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KOORDINATALARI BO'YICHA GEOMETRIK CHEGARALANISHLI SO`NUVCHI BOSHQARILADIGAN OBYEKTNING IXTIYORIY NUQTASIGA YETIB BORISH OPTIMAL VAQTINI TOPISH MASALASI.

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Annotatsiya. *Ushbu maqolada obyekt boshqaruvining koordinatalari o'zaro bog'liqsiz bo'lgan holda optimal boshqaruv nazariyasida obyektning tekislikning istalgan nuqtasiga optimal o'tish masalasi o'rganilgan. Bunda tekislik to'rtta sohaga ajratilib, har bir soha uchun boshqaruv funksiya va eng qisqa o'tish vaqti aniqlanadi. Natijalar umumlashtirilib optimal boshqaruv funksiya va o'tish vaqti topiladi.*

Kalit so'zlar. *Geometrik chegaralanish, obyekt, trayektoriya, o'lchanuvchi funksiya, tezlik, koordinata, optimal boshqaruv funksiya, optimal o'tish vaqti.*

THE PROBLEM OF FINDING OPTIMAL TIME TO REACH ANY POINT OF AN EXTRAORDINARY OBJECT WITH GEOMETRIC CONSTRAINTS BY COORDINATES

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Annotation. *In this article for position coordinates of an object control isn`t related and problem crossing to any point of plain of an object is studied in optimal controls theory. This plain is decided four field. For any field control functon and the least moving time is defined. Results are generalized optimal control function and moving time is found.*

Key words: *Geometric constraints, an object, trajectories, measureable function, speed, coordinates, optimal control function, optimal moving time.*

ЗАДАЧА НАХОЖДЕНИЯ ОПТИМАЛЬНОГО ВРЕМЕНИ ДОСТИЖЕНИЯ ОБЪЕМНОЙ ТОЧКИ ЭКСТРАОРДИНАРНОГО ОБЪЕКТА С ГЕОМЕТРИЧЕСКИМИ ОГРАНИЧЕНИЯМИ ПО КООРДИНАТАМ

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Аннотация. *В этой статье для координат положения объекта управление не связано, и проблема пересечения с любой точкой равнины объекта изучается в теории оптимального управления. Эта равнина решается четырьмя полями. Для любой функции управления полем определяется наименьшее время прохождения. В результате обобщается оптимальная функция управления и определяется время прохождения.*

Ключевые слова: *геометрические ограничения, объект, траектории, измеряемая функция, скорость, координаты, оптимальная функция управления, оптимальное время прохождения.*

1. Introduction

This article explores the optimal control of objects and their scope. It is important to control the transition of such objects from one state to another in the most convenient way in a certain sense. The controlled object is given to us by the following equations $\dot{x}_1 = x(t)$, $\dot{x}_2 = x(t)$ and the initial conditions $x_1(0) = x_{10}$, $x_2(0) = x_{20}$. The field of maturation and the optimal time spent to reach an arbitrary point in that field has been found. The problem of optimal control is characterized by the presence of an object whose movement changes over time. V.I.Blagodatsky described such issues in his book called Introduction to the Theory of Optimal Conrols.

In this article, a control isn`t related to position coordinates of an object and problem crossing to any point of plain of an object is studied in optimal controls theory. This plain is decided four field. For any field control functon and the least moving time is defined. Results are generalized for optimal control function and the fastest moving time is found.

2. Statement of problem

We are given moving object **E** in R^2 and $x=(x_1,x_2)$ means its state in plain. Let dynamics of the object **E** be described the following equations by coordinates

$$\mathbf{E}:\begin{array}{l}\dot{x}_1=u_1,\quad x_1(0)=x_{10},\\ \dot{x}_2=u_2,\quad x_2(0)=x_{20},\end{array}\qquad (1)$$

where $x_1,x_2,u_1,u_2\in R^2$, $n\geq 2$; $x_1(0)=x_{10}$, $x_2(0)=x_{20}$ are object\`s initial positon with respect to coordinates in $t=0$ and $x_{10}\neq x_{20}$

u_1 - the motion speed vector of x_1 coordinate, u_2 - the motion speed of x_2 coordinate, $u(t)=(u_1(t),u_2(t))$ control parameter of an object $\mathbf{E}$ is the mapping $u(\cdot):[0,+\infty)\to R^2$ and it is selected as Lebesgue measurable function. The coordinates of the control function $u(t)$ satisfy the following constraints (G- constraints)

$$|u_1(t)|\leq \alpha e^{-kt},\ \text{for almost everywhere}\quad t\geq 0,\qquad (2)$$

$$|u_2(t)|\leq \beta e^{-kt},\ \text{for almost all everywhere}\quad t\geq 0,\qquad (3)$$

where α, β, k are given positive numbers.

We denote a class of all measurable functions $u_1(t)$ ($u_2(t)$ satisfying (3) satisfying (2) by U_1 (by U_2) respectively.

Definition 1. For the pairs $(x_{10},u_1(\cdot))$, $u_1(\cdot)\in U_1$ and $(x_{20},u_2(\cdot))$, $u_2(\cdot)\in U_2$ the following equalities are called trajectories of an object coordinates

$$x_1(t,u_1(\cdot))=x_{10}+\int_0^t u_1(s)ds,\ t\geq 0,\qquad (4)$$

$$x_2(t,u_2(\cdot))=x_{20}+\int_0^t u_2(s)ds,\ t\geq 0\qquad (5)$$

Definition 2. The following equality is called motion of the object $\mathbf{E}$ in plain

$$x(t)=(t,u_1(t),u_2(t)),\ t\geq 0.\qquad (6)$$

According to Definiton 2, for different controls $u_1(\cdot)\in U_1$ and $u_2(\cdot)\in U_2$ there exists different trajectories starting from the point $x_0=(x_{10},\ x_{20})$.

We find the fastest moving time and optimal control transiting to any point of the plain $P=(p_1,p_2)$ that is given from initial point $x_0=(x_{10},\ x_{20})$.

Firstfull, we study the case which the initial position of the object $\mathbf{E}$ is at the point $x_{10}=0$, $x_{20}=0$. We draw straight line $x_2=\frac{\beta}{\alpha}x_1$, and $x_2=-\frac{\beta}{\alpha}x_1$ through the origin, (x_1,x_2) plain is divided into the following fields:

$$\Pi_1=\left\{(x_1,x_2):x_2>\frac{\beta}{\alpha}x_1,x>-\frac{\beta}{\alpha}x_1\right\},$$

$$\Pi_2=\left\{(x_1,x_2):x_2<\frac{\beta}{\alpha}x_1,x>-\frac{\beta}{\alpha}x_1\right\},$$

$$\varPi_3=\left\{(x_1,x_2):x_2<\frac{\beta}{\alpha}x_1, x<-\frac{\beta}{\alpha}x_1\right\},$$

$$\varPi_4=\left\{(x_1,x_2):x_2>\frac{\beta}{\alpha}x_1, x<-\frac{\beta}{\alpha}x_1\right\}.$$

Suppose that the initial state of the object $\mathbf{E}$ is at the origin.

Case 1: If $P=(p_1,p_2)\in\varPi_1$, then draw a straight line OP and mark the point of intersection OP with the line $x_2=\bar{\beta},\ -\bar{\alpha}\le x_1\le\bar{\alpha}$ by K . In this case, the coordinate of the point K is $(\alpha_1,\bar{\beta})$

From the similarity of the triangles $\vartriangle OMK$ and $\vartriangle OP_1P$ we obtain the following relation:

$$\frac{\alpha_1}{P_1}=\frac{\bar{\beta}}{P_2}\Rightarrow\alpha_1=\frac{p_1}{p_2}\beta e^{-kt},$$

where $\bar{\beta}=\beta e^{-kt}$.

It can be seen that the inequality $\alpha_1<\alpha$ is valid. Consequently, we have the folowing relations :

$$\frac{\bar{\beta}}{p_2}=\frac{\alpha_1}{p_1}<\frac{\bar{\alpha}}{p_1}\quad\Rightarrow\quad\frac{\bar{\alpha}}{p_1}>\frac{\bar{\beta}}{p_2}\quad\Rightarrow\quad\frac{\alpha}{p_1}>\frac{\beta}{p_2}\tag{7}$$

or

$$\frac{p_2}{\beta}>\frac{p_1}{\alpha},\tag{8}$$

where $\bar{\alpha}=\alpha e^{-kt}$.

We choose the following vector function as the control function that the object E moves from the origin to point P.

$$u^1=\overrightarrow{OK}=\left(\frac{p_1}{p_2}\beta e^{-kt},\beta e^{-kt}\right)=\left(\frac{p_1}{p_2}\beta,\beta\right)e^{-kt}\tag{9}$$

Now we show that the control (9) is admissible before applying it, that is, prove that satisfies the constraints (2) and (3).

$$\left|u_1^1\right|=\left|\frac{p_1}{p_2}\beta\right|e^{-kt}=\left|p_1\right|\left|\frac{\beta}{p_2}\right|e^{-kt}<\left|p_1\right|\left|\frac{\alpha}{p_1}\right|e^{-kt}\le\alpha e^{-kt},$$

$$\left|u_2^1\right|\le\beta e^{-kt}.$$

If the object E moves by u^1, then we define the time reaching to the point P.

$$x(t)=\int_0^{T_1}u^1(s)ds=\int_0^{T_1}\left(\frac{p_1}{p_2}\beta,\beta\right)e^{-ks}\,\mathrm{ds}=\left(\frac{p_1}{p_2}\beta,\beta\right)\int_0^{T_1}e^{-ks}ds=$$

$$=\left(\frac{p_1}{p_2}\beta,\beta\right)\frac{1}{k}\left(1-e^{-kT_1}\right)=(p_1,p_2)$$

Now we have the following result by equating the corresponding coordinates from the last equality

$$\frac{\beta p_1}{kp_2}\left(1-e^{-kT_1}\right)=p_1 \quad \Rightarrow \quad T_1=\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}. \tag{10}$$

Similarly, it is not difficult to verify that the result $T_1=\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}$ is obtained by equating the second coordinates.

Lemma1. If $P\in \Pi_1$, then the object E falls into the point P by the control $u^1=\left(\frac{p_1}{p_2}\beta,\beta\right)e^{-kt}$. The time when the object E moves to the point P is equal to $T_1=\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}$.

Now in our problem, for the case $k=0$ we calculate the time when the moves to the point P by that control u^1:

$$\lim_{k\to 0}\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}=\ln\left(\lim_{k\to 0}(1+\frac{kp_2}{\beta-kp_2})^{\frac{1}{k}}\right)=\frac{p_2}{\beta}.$$

<u>**Case 2**</u>: If $P=(p_1,p_2)\in \Pi_2$, then draw a straight line OP and mark the point of intersection OP with the line $x_2=\overline{\beta},\ -\overline{\alpha}\le x_1\le\overline{\alpha}$ by K. In that case, the coordinates of the point K is $(\overline{\alpha},\beta_1)$.

From the similarity of the triangles $\triangle OMK$ *and* $\triangle OP_1P$ we obtain the following relation:

$$\overline{\alpha}=\alpha e^{-kt},\ \overline{\beta}=\beta e^{-kt}$$

$$\frac{\beta_1}{p_2}=\frac{\overline{\alpha}}{p_1} \Rightarrow \beta_1=\frac{p_2}{p_1}\alpha e^{-kt}$$

where $\overline{\alpha}=\alpha e^{-kt}$

It can be seen that the inequality $\beta_1<\beta$ is valid , we have the following relations :

$$\frac{\overline{\alpha}}{p_1}=\frac{\beta_1}{p_2}<\frac{\overline{\beta}}{p_2} \quad \Rightarrow \quad \frac{\overline{\beta}}{p_2}>\frac{\overline{\alpha}}{p_1}\Rightarrow\frac{\beta}{p_2}>\frac{\alpha}{p_1} \tag{11}$$

or

$$\frac{p_1}{\alpha}>\frac{p_2}{\beta} \tag{12}$$

where $\overline{\beta}=\beta e^{-kt}$

We choose the following vector function as the control function that the object E moves from the origin to point P.

$$u^2=\vec{OK}=\left(\alpha e^{-kt},\frac{p_2}{p_1}\alpha e^{-kt}\right)=\left(\alpha,\frac{p_2}{p_1}\alpha\right)e^{-kt} \tag{13}$$

Now we show that the control (13) is admissible before applying , that is, prove that satisfies the constraints (2) and (3).

$$\left|u_2{}^2\right| = \left|\frac{p_2}{p_1}\alpha\right| e^{-kt} = \left|p_2\right|\left|\frac{\alpha}{p_1}\right| e^{-kt} < \left|p_2\right|\left|\frac{\beta}{p_2}\right| e^{-kt} \le \beta e^{-kt}$$

If the object E moves by u^2 then we define the time reaching to the point P.

$$x(\mathrm{T}_2) = \int_0^{T_2} u^2(\mathrm{s})\mathrm{ds} = \int_0^{T_2}\left(\alpha, \frac{p_2}{p_1}\alpha\right) e^{-ks}\,\mathrm{ds} = \left(\alpha, \frac{p_2}{p_1}\alpha\right)\int_0^{T_2} e^{-ks}\,\mathrm{ds} =$$

$$= \left(\alpha, \frac{p_2}{p_1}\alpha\right)\frac{1}{k}\left(1 - e^{-kT_2}\right) = (p_1, p_2)$$

Now we have the following result by equating the corresponding coordinates from the last equality

$$\frac{\alpha}{k}\left(1 - \mathrm{e}^{-kT_2}\right) = p_1 \quad \Rightarrow T_2 = \frac{1}{k}\ln\frac{\alpha}{\alpha - kp_1} \qquad (14)$$

Similarly, it is not difficult to verify that the result $T_2 = \frac{1}{k}\ln\frac{\alpha}{\alpha - kp_1}$ is obtained by equating the second coordinates.

Lemma2. If $P \in \Pi_2$, then the object E falls into the point P by the control $u^2 = \left(\alpha, \alpha\frac{p_2}{p_1}\right)e^{-kt}$. The time when the object E moves to the point P is equal to $T_2 = \frac{1}{k}\ln\frac{\alpha}{\alpha - kp_1}$.

Now in our prolem, for the case $k = 0$ we calculate the time when the object E moves to the point P by that control u^2

$$\lim_{k\to 0}\tfrac{1}{k}\ln\frac{\alpha}{\alpha - kp_1} = \ln\lim_{k\to 0}\left(1 + \frac{kp_1}{\alpha - kp_1}\right)^{\frac{1}{k}} = \frac{p_1}{\alpha}$$

<u>Case 3</u>: If $P = (p_1, -p_2) \in \Pi_3$, then draw a straight line OP and mark the point of intersection OP with the $x_2 = -\overline{\beta},\ -\overline{\alpha} \le x_1 \le \overline{\alpha}$ by K . In that case, the coordinates of the point K is $(\alpha_1, -\overline{\beta})$.

From the similarity of the traingles $\triangle OMK$ and $\triangle OP_1P$ we obtain the following relation :

$$\overline{\alpha} = \alpha e^{-kt},\ -\overline{\beta} = -\beta e^{-kt}$$

$$\frac{\alpha_1}{p_1} = \frac{-\overline{\beta}}{-p_2} \Rightarrow \alpha_1 = \frac{p_1}{p_2}\beta e^{-kt}$$

where, $-\overline{\beta} = -\beta e^{-kt}$

It can be seen that the inequality $\alpha_1 < \alpha$ is valid. Consequently, we have the following relations:

$$\frac{-\overline{\beta}}{-p_2} = \frac{\alpha_1}{p_1} < \frac{\overline{\alpha}}{p_1} \Rightarrow \frac{\overline{\alpha}}{p_1} > \frac{-\overline{\beta}}{-p_2} \Rightarrow \frac{\alpha}{p_1} > \frac{\beta}{p_2} \qquad (15)$$

or

$$\frac{p_2}{\beta} > \frac{p_1}{\alpha} \qquad (16)$$

where, $\overline{\alpha} = \alpha e^{-kt}$

We choose the following vector function as the control function that the object E moves from the origin to point P.

$$u^3 = \overrightarrow{OK} = \left(\frac{p_1}{p_2}\beta e^{-kt}, -\beta e^{-kt}\right) = \left(\frac{p_1}{p_2}\beta, -\beta\right)e^{-kt} \qquad (17)$$

Now we show that the control (17) is admissible before applying it, that is, we prove that satisfies the constraints (2) and (3).

$$\left|u_1^3\right| = \left|\frac{p_1}{p_2}\beta\right|e^{-kt} = \left|p_1\right|\left|\frac{\beta}{p_2}\right|e^{-kt} < \left|p_1\right|\left|\frac{\alpha}{p_1}\right|e^{-kt} \le \alpha e^{-kt}$$

$$\left|u_2^3\right| \le \beta e^{-kt}$$

If the object E moves by u^3, then we define the reaching time to the point P

$$x(T_3) = \int_0^{T_3} u^3(s)\,ds = \int_0^{T_3}\left(\frac{p_1}{p_2}\beta, -\beta\right)e^{-ks}\,ds = \left(\frac{p_1}{p_2}\beta, -\beta\right)\int_0^{T_3} e^{-ks}\,ds =$$

$$= \left(\frac{p_1}{p_2}\beta, -\beta\right)\frac{1}{k}\left(1 - e^{-kT_3}\right) = \left(p_1, -p_2\right)$$

Now we have the following result by equating the corresponding coordinates from the last equality

$$\frac{\beta p_1}{kp_2}\left(1 - e^{-kT_3}\right) = p_1 \Rightarrow T_3 = \frac{1}{k}\ln\frac{\beta}{\beta - kp_2} \qquad (18)$$

Similarly, it is not difficult to verify that the result $T_3 = \frac{1}{k}\ln\frac{\beta}{\beta - kp_2}$ is obtained by equating the second coordinates .

Lemma3. If $P \in \Pi_3$, then the object E falls into the point P by $u^3 = \overrightarrow{OK} = \left(\frac{p_1}{p_2}\beta, -\beta\right)e^{-kt}$. The time when the object E moves to the point P is equal to $T_3 = \frac{1}{k}\ln\frac{\beta}{\beta - kp_2}$.

Now in our prolem, for the case $k=0$ we calculate the time when the object E moves to the point P by that control u^3:

$$\lim_{k\to 0}\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}=\ln\lim_{k\to 0}\left(1+\frac{kp_2}{\beta-kp_2}\right)^{\frac{1}{k}}=\frac{p_2}{\beta}$$

Case 4: If $P=(-p_1,p_2)\in \Pi_4$, then draw a straight line OP and mark the point of intersection OP with the line $x_2=-\overline{\beta},\ -\overline{\alpha}\le x_1\le\overline{\alpha}$ by K. In this case, the coordinates of the point is K $(-\overline{\alpha},\beta_1)$.

From the similarity of the traingles $\triangle OMK$ and $\triangle OP_1P$ we obtain the following relation :

$$\frac{\beta_1}{p_2}=\frac{-\overline{\alpha}}{-p_1}\Rightarrow\ \beta_1=\frac{p_2}{p_1}\alpha e^{-kt}$$

where, $-\overline{\alpha}=-\alpha e^{-kt}$

It can be seen that the inequality $\beta_1<\beta$ is valid. Consequently, we have the following relations :

$$\frac{-\overline{\alpha}}{-p_1}=\frac{\beta_1}{p_2}<\frac{\overline{\beta}}{p_2}\ \Rightarrow\ \frac{\overline{\beta}}{p_2}>\frac{\overline{\alpha}}{p_1}\Rightarrow\frac{\beta}{p_2}>\frac{\alpha}{p_1} \qquad (19)$$

or

$$\frac{p_1}{\alpha}>\frac{p_2}{\beta} \qquad (20)$$

where, $\overline{\beta}=\beta e^{-kt}$

We choose the following vector function as the control function that the object E moves from the origin to point P.

$$u^4=\vec{OK}=\left(-\alpha e^{-kt},\frac{p_2}{p_1}\alpha e^{-kt}\right)=\left(-\alpha,\frac{p_2}{p_1}\alpha\right)e^{-kt} \qquad (21)$$

Now we show that the control (21) is admissible before applying it, that is, prove that satisfies the constraints (2) and (3).

$$|u_1^4|\le\alpha e^{-kt}$$

$$|u_2^4|=\left|\frac{p_2}{p_1}\alpha\right|e^{-kt}=|p_2|\left|\frac{\alpha}{p_1}\right|e^{-kt}<|p_2|\left|\frac{\beta}{p_2}\right|e^{-kt}\le\beta e^{-kt}$$

If the object E moves by u^4, then we define reaching the time to the point P.

$$x(T_4)=\int_0^{T_4}u^4(s)\,ds=\int_0^{T_4}\left(-\alpha,\frac{p_2}{p_1}\alpha\right)e^{-ks}\,ds=\left(-\alpha,\frac{p_2}{p_1}\alpha\right)\int_0^{T_4}e^{-ks}\,ds=$$

$$=\left(-\alpha,\frac{p_2}{p_1}\alpha\right)\frac{1}{k}\left(1-e^{-kT_4}\right)=\left(-p_1,p_2\right)$$

Now we have the following result by equating the corresponding coordinates from the last equality

$$-\frac{\alpha}{k}\left(1-\mathrm{e}^{-kT_4}\right)=-p_1 \quad\Rightarrow\quad T_4=\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1} \qquad (22)$$

Similarly, it is not difficult to verify that the result $T_4=\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}$ is obtained by equating the second coordinates .

Lemma4. If $P\in\Pi_4$, then the object E falls into the point P by $u^4=\left(-\alpha,\alpha\frac{p_2}{p_1}\right)e^{-kt}$.

The time when the object E moves to the point P is equal to $T_4=\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}$.

Now in our prolem, for the case $k=0$ we calculate the time when the objekt E moves to the point P by that control u^4.

$$\lim_{k\to 0}\tfrac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}=\ln\lim_{k\to 0}\left(1+\frac{kp_1}{\alpha-kp_1}\right)^{\frac{1}{k}}=\frac{p_1}{\alpha}$$

Conclusion

If $P\in\Pi_1$ $\left(P\in\Pi_3\right)$, then $T_1=\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}>\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}$

If $P\in\Pi_2$ $\left(P\in\Pi_4\right)$, then $T_2=\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}>\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}$

If $P=\left(p_1,p_2\right)\in\left\{\left(x_1,x_2\right):x_2=\frac{\beta}{\alpha}x_1\right\}$ or $P=\left(p_1,p_2\right)\in\left\{\left(x_1,x_2\right):x_2=-\frac{\beta}{\alpha}x_1\right\}$,

then $T=\frac{1}{k}\ln\frac{\beta}{\beta-kp_2}=\frac{1}{k}\ln\frac{\alpha}{\alpha-kp_1}$.

3. The main result

In the plain **E** of a fixed object $P\in R^2$, $\mathrm{P}=\left(p_1,p_2\right)$, the shortest time of descent to the point is qual to :

$$T^*=\frac{1}{k}\max\left\{\ln\frac{\beta}{\beta-kp_2};\ln\frac{\alpha}{\alpha-kp_1}\right\}$$

Theorem1. If $P\in R^2$, then $u^*=\left(\min\left\{\alpha,\frac{p_1}{p_2}\beta\right\},\min\left\{\frac{p_2}{p_1}\alpha,\beta\right\}\right)e^{-kt}$

by control , $\forall P=\left(p_1,p_2\right)\in R^2$ moves to the point at the following

$$T^{*}=\frac{1}{k}\max\left\{\ln\frac{\beta}{\beta-kp_2};\ln\frac{\alpha}{\alpha-kp_1}\right\}$$

If $x_0 \neq 0$, the following theorem holds.

Theorem 2. If $P\in R^2$, then

$$u^{*}=\left(\min\left\{\alpha,\frac{|p_1-x_{10}|}{|p_2-x_{20}|}\beta\right\},\min\left\{\frac{|p_2-x_{20}|}{|p_1-x_{10}|}\alpha,\beta\right\}\right)e^{-kt}$$

time using control, $\forall P=(p_1,p_2)\in R^2$ moves to the point at the following time

$$T^{*}=\frac{1}{k}\max\left\{\ln\frac{\beta}{\beta-k|p_2-x_{20}|};\ln\frac{\alpha}{\alpha-k|p_1-x_{10}|}\right\}$$

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O'LCHAMI 5 GA TENG NILPOTENT YORDAN ALGEBRALARIDA DIFFERENSIALLASHLAR VA LOKAL DIFFERENSIALLASHLAR

Nuriddinov Olimjon Odiljonovich

Andijon davlat universiteti Matematika kafedrasi tayanch doktoranti

МУНДАРИЖА

ФИЗИКА-МАТЕМАТИКА ФАНЛАРИ

01.00.00 ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ
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