>0$ bo'lsa) va o'ngga (agar $m<0$ bo'lsa) surilgan.

Agar $a>0$ bo'lsa, bu parabolaning tarmoqlari misoldagidek yuqoriga yo'nalgan, agar $a<0$ bo'lsa, pastga (quyiga) yo'nalgan bo'ladi.

Masalan, $y=-\frac{1}{2}(x+1)^2$ va $y=-\frac{1}{2}(x-1)^2$ parabolalarning tarmoqlari pastga yo'nalgan bo'ladi.

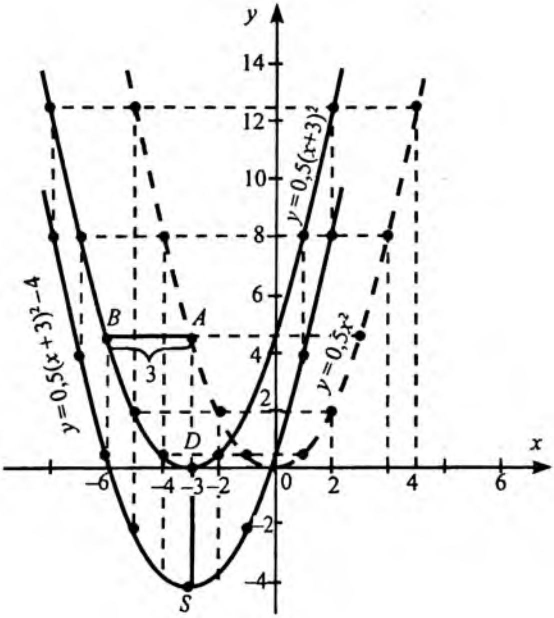
IV. $y=ax^2+bx+c$ ko'rinishdagi kvadrat funksiyaning grafigini ko'rib chiqamiz. Masalan, $y=0{,}5x^2+3x+0{,}5$ va $y=2x^2+3x-2$ funksiyalarning grafiklarini chizamiz. $y=0{,}5x^2+3x+0{,}5$ dan 0,5 ko'paytuvchini qavsdan tashqariga chiqarib, undan ikkihad kvadratini ajratamiz.

$0{,}5x^2+3x+0{,}5=0{,}5(x^2+6x+1)=0{,}5(x^2+2\cdot 3x+9-9+1)=0{,}5((x+3)^2-$ $-8)=0{,}5(x+3)^2-4$, ya'ni $y=0{,}5(x+3)^2-4$.

Bitta koordinatalar sistemasida $y=0{,}5x^2$; $y=0{,}5(x+3)^2$ va $0{,}5(x+3)^2-4$ funksiyalarning grafigini yasaymiz.

x	–5	–4	–3	–2	–1	0	1	2	3	4
$y=0{,}5x^2$	12,5	8	4,5	2	0,5	0	0,5	2	4,5	8
$y=0{,}5(x+3)^2$	2	0,5	0	0,5	2	4,5	8	12,5	18	24,5
$y=0{,}5(x+3)^2-4$	–2	–3,5	–4	–3,5	–2	0,5	4	8,5	14	20,5

Bu grafikni yasash quyidagi uch bosqichda bajariladi (31-chizma).



31-chizma.

1) Jadval bo‘yicha $y=0,5x^2$ funksiyaning grafigini chizib olamiz.

2) $y=0,5(x+3)^2$ funksiyaning grafigi $y=0,5x^2$ funksiya grafigining nuqtalarini 3 birlik chapga ($AB=3$ birlik) surish orqali hosil qilinadi.

3) $y=0,5(x+3)^2-4$ funksiyaning grafigi $y=0,5(x+3)^2$ funksiya grafigining nuqtalarini 4 birlik pastga ($CD=4$ birlik) surish orqali hosil qilinadi.

Natijada, C uchli $y=0,5x^2+3x+0,5=0,5(x+3)^2-4$ funksiyaning grafigi hosil bo‘ladi.

$y=0,5x^2+3x+0,5$ funksiyani qarab chiqishda aytilgan mulohazalarni istalgan $y=ax^2+bx+c$ kvadrat funksiya uchun ham aytish mumkin.

ax^2+bx+c kvadrat uchhadni ikki hadning kvadrat ko‘rinishiga keltiramiz. $ax^2+bx+c=a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=a\left(x^2+2\frac{b}{2a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right)=$

$$=a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right)=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a}.$$

Ya'ni $y=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a}$ formula hosil bo'ldi. Bunda $m=-\frac{b}{2a}$; $-\frac{b^2-4ac}{4a}=\frac{4ac-b^2}{4a}=n$ kabi belgilab, $y=a(x+m)^2+n$ funksiyaning grafigi $y=ax^2$ parabolaga teng bo'lgan parabola bo'ladi. Uning uchi $(m;\ n)=$ $=\left(-\frac{b}{2a};\ -\frac{b^2-4ac}{4a}\right)$ nuqta bo'ladi. Parabola uchini C nuqta bilan belgilasak $S\left(-\frac{b}{2a};\ -\frac{b^2-4ac}{4a}\right)$ kabi bo'ladi.

Ba'zida parabolaning uchi $C(x_0;\ y_0)$ kabi belgilanadi a) $x_0=m=-\frac{b}{2a}$; $y_0=n=-\frac{b^2-4ac}{4a}$ ko'rinishda yoziladi.

$x_0=-\frac{b}{2a}$ parabolaning simmetriya o'qi bo'lib, u OY o'qiga parallel bo'lgan to'g'ri chiziq.

b) $x=0$ da parabola OY o'qini kesadi, u nuqtani A bilan belgilaymiz, ya'ni $y=a\cdot 0^2+b\cdot 0+c=c$ bo'lib, $A(0;\ c)$ topiladi.

d) $x_0=-\frac{b}{2a}$ to'g'ri chiziqqa nisbatan A ga simmetrik bo'lgan A' nuqtani absissasi $2\cdot x_0=2\cdot\left(-\frac{b}{2a}\right)=-\frac{b}{a}$ bo'lgan $A'\left(-\frac{b}{a};\ c\right)$ nuqta bo'ladi.

e) $ax^2+bx+c=0$ tenglamaning ildizlari x_1 va x_2 bo'lsin (agar diskriminant noldan katta bo'lsa). Bu x_1 va x_2 ildizlar parabolani OX o'qi bilan kesishgan nuqtalarining absissalari bo'ladi.

Parabolani OX o'qi bilan kesishgan nuqtalarini B va B' deylik. Bu nuqtalar $B(x_2;\ 0)$ va $B'(x_1;\ 0)$ kabi bo'ladi ($x_1<x_2$ bo'lsin).

$y=ax^2+bx+c$ funksiyaning grafigi $a>0$ bo'lganda parabolaning tarmoqlari yuqoriga va $a<0$ da pastga qaragan bo'ladi.

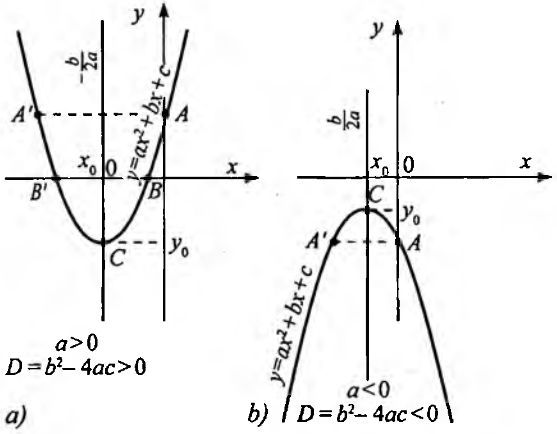
$y=ax^2+bx+c$ funksiyaning grafigini asosiy beshta nuqtalar (C, A, A', B va B') orqali sxematik grafigini chizamiz.

a) Parabolaning uchi – $S\left(-\frac{b}{2a};\ -\frac{b^2-4ac}{4a}\right)$ ni topamiz.

b) Parabola OY o'qini kesib o'tuvchi $A(0;\ c)$ va unga simmetrik bo'lgan $A'\left(-\frac{b}{a};\ c\right)$ nuqtalar topiladi.

d) Parabola OX o'qini kesib o'tuvchi $B(x_2;\ 0)$ va $B'(x_1;\ 0)$ nuqtalar topiladi.

Bunda x_1 va x_2 $ax^2+bx+c=0$ tenglamaning ildizlari (32-*a* chizma).



32-chizma.

Agar $ax^2+bx+c=0$ tenglama ildizga ega bo'lmasa ($D=b^2-4ac<0$), $y=ax^2+bc+c$ parabola asosan uchta nuqta (C, A va A') lar orqali sxematik yasaladi (32-*b* chizma).

Masalan, $y=-0{,}5x^2+2x+6$ funksiyaning grafigini beshta asosiy nuqtalar orqali chizamiz.

1) Parabolaning uchini topamiz; $a=-0{,}5$; $b=2$; $c=6$.

$x_0=-\frac{b}{2a}=-\frac{2}{2\cdot(-0{,}5)}=2$; $y_0=-0{,}5x_0^2+2x_0+6=-0{,}5\cdot 2^2+2\cdot 2+6=-2+4+$ $+6=8$; yoki $y_0=-\frac{b^2-4ac}{4a}=-\frac{2^2-4\cdot(-0{,}5)}{4\cdot(-0{,}5)}=\frac{4+12}{2}=8$. $C(x_0;\ y_0)=C(2;\ 8)$.

2) Parabolani OY o'qini kesgan A nuqtasi va unga simmetrik bo'lgan A' nuqtalarni topamiz:

$A(0;\ C)$ yoki $x=0$ da $y=-0{,}5\cdot 0^2+2\cdot 0+6=6$; $A(0;\ 6)$.

$A'(2x_0;\ c)$ ekanligidan $2\cdot x_0=2\cdot 2=4$; $A'(4;\ 6)$.

3) Parabolani OX o'qini kesadigan nuqtalarini topamiz, chunki $D=b^2-4ac>0$.

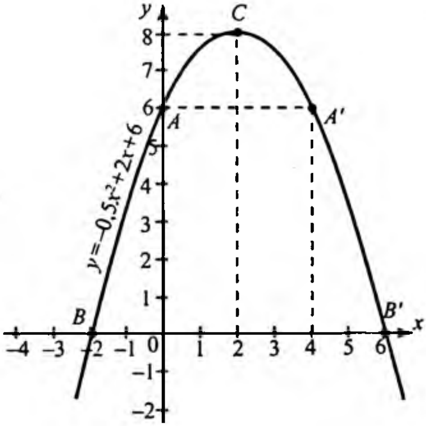
$-0{,}5x^2+2x+6=0;$

$0{,}5x^2-2x-6=0.$

$$x_{1/2}=\frac{1\pm\sqrt{1+0,5\cdot(-6)}}{0,5}=\frac{1\pm2}{0,5};\quad x_1=-2;\quad x_2=6;$$

$B(x_1;\ 0)=B(-2;\ 0)$ va $B'(x_2;\ 0)=B;(6;\ 0)$.

4) Koordinatalar tekisligida topilgan beshta nuqtalar orqali berilgan parabolaning grafigini sxematik chizamiz (33-chizma).



33-chizma.

$y=ax^2+bx+c$ funksiyaning grafigining chizishdagi beshta asosiy nuqtalar:

1) $x_0=-\frac{b}{2a}$, $y_0=ax_0+bx_0+c \quad C(x_0;\ y_0)$;

2) $A(0;\ s)$ va $A'(2x_0;\ s)$;

3) $B(x_1;\ 0)$ va $B'(x_2;\ 0)$ (kvadrat uchhadning ildizlari mavjud bo'lsa).

TAKRORLASH UCHUN SAVOLLAR

1. Umumiy holda kvadrat funksiya qanday ko'rinishda bo'ladi?
2. Kvadrat funksiyada $b=c=0$ bo'lsa, funksiya qanday ko'rinishda bo'ladi? Uning grafigi qanday chiziq bo'ladi?
3. Kvadrat funksiyada $b=0$ bo'lsa, funksiya qanday ko'rinishda bo'ladi? Uning grafigi qanday parabola bo'ladi?
4. $y=ax^2+bc+c$ funksiyadan ikki hadning kvadrati ajratilsa, u qanday ko'rinishda bo'ladi?
5. $y=3x^2$ va $y=-3x^2$ funksiyalarning grafiklari o'zaro qanday joylashgan?

6. $y=3x^2$ va $y=3x^2-2$ funksiyalarning grafiklari o'zaro qanday joylashgan?
7. $y=-2(x+3)^2$ va $y=-2x^2+3$ funksiyalarning grafiklari o'zaro qanday joylashgan?
8. $y=4x^2-3$ va $y=4(x-3)^2+2$ funksiyalarning grafiklari o'zaro qanday joylashgan?
9. Parabolaning grafigini uning qanday beshta nuqtasi orqali sxematik chiziladi?
10. a) $y=4x^2$ funksiya grafigi barcha nuqtalarining ordinatalarini ikki marta kamaytirilsa, qanday funksiyaning grafigi hosil bo'ladi?
    b) ordinatalarini 5 marta orttirilsa-chi?

MASALALARNI YECHING

433. $y=x^2$ parabolaning grafigidan foydalanib:
a) $y=x^2+2$; b) $y=x^2-4{,}5$; d) $y=(x+3)^2$; e) $y=(x-5{,}5)^2$ funksiyaning grafigini yasang.

434. $y=0{,}5x^2$ parabolaning grafigidan foydalanib:
a) $y=-0{,}5x^2$; b) $y=0{,}5x^2+3$; d) $y=-0{,}5(x+2)^2$;
e) $y=0{,}5(x+2)^2-1{,}5$ funksiyaning grafigini chizing.

435. $y=1{,}5x^2$ parabolaning grafigidan foydalanib:
a) $y=1{,}5x^2-2$; b) $y=1{,}5x^2+3$; d) $y=1{,}5(x+2)^2$;
e) $y=1{,}5(x-2{,}5)^2-3$ funksiyaning grafigini chizing.

436. Funksiyaning grafigini yasang:
a) $y=x^2-2x$; b) $y=-2x^2+6$; d) $y=x^2-4x+1$;
e) $y=\frac{1}{3}x^2+2x+1$; f) $y=-0{,}5x^2+2x+7$.

437. Funksiyalarning grafiklarini bitta koordinatalar sistemasida yasang:
a) $y=x^2$; $y=x^2-3$; $y=(x+1{,}5)^2$;
b) $y=-x^2$; $y=-1{,}5x^2+2$; $y=-(x-3)^2$;
d) $y=\frac{1}{3}x^2$; $y=\frac{1}{3}(x-4)^2$; $y=\frac{1}{3}(x-2)+3$.

438. $y=x^2-2a-8$ funksiyaning grafigini yasang va grafik yordamida:

a) funksiyaning $x=2,5$; 0; $-0,5$; -3 bo'lgandagi qiymatini toping;

b) argumentning $y=6$; 0; -2 bo'ladigan qiymatlarini toping.

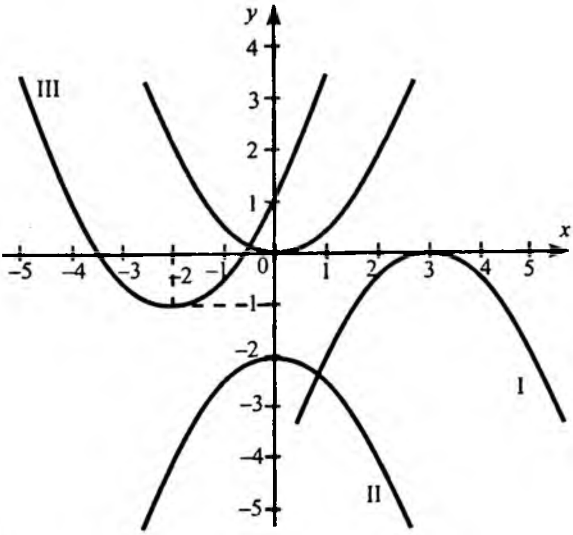
439. Quyidagi funksiyalarning grafiklarini uning beshta asosiy nuqtalari yordamida chizing:

a) $y=x^2-2x-3$; b) $y=-2x^2+4x-6$; d) $y=-4x^2-8x-8$.

440. 1) $y=(x+m)^2$ funksiyaning grafigi OX o'qi bilan (2; 0) umumiy nuqtaga,

2) $y=(x+4)^2+m$ ning grafigi OY o'qi bilan (0; 3) umumiy nuqtaga ega. m ning qiymatini toping.

441. 34-chizmada berilgan parabola grafiklarining funksiyalarini yozing.

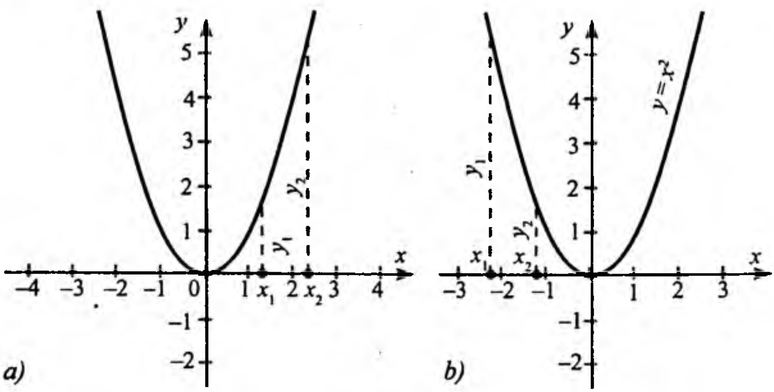


34-chizma.

60-§. Kvadrat funksiyaning xossalari

Kvadrat funksiyaning grafigidan foydalanib, by funksiya argumentning qanday qiymatlarida nolga aylanishini, qanday oraliqlarda u musbat qiymatlar va qandaylarida manfiy qiymatlar olishini, bu funksiyaning eng katta yoki kichik qiymatlari qandayligini bilib olamiz.

$y=x^2$ parabolani ko'zdan kechirib, x argumentning musbat qiymatlarida parabolaning chizig'i «yuqoriga ko'tarilishini» (o'sishini), x ning manfiy qiymatlarida esa «pastga tushishini» (kamayishini) ko'ramiz.



35-chizma.

35-*a* chizmadan, agar argumentning x_1 va x_2 ikkita musbat qiymatlarini olsak, x argumentning katta qiymatiga y funksiyaning katta qiymati mos kelganligini ko'ramiz, ya'ni $x_1<x_2$ bo'lsa, $y_1<y_2$ bo'ladi.

Agar x_1 va x_2 ning manfiy qiymatlarini olib ko'rsak (35-*b*, chizma), y holda $x_1<x_2$ bo'lsa, $y_1>y_2$ ekanligini ko'ramiz.

Umumiy holda: x_1 va x_2 – ikkita musbat son va ular $x_1<x_2$ bo'lsin. Unda $y_1=x_1^2$ va $y_2=x_2^2$.

y_1 va y_2 larni taqqoslash uchun y_2-y_1 ayirmani topamiz. $y_2-y_1=x_2^2-x_1^2=$ $=(x_2-x_1)(x_2+x_1)$ bo'lib, bunda $x_2-x_1>0(x_1<x_2$ edi), $x_2+x_1>0$ bo'lganidan ko'paytma musbat bo'ladi, ya'ni $y_2-y_1>0$. Demak, $x_1<x_2$ bo'lganda $y_1<y_2$ bo'ladi.

1-ta'rif. **Agar argumentning har xil ikki qiymatidan kattasiga funksiyaning katta qiymati mos kelsa, bu funksiya o'suvchi (ortuvchi) funksiya deyiladi.**

Demak, x argumentning musbat qiymatlarida $y=x^2$ funksiya ortib boradi.

Endi x_1 va x_2 ning manfiy qiymatlarini olamiz, bunda $x_1<x_2$ bo'lsin. y_2-y_1 ayirmani topamiz.

$y_2-y_1=(x_2-x_1)(x_2+x_1)$. Bunda $x_2-x_1>0(x_2>x_1$ edi), $x_2+x_1<0$. (x_1 va x_2 – manfiy sonlar).

$(x_2-x_1)(x_2+x_1)$ – ko'paytma manfiy, ya'ni $y_2-y_1<0$-yoki $y_2<y_1$ bo'ladi.

2-ta'rif. **Agar argumentning har xil ikki qiymatidan kattasiga funksiyaning kichik qiymati mos kelsa, bu funksiya kamayuvchi funksiya deyiladi.**

Demak, x argumentning manfiy qiymatlarida $y=x^2$ funksiya kamayadi.

$y=ax^2+bx+c$ kvadrat funksiyaning xossalarini ko'rib chiqishda uning grafigidagi ayrim nuqtalargina muhim ahamiyatga ega.

Bunday nuqtalar:

1. Parabolaning uchi $C(x_0;\ y(x_0))$ topiladi. Agar $a>0$ bo'lsa, parabolaning tarmoqlari yuqoriga, $a<0$ bo'lsa, pastga qaragan bo'ladi.

2. Parabolaning OX o'qini kesib o'tadigan yoki OX o'qiga urinadigan nuqtalari topiladi. ($B(x_1;\ 0)$; $B'(x_2;\ 0)$ yoki urinish nuqtasi $B_1(x;\ 0)$).

3. Topilgan C, B, B' yoki B_1 nuqtalar orqali parabola sxematik chiziladi.

4. Funksiya qiymatining ishora oraliqlari, o‘sish va kamayish oraliqlari, eng katta yoki eng kichik qiymatlari topiladi.

1-m i s o l. x ning qanday qiymatlarida $y=4x^2+x-3$ funksiyaning qiymati nolga teng, noldan katta, noldan kichik ekanini, shuningdek, bu funksiyaning o‘sish va kamayish oraliqlarini toping.

Yechish: 1) $a=4>0$ bo‘lgani uchun parabola tarmoqlari yuqoriga qaragan;

$C(x_0; y(x_0))$ – parabola uchi: $x_0=-\frac{b}{2a}=-\frac{1}{2\cdot 4}=-\frac{1}{8}$; $y(x_0)=4\cdot\left(-\frac{1}{8}\right)^2+$

$+\left(-\frac{1}{8}\right)-3=\frac{1}{16}-\frac{1}{8}-3=-\frac{1}{16}-3=-3\frac{1}{16}$. $S\left(-\frac{1}{8}; -3\frac{1}{16}\right)$ – parabola uchi.

2) Funksiya nol bo‘ladigan qiymatlarni topamiz.

$4x^2+x-3=0$.

$$x_{1/2}=\frac{-0{,}5\pm\sqrt{0{,}25+12}}{4}=\frac{-0{,}5\pm 3{,}5}{4};\ x_1=-1;\ x_2=\frac{3}{4}=0{,}75.$$

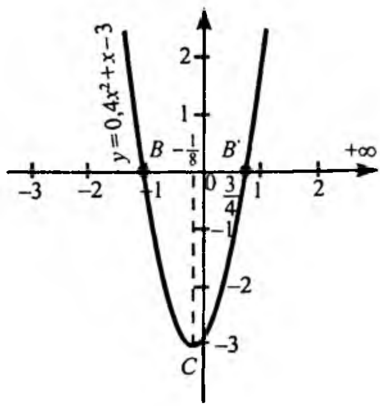
Parabola OX o‘qini $B(-1;\ 0)$ va $B'(0{,}75;0)$ nuqtalarda kesadi, ya’ni $y=0$.

3) C, B va B' nuqtalar orqali parabolani sxematik chizamiz (36-chizma).

4) Funksiya $(-\infty;\ -1)$ va $\left(\frac{3}{4};+\infty\right)$ oraliqlarda musbat, $\left(-1;\ \frac{3}{4}\right)$ oraliqda manfiy;

Funksiya $\left(-\infty;-\frac{1}{8}\right)$ oraliqda kamayadi va $\left(-\frac{1}{8};+\infty\right)$ oraliqda o‘sadi.

Funksiya $x=-\frac{1}{8}$ da eng kichik $y=-3\frac{1}{16}$ qiymatga ega.



36-chizma.

2-m i s o l. $y=3x^2-6x+4$ funksiya x ning qanday qiymatlarida nolga teng, noldan katta, noldan kichik ekanini, uning o‘sish va kamayish oraliqlarini hamda eng kichik qiymatini toping.

Yechish. 1) $a=3>0$ bo'lgani uchun parabolaning tarmoqlari yuqoriga qaragan.

$C(x_0;\ y(x_0))$ – parabolaning uchi: $x_0=-\frac{-6}{2\cdot 3}=1$.

$y(x_0)=3\cdot 1^2-6\cdot 1+4=1$; $C(1;\ 1)$ – parabola uchi.

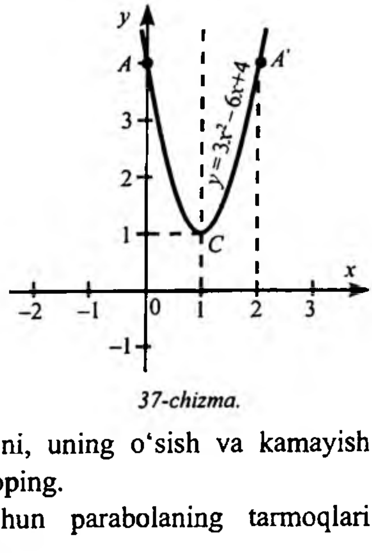
2) Funksiya nol bo'ladigan qiymatlarni topamiz. $3x^2-6x+4=0$.

$x_{1/2}=\frac{3\pm\sqrt{9-12}}{3}=\frac{3\pm\sqrt{-3}}{3}$; tenglama ildizga ega emas, ya'ni B va B' nuqtalar mavjud emas.

Bu holda parabola OY o'qni kesadigan A nuqtani va unga simmetrik bo'lgan A' nuqtalarni topamiz.

$x=0$ da $y=3\cdot 0^2-6\cdot 0+4=4$; $A(0;\ 4)$; $A'(x_0\cdot 2;\ 4)=A'(2;\ 4)$.

3) C, A va A' nuqtalar bo'yicha parabolaning grafigini sxematik chizamiz (37-chizma).



37-chizma.

Funksiya x ning $(-\infty;\ +\infty)$ oralig'ida musbat, manfiy qiymatga ega emas;

Funksiya $(-\infty;\ 1)$ oraliqda kamayadi, $(1;\ +\infty)$ oralikda o'sadi;

Funksiya $x=1$ da eng kichik $y=1$ qiymatga teng.

3-misol. $y=-3x^2-5x+2$ funksiya x ning qanday qiymatlarida nolga teng, noldan katta, noldan kichik ekanini, uning o'sish va kamayish oraliqlarini hamda eng kichik qiymatini toping.

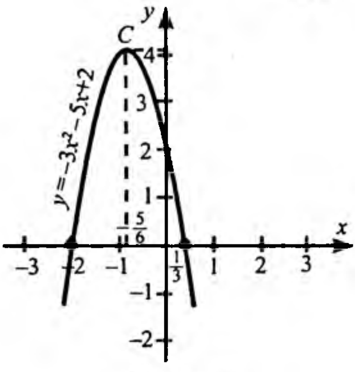
Yechish. 1) $a=-3<0$ bo'lgani uchun parabolaning tarmoqlari pastga qaragan.

$C(x_0;\ y(x_0))$ – parabolaning uchi: $x_0=-\frac{-5}{2\cdot(-3)}=-\frac{5}{6}$.

$y(x_0)=-3\cdot\left(-\frac{5}{6}\right)^2-5\cdot\left(-\frac{5}{6}\right)+2=-\frac{25}{12}+{}^{2/}\frac{25}{6}+{}^{12/}2=\frac{49}{12}=4\frac{1}{12}$; $C\left(-\frac{5}{6};\ 4\frac{1}{12}\right)$.

2) Funksiya nol bo'ladigan qiymatlarini topamiz: $-3x^2-5x+2=0$; $3x^2+5x-2=0$;

$x_{1/2}=\frac{-2,5\pm\sqrt{6,25+6}}{3}=\frac{-2,5\pm3,5}{3}$; $x_1=-2$; $x_2=\frac{1}{3}$. Bunda parabola OX o'qi bilan $B(-2;\,0)$ va $B'\left(\frac{1}{3};\,0\right)$ nuqtalarda kesishadi.



38-chizma.

3) C, B va B' nuqtalar orqali parabolani sxematik chizamiz (38-chizma).

Funksiya x ning $\left(-2;\,\frac{1}{3}\right)$ oralig'ida musbat. Funksiya x ning $(-\infty;\,-2)$ va $\left(\frac{1}{3};\,+\infty\right)$ oralig'ida manfiy bo'ladi; funksiya x ning $\left(-\infty;\,-\frac{5}{6}\right)$ oralig'ida o'sadi va x ning $\left(-\frac{5}{6};\,+\infty\right)$ oralig'ida kamayadi;

Funksiya $x=-\frac{5}{6}$ da eng katta $y=4\frac{1}{12}$ qiymatiga ega.

Ba'zida funksiyaning eng katta qiymatini $\max f(x)$ yoki $\max y(x)$ shaklida yoziladi. Misoldagi funksiyaning eng katta qiymatini $\max y(x)=4\frac{1}{12}$ kabi yoziladi. Funksiyaning eng kichik qiymatini $\min f(x)$ yoki $\min y(x)$ kabi yoziladi.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani o'suvchi funksiya deyiladi?
2. Qanday funksiyani kamayuvchi funksiya deyiladi?
3. Parabolaning uchi – $C(x_0;\,y(x_0))$ nuqta qanday topiladi?
4. Parabolaning OX o'qini kesib o'tadigan nuqtalari B va B' lar qanday topiladi?
5. Parabola sxematik qanday chiziladi?
6. $y=x^2$ parabolani o'sish va kamayish oraliqlarini ayting.

MASALALARNI YECHING

442. Funksiya x ning qanday qiymatlarida musbat qiymatlar va manfiy qiymatlar oladi?

a) $y=3x^2$; b) $y=-2,5x^2$; d) $y=x^2+1,4$;

e) $y=-0,25x^2+4$; f) $y=4x^2+x$; g) $y=-2x^2+7x$.

443. Funksiya x ning qanday qiymatlarida o'sadi va qanday qiymatlarida kamayadi.

a) $y=4x^2+x$; b) $y=6x^2-x-1$;

d) $y=x^2+10x-25$; e) $0,5x^2+4x+10$.

444. x ning $y>0$ bo'ladigan va $y<0$ bo'ladigan qiymatlarini toping:

a) $y=2x^2+x-6$; d) $y=-9x^2+12x-4$;

b) $y=16x^2-8x+1$; e) $y=-0,5x^2+5x-15$.

445. Funksiyani o'sish va kamayish oraliqlarini toping.

a) $y=0,2x^2+1,6x+1,2$; d) $y=-0,7x^2-2,8x+4,8$;

b) $y=4x^2-20x+25$; e) $y=-3,5x^2+2,3$.

446. $y=5x^2$; $y=-5x^2$; $y=5x^2+3$ va $y=5(x-2)^2$ funksiyalarning grafiklarini bitta koordinata tekisligida sxematik chizing va ularni o'sish va kamayish oraliqlarini yozing.

61-§. Ikkinchi darajali tengsizliklar

Ikkinchi darajali bir noma'lumli tengsizliklar deb, quyidagicha ko'rinishdagi tengsizliklarga aytiladi:

$$ax^2+bx+c>0 \qquad (1)$$

va

$$ax^2+bx+c<0 \qquad (2),$$

bunda $a\neq 0$, b va c – haqiqiy sonlar (2) ko'rinishdagi tengsizlikni -1 ga ko'paytirib, uni (1) ko'rinishga keltirish mumkin. Bundan keyin (1) ko'rinishdagi tengsizlikni qarab chiqish bilan cheklanamiz.

Tengsizlikni yechish, x ning qanday qiymatlarida bu tengsizlik to'g'ri ekanini aniqlash demakdir. Buning ma'nosi $ax^2+bx+c>0$ x ning qanday qiymatlarida tengsizlikning chap qismidagi uchhad musbat son bo'lsin degan ma'nodir.

Bir necha misol yechamiz:

1-m i s o l. $2x^2-13x+15>0$ tengsizlikni yeching.

Yechish. Bu tengsizlikni yechish uchun x ning qanday qiymatlarida uchhad musbat bo'lishini topamiz.

Yechishni quyidagi tartibda bajaramiz:

1) Birinchi koeffitsiyent $a=2>0$, parabola tarmoqlari yuqoriga qaragan.

2) Uchhadning diskriminanti $D=b^2=4ac=13^2-4\cdot2\cdot15=49>0$. Uchhad 2 ta ildizga ega.

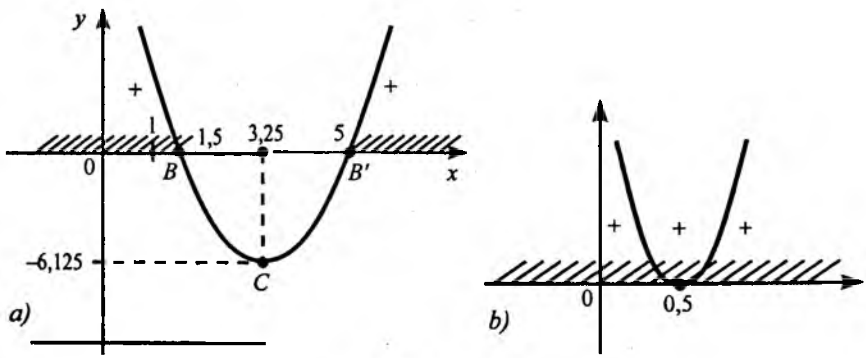
3) $2x^2-13x+15=0$; $x_{1/2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{13\pm\sqrt{49}}{2\cdot2}=\frac{13\pm7}{4}$; $x_1=1,5$; $x_2=5$.

4) Parabola OX o‘qini $B(1,5;\ 0)$ va $B'(5;\ 0)$ nuqtalarda kesadi. Parabola uchining koordinatalari topiladi.

$$x_0=-\frac{b}{2a}=\frac{-(-13)}{2\cdot2}=\frac{13}{4}=3,25;\quad y(x_0)=2\cdot3,25^2-13\cdot3,25+15=-6,125.$$

Parabolaning uchi $C(3,25;\ -6,125)$.

5) Uchta B; B' va C nuqtalar orqali parabolani sxematik chizamiz (39-*a* chizma). Demak, tengsizlik x ning 1,5 dan kichik va 5 dan katta qiymatlarida to‘g‘ri (musbat). *Javob:* $(-\infty;\ 1,5)$ va $(5;\ +\infty)$.



39-chizma.

2-m i s o l. $-4x^2+4x-1<0$ tengsizlikni yeching.

1) Bu tengsizlikning ikkala qismini -1 ga ko‘paytirib $4x^2-4x+1>0$ tengsizlikni hosil qilamiz.

Bunda $a=4>0$.

2) $D=b^2-4ac=(-4)^2-4\cdot4\cdot1=16-16=0$.

3) $4x^2-4x+1=0$ dan $x_1=\frac{4\pm\sqrt{0}}{2\cdot4}=0,5$.

4) Parabola $B(0,5;\ 0)$ nuqtada OX o‘qiga urinib, tarmoqlari yuqoriga qaragan bo‘ladi. Parabolani sxematik chizamiz.

Yechim: $(-\infty;\ 0,5)$ va $(0,5;\ +\infty)$ yoki $x\neq0,5$.

3-misol. $4x^2-4x+15<0$ tengsizlikni yeching.

1) Bu tengsizlikning ikkala qismini -1 ga ko‘paytirib $-4x^2+4x-15>0$ tengsizlikni hosil qilamiz. Bunda $a=-4<0$. Parabolaning tarmoqlari pastga qaragan.

2) $D=b^2-4ac=4^2-4\cdot(-4)\cdot(-15)=$ $=16-260=-244<0$.

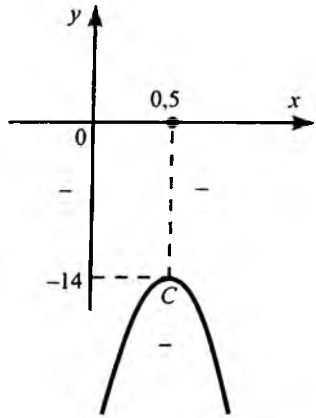
3) $-4x^2+4x-15=0$ tenglama ildizga ega emas, yani OX o‘qini kesmaydi.

4) $x_0=\frac{-b}{2a}=\frac{-4}{2\cdot(-4)}=0,5;$ $y(x)_0=-4\cdot\left(\frac{1}{2}\right)^2+$ $+4\cdot\frac{1}{2}-15=-14;$ $C(0,5;-14)$.

5) Parabolani sxematik tasvirlaymiz.

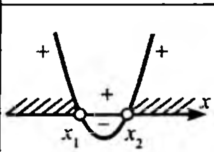
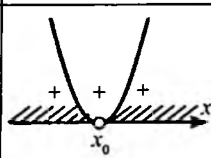
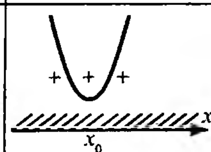
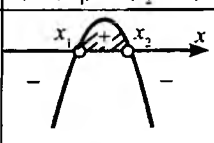
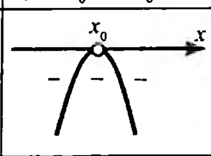
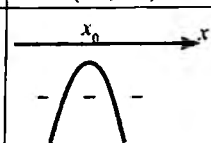
Demak, tengsizlik x ning barcha qiymatlarida manfiy (40-chizma).

Javob: yechimi yo‘q.



40-chizma.

Umumiy holdagi $ax^2+bx+c>0$ tengsizlikning yechimlari jadvalini keltiramiz.

	$D>0$	$D=0$	$D<0$
$ax^2+bx+c>0$ $a>0$			
yechimi	$(-\infty; x_1)$ va $(x_2; +\infty)$	$(-\infty; x_0)$ va $(x_0; +\infty)$	$(-\infty; +\infty)$
$ax^2+bx+c>0$ $a<0$			
yechimi	$(x_1; x_2)$	yechimi yo‘q	yechimi yo‘q

$ax^2+bx+c>0$ tengsizlikni jadval yordamida yechishga misollar keltiramiz.

4-m i s o l. $-4x^2+8x+5>0$ tengsizlikni yeching.

Yechish. 1) $a=-4<0;$ $D=b^2-4ac=8^2-4\cdot(-4)5=64+80=144.$

2) $-4x^2+8x+5=0;$ $4x^2-8x-5=0;$ $x_{1/2}=\frac{8\pm12}{2\cdot4};$ $x_1=-0,5$

$x_2=2,5.$

3) Jadval bo‘yicha yechimni topamiz.

Yechim $(x_1; x_2)$ oraliqda, ya’ni $(-0,5; 2,5)$ bo‘ladi.

Javob: $(-0,5; 2,5)$.

5-m i s o l. $-9x^2+12x-4<0$ tengsizlikni yeching.

Yechish. Tengsizlikni -1 ga ko‘paytirib,

$9x^2-12x+4>0$ ga keltiramiz.

Bunda $a=9>0$; $D=(-12)^2-4\cdot9\cdot4=0$.

$9x^2+12x+4=0$ tenglama bitta $x_0=\frac{-12\pm0}{2\cdot9}=-\frac{2}{3}$; ildizga ega.

Jadvalga asosan yechim $x_0\neq-\frac{2}{3}$ barcha sonlar.

Javob: $\left(-\infty; -\frac{2}{3}\right)$; va $\left(-\frac{2}{3}; +\infty\right)$.

6-m i s o l. $2x(3x-1)>4x^2+5x+9$ tengsizlikni yeching.

Yechish. Qavsni ochib, tengsizlikni sodda ko‘rinishga keltiramiz:

$6x^2-2x-4x^2-5x-9>0$;

$2x^2-7x-9>0$.

1) $a=2>0$; $D=(-7)^2-4\cdot2\cdot(-9)=49+72=121$.

2) $2x^2-7x-9=0$ tenglamani yechamiz.

$$x_{1/2}=\frac{7\pm\sqrt{121}}{2\cdot2}=\frac{7\pm11}{4};\ x_1=\frac{-4}{4}=-1;\ x_2=\frac{18}{4}=4,5.$$

3) Jadval bo‘yicha yechimni topamiz.

Javob: $(-\infty; -1)$ va $(4,5; +\infty)$.

7-m i s o l. $y=\sqrt{6x^2-11x+3}$ funksiyaning aniqlanish sohasini toping.

Yechish. x argumentning (mumkin bo‘lgan) barcha qiymatlari to‘plami funksiyaning aniqlanish sohasi deb ataladi.

Masalan, $y=\sqrt{x}$ da x argument faqat, $[0; +\infty)$ oraliqdagi qiymatlarni qabul qiladi, ya’ni $y=\sqrt{x}$ funksiyaning aniqlanish sohasi $[0; +\infty)$ oraliq bo‘ladi. $y=\sqrt{6x^2-11x+3}$ funksiyaning aniqlanish sohasi $6x^2-11x+3\geq0$ tengsizlikning yechimi bo‘ladi. Bunda: 1) $a=6>0$; $D=11^2-4\cdot6\cdot3=121-72=49$.

2) $6x^2-11x+3=0$ tenglamani yechamiz.

$$x_{1/2}=\frac{11\pm\sqrt{49}}{2\cdot6}=\frac{11\pm7}{12};\quad x_1=\frac{1}{3};\quad x_2=1,5.$$

3) Jadval bo‘yicha yechimni topamiz:

$\left(-\infty; \frac{1}{3}\right]$ va $[1{,}5; +\infty)$. *Javob:* Aniqlanish sohasi. $\left(-\infty; \frac{1}{3}\right]$ va $[1{,}5; +\infty)$.

TAKRORLASH UCHUN SAVOLLAR

1. Ikkinchi darajali tengsizlikning umumiy ko‘rinishi qanday bo‘ladi?
2. Ikkinchi darajali tengsizlikning yechimi deganda nimani tushunamiz?
3. $ax^2+bx+c>0$ va $a>0$ dagi yechimlarini ayting.
4. $ax^2+bx+c>0$ va $a<0$ dagi yechimlarini ayting.
5. Funksiyaning aniqlanish sohasi deb nimaga aytiladi?
6. $y=\sqrt{5x}$ funksiyaning aniqlanish sohasini ayting.

MASALALARNI YECHING

447. Tengsizlikni yeching:

a) $x^2-8x+15>0$;　　e) $8x^2+10x-3>0$;

b) $6x^2-7x+2>0$;　　f) $-x^2-12x-100<0$;

d) $-x^2-2x+48<0$;　　g) $4x^2-4x+15<0$.

448. Tengsizlikning yechimlar to‘plamini toping:

a) $-6x^2+6x+36\geq 0$;　　d) $-2x^2-5x+18\leq 0$;

b) $-9x^2+12x-4<0$;　　e) $3x^2-2x>0$.

449. Tengsizlikni yeching:

a) $0{,}2x^2>1{,}8$;　　d) $3x^2<-2x$;

b) $\frac{1}{3}x^2\geq\frac{1}{9}$;　　e) $-0{,}3x<0{,}6x^2$.

450. Tengsizlikning yechimlari to‘plamini toping:

a) $3x^2+40x+10<-x^2+11x+3$;

b) $9x^2-x+9\geq 3x^2+18x-6$;

d) $2x^2+8x-111<(3x-5)(2x+6)$;

e) $(5x+1)(3x-1)>(4x-1)(x+2)$.

451. Funksiyaning aniqlanish sohasini toping:

a) $y=\sqrt{2x-x^2}$; b) $y=\sqrt{144-9x^2}$;

d) $y=\frac{12}{\sqrt{x^2+4x+4}}$; e) $y=\frac{2x}{\sqrt{x^2-x-42}}$.

452. x ning istalgan qiymatida tengsizlik to‘g‘ri bo‘lishini isbotlang:

a) $4x^2+12x+9\geq 0$; b) $-5x^2+8x-5<0$.

62-§. Ikkinchi darajali tengsizliklar sistemasini yechish

Biz avval 48-§ da bir o‘zgaruvchili tengsizliklar sistemasini o‘rgangan edik.

Bu paragrafda bir o‘zgaruvchili ikkinchi darajali tengsizlik qatnashgan sistemani yechish usullarini o‘rganamiz.

1-misol. $\begin{cases}3x^2+2x-5\geq 0\\ 2x-8,6\geq 0\end{cases}$ tengsizliklar sistemasini yeching.

Yechish. Bu tengsizliklar sistemasini yechish deganda tengsizlikdagi x o‘zgaruvchining tengsizliklarni qanoatlantiradigan qiymatlarini topish tushuniladi.

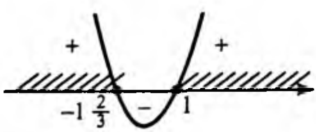
1) Avval $3x^2+2x-5\geq 0$ tengsizlikni yechamiz.

a) $a=3>0$; $D=2^2-4\cdot 3\cdot(-5)=64$.

b) $3x^2+2x-5=0$ tenglamani yechamiz.

$$x_{1/2}=\frac{-2\pm\sqrt{64}}{2\cdot 3}=\frac{-2\pm 8}{6};\quad x_1=\frac{-5}{3}\neq -1\frac{2}{3};\quad x_2=1.$$

d) Parabolani sxematik chizib, yechimni topamiz.



40-chizma.

Tengsizlik yechimi $\left(-\infty; -1\frac{2}{3}\right]$ va $(1; +\infty]$.

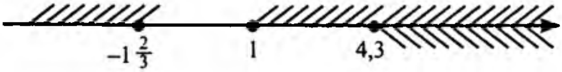
2) $2x-8,6\geq 0$ ni yechamiz; $2x\geq 8,6$

$x\geq 4,3$.

yechim $[4,3; +\infty)$.

Bu ikki tengsizlik yechimlarining umumiy yechimini topamiz:

$$3)\ \begin{cases}\left(-\infty; -1\frac{2}{3}\right] \text{ va } (1; +\infty] \\ [4{,}3; +\infty)\end{cases}$$



Umumiy yechim [4,3; +∞]

Javob: [4,3; +∞).

2-misol. $\begin{cases}3x^2+4x \ge 7 \\ 9x^2-24x+16 \le 0\end{cases}$ tengsizliklar sistemasini yeching.

Yechish. 1) $3x^2+4x \ge 7$ ni yechamiz.

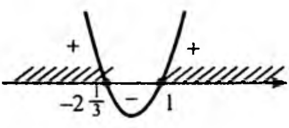
$3x^2+4x-7 \ge 0$. Bunda

a) $a=3>0$; $D=16-4\cdot 3\cdot(-7)=100$.

b) $3x^2+4x-7=0$.

$$x_{1/2}=\frac{-4\pm\sqrt{100}}{2\cdot 3}=\frac{-4\pm 10}{6};\quad x_1=-2\frac{1}{3};\quad x_2=1.$$

d) Parabolani sxematik chizib yechimini topamiz (41-chizma).



41-chizma.

tengsizlik yechimi $\left(-\infty; -2\frac{1}{3}\right]$ va $[1; +\infty)$.

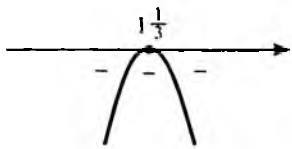
2) $9x^2-24x+16 \le 0$ tengsizlikni yechamiz.

a) $a=9>0$; $D=24^2-4\cdot 9\cdot 16=576-576=0$.

b) Tengsizlikni –1 ga ko'paytirib $-9x^2+24x-16 \ge 0$ hosil qilamiz.

$-9x^2+24x-16=0$ tenglamani yechamiz.

$$x_0=\frac{-24\pm 0}{2\cdot(-9)}=\frac{24}{18}=\frac{4}{3}=1\frac{1}{3}.$$



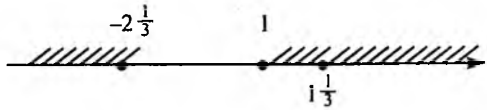
42-chizma.

d) Parabolani sxematik chizib yechimini topamiz.

Yechimi: $x=1\frac{1}{3}$ (42-chizma).

Bu ikki tengsizlik yechimlarining umumiy yechimini topamiz.

$$\begin{cases}\left(-\infty; -2\frac{1}{3}\right] \text{ va } [1; +\infty) \\ x = 1\frac{1}{3}\end{cases}$$



Umumiy yechim: $x = 1\frac{1}{3}$. *Javob:* $1\frac{1}{3}$.

3- m i s o l. Ushbu $\frac{x^2-8x+7}{5x-3} > 0$ tengsizlikni yeching.

Yechish. Bu tengsizlikni yechish quyidagi ikki sistemaning yechilishiga keltiriladi.

1) $\begin{cases} x^2 - 8x + 7 > 0 \\ 5x - 3 > 0 \end{cases}$; 2) $\begin{cases} x^2 - 8x + 7 < 0 \\ 5x - 3 < 0 \end{cases}$.

1. Birinchi sistemadagi ikki tengsizlikni yechib, sistema yechimini topamiz:

$x^2 - 8x + 7 > 0$ da $a = 1 > 0$ va $D = (-8)^2 - 4 \cdot 1 \cdot 7 = 36 > 0$.

$x^2 - 8x + 7 > 0$ tenglamani yechamiz.

$x_{1/2} = 4 \pm \sqrt{16 - 7} = 4 \pm 3$; $x_1 = 1$; $x_2 = 7$.

Tengsizlikning yechimi $(-\infty; 1)$ va $(7; +\infty)$.

$5x - 3 > 0$; $5x > 3$; $x > 0{,}6$; ya'ni $(0{,}6; +\infty)$.

Sistemaning yechimi: $\begin{cases} (-\infty; 1) \text{ va } (7; +\infty) \\ (0{,}6; +\infty) \end{cases}$.

43-chizma.

1-sistemaning yechimi: $(0{,}6; 1)$ va $(7; +\infty)$.

2. $\begin{cases} x^2 - 8x + 7 < 0 \\ 5x - 3 < 0 \end{cases}$ sistemani yechamiz.

$x^2 - 8x + 7 < 0$ tengsizlikni -1 ga ko'paytiramiz.

$-x^2 + 8x + 7 > 0$.

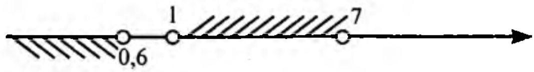
$a=-1<0$ va $D=82-4\cdot(-1)\cdot(-7)=36>0$.

$-x^2+8x-7=0$ tenglamaning ildizi 1 va 7.

$-x^2+8x-7>0$ tengsizlikning yechimi (1; 7).

$5x-3<0$ ning yechimi $5x<3$; $x<0,6$, ya'ni $(-\infty; 0,6)$.

2-sistemaning yechimi: $\begin{cases}(1; 7)\\ -\infty;\ 0,6\end{cases}$



44-chizma.

Bu holda sistema yechimga ega emas.

Javob: (0,6; 1) va $(7; +\infty)$ (1 – sistemaning echimi).

4-misol. Ushbu $\frac{-3x^2-10x+25}{x^2-9x+14}\le 0$ tengsizlikni yeching.

Yechish. Bu tengsizlikni yechish quyidagi ikki sistemani yechishga keltiriladi.

1) $\begin{cases}-3x^2-10x+25\ge 0\\ x^2-9x+14<0\end{cases}$; 2) $\begin{cases}-3x^2-10x+25\le 0\\ x^2-9x+14>0.\end{cases}$

1_a) $-3x^2-10x+25\ge 0$ ni yechamiz.

$a=-3<0$ va $D=(-10)^2-4\cdot(-3)\cdot 25=100+300=400>0$.

$-3x^2-10x+25=0$ tenglamani yechamiz.

$3x^2+10x-25=0$;

$$x_{1/2}=\frac{-5\pm\sqrt{25+75}}{3}=\frac{-5\pm 10}{3};\quad x_1=-5;\ 1\frac{2}{3};$$

Tengsizlikning yechimi: $\left[-5;\ 1\frac{2}{3}\right]$.

1_b) $x^2-9x+14<0$ tengsizlikni yechamiz.

$-x^2+9x-14>0$

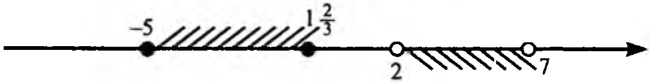
$a=-1>0$; $D=(-9)^2-4\cdot 1\cdot 14=81-56=25>0$.

$x^2-9x+14=0$ tenglamani yechamiz.

$$x_{1/2}=4,5\pm\sqrt{20,25-14}=4,5\pm 2,5;\quad x_1=2;\quad x_2=7.$$

Tengsizlikning yechimi (2; 7).

Sistemaning yechimini son o'qida tasvirlaymiz.



45-chizma.

sistemaning yechimi yo'q.

a) $\begin{cases} -3x^2-10x+25\le 0 \\ x^2-9x+14>0 \end{cases}$ sistemani yechamiz.

$-3x^2-10x+25\le 0$ ni -1 ga ko'paytiramiz.

$3x^2+10x-25\ge 0$. Bunda: $a=3>0$; $D>0$.

$3x^2+10x-25=0$ tenglamaning ildizi $x_1=-5$; $x_2=1\frac{2}{3}$.

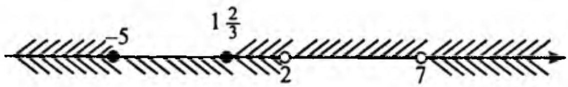
Tengsizlikning yechimi: $(-\infty; -5]$ va $\left[1\frac{2}{3}; +\infty\right)$.

b) $x^2-9x+14>0$; bunda: $a=1>0$; $D>0$.

$x^2-9x+14=0$ tenglamaning ildizi $x_1=2$ va $x_2=7$.

Tengsizlikning yechimi: $(-\infty; 2)$ va $(7; +\infty)$.

Sistemaning yechimi:



46-chizma.

Sistemaning yechimi: $(-\infty; -5]$, $\left[1\frac{2}{3};2\right)$ va $(7; +\infty)$.

Javob: $(-\infty; -5]$, $\left[1\frac{2}{3};2\right)$ va $(7; +\infty)$.

5-m i s o l. $y=\sqrt{\frac{2x^2+13x-7}{5x-x^2}}$ funksiyaning aniqlanish sohasini toping.

Funksiyaning aniqlanish sohasi $\frac{2x^2+13x-7}{5x-x^2}\ge 0$ tengsizlikni yechish orqali topiladi. Bu tengsizlikni yechish quyidagi ikki tengsizliklar sistemasini yechishga keltiriladi.

1) $\begin{cases} 2x^2+13x-7\ge 0 \\ 5x-x^2>0 \end{cases}$; 2) $\begin{matrix} 2x^2+13x-7\le 0 \\ 5x-x^2<0 \end{matrix}$.

a) $2x^2+13x-7\geq 0$.

$a=2>0$ va $D=13^2-4\cdot 2\cdot(-7)=169+56=225>0$.

$2x^2+13x-7=0$ tenglamani yechamiz.

$$x_{1/2}=\frac{-13\pm\sqrt{225}}{2\cdot 2}=\frac{-13\pm 15}{4};\quad x_1=-7;\quad x_2=0,5;$$

tengsizlikning yechimi: $(-\infty; -7]$ va $[0,5; +\infty)$.

b) $5x-x^2>0$ ni yechamiz, ya'ni $-x^2+5x>0$.

Bunda: $a=-1$; $D=25>0$.

$-x^2+5x=0$.

$x(-x+5)=0$; $x_1=0$; $x_2=5$,

tengsizlikning yechimi: $(0; 5)$.

1-sistemaning yechimini topamiz:

$$\begin{cases}(-\infty; -7] \text{ va } [0,5; +\infty)\\(0; 5)\end{cases}$$

47-chizma.

Sistemaning yechimi: $[0,5; 5)$.

2) $\begin{cases}2x^2+13x-7\leq 0;\\5x-x^2<0\end{cases}$ a) $2x^2+13x-7\leq 0$ ni -1 ga ko'paytiramiz.

$-2x^2-13x+7\geq 0$

Bunda: $a=-2<0$ va $D=(-13)^2-4\cdot(-2)\cdot 7=225>0$.

$-2x^2-13x+7=0$ tenglamani yechamiz.

$$x_{1/2}=\frac{13\pm\sqrt{225}}{-2\cdot 2}=\frac{13\pm 15}{-4};\quad x_1=0,5;\quad x_2=-7,$$

tengsizlikning yechimi: $[-7; 0,5]$.

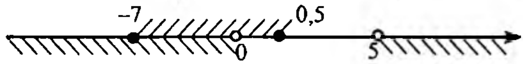
b) $-x^2+5x<0$ ni yechamiz; $x^2-5x>0$.

$a=-1<0$ va $D=225>0$.

$-x^2+5x=0$ tenglamani yechamiz. Bunda:

$x_1=0$ va $x_2=5$. Tengsizlik yechimi: $(-\infty; 0)$ va $(5; +\infty)$.

2-sistemaning yechimini topamiz:



yechim: [–7; 0).

Javob: Funksiyaning aniqlanish sohasi: [–7; 0) va [0,5; 5).

MASALALARNI YECHING

453. Tengsizliklar sistemasini yeching:

a) $\begin{cases} 2x^2-18>0 \\ x<0 \end{cases}$; d) $\begin{cases} -5x^2-3,5x\geq 0 \\ 2x+3>0 \end{cases}$;

b) $\begin{cases} -4x^2+32\leq 0 \\ -3x<0 \end{cases}$; e) $\begin{cases} 3x^2+4x-7\leq 0 \\ 4x^2-3,6x\geq 0 \end{cases}$.

454. Tengsizliklar sistemasini yeching:

a) $\begin{cases} 21x^2+39x-6<0 \\ x<0 \end{cases}$; d) $\begin{cases} x^2-144>0 \\ x-3<0 \end{cases}$;

b) $\begin{cases} x^2-3x-4<0 \\ 3x-12>0 \end{cases}$; e) $\begin{cases} x^2+5x<0 \\ x+7>0 \end{cases}$.

455. Tengsizliklar sistemasini yeching:

a) $\begin{cases} x^2+x-6<0 \\ -x^2+2x+3>0 \end{cases}$; b) $\begin{cases} x^2+4x-5>0 \\ x^2-2x-8<0 \end{cases}$.

456*. Tengsizlikni yeching:

a) $\frac{x^2-5x+8}{x^2-8x+15}>0$; b) $\frac{9x^2-12x+4}{3x^2-8x-3}<0$.

457*. Funksiyaning aniqlanish sohasini toping:

a) $y=\sqrt{\frac{9x^2-24x+16}{x+2}}$; b) $y=\frac{2x-7}{\sqrt{-x^2+x+30}}$.

XIII bob. YUQORI DARAJALI TENGLAMALAR VA TENGLAMALAR SISTEMALARI

63-§. Butun tenglama va uning darajasi

$2(x^2+3)(x-1)=6x-(x-7)$ va

$\frac{x^4-1}{4}-\frac{x^2+1}{2}=3x^2$. Tenglamalarning har birida chap va o'ng qismlar butun ifodalardir. Yoki maxrajlarida o'zgaruvchi (noma'lum son) bo'lmagan tenglamalarni butun tenglamalar deyiladi. Birinchi tenglamaning qavslarini ochib, hamma hadlarini tenglamaning chap qismiga o'tkazamiz va o'xshash hadlarni ixchamlaymiz.

$(2x^2+6)(x-1)-6x+x-7=0$

$2x^3-2x^2+6x-6-5x-7=0$

$2x^3-2x^2+x-13=0.$

Ikkinchi tenglamaning ikkala qismini dastlab 4 ga ko'paytirib, o'xshash shakl almashtirish bajarib, $x^4-14x^2-3=0$ ko'rinishga keltiriladi.

Ko'rilgan misollarning har birida berilgan tenglamaga teng kuchli tenglamaga keltiradigan shakl almashtirishlarni bajardik.

Natijada hamma hadlari chap tomonda bo'lgan umumiy ko'rinishdagi

$P(x)=0$ tenglamani hosil qildik. Bundagi $P(x)$ – standart shakldagi ko'phad bo'ladi.

Agar bir o'zgaruvchili tenglama $P(x)=0$ ko'rinishda yozilgan bo'lsa, bundagi $P(x)$ ko'phadning darajasi tenglamaning darajasi deyiladi. Masalan, $x^3-2x+1=0$ tenglama uchinchi darajali tenglamadir.

$P(x)=0$ tenglamaning darajasi $P(x)$ – standart shakldagi ko'phadning darajasiga teng.

Masalan, $(x^3-1)^2+x^5=x^6-2$ tenglama uchun:

$$x^6-2x^3+1+x^5=x^6-2$$

$$x^5-2x^3+3=0.$$

Hosil bo'lgan tenglamaning darajasi beshga teng. Demak, unga teng kuchli bo'lgan berilgan tenglamaning darajasi ham beshga teng.

Bir o'zgaruvchili yuqori darajali tenglamalarning umumiy ko'rinishlarini ko'rib chiqamiz.

1. Birinchi darajali tenglama $ax+b=0$ ko'rinishda bo'ladi, bunda x – o'zgaruvchi, $a \neq 0$ va b – biror sonlar.

Bundan $ax=-b$; $x=-\frac{b}{a}$ bo'lib, $-\frac{b}{a}$ son tenglamaning ildizi. Har bir birinchi darajali tenglama bitta ildizga ega.

2. Barcha ikkinchi darajali tenglamalar $ax^2+bx+c=0$ ko'rinishga keltiriladi. Buni kvadrat tenglamaning umumiy ko'rinishi deyiladi, bunda x – o'zgaruvchi, $a \neq 0$, b va c – sonlar. Bu $D=b^2-4ac$ ifodani kvadrat tenglamaning diskriminanti deyiladi. Kvadrat tenglamaning ildizlari $x=\frac{-b\pm\sqrt{D}}{2a}$ formula bo'yicha topiladi:

Agar $D>0$ bo'lsa, tenglama ikkita ildizga ega;

Agar $D=0$ bo'lsa, tenglama bitta ildizga ega;

Agar $D<0$ bo'lsa, tenglama ildizga ega emas.

3. Uchinchi darajali tenglama $ax^3+bx+cx+d=0$ ko'rinishga ega bo'ladi. Bunda: x – o'zgaruvchi, $a\neq 0$, b, c, d – sonlar. Bu tenglamaning umumiy yechimi oliy matematika kursida o'rganiladi.

4. To'rtinchi darajali tenglama $ax^4+bx^3+cx^2+dx+e=0$ ko'rinishga ega bo'ladi. Bunda: x – o'zgaruvchi, $a \neq 0$, b, c, d, e – sonlar.

Uchinchi darajali tenglama uchtadan ortiq ildizga ega emasligini, to'rtinchi darajali tenglama to'rttadan ortiq ildizga ega emasligini isbotlash mumkin.

5. n – darajali tenglama $ax^n+bx^{n-1}+...+kx+l=0$ ko'rinishga ega bo'ladi. Bunda: x – o'zgaruvchi (noma'lum son), $a \neq 0$, b,... k, l – sonlar.

Uchinchi va to'rtinchi darajali tenglamalar uchun ildizlar formulasi mavjud, lekin bu formulalar juda murakkabligi tufayli uning bayoni qo'shimcha bo'limda keltirilgan.

Beshinchi va undan yuqori darajali tenglamalar uchun ildizlarning umumiy formulalari mavjud emas.

Shuni aytish kerakki, ba'zan uchinchi yoki yanada yuqori darajali tenglamalarni grafik yoki maxsus usulni qo'llab yechishga muvaffaq bo'linadi. Masalan, ba'zi tenglamalar ko'phadni ko'paytuvchilarga ajratish yordamida yechiladi.

1-m i s o l. $x^3-8x^2-x+8=0$ tenglamani yechamiz.

Tenglamani ko'paytuvchilarga ajratamiz:

$(x^3-8x^2)-(x-8)=0$

$x^2(x-8)-(x-8)=0$

$(x-8)\cdot(x^2-1)=0$

$(x-8)(x-1)(x+1)=0$. Ko'paytmaning nolga teng bo'lishlik shartidan:

$x-8=0$; $x-1=0$ va $x+1=0$ ekanligidan $x_1=8$; $x_2=1$; $x_3=-1$ kelib chiqadi.

Bundan $x^3-8x^2-x+8=0$ tenglama uchta ildizga ega ekanligi kelib chiqadi:

Ildizlari: $x_1=8$; $x_2=1$; $x_3=-1$.

2-m i s o l. $2x^4-18x^2=5x^3-45x$ tenglamani yechamiz.

Yechish. $2x^4-18x^2-5x^3-45x$

$2x^2(x^2-9)-5x(x^2-9)=0$.

$(x^2-9)(2x^2-5x)=0$.

$(x-3)(x+3)\ x(2x-5))=0$.

$x(x-3)(x+3)(2x-5)=0$. Bundan:

$x=0$; $x-3=0$; $x+3=0$; $2x-5=0$.

$x_1=0$; $x_2=3$; $x_3=-3$; $x_4=2,5$.

Ildizlari: 0; –3; 2,5; 3.

MASALALARNI YECHING

458. Tenglamaning darajasi qanday?

a) $2x^2-6x^5+1=0$; d) $(x+8)(x-7)=0$; f) $\frac{x}{2}-\frac{3x}{4}=5$;

b) $x^6-4x^3-3=0$; e) $\frac{1}{7}x^5=0$; g) $5x^3-5x(x^2+4)=1$.

459. Tenglamani yeching:

a) $(8x-1)(2x-3)-(4x-1)^2=38$;

b) $\frac{(15x-1)(1+15x)}{3}-(3x+1)(25x-2)=20\frac{2}{3}$;

d) $9x^2 - \frac{(12x-11)(3x+8)}{4} = 1$; e) $x^4 - x^2 = \frac{(1+2x^2)(2x^2-1)}{4}$.

460. Tenglamani yeching:

a) $y^3-6y=0$;
b) $6x^4+3,6x^2=0$;
d) $x^3-0,1x=0,3x^2$;
e) $9x^3-18x^2-x+2=0$
f) $p^3-p^2=p-1$;
g) $x^4-x^2=3x^3-3x$;
h) $3x^3-x^2-18x-6=0$;
i) $3y^2-2y=2y^3-3$.

461. b ning qanday qiymatlarida tenglama ikkita ildizga ega bo'ladi?

a) $2x^2+6x+b=0$;
b) $5x^2-4x+3b=0$;
d) $3x^2+bx+3=0$;
e) $x^2-bx+5=0$.

Ko'rsatma. Kvadrat tenglama ikkita har xil ildizga ega bo'lishi uchun $D=b^2-4ac>0$ bo'lishi kerak.

462. u ning qanday qiymatlarida tenglama ildizga ega bo'lmaydi?

a) $6x^2+ux+6=0$;
b) $12x^2+4x-u=0$;
d) $2x^2-15x+u=0$;
e) $2x^2+ux+18=0$.

463. $5x^6+6x^4+x^2+4=0$ tenglamaning ildizga ega emasligini isbotlang.

464. $12x^5+7x^3+11x-3=121$ tenglama manfiy ildizlarga ega bo'lishi mumkinmi?

64-§. Kvadrat tenglamaga keltiriladigan tenglamalar

Darajasi ikkidan yuqori bo'lgan tenglamalarni ba'zan yangi o'zgaruvchi kiritish bilan yechish mumkin bo'ladi.

Tenglamalarni ana shu usul bilan yechishga misollar ko'rib chiqamiz.

1-m i s o l. Ushbu $(x^2-5x)^2-30(x^2-5x)-216-0$ tenglamani yechamiz.

Yechish: Bu tenglamaning chap qismini standart shakldagi ko'phadga almashtiramiz: $x^4-10x^3+25x^2-30x^2+150x-216=0$ hosil qilib, uni yechish usulini topish ancha qiyin bo'ladi.

Berilgan tenglamada x^2-5x ifoda ikki marta qatnashgan. U bir marta ikkinchi darajada, bir marta birinchi darajada. Bu hol berilgan tenglamani yangi o'zgaruvchini kiritish yordamida yechishga imkon beradi. x^2-5x ni y bilan belgilaymiz, ya'ni $x^2-5x=y$. Berilgan tenglama $y^2-30y-216=0$ ga keltirildi.

Bu tenglamani yechib, $y_1=-6$ va $y_2=36$ larni topamiz. Bundan:

1) $x^2-5x=-6$ tenglamani yechib, $x_1=2$ va $x_2=3$ topiladi.

2) $x^2-5x=36$ tenglamani yechib, $x_3=-4$ va $x_4=9$ topiladi.

Demak, berilgan tenglama to'rtta ildizga ega:

Javob: 2; 3; –4 va 9.

$ax^4+bx^2+c=0$ ko'rinishdagi to'rtinchi darajali tenglamani yangi o'zgaruvchi kiritish usuli bilan osongina yechiladi.

Bu tenglama x^2 ga nisbatan kvadrat tenglama bo'lgani uchun $\boldsymbol{ax^4+bx^2+c=0}$ ko'rinishdagi tenglama (bunda $a\neq 0$) **Bikvadrat tenglama** deyiladi.

2-m i s o l. $4x^4-5x^2+1=0$ Bikvadrat tenglamani yechamiz.

Yechish. $x^2=y$ bo'lsin, u holda berilgan tenglama u o'zgaruvchiga nisbatan $4y^2-5y+1=0$ hosil bo'ladi.

$$y_{1/2}=\frac{2{,}5\pm\sqrt{6{,}25-4}}{4}=\frac{2{,}5\pm1{,}5}{4};\quad y_1=\frac{1}{4};\, y_2=1.$$

1) $x^2=y_1$ dan $x^2=\frac{1}{4};\, x_{1/2}=\pm\frac{1}{2};\, x_1=-\frac{1}{2};\, x_2=\frac{1}{2}$.

2) $x^2=y_2$ dan $x^2=1;\, x_{3/4}=\pm1;\, x_3=-1;\, x_4=1$.

Demak, berilgan tenglama to'rtta ildizga ega: $-\frac{1}{2}$; $\frac{1}{2}$; -1 va 1.

3-misol. $x^5+x^4-6x^3-6x^2+5x+5=0$ tenglamani yechamiz.

Yechish. Bu tenglamani avval ko'paytuvchilarga ajratib, so'ngra ko'paytuvchilarni nolga tenglab yechamiz:

$(x^5+x^4)+(-6x^3-6x^2)+(5x+5)=0.$

$x^4(x+1)-6x^2(x+1)+5(x+1)=0.$

$(x+1)(x^4-6x^2+5)=0.$

1) $x+1=0.$ 2) $x^4-6x^2+5=0$, bunda $x^2=y$ bo'lsin.

$x_1=-1.$ $y^2-6y+5=0.$

$y_{1/2}=3\pm\sqrt{9-5}=3\pm2;\quad y_1=1;\, y_2=5.$

3) $x^2=y_1$ dan $x^2=1$; $x_{2/3}=\pm1$; $x_2=-1$ va $x_3=1$.

4) $x^2=y_2$ dan $x^2=5$; $x_{4/5}=\pm\sqrt{5}$; $x_4=\sqrt{5}$; $x_4=-\sqrt{5}$ va $x_5=\sqrt{5}$.

Bu ildizlarda $x_1=x_2=-1$ bo'lganidan berilgan tenglama to'rtta ildizga ega:

Javob: -1; 1; $-\sqrt{5}$ va $\sqrt{5}$.

MASALALARNI YECHING

465. Yangi o'zgaruvchini kiritishdan foydalanib, tenglamani yeching.

a) $(5x^2-4)^2+6(5x^2-4)-7=0$;

b) $(x^2-9)^2-8(x^2-9)+7=0$;

d) $2(y^2+2y)^2-7(y^2+2y)=-3$;

e) $(p^2-5)^2-3(p^2-5)=4$.

466. Tenglamani yeching:

a) $(p^2+2p+4)^2-7(p^2+2p+4)=-12$;

b) $(x^2+2x+3)^2-2(x^2+2x+3)=3$.

467. Bikvadrat tenglamani yeching:

a) $x^4+x^2-2=0$;

b) $y^4-7y^2-144=0$;

d) $36z^4-13z^2+1=0$;

e) $16x^4-10x^2+1=0$;

f) $p^4+2p^2+3=0$;

g) $2x^4-9x^2+4=0$.

468. Funksiya grafigining absissalar o'qi bilan kesishish nuqtalarining koordinatalarini toping.

a) $y=x^4-10x+9$;

b) $y=x^4-2x^2-3$;

d) $y=x^4+36x^2$.

469. Tenglamani yeching:

a) $(x^2-1)(x^2+1)-4(x^2-11)=0$;

b) $3x^2(x-1)(x+1)-10x^2+4=0$.

470. Tenglamani yeching:

a) $x^5-x^4-2x^3+2x^2-3x+3=0$

b) $x^5-2x^4-4x^3+8x^2+3x-6=0$.

65-§. Tenglamalar sistemalarini yechishning grafik usuli

Ilgari biz. ikki o'zgaruvchili birinchi darajali tenglamalar sistemalarini yechishni ko'rib o'tgan edik. Endi biz ikkinchi darajali ikkita tenglamadan yoki biri birinchi darajali tenglama, ikkinchisi ikkinchi darajali tenglamadan tuzilgan sistemalarni yechish bilan shug'ullanamiz.

Ikki o'zgaruvchini tenglamalar sistemalarini ***grafik usulda*** yechishga misol keltiramiz.

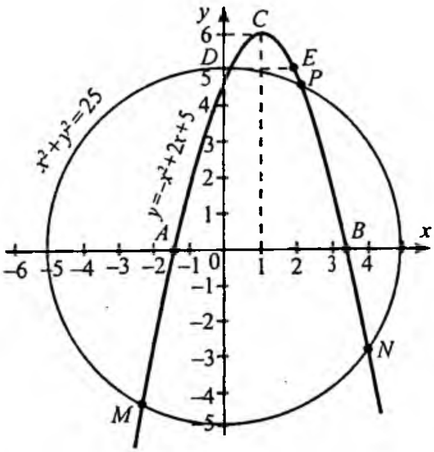
$$\begin{cases} x^2+y^2=25 \\ y=-x^2+2x+5 \end{cases}$$ tenglamalar sistemasini yechamiz:

Bitta koordinatalar sistemasida sistemadagi tenglamalarning grafiklarini yasaymiz: $x^2+y^2=25$ tenglamaning grafigi markazi (0; 0) nuqtada, radiusi, 5 birlik bo'lgan aylanadan iborat;

$y=-x^2+2x+5$ tenglamaning grafigi uchi $C(1;\ 6)$ nuqtada, tarmoqlari pastga qaragan parabola bo'ladi.

Parabola OX o'qini kesadigan nuqtalari $A(\approx-1{,}45;\ 0)$ va $B(3{,}45;\ 0)$. OY o'qini keladigan nuqtasi $D(0;\ 5)$, D nuqtaga parabola o'qiga nisbatan simmetrik nuqta – $E(2;\ 5)$.

48-chizmadan foydalanib, grafiklarning kesishish nuqtalari koordinatalarining taqribiy qiymatlarini topamiz: $M(-2{,}2;\ -4{,}5)$, $D(0;\ 5)$, $P(2{,}2;\ 4{,}5)$, $N(4;\ -3)$.



48-chizma.

Demak, tenglamalar sistemasi to'rtta yechimga ega:

$x_1 \approx -2,2; y_1 \approx -4,5; \quad x_2 \approx 0; y_2 \approx 5;$

$x_3 \approx 2,2; y_3 \approx 4,5; \quad x_4 \approx 4; y_4 \approx -3.$

Bu yechimlardan ikkinchisi va to'rtinchisi aniq, birinchisi va uchinchisi esa taqribiy ekaniga tekshirish yordamida ishonch hosil qilish mumkin.

Tenglamalar sistemasini grafik usulda yechishda grafiklar kesishadigan nuqtalarning ikkala koordinatalari – absissasi va ordinatasi topiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Tenglamalar sistemasini yechish deganda biz nimani tushunamiz?
2. Biz yechadigan sistemamizning tenglamalari qanday bo'lganlarini yechamiz?
3. Sistemani grafik usulda qanday yechiladi?

MASALALARNI YECHING

471. (–1; 3) sonlar jufti quyidagi tenglamaning yechimi bo'ladimi?

a) $x^2-y+2=0$; b) $xy+y=6$.

472. a) (–2; 1) b) (1; –2) sonlar jufti

$\begin{cases} x^2+y^2=5 \\ 6x+5y=-4 \end{cases}$ tenglamalar sistemasining yechimi bo'ladimi?

473. Tenglamalar sistemasini grafik usulda yeching va tekshiring:

$$\begin{cases} y-x^2=0 \\ 2x-y+3=0 \end{cases}.$$

474. Tenglamalar sistemasini grafiklar yordamida yeching:

a) $\begin{cases} xy=6 \\ 2x-3y=6 \end{cases}$; b) $\begin{cases} x^2+y^2=100 \\ y=0,5x^2-10 \end{cases}$.

475. Tenglamalar sistemasini grafik usulda yeching:

a) $\begin{cases} y-x^2=0 \\ x+y=6 \end{cases}$; b) $\begin{cases} (x-4)^2+(y-5)^2=9 \\ y=x \end{cases}$.

476. Tenglamalar sistemasini o'rniga qo'yish usuli bilan yeching:

a) $\begin{cases} 11x-9y=37 \\ x=1+2y \end{cases}$; b) $\begin{cases} 16x-4y=5 \\ 3x-y=2 \end{cases}$.

66-§. Ikkinchi darajali tenglamalar sistemasini yechish

1. Avval bitta ikkinchi darajali tenglama va bitta birinchi darajali tenglamadan tuzilgan ikki o'zgaruvchili tenglamalar sistemasini ko'rib chiqamiz. Bunday sistemani o'rniga qo'yish usulidan foydalanib doimo yechish mumkin.

Buning uchun birinchi darajali tenglamadagi bir o'zgaruvchi ikkinchisi bilan ifodalanadi va topilgan ifoda ikkinchi darajali tenglamaga qo'yiladi. Natijada darajasi ikkidan ortiq bo'lmagan bir o'zgaruvchili tenglama hosil qilinadi. Uni yechib, so'ngra ikkinchi o'zgaruvchining qiymati topiladi.

1-m i s o l. Ushbu $\begin{cases} x^2-3xy-2y^2=2 \\ x+2y=1 \end{cases}$ tenglamalar sistemasini yechamiz.

$x+2y=1$ tenglamadan $x=1-2y$ topib, birinchi tenglamaga qo'yamiz.

$(1-2y)^2-3(1-2y)y-2y^2=2$ va uni soddalashtiramiz.

$1-4y+4y^2-3y+6y^2-2y^2-2=0.$

$8y^2-7y-1=0$. Bu tenglamani yechib, $y_1=-\frac{1}{8}$ va $y_2=1$ lar topildi.

Bu ildizlarni $x=1-2y$ ga qo'yib, $x_1=1-2\left(-\frac{1}{8}\right)=+1\frac{1}{4}$, $x_2=1-2\cdot1=$ $=-1$ lar topildi.

Demak, sistema: $x_1-1\frac{1}{4}$, $y_1=-\frac{1}{8}$ va $x_2=-1$, $y_2=1$.

Javob: $\left(1\frac{1}{4};\ -\frac{1}{8}\right)$, $(-1;\ 1)$.

2. Agar sistema ikkinchi darajali ikki noma'lumli ikkita tenglamadan tuzilgan bo'lsa, uning yechimini topish, odatda ancha murakkab bo'ladi. Ayrim hollarda bunday sistemani o'rniga qo'yish usulidan yoki tenglamalarni qo'shish usulidan foydalanib yechish mumkin. Masalan,

2-m i s o l. Ushbu $\begin{cases} x^2 - y^2 = 5 \\ xy = 6 \end{cases}$ tenglamalar sistemasini yechamiz.

Yechish. $xy=6$ tenglamadan $y = \frac{6}{x}$ ni topamiz va uni birinchi tenglamaga qo'yamiz:

$x^2 - \left(\frac{6}{x}\right)^2 = 5$ hosil qilib, uni yechamiz:

$x^4 - 5x^2 - 36 = 0$ bundan $x^2 = -4$ va $x^2 = 9$ lar topiladi. $x^2 = -4$ tenglama yechimga ega emas.

$x_2 = 9$ tenglama $x_1 = -3$ va $x_2 = 3$ ildizga ega.

Bu ildizlarni $y = \frac{6}{x}$ ga qo'yib, $y_1 = -2$ va $y_2 = 2$ va $x_2 = 3$; $y_2 = 2$.

Javob: (–3; –2), (3; 2).

3-m i s o l. Ushbu $\begin{cases} x^2 + y^2 = 313 \\ x^2 - y^2 = 25 \end{cases}$ tenglamalar sistemasini yechamiz.

Yechish. Sistemaning tenglamalarini hadma-had qo'shib,

$$+\begin{cases} x^2 + y^2 = 313 \\ x^2 - y^2 = 25 \end{cases}$$

$2x^2 = 338$ topiladi.

Bundan $x^2 = \frac{338}{2} = 169$.

$x^2 = 169$

$x_1 = -13$ va $x_2 = 13$ hosil bo'ldi.

Birinchi tenglamadagi x ning o'rniga $x_1 = -13$ qo'yib, $169 + y^2 = 313$ topiladi. Bundan $y^2 = 144$ topilib, $y_1 = -12$ va $y_2 = 12$ topiladi.

Sistemaning ikkita yechimini topdik: (–13; –12) va (–13; 12).

Shunga o'xshash $x_2 = 13$ ni qo'yib, $169 + y^2 = 313$ tenglamadan yana ikkita (13; –12) va (13; 12) yechimni topdik.

Demak, sistema to'rtta yechimga ega:

Javob: (–13; –12), (–13; 12), (13; –12), (13; 12).

4-misol. Ushbu $\begin{cases} 3x + 10y - xy = 4 \\ 2x + 5y - xy = 2 \end{cases}$ tenglamalar sistemasini yechamiz.

Yechish. Ikkinchi tenglamani –1 ga ko'paytirib, tenglamalarni qo'shamiz:

$$+\begin{cases} 3x+10y-xy=4 \\ -2x-5y+xy=-2 \end{cases}$$ bundan $x=2-5y$ topilib, buni ikkinchi tenglamaga

$$x+5y=2,$$

qo'yamiz:

$2(2-5y)+5y-(2-5y)y=2.$

$4-10y+5y-2y+5y^2-2=0$

$5y^2-7y+2-0.$ Bu tenglamani yechib,

$y_1=1$ va $y_2=0{,}4$ topiladi. Bu qiymatlarni

$x=2-5y$ ga qo'yib, $x_1=2-5\cdot 1=-3$

$x_2=2-5\cdot 0{,}4=0$ topiladi.

Demak, sistema ikkita yechimga ega:

$x_1=-3, y_1=1$ va $x_2=0, y_2=0{,}4.$

Javob: (–3; 1), (0; 0,4).

3. Keltirilgan $x^2+pa+q=0$ kvadrat tenglamaning ildizlari x_1 va x_2 bo'lsa, $p=-(x_1+x_2)$ va $q=x_1\cdot x_2$ tenglik o'rinli bo'ladi (Viyet teoremasi).

Masalan, ixtiyoriy ikkita –5 va 7 sonlar biror keltirilgan kvadrat tenglamada $p=-(-5+7)=-2$, $q=-5\cdot 7=-35$ bo'lib, u tenglama $x^2-2x-35=0$ dan iborat bo'ladi.

5-m i s o l. Ushbu $\begin{cases} x^2+y^2=29 \\ x^2y^2=100 \end{cases}$ tenglamalar sistemasini yechamiz.

Yechish: x^2 va y^2 larni biror kvadrat tenglamaning ildizlari bo'lsin deb, Viyet teoremasiga yangi $z^2-29z+100=0$ tenglamani tuzamiz.

$z_{1/2}=14{,}5\pm\sqrt{210{,}25-100}=14{,}5\pm 10{,}5;\quad z_1=4$ va $z_2=25.$

$z_1=x^2=4;\ z_2=y^2=25$, bulardan:

$x_{1/2}=\pm 2;\ y_{1/2}=\pm 5$. ildizlardan (–2; –5), (–2; 5), (2; –5), (2; 5) juftliklar topiladi. Bu juftliklar sistemaning yechimi bo'ladi.

Javob: (–2; –5), (–2; 5), (2; –5) va (2; 5).

Eslatma. Tenglamalarni darajaga ko'tarishga to'g'ri kelganda chet ildizlar paydo bo'lishi mumkin, shuning uchun ildizlarni tekshirish zarur bo'ladi.

TAKRORLASH UCHUN SAVOLLAR

1. Ikkinchi darajali tenglamalar sistemasi o'rniga qo'yish usuli bilan qanday yechiladi?
2. Ikkinchi darajali tenglamalar sistemasi qo'shish usuli bilan qanday yechiladi?
3. Ikkinchi darajali tenglamalar sistemasi yangi kvadrat tenglama tuzish orqali qanday yechiladi?
4. $\begin{cases} x^2+y^2=7 \\ x^2\cdot y^2=6 \end{cases}$ sistemani og'zaki yechimini toping.

MASALALARNI YECHING

477. Tenglamalar sistemasini o'rniga qo'yish usuli bilan yeching:

a) $\begin{cases} x^2-2y^2=7 \\ x=y+2 \end{cases}$; d) $\begin{cases} x^2-5xy=10 \\ x-5y=1 \end{cases}$; f) $\begin{cases} x^2+y^2=100 \\ 3x-2y=2 \end{cases}$;

b) $\begin{cases} x^2-2xy=7 \\ x=3y+2 \end{cases}$; e) $\begin{cases} 5xy-y^2=9 \\ 2x-y=3 \end{cases}$; g) $\begin{cases} 2x^2-3y^2=24 \\ 2x-3y=0 \end{cases}$.

478. Tenglamalar sistemasini yeching:

a) $\begin{cases} x^2+xy-y^2=11 \\ x-2y=1 \end{cases}$; b) $\begin{cases} x^2+xy-3y=9 \\ 3x+2y=-1 \end{cases}$.

479. Tenglamalar sistemasini o'rniga qo'yish usuli bilan yeching:

a) $\begin{cases} x^2+y^2=12 \\ xy=-6 \end{cases}$; b) $\begin{cases} x^2-y^2=-9 \\ xy=20 \end{cases}$.

480. Tenglamalar sistemasini qo'shish usuli bilan yeching:

a) $\begin{cases} x^2-y^2=7 \\ x^2+y^2=25 \end{cases}$; b) $\begin{cases} x^2+2y^2=228 \\ 3x^2-2y^2=172 \end{cases}$.

481. Tenglamalar sistemasini yeching:

a) $\begin{cases} xy+3x-4y=12 \\ xy+2x-2y=9 \end{cases}$; d) $\begin{cases} x^2+3x-4y=20 \\ x^2-2x+y=-5 \end{cases}$;

b) $\begin{cases} 2x-3xy+4y=0 \\ x+3xy-3y=1 \end{cases}$; e) $\begin{cases} y^2+3x-y=1 \\ y^2+6x-2y=1 \end{cases}$.

482. Tenglamalar sistemasini kvadrat tenglama tuzish orqali yeching:

a) $\begin{cases} x^2+y^2=18 \\ x^2y^2=81 \end{cases}$; b) $\begin{cases} x^2+y^2=34 \\ xy=15 \end{cases}$;

d) $\begin{cases} x^2-y^2=7 \\ x^2y^2=144 \end{cases}$; e) $\begin{cases} x^2-y^2=156 \\ xy=160 \end{cases}$.

483. Tenglamalar sistemasini yeching:

a) $\begin{cases} xy+3x-4y=12 \\ xy+2x-2y=9 \end{cases}$; d) $\begin{cases} x^2+3x-4y=20 \\ x^2-2x+y=-5 \end{cases}$;

b) $\begin{cases} -2x+3xy-4y=0 \\ x+3xy-3y=1 \end{cases}$; e) $\begin{cases} y^2+3x-y=1 \\ y^2+6x-2y=1 \end{cases}$.

484. Tenglamalar sistemasini yeching:

a) $\begin{cases} 2x+y=5 \\ \frac{1}{x}+\frac{1}{y}=1,2 \end{cases}$; d) $\begin{cases} \frac{1}{x}-\frac{1}{y}=\frac{1}{3} \\ \frac{1}{x^2}-\frac{1}{y^2}=\frac{1}{4} \end{cases}$;

b) $\begin{cases} y-2x=3 \\ \frac{1}{x}-\frac{1}{y}=0,1 \end{cases}$; e) $\begin{cases} x^2-xy=28 \\ y^2-xy=-12 \end{cases}$.

67-§. Ikkinchi darajali tenglamalar sistemalari yordamida masalalar yechish

1-masala. To‘g‘ri to‘rtburchakning perimetri 80 dm ga teng. Agar to‘g‘ri to‘rtburchakning asosi 8 dm, balandligi 2 dm orttirilsa, to‘g‘ri to‘rtburchakning yuzi bir yarim marta ortadi. To‘g‘ri to‘rtburchakning tomonlarini toping.

Yechish: To‘g‘ri to‘rtburchakning asosi x dm, balandligi y dm bo‘lsin.

Masala shartiga ko‘ra, to‘g‘ri to‘rtburchakning perimetri 80 dm ga teng bo‘lganidan $2x+2y=80$ tenglamani tuzamiz.

To‘g‘ri to‘rtburchakning yuzi xy dm^2 ga teng.

To‘g‘ri to‘rtburchakning tomonlari orttirilgandan so‘ng uning asosi $(x+8)$ dm ga, balandligi $(y+2)$ dm ga, yuzi $(x+8)(y+2)$ dm^2 ga teng bo‘ladi. To‘g‘ri to‘rtburchakning yuzi 1,5 marta ortgani uchun quyidagi tenglamani tuzamiz: $(x+8)(y+2)=1,5\,xy$.

Shunday qilib, ikkita tenglama tuzib, quyidagi sistemaga ega bo‘lamiz: $\begin{cases} 2x+2y=80 \\ (x+8)(y+2)=1,5xy \end{cases}$. Bu sistemani yechib, masalaning javobini topamiz.

$$\begin{cases} x+y=40 \\ -0,5xy+2x+8y+16=0 \end{cases}; \quad \begin{cases} y=40-x \\ -0,5x(40-x)+2x+8(40-x)+16=0 \end{cases}.$$

Ikkinchi tenglamadan $x^2-52x+672=0$ hosil bo‘ladi. Uni yechib, $x_1=24$; $x_2=28$ lar topiladi. $y_1=40-24=16$; $y_2=40-28=12$.

Demak, masala ikkita yechimga ega. To‘g‘ri to‘rtburchakning tomonlari 24 dm va 16 dm yoki 28 dm va 12 dm.

2-masala. Ikki velosipedchi oralaridagi masofa 28 km ga teng bo‘lgan A va B punktlardan bir vaqtda jo‘nashdi va bir soatdan keyin uchrashdi. Agar velosipedchilardan biri B punktga ikkinchisi A ga yetib kelganidan 35 minut keyin yetib kelgan bo‘lsa, har bir velosipedchi qanday tezlik bilan harakatlangan?

Yechish: Birinchi velosipedchining tezligi x km/soat ikkinchi velosipedchining tezligi y km/soat bo‘lsin. Ular bir soatda uchrashganligidan $x+y=28$ tenglamani tuzamiz.

Birinchi velosipedchi 28 km ni $\frac{28}{x}$ soatda, ikkinchi velosipedchi 28 km ni $\frac{28}{y}$ soatda bosib o‘tadi. Birinchi velosipedchi 35 minut$=\frac{35}{60}=\frac{7}{12}$ soat ortiq yurganligidan quyidagicha tenglama tuzamiz $\frac{28}{x}-\frac{28}{y}=\frac{7}{12}$. Bu ikki tenglamalardan sistema tuzib, uni yechamiz:

$$\begin{cases} x+y=28 \\ \frac{28}{x}-\frac{28}{y}=\frac{7}{12} \end{cases}; \quad \begin{cases} y=28-x \\ 28\cdot12y-28\cdot12x=7xy \end{cases}; \quad \begin{cases} y=28-x \\ 336(28-x)-336x=7x(28-x) \end{cases}$$

Ikkinchi tenglamadan $7x^2-868x+9408=$ hosil bo'ladi.

Uni yechib, $x_1=12$ va $x_2=112$ topiladi, ammo $x_2=112$ masala shartini qanoatlantirmaydi. $y=28-12=16$ (km/soat).

Javob: 12 km/soat va 16 km/soat.

MASALALARNI YECHING

485. Ikki sonning yig'indisi 20 ga teng, ularning ko'paytmasi 96 ga teng. Shu sonlarni toping.

486. To'g'ri to'rtburchakning perimetri 82 sm ga teng, uning diagonali 29 sm ga teng. To'g'ri to'rtburchakning tomonlarini toping.

487. To'g'ri to'rtburchakning bir tomoni ikkinchisidan 14 sm uzun. Agar to'g'ri to'rtburchakning diagonali 26 sm ga teng bo'lsa, to'g'ri to'rtburchakning tomonlarini toping.

488. Yuzi 24 a bo'lgan to'g'ri to'rtburchak shaklidagi yer maydonining uzunligi 200 m ga teng bo'lgan devor bilan o'ralgan. Shu maydonning bo'yini va enini toping.

489. To'g'ri burchakli uchburchakning perimetri 84 sm ga teng, uning gipotenuzasi esa 37 sm ga teng. Shu uchburchakning yuzini toping.

490. Bir punktdan ikki otryad bir vaqtda yo'lga chiqdi. Otryadlardan biri shimolga, ikkinchisi sharqqa yo'l oldi. 4 soatdan keyin otryadlar orasidagi masofa 24 km ga teng bo'lib, bunda birinchi otryad ikkinchisidan 4,8 km ortiq yurdi. Har bir otryad qanday tezlik bilan yurgan?

491. To'g'ri burchakli uchburchakning gipotenuzasi 13 sm ga teng. Agar uning katetlaridan biri 4 sm orttirilsa, gipotenuzasi 2 sm ortadi. Uchburchakning katetlarini toping.

492. Ikki ekskavator bir vaqtda ishlab, biror hajmdagi yer ishlarini 3 soatu 45 minutda bajaradi. Bir ekskavator alohida o‘zi ishlab, bu hajmdagi ishni ikkinchisiga qaraganda 4 soat tezroq bajaradi. Shu hajmda yer ishlarini bajarish uchun har bir ekskavatorga alohida qancha vaqt kerak bo‘ladi?

493. Bir kombaynchi bug‘doy hosilini ikkinchi kombaynchidan 24 soat tezroq o‘rib olishi mumkin. Ikkala kombaynchi birgalikda ishlaganda esa, hosilni 35 soatda yig‘ib tugatishadi. Har bir kombaynchi alohida ishlab, hosilni qancha vaqtda o‘rib tugatadi?

494. Oralaridagi masofa 45 km ga teng bo‘lgan M va N punktlardan bir vaqtda ikki poyezd jo‘nadi va 20 minutdan so‘ng uchrashdi. M dan chiqqan poyezd N stansiyaga ikkinchisi M ga yetib kelishidan 9 minut avval keldi. Har bir poyezdning tezligini toping.

1. ARIFMETIK PROGRESSIYA

68-§. Ketma-ketliklar

Musbat juft sonlarni o‘sib borish tartibida yozib chiqamiz. Bunday sonlarning birinchisi 2 ga, ikkinchisi 4 ga, uchinchisi 6 ga teng va hokazo. Biz ushbu **ketma-ketlikni** hosil qilamiz:

2; 4; 6; ... . Bu ketma-ketlikning beshinchi o‘rnida 10 soni, o‘ninchi o‘rnida 20 soni, yuzinchisida 200 soni turadi. Istalgan musbat juft sonni umumiy holda $2n$ formula ko‘rinishida yozish mumkin.

Yana bir ketma-ketlikni ko‘rib chiqamiz. Surati 1 bo‘lgan to‘g‘ri kasrlarni kamayib borish tartibida yozib chiqamiz: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; ... . Istagan n nomer uchun unga mos kasrni ko‘rsatish mumkin; u sonni umumiy ko‘rinishda $\frac{1}{n+1}$ kabi yoziladi (n – natural son).

Masalan, oltinchi o‘rinda $\frac{1}{7}$ kasr, o‘ttizinchi o‘rinda $\frac{1}{31}$ kasr, minginchi o‘rinda $\frac{1}{1001}$ kasr turadi.

Ketma-ketlikni tashkil etuvchi sonlar **ketma-ketlikning hadlari** deyiladi.

Ketma-ketlikning hadlari odatda indeksli harflar bilan belgilanadi. Undagi indekslar hadning tartib nomerini ko‘rsatadi. Masalan, a_1, a_2, a_3, ... va hokazo. Bunda a_1 – birinchi hadi, a_2 – ikkinchi hadi va hokazo.

Umuman ketma-ketlikning n – hadi a_n ko‘rinishda belgilanadi. Ketma-ketlikning o‘zi esa (a_n) kabi belgilanadi.

Ketma-ketlikning hadlari chekli bo‘lishi mumkin. Bunday ketma-ketlikni **chekli ketma-ketlik** deyiladi. Masalan, 10; 11; 12; ..., 99 ketma-ketlik ikki xonali sonlar ketma-ketligi.

Ketma-ketlikning istagan nomeridagi hadni topishga imkon beruvchi usulni ko'rsatish kerak. Ko'pincha ketma-ketlik uning n – hadini n nomerning funksiyasi sifatida ifodalovchi formula yordamida beriladi. Bunday formulani ketma-ketlikning n – hadi formulasi deyiladi. Masalan, $a_n=2n$ – musbat juft sonlar ketma-ketligining formulasi, $b_n=\frac{1}{n+1}$ – surati 1 bo'lgan to'g'ri kasrlar ketma-ketligining formulasi.

1-misol. Ketma-ketlik $y_n=n^2-3n$ formula bilan berilgan bo'lsin. Uning dastlabki beshta hadini topamiz: $n=1$ da $y_1=1^2-3\cdot 1=-2$; $n=2$ da $y_2=2^2-3\cdot 2=-2$; $n=3$ da $y_3=3^2-3\cdot 3=0$; $n=4$ da $y_4=4^2-3\cdot 4=4$; $n=5$ da $y_5=5^2-3\cdot 5=10$ va hokazo.

–2; –2; 0; 4; 10; ... ketma-ketlik hosil bo'ldi.

2-misol. Ketma-ketlik $x_n=(-1)^n\cdot 10$ formula bilan berilgan bo'lsin. Bunda: $x_1=(-1)^1\cdot 10=-10$; $x_2=(-1)^2\cdot 10=10$; $x_3=(-1)^3\cdot 10=-10$; $x_4=(-1)^4\cdot 10=10$ bo'lib, ketma-ketlik –10; 10; –10; 10; ... hosil bo'ladi.

3-misol. $C_n=5$ formula bilan hamma hadlari 5 bo'lgan ketma-ketlik beriladi, ya'ni 5; 5; 5; ... hosil bo'ladi.

4-misol. (a_n) ketma-ketlikning birinchi hadi 3 ga teng, keyingi har bir hadi esa avvalgisining kvadratning teng bo'lsin, ya'ni $a_1=3$; $a_{n+1}=a_n^2$ bo'lsin, ya'ni $a_2=a_1^2=3^2=9$; $a_3=a_2^2=9^2=81$; $a_4=a_3^2=81^2=6561$; hokazo.

Natijada 3; 9; 81; 6561; ... .

Ketma-ketlikning istalgan hadini undan oldingi (bitta yoki bir nechta) hadi bilan ifodalovchi formula **rekurrent formula** deyiladi (lotincha chesichcho – qaytish).

Masalan, 4-misol rekurrent formula orqali topildi.

TAKRORLASH UCHUN SAVOLLAR

1. Sonli ketma-ketliklarga misollar keltiring.
2. Hadlari harflar bilan berilgan ketma-ketliklar qanday yoziladi?
3. Ketma-ketlik qisqacha qanday belgilanadi?
4. Rekurrent formula qanday beriladi?
5. Bitta formula olib, undan ketma-ketlik tuzing.

MASALALARNI YECHING

495. 3 ga karrali bo'lib, o'sish tartibida olingan natural sonlar ketma-ketligidan dastlabki bir nechta hadini yozing. Uning: birinchi, beshinchi, yuzinchi va n – hadlarini yozing.

496. (C_n) – hamma toq nomerli hadlari – 1 ga teng, hamma juft nomerli hadlari esa 0 ga teng bo'lgan ketma-ketlik. Uning C_{10}, C_{25}, C_{200}, C_{2k}, C_{2k+1} hadlarini toping (k – natural son).

497. n – hadi formulasi bilan berilgan ketma-ketlikning dastlabki beshta hadini toping:

a) $x_n=3n-7$; d) $x_n=\frac{n}{2n+1}$;

b) $x_n=n^2+1$; e) $x_n=2^{n-1}$.

498. (b_n) ketma-ketlik $b_n=3n+2$ formula bilan berilgan. Ularni toping:

a) b_5; d) b_{2k}; f) $b_{20}+b_{30}$;

b) b_{10}; e) b_{2k+1}; g) $b_{10}-b_{25}$.

499. $x_n=7-2n$ formula bilan berilgan (x_n) ketma-ketlikning: a) –13; b) –43; d) –61 ga teng hadining nomerini toping (ko'rsatma: $7-2n=-13$ dan n topiladi).

500. (a_n) ketma-ketlikning dastlabki beshta hadini yozib chiqing, bunda:

a) $a_1=1$; $a_{n+1}=a_n+1$; d) $a_1=0$; $a_{n+1}=2a_n+4$;

b) $a_1=1000$; $a_{n+1}=0{,}1\cdot a_n$; e) $a_1=6$; $a_{n+1}=-a_n$.

69-§. Arifmetik progressiyaning ta'rifi. Arifmetik progressiyaning *n* – hadi formulasi

4 ga bo'lganda qoldiqda 1 chiqadigan natural sonlar ketma-ketligini ko'rib chiqamiz: 1; 5; 9; 13; 17; ... .

Bu ketma-ketlikda ikkinchi haddan boshlab har bir hadi avvalgisiga 4 sonini qo'yish natijasida hosil bo'ladi. Bu ketma-ketlik arifmetik progressiyaga misol bo'ladi.

Ta'rif. **Arifmetik progressiya deb shunday ketma-ketlikka aytiladiki, unda ikkinchi haddan boshlab har bir hadi o'zidan oldingi hadiga ayni bir xil sonni qo'shish natijasiga teng.**

Boshqacha aytganda, istalgan natural son n uchun $a_{n+1}=a_n+d$ (bunda d – biror son) shart bajarilsa, ketma-ketlik arifmetik progressiya bo'ladi.

Arifmetik progressiyada d, ikkinchi hadidan boshlab istalgan hadi bilan undan oldingi hadi orasidagi ayirma, ya'ni $d=a_{n+1}-a_n$. d soni arifmetik progressiyaning ayirmasi deyiladi.

Arifmetik progressiya hosil qilish uchun uning birinchi hadi va ayirmasi bo'lishi etarli.

Masalan: 1) $a_1=1$ va $d=3$ bo'lsa, u holda arifmetik progressiya 1; 4; 7; 10; 13; ... hosil bo'ladi.

2) $a_1=-2$ va $d=-5$ bo'lsa, arifmetik progressiya –2; –7; –12; –17; ... hosil bo'ladi.

3) $a_1=8$; va $d=0$ bo'lsa, arifmetik progressiya 8; 8; 8; ... hosil bo'ladi.

Arifmetik progressiyada a_1 – birinchi hadi, d – hadlari ayirmasi bo'lsa, uning istalgan hadini topishni o'rganamiz.

Arifmetik progressiyaning ta'rifidan:

a_1 – birinchi hadi,

$a_2=a_1+d,$

$a_3=a_2+d=a_1+d+d=a_1+2d,$

$a_4=a_3+d=a_1+2d+d=a_1+3d,$

$a_5=a_3+d=a_1+3d+d=a_1+4d.$

Xuddi shunday $a_6=a_1+5d$, $a_7=a_1+6d$ ni topamiz va umuman, a_n ni topish uchun a_1 ga $(n-1)d$ ni qo'shish kerak, ya'ni $\boldsymbol{a_n=a_1+(n-1)d}$.

Biz arifmetik progressiyaning n – hadi formulasini hosil qildik.

Formuladan foydalanib masalalar yechamiz:

1-m i s o l. (C_n) arifmetik progressiya bo'lib, unda $C_1=2{,}3$ va $d=0{,}45$. Shu progressiyaning o'ninchi va yuzinchi hadlarini topamiz:

$$C_{10}=C_1+9d=2{,}3+9\cdot 0{,}45=2{,}3+4{,}05=6{,}35.$$

$$C_{100}=C_1+99d=2{,}3+99\cdot 0{,}45=2{,}3+44{,}55=46{,}85.$$

2-misol. 71 soni (x_n) arifmetik progressiyaning, ya'ni –10; –5,5; –1; 3,5; ... ning hadi yoki hadi emasligini aniqlaymiz.

Bunda $x_1=-10$; $d=x_2-x_1=-5{,}5-(-10)=4{,}5$ bo'lib, n – hadining formulasini yozamiz:

$x_n=x_1+(n-1)d=-10+(n-1)\cdot 4{,}5n-14{,}5=-10+4{,}5n-4{,}5=4{,}5n-14{,}5.$

$x_n=4{,}5n-14{,}5$, bunda n – natural son.

Agar 71 soni (x_n) progressiyaning hadi bo'lsa, $4{,}5n-14{,}5=71$ tenglik bajariladi.

Bundan n ni topamiz:

$4{,}5n=71+14{,}5$

$4{,}5n=85{,}5$

$n=19$. Demak, 71 soni progressiyaning 19-hadi bo'ladi.

TAKRORLASH UCHUN SAVOLLAR

1. Arifmetik progressiya bo'ladigan sonlar ketma-ketligiga misollar keltiring.
2. Arifmetik progressiya deb nimaga aytiladi?
3. Arifmetik progressiyaning ayirmasi nima? U qanday belgilanadi va *y* qanday topiladi?
4. Arifmetik progressiyaning n – hadi formulasini yozing.
5. (C_n) arifmetik progressiyada $C_3=11$ va $C_4=15$ bo'lsa, d ni toping (og'zaki).

MASALALARNI YECHING

501. (a_n) arifmetik progressiyaning dastlab beshta hadini yozib chiqing, bunda:

a) $a_1=10$ va $d=4$; b) $a_1=1{,}2$ va $d=-0;2$; d) $a_1=-3{,}5$ va $d=0{,}4$.

502. (b_n) ketma-ketlik – arifmetik progressiya bo'lib, uning birinchi hadi b_1 ga, ayirmasi d ga teng. Quyidagilarni b_1 va d orqali toping:

a) b_7; b) b_{26}; d) b_{241}; e) b_k; f) b_{k+5}; g) b_{2k}.

503. Agar:

a) $a_1=3$; $d=5$; d) $a_1=7$; $d=1$;

b) $a_1=-9{,}5$; $d=1{,}5$; e) $a_1=8{,}2$; $d=-1$ bo'lsa, (a_n) arifmetik progressiyaning n – hadi formulasini yozing.

504. Arifmetik progressiyada:

a) 3; –1; ... ; b) $\frac{1}{3}$; –2; ... bo'lsa, uning 15; 37 va n – hadini toping.

505. Jism o'z harakatining birinchi sekundida 6 m o'tdi, keyingi har bir sekundda avvalgisidan 4 m ortiq o'tdi. Jism oltinchi sekundda qancha masofani o'tgan?

506. 7 va 35 sonlari orasiga shunday oltita sonni qo'yingki, ular berilgan sonlar bilan birgalikda arifmetik progressiya tashkil etsin.

507. (s_n) arifmetik progressiyaning birinchi hadini va ayirmasini toping, bunda:

a) $c_5=27$ va $c_7=60$; b) $c_{20}=0$ va $c_{66}=-92$.

508. 3; 10; ... arifmetik progressiyada: a) 143; b) 207 sonlar qatnashadimi?

509. Arifmetik progressiyada birinchi hadi 1,7 ga teng, ayirmasi 0,3 ga teng: a) 32 ga; b) 46,7 ga teng hadining nomerini toping.

510. (x_n) arifmetik progressiyada birinchi hadi 8,7 ga teng, ayirmasi esa – 0,3 ga teng. Quyidagi shartlar progressiyaning qaysi hadlari uchun bajariladi:

a) $x_n \geq 0$; b) $x_n>5$; d) $x_n<0$; e) $x_n<-10$.

511. a) 5,3; 5,12; ... arifmetik progressiyaning birinchi manfiy hadini toping.

b) –10,4; –9,65; ... arifmetik progressiyaning birinchi musbat hadini toping.

512. Uchburchak, qavariq to'rtburchak, qavariq beshburchak va hokazolar ichki burchaklarining yig'indisi ketma-ketligi arifmetik progressiya bo'lishini isbotlang. Uning ayirmasi nimaga teng?

70-§. Arifmetik progressiyaning dastlabki *n* ta hadining yig'indisi formulasi

Dastlabki yuzta natural sonlar yig'indisini ularni bevosita qo'shib chiqmasdan qanday topish mumkinligini ko'rsatamiz.

Izlanayotgan yig'indini S bilan belgilaymiz va uni ikki marta yozamiz, birinchisini o'sib borish tartibida, ikkinchisini kamayib borish tartibida joylashtiramiz, ya'ni

$$\begin{array}{l} S=1+2+3+...+98+99+100, \\ +\ S=100+99+98+...+3+2+1 \\ \hline 2S=101+101+101+...+101+101+101=101\cdot 100; \end{array}$$

yozib, ularni qo'shamiz:

$$2S=101\cdot 100$$

$S=\dfrac{101\cdot 100}{2}=5050$. Demak, $1+2+3+...+98+99+100=5050$.

Istalgan arifmetik progressiyaning dastlabki hadlarining yig'indisini topish mumkin.

(a_n) arifmetik progressiyaning dastlabki n ta hadining yig'indisini S_n bilan belgilaymiz va bu yig'indini o'sib borish va kamayib borish tartibida ikki marta yozamiz:

$$\begin{array}{l} S_n=a_1+a_2+a_3+...+a_{n-1}+a_n, \\ +\ S_n=a_n+a_{n-1}+...+a_3+a_2+a_1 \\ \hline 2S_n=(a_1+a_n)+(a_2+a_{n-1})+...+(a_2+a_{n-1})+(a_1+a_n) \end{array}$$

ularni ustun bo'ylab qo'shamiz:

Bundagi har bir yig'indi a_1+a_n ga teng, ya'ni

$$a_2+a_{n-1}=(a_1+d)+(a_n-d)=a_1+d+a_n-d=a_1+a_n;$$

$$a_3+a_{n-2}=(a_1+2d)+(a_n-2d)=a^1+2d+a_n-2d=a_1+a_n;$$

va hokazo. Bunday yig'indilar n ga teng. Shuning uchun $2S_n$ yig'indi

$$2S_n=(a_1+a_n)+(a_1+a_n)+...+(a_1+a_n)+(a_1+a_n)=(a_1+a_n)\cdot n$$

Demak, $2S_n=(a_1+a_n)\cdot n$ bo'lib, bundan

$S_n=\dfrac{(a_1+a_n)\cdot n}{2}$. Bundagi a_n o'rniga $a_n=a_1+(n-1)d$ ni qo'yib,

$S_n=\dfrac{2a_1+(n-1)d}{2}\cdot n$ formula hosil qilinadi.

Bu arifmetik progressiyaning dastlabki n ta hadi yig'indisining formulasi.

Bunda: a_1 – birinchi hadi, a_n – oxirgi hadi, n – hadlar soni.

1-misol. 3; 8; 13; ... arifmetik progressiyaning dastlabki 30 ta hadining yig'indisini topamiz:

Bunda: $a_1=3$; $d=8-3=5$; $n=30$; $a_{30}=a_1+29\cdot d=3+29\cdot 5=148$.

$$S_{30}=\frac{a_1+a_{30}}{2}\cdot 30=(3+148)\cdot 15=151\cdot 15=2265.$$

Agar arifmetik progressiyaning birinchi hadi va ayirmasi berilsa, boshqa ko'rinishdagi yig'indi formulasidan foydalaniladi.

$S_n=\frac{(a_1+a_n)\cdot n}{2}$ dagi a_n ning o'rniga $a_n=a_1+(n-1)d$ ni qo'yib, $S_n=\frac{(a_1+a_1+(n-1)d)}{2}\cdot n=\frac{2a_1+(n-1)d}{2}\cdot n$.

$S_n=\frac{2a_1+(n-1)d}{2}\cdot n$ hosil bo'ladi.

2-misol. –2,6; 0; ...; arifmetik progressiyaning dastlabki 30 ta hadining yig'indisini toping.

Yechish. $a_1=-2,6$; $d=0-(-2,6)=2,6$; $n=30$.

$S_{30}=\frac{2a_1+(n-1)\cdot d}{2}\cdot n=\frac{2\cdot(-2,6)+(30-1)\cdot 2,6}{2}\cdot 30=\frac{-5,2+29\cdot 2,6}{1}\cdot 15=(-5,2+$ $+75,4)\cdot 15=70,2\cdot 15=1053$. $S_{30}=1053$.

3-misol. Qo'shiluvchilari 1 dan n gacha natural sonlar bo'lgan $1+2+3+...+n$ yig'indini topamiz. Bu yig'indi $d=1$ bo'lgan arifmetik progressiya bo'lganidan $S_n=\frac{1+n}{2}\cdot n=\frac{(n+1)n}{2}$ topiladi. Demak, $S_n=\frac{n(n+1)}{2}$.

4-misol. 250 dan katta bo'lmagan oltiga karrali hamma natural sonlar yig'indisini topamiz. Oltiga karrali sonlar $a_n=6n$ formula bilan ifodalanadigan arifmetik progressiya bo'ladi. Bu progressiyada 250 dan katta bo'lmagan oltiga karrali sonlar sonini topamiz: $6n\leq 250$ ni yechib, n topiladi. Bundan $n\leq 41\frac{2}{3}$ bo'lganidan $n=41$ bo'ladi. Bu progressiyada: $a_1=6$; $a_{41}=6\cdot 41=246$, $S_{41}=\frac{6+246}{2}\cdot 41=126\cdot 41=5166$.

Demak, $S_{41}=5166$.

5-masala. Agar arifmetik progressiyada (a_n da) $a_1=2$ va $d=8$ bo'lsa, uning o'n birinchidan yigirma beshinchigacha (yigirma beshinchisi ham kiradi) hadlarining yig'indisini topamiz.

Buning uchun S_{10} va S_{25} yig'indilar topilib, so'ngra S_{11-25} yig'indi topiladi.

$$a_{10}=a_1+9d=2+9\cdot8=74;\ a_{25}=a_1+24d=2+24\cdot8=194.$$

$$S_{10}=\frac{a_1+a_{10}}{2}\cdot10=\frac{2+74}{1}\cdot5=76\cdot5=380.$$

$$S_{10}=\frac{a_1+a_{25}}{2}\cdot25=\frac{2+194}{2}\cdot25=98\cdot25=2450.$$

$$S_{11-25}=S_{25}-S_{10}=2450-380=2070.$$

Javob: 2070.

TAKRORLASH UCHUN SAVOLLAR

1. 1 dan 100 gacha (100 ham kiradi) natural sonlar yig'indisi qanday topiladi?
2. Arifmetik progressiyaning dastlabki n ta hadlarining yig'indisini topish formulasini yozing.
3. Arifmetik progressiyaning birinchi hadi va hadlari ayirmasi orqali uning yig'indisini topish formulasini yozing.
4. 1 dan 150 gacha (50 ham kiradi) natural sonlar yig'indisini toping.

MASALALARNI YECHING

513. Arifmetik progressiyaning dastlabki sakkizta hadining yig'indisini toping:

a) 9; 5; ...; d) –2,6; 0; ...;

b) –23; –20; ...; e) 14,2; 9,6; ... .

514. (x_n) ketma-ketlik (arifmetik progressiya)ning dastlabki ellikta, yuzta va n ta hadining yig'indisini toping, bunda:

a) $x_n=4n+2$; b) $x_n=2n+3$.

515. a) Qo'shiluvchilari 2 dan $2n$ gacha bo'lgan hamma juft natural sonlardan iborat $2+4+6+...+2n$ yig'indini toping.

b) Qo'shiluvchilari 1 dan $2n-1$ gacha bo'lgan hamma toq sonlardan iborat $1+3+5+...+(2n-1)$ yig'indini toping.

516. a) 150 dan katta bo'lmagan hamma natural sonlar yig'indisini toping.

b) 20 dan 120 gacha (120 ham kiradi) hamma natural sonlar yig'indisini toping.

d) hamma ikki xonali sonlarning yig'indisini toping.

e) hamma uch xonali sonlar yig'indisini toping.

517. a) 5 ga karrali bo'lib, 225 dan katta bo'lmagan hamma natural sonlar yig'indisini toping.

b) 100 dan 500 gacha (500 ham kiradi) bo'lgan hamma juft sonlar yig'indisini toping.

d) 6 ga karrali barcha ikki xonali sonlar yig'indisini toping.

518. Agar arifmetik progressiyaning birinchi hadi 7 ga va ayirmasi 15 ga teng bo'lsa, uning o'ninchidan yigirmanchigacha (yigirmanchisi ham kiradi) hadlari yig'indisini toping.

519. Agar $c_7=18{,}5$ va $c_{17}=-26{,}5$ bo'lsa, (c_n) arifmetik progressiyaning dastlabki yigirmata hadining yig'indisini toping.

520. Sharlar uchburchak shaklida shunday joylashganki, birinchi qatorda 1 ta shar, ikkinchi qatorda – 2 ta, uchinchisida – 3 ta va hokazo shar bor. Agar hamma sharlar 120 ta bo'lsa, ular nechta qatorga joylashgan? 30 ta qatorli uchburchak yasash uchun shu sharlardan nechta kerak bo'ladi?

2. GEOMETRIK PROGRESSIYA

71-§. Geometrik progressiyaning ta'rifi. Geometrik progressiyaning *n* – hadi formulasi

Hadlari 2 sonining natural ko'rsatkichli darajalaridan iborat bo'lgan 2; 2^2; 2^3; 2^4; ... ketma-ketlikni ko'rib chiqamiz. Bu ketma-ketlikning ikkinchi hadidan boshlab har bir hadi avvalgi hadini 2 ga ko'paytirish bilan hosil qilinadi. Bu ketma-ketlik ***geometrik progressiyaga*** misol bo'ladi.

Ta'rif. **Geometrik progressiya deb shunday sonlar ketma-ketligiga aytiladiki, unda ikkinchi hadidan boshlab har bir hadi o'zidan oldingi hadni ayni bir songa ko'paytirilganiga teng.**

Boshqacha aytganda, agar istalgan natural, n uchun $b \neq 0$ va $b_{n+1}=b_n q$ (bunda q – biror son) shart bajarilsa, (b_n) ketma-ketlik geometrik progressiya bo'ladi.

Ko'rib o'tilgan misolimizda istalgan natural n uchun $b_{n+1}=b_n \cdot 2$ tenglik to'g'ri, bunda $q=2$.

Geometrik progressiyaning ta'rifidan $\frac{b_{n+1}}{b_n}=q$ tenglikning to'g'riligi kelib chiqadi.

q soni geometrik progressiyaning maxraji deyiladi.

Geometrik progressiyani berilishi uchun uning birinchi hadi va maxrajining berilishi yetarli. Misollar keltiramiz:

1) agar $b_1=1$ va $q=0{,}1$ bo'lsa, u holda 1; 0,1; 0,01; 0,001; 0,0001; ... geometrik progressiya hosil qilamiz.

2) Agar $b_1=-2$ va $q=3$ bo'lsa, u holda –2; –6; –18; –54; ... geometrik progressiya bo'ladi.

3) Agar $b_1=4$ va $q=-3$ bo'lsa, biz 4; –12; 36; –108; ... geometrik progressiyani hosil qilamiz.

Geometrik progressiyaning birinchi hadini va maxrajini bilgan holda uning ketma-ket ikkinchi, uchinchi va umuman istalgan hadini topish mumkin:

$$b_2=b_1q;$$

$$b_3=b_2q=b_1q\cdot q=b_1q^2;$$

$$b_4=b_3q=b_1q^2\cdot q=b_1q^3.$$

Xuddi shunday $b_5 = b_1q_4$; $b_6 = b_1q_5$ va hokazoni topamiz. Umuman, b_n ni topish uchun biz b_1 ni q^{n-1} ga ko'paytirishimiz kerak, ya'ni $b_n = b_1q^{n+1}$. Biz **geometrik progressiyaning** n – **hadi formulasini** hosil qildik.

Bu formuladan foydalanib masalalar yechamiz:

1-m i s o l. Geometrik progressiyada $b_1 = 0{,}8$ va $q = \frac{1}{2}$ bo'lsa, b_{10} ni topamiz.

$$b_{10} = b_1 \cdot q^{n-1} = 0{,}8 \cdot \left(\frac{1}{2}\right)^{10-1} = 0{,}8 \cdot \left(\frac{1}{2}\right)^9 = \frac{2^3}{10} \cdot \frac{1}{2^9} = \frac{1}{10 \cdot 2^6} = \frac{1}{640}.$$

2-m i s o l. Agar $b_1 = 162$ va $b_3 = 18$ bo'lsa, (b_n) geometrik progressiyaning b_8 ni topamiz.

$b_8 = b_1 \cdot q^7$ formuladan topishimiz kerak, ammo bundagi q ni topishimiz kerak.

Buning uchun $b_3 = b_1 \cdot q^2$ dan q ni topamiz:

$18 = 162 \cdot q^2$;

$q^2 = \frac{18}{162} = \frac{1}{9}$.

$q = \pm\frac{1}{3}$.

1) $q = \frac{1}{3}$ bo'lsin, u holda $b_8 = b_1 \cdot q^7 = 162 \cdot \left(\frac{1}{3}\right)^7 = \frac{2 \cdot 3^4}{3^7} = \frac{2}{3^3} = \frac{2}{27}$;

2) $q = -\frac{1}{3}$ bo'lsin, u holda $b_8 = 162 \cdot \left(-\frac{1}{3}\right)^7 = \frac{2 \cdot 3^4}{(-3)^7} = -\frac{2}{27}$.

Demak, masala ikkita yechimga ega:

Javob: $-\frac{2}{27}$ va $\frac{2}{27}$.

3-m i s o l. Siyraklovchi nasos porshenining har bir harakatidan so'ng idishdagi havoning 20% i chiqariladi. Agar idishdagi dastlabki bosim 750 mm sim. ust. ga teng bo'lsa, idish ichidagi havoning porshenni oltita harakatidan keyingi bosimini topamiz.

Yechish. Porshenning har bir harakatidan so'ng idishdagi havoning 20% i chiqib ketgani uchun 80% = 0,8 qism havo qoladi. Porshenning navbatdagi harakatidan so'ng idishda $750 \cdot 0{,}8$ bosimli havo qoladi.

Porshenning ikkinchi harakatidan so'ng idishdagi havoning yana 0,8 qismi qoladi, ya'ni $(750 \cdot 0{,}8) \cdot 0{,}8 = 750 \cdot 0{,}8^2$ bosimli havo qoladi va hokazo.

Porshenning oltita harakatidan so'ng idishda $750 \cdot 0{,}8^6$ ga teng havo qoladi. $750 \cdot 0{,}8^6 = 750 \cdot 0{,}26 \approx 195$ (mm.sim. ust.).

Bunday hisoblash birinchi hadi 750 ga, maxraji 0,8 ga teng bo'lgan geometrik progressiya bo'ladi.

TAKRORLASH UCHUN SAVOLLAR

1. Geometrik progressiya bo'ladigan sonli ketma-ketlikka misol keltiring.
2. Qanday ketma-ketlikni geometrik progressiya deyiladi?
3. Progressiyadagi qanday sonni geometrik progressiyaning maxraji deyiladi?
4. Geometrik progressiyaning maxraji qanday topiladi?
5. Geometrik progressiyaning istalgan hadi (b_n – hadi) formulasini yozing.
6. Geometrik progressiyada $b_1=3$ va $q=2$ bo'lsa, b_4 ni toping (og'zaki).

MASALALARNI YECHING

521. (b_n) geometrik progressiyaning dastlabki beshta hadini toping, bunda:

a) $b_1=6$; $q=2$; d) $b_1=-24$; $q=-1{,}5$;

b) $b_1=-16$; $q=\frac{1}{2}$; e) $b_1=0{,}4$; $q=\sqrt{2}$.

522. (c_n) geometrik progressiyaning:

a) agar $c_1=8$, $c_2=-16$ bo'lsa, uning maxrajini va uchinchi hadini toping;

b) agar $c_2=2\sqrt{2}$; $c_3=4$ bo'lsa, uning maxrajini va to'rtinchi hadini toping.

523. (c_n) ketma-ketlik geometrik progressiya bo'lib, uning birinchi hadi c_1 ga, maxraji q ga teng. Quyidagilarni c_1 va q bilan ifodalang:

a) c_6; b) c_{20}; d) c_{130}; e) c_n; f) c_{n+3}; g) c_{2n}.

524. Geometrik progressiyaning yettinchi va n – hadini toping:

a) 2; –6; ...; d) –0,125; 0,25; ...;

b) –40; –20; ...; e) –10; 10; ... .

525. (b_n) geometrik progressiyaning birinchi hadini toping, bunda:

a) $b_8=384$ va $q=2$; b) $b_6=\frac{32}{81}$ va $q=-\frac{2}{3}$.

526. (c_n) geometrik progressiyaning maxrajini toping, bunda:

a) $c_5=-6$ va $c_7=-54$; b) $c_6=25$ va $c_8=9$.

527. (b_n) geometrik progressiyada:

a) $b_2=25$ va $b_6=\frac{1}{25}$ bo'lsa, b_1 va q ni toping;

b) $b_3=-0,1$ va $b_6=-100$ bo'lsa, b_1 va q ni toping.

528. 60 va $\frac{15}{16}$ sonlari orasiga beshta shunday sonni qo'yingki, ular berilgan sonlar bilan birgalikda geometrik progressiya hosil qilsin.

529. 1 va 16 sonlari orasiga uchta shunday sonni qo'yingki, ular berilgan sonlar bilan birgalikda geometrik progressiya hosil qilsin.

530. O'rmon tajriba uchastkasida yog'ochning yillik ko'payishi 10% ni tashkil yetadi. Agar yog'ochning dastlabki miqdori $2,0\cdot10^4$ m^3 bo'lsa, bu uchastkada, 6-yildan so'ng yog'och qanday miqdorda bo'ladi.

531. Agar kassaga qo'yilgan omonat yiliga 20% ortsa, 1000.000 so'm pul 4-yildan keyin qancha bo'ladi.

532. Tomonlari 16 sm ga teng bo'lgan teng tomonli uchburchakka uning uchlari birinchi uchburchak tomonlarining o'rtasi bo'lgan uchburchak yasalgan. Ikkinchi uchburchakka shunday usul bilan uchinchi uchburchak chizilgan va hokazo. Uchburchakning perimetrlari geometrik progressiya tashkil etishini isbotlang. Sakkizinchi uchburchakning perimetrini toping.

72-§. Geometrik progressiyaning dastlabki *n* ta hadi yig'indisi formulasi

Qadimgi hind afsonasida bunday hikoya qilinadi, shaxmatni ixtiro qilgan kishi o'z ixtirosi uchun shaxmat taxtasining birinchi katagiga bitta bug'doy doni, ikkinchisiga undan ikki marta ortiq don, ya'ni 2 ta don, uchinchisiga undan ikki marta ortiq, ya'ni 4 ta don va hokazo, 64-katakkacha shunday talab qildi. Shaxmat ixtirochisi nechta dona bug'doy olishi kerak?

Gap borayotgan bug'doy donalari soni birinchi hadi 1 ga teng, maxraji esa 2 ga teng bo'lgan geometrik progressiyaning oltmish to'rtta hadining yig'indisidan iborat. Bu yig'indini S bilan belgilaymiz, ya'ni $S=1+2+2^2+2^3+...+2^{62}+2^{63}$. Bu tenglikning ikkala qismini progressiya maxraji 2 ga ko'paytiramiz:

$2S=2+2^2+2^3+2^4+...+2^{63}+2^{64}$ ni hosil qilamiz.

Ikkinchi tenglikdan birinchi tenglikni ayirib, soddalashtiramiz:

$2S-S=(2+2^2+2^3+2^4+...+2^{63}+2^{64})-(1+2+2^2+2^3+...+2^{62}+2^{63})=$ $=2^{64}-1$. yoki $S=2^{64}-1$ dona bug'doy.

Bunday miqdordagi bug'doyni topib berish mumkin emasligi ma'lum bo'lgan, ya'ni $S=2^{64}-1=18446744\ 073709551615$ dona bug'doy.

Bu insoniyat shu vaqtgacha yiqqan bug'doy miqdoridan ancha ko'pdir.

Endi ixtiyoriy geometrik progressiyaning dastlabki n ta hadining yig'indisi formulasini chiqaramiz. Yuqoridagi S yig'indini hisoblashdagi usuldan foydalanamiz.

(b_n) geometrik progressiyaning yig'indisini S_n bilan belgilaymiz:

$S_n=b_1+b_2+b_3+...+b_{n-1}+b_n$. Bu tenglikning ikkala qismini progressiya maxraji q ga ko'paytiramiz:

$S_nq=b_1q+b_2q+b_3q+...+b_{n-1}+b_nq$, bunda:

$b_1q=b_2$; $b_2q=b_3$; $b_3q=b_4$; ...; $b_{n-1}q=b_n$ bo'lgani uchun;

$S_nq=b_2+b_3+b_4+...+b_n+b_nq$ ni hosil qilamiz.

Keyingi tenglikdan birinchi tenglikni ayiramiz:

$S_nq-S_n=(b_2+b_3+b_4+...+b_n+b_nq)-(b_1+b_2+b_3+...+b_{n-1}+b_n)=b_nq-b_1$, bunda $S_n(q-1)=b_nq-b_1$.

$$S_n = \frac{b_n q - b_1}{q-1} \ (q \neq 1)$$

Bu geometrik progressiyaning dastlabki n ta hadining yig'indisi formulasi.

1-m i s o l. $b_1=3$; $q=\frac{1}{2}$ bo'lgan geometrik progressiyaning dastlabki 10 ta hadining yig'indisini topamiz.

Berilgan qiymatlarni $S_n = \frac{b_n q - b_1}{q-1}$ ga qo'yib (avval b_{10} ni topib olib) masala yechiladi.

$$b_{10}=b_1 \cdot q_9 = 3 \cdot \left(\frac{1}{2}\right)^9 = \frac{3}{512};$$

$$S_{10} = \frac{\frac{3}{512} \cdot \frac{1}{2} - 3}{\frac{1}{2} - 1} = \frac{3\left(\frac{1}{1024} - 1\right)}{-\frac{1}{2}} = -\frac{3 \cdot (-1023)}{-512} = \frac{3069}{512} = 5\frac{509}{512}.$$

Demak, $S_{10}=5\frac{509}{512}$.

$S_n = \frac{b_n q - b_1}{q-1}$ formuladagi b_n ni $b_n=b_1 q^{n-1}$ bilan almashtirib, boshqa ko'rinishdagi formulani hosil qilamiz:

$$S_n = \frac{b_1 q^{n-1} \cdot q - b_1}{q-1} = \frac{b_1(q^n - 1)}{q-1}, \text{ ya'ni}$$

$$S_n = \frac{b_1(q^n - 1)}{q-1} = (q \neq 1) \text{ hosil bo'ladi.}$$

2-m i s o l. Qo'shiluvchilari $1; x; x^2; ...; x^{n-1}$ geometrik progressiyaning ketma-ket hadlari bo'lgan $1+x+x^2+...+x^{n-1}$ $(x \neq 1)$ yig'indini topamiz.

Bunda: $b_1=1$; $q=x$, hadlari soni n ta.

$S_n = \frac{b_1(q^n - 1)}{q-1} = \frac{1 \cdot (x^n - 1)}{x-1} = \frac{x^n - 1}{x-1}$. Demak, $1+x+x^2+...+x^{n-1} = \frac{x^n - 1}{x-1}$ buning chap va o'ng qismlarini $x-1$ ga ko'paytirib, $x^n-1=(x-1)(1+x+x^2+...+x^{n-1})$ ni hosil qilamiz.

Bu ayniyatning to'g'riligini $n=2$ va $n=3$ larda tekshirib ko'ramiz:

$x^2-1=(x-2)(x+1)$

$x^3-1=(x-1)(x^2+x+1)$

$n=4$ da ham $x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)=(x-1)(x^3+x^2+$ $+x+1)$ ayniyat to'g'ri ekan.

3-misol. Agar $b_3=12$ va $b_5=48$ bo'lsa, (b_n) geometrik progressiyaning dastlabki oltita hadining yig'indisini topamiz.

$b_3=12$ va $b_5=48$ larga ko'ra, $b_5=b_4q=b_3q\cdot q=b_3q^2$ dan $48=12\cdot q^2$; $q^2=\frac{48}{12}=4$; $q_{1/2}=\pm2$.

Masala shartini qanoatlantiruvchi $q_1=-2$ va $q_2=2$ lar mavjud.

1) Agar $q=-2$ da, $b_3=b_1q^2$, $b_1=\frac{b_3}{q^2}=\frac{12}{(-2)^2}=3$.

$$S_6=\frac{b_1(q^6-1)}{q-1}=\frac{3\cdot((-2)^6-1)}{-2-1}=-63$$

2) Agar $q=2$ da $b_3=b_1q^2$; $b_1=\frac{b_3}{q^2}=\frac{12}{4}=3$ bo'lib, $S_6=\frac{3(2^6-1)}{2-1}=\frac{3\cdot63}{1}=$ $=189$.

4-misol. Geometrik progressiyaning 5-hadi 61, 11-hadi 1647 bo'lsa, uning 7-hadini toping.

Bunda: $b_5=61$ va $b_{11}=1647$.

$b_5=b_1\cdot q^4$

$b_{11}=b_1q^{10}$ lardan $\frac{b_{11}}{b_5}=\frac{b_1q^{10}}{b_1q^4}=q^6$; $\frac{1647}{61}=q^6$;

$q^6=27$; $(q^2)^3=27$; $q^2=3$; $q=\pm\sqrt{3}$ ekanidan,

$b_5=b_1\cdot q^4$; $b_1=\frac{b_5}{q^4}=\frac{61}{(\sqrt{3})^4}=\frac{61}{9}$;

$b_7=b_1\cdot q^6=\frac{61}{9}\cdot27=183$. Demak, $b_7=183$.

TAKRORLASH UCHUN SAVOLLAR

1. Shaxmat ixtirochisi Hindiston podshosidan ixtiro evaziga nima so'ragan?
2. Geometrik progressiyaning dastlabki n ta hadining yig'indisi formulasini yozing.
3. Geometrik progressiyaning yig'indisini topishning boshqa ko'rinishdagi formulasini yozing.
4. $x^n+x^{n-1}+...+x^2+x+1$ ko'phadni ko'paytuvchiga ajratish formulasini yozing.
5. 1; 3; 9; ... geometrik progressiyaning dastlabki beshta hadi yig'indisini toping.

MASALALARNI YECHING

533. Geometrik progressiyaning dastlabki beshta hadining yig'indisini toping:

a) $b_1=8$ va $q=\frac{1}{2}$; b) $b_1=500$ va $q=\frac{1}{5}$.

534. Geometrik progressiyaning dastlabki oltita hadining yig'indisini toping:

a) 3; –6; ...; d) 54; 36; ...; f) $1; -\frac{1}{2}; ...$;

b) –6; –2; ...; e) –32; 16; ...; g) 8; 8; ... .

535. Geometrik progressiyaning dastlabki n ta hadining yig'indisini toping:

a) $1; 3; 3^2; ...$; d) $1; -x; x^2; ...$, bunda $x\neq-1$;

b) $\frac{1}{2}; -\frac{1}{4}; \frac{1}{8}; ...$; e) $1; -x^3; x^6; ...$, bunda $x\neq-1$.

536. (b_n) geometrik progressiyaning dastlabki ettita hadining yig'indisini toping, bunda:

a) $b_7=72{,}9$ va $q=1{,}5$; b) $b_5=\frac{16}{9}$ va $q=\frac{2}{3}$.

537. Geometrik progressiyaning birinchi hadi 2 ga teng, beshinchisi esa 162 ga teng. Uning toq nomerli hadlari musbat, juft nomerli hadlari esa manfiyligi ma'lum bo'lsa, shu progressiyaning dastlabki yettita hadining yig'indisini toping.

538. Geometrik progressiyaning birinchi hadi bilan ikkinchi hadlari orasidagi ayirma 8; ikkinchi va uchinchi hadlarining yig'indisi 12. Shu progressiyani toping.

539. a) To'g'ri burchakli uchburchakning tomonlari geometrik progressiya tashkil qiladimi?

b) Agar n, p va r natural sonlar uchun $n=\frac{p+r}{2}$ tenglik to'g'ri bo'lsa, (b_n) geometrik progressiyada $b_n^2=b_p\cdot b_r$ bo'ladi. Shuni isbotlang.

73-§. $|q|<1$ da cheksiz geometrik progressiyaning yig'indisi

Biz $\frac{1}{3}=0{,}333...=0,(3)$ cheksiz davriy o'nli kasr ekanligini bilamiz.

0,333 ... cheksiz o'nli kasrni cheksiz qo'shiluvchili yig'indi shaklida $0{,}333...=0{,}3+0{,}03+0{,}003+...$ kabi yozish mumkin.

Bu yig'indi $q=0{,}1$ bo'lgan 0,3; 0,03; 0,003; ... ko'rinishdagi geometrik progressiyaning hadlaridan iborat.

Geometrik progressiyaning dastlabki n ta hadining yig'indisi formulasi bo'yicha topamiz:

$$S_n=\frac{b_1(q_n-1)}{q-1}=\frac{0{,}3((0{,}1)^n-1)}{0{,}1-1}=\frac{0{,}3((0{,}1)^n-1)}{-0{,}9}=\frac{1-(0{,}1)^n}{3}=\frac{1}{3}-\frac{(0{,}1)^n}{3}$$

Qo'shiluvchilar soni n ni cheksiz orttirganimizda $\frac{(0{,}1)^n}{3}$ kasr nolga yaqinlashadi.

Haqiqatan,

$n=2$ bo'lsa, u holda $\frac{(0{,}1)^2}{3}=\frac{0{,}01}{3}=\frac{1}{300}$;

$n=3$ bo'lsa, u holda $\frac{(0{,}1)^3}{3}=\frac{0{,}001}{3}=\frac{1}{3000}$;

$n=5$ bo'lsa, u holda $\frac{(0{,}1)^5}{3}=\frac{0{,}00001}{3}=\frac{1}{300000}$ va hokazo.

Shuning uchun n ni cheksiz orttirganimizda $\frac{1}{3}-\frac{(0{,}1)^n}{3}$ ayirma $\frac{1}{3}$ ga istagancha yaqin son, yana $\frac{1}{3}$ bo'ladi.

Demak, $0{,}333...=0{,}3+0{,}03+0{,}003;\ ...=\frac{1}{3}$.

Bundagi $\frac{1}{3}$ soni 0,3; 0,03; 0,003; ... cheksiz geometrik progressiyaning yig'indisi deyiladi.

Endi ixtiyoriy

$$b_1;\ b_1q;\ b_1q^2;\ b_1q^3;\ ...\ (|q|<1)$$

geometrik progressiyani ko'rib chiqamiz.

Bundagi $S_n=\frac{b_1(q^n-1)}{q-1}$ ni quyidagicha shakl almashtiramiz.

$$S_n = \frac{b_1(q^n - 1)}{q-1} = \frac{b_1q^n - b_1}{q-1} = \frac{b_1 - b_1q^n}{1-q} = \frac{b_1}{1-q} - \frac{b_1q^n}{1-q} = \frac{b_1}{1-q} - \frac{b_1}{1-q}q^n.$$

Demak, $S_n = \frac{b_1}{1-q} - \frac{b_1}{1-q} \cdot q^n$.

Bundagi $|q|<1$ bo'lgani uchun, n cheksiz orttirilganda q^n ko'paytuvchi nolga intilishidan $\frac{b_1}{1-q} \cdot q^n$ ko'paytma ham nolga intiladi.

Shuning uchun $S_n = \frac{b_1}{1-q} - \frac{b_1}{1-q} \cdot q^n = \frac{b_1}{1-q}$ hosil bo'ladi.

$\frac{b_1}{1-q}$ **soni** $|q|<1$ **bo'lgan** b_n **cheksiz geometrik progressiyaning yig'indisi deyiladi.**

Ya'ni $b_1 + b_1q + b_1q^2 + ... = \frac{b_1}{1-q}$ yoki $S_n = \frac{b_1}{1-q}$ $(|q|<1)$.

1-m i s o l. 12; –4; $\frac{4}{3}$; ... cheksiz geometrik progressiyaning yig'indisini topamiz.

Bunda: $b_1 = 12$; $q = \frac{-4}{12} = -\frac{1}{3}$; $\left|-\frac{1}{3}\right| = \frac{1}{3} < 1$.

$$S_n = \frac{b_1}{1-q} = \frac{12}{1-\left(-\frac{1}{3}\right)} = \frac{12}{\frac{4}{3}} = 9.$$

2-m i s o l. 3,(12) davriy kasrni oddiy kasr ko'rinishida ifodalaymiz.

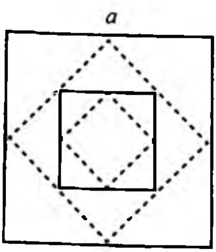
3,(12)=3+0,(12) kabi yozib olib, bundagi
0,(12) ni oddiy kasr ko'rinishida yozamiz:
0,(12)=0,12+0,0012+0,000012+..., bunda $b_1 = 0,12$.

$$q = \frac{0,0012}{0,12} = 0,01 < 1.$$

$S_n = \frac{b_1}{1-q} = \frac{0,12}{1-0,01} = \frac{12}{99} = \frac{4}{33}$; Demak, $0,(12) = \frac{4}{33}$

$$3,(12) = 3 + \frac{4}{33} = 3\frac{4}{33}.$$

3-m i s o l. Tomoni 4 sm ga teng bo'lgan kvadrat berilgan. Uning tomonlari o'rtasi ikkinchi kvadratning uchlaridir, ikkinchi kvadrat tomonining o'rtasi uchinchi kvadratning uchlaridir va hokazo.



49-chizma.

Hamma kvadratlar yuzlarining yig'indisini topamiz (49-chizma).

Yechish. Birinchi kvadratning tomoni a bo'lsin. Uning yuzi a^2 ga teng yoki $S_1 = a^2$.

Ikkinchi kvadratning tomoni a_2 bo'lsin. $a_2 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$. Ikkinchi kvadratning yuzi $S_2 = \left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^2}{2}$.

Uchinchi kvadratning tomoni $a_3^2 = \sqrt{\left(\frac{a_2}{2}\right)^2 + \left(\frac{a_2}{2}\right)^2} = \sqrt{\left(\frac{a}{2\sqrt{2}}\right)^2 + \left(\frac{a}{2\sqrt{2}}\right)^2} = \sqrt{\frac{a^2}{4}} = \frac{a}{2}$. Uchinchi kvadratning yuzi $S_3 = a_3^2 = \left(\frac{a}{2}\right)^2 = \frac{a^2}{4}$; Hosil bo'lgan kvadratlar yuzlarining ketma-ketligi S_1; S_2; S_3; ... yoki a^2; $\frac{a^2}{2}$; $\frac{a^2}{4}$; ... dan iborat. Bunda $q = \frac{a^2}{2} : a^2 = \frac{a^2}{2} : \frac{a^2}{1} = ... = \frac{1}{2}$ bunda $q = \frac{1}{2}$.

Bu kvadratlar yuzlarining ketma-ketligi maxraji $\frac{1}{2}$ bo'lgan cheksiz kamayuvchi geometrik progressiya bo'ladi. Bunda $b_1 = a$; $q = \frac{1}{2}$

Yuzlar yig'indisi: $S_n = \frac{b_1}{1-q} = \frac{a^2}{1-\frac{1}{2}} = 2a^2$.

Bu natijaga ko'ra kvadrat tomoni $a = 4$ ni qo'yib, $S = 2a^2 = 2 \cdot 4^2 = 32$ (kv. birlik). *Javob:* 32 kv. birlik.

TAKRORLASH UCHUN SAVOLLAR

1. 0,444...=0,(4) davriy kasrni cheksiz qo'shiluvchilar bo'lgan yig'indi ko'rinishida yozing.
2. $|q| < 1$ bo'lganda q^n daraja nimaga yaqinlashadi?
3. Cheksiz kamayuvchi geometrik progressiyaga misol keltiring.
4. Cheksiz kamayuvchi geometrik progressiya hadlari yig'indisini topish formulasini yozing.
5. 0,(3) davriy kasrni oddiy kasrga aylantiring.

MASALALARNI YECHING

540. Berilgan geometrik progressiyaning q maxrajini toping va bu progressiyaning yig'indisini toping:

a) $9;\ 3;\ 1;\ ...;$ e) $\sqrt{3};\ -1;\ \frac{1}{\sqrt{3}};\ ...;$

b) $2;\ -\frac{1}{2};\ \frac{1}{8};\ ...;$ f) $2\sqrt{2};\ 2;\ \sqrt{2}\ ...;$

d) $\frac{4}{5};\ \frac{4}{25};\ \frac{4}{125};$ g) $3\sqrt{5};\ 3;\ \frac{3\sqrt{5}}{5};\ ... .$

541. Qo'shiluvchilari cheksiz geometrik progressiyaning hadlari bo'lgan yig'indini toping ($|a|<1$):

a) $1+a+a^2+a^3+...;$ d) $1+a^2+a^4+a^6+...;$

b) $1-a+a^2-a^3+...;$ e) $a-a^4+a^7-a^{10}+... .$

542. Sonlarni oddiy kasr shaklida yozing:

a) 0,(6); e) 1,(81); h) 1,(72);

b) 0,(1); f) 0,2(3); i) 0,43(6);

d) 0,(36); g) 0,32(45); j) 0,017(12).

543. Radiusi 5 sm ga teng bo'lgan aylanaga muntazam uchburchak ichki chizilgan; uchburchakka aylana ichki chizilgan; aylanaga yana muntazam uchburchak ichki chizilgan va hokazo. Aylanalar uzunliklarining va doiralar yuzlarining yig'indilarini toping.

544. 1) Kvadratga doira ichki chizilgan; bu doiraga ikkinchi kvadrat ichki chizilgan; ikkinchi kvadratga yana doira ichki chizilgan va hokazo. Agar birinchi kvadratning tomoni 8 sm ga teng bo'lsa, hamma doiralar yuzlarining yig'indisini toping.

2) Tomoni a bo'lgan muntazam uchburchakka aylana ichki chizilgan; aylanaga muntazam uchburchak ichki chizilgan; bu uchburchakka yana aylanaga ichki chizilgan va hokazo.

a) uchburchaklar perimetrlarining;

b) uchburchaklar yuzlarining;

d) aylanalar uzunliklarining;

e) doiralar yuzlarining yig'indisini toping.

74-§. *n* – darajali ildizning ta'rifi

Ma'lumki, a sonning kvadrat ildizi deb kvadrati a ga teng songa aytiladi. Istalgan natural n – darajali ildiz shunga o'xshash ta'riflanadi.

Ta'rif. **a sonining n darajali ildizi deb shunday songa aytiladiki, u sonning n – darajasi a ga teng bo'ladi.**

Masalan, 32 sonining beshinchi darajali ildizi 2 sonidir, chunki $2^5=32$; 81 ning to'rtinchi darajali ildizi 3 va –3, chunki $3^4=81$ va $(-3)^4=81$. Sonning ikkinchi darajali ildizini **kvadrat ildiz**, uchinchi darajali ildizini **kub ildiz** deb atash qabul qilingan.

Istalgan a sonning n – darajali ildizini $\sqrt[n]{a}$ kabi belgilanadi. U bunday o'qiladi: «a ning n – darajali ildizi».

n – ildiz ko'rsatkichi, a – ildiz ostidagi ifoda.

Masalan, $\sqrt[3]{-125}$ yozuv – **125 ning kub ildizi** deb o'qiladi.

Ta'rifdagi a sonining n – darajali ildizi $\sqrt[n]{a}$ kabi yozilib, $\left(\sqrt[n]{a}\right)^n=a$ tenglik bilan yoziladi.

50-a, chizmada toq ko'rsatkichli darajali funksiyaning grafigi berilgan.

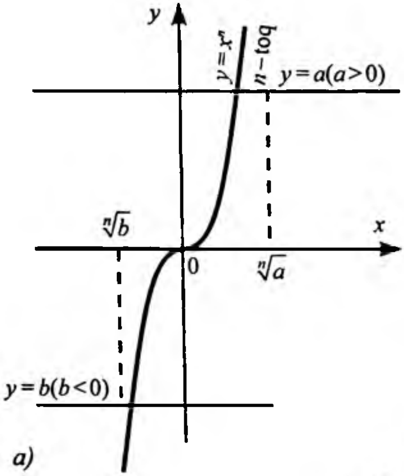
n toq bo'lganda istalgan a soni uchun n – darajasi a ga teng bo'ladigan yagona $\sqrt[n]{a}$ **ildiz** mavjud. Masalan, $\sqrt[5]{32}=2$; $\sqrt[5]{-32}=-2$, chunki $2^5=32$ va $(-2)^5=-32$.

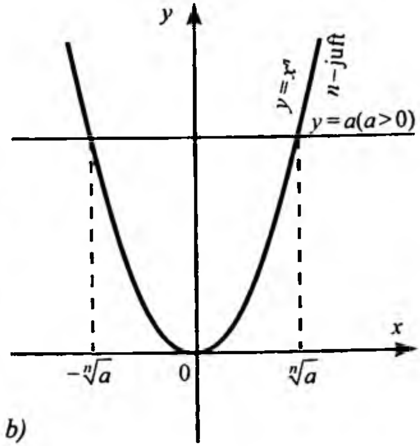
Shunday qilib, **agar n toq son bo'lsa, $\sqrt[n]{a}$ ifoda istalgan a da ma'noga ega.**

Juft ko'rsatkichli darajali funksiyaning xossalaridan:

1) Agar n juft son bo'lsa, $a>0$ da n – darajasi a ga teng bo'lgan ikkita qarama-qarshi son $\left(\sqrt[n]{a} \text{ va } -\sqrt[n]{a}\right)$ mavjud bo'ladi (50-b, chizma).

2) Agar $a=0$ bo'lsa, bunday son (0 soni) bitta bo'ladi.





50-chizma.

3) Agar $a<0$ bo'lsa, n – darajasi a ga teng bo'lgan son $\left(\sqrt[n]{a}\right)$ mavjud emas.

Masalan, $\sqrt[4]{81}=3$, chunki $3^4=81$; $\sqrt[4]{0}=0$, chunki $0^4=0$; $\sqrt[4]{-81}$ mavjud emas, chunki 4-darajasi –81 bo'lgan son mavjud emas.

Shunday qilib, **agar *n* juft bo'lsa,** $\sqrt[n]{a}$ **ifoda faqat** $a\geq 0$ **da ma'noga ega.**

Umuman, $a\geq 0$ da $\sqrt[n]{a}$ ifoda n juft bo'lganda ham, toq bo'lganda ham ma'noga ega bo'ladi va bu ifodaning qiymati nomanfiy son bo'ladi. Uni ***a* sonining *n* – darajali arifmetik ildizi** deyiladi.

Ta'rif. **Nomanfiy *a* sonining *n* – darajali arifmetik ildizi deb *n* – darajasi *a* ga teng bo'lgan nomanfiy songa aytiladi.**

Masalan, $\sqrt[6]{64}=2$; $\sqrt[4]{81}=3$.

Manfiy sonning toq darajali ildizini shu darajadagi arifmetik ildiz bilan ifodalash mumkin. Masalan, $\sqrt[3]{-8}=-\sqrt[3]{8}$; $\sqrt[5]{-29}=-\sqrt[5]{29}$. (Bu yerda $\sqrt[5]{29}$ – arifmetik ildiz.)

1-m i s o l. $x^6=7$ tenglamani yechamiz.

Bu tenglamani ikkita ildizi bo'lib ular $x^6=7$ tenglikda 6 – darajali ildiz chiqarib, ya'ni $\sqrt[6]{x^6}=\pm\sqrt[6]{7}$; $x=\pm\sqrt[6]{7}$ kabi topiladi. Haqiqatan, $\left(\sqrt[6]{7}\right)^6=7$ va $\left(-\sqrt[6]{7}\right)^6=7$.

2-m i s o l. $x^4=81$ tenglamani yechamiz.

Bu tenglamaning ikkita ildizi bo‘lib, ular $x_{1/2}=\pm\sqrt[4]{81}=\pm3$; $x_1=-3$ va $x_2=3$.

3-m i s o l. $x^7=-128$ tenglamani yechamiz. Bu tenglama bitta $x=\sqrt[7]{-128}=$ $=-2$ ildizga ega, chunki $(-2)^7=-128$.

TAKRORLASH UCHUN SAVOLLAR

1. *a* sonining kvadrat ildizi deb nimaga aytiladi?
2. *a* sonining *n* – darajali ildizi deb nimaga aytiladi va qanday yoziladi?
3. *n* toq bo‘lganda *a* sonining *n* – darajali nechta ildizi mavjud?
4. *n* juft bo‘lganda *a* sonining *n*– darajali nechta ildizi mavjud ($a>0$ da, $a=0$ da va $a<0$ da)?
5. Nomanfiy *a* sonining arifmetik ildizi deb nimaga aytiladi?
6. –3 soni 81 ning to‘rtinchi darajali arifmetik ildizi bo‘ladimi?

MASALALARNI YECHING

545. a) $\frac{1}{2}$ soni $\frac{1}{16}$ ning to‘rtinchi darajali arifmetik ildizi ekanligini isbotlang.

b) –2 soni 16 ning to‘rtinchi darajali arifmetik ildizi emasligini isbotlang.

d) 0,1 soni 0,0001 ning beshinchi darajali arifmetik ildizi emasligini ko‘rsating.

546. Tenglikni to‘g‘riligini isbotlang:

a) $\sqrt{361}=19$; f) $\sqrt[10]{1}=1$;

b) $\sqrt[3]{343}=7$; g) $\sqrt[7]{0}=0$;

d) $\sqrt[6]{\frac{1}{64}}=\frac{1}{2}$; h) $\sqrt{7-4\sqrt{3}}=2-\sqrt{3}$;

e) $\sqrt{\frac{32}{243}}=\frac{2}{3}$; i) $\sqrt{9-4\sqrt{5}}=\sqrt{5}-2$.

547. Hisoblang:

a) $\sqrt[9]{512}$; d) $\sqrt[11]{0}$; f) $\sqrt[4]{\frac{16}{625}}$; h) $\sqrt[4]{7\frac{58}{81}}$;

b) $\sqrt[3]{1331}$; e) $\sqrt[7]{-128}$; g) $\sqrt[5]{0,00001}$; i) $\sqrt{7\frac{19}{32}}$.

548. Quyidagi sonlar qanday ikkita ketma-ket butun sonlar orasida yotadi:

a) $\sqrt[3]{3,5}$; b) $\sqrt[3]{20}$; d) $\sqrt[4]{9}$; e) $\sqrt[4]{52}$.

549. Ifoda ma'noga egami:

a) $\sqrt[3]{-19}$; d) $\sqrt[5]{(-3)^3}$; f) $\sqrt[4]{(-5)^4}$;

b) $\sqrt[7]{-0,28}$; e) $\sqrt[8]{(-2)^3}$; g) $\sqrt[10]{(-7)^3}$.

550. Ifodaning qiymatini toping:

a) $\sqrt[5]{-32}$; f) $12-6\sqrt[3]{0,125}$; j) $\sqrt[3]{-125}+5\sqrt[4]{16}$;

b) $-2\sqrt[4]{81}$; g) $\sqrt[7]{-1}+6\sqrt[3]{-27}$; k) $\sqrt[6]{64}-4\sqrt[3]{-64}$;

d) $-4\sqrt[3]{27}$; h) $1+10\sqrt[4]{0,0081}$; l) $\sqrt[3]{-3\frac{3}{8}}+\sqrt{2,25}$;

e) $\sqrt[5]{32}+\sqrt[3]{-8}$; i) $\sqrt[5]{-32}-4\sqrt[3]{64}$; m) $3\sqrt[4]{16}-4\sqrt[3]{-27}$.

551. Hisoblang:

a) $\left(\sqrt{10}\right)^2$; e) $-\sqrt[6]{25^3}$; h) $5\sqrt[3]{(-2)^3}$;

b) $\left(-\sqrt[4]{12}\right)^4$; f) $\sqrt[6]{64^2}$; i) $-2\sqrt[10]{32^2}$;

d) $2\sqrt[4]{(-3)^4}$; g) $\left(\sqrt[7]{-3}\right)^7$; j) $\sqrt[7]{-64}+\sqrt[6]{27^2}$.

552. Tenglamaning ildizlarini toping:

a) $x^3=4$; f) $x^7=-1$; j) $16x^4-1=0$;

b) $x^3=-4$; g) $x^3-27=0$; k) $\frac{1}{2}x^5+4=0$;

d) $x^4=81$; h) $x^{10}+1=0$; l) $0,02x^6-1,28=0$;

e) $x^5=-32$; i) $3x^3-81-0$; m) $\frac{3}{4}x^3-12\frac{3}{4}=0$.

75-§. n – darajali arifmetik ildizning xossalari

Bizga arifmetik kvadrat ildizning quyidagi xossalari ma'lum:

agar $a \geq 0$ va $b \geq 0$ bo'lsa, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$;

agar $a \geq 0$ va $b > 0$ bo'lsa, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ bo'ladi.

n – darajali arifmetik ildiz $n > 2$ bo'lganda ham shunga o'xshash xossalarga ega bo'ladi.

1-teorema. **Agar $a \geq 0$ va $b \geq 0$ bo'lsa, u holda $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ bo'ladi.**

Isbot. O'zgaruvchilarning ko'rsatilgan qiymatlarida: 1) $\sqrt[n]{a} \cdot \sqrt[n]{b} \geq 0$ va 2) $\left(\sqrt[n]{a} \cdot \sqrt[n]{b}\right)^n = ab$ shartlar bajarilishini ko'rsatamiz.

Haqiqatan, $a \geq 0$ va $b \geq 0$ bo'lganida $\sqrt[n]{a}$ va $\sqrt[n]{b}$ nomanfiy bo'lib, $\sqrt[n]{a} \cdot \sqrt[n]{b}$ va $\sqrt[n]{ab}$ larning har biri ma'noga ega, chunki ular arifmetik ildiz bo'lgani uchun $\sqrt[n]{a} \geq 0$ va $\sqrt[n]{b} \geq 0$ edi.

Bundan $\sqrt[n]{a} \cdot \sqrt[n]{b} \geq 0$ kelib chiqadi.

Ko'paytma darajasining xossasiga ko'ra $\left(\sqrt[n]{a} \cdot \sqrt[n]{b}\right)^n = \left(\sqrt[n]{a}\right)^n \cdot \left(\sqrt[n]{b}\right)^n = ab$.

Demak, $\left(\sqrt[n]{a} \cdot \sqrt[n]{b}\right)^n = ab$ dan n – darajali arifmetik ildiz ta'rifiga asosan $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ tenglikni yozamiz.

Ildiz ostidagi ko'paytuvchilar soni ikkitadan ortiq bo'lganda ham yuqoridagi teorema to'g'ri bo'ladi, ya'ni $\sqrt[n]{abc} = \sqrt[n]{(ab)c} = \sqrt[n]{ab} \cdot \sqrt[n]{c} = \sqrt[n]{a} \cdot \sqrt[n]{b} \cdot \sqrt[n]{c}$.

Shunday qilib, **istalgan natural n da nomanfiy ko'paytuvchilardan olingan ildiz bu ko'paytuvchilar ildizlarining ko'paytmasiga teng.**

1-m i s o l. $\sqrt[4]{16 \cdot 0{,}0001} = \sqrt[4]{16} \cdot \sqrt[4]{0{,}0001} = 2 \cdot 0{,}1 = 0{,}2.$

2-teorema. **Agar $a \geq 0$ va $b > 0$ bo'lsa, u holda $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ bo'ladi.**

Bu teoremaning isboti 1-teoremaning isbotiga o'xshash isbotlanadi.

Shunday qilib, n – darajali arifmetik ildizning yana bitta xossasi o'rinli: **istalgan natural n da surati nomanfiy, maxraji musbat**

bo'lgan kasrning ildizi suratining ildizini maxrajining ildiziga bo'linganiga teng. Ya'ni $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

2-m i s o l. $\sqrt[5]{7\frac{19}{32}} = \sqrt[5]{\frac{243}{32}} = \frac{\sqrt[5]{243}}{\sqrt[5]{32}} = \frac{3}{2} = 1,5.$

n – darajali ildizning boshqa xossasiga doir quyidagi misolni ko'rib chiqamiz. $\sqrt[3]{\sqrt{64}}$ bilan $\sqrt[6]{64}$ ifodalarning qiymatlarini taqqoslang.

$\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = 2;$ $\sqrt[6]{64} = \sqrt[6]{2^6} = 2$. Bu ifodalar teng ekan, ya'ni $\sqrt[3]{\sqrt{64}} = \sqrt[6]{64}$.

3-teorema. **Agar** n **va** k **– natural sonlar bo'lib,** $a \geq 0$ **bo'lsa, u holda** $\sqrt[n]{\sqrt[k]{a}} = \sqrt[nk]{a}$ **bo'ladi.**

Isbot. $a \geq 0$ bo'lgani uchun $\sqrt[n]{\sqrt[k]{a}}$ va $\sqrt[nk]{a}$ ifodalar ma'noga ega va nomanfiydir.

Bundan tashqari darajaning xossasiga asosan $\left(\sqrt[n]{\sqrt[k]{a}}\right)^{nk} = \left(\left(\sqrt[n]{\sqrt[k]{a}}\right)^{n}\right)^{k} =$
$= \left(\sqrt[k]{a}\right)^{k} = a.$

Demak, arifmetik ildizning ta'rifiga asosan $\left(\sqrt[n]{\sqrt[k]{a}}\right)^{nk} = a$ dan $\sqrt[n]{\sqrt[k]{a}} = \sqrt[nk]{a}$ kelib chiqadi.

Masalan, $\sqrt{\sqrt[5]{1024}} = \sqrt[10]{2^{10}} = 2.$

4-teorema. **Agar** n, k **va** m **– natural sonlar bo'lib,** $a \geq 0$ **bo'lsa, u holda** $\sqrt[nk]{a^{mk}} = \sqrt[n]{a^m}$ **bo'ladi.**

Isbot. 3-teoremaga asosan $\sqrt[nk]{a^{mk}} = \sqrt[n]{\sqrt[k]{a^{mk}}} = \sqrt[n]{\sqrt[k]{(a^m)^k}} = \sqrt[n]{a^m}$, ya'ni $\sqrt[nk]{a^{mk}} = \sqrt[n]{a^m}$.

Shunday qilib, **agar ildiz ko'rsatkichi va ildiz ostidagi ifodaning daraja ko'rsatkichi ayni bir natural songa ko'paytirilsa yoki bo'linsa, ildizning qiymati o'zgarmaydi.**

Bu xossani ildizning asosiy xossasi deyiladi.

3-misol. $\sqrt[3]{3\sqrt{3}}$ ifodani soddalashtiramiz. 3 ko'paytuvchini kvadrat ildiz ostiga kiritamiz. $\sqrt[3]{3\sqrt{3}} = \sqrt[3]{\sqrt{3^2 \cdot 3}} = \sqrt[3]{\sqrt{3^3}}$ buni ildizning ildizi haqidagi teoremaga asosan (3-teorema).

$\sqrt[3]{\sqrt{3^3}} = \sqrt[6]{3^3}$ buni ildizning asosiy xossasi (4-teorema)ga asosan $\sqrt[6]{3^3} = \sqrt{3}$.

Shunday qilib $\sqrt[3]{3\sqrt{3}} = \sqrt{3}$.

TAKRORLASH UCHUN SAVOLLAR

1. Ko'paytmadan *n* – darajali ildiz qanday chiqariladi (1-teorema)?
2. Kasrdan *n* – darajali ildiz qanday chiqariladi (2-teorema)?
3. Ildizdan *n* – darajali ildiz qanday chiqariladi (3-teorema)?
4. Ildizning asosiy xossasini ayting?
5. Ildizni hisoblang (og'zaki):

a) $\sqrt{25 \cdot 4}$; b) $\sqrt[3]{8 \cdot 27}$; d) $\sqrt[4]{625 \cdot 81}$.

MASALALARNI YECHING

553. Ifodaning qiymatini toping:

a) $\sqrt[3]{27 \cdot 8}$; e) $\sqrt[4]{0,0016 \cdot 81}$; h) $\sqrt[3]{\frac{8}{125}}$;

b) $\sqrt[4]{16 \cdot 0,0001}$; f) $\sqrt[5]{243 \cdot \frac{1}{32}}$; i) $\sqrt[5]{-7\frac{19}{32}}$;

d) $\sqrt[4]{625 \cdot 16}$; g) $\sqrt[6]{64 \cdot \frac{1}{729}}$; j) $\sqrt[4]{3\frac{13}{81}}$.

554. Hisoblang:

a) $\sqrt[4]{7^4 \cdot 2^4}$; d) $\sqrt[5]{3^{10} \cdot 0,5^{15}}$; f) $\sqrt[6]{\frac{4^6}{3^{12}}}$;

b) $\sqrt[6]{2^6 \cdot 3^{12}}$; e) $\sqrt[4]{\frac{3^4}{7^4}}$; g) $\sqrt{\frac{6^8}{11^6}}$.

555. Hisoblang:

a) $\sqrt[3]{24 \cdot 9}$; d) $\sqrt[4]{\frac{125}{0,2}}$; f) $\sqrt[3]{\frac{54}{0,25}}$;

b) $\sqrt[5]{48\cdot 162}$; e) $\sqrt[4]{\frac{16}{0{,}0625}}$; g) $\sqrt[3]{\frac{10800}{0{,}4}}$.

556. Hisoblang:

a) $\sqrt[3]{100}\cdot\sqrt[3]{10}$; d) $\sqrt[3]{9+\sqrt{17}}\cdot\sqrt[3]{9-\sqrt{17}}$; f) $\frac{\sqrt[7]{256}}{\sqrt[7]{2}}$;

b) $\sqrt[5]{3^2 5^3}\cdot\sqrt[5]{3^3\cdot 5^2}$; e) $\sqrt[4]{10-\sqrt{19}}\cdot\sqrt[4]{10+\sqrt{19}}$; g) $\frac{\sqrt[4]{2500}}{\sqrt[4]{4}}$.

557. Ifodani birhad ko'rinishida tasvirlang (harflar nomanfiy sonlar):

a) $\sqrt{25a^2}$; d) $\sqrt[4]{81c^4}$; f) $\sqrt[3]{3\frac{3}{8}y^9}$

b) $\sqrt[3]{8b^3}$; e) $\sqrt[5]{32x^{10}}$; g) $\sqrt[4]{0{,}0625d^8}$.

558. Ko'paytuvchini ildiz belgisi ostidan chiqaring:

a) $\sqrt{4a}$; d) $\sqrt[3]{24c}$; f) $\sqrt[4]{81b^6}$; h) $\sqrt{50x^3}$, bunda $x>0$;

b) $\sqrt{18b}$; e) $\sqrt[5]{7a^6}$; g) $\sqrt[6]{10c^8}$; i) $\sqrt[4]{48a^6}$, bunda $a<0$.

559. Ko'paytuvchini ildiz belgisi ostiga kiriting:

a) $2\sqrt{3}$; d) $3\sqrt[4]{\frac{1}{9}}$; f) $2b\sqrt[4]{3}$, bunda $b<0$;

b) $3\sqrt[3]{2}$; e) $a\sqrt[3]{2}$, bunda $a>0$; g) $3c\sqrt[3]{2c^2}$, bunda $c>0$.

560. Kasr maxrajini ildizdan qutqaring:

a) $\frac{3}{\sqrt{5}}$; d) $\frac{3}{\sqrt[4]{3}}$; f) $\frac{15}{\sqrt[3]{5}}$; h) $\frac{10}{\sqrt[4]{8}}$;

b) $\frac{2}{\sqrt[3]{2}}$; e) $\frac{7}{\sqrt[3]{49}}$; g) $\frac{3}{\sqrt[4]{6}}$; i) $\frac{21}{\sqrt[5]{27}}$.

561. Ifodani soddalashtiring:

a) $\sqrt[5]{\sqrt{3}}$; e) $\sqrt[3]{b\sqrt[3]{b}}$, bunda $b\geq 0$; h) $\sqrt[9]{7^6}$;

b) $\sqrt{\sqrt[3]{7}}$; f) $\sqrt{c\sqrt[3]{c^2}}$, bunda $c\geq 0$; i) $\sqrt[18]{36^3}$;

d) $\sqrt[5]{\sqrt[3]{2}}$; g) $\sqrt[4]{x\sqrt[3]{x}}$, bunda $x\geq 0$; j) $\sqrt{4\sqrt[3]{4}}$.

562. Sonlarni taqqoslang:

a) $\sqrt[3]{3}$ va $\sqrt[6]{5}$; d) $\sqrt[6]{8}$ va $\sqrt{3}$; f) $\sqrt[5]{5}$ va $\sqrt{2\sqrt[3]{3}}$;

b) $\sqrt{2}$ va $\sqrt[6]{7}$; e) $\sqrt[3]{2}$ va $\sqrt[12]{45}$; g) $\sqrt[3]{7}$ va $\sqrt{3\sqrt[3]{2}}$.

563. Ifodaning qiymati ratsional son bo'lishini isbotlang.

a) $\frac{9-4\sqrt{5}}{9+4\sqrt{5}}+\frac{9+4\sqrt{5}}{9-4\sqrt{5}}$; b) $\frac{5+2\sqrt{2}}{5-2\sqrt{2}}+\frac{5-2\sqrt{2}}{5+2\sqrt{2}}$.

564. Ifodaning qiymatini toping:

a) $\sqrt[3]{7+\sqrt{122}}\cdot\sqrt[3]{7-\sqrt{122}}$; b) $\sqrt[4]{7-4\sqrt{3}}\cdot\sqrt{2+\sqrt{3}}$.

565. Ifodani soddalashtiring:

a) $(3+2\sqrt{6})^2+(3-2\sqrt{6})^2$; b) $\left(\sqrt{7+2\sqrt{10}}+\sqrt{7-2\sqrt{10}}\right)^2$.

76-§. Kasr ko'rsatkichli darajaning ta'rifi

a^n ifoda n butun son bo'lganda va $a\neq 0$ da ma'noga ega ekanligini bilamiz.

Masalan, $(-3)^5$ ifoda har biri -3 ga teng bo'lgan beshta ko'paytuvchilarning ko'paytmasini anglatadi. 2^{-6} soni 2^6 darajaga teskari sonni anglatadi.

Endi ko'rsatkichi butun son bo'lmay, kasr son bo'lgan daraja tushunchasini kiritamiz.

$\sqrt[n]{a^m}=a^{\frac{m}{n}}$ tenglik (bunda $a>0$, m – butun son va n – natural son), agar m soni n ga bo'linsa (ya'ni $\frac{m}{n}$ – butun son bo'lsa), to'g'ri bo'ladi.

Masalan, $\sqrt[7]{5^{21}}=5^{\frac{21}{7}}$ tenglikni ko'rib chiqamiz:

1) $\sqrt[7]{5^{21}}=\sqrt[7]{(5^3)^7}=5^3$;

2) $5^{\frac{21}{7}}=5^3$;

lardan $\sqrt[7]{5^{21}}=5^{\frac{21}{7}}$ ekani to'g'ri.

Ta'rif. **Agar a – musbat son, $\frac{m}{n}$ – kasr son (m – butun, n – natural) bo'lsa, u holda $a^{\frac{m}{n}}=\sqrt[n]{a^m}$ bo'ladi.**

Masalan, ta'rifga ko'ra:

$$0,7^{\frac{3}{5}}=\sqrt[5]{0,7^3};\quad \left(\frac{2}{5}\right)^{\frac{3}{7}}=\sqrt[7]{\left(\frac{2}{5}\right)^3}=\sqrt[7]{\frac{8}{125}};\quad 5^{-\frac{2}{3}}=\sqrt[3]{5^{-2}}=\sqrt[3]{\frac{1}{5^2}}=\sqrt[3]{\frac{1}{25}}.$$

Asosi nolga teng bo'lgan daraja faqat musbat kasr ko'rsatkich uchun aniqlanadi: **agar $\frac{m}{n}$ – musbat kasr son bo'lsa, (m va n – natural son), u holda $0^{\frac{m}{n}}=0$ bo'ladi.**

Eslatma. $(-2)^{\frac{3}{4}}$, $(-8)^{\frac{1}{6}}$; $0^{-\frac{1}{2}}$ kabi ifodalar ma'noga ega emas. Kasr ko'rsatkichli darajaning ta'rifiga asosan $2^{\frac{6}{8}}=\sqrt[8]{2^6},=\sqrt[4]{2^3}=2^{\frac{3}{4}}$.

Demak, kasr ko'rsatkichli darajaning daraja ko'rsatkichini qisqartirish mumkin.

Buni umumiy holda ko'rsatamiz. Faraz qilaylik $a>0$, m – butun, n va k – natural son bo'lsin. Kasr ko'rsatkichli darajaning ta'rifidan va ildizning asosiy xossalaridan foydalanib, $a^{\frac{mk}{nk}}=\sqrt[nk]{a^{mk}}=\sqrt[n]{a^m}=a^{\frac{m}{n}}$ hosil bo'ladi.

TAKRORLASH UCHUN SAVOLLAR

1. a^n (n – natural son) daraja nimani bildiradi?
2. Kasr ko'rsatkichli darajaning ta'rifini ayting.
3. Manfiy asosli darajalar nimaga teng?
4. Kasr ko'rsatkichli darajaning darajasini qisqartirish mumkinmi?

MASALALARNI YECHING

566. Kasr ko'rsatkichli darajani ildiz bilan almashtiring:

a) $7^{\frac{2}{3}}$; $6^{\frac{1}{3}}$; $5^{-\frac{1}{2}}$; $10^{-0,5}$;

b) $2,5^{\frac{2}{3}}$; $0,5^{-0,5}$; $\left(\frac{1}{9}\right)^{-\frac{1}{2}}$;

d) $a^{\frac{3}{4}}$; $b^{-\frac{2}{3}}$; $c^{0,6}$;

e) $3x^{\frac{1}{2}}$; $(3x)^{\frac{1}{2}}$; $\frac{1}{5}y^{\frac{1}{5}}$; $-y^{-\frac{2}{3}}$;

f) $(ab)^{\frac{2}{3}}$; $ab^{\frac{2}{3}}$; $(a+b)^{\frac{2}{3}}$; $a^{\frac{2}{3}}+b^{-\frac{2}{3}}$.

567. Arifmetik ildizni kasr ko‘rsatkichli darajalar bilan almashtiring:

a) $\sqrt{5}$; $\sqrt[3]{17^2}$; $\sqrt[5]{3^6}$; $\sqrt[8]{7^{-5}}$; $\sqrt[9]{0{,}125^2}$;

b) $\sqrt[7]{a}$; $\sqrt[8]{a^9}$; $\sqrt[12]{b^{-5}}$; $\sqrt[11]{5c^2}$; $\sqrt[3]{a-b}$.

568. Hisoblang:

a) $100^{\frac{1}{2}}$; d) $9^{-\frac{1}{2}}$; f) $0^{\frac{5}{6}}$; h) $8^{1\frac{1}{3}}$; j) $\left(3\frac{3}{8}\right)^{-\frac{2}{3}}$;

b) $8^{\frac{1}{3}}$; e) $27^{-\frac{1}{3}}$; g) $81^{\frac{3}{4}}$; i) $0{,}25^{-\frac{3}{2}}$; k) $0{,}01^{-2{,}5}$.

569. Ifoda ma’noga egami:

$7^{-\frac{1}{3}}$; $(-7)^{\frac{1}{3}}$; $0^{-\frac{2}{3}}$; $0^{\frac{4}{9}}$.

570. Ifodadagi o‘zgaruvchining qabul qiladigan qiymatlarini ko‘rsating:

a) $x^{\frac{1}{2}}$; b) $(y-1)^{\frac{1}{3}}$; d) $(a+2)^{\frac{3}{5}}$; e) $(c-5)^{-\frac{1}{3}}$.

571. Taqqoslang:

a) $2^{\frac{1}{2}}$ va $3^{\frac{1}{2}}$; d) $0{,}3^{\frac{1}{2}}$ va $5^{\frac{1}{3}}$;

b) $0{,}3^{\frac{1}{2}}$ va $0{,}5^{\frac{1}{2}}$; e) $7^{\frac{1}{3}}$ va $7^{\frac{1}{6}}$.

77-§. Ratsional ko‘rsatkichli darajaning xossalari

Butun ko‘rsatkichli darajaning bizga ma’lum xossalari istagan ratsional daraja uchun ham o‘rinli. Ularni keltirib o‘tamiz.

Istalgan $a>0$ istalgan p va q ratsional sonlar uchun:

1. $a^p \cdot a^q = a^{p+q}$;
2. $a^p : a^q = a^{p-}$;
3. $(a^p)^q = a^{pq}$;

Istalgan $a>0$ va $b>0$ hamda istalgan p ratsional son uchun:

4. $(ab)^p = a^p b^p$;
5. $\left(\frac{a}{b}\right)^p = \frac{a^p}{a^q}$.

Bu xossalarni kasr ko'rsatkichli darajaning ta'rifiga, n – darajali ildizning xossalariga va butun ko'rsatkichli darajaning xossalariga asoslanib isbotlash mumkin.

1. $a^p \cdot a^q = a^{p+q}$ xossani isbotlaymiz.

Bu xossani isbotlash usulini avval xususiy misolda ko'rib chiqamiz:

Masalan, $p=\frac{2}{3}$, $q=\frac{1}{5}$ bo'lsin. $a>0$ da $a^{\frac{2}{3}}a^{\frac{1}{5}}=a^{\frac{2}{3}+\frac{1}{5}}$ ekanini isbotlaymiz. $\frac{2}{3}$ va $\frac{1}{5}$ kasrlarni umumiy maxrajga keltiramiz. U holda $a^{\frac{2}{3}}a^{\frac{1}{5}}=a^{\frac{10}{15}}a^{\frac{3}{15}}$.

$a^{\frac{10}{15}}=\sqrt[15]{a^{10}}$ va $a^{\frac{3}{15}}=\sqrt[15]{a^3}$ bo'lgani uchun arifmetik ildizning ta'rifidan: $\sqrt[15]{a^{10}}\cdot\sqrt[15]{a^3}=\sqrt[15]{a^{10}a^3}=\sqrt[15]{a^{13}}$.

$\sqrt[15]{a^{13}}$ dan kasr ko'rsatkichli darajaga o'tamiz, ya'ni $\sqrt[15]{a^{13}}=a^{\frac{13}{15}}$. Demak, $a^{\frac{2}{3}}a^{\frac{1}{5}}=a^{\frac{13}{15}}$ ekan. Ammo $\frac{2}{3}+\frac{1}{5}=\frac{13}{15}$ bo'lganidan $a^{\frac{2}{3}}a^{\frac{1}{5}}=a^{\frac{2}{3}+\frac{1}{5}}$ kelib chiqadi.

Endi 1-xossani umumiy ko'rinishda isbotlaymiz. p va q ratsional sonlarni bir xil maxrajli kasr ko'rinishida ifodalaymiz:

$p=\frac{k}{n}$ va $q=\frac{m}{n}$ bo'lsin, bunda k va m – butun sonlar, n – natural son. U holda $a>0$ da:

$$a^p a^q = a^{\frac{k}{n}}a^{\frac{m}{n}}=\sqrt[n]{a^k}\cdot\sqrt[n]{a^m}=\sqrt[n]{a^k a^m}=\sqrt[n]{a^{k+m}}=a^{\frac{k+m}{n}}=a^{\frac{k}{n}+\frac{m}{n}}=a^{p+q}.$$

Demak, $a^p q^q = a^{p+q}$ Masalan, $y^{-\frac{2}{3}}\cdot y^{\frac{3}{5}}=y^{-\frac{2}{3}+\frac{3}{5}}=y^{-\frac{1}{15}}$.

$a^p : a^q = a^{p-q}$ va $(a^p)^q = a^{pq}$ xossalar ham 1-xossaning isboti kabi isbotlanadi. 4-xossani $a>0$ va $b>0$ hamda istalgan p ratsional bo'lganda $(ab)^p = a^p b^p$ ekanini isbotlaymiz.

Faraz qilaylik, $p=\frac{m}{n}$ bo'lsin, bunda m – butun son va n – natural son. U holda $(ab)^p=(ab)^{\frac{m}{n}}=\sqrt[n]{(ab)^m}=\sqrt[n]{a^m b^m}=\sqrt[n]{a^m}\cdot\sqrt[n]{b^m}=a^{\frac{m}{n}}\cdot b^{\frac{m}{n}}=$ $=a^p b^p$. Demak, $(ab)^p = a^p b^p$

Masalan, $(27\cdot 8)^{-\frac{1}{3}}=27^{-\frac{1}{3}}\cdot 8^{-\frac{1}{3}}=\frac{1}{\sqrt[3]{27}}\cdot\frac{1}{\sqrt[3]{8}}=\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{6}$.

5-xossani $\frac{a}{b}$ kasrni ab^{-1} ko‘paytma ko‘rinishida ifodalab, 4-xossani qo‘llab isbotlanadi.

Masalan: 1) $\left(\frac{27^3}{125^6}\right)^{\frac{1}{9}} = \frac{27^{\frac{1}{3}}}{125^{\frac{2}{3}}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125^2}} = \frac{3}{5^2} = \frac{3}{25} = 0{,}12.$

2) $p^{-3}q^{2\frac{1}{4}}\left(p^{-\frac{2}{7}}q^{\frac{1}{14}}\right)^{-3,5} = p^{-3}q^{2\frac{1}{4}}\cdot p^{-\frac{2}{7}\left(-\frac{7}{2}\right)}\cdot q^{\frac{1}{14}\left(-\frac{7}{2}\right)} = p^{-3}q^{2\frac{1}{4}}\cdot p^{1}\cdot q^{-\frac{1}{4}} = p^{-2}q^{2}.$

TAKRORLASH UCHUN SAVOLLAR

1. Istalgan $a>0$ va istalgan ratsional p va q sonlar uchun:

   a) $a^p \cdot a^q =$; b) $a^p : a^q =$; $(a^p)q =$ formulalarni yozing.

2. Istalgan $a>0$ va $b>0$ hamda istalgan ratsional p son uchun:

   a) $(ab)^p =$; b) $\left(\frac{a}{b}\right)^p =$ formulalarni yozing.

3. a sonining n – darajali ildizi deb nimaga aytiladi?
4. Og‘zaki hisoblang:

   a) $\sqrt[3]{125}$; b) $\sqrt[4]{64}$; d) $\sqrt[5]{0{,}00001}$; e) $\sqrt[4]{5\frac{1}{16}}$.

MASALALARNI YECHING

572. Ratsional ko‘rsatkichli daraja shaklida ifodalang:

a) $c^{\frac{1}{2}}c^{\frac{1}{3}}$; e) $x^2 : x^{\frac{3}{2}}$; h) $\left(b^{\frac{1}{2}}\right)^{\frac{1}{3}}$;

b) $b^{-\frac{1}{3}}b^{\frac{1}{2}}$; f) $y^{\frac{5}{6}} : y^{\frac{1}{3}}$; i) $\left(a^{\frac{3}{2}}\right)^{\frac{4}{9}}$;

d) $a^{\frac{2}{3}}a^{\frac{1}{6}}$; g) $z^{\frac{1}{5}} : z^{-\frac{1}{2}}$; j) $\left(c^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$.

573. Daraja shaklida ifodalang:

a) $x^{0,2}x^{-1}x^{0,6}$; b) $a^{\frac{2}{3}}a^{\frac{1}{6}}a^{\frac{5}{3}}$; d) $y^{0,8}y^{-5}y^{7,2}$; e) $b^{\frac{3}{8}}b^{-\frac{5}{24}}b^{\frac{1}{3}}$.

574. Soddalashtiring:

a) $\left(x^{\frac{2}{3}}\right)^{0,6}\cdot x^{-\frac{2}{5}}$; d) $\left(a^{-\frac{4}{3}}\right)^{-\frac{1}{2}}:\left(a^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$;

b) $\left(y^{-\frac{5}{8}}\right)^{0,4}\cdot y^{0,25}$; e) $\left(c^{\frac{5}{12}}\right)^{1,2}\cdot\left(c^{-\frac{1}{3}}\right)^{-1,5}$.

575. Hisoblang:

a) $5^{\frac{1}{5}}\cdot 5^{-0,25}\cdot 5^{\frac{4}{5}}\cdot 5^{-0,75}$; d) $4^{0,7}\cdot 2^{-0,4}\cdot 16^{0,75}$;

b) $9^{-\frac{4}{3}}\cdot 27^{\frac{4}{3}}\cdot 3^{\frac{2}{3}}$; e) $25^{0,8}\cdot 5^{1,4}:(5^{0,4})^{5}$.

576. Ifodani soddalashtiring:

a) $(125x^{6})^{-\frac{2}{3}}$; d) $(x^{-2})^{-\frac{1}{2}}$; f) $(16a^{-\frac{1}{4}})^{-\frac{1}{2}}$;

b) $(27x^{3})^{-\frac{1}{3}}$; e) $(64c^{12})^{-\frac{2}{3}}$; g) $(27b^{-6})^{-\frac{1}{3}}$.

577. Ifodani soddalashtiring:

a) $a^{\frac{5}{3}}b^{-\frac{1}{6}}\cdot\left(a^{-\frac{1}{3}}b^{\frac{1}{3}}\right)^{4}$; d) $\left(a^{\frac{1}{4}}x^{-\frac{2}{3}}\right)^{1\frac{1}{5}}\cdot a^{0,7}x^{0,8}$;

b) $\left(c^{-\frac{3}{7}}y^{-0,4}\right)^{3}\cdot c^{\frac{2}{7}}y^{0,2}$; e) $p^{-1}q^{\frac{5}{4}}\left(p^{-\frac{2}{7}}q^{\frac{1}{14}}\right)^{-3,5}$.

578. Ifodani kasr ko‘rsatkichli daraja ko‘rinishida yozing:

a) $\sqrt[10]{x}\cdot\sqrt[15]{x}$; d) $\sqrt[7]{y^{2}}\cdot\sqrt[3]{y^{-1}}$; f) $\sqrt[10]{y\sqrt[3]{y^{2}}}$;

b) $\sqrt[8]{a^{3}}\cdot\sqrt[12]{a}$; e) $\sqrt[3]{b^{2}\sqrt{b}}$; g) $\sqrt[5]{x^{2}\sqrt[4]{x^{-3}}}$.

579. Istalgan $a>0$ da tenglikning to‘g‘riligini isbotlang:

a) $\dfrac{\sqrt[3]{a^{2}\sqrt{a}}}{\sqrt[4]{a^{3}\sqrt[3]{a}}}=1$; b) $\dfrac{\sqrt[4]{a^{3}\sqrt[3]{a^{2}}}}{\sqrt[6]{a^{5}\sqrt{a}}}=1$.

580. Tenglamani yeching:

a) $x^{\frac{1}{2}}=5$; d) $x^{1,5}=27$; f) $x^{0,2}\cdot x^{1,8}=16$;

b) $x^{\frac{2}{3}}=4$; e) $x^{-0,8}=16$; g) $x^{\frac{5}{8}}\cdot x^{\frac{3}{8}}=-25$.

78-§. Kasr ko'rsatkichli daraja qatnashgan ifodalarni shakl almashtirish

Kasr ko'rsatkichli darajalar qatnashgan ifodalarni aynan shakl almashtirish bajariladigan misollarda ko'rib chiqamiz.

1-misol. Quyidagi ifodaning qiymatini $x=12,25$ bo'lganda topamiz: $\left(x^{\frac{1}{4}}-6\right)^2-12x^{\frac{1}{4}}\left(x^{-\frac{1}{4}}-1\right)$ bu ifodani dastlab soddalashtiramiz:

$$\left(x^{\frac{1}{4}}-6\right)^2-12x^{\frac{1}{4}}\left(x^{-\frac{1}{4}}-1\right)=x^{\frac{1}{2}}-12x^{\frac{1}{4}}+36-12x^0+12x^{\frac{1}{4}}=x^{\frac{1}{2}}+24.$$

Bu natijaga $x=12,25$ ni qo'yib hisoblaymiz: $x^{\frac{1}{2}}+24=12,25^{\frac{1}{2}}+24=$ $=(3,5^2)^{\frac{1}{2}}+24=3,5+24=27,5.$

2-misol. $\dfrac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}$ ifodani soddalashtiramiz. $a^{\frac{1}{2}}-b^{\frac{1}{2}}$ ni kvadratlar ayirmasi ko'rinishida ifodalab, ko'paytuvchilarga ajratamiz:

$$\frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}=\frac{\left(a^{\frac{1}{4}}\right)^2-\left(b^{\frac{1}{4}}\right)^2}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}=\frac{\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}\right)\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}\right)}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}=a^{\frac{1}{4}}+b^{\frac{1}{4}}$$

3-misol. $\dfrac{x^{\frac{3}{4}}-25x^{\frac{1}{4}}}{x^{\frac{1}{2}}+5x^{\frac{1}{4}}}$ kasrni qisqartiramiz.

Kasrning surat va maxrajidan umumiy ko'paytuvchilarni qavsdan chiqaramiz.

$$\frac{x^{\frac{3}{4}}-25x^{\frac{1}{4}}}{x^{\frac{1}{2}}+5x^{\frac{1}{4}}}=\frac{x^{\frac{1}{4}}\left(x^{\frac{1}{2}}-25\right)}{x^{\frac{1}{4}}\left(x^{\frac{1}{4}}+5\right)}=\frac{\left(x^{\frac{1}{4}}-5\right)\left(x^{\frac{1}{4}}+5\right)}{x^{\frac{1}{4}}+5}=x^{\frac{1}{4}}-5.$$

4-m i s o l. $\left(\frac{1-y^{1,5}}{1-y^{0,5}}+y^{0,5}\right)\left(\frac{1+y^{1,5}}{1+y^{0,5}}-y^{0,5}\right)$ ifodani soddalashtiring.

Har bir qavsdagi ifodalarni alohida soddalashtirib, natijalarni ko'paytiramiz:

$$1)\ \frac{1-y^{1,5}}{1-y^{0,5}}+y^{0,5}=\frac{1^3-\left(y^{\frac{1}{2}}\right)^3}{1-y^{\frac{1}{2}}}+y^{\frac{1}{2}}=\frac{\left(1-y^{\frac{1}{2}}\right)\left(1+y^{\frac{1}{2}}+y\right)}{1-y^{\frac{1}{2}}}+y^{\frac{1}{2}}=1+y^{\frac{1}{2}}+y+y^{\frac{1}{2}}=$$

$$=1+2y^{\frac{1}{2}}+y=\left(1+y^{\frac{1}{2}}\right)^2;$$

$$2)\ \frac{1+y^{1,5}}{1+y^{0,5}}-y^{0,5}=\frac{1^3+\left(y^{\frac{1}{2}}\right)^3}{1+y^{\frac{1}{2}}}-y^{\frac{1}{2}}=\frac{\left(1+y^{\frac{1}{2}}\right)\left(1-y^{\frac{1}{2}}+y\right)}{1+y^{\frac{1}{2}}}-y^{\frac{1}{2}}=1-y^{\frac{1}{2}}+y-y^{\frac{1}{2}}=$$

$$=1-2y^{\frac{1}{2}}+y=\left(1-y^{\frac{1}{2}}\right)^2;$$

$$3)\ \left(1+y^{\frac{1}{2}}\right)^2\cdot\left(1-y^{\frac{1}{2}}\right)^2=\left(1^2-\left(y^{\frac{1}{2}}\right)^2\right)^2=(1-y)^2.$$

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581. Ifodani yig'indi shaklida ifodalang:

a) $a^{\frac{1}{2}}x^{\frac{1}{2}}\left(a^{\frac{1}{2}}+x^{\frac{1}{2}}\right)$; f) $\left(1+b^{\frac{1}{2}}\right)\left(1-b^{\frac{1}{2}}\right)$;

b) $c^{-\frac{1}{3}}y^{\frac{1}{3}}\left(c^{\frac{1}{3}}-y^{\frac{1}{3}}\right)$; g) $\left(p^{\frac{1}{2}}-q^{\frac{1}{2}}\right)^2$;

d) $\left(a^{\frac{2}{3}}-1\right)\left(a^{\frac{1}{3}}+2\right)$; h) $\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)$;

e) $\left(x^{-\frac{3}{4}}+2\right)\left(x^{-\frac{1}{4}}-3\right)$; i) $\left(x^{\frac{1}{2}}+1\right)\left(x-x^{\frac{1}{2}}+1\right)$.

582. Amallarni bajaring:

a) $x^{0,5}y^{0,5}(x^{-0,5}-y^{1,5})$; f) $\left(y^{\frac{1}{3}}-z^{\frac{1}{2}}\right)^3$;

b) $(3p^{0.5}+q^{-1})(3p^{0.5}-q^{-1})$; g) $\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}\right)$;

d) $\left(a^{\frac{1}{2}}+2b^{\frac{1}{2}}\right)^2$; h) $\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right)^2$;

e) $\left(1+b^{\frac{1}{3}}\right)^3$; i) $\left(x^{\frac{1}{4}}-x^{\frac{1}{3}}\right)^2+2x^{\frac{7}{12}}$.

583. Umumiy ko‘paytuvchini qavsdan tashqariga chiqaring:

a) $x-2x^{\frac{1}{2}}$; d) $b^{\frac{3}{4}}-2b^{\frac{1}{4}}$; f) $(ab)^{\frac{1}{3}}-(ac)^{\frac{1}{3}}$;

b) $a^{\frac{1}{2}}-5a^{\frac{1}{4}}$; e) $c^{\frac{5}{3}}+6c^{\frac{2}{3}}$; g) $15^{\frac{1}{3}}+20^{\frac{1}{3}}$.

584. a) $a-b$ ayirmani kvadratlar ayirmasi ko‘rinishida ifodalab; b) kublarning ayirmasi ko‘rinishida ifodalab, ko‘paytuvchilarga ajrating, bunda $a \geq 0$ va $b \geq 0$.

585. $a^2-b^2=(a-b)(a+b)$ ayniyatdan foydalanib ko‘paytuvchilarga ajrating:

a) $x^{\frac{1}{2}}-y^{\frac{1}{2}}$; d) $a^{\frac{1}{8}}-b^{\frac{1}{8}}$; f) $9x^{\frac{1}{5}}-4y^{\frac{1}{5}}$;

b) $x^{\frac{1}{4}}-y^{\frac{1}{4}}$; e) $a^{\frac{1}{3}}-b^{\frac{1}{3}}$; g) $5c^{\frac{1}{7}}-7d^{\frac{1}{7}}$.

586. $a^3 \pm b^3=(a \pm b)(a^2 \mp ab+b^2)$ ayniyatdan foydalanib, ifodani ko‘paytuvchilarga ajrating:

a) $\left(x^{\frac{1}{2}}\right)^3-8$; b) $y^{\frac{3}{2}}+27$; d) $n^{\frac{6}{7}}-0{,}001$; e) $\left(m^{\frac{9}{5}}\right)^3+0{,}125$.

587. Kasrni qisqartiring:

a) $\dfrac{5+5^{\frac{1}{2}}}{3\cdot 5^{\frac{1}{2}}}$; d) $\dfrac{x+2x^{\frac{1}{2}}}{2x}$; f) $\dfrac{a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}{b-a}$;

b) $\dfrac{n}{n-n^{\frac{1}{2}}}$; e) $\dfrac{b^{\frac{2}{3}}-b^{\frac{1}{2}}}{b^{\frac{1}{2}}-b^{\frac{1}{3}}}$; g) $\dfrac{m+n}{m^{\frac{2}{3}}-m^{\frac{1}{3}}n^{\frac{1}{3}}+n^{\frac{2}{3}}}$.

588. Ifodaning qiymatini toping:

a) $\dfrac{a-4a^{\frac{1}{2}}}{a^{\frac{2}{4}}+2a^{\frac{1}{2}}}$ bunda $a=81$;

d) $\dfrac{x}{x^{\frac{1}{3}}-2}-\dfrac{2x+16}{x^{\frac{2}{3}}-4}$ bunda $x=27$;

b) $\dfrac{x^{\frac{1}{2}}-9x^{\frac{1}{6}}}{x^{\frac{1}{3}}-3x^{\frac{1}{6}}}$ bunda $x=64$;

e) $\dfrac{8}{y^{\frac{1}{4}}+2}+\dfrac{y-8y^{\frac{1}{4}}}{y^{\frac{1}{2}}-4}$ bunda $y=25$.

589. Ifodani soddalashtiring:

a) $\dfrac{a}{a^{\frac{1}{2}}b^{\frac{1}{2}}+b}+\dfrac{b}{a^{\frac{1}{2}}b^{\frac{1}{2}}-a}-\dfrac{a+b}{\sqrt{ab}}$;

b) $\dfrac{\sqrt{x}}{x^{\frac{1}{2}}-6}-\dfrac{3}{x^{\frac{1}{2}}+6}+\dfrac{x}{36-x}$;

d) $\dfrac{2}{p^{\frac{1}{2}}-q^{\frac{1}{2}}}-\dfrac{2p^{\frac{1}{2}}}{p^{\frac{3}{2}}+q^{\frac{3}{2}}}\cdot\dfrac{p-p^{\frac{1}{2}}q^{\frac{1}{2}}+q}{p^{\frac{1}{2}}-q^{\frac{1}{2}}}$;

e) $\dfrac{\sqrt{x}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}+\dfrac{\sqrt{y}}{x^{\frac{1}{2}}-y^{\frac{1}{2}}}$.

590. Ifodani soddalashtiring:

a) $\dfrac{a^{\frac{1}{2}}+b^{\frac{1}{2}}}{a^{\frac{1}{2}}}-\dfrac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}+\dfrac{b}{a-a^{\frac{1}{2}}b^{\frac{1}{2}}}$;

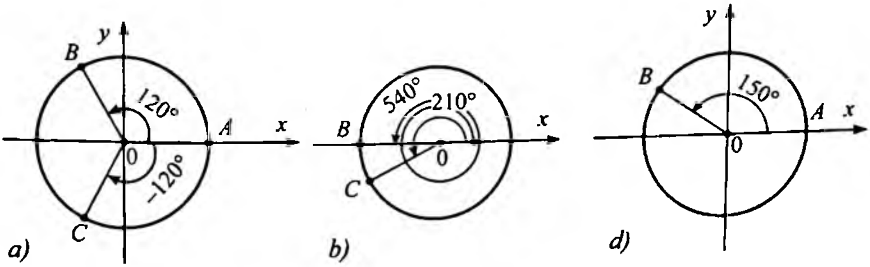
b) $\left(\dfrac{q^{\frac{1}{2}}}{p-p^{\frac{1}{2}}q^{\frac{1}{2}}}+\dfrac{p^{\frac{1}{2}}}{q-p^{\frac{1}{2}}q^{\frac{1}{2}}}\right)\cdot\dfrac{pq^{\frac{1}{2}}+p^{\frac{1}{2}}q}{p-q}$;

d) $\left[4^{-\frac{1}{2}}+\left(\dfrac{1}{2^{-\frac{3}{2}}}\right)^{-\frac{4}{3}}\right]\cdot\left[4^{-0,25}-\left(2\sqrt{2}\right)^{-\frac{4}{3}}\right]$;

e) $\left[\left(a^{\frac{1}{3}}-x^{\frac{1}{3}}\right)^{-1}(a-x)-\dfrac{a+x}{a^{\frac{1}{3}}+x^{\frac{1}{3}}}\right]\cdot 2(ax)^{-\frac{1}{3}}$.

79-§. Burchaklar. Istalgan burchak trigonometrik funksiyalarining ta'rifi

OX o'qida koordinatalar boshidan o'ngda *A* nuqtani belgilaymiz (51-*a*, chizma) va bu nuqta orqali markazi *O* nuqtada bo'lgan aylana o'tkazamiz. *OA* radiusni **boshlang'ich radius** deb ataymiz.



51-chizma.

Boshlang'ich radiusni *O* nuqta atrofida soat strelkasi harakatiga qarshi 120° buramiz. Bunda u *OB* radiusga o'tadi. Bu holda burish burchagi 120° ga teng deyiladi. Agar boshlang'ich radius *O* nuqta atrofida soat strelkasi harakati bo'yicha 120° burilsa, u *OC* radiusga o'tadi. Bu holda burish burchagi – 120° ga teng deyiladi (51-*a*, chizma).

Umuman, soat strelkasi harakatiga qarshi burishda burish burchagi **musbat**, soat strelkasi harakati bo'yicha burishda esa **manfiy burchak** hisoblanadi.

Biz geometriya kursidan burchak o'lchovlari 0° dan 180° gacha sonlar bilan ifodalanishini bilamiz. Agar boshlang'ich radius soat strelkasi harakatiga qarshi 180° ga burilsa, keyin yana 30° ga burilsa, burish burchagi 210° ga teng bo'ladi (51-*b*, chizma).

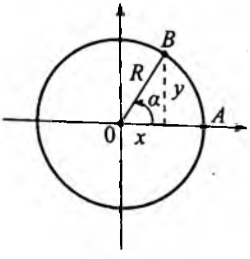
Agar boshlang'ich radiusni soat strelkasi harakatiga qarshi to'liq bir marta aylantirilsa (burilsa), burish burchagi 360° ga teng bo'ladi.

Bu burchakni **to'liq burchak** deyiladi. Agar boshlang'ich radius soat strelkasi harakatiga qarshi bir yarim marta aylansa, u holda burish burchagi 540° ga teng – bo'ladi (51-*b*, chizma).

OA va *OB* ridiuslarni qaraymiz (51-*d*, chizma). *OA* boshlang'ich radius *OB* radiusga o'tadigan istagancha burish burchaklari mavjud, masalan, agar $\angle AOB=\alpha$ bo'lsa, u holda mos burish burchaklari $\boldsymbol{\alpha+360°n}$ ga teng bo'ladi, bu yerda n – butun son.

Masalan, $n=0; \pm1; \pm2; ...$ bo'lganda $150°+360°\cdot0=150°$; $150°+360°\cdot1=510°$; $150°+360°\cdot(-1)=-210°$; $150°+360°\cdot2=870°$; $150+360°\cdot(-2)=-570°$; va hokazo.

Geometriya kursida α burchakning $0°\leq\alpha\leq180°$ bo'lgandagi sinus, kosinus, tangens va kotangensiga ta'rif berilgan. Endi biz bu ta'riflarni α ixtiyoriy burchak bo'lgan hol uchun tatbiq etamiz. Aylananing har bir nuqtaga biror burchak mos keladi. Bu burchak boshlang'ich radius bilan aylana nuqtasiga tegishli radius orasidagi burchakdan iborat (52-chizma).



52-chizma.

Bunda: *OA* – boshlang'ich radius, *B* – aylananing ixtiyoriy nuqtasi, α – aylananing *B* nuqtasiga mos keluvchi burchak.

O nuqta atrofida ixtiyoriy α burchakka burishda *OA* boshlang'ich radius *OB* radiusga o'tsin. *B* nuqtaning koordinatalari *X* va *Y*, boshlang'ich radius uzunligi *R* bo'lsin.

TA'RIFLAR:

1. **B nuqta ordinatasining radiusga nisbati α burchakning sinusi deyiladi, ya'ni $\frac{y}{R}=\sin\alpha$.**

2. **B nuqta absissasining radiusga nisbati α burchakning kosinusi deyiladi, ya'ni $\frac{x}{R}=\cos\alpha$.**

3. **B nuqta ordinatasining shu nuqta absissasiga nisbati α burchakning tangensi deyiladi, ya'ni $\frac{y}{x}=\mathrm{tg}\alpha$.**

4. *B* nuqta absissasining shu nuqta ordinatasiga nisbati *α* burchakning kotangensi deyiladi, ya'ni $\frac{x}{y}=\text{ctg}\alpha$.

5. Radiusning *B* nuqta absissasiga nisbati *α* burchakning sekansi deyiladi, ya'ni $\frac{R}{x}=\sec\alpha$.

6. Radiusning *B* nuqta ordinatasiga nisbati *α* burchakning kosekansi deyiladi, ya'ni $\frac{R}{y}=\text{cosec}\alpha$.

sin*α*, cosα tg*α*, ctg*α*, sec*α*, cosec*α* larning qiymatlari boshlang'ich radiusning qiymatiga bog'liq bo'lmay, ularning qiymatlari *α* burchakka bog'liq bo'ladi.

Teorema. ***Trigonometrik funksiyaning qiymatlari aylana radiusiga bog'liq bo'lmay, faqat burchak kattaligiga bog'liq.***

Berilgan:

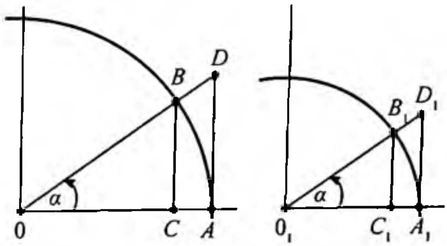
$\angle AOB=\angle A_1O_1B_1=\alpha.$

$OA\neq O_1A_1$ – radiuslar.

$OA=R$; $O_1A_1=R_1$, ya'ni $R\neq R_1$.

$\cup AB\neq\cup A_1B_1$ (53-chizma).

Isbot qilish kerak: $\sin\alpha=\sin_1\alpha$; $\cos\alpha=\cos_1\alpha$; $\text{tg}\alpha=\text{tg}_1\alpha$ va hokazo.



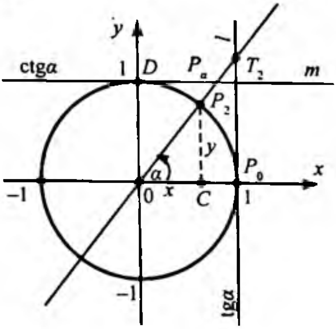
53-chizma.

Isbot. *R* radiusli trigonometrik funksiyalarni sin*α*, cos*α*, tg*α*, ..., R_1 radiusli trigonometrik funksiyalarni $\sin_1\alpha$, $\cos_1\alpha$, $\text{tg}_1\alpha$, ... kabi belgilaymiz. $OBC\backsim O_1B_1C_1$; $ODA\backsim O_1D_1A_1$ o'xshashliklardan: $\frac{BC}{OB}=\frac{B_1C_1}{O_1B_1}$; $\frac{OC}{OB}=\frac{O_1C_1}{O_1B_1}$; $\frac{AD}{OA}=\frac{A_1D_1}{O_1A_1}$. Bu nisbatlar trigonometrik funksiya ta'rifiga asosan $\sin\alpha=\sin_1\alpha$, $\cos\alpha=\cos_1\alpha$; $\text{tg}\alpha=\text{tg}_1\alpha$; ... kelib chiqadi.

Shunday qilib, teng burchakning trigonometrik funksiyalari radius har qanday bo'lganda ham bitta qiymatga ega bo'ladi.

Trigonometrik funksiyalarning qiymatlari boshlang'ich radiusning uzunligiga bog'liq bo'lmagani uchun, bu radiusni har doim bir xil uzunlikda olish mumkin. Odatda, $R=1$ deb hisoblaniladi. U vaqtda boshlang'ich radiusning uchi birlik aylanadi yotadi; burchak (*α*) esa uning ikki radiusi bilan hosil bo'ladi. Bu holda trigonometrik funksiyalar: $\cos\alpha=x$; $\sin\alpha=y$, $\text{tg}\alpha=\frac{y}{x}$; $\text{ctg}\alpha=\frac{x}{y}$; $\sec\alpha=\frac{1}{x}=\frac{1}{\cos\alpha}$;

$\operatorname{coses}\alpha=\frac{1}{y}=\frac{1}{\sin\alpha}$ ko'rinishga ega bo'ladi. $\operatorname{tg}\alpha=\frac{y}{x}=\frac{\sin\alpha}{\cos\alpha}$ va $\operatorname{ctg}\alpha=\frac{x}{y}=\frac{\cos\alpha}{\sin\alpha}$, ya'ni **$\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$** va **$\operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$** kabi ko'rinishga ega bo'ladi.



54-chizma.

Bir qator masalalarni yechish uchun tangenslar chizig'i haqida tasavvurga ega bo'lish foydali. Birlik aylanaga P_0 nuqtada l urinma o'tkazamiz (54-chizma). α ixtiyoriy burchak (ixtiyoriy son), bunda $\cos\alpha\neq0$ bo'lsin. Bundagi $P_2C=y=\sin\alpha$; $OC=x=\cos\alpha\ \Delta OP_2C \backsim \Delta OTP_0$ dan $\frac{P_2C}{OC}=\frac{T_2P_0}{OP_0}$; $\frac{\sin\alpha}{\cos\alpha}=\frac{T_2P_0}{1}$; **$\operatorname{tg}\alpha=T_2P_0$.**

Demak, T_2P_0 kesma l urinmada yotadi. Shu sababli l to'g'ri chiziqni tangenslar chizig'i deyiladi. $D(0;\ 1)$ nuqtadan OY o'qiga perpendikulyar qilib, o'tkazilgan m to'g'ri chiziq ($m=\operatorname{ctg}\alpha$) kotangenslar chizig'i deyiladi. Buning isboti yuqoridagi tangens chizig'i kabi isbotlanadi (54-chizma).

TAKRORLASH UCHUN SAVOLLAR

1. Qanday burchakni musbat burchak va qanday burchakni manfiy burchak deyiladi?
2. OX o'qida boshlang'ich radius olib, 200° va –200° li burchaklarni chizing.
3. Qanday burchakni yoyiq burchak deyiladi?
4. Qanday burchakni to'liq burchak deyiladi?
5. Berilgan α burchakni to'liq burchak deyiladimi?
6. Aylananing istalgan nuqtasiga mos keluvchi trigonometrik funksiyalar ta'rifini ayting va u nisbatni yozing.
7. Qanday aylanani birlik aylana deyiladi va uning istalgan bir nuqtasining trigonometrik funksiyasini yozing.
8. Trigonometrik funksiyaning qiymati haqidagi teoremani ayting.
9. Qanday chiziqni tangenslar chizig'i va qaysi chiziqni kotangenslar chizig'i deyiladi?

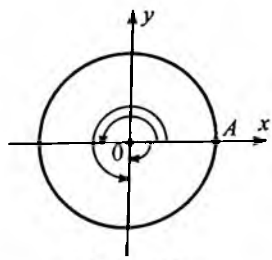
MASALALARNI YECHING

591. a) Markazi koordinatalar boshida bo'lgan aylana chizing va 150°, 210°, 540°, –45°, –135°, –720 ga teng bo'lgan burish burchaklarini tasvirlang.

b) 55-chizmada strelkalar bilan ko‘rsatilgan burish burchaklari nimaga teng?

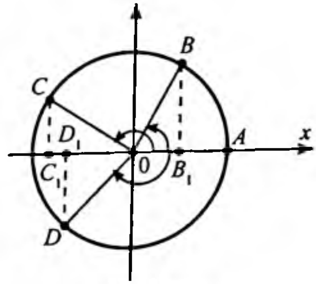
592. Burchak, qaysi chorak burchagi bo‘ladi?

a) $\alpha=100°$; b) $\alpha=190°$; d) $\alpha=285°$;
e) $-25°$; f) $\alpha=-110°$; g) $\alpha=1200°$;
h) $\alpha=-2000°$; i) $\alpha=4500°$.



55-chizma.

593. Markazi koordinatalar boshida va radiusi 2 sm bo‘lgan aylanadagi $\angle AOB$, $\angle AOC$ va $\angle AOD$ burchaklarning sinusi, kosinusi, tangensi va kotangenslarini yozing (56-chizma).



56-chizma.

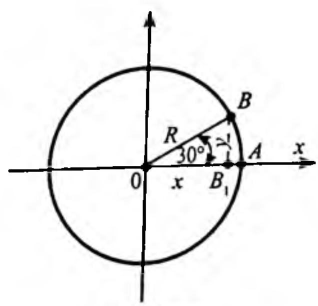
594. Markazi koordinatalar boshida bo‘lgan birlik aylana chizing. Unda 150° va –120° burchaklar chizib, shu burchaklarning trigonometrik funksiyalarining taqribiy qiymatini toping.

80-§. Ba’zi burchaklar trigonometrik funksiyalarining qiymatlari

Markazi koordinatalar boshida va radiusi $OA=R$ bo‘lgan aylana chizamiz. OA – boshlang‘ich radius; OB – qo‘zg‘aluvchi radius (57-chizma). A nuqtaning koordinatasi $(R;\ O)$, B nuqta koordinatasi $(x;\ y)$ bo‘lsin.

1) $\angle AOB=30°$ bo‘lsin. Geometriyadan ma'lumki, $y=\frac{R}{2}$ (gipotenuzaning yarmiga teng). $x=OB_1$, ni R va $\frac{R}{2}$ lar orqali topamiz.

$$x=OB_1=\sqrt{R^2-y^2}=\sqrt{R^2-\left(\frac{R}{2}\right)^2}=\frac{\sqrt{3}R}{2};$$



57-chizma.

Trigonometrik funksiyalarning ta’rifiga asosan:

$$\sin 30° = \frac{y}{R} = \frac{\frac{R}{2}}{R} = \frac{1}{2}; \ \sin 30° = \frac{1}{2}.$$

$$\cos 30° = \frac{x}{R} = \frac{\frac{\sqrt{3}R}{2}}{R} = \frac{\sqrt{3}}{2}; \quad \cos 30° = \frac{\sqrt{3}}{2};$$

$$\mathrm{tg}\,30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}; \quad \mathrm{tg}\,30° = \frac{\sqrt{3}}{3};$$

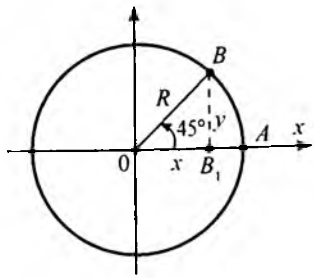
$$\mathrm{ctg}\,30° = \frac{\cos 30°}{\sin 30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

2) ΔAOB da $\angle B = 60°$ ga teng. Shu $60°$ li burchak trigonometrik funksiyalarini topamiz:

$$\sin 60° = \frac{x}{R} = \frac{\frac{\sqrt{3}R}{2}}{R} = \frac{\sqrt{3}}{2}; \quad \sin 60° = \frac{\sqrt{3}}{2};$$

$$\cos 60° = \frac{y}{R} = \frac{\frac{1}{2}R}{R} = \frac{1}{2}; \quad \cos 60° = \frac{1}{2};$$

$$\mathrm{tg}\,60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}; \quad \mathrm{tg}\,60° = \sqrt{3};$$



58-chizma.

$$\mathrm{ctg}\,60° = \frac{\cos 60°}{\sin 60°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{1}; \quad \mathrm{ctg}\,60° = \frac{\sqrt{3}}{3}.$$

3. Markazi koordinatalar boshida va radiusi $OA = R$ bo'lgan markaziy burchagi $45°$ bo'lgan teng yonli BOB_1 ni chizamiz (58-chizma). Bunda $BB_1 = OB_1 = x = y$ bo'ladi.

BOB_1 dan $x^2 + y^2 = R^2$; $\quad 2x^2 = R^2 \quad x = \frac{R}{\sqrt{2}}; \quad y = \frac{R}{\sqrt{2}}.$

$$\sin 45° = \frac{y}{R} = \frac{\frac{R}{\sqrt{2}}}{R} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}; \quad \sin 45° = \frac{\sqrt{2}}{2}.$$

$$\cos 45° = \frac{x}{R} = \frac{\frac{R}{\sqrt{2}}}{R} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}; \quad \cos 45° = \frac{\sqrt{2}}{2}.$$

$$\operatorname{tg} 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1; \quad \operatorname{tg} 45° = 1.$$

$$\operatorname{ctg} 45° = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1; \quad \operatorname{ctg} 45° = 1.$$

4. Agar B nuqta A nuqta ustiga tushsa, $AOB=0°$ bo'ladi. U holda $x=R$; $y=0$ bo'ladi.

$$\sin 0° = \frac{y}{R} = \frac{O}{R} = 0; \quad \sin 0° = 0;$$

$$\cos 0° = \frac{x}{R} = \frac{R}{R} = 1; \quad \cos 0° = 1.$$

$$\operatorname{tg} 0° = \frac{\sin 0°}{\cos 0°} = \frac{0}{1} = 0; \quad \operatorname{tg} 0° = 0; \ \operatorname{ctg} 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$$ – mavjud emas.

5. Agar $\angle AOB=90°$ bo'lsa, B nuqtaning koordinatalari $(0; R)$ ga teng bo'ladi. U holda:

$$\sin 90° = \frac{y}{R} = \frac{R}{R} = 1; \quad \sin 90° = 1;$$

$$\cos 90° = \frac{x}{R} = \frac{O}{R} = 1; \quad \cos 90° = 0;$$

$$\operatorname{tg} 90° = \frac{\sin 90°}{\cos 90°} = \frac{1}{0}$$ – mavjud emas;

$$\operatorname{ctg} 90° = \frac{\cos 90°}{\sin 90°} = \frac{0}{1} = 0.$$

6. Agar $\angle AOB=180°$ bo'lsa, B nuqtaning koordinatalari $(-R; 0)$ ga teng bo'ladi. U holda:

$$\sin 180° = \frac{y}{R} = \frac{O}{R} = 0; \quad \sin 180° = 0;$$

$$\cos 180° = \frac{x}{R} = \frac{-R}{R} = -1; \quad \cos 180° = -1;$$

$$\operatorname{tg} 180° = \frac{\sin 180°}{\cos 180°} = \frac{0}{-1} = 0; \quad \operatorname{tg} 180° = 0;$$

$$\operatorname{ctg} 180° = \frac{\cos 180°}{\sin 180°} = \frac{-1}{0}$$ – mavjud emas.

7. Agar $\angle AOB=270°$ bo'lsa, B nuqtaning koordinatalari $(0; -R)$ ga teng bo'ladi. U holda:

$$\sin 270° = \frac{y}{R} = \frac{-R}{R} = -1; \quad \sin 270° = -1;$$

$$\cos 270° = \frac{x}{R} = \frac{O}{R} = 0; \quad \cos 270° = 0.$$

$$\mathrm{tg}\, 270° = \frac{\sin 270°}{\cos 270°} = \frac{-1}{0} \text{ – mavjud emas.}$$

$$\mathrm{ctg}\, 270° = \frac{\cos 270°}{\sin 270°} = \frac{0}{-1} = 0. \ \mathrm{ctg}\, 270° = 0.$$

Trigonometrik funksiyalar ba'zi burchaklari qiymatlarining jadvalini tuzamiz.

α	0°	30	45°	60°	90°	180°	270°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0
$\mathrm{tg}\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–
$\mathrm{ctg}\alpha$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0

Jadvaldagi (–) ishora ifodaning qiymati mavjud emasligini anglatadi.

1-misol. $2\sin 30° + 5\cos 45° - 6\mathrm{tg}\, 45°$ ifodaning qiymatini topamiz.

$$2\sin 30° + 5\sin 45° - 6\mathrm{tg}45° = 2 \cdot \frac{1}{2} + 5 \cdot \frac{\sqrt{2}}{2} - 6 \cdot 1 = 1 + \frac{5\sqrt{2}}{2} - 6 = \frac{5\sqrt{2}}{2} - 5 = 5\left(\frac{\sqrt{2}}{2} - 1\right).$$

2-misol. $\cos\alpha + \sin 2\alpha + \cos 3\alpha$ ifodaning qiymatini $\alpha = 30°$ da hisoblang.

$$\cos\alpha + \sin 2\alpha + \cos 3\alpha = \cos 30° + \sin(2 \cdot 30°) + \cos(3 \cdot 30°) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 0 = \frac{2\sqrt{3}}{2} = \sqrt{3}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Trigonometrik funksiyalarning 0° dagi qiymatlarini ayting.
2. Trigonometrik funksiyalarning 30° dagi qiymatlarini ayting.
3. Trigonometrik funksiyalarning 45° dagi qiymatlarini ayting.
4. Trigonometrik funksiyalarning 60° dagi qiymatlarini ayting.
5. Jadvaldagi (–) ishora nimani anglatadi?
6. Jadvalni yod olib, so'ngra aytib bering.

MASALALARNI YECHING

595. Hisoblang:

a) $\sin 30° + \cos 60° + \operatorname{tg} 45°$; b) $4\sin 45° \cdot \operatorname{tg} 45° \cdot \cos 45°$;

d) $2\sin 60° + 4\operatorname{tg} 60°$; e) $4\sin 60° \cdot \operatorname{tg} 60° \cdot \cos 60°$.

596. Ifodaning qiymatini toping:

a) $2\cos 60° + \sqrt{3} \cdot \cos 30°$; d) $2\sin 30° + 6\cos 60° - 4\operatorname{tg} 45°$;

b) $5\sin 30° - \operatorname{ctg} 30°$; e) $12\sin 60° \cdot \cos 60°$.

597. α ning: a) $\sin\alpha = 1$; b) $\cos\alpha = -1$; d) $\sin\alpha = 0$; e) $\operatorname{tg}\alpha = 0$ bo'ladigan bir nechta qiymatini ko'rsating.

598. Quyidagi ifodaning eng katta va eng kichik qiymatlari qanday:

a) $1 + \sin\alpha$; b) $2 - \cos\alpha$.

599. $\sin\alpha$ quyidagi qiymatlarni qabul qilishi mumkinmi?

a) $\sqrt{2}$; b) $\frac{1}{\sqrt{2}}$; d) $\frac{1+\sqrt{3}}{2}$; e) $\frac{1-\sqrt{3}}{2}$.

600. Ifodaning qiymatini toping:

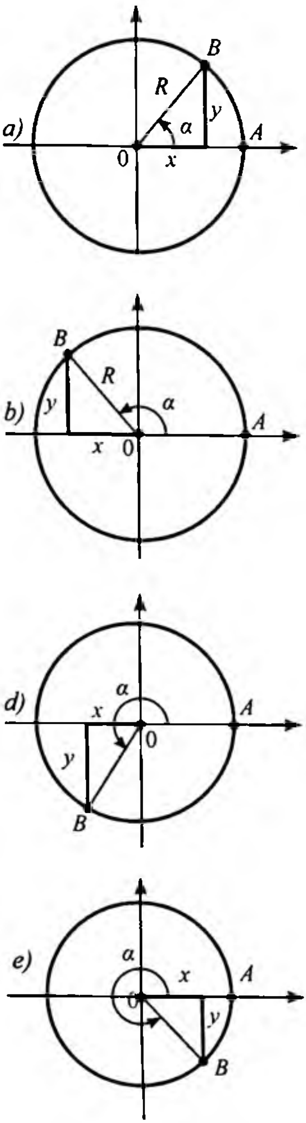
a) $2\cos 0° - 4\sin 90° + 5\operatorname{tg} 180°$; d) $\operatorname{tg} 360° - \frac{3}{4}\sin 270° - \frac{1}{4}\cos 180°$;

b) $2\operatorname{ctg} 90° - 3\sin 270° + 5\sin 0°$; e) $3\operatorname{ctg} 90° - 3\sin 270° + \frac{3}{5}\operatorname{tg} 360°$.

601. α ning quyidagi qiymatlarida $\cos 2\alpha + \cos 3\alpha$ ifodaning qiymatini toping: a) $\alpha = 15°$; b) $\alpha = 30°$; d) $\alpha = 90°$.

81-§. Trigonometrik funksiyalarning ishoralari

Choraklarning har birida sinus, kosinus, tangens va kotangens qanday ishoraga ega bo'lishini aniqlaymiz.



59-chizma.

Markazi koordinatalar boshida, radiusi R ga teng bo'lgan OA radiusni α burchakka burishda A nuqtaning koordinatalari x va y bo'lgan B nuqtaga o'tgan bo'lsin (59-*a*, chizma).

1. α burchak **I** chorakda bo'lsin (59-*a*, chizma).

Bunda: $x>0$; $y>0$, $R>0$.

$\sin\alpha=\frac{y}{R}>0$; $\cos\alpha=\frac{x}{R}>0$, $\operatorname{tg}\alpha=\frac{y}{x}>0$; $\operatorname{ctg}\alpha=\frac{x}{y}>0$.

2. α burchak **II** chorakda bo'lsin (59-*b*, chizma).

Bunda: $x<0$; $y>0$, $R>0$.

$\sin\alpha=\frac{y}{R}>0$; $\cos\alpha=\frac{x}{R}<0$, $\operatorname{tg}\alpha=\frac{y}{x}<0$; $\operatorname{ctg}\alpha=\frac{x}{y}<0$.

3. α burchak **III** chorakda bo'lsin (59-*d*, chizma).

Bunda: $x<0$; $y<0$, $R>0$.

$\sin\alpha=\frac{y}{R}<0$; $\cos\alpha=\frac{x}{R}<0$, $\operatorname{tg}\alpha=\frac{y}{x}>0$; $\operatorname{ctg}\alpha=\frac{x}{y}>0$.

4. α burchak **IV** chorakda bo'lsin (59-*e*, chizma).

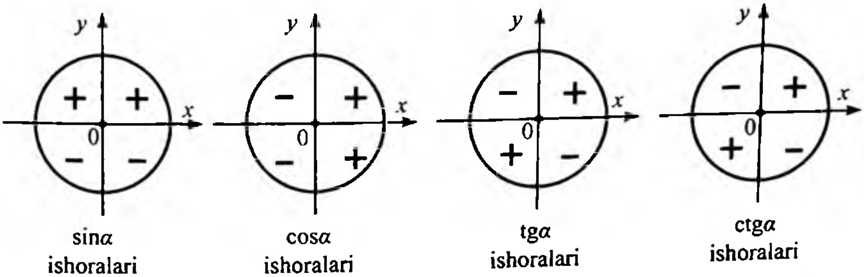
Bunda: $x>0$; $y<0$, $R>0$.

$\sin\alpha=\frac{y}{R}<0$; $\cos\alpha=\frac{x}{R}>0$, $\operatorname{tg}\alpha=\frac{y}{x}<0$; $\operatorname{ctg}\alpha=\frac{x}{y}<0$.

Agar OA radiusni α burchakka burganda OB radius hosil qilgan burchak bo'lsa, OA ni α dan butun aylanishlar soniga farq qiluvchi burchakka burganda ham xuddi shu radius hosil bo'ladi. Bu istagan burchakning sinusi, kosinusi, tangensi va kotangensi qiymatlarini topishni ularning 360° dan kichik nomanfiy burchaklar uchun qiymatlarini topishga imkon beradi.

Masalan, $\sin 785° = \sin(2 \cdot 360° + 65) = \sin 65°$.

Sinus, kosinus, tangens va kotangensning har bir choraklardagi ishoralari 60-chizmada ko'rsatilgan.



60-chizma.

α burchakka qarama-qarshi bo'lgan $-\alpha$ burchaklarning sinus, kosinus, tangens va kotangenslari orasidagi bog'lanishni ifodalovchi formulalarni keltirib chiqaramiz. OA radius α burchakka burilganda OB radiusga o'tsin, $-\alpha$ burchakka burilganda OC radiusga o'tsin. B va C nuqtalar OX o'qiga nisbatan simmetrik bo'ladi.

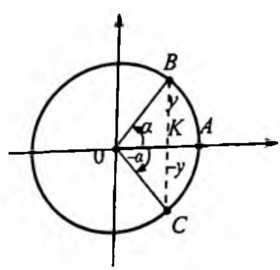
B nuqtaning koordinatalari x va y ga teng bo'ladi. Bular orqali quyidagilarni topamiz (61-chizma):

$$\sin(-\alpha) = \frac{-y}{R} = -\frac{y}{R} = -\sin\alpha;$$

$$\cos(-\alpha) = \frac{x}{R} = \cos\alpha.$$

$$\mathrm{tg}(-\alpha) = \frac{-y}{x} = -\frac{y}{x} = -\mathrm{tg}\alpha;$$

$$\mathrm{ctg}(-\alpha) = \frac{x}{-y} = -\frac{x}{y} = -\mathrm{ctg}\alpha.$$



61-chizma.

Natijada quyidagi formulalar hosil bo‘ldi:

$\sin(-\alpha)=-\sin\alpha$; **$tg(-\alpha)=-tg\alpha$;**

$\cos(-\alpha)=\cos\alpha$; **$ctg(-\alpha)=ctg\alpha$.**

Bunda $\cos(-\alpha)=\cos\alpha$ bo‘lganidan kosinus juft funksiya bo‘ladi, ya’ni $\cos(-60°)=\cos60°=\frac{1}{2}$, $\cos(-270°)=\cos270°=0$.

Sinus, tangens va kotangenslarda $\sin(-\alpha)=-\sin\alpha$; $tg(-\alpha)=-tg\alpha$; $ctg(-\alpha)=-ctg\alpha$ bo‘lgani uchun bularni toq funksiyalar deymiz.

Masalan: $\sin(-30°)=-\sin30°=-\frac{1}{2}$, $tg(-45°)=-tg45°=-1$; $ctg(-60°)=$ $=-ctg60°=-\frac{\sqrt{3}}{3}$.

1-misol. Ifodaning ishorasini aniqlang:

a) $\sin100°\cdot\cos300°$; b) $tg220°:\cos440°$.

Yechish. a) $\sin100°\cdot\cos300°$ da $\sin100°>0$ (2-chorakda);

$\cos300°>0$ (4-chorakda) bo‘lgani uchun $\sin100°\cdot\cos300°>0$.

b) $tg220°\cdot\cos440°$ da $tg220°>0$ (3-chorakda);

$\cos440°=\cos(360°+80°)=\cos80°>0$ (1-chorakda) bo‘lgani uchun $tg220°:\cos440°>0$.

2-misol. $\sin(-405°)$ni hisoblaymiz.

$\sin(-405°)=-\sin405°=-\sin(360°+45°)=-\sin45°=-\frac{\sqrt{2}}{2}$.

TAKRORLASH UCHUN SAVOLLAR

1. I chorakda trigonometrik funksiyalar qanday ishorali bo‘ladi?
2. II chorakda trigonometrik funksiyalar qanday ishorali bo‘ladi?
3. III chorakda trigonometrik funksiyalar qanday ishorali bo‘ladi?
4. IV chorakda trigonometrik funksiyalar qanday ishorali bo‘ladi?
5. α burchakka qarama-qarshi bo‘lgan $-\alpha$ burchakka burilganda hosil bo‘lgan trigonometrik funksiyalarning formulalarini yozing.

MASALALARNI YECHING

602. Quyidagi funksiyalarning ishoralarini ayting:

a) $\sin48°$; $\cos°48°$; $tg48°$; $ctg48°$;

b) $\sin137°$; $\cos137°$; $tg137°$; $ctg137°$;

d) $\sin200°$; $\cos200°$; $tg200°$; $ctg200°$;

e) $\sin310°$; $\cos310°$; $tg310°$; $ctg310°$.

603. Ishorasini aniqlang:

a) sin 280°; b) ctg 359°; d) tg 500°; e) sin(–80°); f) cos(–116°).

604. a) $\sin\alpha>0$ va $\cos\alpha>0$; d) $\sin\alpha<0$ va $\cos\alpha<0$;
b) $\sin\alpha<0$ va $\cos\alpha>0$; e) $\mathrm{tg}\alpha<0$ va $\cos\alpha>0$
bo'lsa, α qaysi chorak burchak bo'ladi.

605. Ifodaning ishorasini aniqlang:
a) $\sin 120°\cdot\cos 200°$; d) $\cos 320°\cdot\mathrm{ctg}\,100°$;
b) $\mathrm{tg}\,150°\cdot\sin 190°$; e) $\mathrm{tg}\,170°\cdot\sin 300°$.

606. Qaysi chorakda quyidagilarning ishoralari bir xil bo'ladi?
a) $\sin\alpha$ va $\cos\alpha$; b) $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$; d) $\cos\alpha$ va $\mathrm{tg}\alpha$; e) $\sin\alpha$ va $\mathrm{tg}\alpha$.

607. Ifodaning qiymatini toping:
a) sin(–60°); b) cos(–30°); d) tg(–45°); e) ctg(–60°);
f) sin(–420°); g) cos(–390°); h) tg(–720°); i) ctg(–450°).

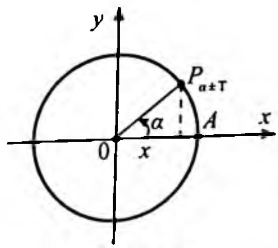
608. Hisoblang:
a) $\sin(-180°)+\mathrm{tg}(-60°)+\cos(-405°)$;
b) $\cos(-420°)+\sin(-450°)-\mathrm{tg}(-765°)$.

82-§. Trigonometrik funksiyalarning davriyligi

Ta'rif. **Shunday $T\neq 0$ son mavjud bo'lib, $f(x)$ funksiyaning aniqlanish sohasida bo'lgan barcha x larda $(x\pm T)$ sonlar ham shu sohaga tegishli bo'lsa va $f(x\pm T)=f(x)$ tenglik bajarilsa, $f(x)$ davriy funksiya deyiladi.**

Teorema. **Sinus, kosinus, tangens va kotangens davriy funksiyalardir.**

Isbot. α soni trigonometrik funksiyaning aniqlanish sohasiga tegishli bo'lsin, u holda $\alpha+2\pi$, $\alpha-2\pi$ sonlar ham uning aniqlanish sohasiga tegishli ekanligini ko'rsatamiz (62-chizma).

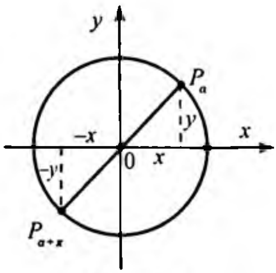


62-chizma.

Birlik aylanadagi P_α, $P_{\alpha+2\pi}$, $P_{\alpha-2\pi}$ nuqtalar ustma-ust tushadi.

Ularning koordinatalari $(x;\ y)$ dan iborat bo‘ladi. Ularning mos trigonometrik funksiyalari teng bo‘ladi, ya’ni $\sin(\alpha+2\pi)=\sin\alpha$; $\cos(\alpha\pm2\pi)=\cos\alpha$; $\mathrm{tg}(\alpha\pm2\pi)=\mathrm{tg}\alpha$ va $\mathrm{ctg}(\alpha\pm2\pi)=\mathrm{ctg}\alpha$.

Shunday qilib, 2π soni trigonometrik funksiyalarning davri bo‘ladi.



63-chizma.

Tangens va kotangens funksiyalar uchun π soni eng kichik musbat davri bo‘ladi (63-chizma).

$$\left.\begin{aligned}&\mathrm{tg}\alpha=\frac{y}{x}\\&\mathrm{tg}(\alpha+\pi)=\frac{-y}{-x}=\frac{y}{x}\end{aligned}\right\}\mathrm{tg}(\alpha+\pi)=\frac{y}{x}=\mathrm{tg}\alpha$$

Shu kabi $\mathrm{ctg}(\alpha+\pi)=\mathrm{ctg}\alpha$ kelib chiqadi.

Masalan: $\mathrm{tg}210°=\mathrm{tg}(30°+180°)=\mathrm{tg}30°=\frac{\sqrt{3}}{3}$; $\mathrm{ctg}(-225°)=-\mathrm{ctg}(45°+180°)=$ $=-\mathrm{ctg}45°=-1$.

Trigonometrik funksiyalarning umumiy davri quyidagicha yoziladi:

$\sin(\alpha\pm2n\pi)=\sin\alpha$; $\quad\mathrm{tg}(\alpha\pm n\pi)=\mathrm{tg}\alpha$

$\cos(\alpha\pm2n\pi)=\cos\alpha$; $\quad\mathrm{ctg}(\alpha\pm n\pi)=\mathrm{ctg}\alpha$; bularda n – butun son.

Masalan: $\sin1500°=\sin(4\cdot360°+60°)=\sin60°=\frac{\sqrt{3}}{2}$;

$$\cos\frac{17\pi}{4}=\cos\left(4\pi+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani davriy funksiya deyiladi?
2. Davriy funksiya haqidagi teoremani ayting?
3. Trigonometrik funksiyalarning eng kichik musbat davri nimaga teng?
4. sin 390°, cos 405° larni og‘zaki toping.

MASALALARNI YECHING

609. Trigonometrik funksiyalarning davriyligidan, juft va toqligidan foydalanib, berilgan funksiyalarning qiymatlarini toping:

a) $\sin415°$; $\quad\cos415°$; $\quad\mathrm{tg}415°$; $\quad\mathrm{ctg}415°$;

b) $\sin\frac{13\pi}{2}$; $\quad\cos\frac{13\pi}{2}$; $\quad\mathrm{tg}\frac{13\pi}{2}$; $\quad\mathrm{ctg}\frac{13\pi}{2}$;

d) $\sin(-1110°)$; $\cos(-1110°)$; $\mathrm{tg}(-1110°)$; $\mathrm{ctg}(-1110°)$;

e) $\sin\left(-\frac{17\pi}{3}\right)$; $\cos\left(-\frac{17\pi}{3}\right)$; $\mathrm{tg}\left(-\frac{17\pi}{3}\right)$; $\mathrm{ctg}\left(-\frac{17\pi}{3}\right)$.

610. Argumentni graduslar yoki radianlarning eng kichik musbat son bilan ifodalang:

a) $\sin 1460°$; b) $\cos(-1450°)$; d) $\sin 14000$; e) $\cos(-2110°)$.

611. a) $\sin\frac{21\pi}{5}$; b) $\cos\frac{19\pi}{5}$; d) $\mathrm{tg}\frac{8\pi}{7}$; e) $\mathrm{ctg}\frac{5\pi}{7}$.

612. Quyidagi davriy funksiyalarning eng kichik musbat davrini toping:

a) $\sin 2x$; b) $\cos\frac{x}{2}$; d) $\mathrm{tg}\frac{x}{4}$; e) $\mathrm{ctg}\frac{2x}{3}$.

Izoh. $\sin(kx+b)$ va $\cos(kx+b)$ funksiyalarning davri T ni topish uchun $\sin x$, $\cos x$ ning davri 2π ni k ga bo‘lish lozim, ya’ni $T=\frac{2\pi}{k}$. $\mathrm{tg}(kx+b)$ va $\mathrm{ctg}(kx+b)$ funksiyalarning davri T ni topish uchun $\mathrm{tg}x$, $\mathrm{ctg}x$ ning davri π ni k ga bo‘lish lozim, ya’ni $T=\frac{\pi}{k}$.

Masalan: 1) funksiyaning davrini topamiz: $y=\sin\left(2{,}5x-\frac{\pi}{4}\right)$.

Yechish. Bunda $k=2{,}5$; davri 2π. $T=\frac{2\pi}{2{,}5}=\frac{4}{5}\pi$.

2) $y=\mathrm{tg}\frac{x}{4}$ ning davrini topamiz:

Yechish. Bunda $k=\frac{1}{4}$, davri π.

$T=\pi:\frac{1}{4}=4\pi$.

3) $y=\sin x\cdot\cos x\cdot\cos 2x$ funksiyaning davrini topamiz.

Yechish. Avval $y=\sin x\cdot\cos x\cdot\cos 2x$ funksiyani bitta funksiya ko‘rinishiga keltiramiz ($2\sin\alpha\cdot\cos\alpha=\sin 2\alpha$ formula 88-§ da bayon etiladi):

$y=\sin x\cdot\cos x\cdot\cos 2x=\frac{1}{2}(2\sin x\cdot\cos x)\cdot\cos 2x=\frac{1}{2}\sin 2x\cdot\cos 2x=$

$=\frac{1}{4}(2\sin 2x\cdot\cos 2x)=\frac{1}{4}\sin 4x$. Bunda $k=4$; davri 2π.

$$T=\frac{2\pi}{k}=\frac{2\pi}{4}=\frac{\pi}{2};\ Javob:\ \frac{\pi}{2}.$$

613. Quyidagi davriy funksiyalarning eng kichik musbat davrini toping:

a) $\sin(2x-\pi)$; b) $\cos=\left(\frac{3x-0{,}5\pi}{4}\right)$;

d) $\mathrm{tg}(4x-\pi)$; e)* $\sin^2\frac{x}{2}\cdot\cos^2\frac{x}{2}$.

614*. Trigonometrik funksiyalarning eng kichik musbat davrini toping (trigonometrik ifodalarni soddalashtirish davrida yechiladi):

a) $y=0{,}1\ \sin 10x\cdot\cos 10x$; b) $y=\sin^2\frac{x}{4}-\cos^2\frac{x}{4}$;

d) $y=\sin^4x+\cos^4x$; e) $\sin^6x+\cos^6x-\sin^2x\cdot\cos^2x$.

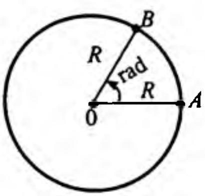
83-§. Burchaklarning radian o'lchovi

Burish burchagi, gradus bilan, minut bilan va sekund bilan o'lchanadi. Bunda: $1°=\frac{2\pi R}{360°}=\frac{\pi R}{180°}$ yoyli markaziy burchak; $1°=60'$ (minut); $1'=60''$ (sekund).

Bu birliklar qatorida ***radian*** deb ataluvchi yana bir burchak birligi qo'llaniladi.

R radiusli biror aylana chizib, ikkita radius tuzilgan musbat $\angle AOB$ markaziy burchakni hosil qilamiz (64-chizma). Burchaklarni radian bilan o'lchashda o'lchov birligi qilib, uzunligi radius uzunligiga teng bo'lgan yoy ($\smile AB$) tiraluvchi musbat markaziy burchak ($\angle AOB$) olinadi.

Ta'rif. **Burchakning radian o'lchovi deb, shu burchak yoyi uzunligining o'sha yoy radiusi uzunligiga nisbati aytiladi.**



64-chizma.

Bitta to'la (musbat) aylanishning radian o'lchovi, aylana uzunligini radiusga bo'lish natijasiga teng, ya'ni 1 to'liq burchak$=\frac{2\pi R}{R}=2\pi=6{,}283185...$ radian. 2π radian$=360°$, π radian $=180°$; 1 radian$=$

$$=\frac{180°}{\pi}=\frac{180°}{3{,}1416}\approx 57°17'45''.$$

Gradusdan radian o‘lchoviga o‘tish:

$1° = \frac{\pi}{180°} = 0{,}017453...$. Masalan: $36° = \frac{\pi}{180°} \cdot 36° = \frac{\pi}{5} \approx 0{,}628$;

$108° = \frac{\pi}{180°} \cdot 108° = \frac{3\pi}{5} \approx 1{,}884$.

Radiandan gradus o‘lchoviga o‘tish:

2,5 radian $= \frac{180°}{\pi} \cdot 2{,}5 \approx 143{,}31° = 143°18{,}6' = 143°18'36''$.

$(0{,}31° = 0{,}31 \cdot 60' = 18{,}6'$; $0{,}6 \cdot 60'' = 36''$; $0{,}31° = 18'36'')$.

Trigonometrik ifodalarda ishlatiladigan ba’zi gradus o‘lchovlarni radian orqali ifodalari:

$30° = \frac{\pi}{180°} \cdot 30 = \frac{\pi}{6}$; $\quad 90° = \frac{\pi}{180°} \cdot 90 = \frac{\pi}{2}$;

$45° = \frac{\pi}{180°} \cdot 45 = \frac{\pi}{4}$; $\quad 180° = \frac{\pi}{180°} \cdot 180 = \pi$;

$60° = \frac{\pi}{180°} \cdot 60 = \frac{\pi}{3}$; $\quad 270° = \frac{\pi}{180°} \cdot 270 = \frac{3}{2}\pi$;

$360° = \frac{\pi}{180°} \cdot 360 = 2\pi$.

Ko‘pincha trigonometrik ifodalarda radian o‘lchovlar ishlatiladi. Masalan sin1,5-yozuvi «1,5 radianga teng burchakning sinusini bildiradi».

$\sin 1{,}5 \approx 0{,}9975$ (jadvaldan topildi). Bu 1,5 rad $85°58'$ olinib, $\sin 1{,}5 = \sin 85°58' \approx 0{,}9975$ topiladi.

$\sin 12 = \sin(3{,}14 \cdot 4 - 0{,}56) = \sin(-0{,}56) = -\sin 0{,}56 = -0{,}5312$.

1-misol. $130°26'$ burchakni radian o‘lchovida toping.

Yechish. $130°26' = 90° + 40°26'$ ko‘rinishda yozib olinadi va ularning radian o‘lchovida yozib olinib, natijalar qo‘shiladi. Buning uchun V.M. Bradisning XVI jadvalidan foydalaniladi.

$$\begin{array}{r} 90° - 1{,}5708 \\ 40°24' - 0{,}7051 \\ \underline{2' - \quad 6} \\ 130°46' - 2{,}2765 \end{array}$$

kabi topiladi.

2-misol. 1,25 radianga (taqriban) teng burchakning gradus o‘lchovini toping.

Yechish. V.M. Bradis jadvalidan (XI jadvaldan) 1,2500 ga yaqin bo'lgan 1,2497 ni topamiz. Bunga $71°36'$ li burchak topiladi.

Umuman sin$x$ yozuv (bunda x – ixtiyoriy haqiqiy son) x radianga teng burchakning sinusini anglatadi. Shunday qilib, sinusning aniqlanish sohasi $(-\infty; +\infty)$ bo'lgan funksiyadir. Xuddi shunday kosinusning aniqlanish sohasi $(-\infty; +\infty)$ bo'lgan funksiyadir. Sinusning ham, kosinusning ham qiymatlar sohasi $[-1; 1]$ oraliq bo'ladi.

Tangens – aniqlanish sohasi $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2},...$ lardan boshqa hamma sonlardan iborat funksiyadir. Bu funksiyaning aniqlanish sohasi haqiqiy sonlar to'plami bo'ladi.

Kotangens – aniqlanish sohasi $0, \pm\pi, 2\pi, \pm3\pi, ...$ dan boshqa hamma sonlardan iborat bo'lgan funksiyadir. Uning qiymatlari sohasi haqiqiy sonlar to'plamidan iborat.

TAKRORLASH UCHUN SAVOLLAR

1. Burchakning radian o'lchovi deb nima qabul qilingan?
2. Burchakning radian o'lchovi deb nimaga aytiladi?
3. To'liq burchak necha radianga teng?
4. 1 radian qancha gradus o'lchoviga teng?
5. 1° burchak qancha radianga teng?
6. Radian o'lchovida sinus va kosinuslarning aniqlanish sohasi nimadan iborat?
7. Radian o'lchovida sinus va kosinuslarning qiymatlari sohasi nimaga teng?
8. Radian o'lchovida tangens va kotangenslarning aniqlanish sohasi nimadan iborat?
9. Radian o'lchovida tangens va kotangenslarning qiymatlari sohasi nimaga teng?

MASALALARNI YECHING

615. Burchakning quyidagi radian o'lchoviga ko'ra gradus o'lchovini toping:

a) 0,5; d) $\frac{\pi}{5}$; f) $\frac{3}{4}\pi$; h) $-\frac{9}{2}\pi$;

b) 10; e) $\frac{\pi}{9}$; g) $-\frac{5}{6}\pi$; i) 12π.

616. Burchakning radian o'lchovini toping:

a) $135°$; d) $36°$; f) $240°$; h) $-120°$;

b) $210°$; e) $150°$; g) $300°$; i) $-225°$.

617. α burchakka qo'shni burchakni radian bilan ifodalang, bunda:

a) $\alpha = \frac{5\pi}{6}$; b) $\alpha = \frac{11\pi}{12}$; d) $\alpha = 0,3\,\pi$.

618. α burchak qaysi chorak burchagi?

a) $\alpha = \frac{3\pi}{4}$; b) $\alpha = 1,8\,\pi$; d) $\alpha = 0,6\,\pi$; $\alpha = 1$.

619. Ifodaning ishorasini aniqlang:

a) $\sin\frac{5\pi}{6}$; d) $\sin 1$; f) $\mathrm{tg}\,\frac{\pi}{4}$; h) $\mathrm{ctg}\,\frac{2\pi}{3}$;

b) $\cos\frac{3\pi}{4}$; e) $\cos 0,9$; g) $\mathrm{tg}\,3$; i) $\mathrm{ctg}\,0,2$.

620. Ifodaning qiymatini toping:

a) $2\sin\pi - 2\cos\frac{3\pi}{2} + 3\mathrm{tg}\,\frac{\pi}{4} - \mathrm{ctg}\,\frac{\pi}{2}$;

b) $\sin\left(-\frac{\pi}{4}\right) + 3\cos\frac{\pi}{3} - \mathrm{tg}\,\frac{\pi}{6} + \mathrm{ctg}\,\frac{\pi}{3}$;

d) $2\sin\frac{\pi}{4} - 3\mathrm{tg}\,\frac{\pi}{6} + \mathrm{ctg}\left(-\frac{3\pi}{2}\right) - \mathrm{tg}\,\pi$;

e) $3\mathrm{tg}\left(-\frac{\pi}{4}\right) + 2\sin\frac{\pi}{4} - 3\mathrm{tg}\,0 - 2\mathrm{ctg}\,\frac{\pi}{4}$.

Namuna. $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{3}\cdot\mathrm{tg}^2\frac{\pi}{4} + \mathrm{ctg}^2\frac{\pi}{6}$ ni hisoblang.

Hisoblash: $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{3}\cdot\mathrm{tg}^2\frac{\pi}{4} + \mathrm{ctg}^2\frac{\pi}{6} = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\cdot 1^2 +$

$+ (\sqrt{3})^2 = \frac{1}{2} - \frac{3}{4} = 2\frac{3}{4}$.

621. Ifodaning qiymatini toping:

a) $\sin 2,5\pi$; b) $\cos\left(-\frac{9\pi}{4}\right)$; d) $\mathrm{tg}\,\frac{13\pi}{6}$; e) $\sin\left(-\frac{19\pi}{2}\right)$.

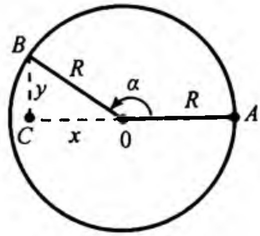
84-§. Ayni bir argumentning trigonometrik funksiyalari orasidagi munosabatlar

Faraz qilaylik, $OA=R$ radiusni α burchakka burishda OB radius hosil qilingan bo'lsin (65-chizma). U holda ta'rifga ko'ra $\sin\alpha=\frac{y}{R}$; $\cos\alpha=\frac{x}{R}$.

Bunda x B nuqtaning absissasi, y uning ordinatasi.

Bundan $\boldsymbol{x=R\cos\alpha}$; $\boldsymbol{y=R\sin\alpha}$ hosil bo'ladi.

ΔOBC to'g'ri burchakli bo'lgani uchun Pifagor teoremasiga asosan $x^2+y^2=R^2$ tenglikni yozamiz. Bunga yuqoridagi $x=R\cos$; $y=R\sin$ larni qo'yib, $(R\cos\alpha)^2+(R\sin\alpha)^2=R^2$ ni hosil qilamiz. Bundan:



65-chizma.

$\boldsymbol{\sin^2\alpha+\cos^2\alpha=1}$ **(1)** formula kelib chiqadi.

Bu formula bir argumentning sinusi va kosinusi orasidagi munosabatni ifodalaydi.

Endi ayni bir argumentning tangensi, kotangensi, sinusi va kosinusi o'zaro qanday munosabatda ekanligini ko'rib chiqamiz:

$$\operatorname{tg}\alpha=\frac{y}{x}=\frac{R\sin\alpha}{R\cos\alpha}=\frac{\sin\alpha}{\cos\alpha},\quad \text{ya'ni } \operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha} \qquad (2).$$

$$\operatorname{ctg}\alpha=\frac{x}{y}=\frac{R\cos\alpha}{R\sin\alpha}=\frac{\cos\alpha}{\sin\alpha},\quad \text{ya'ni } \operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha} \qquad (3).$$

$$\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}=1,\quad \text{ya'ni } \operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1 \qquad (4).$$

Ayni argumentning tangensi bilan kosinusi va kotangensi bilan sinusi orasidagi munosabatni ifodalovchi formulalarni keltirib chiqaramiz:

(1) tenglikning ikkala qismini $\cos^2\alpha$ ga bo'lamiz:

$$\frac{\sin^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha};\quad 1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha} \qquad (5)$$

Agar (1) tenglikning ikkala qismini $\sin^2\alpha$ ga bo'lamiz:

$$\frac{\sin^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\sin^2\alpha}=\frac{1}{\sin^2\alpha}; \quad \mathbf{1+ctg^2\alpha=\frac{1}{\sin^2\alpha}} \qquad (6)$$

Bulardagi (5) tenglik $\cos\alpha \neq 0$ da, (6) tenglik esa $\sin\alpha \neq 0$ da to‘g‘ri bo‘ladi.

(1) – (6) tengliklarni **asosiy trigonometrik ayniyatlar** deyiladi.

Bu ayniyatlardan birining berilgan qiymatiga ko‘ra boshqa funksiyaning qiymatini topishda foydalaniladi.

1-misol. Agar $\sin\alpha=\frac{5}{13}$ va $\frac{\pi}{2}<\alpha<\pi$ bo‘lsa, $\cos\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni topamiz.

Yechish. Avval $\cos\alpha$ ni $\sin^2\alpha+\cos^2\alpha=1$ ayniyatdan $\cos^2\alpha=1-\sin^2\alpha$ kabi topiladi.

$$\cos\alpha=\pm\sqrt{1-\sin^2\alpha}=\pm\sqrt{1-\left(\frac{5}{13}\right)^2}=\pm\sqrt{\frac{144}{169}}=\pm\frac{12}{13}$$

α burchak II chorakda bo‘lgani uchun $\pm\frac{12}{13}$ qiymatning $-\frac{12}{13}$ qiymati olinadi. Demak, $\cos\alpha=-\frac{12}{13}$.

Sinus va kosinuslarni bilgan holda uning tangensi:

$$\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{5}{13}}{-\frac{12}{13}}=-\frac{5}{12} \text{ kabi topiladi.}$$

α burchakning kotangensi $\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1$ ayniyatdan topiladi:

$$\operatorname{ctg}\alpha=\frac{1}{\operatorname{tg}\alpha}=\frac{1}{-\frac{5}{12}}=-\frac{12}{5}=-2,4.$$

Javob: $\cos\alpha=-\frac{12}{13}$; $\operatorname{tg}\alpha=-\frac{5}{12}$; $\operatorname{ctg}\alpha=-2,4$.

2-misol. $\operatorname{tg}\alpha=2$ va $0<\alpha<\frac{\pi}{2}$ bo‘lsa, $\sin\alpha$, $\cos\alpha$ va $\operatorname{ctg}\alpha$ ni topamiz.

Yechish: $1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha}$ ayniyatdan foydalanib, $\cos\alpha$ ni topamiz.

$\cos^2\alpha=\frac{1}{1+\operatorname{tg}^2\alpha}=\frac{1}{1+2^2}=\frac{1}{5}$, bundan $\cos\alpha=\pm\sqrt{\frac{1}{5}}=\pm\frac{\sqrt{5}}{5}$, α burchak I chorakda bo‘lganidan $\cos\alpha=\frac{\sqrt{5}}{5}$ ga teng.

$$\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}.$$

Kotangensning qiymati tangens orqali $\mathrm{ctg}\alpha = \frac{1}{\mathrm{tg}\alpha} = \frac{1}{2}$ topiladi.

Javob: $\sin\alpha = \frac{2\sqrt{5}}{5}$; $\cos\frac{\sqrt{5}}{5}$; $\mathrm{ctg}\alpha = \frac{1}{2}$.

TAKRORLASH UCHUN SAVOLLAR

1. Bir argumentning sinusi va kosinuslari orasidagi munosabatni ifodalovchi ayniyatni yozing.
2. $\mathrm{tg}\alpha$ bilan $\mathrm{ctg}\alpha$ orasidagi munosabatni ifodalovchi ayniyatni yozing.
3. $\cos\alpha$ bilan $\mathrm{tg}\alpha$ orasidagi munosabatni ifodalovchi ayniyatni yozing.
4. $\sin\alpha$ bilan $\mathrm{ctg}\alpha$ orasidagi munosabatni ifodalovchi ayniyatni yozing.

MASALALARNI YECHING

622. $\frac{\pi}{2} < \alpha < \pi$ ekani ma'lum. Agar:

a) $\cos\alpha = -0,6$ bo'lsa, $\sin\alpha$ ni toping;

b) $\sin\alpha = \frac{1}{3}$ bo'lsa, $\cos\alpha$ ni toping;

d) $\cos\alpha = -\frac{15}{17}$ bo'lsa, $\mathrm{tg}\alpha$ ni toping;

e) $\mathrm{ctg}\alpha = -2$ bo'lsa, $\sin\alpha$ ni toping.

623. Biror β burchak uchun quyidagi shart bajarilishi mumkinmi?

a) $\sin\beta = \frac{9}{41}$ va $\cos\beta = \frac{40}{41}$;

b) $\sin\beta = \frac{3}{4}$ va $\cos\beta = \frac{1}{4}$;

d) $\mathrm{tg}\beta = \frac{5}{9}$ va $\mathrm{ctg}\beta = 1,8$;

e) $\mathrm{tg}\beta = \sqrt{2}-1$ va $\mathrm{ctg}\beta = \sqrt{2}+1$.

624. a) Agar $\sin\alpha = \frac{9}{41}$ va $\frac{\pi}{2} < \alpha < \pi$ bo'lsa, $\mathrm{tg}\alpha$ ni toping;

b) agar $\mathrm{ctg}\alpha = \frac{1}{3}$ va $\pi < \alpha < \frac{3\pi}{2}$ bo'lsa, $\cos\alpha$ ni toping.

625. Quyidagi berilganlarga ko'ra α burchakning qolgan trigonometrik funksiyalari qiymatlarini toping:

a) $\sin\alpha = \frac{3}{5}$ va $0 < \alpha < \frac{\pi}{2}$;

b) $\cos\alpha = \frac{8}{17}$ va α – II chorak burchagi;

d) $\operatorname{tg}\alpha = -\frac{\sqrt{3}}{3}$ va $\frac{\pi}{2} < \alpha < \pi$;

e) $\operatorname{ctg}\alpha = -2{,}5$ va α IV chorak burchagi.

626. Quyida berilganlarga ko'ra α burchakning trigonometrik funksiyalari qiymatlarini toping:

a) $\sin\alpha = \frac{8}{17}$; b) $\cos\alpha = -\frac{\sqrt{3}}{2}$.

85-§. Asosiy trigonometrik formulalarning ifodalarni almashtirishda qo'llanilishi

Bir argumentning trigonometrik funksiyalari orasidagi o'rnatilgan munosabat formulalari trigonometrik ifodalarni soddalashtirishga va trigonometrik ayniyatlarni isbotlashga yordam beradi.

1-m i s o l. $\operatorname{ctg}^2\alpha\,(\cos^2\alpha - 1)$ ifodani soddalashtiramiz.

$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$ va $\sin^2\alpha + \cos^2\alpha = 1$ formulalaridan foydalanib, quyidagini hosil qilamiz.

$$\operatorname{ctg}^2\alpha(\cos^2\alpha - 1) = \frac{\cos^2\alpha}{\sin^2\alpha}(-\sin^2\alpha) = -\cos^2\alpha.$$

2-m i s o l. $\frac{\sin}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha}$ ifodani soddalashtiramiz.

Yechish. $\frac{\sin}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha(1+\cos\alpha)} = \frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{\sin\alpha(1+\cos\alpha)} =$

$$= \frac{\sin^2\alpha + \cos^2\alpha + 1 + 2\cos\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{2+2\cos\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{2(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)} = \frac{2}{\sin\alpha}.$$

3-m i s o l. $\operatorname{tg}^2\alpha - \sin^2\alpha = \operatorname{tg}^2\alpha \cdot \sin^2\alpha$ ayniyatni isbotlaymiz.

Isbot. Berilgan tenglikning chap qismini shakl almashtiramiz:

$$\mathrm{tg}^2\alpha-\sin^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}-\sin^2\alpha=\sin^2\alpha\left(\frac{1}{\cos^2\alpha}-1\right)=\sin^2\alpha\,(1+\mathrm{tg}^2\alpha-1)=$$

$=\sin^2\alpha\cdot\mathrm{tg}^2\alpha$. Biz tenglikning o'ng tomonidagi ifodani hosil qildik. Ayniyat isbotlandi.

4-m i s o l. $\frac{\cos^2\alpha-\sin^2\alpha}{\mathrm{ctg}^2\alpha-\mathrm{tg}^2\alpha}=\sin^2\alpha\cdot\cos^2\alpha$ ayniyatni isbotlaymiz.

Isbot. Bunda $1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha}$ va $1+\mathrm{ctg}^2\alpha=\frac{1}{\sin^2\alpha}$ formulalardan foydalanamiz:

$$\frac{\cos^2\alpha-\sin^2\alpha}{\mathrm{ctg}^2\alpha-\mathrm{tg}^2\alpha}=\frac{\cos^2\alpha-\sin^2\alpha}{\frac{1}{\sin^2\alpha}-1-\left(\frac{1}{\cos^2\alpha}-1\right)}=\frac{\cos^2\alpha-\sin^2\alpha}{\frac{1}{\sin^2\alpha}-\frac{1}{\cos^2\alpha}}=\frac{\cos^2\alpha-\sin^2\alpha}{\frac{\cos^2\alpha-\sin^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}}=$$

$=\sin^2\alpha\cdot\cos^2\alpha$. Ayniyat isbotlandi.

MASALALARNI YECHING

627. Ifodani soddalashtiring:

a) $1-\cos^2\alpha$; e) $\sin\alpha\cdot\mathrm{ctg}\alpha$;

b) $2-\sin^2\alpha-\cos^2\alpha$; f) $\frac{1}{\sin^2\alpha}-1$;

d) $(1-\sin\alpha)(1+\sin\alpha)$; g) $\sin^2\alpha-\mathrm{tg}\alpha\cdot\mathrm{ctg}\alpha$.

628. Ifodani soddalashtiring:

a) $1-\sin^2\alpha-\cos^2\alpha$; e) $\mathrm{ctg}^2\alpha(\cos^2\alpha-1)+1$;

b) $\sin\beta+\cos\beta\cdot\mathrm{tg}\beta$; f) $(\sin\beta+\cos\beta)^2+(\sin\beta-\cos\beta)^2$;

d) $\frac{\sin^2\alpha-1}{1-\cos^2\alpha}$; g) $\frac{1}{1+\cos\alpha}+\frac{1}{1-\cos\alpha}$.

629. Ifodani almashtiring:

a) $\sin^2\beta+\cos^2\beta+\mathrm{tg}^2\beta$; d) $\cos^2\gamma-(\mathrm{ctg}^2\gamma+1)\sin^2\gamma$;

b) $\frac{\sin^2x-1}{\cos^2x-1}+\mathrm{tg}x\cdot\mathrm{ctg}x$; e) $\frac{1-\mathrm{ctg}\beta}{\mathrm{tg}\beta-1}$.

630. β ning hamma qabul qiladigan qiymatlarida ifodaning qiymati β ga bog'liq bo'lmasligini isbotlang.

a) $\frac{\sin^2\beta-\cos^2\beta+1}{\sin^2\beta}$;   d) $\frac{1}{1+\operatorname{tg}^2\beta}+\frac{1}{1+\operatorname{ctg}^2\beta}$;

b) $\frac{1+2\sin\beta\cdot\cos\beta}{(\sin\beta+\cos\beta)^2}$;   e) $\frac{1+\sin\beta}{\cos\beta}\cdot\frac{1-\sin\beta}{\cos\beta}$.

631. Ifodani soddalashtiring:

a) $\operatorname{tg}(-\alpha)\cdot\cos\alpha+\sin\alpha$;   d) $\cos^2\alpha\cdot\operatorname{tg}^2(-\alpha)-1$;

b) $\frac{\operatorname{ctg}(-\alpha)\cdot\sin\alpha}{\cos\alpha}$;   e) $\frac{1-\operatorname{tg}(-\alpha)}{\sin\alpha+\cos(-\alpha)}$.

632. Ifodani soddalashtiring:

a) $\frac{\cos\alpha}{1+\sin\alpha}+\operatorname{tg}\alpha$;   d) $\frac{\cos\alpha}{1-\sin\alpha}+\frac{\cos\alpha}{1+\sin\alpha}$;

b) $\frac{\operatorname{tg}^2\varphi-1}{\operatorname{tg}^2\varphi+1}+\cos^2\varphi$;   e) $\frac{\sin^3\alpha+\cos^3\alpha}{\sin\alpha+\cos\alpha}+\sin\alpha\cdot\cos\alpha$.

633. Ayniyatni isbotlang:

a) $(\operatorname{tg}\alpha+\operatorname{ctg}\alpha)^2-(\operatorname{tg}\alpha-\operatorname{ctg}\alpha)^2=4$;

b) $(2+\sin\beta)(2-\sin\beta)+(2+\cos\beta)(2-\cos\beta)=7$;

d) $\operatorname{ctg}\alpha+\frac{\sin\alpha}{1+\cos\alpha}=\frac{1}{\sin\alpha}$;

e) $\frac{1-2\sin x\cdot\cos x}{\sin x-\cos x}=\sin x-\cos x$.

634. Ayniyatni isbotlang:

a) $\frac{\cos^3\alpha-\sin^3\alpha}{1+\sin\alpha\cos\alpha}=\cos\alpha-\sin\alpha$;   d) $\frac{\cos\beta}{1-\sin\beta}-\frac{\cos\beta}{1+\sin\beta}=2\operatorname{tg}\beta$;

b) $(1+\operatorname{tg})^2+(1-tg\alpha)^2=\frac{2}{\cos^2\alpha}$;   e) $\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}=\operatorname{tg}\alpha\cdot\operatorname{tg}\beta$.

635. Ifodani soddalashtiring va uning qiymatini toping:

a) $1-\sin\alpha\cdot\cos\alpha\cdot\operatorname{tg}\alpha$, bunda $\sin\alpha=0{,}7$;

b) $\cos^4\alpha+\sin^2\alpha\cdot\cos^2\alpha$, bunda $\operatorname{tg}\alpha=2$;

d) $\frac{\sin\alpha}{1+\cos\alpha}+\frac{\sin\alpha}{1-\cos\alpha}$, bunda $\sin\alpha=-\frac{1}{8}$.

86-§. Keltirish formulalari

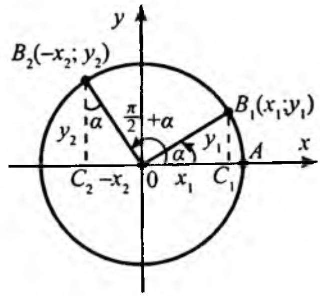
Keltirish formulalari deb, ushbu:

$-\alpha$; $\frac{\pi}{2}\pm\alpha$; $\pi\pm\alpha$; $\frac{3\pi}{2}\pm\alpha$; $2\pi\pm\alpha$ argumentlarning trigonometrik funksiyalarini α argument funksiyalari orqali ifodalovchi formulalarga aytiladi; bunda α argumentning ixtiyoriy (mumkin bo'lgan) qiymatidir.

Avval sinus va kosinus uchun keltirish formulalarini chiqaramiz.

Istalgan α uchun $\sin\left(\frac{\pi}{2}\pm\alpha\right)=\cos\alpha$ va $\cos\left(\frac{\pi}{2}\pm\alpha\right)=\mp\sin\alpha$ ekanini isbotlaymiz.

$OA=R$ radiusni α burchakka va $\frac{\pi}{2}+\alpha$ burchakka buramiz, bunda OA radius mos ravishda OB_1 va OB_2 radiuslarga o'tadi (66-chizma).



66-chizma.

B_1 nuqtaning koordinatalarini x_1 va y_1, B_2 nuqtaning koordinatalarini x_2 va y_2 deylik. Bundagi $\Delta OB_1C_1=\Delta OB_2C_2$, chunki $OB_1=OB_2=R$; $\angle B_1OC_1=\angle OB_2C_2$ va uchburchak-lar to'g'ri burchakli. Ularda teng burchaklar qarshisida teng tomonlar yotganidan:

$y_2=x_1$ va $-x_2=y_1$. Bu tengliklarni R ga bo'lib, $\frac{y_2}{R}=\frac{x_1}{R}$ va $-\frac{x_2}{R}=\frac{y_1}{R}$ larni hosil qilamiz. Bu nisbatlardan:

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$ va $-\cos\left(\frac{\pi}{2}+\alpha\right)=\sin\alpha$ ($x_2=-y_1$ bo'lgani uchun) $\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$.

Demak, $\mathbf{\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha}$ va $\mathbf{\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha}$.

$\sin\left(\frac{\pi}{2}-\alpha\right)$ va $\cos\left(\frac{\pi}{2}-\alpha\right)$ larni nimaga teng ekanligini topamiz.

Haqiqatan, $\frac{\pi}{2}-\alpha=\frac{\pi}{2}+(-\alpha)$ ekanligidan:

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\sin\left(\frac{\pi}{2}+(-\alpha)\right)=\cos(-\alpha)=\cos\alpha;$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\frac{\pi}{2}+(-\alpha)\right)=-\sin(-\alpha)=-(-\sin\alpha)=\sin\alpha$$

Demak, $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$ va $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$.

Endi $\pi+\alpha$ burchakning sinusi va kosinusi uchun keltirish formulalarini topamiz:

$\pi+\alpha=\frac{\pi}{2}+\left(\frac{\pi}{2}+\alpha\right)$ tenglikdan foydalanamiz.

$$\left.\begin{aligned}\sin(\pi+\alpha)=\sin\left(\frac{\pi}{2}+\left(\frac{\pi}{2}+\alpha\right)\right)=\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha.\\ \cos(\pi+\alpha)=\cos\left(\frac{\pi}{2}+\left(\frac{\pi}{2}+\alpha\right)\right)=-\sin\left(\frac{\pi}{2}+\alpha\right)=-\cos\alpha.\end{aligned}\right\}\begin{aligned}\sin(\pi+\alpha)=-\sin\alpha\\ \cos(\pi+\alpha)=-\cos\alpha\end{aligned}\quad(1)$$

(1) formuladagi α ni $-\alpha$ bilan almashtirib,

$\sin(\pi-\alpha)=-\sin(-\alpha)=\sin\alpha$

$\cos(\pi-\alpha)=-\cos(-\alpha)=\cos\alpha.$

Demak, $\sin(\pi-\alpha)=\sin\alpha$ va $\cos(\pi-\alpha)=-\cos\alpha.$ $\frac{3\pi}{2}+\alpha$ burchakning keltirish formulalarini $\frac{3\pi}{2}+\alpha=\pi+\left(\frac{\pi}{2}+\alpha\right)$ almashtirishdan foydalanib, $\sin\left(\frac{3\pi}{2}+\alpha\right)=\sin\left(\pi+\left(\frac{\pi}{2}+\alpha\right)\right)=-\sin\left(\frac{\pi}{2}+\alpha\right)=-\cos\alpha;$ $\cos\left(\frac{3\pi}{2}+\alpha\right)=\cos\left(\pi+\left(\frac{\pi}{2}+\alpha\right)\right)=-\cos\left(\frac{\pi}{2}+\alpha\right)=-(-\sin\alpha)=\sin\alpha.$

Demak, $\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$ va $\cos\left(\frac{3\pi}{2}+\alpha\right)=\sin\alpha.$

Bu formuladagi α ni $-\alpha$ bilan almashtirib, $\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$ va $\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$ formulani hosil qilamiz.

Nihoyat, $2\pi+\alpha$ burchakning sinus va kosinusi uchun keltirish formulalari α burchakni butun aylanishlar soniga o'zgartirganda sinus va kosinuslarning qiymati o'zgarmasligidan $\sin(2\pi+\alpha)=\sin\alpha$ va $\cos(2\pi+\alpha)=\cos\alpha$ tengliklarni yozamiz.

Demak, $\sin(2\pi+\alpha)=\sin\alpha$ va $\cos(2\pi+\alpha)=\cos\alpha.$

Nihoyat, $\sin(2\pi-\alpha)=-\sin\alpha$ va $\cos(2\pi-\alpha)=\cos\alpha$ formula ham o'rinli bo'ladi.

Tangens va kotangens uchun keltirish formulalari sinus va kosinus uchun keltirish formulalari yordamida hosil qilinadi.

Masalan: $$\operatorname{tg}\left(\frac{\pi}{2}+\alpha\right)=\frac{\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{\pi}{2}+\alpha\right)}=\frac{\cos\alpha}{-\sin\alpha}=-\operatorname{ctg}\alpha.$$

$$\operatorname{ctg}(\pi+\alpha)=\frac{\cos(\pi+\alpha)}{\sin(\pi+\alpha)}=\frac{-\cos\alpha}{-\sin\alpha}=\operatorname{ctg}\alpha.$$

α	$-\alpha$	$\frac{\pi}{2}+\alpha$ $(90°+\alpha)$	$\frac{\pi}{2}-\alpha$ $(90°-\alpha)$	$\pi+\alpha$ $(180°+\alpha)$	$\pi-\alpha$ $(180°-\alpha)$	$\frac{3\pi}{2}+\alpha$ $(270°+\alpha)$	$\frac{3\pi}{2}-\alpha$ $(270°-\alpha)$	$2\pi+\alpha$ $(360°+\alpha)$	$2\pi-\alpha$ $(360°-\alpha)$
$\sin\alpha$	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$\sin\alpha$	$-\sin\alpha$
$\cos\alpha$	$\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$
$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$
$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$

Jadvaldan keltirish formulalari uchun o'rinli bo'lgan qonuniyatlarni sanab o'tamiz:

1. α burchak I chorak burchagi bo'lsa, dastlabki funksiya qanday ishorali bo'lsa, tenglikning o'ng tomonidagi funksiya ham shunday ishorali bo'ladi;

2. $\pi\pm\alpha$ va $2\pi\pm\alpha$ burchaklar uchun dastlabki funksiya nomi saqlanadi.

3. $\frac{\pi}{2}\pm\alpha$ va $\frac{3\pi}{2}\pm\alpha$ burchaklar uchun dastlabki funksiya nomi o'zgaradi (sinus kosinusga, kosinus sinusga, tangens kotangensga va kotangens tangensga).

Masalan, $\operatorname{tg}(\pi-\alpha)$ ni α burchakning funksiyasi orqali ifodalaymiz. Agar α ni I chorak burchagi deb hisoblasak, u holda $\pi-\alpha$ II chorak burchagi bo'ladi. Ikkinchi chorakda tangens manfiy, demak tenglikning o'ng tomoniga «minus» ishorasini qo'yish kerak. α burchak π dan ayrilgani uchun «tangens» nomi saqlanadi. Shuning uchun $\operatorname{tg}(\pi-\alpha)=-\operatorname{tg}\alpha$.

Keltirish formulalari yordamida istalgan burchak trigonometrik funksiyalari qiymatlarini topishni O dan $\frac{\pi}{2}$ gacha burchakning trigonometrik funksiyalari qiymatlarini topishga keltirish mumkin. Misollar keltiramiz:

1-m i s o l. $\cos\frac{8\pi}{3}$ ifodaning qiymatini topamiz.

$$\cos\frac{8\pi}{3}=\cos\left(2\pi+\frac{2\pi}{3}\right)=\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}.$$

2-m i s o l. $\sin(-600°) = -\sin 600° = -\sin(360° + 240°) = -\sin 240° =$
$= -\sin(180° + 60°) = -(-\sin 60°) = \sin 60° = \frac{\sqrt{3}}{2}.$

3-m i s o l. $\frac{\operatorname{tg}(\pi-\alpha)}{\cos(\pi+\alpha)} \cdot \frac{\sin\left(\frac{3\pi}{2}+\alpha\right)}{\operatorname{tg}\left(\frac{3\pi}{2}+\alpha\right)} = \operatorname{tg}^2\alpha$ ayniyatni isbotlaymiz.

Isbot. $$\frac{\operatorname{tg}(\pi-\alpha)}{\cos(\pi+\alpha)} \cdot \frac{\sin\left(\frac{3\pi}{2}+\alpha\right)}{\operatorname{tg}\left(\frac{3\pi}{2}+\alpha\right)} = \frac{-\operatorname{tg}\alpha}{-\cos\alpha} \cdot \frac{-\cos\alpha}{-\operatorname{ctg}\alpha} = \frac{\operatorname{tg}\alpha}{\operatorname{ctg}\alpha} = \frac{\operatorname{tg}\alpha}{\frac{1}{\operatorname{tg}\alpha}} = tg^2\alpha.$$

Ayniyat isbotlandi.

TAKRORLASH UCHUN SAVOLLAR

1. Qaysi burchaklarning trigonometrik funksiyalari α o'tkir burchakning trigonometrik funksiyalariga keltiriladi.
2. a) $\sin\left(\frac{\pi}{2}+\alpha\right)$ va $\cos\left(\frac{\pi}{2}+\alpha\right)$ larni;

   b) $\cos\left(\frac{\pi}{2}+\alpha\right)$ va $\cos\left(\frac{\pi}{2}-\alpha\right)$ larni α burchakning trigonometrik funksiyalariga keltiring.
3. a) $\sin(\pi+\alpha)$ va $\cos(\pi+\alpha)$ larni;

   b) $\sin(\pi-\alpha)$ va $\cos(\pi-\alpha)$ larni α burchakning trigonometrik funksiyalariga keltiring.
4. a) $\sin\left(\frac{3\pi}{2}+\alpha\right)$ va $\cos\left(\frac{3\pi}{2}+\alpha\right)$ larni;

   b) $\sin\left(\frac{3\pi}{2}-\alpha\right)$ va $\cos\left(\frac{3\pi}{2}-\alpha\right)$ larni α burchakning trigonometrik funksiyalariga keltiring.
5. a) $\sin(2\pi+\alpha)$ va $\cos(2\pi-\alpha)$ larni;

   b) $\sin(2\pi-\alpha)$ va $\cos(2\pi-\alpha)$ larni α burchakning trigonometrik funksiyalariga keltiring.
6. Tangens va kotangens funksiyalar uchun keltirish formulalari qanday topiladi?
7. Keltirish formulalarni qanday qonuniyatlari bor?

MASALALARNI YECHING

636. α burchakning trigonometrik funksiyalariga almashtiring:

a) $\sin\left(\frac{\pi}{2}-\alpha\right)$; f) $\cos\left(\frac{3\pi}{2}-\alpha\right)$; j) $\sin(2\pi+\alpha)$;

b) $\cos\left(\frac{\pi}{2}+\alpha\right)$; g) $\sin\left(\frac{3\pi}{2}+\alpha\right)$; k) $\cos(2\pi-\alpha)$;

d) $\mathrm{tg}(\pi-\alpha)$; h) $\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right)$; l) $\mathrm{tg}(180°+\alpha)$;

e) $\mathrm{ctg}(\pi+\alpha)$; i) $\mathrm{ctg}\left(\frac{3\pi}{2}-\alpha\right)$; m) $\mathrm{ctg}(270°-\alpha)$.

637. $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni 0° dan 90° gacha oraliqdagi burchakning trigonometrik funksiyasi orqali ifodalang, bunda:

a) 130°; b) 190°; d) $\alpha=-300°$; e) $\alpha=580°$.

638. $\left(0;\frac{\pi}{2}\right)$ oraliqdagi burchakning trigonometrik funksiyasiga keltiring:

a) $\cos 0{,}7\pi$; b) $\mathrm{ctg}\left(-\frac{3\pi}{5}\right)$; d) $\sin 1{,}6\pi$; e) $\mathrm{tg}\left(-\frac{9\pi}{5}\right)$.

639. Ifodaning qiymatini toping:

a) sin 240°; d) tg 300°; f) ctg(–225°);

b) cos(–210°) e) sin 330°; g) sin 840°.

640. $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni toping, bunda:

a) $\alpha=\frac{2}{3}\pi$; b) $\frac{3\pi}{4}$; d) $\frac{5\pi}{6}$; e) $\frac{17\pi}{4}$.

641. Ifodani soddalashtiring:

a) $\sin\left(\alpha-\frac{\pi}{2}\right)$; d) $\mathrm{tg}(-\alpha+270°)$; f) $\cos\left(\alpha-\frac{3\pi}{2}\right)$;

b) $\cos(\alpha-\pi)$; e) $\mathrm{ctg}(\alpha-360°)$; g) $\mathrm{tg}(\alpha-4\pi)$.

642. a) Agar A, B, C – uchburchakning burchaklari bo‘lsa, y holda $\sin\frac{A+B}{2}=\cos\frac{C}{2}$ ekanini isbotlang.

b) Agar $\alpha+\beta+\gamma=180°$ bo‘lsa, u holda $\mathrm{tg}\frac{\alpha+\beta}{2}=\mathrm{ctg}\frac{\gamma}{2}$ ekanini isbotlang.

643. Ifodani soddalashtiring:

a) $\sin(90°-\alpha)+\cos(180°+\alpha)+\mathrm{tg}(270°+\alpha)+\mathrm{ctg}(360°+\alpha)$;

b) $\sin\left(\frac{\pi}{2}+\alpha\right)-\cos(\alpha-\pi)+\mathrm{tg}(\pi-\alpha)+\mathrm{ctg}\left(\frac{5\pi}{2}-\alpha\right)$;

d) $\frac{\cos(-\alpha)\cdot\cos(180°+\alpha)}{\sin(-\alpha)\cdot\sin(90°+\alpha)}$; e) $\frac{\sin(\pi+\alpha)\cdot\cos(2\pi-\alpha)}{\mathrm{tg}(\pi-\alpha)\cdot\cos(\alpha-\pi)}$.

644. Isbotlang:

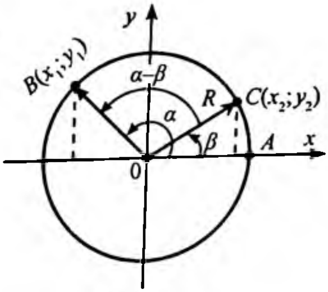
a) $\sin(\pi+x)\cos\left(\frac{\pi}{2}+x\right)-\cos(2\pi+x)\sin\left(\frac{3\pi}{2}-x\right)=1.$

b) $\frac{\sin(\pi-\alpha)}{\mathrm{tg}(\pi+\alpha)}\cdot\frac{\mathrm{ctg}\left(\frac{\pi}{2}-\alpha\right)}{\mathrm{tg}\left(\frac{\pi}{2}+\alpha\right)}\cdot\frac{\cos(2\pi-\alpha)}{\sin(-\alpha)}=\sin\alpha.$

87-§. Ikki burchak yig'indisi va ayirmasining formulalari

Ikki burchak yig'indisi va ayirmasining trigonometrik funksiyalarini shu burchakning trigonometrik funksiyalari orqali ifodalashga imkon beruvchi formulalarni keltirib chiqaramiz.

R ga teng bo'lgan OA radiusni O nuqta atrofida α burchakka va β burchakka buramiz (67-chizma). OB va OC radiuslarni hosil qilamiz.



67-chizma.

B nuqtaning koordinatalarini x_1 va y_1, C nuqtaning koordinatalarini x_2 va y_2 bo'lsin. $\overline{OB}$ va $\overline{OC}$ radiuslarni va ham mos ravishda $(x_1; y_1)$ va $(x_2; y_2)$ koordinatalarga ega bo'ladi. Vektorlarning skalyar ko'paytmasining ta'rifiga asosan $\overline{OB}\cdot\overline{OC}=x_1\cdot x_2+y_1\cdot y_2$ tenglikni yozamiz.

$\overline{OB}\cdot\overline{OC}$ skalyar ko'paytmani va burchaklarning trigonometrik funksiyalari orqali ifodalaymiz. Sinus va kosinuslar ta'rifidan: $x_1=R\cos\alpha$, $y_1=R\sin\alpha$, $x_2=R\cos\beta$, $y_2=R\sin\beta$ larni yozamiz.

x_1, x_2, y_1, y_2 ning qiymatlarini $\overline{OB} \cdot \overline{OC} = x_1 \cdot x_2 + y_1 \cdot y_2$ ga qo'yib, $\overline{OB} \cdot \overline{OC} = R^2(\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta)$ ni hosil qilamiz. Demak,

$$\overline{OB} \cdot \overline{OC} = R^2(\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta) \quad (1)$$

Vektorlarning skalyar ko'paytmasi haqidagi teoremaga asosan (vektorlar orasidagi burchak orqali):

$$\overline{OB} \cdot \overline{OC} = |\overline{OB}| \cdot |\overline{OC}| \cos(\alpha-\beta) = R^2\cos(\alpha-\beta)$$

$\overline{OB}$ va $\overline{OC}$ vektorlar orasidagi COB$=\alpha-\beta$ ga teng.

Bulardan: $\overline{OB} \cdot \overline{OC} = |\overline{OB}| \cdot |\overline{OC}| \cos(\alpha-\beta) = R^2\cos(\alpha-\beta)$ kelib chiqadi. Demak,

$$\overline{OB} \cdot \overline{OC} = R^2\cos(\alpha-\beta) \quad (2)$$

(1) va (2) formulalarni tenglashtirib,

$R^2\cos(\alpha-\beta) = R^2(\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta)$ hosil qilamiz.

Demak, $$\mathbf{\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta} \quad \mathbf{(I)}$$

Ikki burchak ayirmasining kosinusi shu burchaklar kosinuslari ko'paytmasi bilan shu burchaklar sinuslari ko'paytmasining yig'indisiga teng.

I formula yordamida yig'indining kosinusi formulasini hosil qilamiz:
$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) = \cos\alpha \cdot \cos(-\beta) + \sin\alpha \cdot \sin(-\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta.$

Demak, $$\mathbf{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta} \quad \mathbf{(II)}$$

Ikki burchak yig'indisining kosinusi shu burchaklar kosinuslari ko'paytmasidan shu burchaklar sinuslari ko'paytmasining ayirmasiga teng.

Endi shu burchaklar yig'indisining sinusi va ayirmasining sinusi formulasini keltirib chiqaramiz.

Keltirish formulasi va I formuladan foydalanib quyidagini hisoblaymiz:

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)=\cos\left(\frac{\pi}{2}-\alpha\right)\cdot\cos\beta+$$

$$+\sin\left(\frac{\pi}{2}-\alpha\right)\cdot\sin\beta=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta.$$

Demak, $\quad \mathbf{\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha+\sin\beta} \quad$ **(III)**

Ikki burchak yig'indisining sinusi birinchi burchak sinusi bilan ikkinchi burchak kosinusi ko'paytmasiga, birinchi burchak kosinusi bilan ikkinchi burchak sinusi ko'paytmasini qo'shilganiga teng.

Ayirmaning sinusi:

$$\sin(\alpha-\beta)=\sin(\alpha+(-\beta))=\sin\alpha\cdot\cos(-\beta)+\cos\alpha\cdot\sin(-\beta)=$$
$$=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta.$$

Demak, $\quad \mathbf{\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta} \quad$ **(IV)**

Ikki burchak ayirmasining sinusi birinchi burchak sinusi bilan ikkinchi burchak kosinusi ko'paytmasidan, birinchi burchak kosinusi bilan ikkinchi burchak sinusi ko'paytmasining ayirmasiga teng.

1-m i s o l. $\cos15°$ va $\sin15°$ ni hisoblaymiz.

15° ni 45°–30° ayirma ko'rinishida yozib olib, $\cos15°=\cos(45-30°)==$ $\cos45°\cdot\cos30°+\sin45°\cdot\sin30°=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}$;

$\sin15°=\sin(45°-30°)=\sin45°\cdot\cos30°-\cos45°\cdot\sin30°=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-$ $-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$.

15° ni 60°–45° ko'rinishda yozish ham mumkin.

2-m i s o l. $\cos(\alpha+\beta)+\cos(\alpha-\beta)$ ni soddalashtiramiz:

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta+\cos\alpha\cdot\cos\beta+$$
$$+\sin\alpha\cdot\sin\beta=2\cos\alpha\cdot\cos\beta.$$

I–IV formulalardan foydalanib, tangens va kotangenslar uchun formulalar chiqaramiz.

$\operatorname{tg}(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta}$ bu kasrning surat va maxrajini $\cos\alpha\neq0$ va $\cos\beta\neq0$ deb faraz qilib, $\cos\alpha\cdot\cos\beta$ ga bo'lamiz:

$$\operatorname{tg}(\alpha+\beta)=\frac{\dfrac{\sin\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta}+\dfrac{\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}}{\dfrac{\cos\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta}-\dfrac{\sin\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}}=\frac{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}}{1+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}}=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$

Demak, $$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta} \qquad \text{(V).}$$

Shunga o'xshash $$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta} \qquad \text{(VI)}$$
formula topiladi.

Yig'indining kotangensi formulasini chiqaramiz. Bunda $\sin\alpha\neq0$ va $\sin\beta\neq0$ deb faraz qilib, $\operatorname{ctg}(\alpha+\beta)$ ning surat va maxrajini $\sin\alpha\cdot\sin\beta$ ga bo'lamiz:

$$\operatorname{ctg}(\alpha+\beta)=\frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}=\frac{\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta}{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}=$$

$$=\frac{\dfrac{\cos\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta}-\dfrac{\sin\alpha\cdot\sin\beta}{\sin\alpha\cdot\sin\beta}}{\dfrac{\sin\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta}+\dfrac{\cos\alpha\cdot\sin\beta}{\sin\alpha\cdot\sin\beta}}=\frac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta-1}{\operatorname{ctg}\beta+\operatorname{ctg}\alpha}$$

Demak, $$\operatorname{ctg}(\alpha+\beta)=\frac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta-1}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta} \qquad \text{(VII)}$$

Shunga o'xshash

$$\operatorname{ctg}(\alpha-\beta)=\frac{1+\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta}{\operatorname{ctg}\beta-\operatorname{ctg}\alpha} \qquad \text{(VIII)}$$

formula topiladi.

V va VII formuladagi β ni $-\beta$ bilan almashtirib, VI va VIII formulalar topiladi.

3-misol. $\operatorname{tg}\alpha=\frac{1}{3}$ va $\operatorname{tg}\beta=\frac{1}{4}$ bo'lsa, $\operatorname{tg}(\alpha+\beta)$ va $\operatorname{tg}(\alpha-\beta)$ ni hisoblaymiz.

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}=\frac{\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{3}\cdot\frac{1}{4}}=\frac{1}{13};$$

$$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}=\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{3}\cdot\frac{1}{4}}=\frac{7}{11}.$$

4-m i s o l. $\frac{2\sin\alpha\cdot\cos\beta-\sin(\alpha-\beta)}{\cos(\alpha-\beta)-2\sin\alpha\cdot\sin\beta}=\operatorname{tg}(\alpha+\beta)$ ekanligini ko'rsatamiz.

Isbot. $$\frac{2\sin\alpha\cdot\cos\beta-\sin(\alpha-\beta)}{\cos(\alpha-\beta)-2\sin\alpha\cdot\sin\beta}=\frac{2\sin\alpha\cdot\cos\beta-\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta-2\sin\alpha\cdot\sin\beta}=$$

$$=\frac{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta}=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\operatorname{tg}(\alpha+\beta).$$

Tenglik isbotlandi.

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1. $\cos(\alpha-\beta)$ formulani yozing va mazmunini ayting.
2. $\cos(\alpha+\beta)$ formulani yozing va mazmunini ayting.
3. $\sin(\alpha+\beta)$ formulani yozing va mazmunini ayting.
4. $\sin(\alpha-\beta)$ formulani yozing va mazmunini ayting.
5. $\operatorname{tg}(\alpha+\beta)$ va $\operatorname{tg}(\alpha-\beta)$ formulani yozing.
6. $\operatorname{ctg}(\alpha+\beta)$ va $\operatorname{ctg}(\alpha-\beta)$ formulalarni yozing.

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645. Quyidagilarni hisoblang.

a) $\sin75°$; b) $\cos75°$; d) $\operatorname{tg}105°$; e) $\operatorname{ctg}105°$.

646. Qo'shish formulalari yordamida soddalashtiring:

a) $\sin\left(\frac{\pi}{4}+\alpha\right)$; d) $\sin\left(\frac{\pi}{3}+\alpha\right)$; f) $\sin\left(\frac{3\pi}{2}+\alpha\right)$;

b) $\cos\left(\frac{\pi}{4}+\varphi\right)$; e) $\cos\left(\frac{\pi}{3}-\alpha\right)$; g) $\operatorname{tg}(\pi+\alpha)$.

647. Ifodani soddalashtiring:

a) $\sqrt{2}\sin\left(\frac{\pi}{4}+\alpha\right)-\cos\alpha$; d) $2\cos\left(\frac{\pi}{3}-\alpha\right)-\sqrt{3}\sin\alpha$;

b) $\sqrt{2}\sin\left(\alpha-\frac{\pi}{4}\right)-\sin\alpha$; e) $\sqrt{3}\cos-2\cos\left(\alpha-\frac{\pi}{6}\right)$.

648. $\sin\alpha = \frac{8}{17}$; $\cos\beta = \frac{4}{5}$ va α va β lar I chorak burchaklari bo'lsa, ifodaning qiymatini toping:

a) $\sin(\alpha+\beta)$; b) $\cos(\alpha+\beta)$; d) $\cos(\alpha-\beta)$.

649. Agar $\sin\alpha = \frac{9}{41}$, $\sin\beta = -\frac{40}{41}$ α II chorak burchagi, β esa IV chorak burchagi bo'lsa:

a) $\sin(\alpha+\beta)$; b) $\cos(\alpha-\beta)$ larni toping.

650. Ifodaning qiymatini toping:

a) $\cos107° \cdot \cos17° + \sin107° \cdot \sin17°$;

b) $\cos36° \cdot \cos24° - \sin36° \cdot \sin24°$;

d) $\sin63° \cdot \cos27° + \cos63° \cdot \sin27°$;

e) $\sin51° \cdot \cos21° - \sin51° \cdot \sin21°$;

f) $\cos63° \cdot \sin18° - \sin63° \cdot \cos18°$;

g) $\sin15° \cdot \sin45° - \cos15° \cdot \cos45°$.

651. Ifodani soddalashtiring:

a) $\sin3\gamma \cdot \cos\gamma - \cos3\gamma \cdot \sin\gamma$;

b) $\sin\left(\alpha+\frac{\pi}{6}\right)\cos\left(\alpha-\frac{\pi}{6}\right) + \cos\left(\alpha+\frac{\pi}{6}\right) \cdot \sin\left(\alpha-\frac{\pi}{6}\right)$;

d) $\cos(60°-\alpha) + \cos(60°+\alpha)$;

e) $\cos(30°+\alpha) - \cos(30°-\alpha)$.

652. Soddalashtiring:

a) $\frac{\sin(\alpha+\beta) - \cos\alpha \cdot \sin\beta}{\sin(\alpha-\beta) + \cos\alpha \cdot \sin\beta}$; b) $\frac{\cos(\alpha-\beta) - 2\sin\alpha \cdot \sin\beta}{2\sin\alpha \cdot \cos\beta - \sin(\alpha-\beta)}$.

653. Agar α, β va γ uchburchakning burchaklari bo'lsa, $\sin\gamma = \sin\alpha \times \cos\beta + \cos\alpha \cdot \sin\beta$ ekanini isbotlang.

88-§. Ikkilangan burchak formulalari

Qo'shish formulalari yordamida $\sin 2\alpha$, $\cos 2\alpha$, $\operatorname{tg} 2\alpha$ va $\operatorname{ctg} 2\alpha$ larni α burchakning trigonometrik funksiyalari orqali ifodalashga imkon beradi.

Qo'shish formulalaridagi β ni α ga teng deb olib, quyidagi formulalarni hosil qilamiz:

1) $\sin(\alpha+\alpha)=\sin\alpha\cdot\cos\alpha+\cos\alpha\cdot\sin\alpha=2\sin\alpha\cdot\cos\alpha$.

Demak, $$\mathbf{\sin 2\alpha=2\sin\alpha\cdot\cos\alpha} \qquad (1).$$

2) $\cos(\alpha+\alpha)=\cos\alpha\cdot\cos\alpha-\sin\alpha\cdot\sin\alpha=\cos^2\alpha-\sin^2\alpha$;

Demak, $$\mathbf{\cos 2\alpha=\cos^2\alpha-\sin^2\alpha} \qquad (2).$$

3) $\operatorname{tg}(\alpha+\alpha)=\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\alpha}=\dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$.

Demak, $$\mathbf{\operatorname{tg}2\,\alpha=\frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}}. \qquad (3).$$

4) $\operatorname{ctg}(\alpha+\alpha)=\dfrac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\alpha-1}{\operatorname{ctg}\alpha+\operatorname{ctg}\alpha}=\dfrac{\operatorname{ctg}^2\alpha-1}{2\operatorname{ctg}\alpha}$.

Demak, $$\mathbf{\operatorname{ctg}2\alpha=\frac{\operatorname{ctg}^2\alpha-1}{2\operatorname{ctg}\alpha}} \qquad (4).$$

Bu **formulalarni ikkilangan burchak formulalari** deyiladi.

1-m i s o l. $\sin\alpha=0{,}6$ va α–II chorak burchagi ekanini bilgan holda $\sin 2\alpha$ va $\cos 2\alpha$ larning qiymatlarini topamiz.

Yechish. Avval $\cos\alpha$ ni topamiz:

$$\cos\alpha=\pm\sqrt{1-\sin^2\alpha}=-\sqrt{1-0{,}6^2}=-0{,}8.$$

$\sin 2\alpha=2\sin\alpha\cdot\cos\alpha=2\cdot 0{,}6\cdot(-0{,}8)=-0{,}96$.

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=(-0{,}8)^2-(0{,}6)^2=0{,}64-0{,}36=0{,}28$.

2-m i s o l. $\sin\alpha\cdot\cos^3\alpha-\sin^3\alpha\cdot\cos\alpha$ ni soddalashtiramiz.

Buning uchun ifodadan $\sin\alpha\cdot\cos\alpha$ ni qavsdan tashqariga chiqaramiz va ikkilangan burchak formulalaridan foydalanamiz:

$\sin\alpha \cdot \cos^3\alpha - \sin^3\alpha \cdot \cos\alpha = \sin\alpha \cdot \cos\alpha(\cos^2\alpha - \sin^2\alpha) =$
$= \frac{1}{2}(2\sin\alpha \cdot \cos\alpha \cdot \cos2\alpha) = \frac{1}{2}\sin2\alpha \cdot \cos2\alpha = \frac{1}{4}(2\sin2\alpha \cdot \cos2\alpha) = \frac{1}{4}\sin4\alpha.$

Javob: $\frac{1}{4}\sin4\alpha$.

$1-\cos2\alpha$ va $1+\cos2\alpha$ larni $\sin\alpha$ va $\cos\alpha$ lar orqali ifodalangan formulalarini topamiz:

$1-\cos2\alpha = \sin^2\alpha + \cos^2\alpha - (\cos^2\alpha - \sin^2\alpha) = 2\sin^2\alpha.$

$1+\cos2\alpha = \sin^2\alpha + \cos^2\alpha + \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha.$

Demak, $\mathbf{1-\cos2\alpha = 2\sin^2\alpha}$ yoki $\frac{1-\cos2\alpha}{2}$. (5).

Demak, $\mathbf{1+\cos2\alpha = 2\cos^2\alpha}$ yoki $\frac{1+\cos2\alpha}{2}$. (6).

3-m i s o l. $\frac{1-\cos\alpha}{1+\cos\alpha}$ ifodani soddalashtiramiz.

Yechish. $\frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \mathrm{tg}^2\frac{\alpha}{2}$ (bu yerda $\alpha = 2\cdot\frac{\alpha}{2}$ ko'rinishda olingan).

4-m i s o l. $2(1-\cos\alpha) - \sin^2\alpha = 4\sin^4\frac{\alpha}{2}$ ayniyatni isbotlaymiz.

Isbot. Buning isbotida $1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$ va $\sin\alpha = 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$ formulalardan foydalanamiz:

$$2(1-\cos\alpha) - \sin^2\alpha = 2\cdot2\cdot\sin^2\frac{\alpha}{2} - \left(2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}\right)^2 = 4\sin^2\frac{\alpha}{2} -$$

$$-4\sin^2\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2} = 4\sin^2\frac{\alpha}{2}\left(1-\cos^2\frac{\alpha}{2}\right) = 4\sin^2\frac{\alpha}{2}\cdot\sin^2\frac{\alpha}{2} = 4\sin^4\frac{\alpha}{2}.$$

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1. $\sin2\alpha$ ifodani α burchak trigonometrik funksiyalari orqali ifodalang.
2. $\cos2\alpha$ ni α burchak trigonometrik funksiyalari orqali ifodalang.
3. $\mathrm{tg}2\alpha$ ni α burchak trigonometrik funksiyalari orqali ifodalang.
4. $\mathrm{ctg}2\alpha$ ni α burchakning trigonometrik funksiyasi orqali ifodalang.
5. $1-\cos2\alpha$ ni α burchakning trigonometrik funksiyasi orqali ifodalang.
6. $1+\cos2\alpha$ ni α burchakning trigonometrik funksiyasi orqali ifodalang.

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654. Ifodani soddalashtiring:

a) $\frac{\sin 2\alpha}{\sin \alpha}$;

b) $\frac{\cos 2\alpha}{\cos\alpha - \sin\alpha}$;

d) $\cos 2\beta + \sin^2\beta$;

e) $\frac{\sin \beta}{2\cos^2 \frac{\beta}{2}}$.

655. Kasrni qisqartiring:

a) $\frac{\sin 40°}{\sin 20°}$;

b) $\frac{\sin 100°}{\cos 50°}$;

d) $\frac{\cos 80°}{\cos 40° + \sin 40°}$;

e) $\frac{\cos 36° + \sin^2 18°}{\cos 18°}$.

656. $\mathrm{tg}\alpha = \frac{3}{4}$ va $180° < \alpha < 270°$ bo'lsa, quyidagilarni toping:

a) $\sin 2\alpha$; b) $\cos 2\alpha$; d) $\mathrm{tg} 2\alpha$.

657. Ifodani soddalashtiring:

a) $2\sin 20° \cdot \cos 20°$;

b) $\cos^2 \frac{\pi}{10} - \sin^2 \frac{\pi}{10}$;

d) $\frac{2\mathrm{tg} 5°}{1 - \mathrm{tg}^2 5°}$;

e) $\sin \frac{\pi - \alpha}{2} \cdot \cos \frac{\pi - \alpha}{2}$;

f) $2\cos^2 \frac{\pi + \alpha}{4} - 2\sin^2 \frac{\pi + \alpha}{4}$;

g) $\frac{4\mathrm{tg}\frac{3\pi - \alpha}{2}}{1 - \mathrm{tg}^2 \frac{3\pi - \alpha}{2}}$.

658. Hisoblang:

a) $2\sin 15° \cdot \cos 15°$;

b) $\cos^2 15° - \sin^2 15°$;

d) $\frac{2\mathrm{tg} 15°}{1 - \mathrm{tg}^2 15°}$;

e) $2\sin 105° \cdot \cos 105°$;

f) $\cos^2 \frac{7\pi}{2} - \sin^2 \frac{7\pi}{2}$;

g) $\frac{2\mathrm{tg} 75°}{1 - \mathrm{tg}^2 75°}$.

659. Ayniyatni isbotlang:

a) $1 - (\sin\alpha - \cos\alpha)^2 = \sin 2\alpha$;

b) $\cos^4\alpha - \sin^4\alpha = \cos 2\alpha$;

d) $4\sin\alpha \cdot \cos\alpha \cdot \cos 2\alpha = \sin 4\alpha$;

e) $(\mathrm{tg}\alpha + \mathrm{ctg}\alpha)\sin 2\alpha = 2$.

660. Ifodani soddalashtiring:

a) $1+\cos 4\alpha$; d) $\frac{1-\cos 2\alpha}{\sin 2\alpha}$; f) $\frac{2\sin\alpha+\sin 2\alpha}{2\sin\alpha-\sin 2\alpha}$;

b) $1-\cos 4\alpha$; e) $\operatorname{tg}\alpha(1+\cos 2\alpha)$; g) $\frac{1-\cos 2\alpha+\sin 2\alpha}{1+\sin\alpha-\sin 2\alpha}$.

661. Ayniyatni isbotlang:

a) $1+\sin\alpha=2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$; d) $1-\sin\alpha=2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$;

b) $\frac{1+\cos 2\alpha}{1-\cos 2\alpha}=\operatorname{ctg}^2\alpha$; e) $\frac{1-\sin 2\alpha}{1+\sin 2\alpha}=\operatorname{tg}^2\left(\frac{\pi}{4}-\alpha\right)$.

662. Teng yonli uchburchak asosidagi burchagining kosinusi 0,8 ga teng. Shu uchburchak uchidagi burchagining sinusi va kosinusini toping.

663. Quyidagi tenglikni qanoatlantiradigan x burchak mavjudmi?

a) $\sin x\cdot\cos x=\frac{3}{7}$; b) $\sin x\cdot\cos x=\frac{7}{3}$.

89-§. Trigonometrik funksiyalarning yig'indisi va ayirmasining formulalari

Sinuslar yoki kosinuslarning yig'indisini va ayirmasini ko'paytma shaklida ifodalash mumkin. Bunday shakl almashtirishlarga asoslangan formulalar qo'shish formulalaridan keltirib chiqariladi.

$\sin\alpha+\sin\beta$ yig'indini ko'paytma shakliga keltirish uchun $\alpha=x+y$ va $\beta=x-y$ deb faraz qilamiz va sinuslarning yig'indisi hamda sinuslarning ayirmasi formulalaridan foydalanamiz.

Bunda:

$$\sin\alpha+\sin\beta=\sin(x+y)+\sin(x-y)=\sin x\cos y+\cos x\sin y+$$
$$+\sin x\cos y-\cos x\sin y=2\sin x\cos y.$$

$$+\begin{array}{l} x+y=\alpha \\ x-y=\beta \\ \hline 2x=\alpha+\beta \end{array};\quad x=\frac{\alpha+\beta}{2} \qquad -\begin{array}{l} x+y=\alpha \\ x-y=\beta \\ \hline 2y=\alpha-\beta \end{array}\quad y=\frac{\alpha-\beta}{2}$$

ekanligi topilib, $\sin\alpha+\sin\beta$
$2\sin x+\operatorname{sos} y$

ga qo'yamiz:

$$\sin\alpha+\sin\beta=2\sin x\cos y=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}.$$

Demak,
$$\mathbf{\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}} \qquad (1).$$

Ikki burchak sinuslarining yig‘indisi shu burchaklar yig‘indisi yarmining sinusi bilan ular ayirmasi yarmining kosinusi ko‘paytmasining ikkilanganiga teng.

Ikki burchak sinuslarining yig‘indisi formulasidagi β ni $-\beta$ bilan almashtirib,

$$\sin\alpha+\sin(-\beta)=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-(-\beta)}{2}$$ bundan

$$\mathbf{\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}} \qquad (2)$$

hosil bo‘ladi.

Ikki burchak sinuslarining ayirmasi shu burchaklar ayirmasi yarmining sinusi bilan ular yig‘indisi yarmining kosinusiga ko‘paytmasining ikkilanganiga teng.

Ikki burchak kosinuslarining yig‘indisi va ayirmasining formulalari ham xuddi sinuslar yig‘indisi va ayirmasining formulalari kabi chiqariladi.

Kosinuslar yig‘indisining formulasi:

$$\mathbf{\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}} \qquad (3)$$

Ikki burchak kosinuslarining yig‘indisi shu burchaklar yig‘indisi yarmining kosinusi bilan ularning ayirmasi yarmining kosinusiga ko‘paytmasining ikkilanganiga teng.

Kosinuslar ayirmasining formulasi:

$$\mathbf{\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}} \qquad (4)$$

Ikki burchak kosinuslarining ayirmasi shu burchaklar yig‘indisi yarmining sinusi bilan ular ayirmasi yarmi sinusining qarama-qarshi ishorasi bilan olingan ko‘paytmasining ikkilanganiga teng.

$$\operatorname{tg}\alpha+\operatorname{tg}\beta=\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}=\frac{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$$

Demak,
$$\operatorname{tg}\alpha+\operatorname{tg}\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta} \qquad (5)$$

Bu formuladagi β ning o‘rniga $-\beta$ qo‘yib,

$$\operatorname{tg}\alpha-\operatorname{tg}\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta} \qquad (6)$$

formulani hosil qilamiz.

$$\operatorname{ctg}\alpha+\operatorname{ctg}\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta} \qquad (7)$$

va
$$\operatorname{ctg}\alpha-\operatorname{ctg}\beta=\frac{\sin(\beta-\alpha)}{\cos\alpha\cdot\cos\beta} \qquad (8)$$

larni mustaqil isbotlang.

1-misol. $\cos 0{,}8\pi-\sin 0{,}7\pi$ ayirmani ko‘paytma shakliga keltiramiz.

$$\cos 0{,}8\pi-\sin 0{,}7\pi=\cos 0{,}8\pi-\sin(0{,}5\pi+0{,}2\pi)=\cos 0{,}8\pi-\cos 0{,}2\pi=$$
$$=-2\sin\frac{0{,}8\pi+0{,}2\pi}{2}\sin\frac{0{,}8\pi-0{,}2\pi}{2}=-2\sin 0{,}3\pi.$$

2-misol. $1-\sin\alpha$ ifodani ko‘paytma ko‘rinishda ifodalaymiz.

Buning uchun $1=\sin\frac{\pi}{2}$ tenglikdan foydalanamiz.

$$1=\sin\alpha=\sin\frac{\pi}{2}-\sin\alpha=2\sin\frac{\frac{\pi}{2}-\alpha}{2}\cos\frac{\frac{\pi}{2}+\alpha}{2}=2\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right).$$

3-misol. $\cos 2y-\cos 4y-\cos 6y+\cos 8y$ yig‘indini ko‘paytma ko‘rinishida yozamiz.

$$\cos 2y-\cos 4y-\cos 6y+\cos 8y=(\cos 8y+\cos 2y)-(\cos 6y+\cos 4y)=$$
$$=2\cos\frac{8y+2y}{2}\cdot\cos\frac{8y-2y}{2}-2\cos\frac{6y+4y}{2}\cdot\cos\frac{6y-4y}{2}=$$
$$=2\cos 5y\cdot\cos 3y-2\cos 5y\cdot\cos y=2\cos 5y(\cos 3y-\cos y)=$$
$$=2\cos 5y\cdot\left(-2\sin\frac{3y+y}{2}\cdot\sin\frac{3y-y}{2}\right)=-4\cos 5y\cdot\sin 2y\cdot\sin y.$$

TAKRORLASH UCHUN SAVOLLAR

1. Ikki burchak sinuslar yig'indisini ko'paytma ko'rinishidagi formulani yozing.
2. Ikki burchak sinuslari ayirmasini ko'paytma ko'rinishidagi formulani yozing.
3. Ikki burchak kosinuslari yig'indisini ko'paytma ko'rinishidagi formulani yozing.
4. Ikki burchak kosinuslari ayirmasini ko'paytma ko'rinishidagi formulani yozing.

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664. Yig'indini ko'paytma ko'rinishida yozing:

a) $\sin 45° + \sin 15°$; d) $\sin 45° - \sin 15°$;

b) $\cos 45° + \cos 15°$; e) $\cos 45° - \cos 15°$.

665. Ko'paytma shakliga keltiring:

a) $\sin 40° - \sin 20°$; e) $\cos 46° - \cos 74°$;

b) $\sin 20° - \sin 40°$; f) $\cos\frac{11\pi}{2} + \cos\frac{3\pi}{4}$;

d) $\cos 15° + \cos 30°$; g) $\sin\left(\frac{\pi}{6} + \alpha\right) - \sin\left(\frac{\pi}{6} - \alpha\right)$.

666. Ko'paytma shakliga keltiring:

a) $\sin 3\alpha + \sin\alpha$; e) $\cos y - \cos 5y$;

b) $\sin 3\alpha - \sin 5\alpha$; f) $\sin 3y - \sin 4y$;

d) $\cos 2x + \cos 3x$; g) $\cos 11\beta - \cos 20\beta$.

667. Ko'paytma shakliga keltiring:

a) $\frac{1}{2} + \sin\alpha$; d) $1 + \sin\alpha$; f) $2\sin x + 1$;

b) $\frac{1}{2} - \cos\alpha$; e) $\cos x - 1$; g) $1 - 2\cos x$.

668. Isbotlang:

a) $\frac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \mathrm{tg}\, 4\alpha$; b) $\frac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \mathrm{tg}\, 3\alpha \cdot \mathrm{tg}\,\alpha$.

669. Ko'paytuvchilarga ajrating:

a) $\sin x + \sin 2x + \sin 3x + \sin 4x$;

b) $\cos x + \cos 2x + \cos 3x + \cos 4x$.

670. Tenglikning to‘g‘riligini isbotlang.

a) $\sin 10° + \sin 50° - \cos 20° = 0$;

b) $\sin 87° - \sin 59° - \sin 93° + \sin 61° = \sin 1°$.

671. Ko‘paytma ko‘rinishida ifodalang:

a) $\operatorname{tg} 2\alpha + \operatorname{tg} \alpha$;

b) $\operatorname{tg} 3\beta - \operatorname{tg} \beta$;

d) $\operatorname{tg} \frac{\pi}{12} + \operatorname{tg} \frac{\pi}{3}$;

e) $\operatorname{tg} \frac{4\pi}{5} - \operatorname{tg} \frac{3\pi}{5}$;

f) $\operatorname{tg} 4x + \operatorname{ctg} 2x$;

g) $\operatorname{tg} \frac{5\pi}{8} - \operatorname{ctg} \frac{\pi}{8}$.

672. Tenglikni isbotlang:

a) $\frac{\operatorname{tg} 2\alpha + \operatorname{tg} 2\beta}{\operatorname{ctg} 2\alpha + \operatorname{ctg} 2\beta} = \operatorname{tg} 2\alpha \cdot \operatorname{tg} 2\beta$;

b) $\frac{\operatorname{tg}(\alpha+\beta) + \operatorname{tg}(\alpha-\beta)}{\operatorname{tg}(\alpha+\beta) - \operatorname{tg}(\alpha-\beta)} = \frac{\sin 2\alpha}{\sin 2\beta}$.

90-§. Argumentni teng ikkiga bo‘lish formulalari

Argumentni teng ikkiga bo‘lish formulalari yarim argumentning, ya’ni $\frac{\alpha}{2}$ argumentning trigonometrik funksiyalarini α argumentning trigonometrik funksiyalari orqali ifodalaydi.

Ikkilangan argumentning kosinusi formulasidagi α ni $\frac{\alpha}{2}$ argument bilan almashtiramiz; ya’ni $\cos\alpha 2 = \cos^2\alpha - \sin^2\alpha$ dan:

$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$. Bu ayniyatni asosiy $1 = \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}$ ayniyat bilan birgalikda olib, ularni hadlab avval qo‘shamiz; so‘ngra ayiramiz:

$$1 + \cos\alpha = 2\cos^2\frac{\alpha}{2} \qquad 1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}; \qquad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}.$$

Bu ayniyatlarni hadlab bo‘lib,

$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$; va $\operatorname{ctg}\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$ formulalarni hosil qilamiz.

1-misol. 15° ning sinus va kosinuslarini topamiz.

$$\sin 15^\circ = \pm\sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}; \quad \cos 15^\circ = \pm\sqrt{\frac{1+\cos 30^\circ}{2}} =$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}.$$

2-misol. $\sin\alpha=-\frac{3}{5}$ va $\pi<\alpha<\frac{3\pi}{2}$ bo'lgan $\cos\frac{\alpha}{2}$; $\sin\frac{\alpha}{2}$; $\mathrm{tg}\frac{\alpha}{2}$ larni topamiz.

Yechish. $\cos\alpha = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$.

$\frac{\alpha}{2}$ burchak $\frac{\pi}{2}<\frac{\alpha}{2}<\frac{3\pi}{4}$ (ikkinchi chorakda yotadi).

Shuning uchun $\cos\frac{\alpha}{2}<0$; $\sin\frac{\alpha}{2}>0$ va $\mathrm{tg}\frac{\alpha}{2}<0$ bo'ladi.

$$\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\left(-\frac{4}{5}\right)}{2}} = -\sqrt{\frac{1}{10}} = -\frac{\sqrt{10}}{10};$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10};$$

$$\mathrm{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\frac{3\sqrt{10}}{10}}{-\frac{\sqrt{10}}{10}} = -3.$$

$\mathrm{tg}\frac{\alpha}{2} = \dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$ tenglikni surat va maxrajini

$2\cos\frac{\alpha}{2}$ ga ko'paytiramiz.

$$\mathrm{tg}\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}\cdot 2\cos\frac{\alpha}{2}}{2\cdot\cos\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}} = \frac{\sin\alpha}{2\cos^2\frac{\alpha}{2}} = \frac{\sin\alpha}{1+\cos\alpha}.$$

Demak, $\mathbf{tg}\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$

Xuddi shunday $\mathrm{tg}\frac{\alpha}{2}=\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$ ni $2\sin\frac{\alpha}{2}$ ga ko'paytirib, $\mathrm{tg}\frac{\alpha}{2}=$

$$=\frac{2\sin^2\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}\cdot\sin\frac{\alpha}{2}}=\frac{1-\cos\alpha}{\sin\alpha}.$$

Demak, $\mathrm{tg}\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}$.

3-misol. Agar $\cos\alpha=0{,}8$ va α – o'tkir burchak bo'lsa, $\mathrm{tg}\frac{\alpha}{2}$ ni topamiz.

Yechish. $\sin\alpha=\sqrt{1-0{,}8^2}=0{,}6$.

$$\mathrm{tg}\alpha=\frac{\sin\alpha}{1+\cos\alpha}=\frac{0{,}6}{1+0{,}8}=\frac{1}{3}.$$

$\sin3\alpha$ ni α burchakning trigonometrik funksiyasi orqali ifodalaymiz:

$$\sin3\alpha=\sin(\alpha+2\alpha)=\sin\alpha\cdot\cos2\alpha+\cos\alpha\cdot\sin2\alpha=\sin\alpha(\cos^2\alpha-\sin^2\alpha)+\cos\alpha\times$$
$$\times2\cdot\sin\alpha\cdot\cos\alpha=\sin\alpha(1-2\sin^2\alpha)+2\sin\alpha(1-\sin^2\alpha)=\sin\alpha-2\sin^3\alpha+2\sin\alpha-$$
$$-2\sin^3\alpha=3\sin\alpha-4\sin^3\alpha.$$

$\cos3\alpha$ ham yuqoridagidek topiladi.

Demak, $\mathbf{\sin3\alpha=3\sin\alpha-4\sin3\alpha}$ va $\mathbf{\cos3\alpha=4\cos3\alpha-3\cos\alpha}$;

$\mathbf{tg3\alpha=\frac{3tg\alpha-tg^3\alpha}{1-3tg^2\alpha}}$ lar ham topiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Yarim burchakning trigonometrik funksiyalarini topishda qaysi formulalardan foydalanildi?
2. $\sin\frac{\alpha}{2}$ va $\cos\frac{\alpha}{2}$ larni $\cos\alpha$ orqali topish formulalarini yozing.
3. $\mathrm{tg}\frac{\alpha}{2}$ ni α burchak orqali ifodalash formulalarini yozing.
4. $\sin15°$ va $\cos15°$ lar qanday topiladi?

MASALALARNI YECHING

673. a) $\sin 22°30'$; b) $\cos 22°30'$; d) $\operatorname{tg} 22°30'$ larni hisoblang.

674. a) $\sin 75°$; b) $\cos 75°$; d) $\operatorname{tg} 75°$ larni hisoblang.

675. $\operatorname{tg}\alpha=-\frac{3}{4}$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo'lsa, $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$ va $\operatorname{tg}\frac{\alpha}{2}$ larni toping.

676. Hisoblang:

a) $2\cos22°30'$; b) $2\cos11°15'$.

677. Ifodani soddalashtiring:

a) $\frac{1+\cos\alpha}{\cos\frac{\alpha}{2}}$; b) $\frac{1-\cos2\alpha}{\sin\alpha}$; d) $\frac{1-\cos\alpha}{1+\cos\alpha}$.

678. Ayniyatni isbotlang:

a) $\frac{1-\sin2\alpha}{\cos2\alpha}=\frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$; b) $\frac{\cos2\alpha}{1+\sin2\alpha}=\frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$;

d) $(\sin\alpha+\cos\alpha)^2-(\sin\alpha-\cos\alpha)^2=2\sin\alpha2$;

e) $(\sin\alpha+\cos\alpha)^2-2\sin^2\alpha=\sin2\alpha+\cos2\alpha$.

679. Isbotlang:

a) $\cos20°\cdot\cos70°=\frac{1}{2}\sin40°$; d) $4\sin\alpha\cdot\cos\alpha(\cos^2\alpha-\sin^2\alpha)=\sin4\alpha$;

b) $\cos40°\cdot\sin50°=\frac{1}{2}(1+\sin10°)$; e) $\cos^4\frac{\alpha}{2}-\sin^4\frac{\alpha}{2}=\cos\alpha$.

680. Ayniyatni isbotlang:

a) $\frac{1+\sin2\alpha}{1-\sin2\alpha}=\operatorname{ctg}^2(45°-\alpha)$;

b) $\frac{\sin^2\alpha-\sin^2\beta}{\sin\alpha\cdot\cos\beta-\sin\beta\cdot\cos\alpha}=\sin(\alpha+\beta)$.

91-§. Trigonometrik funksiyalarning ko'paytmasini yig'indi shakliga keltirish formulalari

Ushbu

$$\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta \qquad \text{(I) va}$$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta \qquad \text{(II)}$$

formulalarni hadlab qo'shib, hosil bo'lgan tenglikning chap qismini o'ngga va o'ng qismini chapga ko'chirib 2 ga bo'lamiz:

$$\cos\alpha\cdot\cos\beta=\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

Natijada ikki kosinus ko'paytmasini yig'indi shakliga keltirish formulasini hosil qildik.

Demak, ikki kosinusning ko'paytmasi, argumentlar ayirmasi va yig'indisi kosinuslari yig'indisining yarmiga teng.

(I) tenglikdan (II) tenglikni hadlab ayirsak, sinuslar ko'paytmasini ayirma shakliga keltirish formulasini hosil qilamiz:

$$\sin\alpha\cdot\sin\beta=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$$

Demak, ikki sinusning ko'paytmasi, argumentlar ayirmasi kosinusidan argumentlar yig'indisi kosinusini ayirish natijasining yarmiga teng.

Ushbu

$$\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta \qquad \text{va}$$

$$\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta$$

formulalarni hadlab qo'shib,

$$\sin\alpha\cdot\cos\beta=\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

formulani hosil qilamiz.

Demak, sinus va kosinusning ko'paytmasi, argumentlar yig'indisi va ayirmasi sinuslari yig'indisining yarmiga teng.

1-misol. $\cos2\alpha\cdot\cos4\alpha$ ni yig'indi shakliga keltiramiz.

Yechish. $\cos 4\alpha \cdot \cos 2\alpha = \frac{\cos(4\alpha+2\alpha)+\cos(4\alpha-2\alpha)}{2} = \frac{1}{2}(\cos 6\alpha + \cos 2\alpha)$

2-misol. $\sin^4\alpha \cdot \cos^2\alpha$ ni yig'indi shakliga keltiramiz.

Yechish. $\sin^4\alpha \cdot \cos^2\alpha = \sin^2\alpha(\sin\alpha \cdot \cos\alpha)^2 = \frac{1-\cos 2\alpha}{2} \cdot \frac{\sin^2 2\alpha}{4} = \frac{1}{8}(1-\cos 2\alpha) \times$

$\times \frac{1-\cos 4\alpha}{2} = \frac{1}{16}(1-\cos 2\alpha)(1-\cos 4\alpha) = \frac{1}{16}(1-\cos 2\alpha - \cos 4\alpha + \frac{1}{2}(\cos 6\alpha + \cos 2\alpha) =$

$= \frac{1}{16} - \frac{1}{32}\cos 2\alpha - \frac{1}{16}\cos 4\alpha + \frac{1}{32}\cos 6\alpha.$

TAKRORLASH UCHUN SAVOLLAR

1. Trigonometrik funksiyalarning ko'paytmasini yig'indi shakliga keltirish uchun qaysi formulalar ishlatiladi?
2. Ikki kosinusning ko'paytmasini yig'indi shakliga keltirish formulasini yozing.
3. Ikki sinusning ko'paytmasini yig'indi shakliga keltirish formulasini yozing.
4. Sinus va kosinus ko'paytmasini yig'indi shakliga keltirish formulasini yozing.

MASALALARNI YECHING

681. Ushbu ifodalarning qiymatini jadvaldan foydalanmasdan toping:

a) $\cos 45° \cdot \cos 15°$; b) $\sin 105° \cdot \sin 75°$; d) $\sin\frac{\pi}{24} \cdot \cos\frac{5\pi}{24}$.

682. Ko'paytmalarni yig'indi shakliga keltiring:

a) $\cos 20° \cdot \cos 10°$; e) $2\sin 6° \cdot \sin 24°$;

b) $\sin 20° \cdot \sin 5°$; f) $\cos\frac{\pi}{5} \cdot \sin\frac{\pi}{8}$;

d) $\sin 12° \cdot \cos 48°$; g) $4\sin\frac{\pi}{6} \cdot \cos\frac{\pi}{12}$.

683. Ko'paytmani yig'indiga keltiring.

a) $2\cos\alpha \cdot \cos 3\alpha$; e) $\sin(\alpha-\beta) \cdot \cos(\beta-\alpha)$;

b) $\sin 5\alpha \cdot \sin 3\alpha$; f) $4\cos\alpha \cdot \cos 3\alpha \cdot \cos 4\alpha$;

d) $\cos(\alpha+\beta) \cdot \cos\alpha$; g) $2\sin\alpha \cdot \sin 2\alpha \cdot \sin 3\alpha$.

684. Quyidagilarni 1-darajali trigonometrik funksiyalarning yig'indisiga keltiring:

a) $\sin^2\alpha$; b) $\sin^3\alpha$; d) $\cos^4\alpha$;

e) $\sin^4\alpha$; f) $\cos^2\alpha\cdot\sin^2\alpha$; g) $\cos^3\alpha\cdot\sin^2\alpha$.

92-§. Trigonometrik funksiyalarni yarim argumentning tangensi orqali ifodalovchi formulalar

Argumentni ikkilash formulalarini yozamiz:

$\sin\alpha=2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$ va $\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$

$\sin\alpha=\frac{\sin\alpha}{1}$ va $\cos\alpha=\frac{\cos\alpha}{1}$ deb olib, quyidagicha shakl almashtiramiz: $\sin\alpha=\frac{\sin\alpha}{1}=\frac{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$ bu kasrning surat va maxrajini $\cos^2\frac{\alpha}{2}\neq0$ deb faraz qilib, $\cos^2\frac{\alpha}{2}$ ga bo'lamiz.

$$\sin\alpha=\frac{\dfrac{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\cos2\frac{\alpha}{2}}}{\dfrac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}=\frac{2\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}.\quad \text{Demak,}\quad \sin\alpha=\frac{2\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}.$$

$$\cos\alpha=\frac{\dfrac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}.\quad \text{Demak,}\quad \cos\alpha=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}.$$

$$\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\dfrac{2\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}}{\dfrac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}}=\frac{2\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}^2\frac{\alpha}{2}}.\quad \text{Demak,}\quad \mathrm{tg}\alpha=\frac{2\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}^2\frac{\alpha}{2}}.$$

Misol. Agar $\operatorname{tg}\frac{\alpha}{2}=\frac{1}{2}$ bo'lsa, $\frac{\sin\alpha}{2-3\cos\alpha}$ ning qiymatini topamiz.

Yechish. $$\frac{\sin\alpha}{2-3\cos\alpha}=\frac{\frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}}{2-3\cdot\frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}}=\frac{\frac{2\cdot\frac{1}{2}}{1+\frac{1}{4}}}{2-3\cdot\frac{1-\frac{1}{4}}{1+\frac{1}{4}}}=\frac{\frac{4}{5}}{2-3\cdot\frac{3}{5}}=\frac{\frac{4}{5}}{\frac{1}{5}}=4.$$

Javob: 4.

TAKRORLASH UCHUN SAVOLLAR

1. $\sin 2\alpha$, $\cos 2\alpha$ va $\operatorname{tg}2\alpha$ larni α argumentli funksiyalar orqali ifodalovchi formulalarni yozing.
2. $\sin\alpha$ va $\cos\alpha$ ni $\operatorname{tg}\frac{\alpha}{2}$ orqali ifodalovchi formulalarni yozing.
3. $\operatorname{tg}\alpha$ ni $\operatorname{tg}\frac{\alpha}{2}$ orqali ifodalovchi formulani yozing.
4. $\operatorname{ctg}\alpha$ ni $\operatorname{tg}\frac{\alpha}{2}$ orqali ifodalovchi formulani yozing.

MASALALARNI YECHING

685. Agar $\operatorname{tg}\frac{\alpha}{2}=\frac{2}{3}$ bo'lsa, $\sin\alpha$, $\cos\alpha$ va $\operatorname{tg}\alpha$ larni toping.

686. Agar $\operatorname{tg}\frac{\alpha}{2}=3$ bo'lsa, a) $\frac{12}{\sin\alpha}$; b) $-\frac{8}{\cos\alpha}$ larni toping.

687. Agar $\operatorname{tg}\frac{\alpha}{2}=\frac{2}{15}$ bo'lsa, $\frac{\operatorname{tg}\alpha-\sin\alpha}{\operatorname{tg}\alpha+\sin\alpha}$ ifodaning qiymatini toping.

688. Tenglikning to'g'ri ekanligini isbotlang:

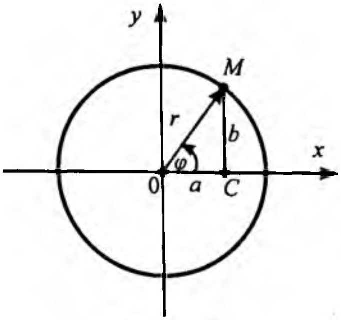
a) $\frac{2}{\operatorname{ctg}\frac{\alpha}{2}+\operatorname{tg}\frac{\alpha}{2}}=\sin\alpha$; b) $\frac{\operatorname{ctg}\frac{\alpha}{2}-\operatorname{tg}\frac{\alpha}{2}}{\operatorname{ctg}\frac{\alpha}{2}+\operatorname{tg}\frac{\alpha}{2}}=\cos\alpha$;

d) $\frac{\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}\frac{\alpha}{2}}+\frac{\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}\frac{\alpha}{2}}=\operatorname{tg}\alpha$; e) $\frac{1}{1-\operatorname{tg}\frac{\alpha}{2}}-\frac{1}{1+\operatorname{tg}\frac{\alpha}{2}}=\operatorname{tg}\alpha$.

93-§. $a\sin\alpha + b\cos\alpha$ ifodani ko‘paytmaga aylantirish

a va b nolga teng emas deb faraz qilamiz. Tekislikda absissasi a ga va ordinatasi b ga teng bo‘lgan $M(a;\ b)$ nuqta olamiz. $\overline{OM}$ radius vektorning uzunligi $r=\sqrt{a^2+b^2}$ bo‘ladi.

$\overline{OM}$ bilan absissa o‘qi orasida hosil bo‘lgan φ burchakning kosinusi va sinusi $\cos\varphi=\frac{a}{\sqrt{a^2+b^2}}$; $\sin\varphi=\frac{b}{\sqrt{a^2+b^2}}$; $\operatorname{tg}\varphi=\frac{b}{a}$ (68-chizma).



68-chizma.

Berilgan ifodani quyidagicha shakl almashtiramiz:

$$a\sin\alpha+b\cos\alpha=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin\alpha+\right.$$

$$\left.+\frac{b}{\sqrt{a^2+b^2}}\cos\alpha\right)=\sqrt{a^2+b^2}\sin(\alpha+\varphi).$$

Demak, $\boldsymbol{a\sin\alpha+b\cos\varphi=\sqrt{a^2+b^2}\sin(\alpha+\varphi)}$.

Bunda φ – yordamchi burchak.

1-misol. $\sin\alpha+\cos\alpha$ ni ko‘paytmaga almashtiramiz.

Bunda: $a=1$; $b_1=1$; $\sqrt{a^2+b^2}=\sqrt{1+1}=\sqrt{2}$, $\frac{\sqrt{2}}{2}=\sin\frac{\pi}{4}=\cos\frac{\pi}{4}$.

$$\sin\alpha+\cos\alpha=\sqrt{2}\left(\sin\alpha\cdot\frac{\sqrt{2}}{2}+\cos\alpha\frac{\sqrt{2}}{2}\right)=$$

$$=\sqrt{2}\left(\sin\alpha\cdot\cos\frac{\pi}{4}+\cos\alpha\cdot\sin\frac{\pi}{4}\right)=\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right).$$

Demak, $\sin\alpha+\cos\alpha=\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)$.

2-misol. $5\sin\alpha-4\cos\alpha$ ni ko‘paytmaga almashtiramiz.

Bunda: $a=5$; $b=-4$; $r=\sqrt{a^2+b^2}=\sqrt{5^2+(-4)^2}=\sqrt{41}$;

$$\sin\varphi=\frac{b}{\sqrt{a^2+b^2}}=\frac{-4}{\sqrt{41}};\quad \cos\varphi=\frac{a}{\sqrt{a^2+b^2}}=\frac{5}{\sqrt{41}};$$

$$5\sin\alpha - 4\cos\varphi = \sqrt{41}\left(\sin\alpha\cdot\frac{5}{\sqrt{41}} - \cos\alpha\cdot\frac{4}{\sqrt{41}}\right) = \sqrt{41}\sin(\alpha-\varphi).$$

Bunda: $\cos\varphi = \frac{5}{\sqrt{41}} = \frac{5}{6{,}403} = 0{,}781$. $\varphi = \arccos 0{,}781 \approx 51°21'$.

Demak, $5\sin\alpha - 4\cos 2 \approx 6{,}403\sin(\alpha - 51°21')$

TAKRORLASH UCHUN SAVOLLAR

1. Tekislikda absissasi a va ordinatasi b bo'lgan $\overline{OM}$ vektorni absissa o'qi bilan hosil qilgan φ burchakni yasang. $\overline{OM}$ uzunligini toping.
2. Shu φ burchakning sinusi va kosinusini yozing.
3. $a\sin\alpha + b\cos\alpha$ ni ko'paytma ko'rinishida yozing.

MASALALARNI YECHING

689. Quyidagilarni ko'paytma shakliga keltiring:

a) $2\sin\alpha + \cos\alpha$;
b) $\sin\alpha - \sqrt{3}\cos\alpha$;
d) $2\cos\alpha - \sqrt{2}\sin\alpha$;
e) $\sqrt{3}\sin\alpha + \cos\alpha$;
f) $3\sin\alpha - \sqrt{3}\cos\alpha$;
g) $3\sin\alpha + 5\cos\alpha$.

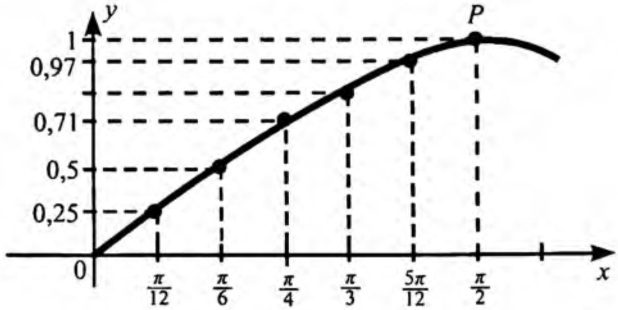
94-§. Trigonometrik funksiyalarning grafiklari

1. $y=\sin x$ funksiyaning grafigi

Trigonometrik funksiya argumenti α ni x bilan belgilab, $y=\sin x$ funksiya grafigini chizamiz. $y=\sin x$ funksiya grafigini chizamiz. $y=\sin x$ funksiya $0 \le x \le \frac{\pi}{2}$ oraliqda 0 dan 1 gacha ortadi. Demak, chiziq $O(0;\ 0)$ dan $P\left(\frac{\pi}{2};\ 1\right)$ nuqtagacha yuqoriga ko'tariladi (69-chizma).

Grafikning $\left[0;\ \frac{\pi}{2}\right]$ oraliqdagi nuqtalarini yasash uchun, sinus qiymatlarining quyidagi jadvalini tuzamiz.

x	0	$\frac{\pi}{12}$ (15°)	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{5\pi}{12}$ (75°)	$\frac{\pi}{2}$ (90°)
$y=\sin x$	0	$\approx 0{,}26$	$\approx 0{,}5$	$\approx 0{,}71$	$\approx 0{,}87$	$\approx 0{,}97$	1



69-chizma.

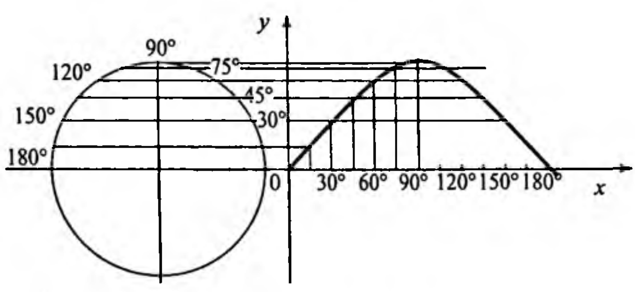
Sinusning qiymatlari jadvaldan ikkita kasr xonasi aniqligida olingan.

$y=\sin x$ funksiyaning grafigini istalgan aniqlikda geometrik usulda yasash mumkin.

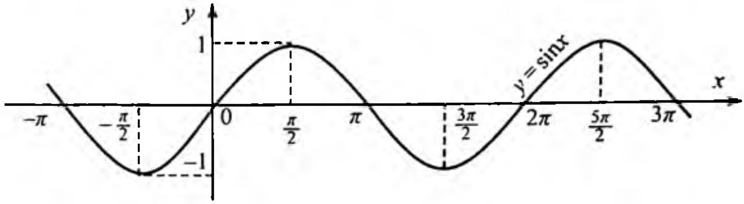
Birlik aylana olib, uning yuqorigi va absissa o'qining yarim aylanaga teng uzunlikdagi $0\leq x\leq\pi$ kesmasi bir xil miqdorlarda teng bo'laklarga bo'lingan (70-chizmada 12 bo'lakka bo'lingan).

Aylananing bo'linish nuqtalaridan OX o'qiga parallel qilib o'tkazilgan to'g'ri chiziqlar absissa o'qining $0\leq x\leq\pi$ oraliqdagi bo'linish nuqtalaridan o'tkazilgan perpendikulyarlar bilan kesishib sinus funksiyaning grafigiga tegishli bo'lgan nuqtalari hosil qiladi. Bu nuqtalar orqali lekalo yordamida tekis egri chiziq hosil qilinadi.

Bu egri chiziq $y=\sin x$ funksiyaning $0\leq x\leq\pi$ oraliqda chizilgan grafigi bo'ladi. Bu egri chiziqni **sinusoida** deyiladi. Sinus toq funksiya bo'lgani uchun uning grafigi koordinatalar boshiga nisbatan simmetrik joylashadi. Uning $-\pi\leq x\leq 0$ oraliqdagi grafigi $0\leq x\leq\pi$ oraliqdagi grafigiga simmetrik bo'lib joylashadi. Natijada $-\pi\leq x\leq\pi$ oraliq uzunligi jihatidan sinusning davriga teng.



70-chizma.



71-chizma.

Shu sababli sinusoidaning hamma soni yasash uchun uning $-\pi \le x \le \pi$ oraliq uchun yasalgan qismining ketma-ket o'ngga va chapga 2π; 4π; 6π; ... masofaga surib yasaladi (71-chizma).

$y=\sin x$ funksiya $\left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$ oraliqda −1 dan 1 gacha o'sadi, $\left(\frac{\pi}{2};\ \frac{3\pi}{2}\right)$ oraliqda 1 dan −1 gacha kamayadi.

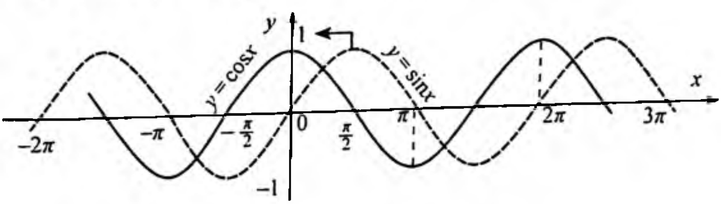
2. $y=\cos x$ funksiyaning grafigi

Keltirish formulasiga muvofiq $y=\cos x=\sin\left(\frac{\pi}{2}+x\right)$. Bu formuladan kosinus grafigining x absissali nuqtadagi ordinatasi sinusoidaning $x+\frac{\pi}{2}$ absissali nuqtasidagi ordinatasi bilan bir xil ekanligi kelib chiqadi. Masalan, $x=0$ da $y=\cos 0=\sin\left(0+\frac{\pi}{2}\right)=1$ va $x=\frac{\pi}{4}$ da

$y=\cos\frac{\pi}{4}=\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$ va hokazo.

$y=\sin\left(\frac{\pi}{2}+x\right)=\cos x$ bo'lgani uchun $y=\cos x$ funksiyaning grafigi $y=\sin x$ funksiya grafigini absissa o'qi bo'ylab $\frac{\pi}{2}$ masofa chapga ko'chirishdan hosil qilinadi (72-chizma).

$y=\cos x$ funksiyaning grafigini **kosinusoida** deyiladi.



72-chizma.

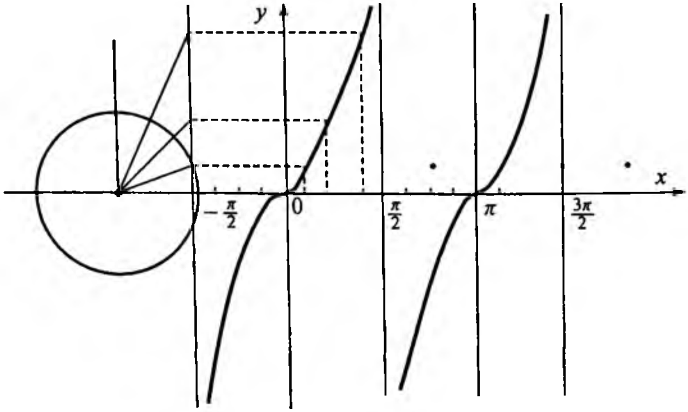
$y=\cos x$ funksiya $(0;\ \pi)$ oraliqda 1 dan -1 gacha kamayadi, $(\pi;\ 2\pi)$ oraliqda -1 dan 1 gacha o'sadi.

3. $y=\operatorname{tg}x$ funksiyaning grafigi

Tangens funksiya $0\leq x\leq\frac{\pi}{2}$ oraliqda 0 dan ga ∞ cha ortib boradi. $0\leq x\leq\frac{\pi}{2}$ oraliqda $y=\operatorname{tg}x$ funksiyaning geometrik usulda yasalishini ko'rib chiqamiz.

Birlik aylananing birinchi choragi va absissa o'qining 0 dan $\frac{\pi}{2}$ gacha kesmasi bir necha teng bo'lakka (73-chizmada 4 bo'lakka) bo'linmagan.

Aylananing bo'linishi nuqtalari doira markazidan tangens o'qigacha birlashtiriladi, so'ngra tangens o'qining kesmalari, absissa o'qidagi mos nuqtalardan o'tkazilgan perpendikulyarlar bilan kesishgan nuqtalar olinadi. Bu nuqtalar tekis egri chiziq bilan birlashtiriladi. Bu egri chiziq $y=\operatorname{tg}x$ funksiyaning $0\leq x\leq\frac{\pi}{2}$ oraliqdagi grafigi bo'ladi. Tangens funksiyaning toqlik xossasidan uning grafigi koordinatalar boshiga nisbatan simmetrik ekanligi kelib chiqadi.



73-chizma.

Shuning uchun $0 \le x \le \frac{\pi}{2}$ oraliqdagi grafik simmetriyadan foydalanib, uni $-\frac{\pi}{2} \le x \le 0$ oraliqqa (IV chorakka) o'tkaziladi.

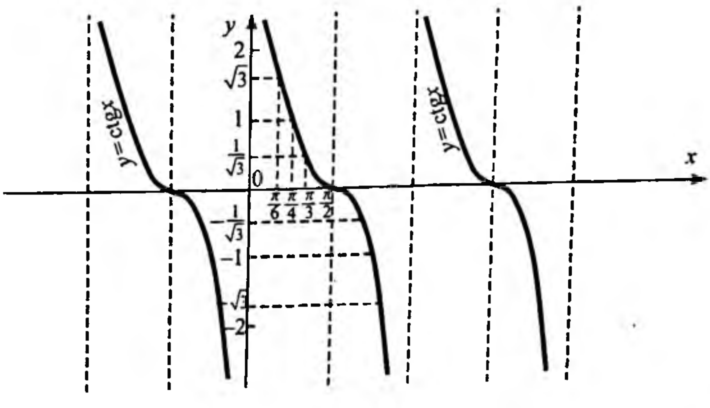
$\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ oraliq o'zining uzunligi jihatidan tangensning davriga teng bo'lgani uchun shu oraliqdagi grafik $y = \mathrm{tg}x$ funksiyaning grafigi bo'ladi. Bu grafikni π; 2π; 3π; ... masofa ko'chirib tangensning barcha grafiklari yasaladi. Bu grafikni **tangensoida** deyiladi.

4. $y = \mathrm{ctg}x$ funksiyaning grafigi

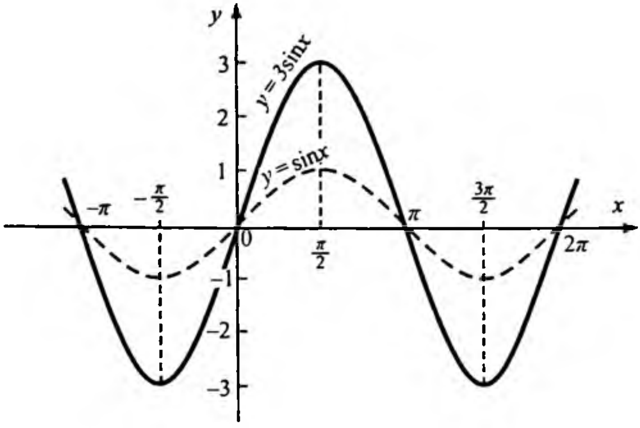
$y = \mathrm{ctg}x$ funksiyaning grafigi 72-chizmada tasvirlangan. Grafikni yasash uchun uning umumiy davri $k\pi$ ekanligini va funksiya $0 < x < \pi$ oraliqda ∞ dan $-\infty$ gacha kamayishini esda tutish yetarli. Ayrim nuqtalarni yasash uchun qiymatlar jadvalini tuzib olamiz.

x	$0 = 0°$	$\frac{\pi}{6} = 30°$	$\frac{\pi}{4} = 45°$	$\frac{\pi}{3} = 60°$	$\frac{\pi}{2} = 90°$	$\frac{2\pi}{3} = 120°$	$\frac{3\pi}{4} = 135°$	$\frac{5\pi}{6} = 150°$	$\pi = 180°$
ctg$x$	$+\infty$	$\sqrt{3} = 1,73$	$\frac{\pi}{4} = 1$	$\frac{\sqrt{3}}{3} = 0,57$	0	$-\frac{\sqrt{3}}{3} = -0,57$	-1	$-\sqrt{3} = -1,73$	$-\infty$

$y = \mathrm{ctg}x$ funksiya grafigini keltirish formulasiga asosan $\mathrm{ctg}x = -\mathrm{tg}\left(x + \frac{\pi}{2}\right)$ yozib, tangens grafigini OX o'qi bo'ylab $\frac{\pi}{2}$ birlik chapga surish va uni shu o'qqa simmetrik qilib tasvirlashdan hosil qilish mumkin.



74-chizma.



75-chizma.

1-m i s o l. $y=3\sin x$ funksiyaning grafigini yasaymiz.

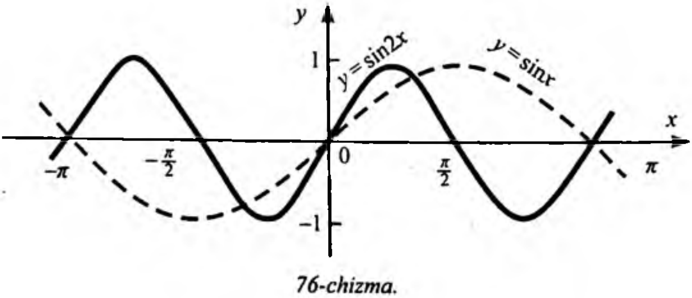
Yechish. Buni yasash uchun x ning berilgan qiymatida $y=3\sin x$ grafikning ordinatasi oddiy sinusoida ordinatasining uchlanganiga teng bo'ladi. Bu grafik oddiy sinusoida ordinatalarining har birini uch martadan uzaytirish yo'li bilan hosil qilinadi (75-chizma).

$y=3\sin x$ funksiya ham, $y=\sin x$ funksiyadek o'zgarmas ishora oraliqqa va 2π davrga ega. ± 3 – uning eng katta va eng kichik qiymati.

2-m i s o l. $y=\sin 2x$ funksiyaning grafigini yasaymiz.

Yechish. $y=\sin 2x$ funksiyaning eng kichik davri 2π bo'lsa, $y=\sin 2x$ funksiyaning eng kichik musbat davri π ga teng, chunki $2x=2\pi$ dan $x=\pi$ ga teng.

Demak, $y=\sin 2x$ ning grafigi $y=\sin x$ funksiya grafigini absissa o'qi bo'ylab ikki marta «siqish» yo'li bilan hosil qilish mumkin (76-chizma).



76-chizma.

TAKRORLASH UCHUN SAVOLLAR

1. $y=\sin x$ funksiya qaysi oraliqda o'sadi va qaysi oraliqda kamayadi?
2. $y=\cos x$ funksiya qaysi oraliqda o'sadi va qaysi oraliqda kamayadi?
3. $y=\sin x$ funksiyaning grafigi qanday egri chiziq bo'ladi?
4. $y=\cos x$ funksiyaning grafigi qanday egri chiziq bo'ladi?
5. $y=\mathrm{tg}x$ funksiyaning grafigi qanday egri chiziq bo'ladi?
6. $y=\mathrm{ctg}x$ funksiyaning grafigi qaysi oraliqda kamayadi va grafigi qanday chiziq bo'ladi?

MASALALARNI YECHING

690. $y=\sin x$ funksiyaning grafigidan foydalanib, $\sin\frac{\pi}{4}$, $\sin\frac{\pi}{8}$; $\sin 2$; $\sin(-3)$ larni toping.

691. $y=\cos x$ funksiyaning grafigidan foydalanib, $\cos\frac{\pi}{3}$, $\cos\frac{\pi}{6}$, $\cos 3$ va $\cos(-4)$ larni toping.

692. $y=\sin x$ va $y=\cos x$ grafiklaridan foydalanib,

a) $\sin 10°$ va $\cos 10°$; b) $\sin(-20°)$ va $\cos 20°$; d) $\sin 0,7$ va $\cos 0,7$ qiymatlarini toping.

693. $y=\mathrm{tg}x$ va $y=\mathrm{ctg}x$ funksiyalarning grafiklaridan foydalanib:

a) $\mathrm{tg}\frac{\pi}{8}$ va $\mathrm{ctg}\frac{\pi}{8}$; b) $\mathrm{tg}0,6$; $\mathrm{ctg}0,6$; d) tg va ctg larni toping.

694. $y=\sin x$, $y=\cos x$, $y=\mathrm{tg}x$ va $y=\mathrm{ctg}x$ funksiyalarning grafiklaridan foydalanib, tenglamalarning boshlang'ich ildizlarini toping:

a) $\sin x=\frac{1}{3}$; $\cos x=-\frac{1}{3}$; b) $\mathrm{tg}x=-3$; $\mathrm{ctg}x=3$.

d) $\sin x=\frac{1}{\sqrt{3}}$; $\cos x=-\frac{1}{\sqrt{3}}$; $\mathrm{tg}x=-\frac{4}{\sqrt{5}}$.

Quyidagi funksiyalarning grafiklarini bitta koordinata tekisligida chizing:

695. $y=\sin\left(x+\frac{\pi}{3}\right)$ va $y=\sin\left(x-\frac{\pi}{3}\right)$.

696. $y=\sin\left(2x-\frac{\pi}{6}\right)$ va $y=\sin\left(2x+\frac{\pi}{6}\right)$.

697. $y=2\cos(x+\pi)$ va $y=-2\cos x$;

698. $y=0{,}5\sin\left(2x-\frac{\pi}{4}\right)$ va $y=-0{,}5\sin\left(2x+\frac{\pi}{4}\right)$.

MATEMATIKA (arifmetika, algebra) TESTLARIDAN NAMUNALAR

1. Ushbu 31 32 33...7980 sonda nechta raqam bor?
A) 96; B) 98; C) 100; D) 112.

2. Ushbu 313233... 6970 sonning raqamlari yig‘indisini toping.
A) 360; B) 364; C) 480; D) 500.

3. m raqamining qanday qiymatlarida 2134 m soni 4 ga qoldiqsiz bo‘linadi.
A) 6 va 4; B) 9 va 8; C) 4 va 8; D) 0,4 va 8.

4. $357+2x^2$ yig‘indi x raqamining qanday qiymatlarida 3 ga qoldiqsiz bo‘linadi?
A) 2, 6, 9; B) 2, 5, 9; C) 2, 5, 8; D) 3, 6, 9.

5. 312 va 264 larning umumiy bo‘luvchilari nechta?
A) 5; B) 6; C) 8; D) 7.

6. 9, 10, 22 va 25 sonlarning o‘zaro tublari nechta?
A) 6; B) 5; C) 4; D) 3.

7. Ikki sonning nisbati 11:13 kabi, ularning eng katta umumiy bo‘luvchisi 7 ga teng. Bu sonlarning yig‘indisini toping.
A) 161; B) 168; C) 175; D) 176.

8. $2^{20}+3^{20}$ yig‘indining oxirgi raqamini toping.
A) 5; B) 6; C) 7; D) 11.

9. 3^{2015} daraja qanday raqam bilan tugaydi?
A) 1; B) 3; C) 7; D) 9.

10. $4-7+8-11+...+96-99$ yig‘indini hisoblang.
A) –75; B) –72; C) –80; D) –60.

11. Ifodani hisoblang.

$5\frac{5}{7}\cdot\frac{3}{8}+5\frac{1}{4}:2\frac{1}{3}+5\frac{17}{28}.$

A) $4\frac{3}{28}$; B) $8\frac{11}{28}$; C) 10; D) 12.

12. $\left(\frac{1}{8}+\frac{1}{24}+\frac{1}{48}+\frac{1}{80}\right):\frac{1}{15}$ ni hisoblang.

A) 3; B) $2\frac{1}{5}$; C) $5\frac{1}{2}$; D) $3\frac{3}{4}$.

13. $\frac{100^5}{(80+20)^{10}}\cdot 50^5$ ni hisoblang.

A) 8; B) 16; C) $\frac{1}{32}$; D) $\frac{1}{64}$.

14. $\frac{3}{17}$; $\frac{8}{13}$ va $\frac{16}{19}$ sonlarga bo‘linganda bo‘linmalar eng kichik natural sonlar chiqadigan sonni toping.

A) 24; B) 36; C) 42; D) 48.

15. $\frac{1}{10\cdot13}+\frac{1}{13\cdot16}+...+\frac{1}{67\cdot70}$ yig‘indini hisoblang.

A) $\frac{3}{70}$; B) $\frac{1}{35}$; C) $\frac{3}{35}$; D) $\frac{3}{75}$.

16. $2+\frac{1}{10\cdot11}+\frac{1}{11\cdot12}+...+\frac{1}{19\cdot20}$ yig‘indini hisoblang.

A) $2\frac{3}{40}$; B) $2\frac{1}{10}$; C) $2\frac{3}{10}$; D) $2\frac{1}{20}$.

17. Agar $\frac{11}{13}+\frac{24}{29}+\frac{43}{51}+\frac{47}{59}=c$ bo‘lsa, $\frac{2}{13}+\frac{5}{29}+\frac{8}{51}+\frac{12}{59}$ ni hisoblang.

A) $2-s$; B) $4-c$; C) $4{,}5-c$; D) $10-c$.

18. $\frac{6}{5\cdot7}+\frac{6}{7\cdot9}+\frac{6}{9\cdot11}+...+\frac{6}{73\cdot75}$ ni hisoblang.

A) $\frac{14}{25}$; B) $\frac{16}{25}$; C) $\frac{12}{75}$; D) $\frac{28}{75}$.

19. $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{182}$ ni hisoblang.

A) $\frac{1}{4}$; B) $\frac{10}{33}$; C) $\frac{11}{42}$; D) $\frac{15}{56}$.

20. $\frac{3^9\cdot 2^{19}+15\cdot 4^9\cdot 9^4}{6^9\cdot 2^{10}+12^{10}}\cdot\left(1\frac{1}{2}\right)^{-1}$ ni hisoblang.

A) 1; B) $\frac{1}{2}$; C) $\frac{2}{3}$; D) $\frac{1}{3}$.

21. $32\cdot 0{,}99\cdot 25\cdot 1{,}25+411+57\cdot 5\cdot 0{,}4\cdot 25\cdot\frac{4}{19}$ ni hisoblang.

A) 2000; B) 2001; C) 2012; D) 2021.

22. $\frac{0{,}13}{0{,}0013}+\frac{0{,}02}{0{,}0005}+\frac{0{,}7}{0{,}0014}-\frac{0{,}42}{0{,}003}$ ni hisoblang.

A) 340; B) 380; C) 500; D) 540.

23. Qaysi juft kasrlarning maxrajlarini eng kichik karralisi kichik bo'ladi?

1) $\frac{7}{20}$; 2) $\frac{11}{160}$; 3) $\frac{5}{48}$; 4) $\frac{7}{32}$.

A) 2 va 3; B) 1 va 4; C) 1 va 3; D) 3 va 4.

24. $2,(7)+2,7(6)$ ni hisoblang.

A) $5\frac{39}{90}$; B) $5\frac{43}{99}$; C) $5\frac{49}{90}$; D) $6\frac{49}{90}$.

25. $\left(1\frac{3}{4}:1{,}125+1{,}75:0,(6)\right)\cdot 1\frac{5}{7}+2{,}8\ (3)$.

A) $2\frac{1}{7}$; B) $2\frac{2}{7}$; C) $6\frac{2}{3}$; D) 10.

26. $\frac{1\frac{2}{13}\cdot 0{,}4(3)+2:1,(3)}{\frac{3}{8}+0{,}125}-\sqrt{\sqrt{256}}$ ni hisoblang.

A) 2; B) 0,4; C) 0; D) 0,2.

27. Kitob betlarini sahifalash uchun 531 ta raqam kerak bo'ladi. Bu kitob necha betlik bo'ladi?

A) 203; B) 212; C) 213; D) 223.

28. $(x^2+9)^2-36x^2$ ni ko'paytuvchilarga ajrating.

A) $(x-3)^2(x+3)^2$; B) $(x^2-3)(x^2+3)$;
C) $(x^2-9)(x^2+9)$; D) $x^2(x^2-6)$.

29. $x^3-3x^2-4x+12$ ko‘phad quyidagilardan qaysilariga bo‘linmaydi?

A) $x-3$; B) $x+2$; C) x^2-x-6; D) $x+3$.

30. $(a+b+c)(ab+ac+bc)-abc$ ni ko‘paytuvchilarga ajrating.

A) $(a+b)(b+c)(b-c)$; B) $(a+b)(a+c)(b+c)$;
C) $(a+b)(a-c)(b+c)$; D) $(a-b)(b+c)(a-c)$.

31. $\frac{n^3+7n^2+6n}{n^3-n}$ kasrni qisqartiring.

A) $n+6$; B) $n-1$; C) $\frac{n+6}{n-1}$; D) $\frac{n+1}{n+6}$.

32. $\frac{x^3+y^3}{x^2-xy+y^2}-\frac{x^2-y^2}{x+y}$ ni soddalashtiring.

A) $2y$; B) $2x$; C) $-2x$; D) $2x-2y$.

33. Ushbu $\frac{a^3+b^3}{a^2-ab+b^2}\cdot(a-b)\cdot\frac{a^3-b^3}{a^2+ab+b^2}\cdot(a+b)$ ning $a=\sqrt{8}$ va $b=\sqrt{2}$ bo‘lgandagi qiymatini hisoblang.

A) 30; B) 32; C) 34; D) 36.

34. $\frac{12-3n}{n}$ ifoda n ning nechta natural qiymatlarida natural son bo‘ladi.

A) 4 ta; B) 3 ta; C) 2 ta; D) 6 ta.

35. $(3z-x)^3+(x-2y)^3-(3z-2y)^3$ ni ko‘paytuvchilarga ajrating.

A) $3(3z-x)(x-2y)(3z-2y)$; B) Ko‘paytuvchilarga ajralmaydi;
C) $-3(3x-2y)(3z-x)(x-2y)$; D) $3(3z-2y)(3z-x)(x-2y)$.

36. $\frac{m^4-16}{m^4+2m^3+4m^2+8m}$ kasrni qisqartiring.

A) $(m-2)(m+2)^{-1}$; B) $(m-2)m^{-1}$;
C) $(m-2)(m+2)^{-1}$; D) $(m+2)m^{-1}$.

37. $\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{3}{\sqrt{6}-\sqrt{3}}-\frac{4}{\sqrt{7}+\sqrt{3}}$ ni hisoblang.

A) 0; B) -1; C) $\sqrt{3}$; D) $-\sqrt{7}$.

38. Ayirmaning qiymatini toping $\sqrt{9-2\sqrt{20}}-\sqrt{9+2\sqrt{20}}$.

A) -5; B) -6; C) -2; D) -4.

39. Yig'indining qiymatini toping $\sqrt{11+6\sqrt{2}}+\sqrt{11-6\sqrt{2}}$

A) $4\sqrt{2}$; B) 5; C) 6; D) 8.

40. $\sqrt{\sqrt{17-12\sqrt{2}}}$ ni hisoblang.

A) $3-2\sqrt{2}$; B) $2-\sqrt{2}$; C) $2\sqrt{2}-1$; D) $\sqrt{2}-1$.

Ko'rsatma. $\sqrt{\sqrt{28-16\sqrt{3}}}$ ni hisoblaymiz.

$$\sqrt{\sqrt{28-16\sqrt{3}}}=\sqrt{\sqrt{4^2-2\cdot4\cdot2\sqrt{3}+(2\sqrt{3})^2}}=\sqrt{\sqrt{(4-2\sqrt{3})^2}}=$$

$$=\sqrt{4-2\sqrt{3}}=\sqrt{\left(\sqrt{3}\right)^2-2\sqrt{3}+1}=\sqrt{\left(\sqrt{3}-1\right)^2}=\sqrt{3}-1.$$

41. $1998x^2-2000x+2=0$ tenglamani yeching.

A) -1 va $\frac{1}{999}$; B) $\frac{1}{999}$ va 1; C) -1 va $\frac{1}{999}$; D) -1 va 1.

42. $x^2+px+q=0$ tenglamaning ildizlari $x^2-3x-10=0$ ning ildizlaridan ikki marta katta. $p+q$ yig'indini toping.

A) 2; B) -7; C) -40; D) -46.

43. Ildizlari $4-\sqrt{7}$ va $4+\sqrt{7}$ bo'lgan kvadrat tenglamani tuzing.

A) $x^2+8x+9=0$; B) $x^2+8x-9=0$;
C) $x^2-8x+9=0$; D) $x^2-8x-9=0$.

44. $\frac{x^2-x-2}{x^2+x}$ tenglama nechta ildizga ega?

A) 1 ta; B) 2 ta; C) 3 ta; D) mavjud emas.

45. Agar $\frac{x^2-y^2}{x-y}:\left(\frac{x^4+xy^3}{x^2+xy}+3xy\right)$ bo'lsa, ifodani $x=-6,37$ va $y=-3,63$ dagi qiymatini toping:

A) -1; B) $-0,1$; C) 0,1; D) 1.

46. p ning qanday qiymatlarida $4x-7p=5$ tenglama manfiy ildizga ega?

A) $p<\frac{5}{7}$; B) $p>\frac{5}{7}$; C) $p<-\frac{5}{7}$; D) $p>-\frac{5}{7}$.

47. $5x^2+bx-28=0$ tenglamaning ildizlari x_1 va x_2 uchun $5x_1+2x_2=1$ munosabat o'rinli. Agar b butun son ekanligi ma'lum bo'lsa, uning qiymatini toping.

A) 9 va 13; B) –13; C) 13; D) –9.

48. Agar $\begin{cases} x^3+2x^2y+xy^2-x-y=2 \\ y^3+2xy^2+x^2y+x+y=6 \end{cases}$ bo'lsa, $x+y$ ning qiymatini toping.

A) 1; B) 2; C) –1; D) –2.

49. $\begin{cases} x^2+y^2=16 \\ y-x=4 \end{cases}$ tenglamalar sistemasi nechta yechimga ega.

A) 2 ta; B) 1 ta; C) 3 ta; D) yechimi yo'q.

50. Agar $\begin{cases} x^2+y^2+xy=7 \\ x+y=3 \end{cases}$ bo'lsa, $2xy$ ning qiymatini toping.

A) 1; B) 3; C) 4; D) –4.

51. Agar $x+y=\sqrt{2+\sqrt{31}}$ va $xy=1$ bo'lsa, x^5y+xy^5 ning qiymatini toping.

A) 18; B) 29; C) 31; D) 51.

52. $5<x<109$ tengsizlikni qanoatlantiruvchi, 12 ga karrali nechta natural son mavjud.

A) 9; B) 10; C) 11; D) 12.

53. Quyida keltirilgan tengsizliklardan qaysi biri $3x-a>b-2x$ tengsizlikka teng kuchli emas.

A) $5x-a>b$; B) $a-3x<2x-b$;
C) $5x>a+b$; D) $3x<a+b-2x$.

54. $\begin{cases} 5x+1\geq 3\left(x-2\frac{1}{2}\right) \\ 12-3x\geq 11 \end{cases}$ tengsizliklar sistemasi nechta butun yechimga ega?

A) 4 ta; B) 5 ta; C) 3 ta; D) mavjud emas.

55. k ning qanday qiymatlarida $\frac{4x-1}{x-1}=-k+2$ tenglama manfiy yechimga ega bo'ladi?

A) $(1;\ \infty)$; B) $(-1;\ 2)$; C) $(-2;\ 1)$; D) $(-\infty;\ -2)$.

56. $\frac{(x-4)(x+2)}{(x-1)^2}<0$ tengsizlikning eng katta va eng kichik butun yechimlari ayirmasini toping.

A) 4; B) 5; C) 6; D) 8.

57. $\frac{(-x^2+x-1)(x^2+x-2)}{x^2-7x+12}\geq 0$ tengsizlikning butun yechimlari nechta?

A) 3 ta; B) 4 ta; C) 5 ta; D) cheksiz ko'p.

58. $y=\frac{\sqrt{x^2-x-30}}{\sqrt{|x^2-x-42|}}$ funksiyaning aniqlanish sohasini toping.

A) $(-\infty;\ -5]\cup(6;\ \infty)$; B) $(-\infty;\ -6]\cup(-6;\ 7)$;

C) $(-\infty;\ -6)\cup(-6;\ -5]\cup[6;\ 7)\cup(7;\ \infty)$; D) $(-\infty;\ -5]\cup(6;\ \infty)$.

59. Agar $|x-2|+3x=6$ bo'lsa, $|x|$ ni toping.

A) 6; B) 4; C) 3; D) 2.

60. $\frac{(2|x|-3)^2-|x|-6}{4x+1}=0$ tenglama ildizlarining ko'paytmasini toping.

A) $\frac{9}{16}$; B) $-\frac{9}{16}$; C) $\frac{3}{4}$; D) $\frac{3}{16}$.

61. $\sqrt[3]{x^3+19}=x+1$ tenglama katta ildizining eng kichik ildiziga nisbatini toping.

A) $-\frac{2}{3}$; B) $\frac{2}{3}$; C) $-\frac{1}{2}$; D) $\frac{2}{3}$.

62. $\sqrt{25-x^2}+\sqrt{15-x^2}=5$ bo'lsa, $\sqrt{25-x^2}-\sqrt{15-x^2}$ ning qiymatini toping.

A) 5; B) 3; C) 2; D) 10.

63. $\sqrt{\sqrt{11x^2+1}-2x}=1-x$ tenglamaning turli ildizlari sonini aniqlang.

A) 0; B) 1; C) 3; D) 4.

64. $(x+3)\sqrt{x^2-x-2}\geq 0$ tengsizlikning yechimini toping.

A) $[-3;-1]\cup[2;\infty)$; B) $[-3;\infty)$; C) $[-1;2]$; D) $(-\infty;-1)$.

65. $x(x^2+4x+4)\sqrt{25-x^2}\geq 0$ tengsizlikning butun sonlardan iborat yechimlari yig'indisini toping.

A) –5; B) 5; C) 3; D) 0.

66. Agar arifmetik progressiyada $a_2+a_5-a_3=10$ va $a_1+a_6=17$ bo'lsa, uning o'ninchi hadini toping.

A) 20; B) 28; C) 24; D) 32.

67. 7 ta sonning o'rta arifmetigi 13 ga teng. Bu sonlarga qaysi son qo'shilsa, ularning o'rta arifmetigi 18 bo'ladi?

A) 53; B) 50; C) 58; D) 61.

68. $\frac{1}{100}+\frac{2}{100}+...+\frac{N}{100}=100N$ tenglikni qanoatlantiruvchi natural N sonni toping.

A) 21999; B) 20001; C) 19999; D) 19991.

69. Arifmetik progressiyaning o'nta va o'ttizta hadlarining yig'indisi $S_{10}=100$ va $S_{30}=900$ bo'lsa?

A) 1400; B) 1500; C) 1600; D) 1800.

70. O'suvchi geometrik progressiyaning birinchi hadi 3 ga, yettinchi va to'rtinchi hadlarining ayirmasi 168 ga teng. Shu progressiyaning maxrajini toping.

A) $\frac{3}{2}$; B) 2; C) 3; D) $2\sqrt{2}$.

71. Geometrik progressiyada $S_n-S_{n-1}=64$ va $S_{n+1}-S_n=128$ bo'lsa, uning maxrajini toping.

A) –3; B) –2; C) 2; D) 3..

72. $1-3x+9x^2-...-3^9x^9=0$ tenglamani yeching.

A) $\pm\frac{1}{3}$; B) $\frac{1}{3}$; C) $-\frac{1}{3}$; D) ±1.

73. $\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{9}+\frac{1}{8}\cdot\frac{1}{27}-$ ... cheksiz kamayuvchi geometrik progressiyaning yig'indisini toping.

A) 1,2; B) 0,75; C) 0,2; D) 0,5.

74. Hadlari $b_n=3n-10,5(n\in N)$ formula bilan berilgan ketma-ketlikning dastlabki 40 ta hadining yig'indisini toping.

A) 2020; B) 2040; C) 2440; D) 2420.

75. Kitobning narxi 20000 so'm turadi. Uning narxi 18% ga arzonlashdi. Kitob qayta ishlangandan keyin narxi 15% ga ko'tarildi. Kitobning narxi qancha bo'ldi?

A) 18200; B) 18800; C) 18860; D) 19600.

76. x ga teskari bo'lgan son x ning 36% ini tashkil qiladi. x ning qiymatini toping.

A) $2\frac{1}{3}$; B) $1\frac{2}{3}$; C) $1\frac{1}{3}$; D) $2\frac{1}{4}$.

77. To'g'ri to'rtburchakning bo'yi 20% ga orttirildi. Uning yuzi o'zgarmasligi uchun enini necha foizga kamaytirish kerak?

A) 25%; B) ≈18%; C) 20%; D) ≈17%.

78. Birinchi quvur hovuzni 3 soatda, ikkinchi quvur esa 5 soatda to'ldiradi. Ikkala quvur birgalikda hovuzni qancha vaqtda to'ldiradi?

A) $1\frac{7}{8}$ soat; B) $2\frac{1}{5}$ soat; C) $2\frac{1}{2}$ soat; D) 2 soat.

79. Sement va qumdan iborat 30 kg qorishmaning 60% ini sement tashkil qiladi. Qorishmaning 40% i sementdan iborat bo'lishi uchun qorishmaga qancha qum qo'shish kerak?

A) 10 kg; B) 12 kg; C) 15 kg; D) 18 kg.

80. Massasi 72 kg bo'lgan mis va rux qotishmasining tarkibida 45% mis bor. Qotishma tarkibida 60% mis bo'lishi uchun unga qancha mis qo'shish kerak?

A) 26 B) 27, C) 28, D) 30.

81. Quyidagi oddiy kasr ko‘rinishida berilgan sonlardan qaysilarini chekli o‘nli kasr ko‘rinishiga keltirib bo‘lmaydi?

1) $\frac{35}{88}$; 2) $\frac{4}{125}$; 3) $\frac{34}{75}$; 4) $\frac{11}{80}$.

A) 1; 2; B) 3; 4; C) 1; 3; D) 2; 4.

82. Hisoblang:

$\dfrac{\frac{2}{9}+3{,}6(1)}{1{,}91(6)-1\frac{5}{6}}$; A) 46, B) (51), C) $\frac{23}{72}$, D) 42.

83. Ko‘nhadlar ayirmasini toping.

$P=\frac{1}{3}x-\frac{1}{3}y-(x+2y)$

$Q=\frac{1}{3}x+\frac{1}{3}y-(x-y)$

A) $-3\frac{2}{3}y$; B) $4\frac{1}{3}y$; C) $-4y$; D) $42y$.

84. 7^6-27 soni qo‘yidagilarning qaysi biriga qoldiqsiz bo‘linadi?

A) 51; B) 49; C) 23; D) 13.

85. $x^3-3x^2-4x+12$ ko‘phad quyidagilarning qaysi biriga bo‘linmaydi?

A) $x-3$; B) $(x+3)$; C) $(x-2)$; D) $(x+2)$.

86. $\sqrt{19-8\sqrt{3}}+5\sqrt{3}$ ni hisoblang.

A) $4-\sqrt{3}$; B) $5\sqrt{3}$; C) $4(1+\sqrt{3})$; D) $6-\sqrt{3}$.

87. $\sqrt{11+6\sqrt{2}}-\sqrt{11-6\sqrt{2}}$ ni hisoblang.

A) 6; B) $3\sqrt{2}$; C) $-2\sqrt{2}$; D) $2\sqrt{2}$.

88. $\sqrt{9+4\sqrt{2}}-2\sqrt{8}$ ni hisoblang.

A) $1-2\sqrt{2}$; B) 9; C) 8; D) $6\sqrt{2}$.

89. $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$ $(1\le x\le 2)$ni hisoblang.

A) $-2\sqrt{x-1}$; B) $4\sqrt{x-1}$; C) 4; D) $2\sqrt{x-1}$.

90. $\left(\frac{a-b}{\sqrt[3]{a}-\sqrt[3]{b}}+\sqrt[3]{ab}\right):\left(\sqrt[3]{a}+\sqrt[3]{b}\right)+\left(\sqrt[3]{a^2}-\sqrt[3]{b^2}\right):\left(\sqrt[3]{a}+\sqrt[3]{b}\right)$ ni $a=27$ da hisoblang.

A) 5; B) 6; C) 9; D) 12.

91. $\sqrt{a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}-\frac{a-b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}$ ni (a>b) soddalashtiring.

A) $2b^{\frac{1}{2}}$; B) $2a^{\frac{1}{2}}$; C) $-2b^{\frac{1}{2}}$; D) $-2a^{\frac{1}{2}}$.

92. Agar $a-b=1$ va $(a^2-b^2)(a-b)=9$ bo'lsa, ab ning qiymatini toping.

A) 19; B) 20; C) 21; D) 24.

93. $\begin{cases} xy+x+y=11 \\ x^2y+y^2x=30 \end{cases}$ tenglamalar sistemasi uchun $x+y$ ning eng katta qiymatini toping.

A) 5; B) 6; C) 7; D) 8.

94. $x^2+y^2=17$; $x^3y^3=343$ bo'lsa, x^4+y^4 ning qiymatini toping.

A) 176; B) 187; C) 191; D) 207.

95. $\begin{cases} x+y=\sqrt{xy}+7 \\ x^2+y^2+xy=133 \end{cases}$ bo'lsa, xy ning qiymatini toping.

A) 42; B) 16; C) 25; D) 36.

96. $(x+1)^2>(x+2)^2$ tengsizlikni qansatlantiruvchi eng katta butun sonni toping.

A) –4; B) –3; C) –2; D) –1.

97. $\begin{cases} \frac{x-1}{4}\le\frac{x}{5} \\ \frac{x}{3}>\frac{x+4}{7} \end{cases}$ Tenglamalar sistemasi butun echimlarining yig'indisini toping.

A) 7; B) 9; C) 10; D) 12.

98. Ushbu $|x^2+5x-4|=3x-1$ tenglamaning ildizlari yig'indisini toping.

A) –8; B) –10; C) $-3+\sqrt{21}$; D) $-6+\sqrt{21}$.

99. $\frac{(x+3)(x-5)}{x+1} \geq 0$ tengsizlikni eching

A) $(3; -1]\cup[5; \infty)$; B) $(-3; -1)\cup[5; \infty)$;

C) $[-3; -1)\cup[5; \infty)$; D) $[5; \infty)$.

100. $y=\frac{\sqrt{x^2-3x+2}+\sqrt{x}}{\sqrt{3-x}}$ funktsiyaning aniqlanish sohasini toping.

A) $(0; 1)\cup(2; 3)$; B) $[0; 1]\cup[2; 3)$;

C) $[0; 1)\cup(2; 3)$; D) $[0; 1)\cup[2; 3]$.

101. Arifmetik progressiyada $a_2-a_1=6$ bo'lsa, a_8-a_6 ni toping.

A) 10; B) 14; C) 8; D) 12.

102. 150 dan katta bo'lmagan 7 ga karrali barcha natural sonlarning yig'indisini toping.

A) 1617; B) 1450; C) 1807; D) 1517.

103. Geometrik progressiyaning dastlabki 6 ta hadi 2, b_2, b_3, b_4, b_5, 486 bo'lsa, $b_2+b_3+b_4+b_5$ ni hisoblang.

A) 200; B) 210; C) 220; D) 230.

104*. Agar geometrik progressiyada $b_1=2$; $b_n=\frac{1}{8}$ va $S_n=3\frac{7}{8}$ bo'lsa, uning nechta hadi bor.

A) 6; B) 5; C) 8; D) 10.

105. Cheksiz kamayuvchi geometrik progressiyaning birinchi hadi ikkinchi hadidan 8 ga ortiq, hadlarining yig'indisi 18 ga teng. Progressiyaning uchinchi hadini toping.

A) $-1\frac{1}{3}$; B) $-3\frac{1}{3}$; C) $1\frac{1}{3}$; D) $1\frac{2}{3}$.

106. $1+\sin 60°+\sin^2 60°+\sin^3 60°+...$ cheksiz geometrik progressiyaning yig'indisini toping.

A) $2(2-\sqrt{3})$; B) $2(2+\sqrt{3})$; C) $-2+\sqrt{3}$; D) $2-\sqrt{3}$.

107. $\frac{\sin(\pi+\alpha)}{\sin\left(\frac{3\pi}{2}+\alpha\right)}+\frac{\cos(\pi-\alpha)}{\cos\left(\frac{\pi}{2}+\alpha\right)-1}$ ni soddalashtiring.

A) $\sin\alpha$; B) $\cos\alpha$; C) $\frac{1}{\sin\alpha}$; D) $\frac{1}{\cos\alpha}$.

108. $\frac{\cos 18°\cdot\cos 28°+\cos 108°\cdot\sin 208°}{\sin 34°\cdot\sin 146°+\sin 236°\cdot\sin 304°}$ ni soddalashtiring.

A) $\cos 10°$; B) $\sin 10°$; C) $-\sin 10°$; D) $\cos 46°$.

109. $\frac{1+\cos 2\alpha+\cos^2\alpha}{\sin^2\alpha}$ ifodani soddalashtiring.

A) $3\mathrm{tg}^2\alpha$; B) $\mathrm{ctg}^2\alpha$; C) $3\mathrm{ctg}^2\alpha$; D) $1,5\mathrm{tg}^2\alpha$.

6. a) barcha qiymatida; b) $y \neq 0$; d) $x \neq 11$; e) barcha qiymatida; f) $a \neq -3$; g) $b \neq 10$. **10.** $2n-1$. **18.** d) $3a-6$; e) $1,5b+10$. **19.** a) $11-6,5x$; e) $6b-5$; f) $y-8$; g) $8x-8$. **20.** a) $8+2x$; b) $46-5y$; d) 5; e) $36,8\ s-8$. **21.** d) $-3,1$; e) 276. **34.** e) -70; f) -210; g) -9; h) 25; i)78,5; j) 1,2. **35.** 667; 1887. **38.** d) 6^{17}; e) 2^{14}; f) $0,4^{7}$; g) $0,1^{10}$. **39.** d) 2^{10}; e) 2^{11}. **42.** b) 100; e) $1\frac{7}{9}$; g) $-\frac{8}{27}$. **43.** d) 25; e) 0,36. **44.** d) -125; e) -28. **47.** f) $\frac{5}{7}$; g) 5000 0000. **51.** a) 2^{30}; b) 8^{20}; d) 16^{15}; e) 32^{12}. **53.** a) x^{13}; b) a^{11}; d) a^{14}; e) m^{35}; f) a^{8}; g) $32b^{15}$. **54.** d) x^{22}; e) y^{21}; f) y^{18}. **55.** a) 16; b) 5; d) 125; e) 3. **56.** 4; 9; 100; n^2. **66.** a) x^2; b) $-0,2c^3-0,1c^2$; d) $8ab-8a^2b^2-9ab^2$; e) $-2\frac{2}{3}ab^3+a^3b-5a^2b$. **69.** a) $7ab-\frac{1}{6}a^2b$; b) $\frac{3}{8}x^2y$. **70.** a) $-2pq-3,9p^2-q$; b) $-2,6ab-5,2a^2+1,4b$. **72.** a) $13a+3b$; b) $3x+3$; d) $-\frac{1}{2}x+3\frac{1}{4}$; e) $1,6y-0,1$. **73.** a) $3a^2b-ab^2$; b) $7a^2-2ab-8b^2$; d) $2x^2+x-3$; e) m^2+6. **74.** a) $3ax$; b) $2a^4-10a^3b+10ab^3$. **79.** d) $28x-21y+25z$; e) $5m^2-3mn-3n^2$. **80.** a) $0,7y^2+0,25y-7,2$; b) $3xy-6x^2+7$; d) $-4ab^2-4ab-5b$; e) $\frac{5}{6}x^2y^2-\frac{7}{12}ab-1\frac{5}{6}a^2b^2-1\frac{1}{4}$. **81.** a) $1,67a^2-0,51ab+3\frac{1}{2}ac-5bc$; b) $12,15x^2+2,865xy$. **87.** a) $-108a^9$; b) $392\ m^{11}$; d) $8x^{2n+3}$; e) $108a^{2m+9}$. **91.** d) $-18n$; e) $32q$. **92.** a) $3ab+b^2$;

b) $-m^2-20$ mn; d) $8x^3-103xy$; e) $38ab-42a^2$. **96.** b) $1{,}728p^3-0{,}125q^3$; d) $\frac{8}{27}x^3+3\frac{3}{8}y^3$. **97.** a) 15; b) 98. **103.** h) $0{,}008x^6-0{,}6x^4y^2+15x^2y^4-125y^6$; i) $\frac{8}{27}b^6-b^4c+\frac{9}{8}b^2c^2-\frac{27}{64}c^3$; j) $2{,}744b^9c^3+29{,}4b^8c^3+105b^7c^3+125b^6c^3$. **104.** f) $(d+0{,}3c)(d^2-0{,}3dc+0{,}09c^2)$; g) $(5-0{,}4\ p^3)\cdot(25+2p^3+0{,}16p^6)$. **105.** f) $5a^4(a-5)(a+5)$; g) $a^2(a+0{,}4)\ (a^2-0{,}4a+0{,}16)$. **106.** f) $(x+y)(x^2+xy+y^2)$; g) $(a-b)(a^2+6ab+b^2)$. **107.** a) $(x^2-xy+y^2)(x+y+2)$; b) $(a^2+ab+b^2)(a-b+3)$; d) $(a-b)(a+b)\times(a^2-ab+b^2)$; e) $(x-y)(x+y)\times\times(x^2+xy+y^2)$. **115.** a) $9a^4-14a^3b+3b^2$; b) $b^2-7a^2b^3$. **116.** a) $m-n$; b) $2\frac{1}{2}-6x$. **117.** a) $1-2t$; b) $-30b-5a$. **121.** d) $\frac{2}{3}$; e) 2. **122.** d) 1; e) 2. **123.** a) 1; b) 5; d) 30; e) $-2\frac{2}{11}$. **124.** a) 4; b) $-6\frac{2}{3}$; d) $2\frac{4}{7}$; e) 2,22. **125.** a) 1; b) –12. **126.** a) 19; b) 6; d) 4; e) $2\frac{5}{6}$. **127.** a) yechimi yo'q; b) 8; d) 3; e) 4. **128.** a) 3; b) $-\frac{2}{3}$. **129.** a) 10; b) 3; d) –1; e) –3. **130.** 24 ta. **131.** 350; 420; 504. **132.** 80; 75; 400; 504. **133.** 5. **134.** 2 kg. **135.** 40 kg; 20 kg. **136.** 20 km/soat. **137.** 150 kg; 250. **138.** 141 1000; 1079 000. **139.** 460 000 so'm; 410 000 so'm. **140.** 1100 km; 2000 km. **141.** 640 km; 60 km/soat. **142.** 1320 km 230 km/soat. **143.** 243; 162. **154.** a) (–8; 1); b) yechimi yo'q; d) cheksiz yechim. **156.** d) $\left(4\frac{1}{3};\ -1\frac{1}{9}\right)$; e) (2,25; –3,5). **157.** a) (3; –0,5); b) $\left(-\frac{1}{3};\ 2\right)$. d) (–166; 34); e) $\left(5\frac{1}{3};-\frac{1}{9}\right)$. **158.** a) (–6; 4); b) (12; –2); d) (5; –3); e) (–1; –5). **159.** d) (60; 30); e) (2; –0,25). **160.** a) (0,25; 0);

b) $(-0{,}6;\ -2)$; d) $(2;\ 1)$; e) $\left(\frac{1}{2};\ -\frac{1}{3}\right)$. **161.** a) $(100;\ 1)$; b) $(6;\ 5)$; d) $(0{,}4;\ -0{,}2)$; e) $(0{,}1;\ 0{,}3)$. **162.** a) $(7;\ -2)$; b) $(2;\ 1)$. **163.** a) $(9;\ 8)$; b) $(-0{,}8;\ -0{,}8)$; d) $(15;\ 12)$; e) $(-8;\ 6)$. **164.** a) cheksiz yechimga ega; b) yechimi yo‘q. **165.** 5,5; 7,5. **166.** 7 m; 8 m. **167.** 380 ga; 320 ga. **168.** 9 kg; 6 kg. **169.** 5000 so‘m; 3000 so‘m. **170.** 18,8 t; 88,5 t. **171.** 60 km/soat. **172.** 80 km/soat; 60 km/soat. **173.** 18 km/soat; 2 km/soat. **174.** 460 m^3; 560 m^3. **175.** 9,6 kg; 6 kg. **176.** 4,5 m^3; 3 m^3. **177.** 320 ta; 360 ta. **184.** d) $y\neq 0$ va $y\neq 3$; e) barcha sonlar. **185.** e) $x\neq\frac{7}{3}$; f) $x\neq 0$ va $x\neq -1$; g) $x\neq 0$. **190.** a) $\frac{y-4}{3}$; b) $\frac{5}{x+3y}$; d) $\frac{c+2}{7c}$; e) $\frac{6c}{d-3}$; f) $\frac{a+5}{a-5}$; g) $\frac{y+3}{y-3}$. **191.** a) $3x-y$; b) $\frac{a}{2b-1}$; d) $\frac{1}{x-2}$; e) $1-a+a^2$. **192.** a) x^2; b) $-y^4$; d) $-b^5$; e) c^3-c^2. **196.** e) $\frac{2p-11q}{5p}$; f) $-\frac{d}{c}$; g) 1. **197.** a) $-(x+4)$; b) $5-a$; d) $\frac{3}{a+b}$; e) $\frac{1}{x-8}$; f) $-\frac{3}{a-b}$; g) $\frac{2}{x+y}$. **198.** a) $\frac{a-6}{c-3}$; b) $\frac{x+5}{y-1}$; d) -5; e) $a-4$; f) $\frac{x+4}{x-4}$; g) $\frac{x+5}{x-5}$. **200.** d) $\frac{41a+13b}{36a}$; e) $\frac{9x+16}{24y}$; f) $\frac{a^2+b^2}{a^2b}$; g) $\frac{4a^2-3ab-3b^2}{a^2b^2}$. **201.** a) $\frac{15b^2+4c^2}{18b^2c^2}$; b) $\frac{16x^2-35y^2}{10x^2y^2}$; d) $\frac{a^3+b^3}{a^2b^2}$; e) 6. **202.** a) $-\frac{1}{2p}$; b) 10; d) $-\frac{(a-b)^2}{ab}$; e) $\frac{5b-3a}{4}$; f) $\frac{a+b}{12}$; g) $\frac{b^2+5b-1}{b}$. **203.** f) $\frac{2p}{9p^2-1}$; j) $\frac{px-3p}{6x^2-x-2}$. **204.** a) $\frac{a+x}{x}$; b) $\frac{2y-b}{y}$; d) $-\frac{2a+b}{ab}$; e) $\frac{2y-3x}{xy}$. **205.** a) $\frac{x-y}{x+y}$; b) $\frac{b-c}{b+c}$; d) $\frac{a+1}{a^2-a}$; e) $\frac{b-5a}{ab+5a^2}$. **206.** a) $\frac{ab}{a^3+b^3}$;

b) $\frac{p-q}{p^2+pq+q^2}$; d) $\frac{1}{a^3+1}$; e) $\frac{3a}{a+4}$. **207.** a) $\frac{a^2}{b(a-b)^2}$; b) $\frac{36}{(a-3)^2(a+3)^2}$;

d) $\frac{2x-4}{x^2+2x+4}$; e) $\frac{1}{a-1}$. **210.** f) $\frac{13mx}{3n}$; g) $\frac{11x^2}{3ab}$. **211.** a) $\frac{a^2x^2}{5b^3}$; b) $\frac{m^2}{p^4}$.

212. f) $-\frac{1000m^6}{n^6p^3}$; g) $\frac{b^{12}c^8}{4096a^{12}}$. **213.** d) $\frac{1}{axy}$; e) $\frac{4}{x^2}$. **214.** a) 1; b) $-1\frac{1}{9}$ va $-\frac{2}{21}$.

215. a) $\frac{x^2-9x+20}{6}$; b) $\frac{(a+1)(a+2b)}{12}$. **218.** d) $\frac{4c^2d^2}{27a^2b}$; e) $\frac{a^2}{x^2y^2}$. **219.** d) $\frac{a^2}{a^2-2a-15}$;

e) $\frac{m+n}{2m}$. **220.** a) $\frac{10}{11}$ va -1; b) 0,42. **221.** d) $\frac{x+1}{a-x}$; e) $\frac{2a(p-3)}{p^2+2p+4}$; f) x; g) $\frac{x+1}{x-1}$.

222. d) $\frac{(a+b)^2}{b^2}$; e) 0. **223.** a) $-a$; b) $-x$; d) $\frac{10}{2m+1}$; e) $\frac{2}{x-3}$. **224.** a) $-2x$;

b) $\frac{1}{3(q-p)}$; d) $\frac{x^2}{a+x}$; e) $-\frac{a}{2(x+1)}$. **225.** a) $2x(x+y)$; b) $\frac{x-2y}{2xy}$; d) $\frac{a(n-a)}{a+n}$;

e) $\frac{2a(b-2a)}{2a+b}$. **226.** a) $\frac{1}{a^2+a+1}$; b) 1; d) $\frac{x-1}{x(x+1)}$; e) $\frac{x^2+a^2}{x+1}$. **227.** a) 0,75; b) -3.

228. a) 1; b) $\frac{ab+bc+ac}{a+b+c}$; d) $\frac{a-b+3c}{a+b-c}$. **229.** a) $\frac{a^2}{b^2}$; b) $a+b$. **233.** h) 10. i) 6,2.
234. h) -16; i) -10; l) 3; m) $-0,4$. **236.** f) $\pm0,6$; g) ±8. **250.** f) 14; g) 4,8.
251. d) 12; e) 12; f) 6; g) $\frac{15}{16}$. **252.** f) 15; g) 2. **253.** f) 1,5; g) 2,4. **254.**

d) 1,583; e) 0,633. **261.** d) $\frac{4x}{y}$; e) $-\frac{c^5}{d}$. **262.** a) 24; b) 48; d) 42; e) 65.

268. f) $0,1b^2\sqrt{b}$; g) $-3c\sqrt[3]{3}$. **269.** d) $\sqrt{2x}$; e) $\sqrt{-2x}$; g) $\sqrt{28b^2}$. **272.** h) 6;

i) $-0,5$. **273.** f) 32; g) 60; h) 6; i) -19. **275.** f) $-\sqrt{7}$; g) $\frac{1}{\sqrt{x}}$; h) $\frac{1}{\sqrt{x}}$; i) $\frac{\sqrt{2}}{3}$.

276. k) $9(3+2\sqrt{2})$; l) $\frac{2(5\sqrt{2}-1)}{7}$; m) $4(\sqrt{6}-\sqrt{3})$. **277.** a) $\sqrt{2}$; b) $\frac{\sqrt{5}}{\sqrt{2}}$; d) $\frac{\sqrt{5}}{\sqrt{2}}$;

e) $\frac{\sqrt{3}}{2}$; f) $\sqrt{3}$; g) $\sqrt{10}-\sqrt{3}-1$. **278.** a) $\frac{2+\sqrt{2}-\sqrt{6}}{4}$; b) $\frac{2\sqrt{15}+\sqrt{5}-3\sqrt{3}+4}{22}$.
281. 2) 0 va –2,5. **282.** 2) ±8; 3) 0 va 0,4; 4) ±1. **283.** 1) ±9; 2) ±2; 3) ±2; 4) ±6. **284.** a) –16; 4, b) –5; 9, d) –12; –2, e) –4; 15. **285.** a) –3; 4), b) 3; 4, d) 1,5; 3, e) $-2\frac{2}{3}$; $-\frac{3}{4}$. **286.** a) 2,6; –5; b) –2; $1\frac{1}{3}$; d) 1,6; 4; e) –1,2; 2; 2. **287.** –1 va –5. **288.** –7; –19 va 7; 19. **289.** 11 va 12. **291.** a) –8; 12; b) $\frac{1}{3}$; 3, d) $20\pm\sqrt{5}$; e) ildizi yo'q. **292.** a) –8; 3, b) 1,75; 4, d) –91; 87, e) –59; 53. **293.** a) 0,5; 1, b) $-1\frac{4}{7}$; 2, d) $-1\frac{11}{13}$; 3, e) $-\frac{7}{9}$; 7. **294.** a) –2,8; $\frac{1}{3}$, b) $-6\frac{3}{7}$; 2, d) 1,8, e) –5; 4. **295.** a) –9; 2, b) –8,5; –2, d) –4; 9, e) 2. **296.** a) 1,4; 3, b) $-(3+\sqrt{3})$; $1-\sqrt{3}$, d) $14\pm8\sqrt{3}$. e) 5,2; 10. **297.** 32 sm. **298.** 140. **299.** 10 sm. **300.** 22 ta. **301.** 30 ta. **302.** 26 ta. **303.** 16; 17; 18 va –18; –17; –16. **304.** 70 km/soat; 80 km/soat. **305.** 400 km/soat; 320 km/soat. **306.** 21 ta. **307.** 50 km/soat. **308.** 60 km/soat. **309.** 20%. **310.** 5%. **313.** –5; $p=-2$. **314.** 0,5; $q=6,25$. **315.** $\frac{3}{5}$; $b=-43$. **316.** $q=35$. **319.** e) $(x-5)(x-6)$; f) $(1-y)(y-5)$; g) $(x+1)(7-2x)$. **320.** a) $2(x-0,5)^2$; b) $-(3x-2)^2$; d) $(4a+3)^2$; e) $(0,5m-2)^2$. **323.** a) $x+1\frac{1}{7}$; b) $\frac{5}{2a+9}$; d) $\frac{b-3}{b-5}$; e) $-\frac{y+4}{y+9}$; f) $-\frac{c+10}{c+2}$; g) $\frac{m-3}{m-2}$. **324.** a) $\frac{3}{x+5}$; b) $\frac{2x+1}{x}$; d) $\frac{5a+3}{14-11a}$.
325. ustma-ust tushadi. **326.** e) –27 va –1; f) –0,2; g) $\frac{2}{11}$. **327.** a) –12,5; b) $-2\frac{2}{3}$; d) –1 va 3,5; e) 0 va –8; f) –3,25 va 1; g) 2. **328.** a) $3\pm\sqrt{5}$; b) $-4\frac{1}{3}$; d) –1 va 2; e) 4; f) 2; g) $\frac{5\pm\sqrt{19}}{3}$. **329.** a) –3; b) $-1\frac{2}{3}$ va 0; d) –3 va 3;

e) 9 va 13; f) $1\frac{1}{3}$ va $2\frac{1}{3}$; g) 1 va 7. **330.** a) $\frac{5\pm\sqrt{17}}{4}$; b) $1\pm\sqrt{14}$; d) $-\frac{1}{9}$ va 1; e) $\frac{7\pm3\sqrt{6}}{10}$. **331.** a) $2\pm\sqrt{35}$; b) –1,5; 0; d) $\frac{2}{3}$; 1; e) 0,4; 0,5. **332.** $\frac{4}{3}$. **333.** $\frac{11}{15}$. **334.** 15 km/soat. **335.** 12 km/soat. **336.** 27 km/soat. **337.** 80 km/soat. **338.** 10 kun va 15 kun. **339.** 10 kun va 15 kun. **340.** 2,5 km/soat. **341.** 90 km/soat. **360.** a) $3{,}1<\sqrt{2}+\sqrt{3}<3{,}3$; b) $0{,}2<\sqrt{3}-\sqrt{2}<0{,}4$; d) $2{,}38<\sqrt{6}<2{,}7$; e) $1{,}1(3)<\sqrt{1{,}5}<1{,}286$. **361.** $108\le p\le 114$. **362.** yaroqli. **369.** a) 0 va 7; b) –11 va –3; d) –2 va 5; e) yo‘q va 4. **375.** (–∞; 3,2). **376.** a) $\left(-\infty; -\frac{7}{8}\right]$; b) (9; +∞); d) (–∞; –3,1]; e) (–∞; 22,5). **377.** a) (–∞; –0,8); b) [0; +∞); d) (1; +∞); e) (4,8; +∞). **378.** a) (6; +∞); b) (0; +∞); d) (–∞; –5]; e) (–3; +∞). **379.** e) (1,8; +∞); f) [0,5; +∞); g) $\left(-\infty; 1\frac{5}{7}\right]$; h) $\left(-\infty; -\frac{2}{3}\right)$. **380.** a) $\left(-\infty; \frac{1}{6}\right)$; b) [–5; +∞); d) (–∞; –0,6]; e) $\left(-\infty; -3\frac{5}{6}\right)$. **381.** a) 1; 2; 3; 4. b) 1 va 2. **384.** a) (6; ∞); b) (–∞; –1); d) $\left(0; 3\frac{1}{3}\right)$; e) yechimi yo‘q. **385.** a) (2; +∞); b) [5; +∞); d) (–∞; –1,5); e) (–1; 0,8). **386.** a) (–∞; 2,4); b) yechimi yo‘q; d) $\left[-8; 1\frac{1}{3}\right)$; e) [1,5; ∞). **387.** a) (–∞; –3); b) yechimi yo‘q; d) (0,1; ∞); e) (–0,24; +∞). **388.** a) [0,6; 5]; b) (2; 16]; d) $\left(\frac{1}{13}; 9\right)$; e) yechimi yo‘q. **389.** a) (10; 21); b) (0,8; 2); d) (1; 3). **398.** h) 1; i) 81; j) 15625. **400.** e) $2y^{-4}$; f) $\frac{1}{12}pq^{-8}$; g) $2a^4b^2$. **401.** a) –1; b) 6. **402.** a) $\frac{1}{3}x^4y^7$; b) $81ab^7$; d) $15x^5y^5$; e) $\frac{5}{4}p^5q^{10}$. **403.** a) $4x$; b) $3b$. **405.** d) $x\neq1{,}25$; e) $x\neq35$. **406.** d) (–4; +∞); e) (–∞; 5). **407.** a) $x\neq0$; $x\neq4$; b) (0; +∞); d) [0; +∞); e) $(-\infty; 0]\cup(2; +\infty)$.

409. $y=180°-2x$ va $0°<x<90°$. **412.** $-\frac{22\sqrt{2}-13}{17}$. **440.** 1) –2; 2) –13. **441.** $y_{I}=-0{,}5\cdot(x-3)^2$; $y_{II}=-0{,}5x^2-2$; $y_{III}=0{,}5(x+2)^2-1$. **442.** e) [–4; 4] va (–∞; –4] va (4; +∞); g) [0; 3,5) va (–∞; 0]; [3,5; +∞). **443.** b) $\left(-\infty; \frac{1}{12}\right]$ va $\left[\frac{1}{12}; +\infty\right)$; d) (–∞; 5] va [5; +∞). **444.** a) (–∞; 2] va [1,5; +∞) da $y>0$; (–2; 1,5) da $y<0$; b) $\left(-\infty; \frac{1}{4}\right)$ va $\left(\frac{1}{4}; +\infty\right)$ da $y>0$, $y<0$-yo'q. e) $y>0$-yo'q, (–∞; +∞) da $y<0$. **445.** a) [–4; +∞) va (–∞; –4); b) [2,5; +∞) va (–∞; 2,5]; d) (–∞; –2] va [–2; +∞); e)[0; +∞) va (–∞; 0]; [0; ∞) va (–∞; 0]. **447.** a) (–∞; 3) va (5; +∞); b) (–∞; 0,5) va $\left(\frac{2}{3}; +\infty\right)$; d) (–∞; –6) va (8; +∞); e) (–∞; –1,5) va $\left(\frac{1}{4}; +\infty\right)$; f) (–∞; +∞); i) yechimi yo'q. **448.** a) [–2; 3]; b) $x\neq\frac{2}{3}$; d) (–∞; –4,5] va [2; +∞); e) (–∞; 0) va $\left(\frac{2}{3}; +\infty\right)$; **449.** a) (–∞; –3) va (3; +∞); b) $\left(-\infty; -\frac{\sqrt{3}}{3}\right]$ va $\left[\frac{\sqrt{3}}{3}; +\infty\right)$; d) $\left(-\frac{2}{3}; 0\right)$; e) (–∞; –0,5) va (0; +∞). **450.** a) $\left(-7; -\frac{1}{4}\right)$; b) (–∞; 1,5] va $\left[1\frac{2}{3}; +\infty\right)$; d) (–∞; +∞); e) $\left(-\infty; \frac{9-\sqrt{37}}{22}\right)$ va $\left(\frac{9+\sqrt{37}}{2}; +\infty\right)$. **451.** a) [0; 2]; b) [–4; 4]; d) $x\neq-2$; e) (–∞; –6) va (7; +∞). **453.** a) (–∞; –3); b) $[2\sqrt{2}; +\infty)$; d) [–0,7; 0]; e) $\left[-2\frac{1}{3}; 0\right]$ va [0,9; 1]. **454.** a) (–2; 0); b) yechimi yo'q; d) (–∞; –12); e) (–5; 0). **455.** a) (–1; 2); b) (1; 4). **456.** a) (–∞; 3) va (5; +∞); b) $\left(-\frac{1}{3}; \frac{2}{3}\right)$ va $\left(\frac{2}{3}; 3\right)$. **457.** a) (–2; +∞); b) (–5; 6). **459.** a) –2; b) –1; d) $1\frac{1}{3}$; e) –0,5 va 0,5. **460.** a) 0; $\pm\sqrt{6}$; b) 0; d) 0; –0,2; 0,5; e) $-\frac{1}{3}$; $\frac{1}{3}$; 2; f) –1; 1; g) –1; 0; 1; 3; h) $\frac{1}{3}$; i) 1,5.

461. a) $b<4{,}5$; b) $b<\frac{4}{15}$; d) $b<-6$ va $b>6$; e) $b<-2\sqrt{5}$ va $b>2\sqrt{5}$. **462.** a) $-12<i<12$; b) $u<-\frac{1}{3}$; d) $u>28\frac{1}{8}$; e) $-12<u<12$. **464.** mumkin emas. **465.** a) -1 va 1; b) $-\sqrt{10}$; $\sqrt{10}$; -4; 4. d) $\frac{-2-\sqrt{6}}{2}$; $\frac{-2+\sqrt{6}}{2}$; -3; 1; e) -2; 2; -3; 3. **466.** a) -1; 0; -2; b) -2; 0. **467.** a) -1 va 1; b) -4 va 4; d) $-\frac{1}{3}$; $\frac{1}{3}$; $-\frac{1}{2}$; $\frac{1}{2}$; e) $-\frac{\sqrt{2}}{4}$; $\frac{\sqrt{2}}{4}$; $-\frac{\sqrt{2}}{2}$; $\frac{\sqrt{2}}{2}$; f) ildizi yo'q; g) $-\frac{\sqrt{2}}{2}$ va $\frac{\sqrt{2}}{2}$; ± 2. **468.** a) $(-3;\ 0)$, $(-1;\ 0)$, $(1;\ 0)$, $(3;\ 0)$; b) $(-\sqrt{3};\ 0)$ va $(\sqrt{3};\ 0)$; d) $(0;\ 0)$. **469.** a) ildizi yo'q; b) $-\frac{\sqrt{3}}{3}$; $\frac{\sqrt{3}}{3}$; -2; 2. **470.** a) 1; $-\sqrt{3}$; $\sqrt{3}$; b) $-\sqrt{3}$; -1; 1; $\sqrt{3}$; 2. **476.** a) $(5;\ 2)$, b) $\left(-\frac{3}{4};-4\frac{1}{4}\right)$. **477.** a) $(3;\ 1)$, $(5;\ 3)$; b) $(-7;\ -3)$, $(3;\ \frac{1}{3})$; d) $(10;\ 1{,}8)$; e) $(-1{,}5;\ -6)$ va $(2;\ 1)$; f) $(6;\ 8)$, $\left(-5\frac{1}{13};-8\frac{8}{13}\right)$; g) $(6;\ 4)$, $(-6;\ -4)$. **478.** a) $(-3;\ -2)$, $(3;\ 1)$; b) $(3;\ -5)$, $(5;\ -8)$. **479.** a) $(-\sqrt{6};\sqrt{6})$; $(\sqrt{6};-\sqrt{6})$; b) $(-5;\ -4)$, $(5;\ 4)$. **480.** a) $(-4;\ -3)$, $(-4;\ 3)$, $(4;\ -3)$, $(4;\ 3)$; b) $(-10;\ -8)$, $(-10;\ 8)$, $(10;\ -8)$, $(10;\ 8)$. **481.** a) $(-3;\ -3)$, $(4;\ 0{,}5)$; b) $\left(\frac{4}{9};-\frac{1}{3}\right)$, $(1;\ -2)$; d) $(0;\ -5)$, $(1;\ -4)$; e) $\left(-\frac{1}{3};-1\right)$, $\left(\frac{1}{3};1\right)$. **482.** a) $(\pm 3;\ \mp 3)$; b) $(-3;\ -5)$, $(3;\ 5)$; d) $(\pm 4;\ \mp 3)$, $(\mp 4;\ \pm 3)$; e) $(\pm 16;\ \pm 10)$. **483.** a) $(-3;\ -3)$, $(4;\ 0{,}5)$; b) $\left(\frac{4}{9};-\frac{1}{3}\right)$, $(1;\ -2)$; d) $(0;\ -5)$, $(1;\ -4)$; e) $\left(\pm\frac{1}{3};\ \pm 1\right)$; **484.** a) $\left(1\frac{1}{4};2\frac{1}{2}\right)$, $\left(1\frac{2}{3};1\frac{2}{3}\right)$; b) $(-2{,}5;\ -2)$, $(6;\ 15)$; d) $\left(\frac{24}{13};\ \frac{24}{5}\right)$; e) $(7;\ 3)$, $(-7;\ -3)$. **485.** 8 va 12. **486.** 20 sm va 21 sm. **487.** 10 sm va 24 sm. **488.** 40 m va 60 m. **489.** 210 sm^2. **490.** 3,6 km/soat, 4,8 km/soat. **491.** 5 sm, 12 sm. **492.** 6 soat, 10 soat. **493.** 60 soat, 84 soat. **494.** 60 km/soat, 75 km/soat. **499.** b) 25, d) 34. **500.** d) 0, 4, 12, 28, 60; f) 6, –6, 6, –6, 6. **504.** a) –53, –141, 7–4n; b) $-32\frac{1}{3}$, $-83\frac{2}{3}$, $2\frac{2}{3}-2\frac{1}{3}n$. **505.** 26 m. **506.** 7, 11, 15, 19, 23, 27, 31, 35. **507.** a) $c_1=21$, $d=1{,}5$; b) $c_1=38$, $d=-2$. **508.** a) ha; b) yo'q. **509.** a) 102; b) 151. **510.** a) $1\le n\le 30$; b) $1\le n\le 13$; d) $n\ge 31$; e) $n\ge 64$. **511.** a) $-0{,}1\cdot(n\ge 31)$; b) $0{,}1(n\ge 15)$. **512.** $2d$; $a_n=2d(n-2)$. **515.** a) $n\cdot(n+1)$; b) n^2. **516.** a) 11325; b) 7070; d) 4905; e) 494550. **517.** a) 5175; b) 60300; d) 810. **518.** 2387. **519.** 55. **520.** 15 qator, 465 shar. **524.** d) –8, $-(-2)^{n-4}$; e) –10; –10; $-10(-1)^{n-1}$. **525.** a) 3; b) –3. **526.** a) ±3; b) ±0,6. **527.** a) $b_1=\pm125$, $q=\pm\frac{1}{5}$; b) –0,001, $q=10$. **528.** 30; 15; $\frac{15}{2}$; $\frac{15}{4}$; $\frac{15}{8}$. **529.** 2; 4; 8. **530.** $3{,}5\cdot10^4$ m³. **531.** 2 073600 so'm. **532.** 0,375 sm. **535.** b) $\frac{1-\left(-\frac{1}{2}\right)^n}{3}$; e) $\frac{(-x)^{3n}-1}{1+x^3}$. **536.** a) 205,9; b) $25\frac{34}{81}$. **537.** 1094. **538.** 2; –6; 18; ... va 16; 8; 4; ... . **539.** yo'q. **541.** d) $\frac{1}{1-a^2}$; e) $\frac{a}{1+a^3}$. **542.** a) $\frac{2}{3}$; e) $1\frac{9}{11}$; f) $\frac{7}{30}$; g) $\frac{357}{1100}$; h) $1\frac{8}{11}$; i) $\frac{131}{300}$; j) $\frac{131}{6600}$. **543.** 20π sm², $\frac{100\pi}{3}$ sm². **544.** 1) 32 sm²; 2) a) 6 a; b) $\frac{a^2\sqrt{3}}{3}$; d) $\frac{2\sqrt{3}}{3}\pi a$; e) $\frac{\pi a^2}{9}$. **550.** j) 5; k) 18; l) 0; m) 18. **551.** h) –10; i) –4; j) –1). **552.** j) ±0,5; k) –2; l) ±2; m) $\pm\sqrt{17}$. **553.** h) $\frac{2}{5}$; i) –1,5; j) $1\frac{1}{3}$. **554.** f) $\frac{4}{9}$; g) $\frac{1296}{1331}$. **555.** f) 6; g) 30. **556.** f) 2; g) 5. **561.** h) $\sqrt[3]{49}$; i) $\sqrt[3]{6}$; j) $\sqrt[3]{16}$.

564. a) 3; b) 1. **565.** a) 66; b) 20. **568.** j) $\frac{4}{9}$; k) 10^5. **570.** e) $s \neq 5$. **574.** d) a; e) s. **575.** d) 16; e) 0. **576.** f) $\frac{1}{4}a^{\frac{1}{8}}$; g) $\frac{1}{3}b^2$. **577.** d) a; e) q. **578.** f) $y^{\frac{1}{6}}$; g) $x^{\frac{1}{4}}$. **580.** d) 9; e) $\frac{1}{32}$; d) 9; g) mavjud emas. **581.** h) $a-b$; i) $x+1$.

582. h) $4\sqrt[3]{ab}$; i) $x^{\frac{1}{2}}+x^{\frac{2}{3}}$. **585.** f) $\left(3x^{\frac{1}{10}} \pm 2y^{\frac{1}{10}}\right)$; g) $\left(\sqrt{5}c^{\frac{1}{14}} \pm \sqrt{7}d^{\frac{1}{14}}\right)$.

586. e) $\left(m^{\frac{3}{5}}+0,5\right)\times\left(m^{\frac{6}{5}}-0,5m^{\frac{3}{5}}+0,25\right)$. **587.** f) $\frac{b^{\frac{1}{2}}-a^{\frac{1}{2}}}{b^{\frac{1}{2}}+a^{\frac{1}{2}}}$; g) $m^{\frac{1}{3}}+n^{\frac{1}{3}}$.

588. a) 1; b) 5; d) 13; e) 9. **589.** a) $\frac{b+a}{b-a}$; b) $\frac{3}{x^{\frac{1}{2}}-6}$; d) $\frac{2q^{\frac{1}{2}}}{p-q}$; e) $\frac{x+y}{x-y}$.

590. a) 0; b) $\frac{p^{\frac{1}{2}}+q^{\frac{1}{2}}}{q^{\frac{1}{2}}-p^{\frac{1}{2}}}$; d) $\frac{7}{16}$; e) $4(|a| \neq |x|)$. **598.** a) 2 va 0; b) 3 va 1.

600. a) –2; b) 3; d) 1; e) 3. **601.** a) $\frac{\sqrt{3}+\sqrt{2}}{2}$; b) $\frac{1}{2}$; d) –1. **607.** f) $-\frac{\sqrt{3}}{2}$; g) $\frac{\sqrt{3}}{2}$; h) 0; i) 0. **608.** a) $-\frac{\sqrt{2}}{2}-\sqrt{3}$; b) $\frac{1}{2}$. **610.** a) sin20°; b) cos10°; d) –sin40°; e) cos50°. **611.** b) $\cos\frac{\pi}{5}$; d) $\mathrm{tg}\,\frac{\pi}{7}$; e) $-\mathrm{ctg}\,\frac{2\pi}{7}$. **612.** a) π; b) 4π; d) 4π; e) $1,5\pi$. **613.** a) π; b) $\frac{8}{3}\pi$; d) $\frac{\pi}{4}$; e) π. **614.** a) $0,1\pi$; b) 4π; d) $\frac{\pi}{2}$; e) $\frac{\pi}{2}$. **616.** g) $\frac{5\pi}{3}$; h) $-\frac{4\pi}{3}$; i) $-\frac{5\pi}{4}$. **617.** d) $0,7\pi$. **620.** a) 3; b) $\frac{3-\sqrt{2}}{2}$; d) $\sqrt{2}-\sqrt{3}$; e) $-5+\sqrt{2}$. **621.** d) $\frac{\sqrt{3}}{3}$; e) 1. **624.** a) $-\frac{9}{40}$; b) $-\frac{\sqrt{10}}{10}$.

625. e) $\sin\alpha=-\frac{2\sqrt{29}}{29}$; $\cos\alpha=\frac{5\sqrt{29}}{29}$; $\mathrm{tg}\alpha=-\frac{2}{5}$. **626.** b) $\cos\alpha=\pm\frac{15}{17}$;

d) $\operatorname{tg}\alpha = \pm\frac{8}{15}$; e) $\operatorname{ctg}\alpha = \pm 1\frac{7}{8}$. **627.** f) $\operatorname{ctg}^2\alpha$; g) $-\cos^2\alpha$. **628.** e) $\sin^2\alpha$; f) 2; g) $\frac{2}{\sin^2\alpha}$. **629.** a) $\frac{1}{\cos^2\beta}$; b) $\frac{1}{\sin^2 x}$; d) $-\sin 2\gamma$; e) $\operatorname{ctg}\beta$. **630.** a) 2; b) 1; d) 1; e) 1. **631.** d) $-\cos^2\alpha$; e) $\frac{1}{\cos\alpha}$. **632.** a) $\frac{1}{\cos\alpha}$; b) $\sin^2\varphi$; d) $\frac{2}{\cos\alpha}$; e) 1. **635.** a) 0,51; b) 0,2; d) -16. **638.** a) $-\sin 0{,}2\pi$; b) $\operatorname{tg} 0{,}1\pi$; d) $-\cos 0{,}1\pi$; e) $\operatorname{tg} 0{,}2\pi$. **641.** d) $\operatorname{ctg}\alpha$; e) $\operatorname{ctg}\alpha$; f) $-\sin\alpha$; g) $\operatorname{tg}\alpha$. **643.** a) 0; b) $2\cos\alpha$; d) $\operatorname{ctg}\alpha$; e) $-\cos\alpha$. **647.** a) $\sin\alpha$; b) $-\cos\alpha$; d) $\cos\alpha$; e) $-\sin\alpha$. **649.** a) 1; b) $-\frac{720}{1681}$. **650.** a) 0; b) $\frac{1}{2}$; d) 1; e) $\frac{1}{2}$; f) $-\frac{\sqrt{2}}{2}$; g) $-\frac{1}{2}$. **651.** a) $\sin 2\gamma$; b) $\sin 2\alpha$; d) $\cos\alpha$; e) $-\sin\alpha$. **652.** a) 1; b) $\operatorname{ctg}(\alpha+\beta)$. **654.** d) $\cos^2\beta$; e) $\operatorname{tg}\frac{\beta}{2}$. **655.** d) $\cos 40° - \sin 40$; e) $\cos 18°$. **656.** a) 0,96; b) 0,28; d) $3\frac{3}{7}$. **657.** e) $\frac{1}{2}\sin\alpha$; f) $-2\sin\frac{\alpha}{2}$; g) $-2\operatorname{tg}\alpha$. **658.** e) $-\frac{1}{2}$; f) $-\frac{\sqrt{3}}{2}$; g) $-\frac{\sqrt{3}}{3}$. **660.** f) $\operatorname{ctg}^2\frac{\alpha}{2}$; g) $\operatorname{tg}\alpha$. **662.** 0,96 va $-0{,}28$. **663.** a) mavjud emas. **667.** e) $-2\sin^2\frac{x}{2}$; f) $4\sin\left(\frac{\pi}{12}+\frac{x}{2}\right)\cos\left(\frac{\pi}{12}-\frac{x}{2}\right)$; g) $-4\sin\left(\frac{\pi}{6}+\frac{x}{2}\right)\sin\left(\frac{\pi}{6}-\frac{x}{2}\right)$; **669.** a) $4\sin\frac{5x}{2}\cos x\cdot\cos\frac{x}{2}$; b) $4\cos\frac{5x}{2}\cos x\times$ $\times\cos\frac{x}{2}$. **671.** d) $\frac{2\sin\frac{5\pi}{12}}{\cos\frac{\pi}{12}}$; e) $\frac{\operatorname{tg}\frac{\pi}{5}}{\cos\frac{2\pi}{5}}$; f) $\frac{\operatorname{ctg}2x}{\cos 4x}$; g) $-\frac{\sqrt{2}}{2\cos^2\frac{3x}{8}}$. **673.** a) $\frac{1}{2}\left(\sqrt{2-\sqrt{2}}\right)$; d) $\sqrt{2}-1$; **674.** a) $\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$; b) $\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$; d) $2+\sqrt{3}$. **675.** $\frac{\sqrt{10}}{10}$; $-\frac{3\sqrt{10}}{10}$; $-\frac{1}{3}$. **676.** a) $\sqrt{2+\sqrt{2}}$; b) $\sqrt{2+\sqrt{2+\sqrt{2}}}$. **677.** d) $\operatorname{tg}^2\frac{\alpha}{2}$. **681.** d) $\frac{\sqrt{2}-1}{4}$.

682. e) $\cos 18° - \frac{\sqrt{3}}{2}$; f) $\frac{1}{2}\left(\sin\frac{13\pi}{40} - \sin\frac{3\pi}{40}\right)$; g) $\sqrt{2} + 2\sin\frac{\pi}{12}$. **683.** e) $\frac{1}{2}\sin(2\alpha - 2\beta)$; f) $1 + \cos\alpha 8 + \cos 6\alpha + \cos 2\alpha$; g) $\frac{1}{2}(\sin 4\alpha + \sin 2\alpha - \sin 6\alpha)$. **684.** d) $\frac{1}{8}(3 + 4\cos\alpha 2 + \cos 4\alpha)$; e) $\frac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$; f) $\frac{1}{8}(1 - \cos 4\alpha)$; g) $\frac{1}{16}(2\cos\alpha - \cos 5\alpha - \cos 3\alpha)$. **686.** a) 20; b) 10. **687.** $\frac{4}{225}$. **689.** a) $\sqrt{5}\sin(\alpha + \varphi)$, $\varphi \approx 26°34'$; b) $2\sin\left(\alpha - \frac{\pi}{3}\right)$; e) $2\sin\left(\alpha + \frac{\pi}{6}\right)$; f) $2\sqrt{3}\sin\left(\alpha - \frac{\pi}{6}\right)$; g) $\sqrt{34}\sin(\alpha + \varphi)$, $\varphi \approx 59°2'$.

Matematika (arifmetika, algebra) testining javoblari

	0	1	2	3	4	5	6	7	8	9
0		C	B	D	C	C	C	B	C	C
1	B	C	A	C	D	B	D	B	A	C
2	D	B	C	D	C	D	C	C	A	D
3	B	C	A	D	B	C	B	A	D	C
4	D	B	D	C	A	B	C	B	B	A
5	C	B	A	D	B	C	A	B	C	D
6	B	A	C	C	A	D	B	A	C	C
7	B	C	A	C	B	C	B	D	A	C
8	B	C	A	A	C	B	C	D	A	D
9	B	C	B	B	C	D	C	B	D	C
10	B	D	A	D	B	C	B	D	A	C

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MUNDARIJA

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VII bob. Kvadrat tenglama va uning ildizlari

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1. Arifmetik progressiya

2. Geometrik progressiya

XV bob. **Ratsional ko'rsatkichli daraja**

XVI bob. **Trigonometrik ifodalar va ularda shakl almashtirish**

M. ORTIQOV, Sh.M. YUSUPJONOVA

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(qo'llanma)

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