Моя профессиональная карьера

Научно-образовательный электронный журнал

ОБРАЗОВАНИЕ И НАУКА В XXI ВЕКЕ

Выпуск №21 (том 3)
(декабрь, 2021)

Международный научно-образовательный электронный журнал «ОБРАЗОВАНИЕ И НАУКА В XXI ВЕКЕ»

УДК 37

ББК 94

Международный научно-образовательный электронный журнал «ОБРАЗОВАНИЕ И НАУКА В XXI ВЕКЕ». Выпуск №21 (том 3) (декабрь, 2021). Дата выхода в свет: 31.12.2021.

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Название публикации: «QUVLOVCHI VA QOCHUVCHILARNING UCHRASHISH NUQTALARI TO'PLAMINING DINAMIKASINI QURISH»

Ushbu maqolada uchrashish nuqtalari to'plamining dinamikasi o'zgarishi ko'p qiymatli akslantirishlar nazaryasi xossalariga asosan tekshiriladi va asosiy lemmaning sodda isboti keltiriladi. Maqola bevosita Ayzeks, Pshenichniy, Petrosyan va Azamovlarning ishlarini rivojlantiradi.

Kalit so`zlar: quvlovchi, qochuvchi, geometrik chegaralanish, integral chegaralanish.

1. Kirish

R^n fazodagi P quvlovchi va E qochuvchi berilgan. Ularning harakati quyidagi tenglamalar bilan ifodalanadi.

$$P: \ \dot{x} = u \ , \qquad x(0) = x_0, \tag{1}$$

$$E: \ \dot{y} = v \ , \qquad y(0) = y_0 . \tag{2}$$

$x - P$ ning fazodagi holati $x \in R^n$; y – E ning fazodagi holati $y \in R^n$.

P quvlovchi u tezlik bilan harakat qiladi va bu parameter o'lchanuvchi funksiya sifatida tanlanib $u(\cdot):[0;\infty) \to R^n$ uning uchun quyidagicha chegaralanish bajarilsin

$$|u(t)| \le \alpha \ \text{deyarli barch} \ t \ge 0 \ . \tag{3}$$

Xuddi shunday E qochuvchi v tezlik bilan harkatlanadi va quyidagi $v(\cdot):[0;\infty) \to R^n$ o'lchanuvchi funksiya sifatida tanlanadi va uning uchun quyidagi tengsizlikning bajarilishi talab etiladi

$$|v(t)| \le \beta \ \text{deyarli barcha} \ t \ge 0 \tag{4}$$

P obyektning maqsadi E obyekt avvaldan berilgan biror M to'plamga tushguncha u bilan ustma-ust tushish, ya'ni $x(t)=y(t)$; E ning maqsadi esa M to'plamga yetguncha P bilan to'qnashmaslik, ya'ni $x(t)\neq y(t), \quad 0\le t\le \overline{t}$ va $y(\overline{t})\in M$.

Ta'rif 1. P quvlovchining $\Pi-$strategiyasi deb quyidagi funksiyaga aytiladi.

$$u_{\Pi}(v)=v-\lambda(v,z_0)\xi_0, \tag{5}$$

bunda

$$\lambda(v,\xi_0)=<v,\xi_0>+\sqrt{<v,\xi_0>^2+\alpha^2-|v|^2}. \tag{6}$$

Ta'rif 2.. (4) tengsizlikni qanoatlantiruvchi $v_t(\cdot)=\{v(s):0\le s\le t\}$ funksiyaga $[0,t]$ oraliqdagi quvlovchining boshqaruv funksiyasi deb ataladi. (4) shartni qanoatlantiruvchi barcha qochuvchining boshqaruvlari to'plamini V sifatida belgilaymiz, (3) shartni qanoatlantiruvchi barcha quvlovchining boshqaruvlari to'plamini U sifatida belgilaymiz.

Teorema 1. Agar ixtiyoriy $v_t(\cdot)\in V$ uchun quvlovchi $u\left(v(t)\right)$, yani (5) strategiyani amalga oshirsa o'yinni biror chekli t^* vaqtda yakunlash mumkin va $t^*\in[0;T]$ bo'ldi, bunda $T=\dfrac{|z_0|}{\alpha-\beta}$ – o'yin tugashining kafolatlovchi vaqti

Teorema 1 ning isboti Azamov, Pshenichniy, Petrosiyan ishlarida keltirilgan.

Ta'rif 3. Qutilish chizig'i masalasida o'yin quvlovchi foydasiga hal bo'ladi deyiladi, agar $x(t^*)=y(t^*)$ bajarilsa va $0\le t\le t^*$, $y(t)\notin M$.

Ta'rif 4. Qutilish chizig'i masalasi qochuvchi foydasida hal bo'ladi deyiladi, agar biror t^* vaqtda . $y(t^*)\in M$ bo'lsa va $0\le t\le t^*$, oraliqda quyidagi tengsizlik bajarilsa $x(t)\neq \text{y(t)}$.

2. Asosiy lemmalar

Teorema1 ga asosan $\alpha>\beta$ bo'lsa biror w nuqtada ob'yetlar uchrashishadi.

Ushbu bo’limda M to’plamni e’tiborga olmagan holda barcha uchrashish nuqtalarini to’plamini quramiz va bu to’plamni vaqtga bo’gliq holda dinamikasini tekshiramiz.

Faraz qilamiz qochuvchi har bir t da biror $v_t(\cdot)\in \mathrm{V}$ boshqaruvni tanlasin. Unga mos ravishda quvlovchi $u_\Pi(v(t))$ Π-strategiyaning amalga oshirsin. Natijada ularning harakat trektoriyalari quyidagicha bo’ladi:

$$y(t)=y_0+\int_0^t v(s)ds, \qquad x(t)=x_0+\int_0^t u_\Pi(v(s))ds$$

$x(t)$ va $y(t)$ ga mos ravishda quyidagicha to’plamni quramiz.

$$W(t)=\{\omega:\beta|\omega-x(t)|\geq\alpha|\omega-y(t)|\}. \tag{7}$$

Bu erda tekshirish mumkinki to obyektlar uchrashguncha qochuvchining har bir t dagi $y(t)$ holati $W(t)$ to’plamga tegishli bo’ladi.

Lemma 1. $0\leq t\leq t^*$ oralikda quyidagi formula o’rinlik bo’ladi

$$W(t)=x(t)+\Lambda(t,v_t(\cdot))(\mathrm{R}(\mathrm{z}_0)\mathrm{S}-c(\mathrm{z}_0)) \tag{8}$$

bunda $\mathrm{c}(z_0)=\dfrac{\alpha^2}{\alpha^2-\beta^2}z_0$, $R(z_0)=\dfrac{\alpha\beta}{\alpha^2-\beta^2}|z_0|$.

Agar quvlovchi П-strategiyani amalga oshirsa va yaqinlashish funksiyasi quyidagicha o’zgarishini ko’rilgan

$$z(t)=z_0\Lambda(t,v_t(\cdot)) \tag{9}$$

bunda

$$\Lambda(t,v_t(\cdot))=1-\frac{1}{|z_0|}\int_0^t\lambda(v(s))ds$$

ushbu funksiya scalar yaqinlashish funksiyasi deb atalgan. (7) ko’rinishdan quyidagilarni topamiz

$$W(t)=x(t)+\{\omega:\beta|\omega|\geq\alpha|\omega+z(t)|\}. \tag{10}$$

Avval quyidagi tenglikni ko'rsatamiz

$$W(t) = x(t) + R(z(t))S - c(z(\mathrm{t})), \qquad (11)$$

bunda $\mathrm{R}(z(t)) = \dfrac{\alpha\beta}{\alpha^2 - \beta^2}(z(t))$, $\qquad c(z(t)) = \dfrac{\alpha^2}{\alpha^2 - \beta^2}|z(t)|$.

(10) ko'rinishga ko'ra

$$\beta^2 |\omega|^2 \geq \alpha^2 |\omega + z(t)|^2$$

$$\beta^2 |\omega|^2 \geq \alpha^2 (|\omega^2| + 2 < \omega, z(t) > + |z|^2)$$

$$\beta^2 |\omega|^2 \geq \alpha^2 |\omega^2| + 2\alpha^2 < \omega, z(t) > + \alpha^2 |z|^2)$$

$$0 \geq (\alpha^2 - \beta^2)|\omega|^2 + 2\alpha^2 < \omega, z(t) > + \alpha^2 |z|^2.$$

$\alpha > \beta$ bo'lgani uchun oxirgi tengsizlikdan

$$0 \geq |\omega|^2 + 2 < \omega, \frac{\alpha^2}{\alpha^2 - \beta^2} z(t) > + \frac{\alpha^2}{\alpha^2 - \beta^2}|z|^2$$

yoki

$$0 \geq |\omega|^2 + 2 < \omega, \frac{\alpha^2}{\alpha^2 - \beta^2} z(t) > + \left(\frac{\alpha^2}{\alpha^2 - \beta^2}\right)^2 |z|^2 + \frac{\alpha^2}{\alpha^2 - \beta^2}|z|^2 - \left(\frac{\alpha^2}{\alpha^2 - \beta^2}\right)^2 |z|^2$$

$$0 \geq \left|\omega + \frac{\alpha^2}{\alpha^2 - \beta^2} z(t)\right|^2 + \left[\frac{\alpha^2}{\alpha^2 - \beta^2} - \left(\frac{\alpha^2}{\alpha^2 - \beta^2}\right)^2\right](z(t))^2.$$

Bundan

$$\frac{\alpha^2 \beta^2}{\left(\alpha^2 - \beta^2\right)^2}|z(t)|^2 \geq \left|\omega + \frac{\alpha^2}{\alpha^2 + \beta^2} z(t)\right|^2$$

Yoki

$$\left|\omega+\frac{\alpha^2}{\alpha^2-\beta^2}z(t)\right|\le\frac{\alpha\beta\left|z(t)\right|^2}{\alpha^2-\beta^2}\ .$$ (12) Bu

esa (11) tenglik bajarilishini ko'rsatadi.

Agar (9) tenglikni hisobga olsak

$$W(t)=x(t)+\frac{\alpha\beta}{\alpha^2-\beta^2}\left|z_0\right|\Lambda(t,V_t(\cdot))S+\frac{\alpha^2}{\alpha^2-\beta^2}z_0\Lambda(t,V_t(\cdot))=$$
$$=x(t)+\Lambda(t,V_t(\cdot))\left[\frac{\alpha\beta}{\alpha^2-\beta^2}\left|z_0\right|S-\frac{\alpha^2}{\alpha^2-\beta^2}z_0\right]$$

yoki (8) formulaning o'rinliligi hosil bo'ladi. Lemma isbot.

Lemma2 (Asosiy lemma). $W(t)$ bunda ko'p qiymatli akslantirish ixtiyoriy $v_t(\cdot)\in V$ uchun $0\le t\le t^*$ oralikda monoton kamayuvchi, yani $t_2>t_1\Rightarrow W(t_2)\subset W(t_1),\ 0\le t_1\le t_2\le t^*$.

Isbot. $W(t)$ ni monotonligi quyidagi tengsizlikni o'rinliligidan hosil bo'ladi

$$\frac{d}{dt}F(W(t,v_t(\cdot),\psi)\le 0$$ (13)

$F(\mathrm{W},\psi)-\max_{w\in W}<\mathrm{w},\psi>$ W-to'plamning tirgak funksiyasi.

Ma'lumki $|\,\mathrm{v}\,|\le\beta$ yoki bu yerda

$$|\,\mathrm{v}\,|^2\le\frac{\beta^2}{\alpha^2-\beta^2}(\alpha^2-|\,\mathrm{v}\,|^2)$$ (14)

$$\alpha^2-|\,\mathrm{v}\,|^2=\lambda(\mathrm{v},\mathrm{z}_0)(\lambda(\mathrm{v},\mathrm{z}_0)-2<\mathrm{v},\xi_0>)$$

$$\lambda(\mathrm{v},\mathrm{z}_0)=<\mathrm{v},\xi_0>+\sqrt{<\mathrm{v},\xi_0>^2+\alpha^2-|\,\mathrm{v}\,|^2}$$

Buni (14) ga qo'yamiz.

$$|\mathrm{v}|^2 \le \frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)(\lambda(\mathrm{v},\mathrm{z}_0)-2\lambda(\mathrm{v},\mathrm{z}_0)<\mathrm{v},\xi_0>)$$

$$|\mathrm{v}|^2 \le \frac{\beta^2}{\alpha^2-\beta^2}\lambda^2(\mathrm{v},\mathrm{z}_0)-2\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)<\mathrm{v},\xi_0>$$

Yoki

$$|\mathrm{v}|^2+2\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)<\mathrm{v},\xi_0> \le \frac{\beta^2}{\alpha^2-\beta^2}\lambda^2(\mathrm{v},\mathrm{z}_0)$$

$$|\mathrm{v}|^2+2\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)<\mathrm{v},\xi_0>+[\frac{\beta^2}{\alpha^2-\beta^2}\lambda^2(\mathrm{v},\mathrm{z}_0)\xi_0]^2 \le$$

$$\le \frac{\beta^2}{\alpha^2-\beta^2}\lambda^2(\mathrm{v},\mathrm{z}_0)+[\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0]^2$$

$$|\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0|^2 \le \lambda^2(\mathrm{v},\mathrm{z}_0)[\frac{\beta^2}{\alpha^2-\beta^2}+\frac{\beta^4}{(\alpha^2-\beta^2)^2}]$$

Demak,

$$|\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0|^2 \le \frac{\alpha^2\beta^2\lambda^2(\mathrm{v},\mathrm{z}_0)}{(\alpha^2-\beta^2)^2}$$

va ildiz olsak ikki tomonidan

$$|\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0| \le \frac{\alpha\beta\lambda(\mathrm{v},\mathrm{z}_0)}{(\alpha^2-\beta^2)} \qquad (15)$$

$\forall \psi \in R, |\psi|=1$ uchun quyidagi tenglik bajarilsa,

$$|\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0| \ge (\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0,\psi)$$

Bundan va (15) dan:

$$\frac{\alpha\beta}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0) \ge (\mathrm{v}+\frac{\beta^2}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0)\xi_0,\psi)$$
$$\forall\,\psi \in R^n, |\psi| = 1$$

uchun. Bundan:

$$(v-\lambda(\mathrm{v},\mathrm{z}_0)\xi_0+\lambda(\mathrm{v},\mathrm{z}_0)\frac{\alpha^2}{\alpha^2-\beta^2}\xi_0,\psi) \le \frac{\alpha\beta}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0) \tag{16}$$

Ma'lumki, $\dot{x} = u_\Pi(v) = v-\lambda(v,\mathrm{z}_0)\xi_0$, budan

$$\left(\dot{\mathrm{x}}(\mathrm{t})+\lambda(\mathrm{v}(\mathrm{t}),\mathrm{z}_0)\frac{\alpha^2}{\alpha^2-\beta^2}\xi_0,\psi\right) \le \frac{\alpha\beta}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0) \Rightarrow$$

$$(\dot{\mathrm{x}}(\mathrm{t}),\psi)+\lambda(\mathrm{v}(\mathrm{t}),\mathrm{z}_0)\frac{\alpha^2}{\alpha^2-\beta^2}(\xi_0,\psi)-\frac{\alpha\beta}{\alpha^2-\beta^2}\lambda(\mathrm{v},\mathrm{z}_0) \le 0 \Rightarrow$$

$$(\dot{\mathrm{x}}(\mathrm{t}),\psi)-\lambda(\mathrm{v}(\mathrm{t}),\mathrm{z}_0)\left(\frac{\alpha\beta}{\alpha^2-\beta^2}-\frac{\alpha^2}{\alpha^2-\beta^2}(\xi_0,\psi)\right) \le 0 \tag{17}$$

Lemma1 va oxirgi tengsizlikka asosan

$$\frac{d}{dt}F(\mathrm{W}(\mathrm{t}),\psi)) = \frac{d}{dt}F\left(x(\mathrm{t})+\left(1-\frac{1}{|\mathrm{z}_0|}\int_0^t \lambda(\mathrm{v}(\mathrm{s}))\,\mathrm{ds}\right)\left(R(\mathrm{z}_0)S-c(\mathrm{z}_0)\right),\psi\right) =$$

$$(\dot{\mathrm{x}}(\mathrm{t}),\psi)-\lambda(\mathrm{v}(\mathrm{t}),\mathrm{z}_0)\left(\frac{\alpha\beta}{\alpha^2-\beta^2}-\frac{\alpha^2}{\alpha^2-\beta^2}(\xi_0,\psi)\right) \le 0,$$

Ya'ni (13) tengsizlik o'rinli. Lemma 2 isbot.

Hulosa 1. Lemma 2 ga asosan $W(t) \subset W(0)$ ekanligi kelib chiqadi, ya'ni qochuvchi ixtiyoriy harakat qilganda ham

$$W(0) = \{\mathrm{w} : \beta|w-x_0| \ge \alpha|w-y_0|\} = x_0+R(\mathrm{z}_0)\mathrm{S}-c(\mathrm{z}_0)$$

bo`lgan shar ichidan chiqib ketmaydi.

Teorema 2. Agar $W(0) \cap \mathrm{M} = \varnothing$ bo`lsa qutilish cizig`i o'yni quvlovchi foydasiga hal bo`ladi.

Bu teoremaning isboti bevosita Lemma 2 dan kelib chiqadi.

Teorema 3. Agar $W(0) \cap \mathrm{M} \neq \varnothing$ bo`lsa uholda qutilish chizig`i masalasi qochuvchi foydasiga hal bo`ladi.

Bu teoremaning isboti [Azamov, Samatov] ishida keltirilgan.

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