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Shadow and massless particles around regular Bardeen black holes in 4D Einstein Gauss–Bonnet gravity

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In this paper, we study dynamics of massless (photon and neutrino-like) particles in the spacetime of regular Bardeen black hole (BH) in novel four-dimensional Einstein–Gauss–Bonnet (4D EGB) theory. First, we have analyzed the hor properties of the spacetime of the Bardeen BH with respect to the Gauss–Bonnet (GB) coupling parameter α. Our detailed analysis has shown that the GB coupling parameter is limited by maximum and minimum values at, $\alpha \in (-0.15 \div 1)$ and the extreme value of magnetic charge of the BH depends on the parameter α. We have explored motion of neutrino-like particles and photons around the Bardeen BH. We have also explored the possibility of mimicking cases of the spin of Kerr BH and charge of Reissner–Nordström BH by the Bardeen charge, providing the same values of impact parameter for photons corresponding to the

BH shadow size. Finally, we have investigated spherically infalling accretion and shadow of the Bardeen BHs in 4D EGB theory.

Keywords: Black holes; magnetic fields; magnetized particles; relativistic stars; orbits.

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1. Introduction

Solar system observations (Mercury perihelion shift, light bending due to Sun, etc.) solar system tests and observation of gravitational waves[1,2] and shadow of the central black hole M87[3,4] have provided very precise tests of the general relativity proposed by Einstein in both weak-field regime and strong-field regime. At the same time, the current resolutions of the experimental and observational data allow us to consider the alternative and modified theories of gravity. The study of the alternative and modified of gravity has been motivated with the several existing fundamental problems which general relativity met. These problems, particularly, include the appearance of singularity during the collapse, incompatibility of the classical general relativity with quantum field theory, etc. In this context, one may study the modifications of the gravity theories and construct the tests of the latter using observational and/or experimental data.

Namely, Gauss–Bonnet (GB) term is the modification to Einstein's gravity, which has very interesting features. Particularly, one of the interesting features according to Lovelock theorem[5] is that GB term does not affect the gravity in $D = 4$ dimensional spacetime. However, authors of Ref. 6 have proposed alternative approach avoiding the Lovelock restriction and obtained solution of Einstein–Gauss–Bonnet (EGB) in four-dimensional spacetime. This alternative approach based on rescaling the GB term by the factor $1/(D - 4)$ and keeping the modification in $D = 4$ spacetime. After this approach, several properties of the solution of 4D EGB theory have been intensively explored in Refs. 7–13 including the stability of the solution and thermodynamic properties.

Authors of Refs. 14–17 have investigated the rotating black hole solution in 4D EGB gravity and corresponding spacetime structure. Particularly, photon motion around BH in 6D EGB theory has been extensively studied in Refs. 18–21. Properties of the BH solutions in 4D EGB gravity have been widely explored in Refs. 22–27.

As we discussed above, the regularization of the black hole solution and avoiding the singularity problem in spacetime in classical theories of gravity are one of the main problems of theoretical astrophysics. Indeed, there are several approaches allowing the regularization of the solution and corresponding spacetime. One of the pioneering works on obtaining the regular BH solution has been performed by Bardeen in 1972[28] introducing the nonlinear electrodynamics coupled with general relativity. Similar regular Bardeen BH solution within the 4D EGB gravity has been obtained by the authors of Ref. 29. Here we plan to explore the photon motion around regular Bardeen BH in 4D EGB gravity.

It is worth noting that there are some critics on the validity of these modifications to the theory and the corresponding solutions proposed in Ref. 6. Particularly, authors of Refs. 30–33 have addressed few points of the modifications and the solution and existence of the latter in 4D spacetime. However, any theory/model/modification has to be tested using observational/experimental data. Here we aimed to construct to test the EGB theory and corresponding regular solution in 4D spacetime using the study of massless particles motion.

Study of massless particles, especially photon dynamics around compact relativistic objects, may lead to explore so-called shadow of BH. The shadow of BH appears due to strong light bending and consequently capturing the latter by the compact gravitating object. As a result, an observer may detect the black spot on the celestial plane. The shape and form of the shadow of the BH strongly depend on parameters of the solution and gravity models. The observation of first ever image of M87[34,35] might be used to test the gravity theories using the photon motion around BH in modified and alternative theories of gravity. Particularly, shadow formed by the BH in different gravity models have been widely explored in Refs. 36–59. The photon motion and gravitational lensing have been widely studied in Refs. 60–77.

In this paper, we plan to study motion of massless particles and shadow cast by the regular Bardeen BH in 4D EGB theory. This paper is organized as follows. We start with the review of the regular Bardeen BH solution in 4D EGB gravity in Sec. 2. In Sec. 3, we study the massless particles motion in the spacetime around regular Bardeen BH in 4D EGB gravity. Section 4 is devoted to study the shadow of the regular Bardeen BH in 4D EGB gravity.

We conclude and summarize the results of the paper in Sec. 5. In this paper, we use a spacelike signature $(-,+,+,+)$ for the spacetime and geometrized unit of the system where $G = 1 = c$. Latin (Greek) indices run from 1 (0) to 3.

2. Bardeen BH in 4D EGB Theory

The spacetime of regular Bardeen BH in the EGB gravity can be found re-scaling it with the GB coupling constant $\alpha/(D-4)$ together with the minimally coupled term which includes the nonlinear electrodynamics (NED) in D-dimensional spacetime using the following action[29]:

$$\mathcal{I}_G = \frac{1}{16\pi}\int d^D x\sqrt{-\mathbf{g}}\left(R + \frac{\alpha}{D-4}L_{GB} + L(F)\right), \tag{1}$$

where $\mathbf{g} = \det|g|$, $L(F)$ is the Lagrangian density being function of the electromagnetic field invariant $4F = F_{\mu\nu}F^{\mu\nu}$, which defines the electromagnetic field tensor for the gauge potential A_μ: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $L_{\rm GB}$ the Lagrangian corresponds to the GB theory being correction to GR has the following form:

$$L_{\rm GB} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \tag{2}$$

and Ricci scalar, Ricci and Reimann tensors are defined as R, $R_{\mu\nu}$ and $R^{\mu}_{\nu\gamma\sigma}$, respectively. The gravitational field equations for the action (1) find as

$$G_{\mu\nu} + \frac{\alpha}{D-4} H_{\mu\nu} = T_{\mu\nu}, \tag{3}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{4}$$

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\sigma}R^{\nu}_{\sigma} - 2R_{\mu\nu\sigma\rho}R^{\sigma\rho} - R_{\mu\sigma\delta\rho}R^{\sigma\delta\rho}_{\nu}) - \frac{1}{2} g_{\mu\nu} L(F), \tag{5}$$

$$T_{\mu\nu} = 2\left[L_F F_{\mu\sigma} F^{\sigma}_{\nu} - \frac{1}{4} g_{\mu\nu} L(F)\right]. \tag{6}$$

The Lagrangian for the NED field in the D-dimensional spacetime assuming the field is generated by the magnetic charge Q which has the form[29]

$$L(F) = \frac{(D-1)(D-2)\mu^{D-3}}{4Q^{D-3}} \left(\frac{\sqrt{2Q^2F}}{1+\sqrt{2Q^2F}}\right)^{\frac{2D-3}{D-2}}, \tag{7}$$

where

$$F = \frac{Q^{2(D-3)}}{2r^{2(D-2)}}. \tag{8}$$

The solution of Eq. (3) in the case of $D = 4$ together with the NED field Lagrangian (7) is obtained in Ref. 29 as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + d\Omega^2, \tag{9}$$

where the lapse function

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 \pm \sqrt{1 + \frac{8M\alpha}{(r^2+Q^2)^{\frac{3}{2}}}}\right), \tag{10}$$

and α and M are the GB parameter and the mass of the BH. The "-" sign in Eq. (10) is a real solution, because, in the GR limit ($\alpha \to 0$) it turns to be pure Bardeen BH solution,[78] in turn, it turns to the Schwarzschild BH one when $Q = \alpha = 0$. It is seen from the lapse function, the unit of the GB parameter is M^2. Therefore, we use in further calculations and plots, α as α/M^2.

Now, we are interested in which values of α and Q the spacetime given in Eq. (10) reflects a BH spacetime. To find the relationship between these two GB and magnetic charge parameters, we use the solutions of the following standard system of equations:

$$f(r) = 0, \quad f'(r) = 0.$$

The relationship between parameters α and Q corresponding to the BH horizon existence is shown in Fig. 1. Here, we have divided the space $\alpha - Q$ by two BH and no

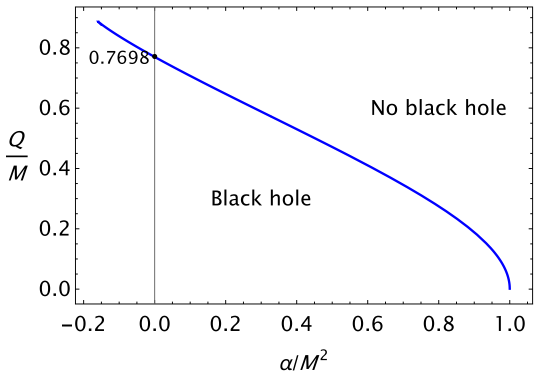

Fig. 1. Relations between the parameter α and the BH charge Q for the existence of the Bardeen BH horizon.

BH regions. We note that no BH means an object without event horizon. Using the numerical calculations, we have obtained that the parameter α takes the values in the range $\alpha/M^2 \in (-0.15869, 1)$ and the magnetic charge does $Q/M \in (0, 0.88625)$. Thus, the corresponding event horizon can be in the range $r_h/M \in (0.840291, 2)$ (see more details in Ref. 79).

3. Motion of Massless Particles

In this section, we explore the massless particle motion, namely, dynamics of photon and neutrino or neutrino-like particles around regular Bardeen BH in 4D EGB gravity. The interesting feature of such particle motion is that photons do not follow the null geodesic due to the interaction of the photon with the NED field of the regular Bardeen BH. However, neutrino-like massless particles do not feel the nonlinearity of the electromagnetic field around the regular BH. The photon motion is governed by null geodesics of the effective geometry[80,81] which includes the NED effects coupled to the spacetime structure.

3.1. *The motion of neutrino-like particles*

First, we explore the equations of motion for massless (neutrino-like) particles using Euler–Lagrange equations in the background spacetime (9)

$$L_{\rm p} = \frac{1}{2} g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}. \tag{11}$$

The conserved quantities of motion read

$$g_{tt}\dot{t} = -\mathcal{E}, \quad g_{\phi\phi}\dot{\phi} = \mathcal{L}, \tag{12}$$

where $\mathcal{E}$ and $\mathcal{L}$ are the specific energy and angular momentum of the particle, respectively. As we discussed above, since there is no interaction between such particles and NED field, one can use the spacetime (10). Equations of motion for the particle are governed by the normalization condition

$$g_{\mu\nu}u^{\mu}u^{\nu} = \epsilon, \tag{13}$$

where ϵ takes 0 and, -1 corresponding to massless and massive particles, respectively.

Taking into account Eq. (12) one can find the equation for radial motion of massless particles ($\epsilon = 0$) at the equatorial plane where $\theta = \pi/2$ and $\dot{\theta} = 0$ in the following form:

$$\frac{\dot{r}^2}{\mathcal{L}^2} + V_{\rm eff}(r) = \frac{\mathcal{E}^2}{\mathcal{L}^2}, \tag{14}$$

where

$$V_{\rm eff}(r) = \frac{f(r)}{r^2} \tag{15}$$

is the effective potential of radial motion of photons at the equatorial plane. Circular orbits for massless particle can be found using condition $V'_{\rm eff} = 0$ which implies

$$f'(r) - \frac{2}{r}f(r) = 0. \tag{16}$$

For the impact parameter of the circular null geodesics of the spacetime, one may get the following relationv[82]:

$$b^2_{c({\rm neu})} = \left.\frac{r^2}{f(r)}\right|_{r=r_{\rm neu}}, \tag{17}$$

where $r_{\rm neu}$ is the minimum radius of circular orbits for massless neutrino-like particles.

Figure 2 shows the dependence of the radius of minimum circular orbits and the impact parameter for the neutrino-like particles from the magnetic charge Q for the different values of the GB parameter α. One can easily see from Fig. 2 that

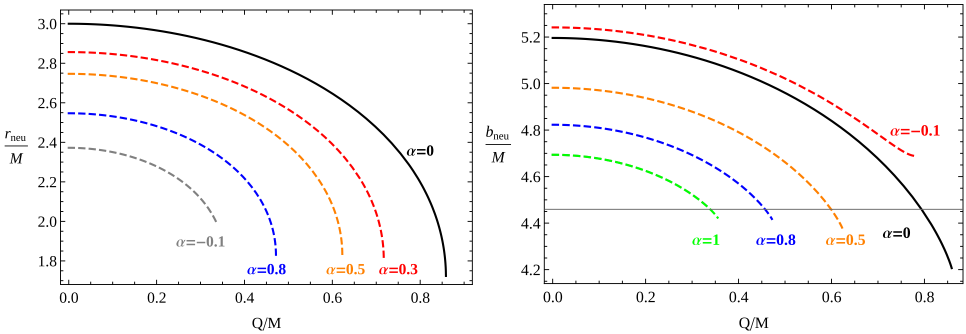

Fig. 2. The radius of circular orbits (left panel) and impact parameter (right panel) of massless neutrino-like particles around Bardeen regular BHs as a function of magnetic parameter g for the different values of the EGB parameter α.

both the impact parameter and the minimum radius of neutrino-like particles at the circular orbits decrease with the increase of both magnetic and GB parameters.

3.2. *Photon orbits*

The photon motion in the spacetime of a Bardeen BH given in Eq. (10) in 4D EGB gravity was determined by the modified null geodesics of the effective geometry due to interaction between photons and NED field, as we mentioned above, defined as[81,83–85]

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - 4\frac{L_{FF}}{L_F}F_{\mu}^{\lambda}F^{\mu\nu}, \tag{18}$$

$$\tilde{g}_{\mu\nu} = 16\frac{L_{\rm FF}F_{\mu\eta}F_{\nu}^{\eta} - (L_{\rm F} + 2FL_{\rm FF})g_{\mu\nu}}{F^2L_{\rm FF}^2 - 16(L_{\rm F} + FL_{\rm FF})^2}, \tag{19}$$

$$\mathrm{d}\tilde{s}^2 = -\frac{f(r)}{L_F}\,\mathrm{d}t^2 + \frac{1}{L_F f(r)}\mathrm{d}r^2 + \frac{r^2}{\Phi}\mathrm{d}\theta^2 + \frac{r^2}{\Phi}\mathrm{d}\phi^2, \tag{20}$$

where

$$L_{\rm F} = \frac{\partial L(F)}{\partial F}, \quad L_{\rm FF} = \frac{\partial^2 L(F)}{\partial F^2}, \tag{21}$$

$$\Phi = L_F + 2L_{\rm FF}F. \tag{22}$$

The motion of photons in the spacetime around regular BHs in GR coupled to NED can be described in the so-called, eikonal equation

$$\tilde{g}_{\mu\nu}k^{\mu}k^{\nu} = 0, \tag{23}$$

where k^{μ} is the four-wave vector related to the four-momentum of photons by the relation $p^{\mu} = k^{\mu}$ ($\hbar = 1$). The effective potential for the radial motion of photons around regular BH with the NED Lagrangian has the following form[81,82,86]:

$$V_{\rm eff}(r) = \frac{f(r)}{r^2}\left(1 + 2\frac{L_{\rm FF}}{L_{\rm F}}F\right). \tag{24}$$

For LED ($L(F) = F$) the effective potential turns the standard form (see, e.g. Refs. 87–89).

The local minimum of the radial profile of the effective potential corresponds to the photon circular orbits. We call b as impact parameter of a light that reaches infinity, it can be defined as follows:

$$b \equiv \frac{\mathcal{L}}{\mathcal{E}}, \tag{25}$$

where both quantities $\mathcal{L}$ and $\mathcal{E}$ are conserved because the effective metric (20) does not depend on time t and azimuthal angle ϕ. The radius of the photosphere and impact parameter of circular orbits of photons can be found by setting the condition $V'_{\rm eff} = 0$, where the prime $'$ denotes the derivative with respect to the radial coordinate.

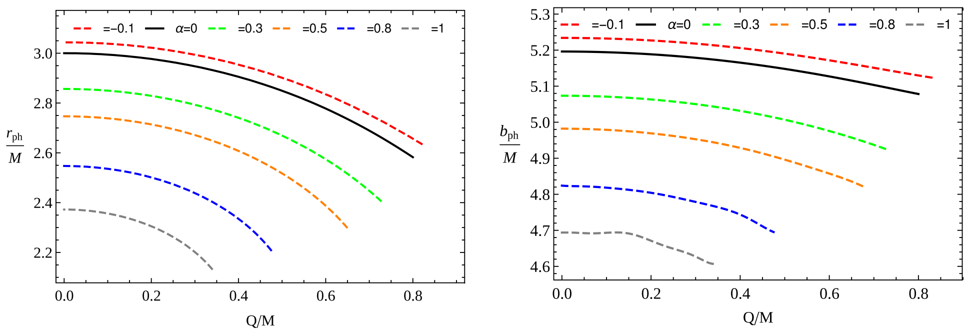

Fig. 3. The dependence of the photon circular orbits radii from the magnetic charge g for the representative values of coupling parameter α.

The value of the impact parameter defines the radius of the static BH shadow. The impact parameter of photons in the spacetime around regular BH with NED charge can be calculated using the following reduced expression[81,82,85,86]:

$$b_c^2 = \frac{L_{\rm F}}{L_{\rm F} + 2FL_{\rm FF}} \frac{r^2}{f(r)}\bigg|_{r=r_{\rm ph}}, \tag{26}$$

where $r_{\rm ph}$ is photon's circular orbit radius.

Figure 3 illustrates the dependence of the impact parameter for photons and innermost circular photon orbits from the magnetic charge Q for the different values of α. From this dependence, one can easily see that increase of the magnetic charge results in decrease of shadow size. It is also worth paying attention to the cases when a static BH with nonzero magnetic charge Q and GB parameter α may produce the shadow of the same size as those produced by other types of black holes. Below, we make the comparison of the photon impact parameter around different BHs, namely, rotating Kerr and RN BHs.

3.3. *Impact parameter of photons for Kerr BH*

We start with the photon impact parameter for Kerr BH and find the dependence of the latter on spin parameter. The effective potential of the photons around Kerr BH reads

$$V_{\rm eff}(r) = \frac{1}{r^2}\left[1 - \frac{2M}{r}\left(1 - \sigma\frac{a}{b}\right) - \left(\frac{a}{b}\right)^2\right], \tag{27}$$

where M is the total mass of the BH, a is the spin parameter, b is the impact parameter, $\sigma = 1$ and $\sigma = -1$ correspond to the co-rotating and contour-rotating photons, respectively. Using the following condition[90]:

$$\frac{1}{b^2} - V_{\rm eff}(r)|_{r=r_{\rm ph}} = 0 \tag{28}$$

and from Eqs. (27) and (28) one can immediately have an expression for the impact parameter of photons in the following form:

$$2b_*^{\pm} = a_*^2 \pm a_* + 3\left(\mathcal{A} + \frac{9}{\mathcal{A}} + 6\right) + \sqrt{2a_*^2 - 3\mathcal{A} - \frac{27}{\mathcal{A}} + \frac{2a_*[a^2 - 27]}{a_*^2 + 3\left(\mathcal{A} + \frac{9}{\mathcal{A}} + 6\right)} + 36}, \tag{29}$$

with

$$\mathcal{A}^3 = 2a_*^2\sqrt{a_*^4 + 27} - 2a_*^4 - 27, \tag{30}$$

where "+" and "-" signs correspond to co-and counter-rotating photons, respectively, notation * stands for quantities normalized to the BH mass as $b_* = b/M$, $a_* = a/M$.

3.4. *Impact parameter of photons for RN BH*

Now, we calculate impact parameter of photons in the spacetime around RN BH and compare it with the impact parameter in the spacetime of the Kerr BH. In fact, there is no interaction between electromagnetic fields created by LED and photons. The effective potential of photons around RN BH has the following form[82,83,86]:

$$V_{\text{eff}} = \frac{f(r)}{r^2}, \tag{31}$$

where radial lapse function for RN BH

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{32}$$

where Q is the magnetic charge of RN BH.

Now implying the condition (28) one can easily find the expression for the impact parameter of photons around RN black hole taking the form

$$b_* = \frac{(3 + \sqrt{9 - 8Q_*^2})^2}{\sqrt{8(3 - 2Q_*^2 + \sqrt{9 - 8Q_*^2})}}, \tag{33}$$

where $Q_* = Q/M$

Figure 4 presents the impact parameters for photons as a function of spin of Kerr BH and magnetic charge of RN BH. One can see that both parameters cause decreasing in the impact parameter. However, when $a = Q = 0.8478M$ both BH provide the same impact parameter $b = 4.4454M$.

Now we compare Eqs. (26), (29) and (33) to show how the charge of Bardeen regular BH can mimic spin of rotating Kerr and magnetic charge of RN BH through the same values of impact parameter of photons at circular orbits (size of BH shadow).

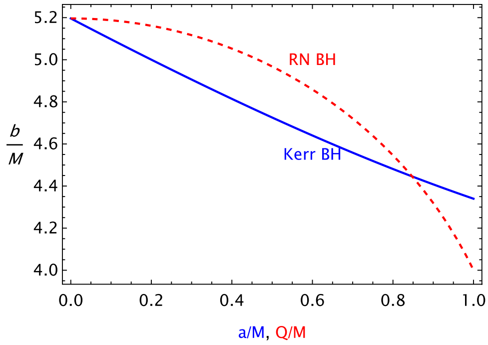

Fig. 4. Same figure as Fig. 3 but for Kerr and RN BHs.

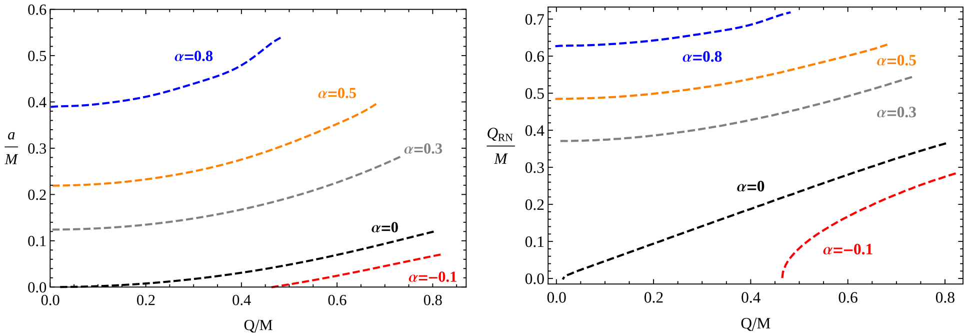

Fig. 5. Mimicking plots of magnetic charge of Bardeen regular BH, Q in 4D EGB and rotating parameter a of Kerr BH (top panel) and magnetic charge of RN BH (bottom) through the same impact parameters for photons at circular orbits for different values of GB parameter α.

In Fig. 5, we present relations between degeneracy values of spin of Kerr BH, magnetic charge of RBH BH, and magnetic charge of Berdeen BH at different values of the GB coupling parameter. One can see from the figure that the mimic range of the spin parameter shifts up with increasing parameter, α while the presence of negative α shifts right to the mimic value of the magnetic charge parameter of Bardeen BH. In this figure, we have considered $\sigma = +1$ and obtained the results as follows:

In Table 1, we have provided the range of mimicking values of the spin of Kerr BH and the charge of RN BH at the values of the Bardeen charge from 0 to its extreme. One can see from the table that the negative values of the GB parameter cause to decrease the mimic range of the spin and charge parameters. An increase of the parameter α increases the lower and upper values of the mimic values of them.

Moreover, our numerical calculations show that when $\sigma = -1$ (retrograde orbits), the magnetic charge of Bardeen RBH can mimic spin parameter up to $a/M = 0.04$ at $\alpha/M^2 = -0.15$.

Table 1. The range of mimic values of the spin of Kerr BH a and magnetic charge of RN BH for different values of the GB coupling parameter at $Q/M \in (0, Q_{\max})$.

α	a/M	Q_{RN}/M
-0.1	$0 \div 0.07$	$0 \div 0.282$
0	$0 \div 0.119$	$0 \div 0.363$
0.3	$0.124 \div 0.280$	$0.371 \div 0.543$
0.5	$0.218 \div 0.394$	$0.484 \div 0.631$
0.8	$0.389 \div 0.537$	$0.627 \div 0.717$

4. Spherically Infalling Accretion and Shadows of Bardeen BH in 4D EGB Theory

Photons' beam consisting of the same frequency $\nu_{\rm obs}$ can be emitted by the falling particles as electromagnetic radiation. Their intensity detected by an observer located at infinity $I_{\rm obs}$ can be evaluated, integrating the specific emissivity of the falling particles along null geodesics γ, as follows:

$$I_{\rm obs}(\nu_{\rm obs}, X, Y) = \int_{\gamma} g^3 j(\nu_{\rm e}) dl_{\rm prop}, \tag{34}$$

where $g = \nu_{\rm obs}/\nu_{\rm e}$ is known as the redshift factor, $\nu_{\rm e}$ is the photon frequency as measured in the rest-frame of the emitting particles, $j(\nu_{\rm e})$ is the emissivity per unit volume in the rest-frame of the emitter, and $dl_{\rm prop}$ is the infinitesimal proper length as measured in the rest-frame of the emitter. The redshift factor is

$$g = \frac{k_\alpha u^\alpha_{\rm obs}}{k_\beta u^\beta_{\rm e}}, \tag{35}$$

where $u^\mu_{\rm obs} = (1, 0, 0, 0)$ is the four-velocities of the distant observer, while $u^\mu_{\rm e}$ is the four-velocity of the accreting gas-like particles emitting the electromagnetic radiation. The 4-velocity of the accretion matter u^μ_e is

$$u^\mu_e = \left(\frac{1}{f(r)}, -\sqrt{1 - f(r)}, 0, 0\right). \tag{36}$$

Now, in order to find k^t, and k^r components of 4-velocity of photons, we use Eq. (20), and we have

$$k^r = \mathcal{L}_F \sqrt{1 - f(r) \frac{\Phi}{\mathcal{L}_F} \frac{\tilde{l}^2}{r^2}}, \tag{37}$$

$$k^t = \frac{\mathcal{L}_F}{f(r)}. \tag{38}$$

For the emissivity, we concern a simple model following Ref. 91, assuming the emission is monochromatic, and it has power-law $-1/r^2$ radial dependence

$$j(\nu_{\rm e}) \propto \frac{\delta(\nu_{\rm e} - \nu_\star)}{r^2}. \tag{39}$$

We also assume that $dl_{\text{prop}} = k_\alpha u_{\text{e}}^\alpha d\lambda$ and taking into account all the above-mentioned assumptions, one can get

$$dl_{\text{prop}} = \frac{k_t}{g|k_r|} dr. \tag{40}$$

Finally, we have

$$F_{\text{obs}}(X, Z) \propto \int_\gamma \frac{g^3 k_t dr}{r^2 |k_r|}. \tag{41}$$

Below, we show the intensity map of the images for a Bardeen BH in 4D EGB for different values of the GB parameter α and the magnetic charge of the Bardeen BH g.

Figure 6 illustrates the BH shadow in celestial coordinates and intensity of radiation by falling gas into the central Bardeen BH in 4D EGB gravity for positive and the negative values of the GB parameter $\alpha/M^2 = -0.15$ and 0.6 and the values

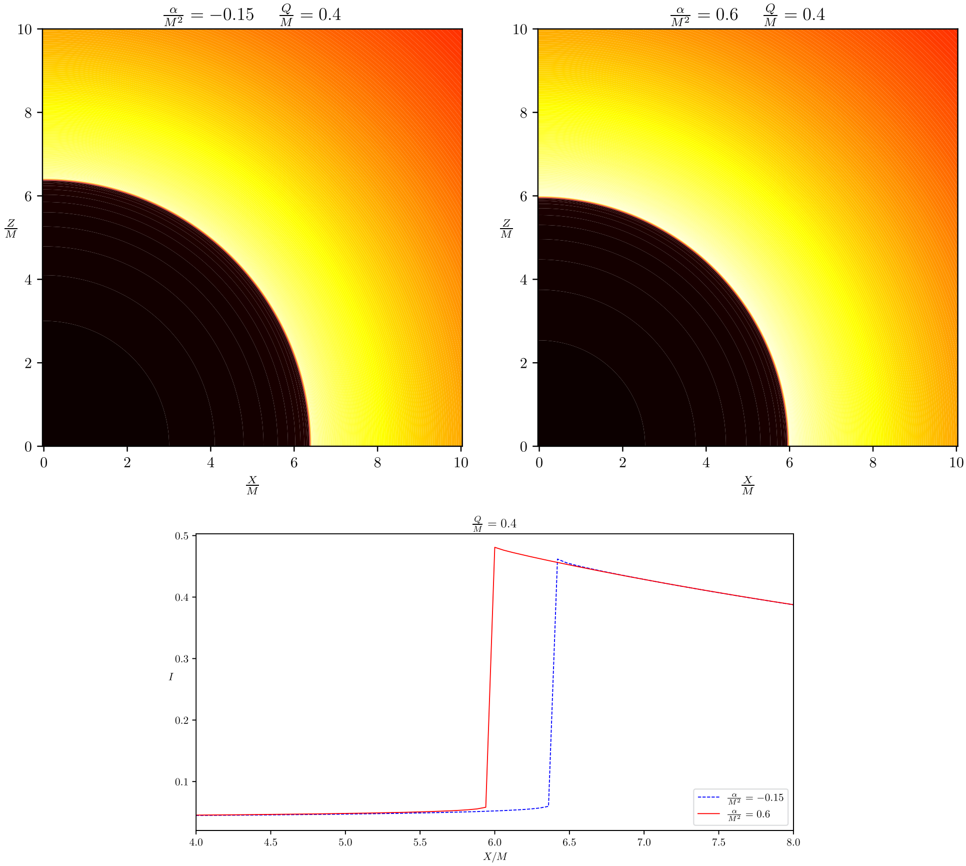

Fig. 6. Upper panel: images of optically thin emission regions surrounding the Bardeen BH in 4D EGB gravity. Lower panel: Intensity of gas emission as function of impact parameter.

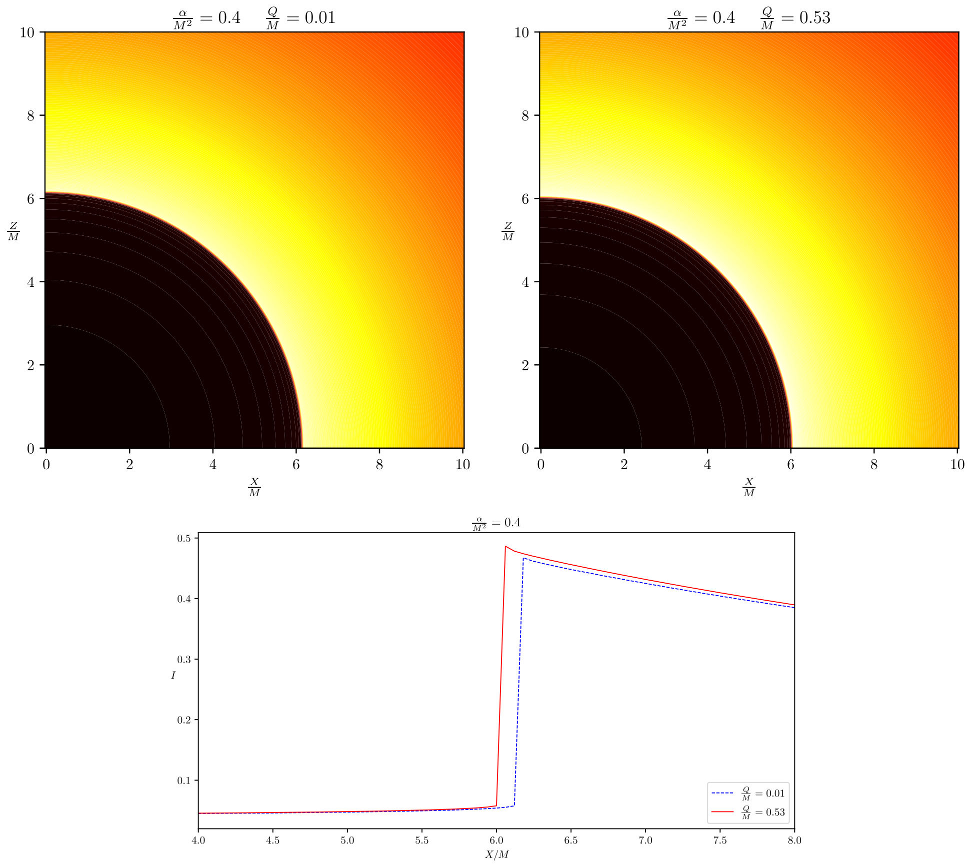

Fig. 7. The same figure with Fig. 6, but for fixed value of the GB parameter.

of the BH charge $Q/M = 0.4$. It is observed from this figure, the positive values of the GB parameter make quite intense radiation of the falling particles, and the point where the radiation intensity reaches its maximum come close towards the central BH.

However, the effects of the magnetic charge of the BH is much weaker than the GB parameter effects on the radiation intensity (see Fig. 7).

Figure 8 shows contours of the Bardeen BH shadow in 4D EGB theory for the different values of the GB and magnetic charge parameters. We use ray tracing code to determine the geometric structure of the BH shadow. The detailed algorithm of this code has been described in Refs. 92 and 93. Since, in our case, the electrodynamics around the BH has nonlinear character and taking account the interaction between NED field and photons, we used the effective metric (Eq. (20)) to solve the null-geodesic equation.

We can see the previous results for the impact parameter (Fig. 2) and the shadow radius changes in exactly the same way (increasing or decreasing, respectively) when

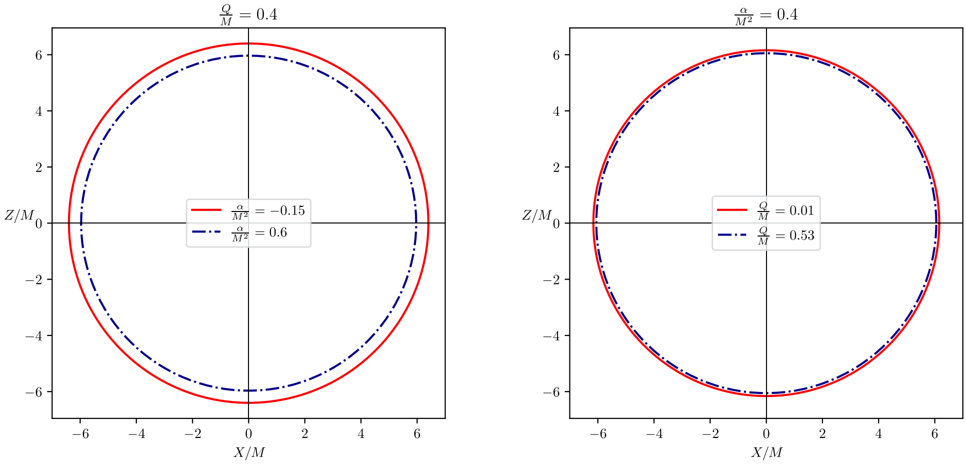

Fig. 8. The black hole shadow contours for different values of the GB parameter α and the BH charge Q.

changing the GB, α, and magnetic charge Q parameters of BH. It is seen from the figure that the increase of both GB and magnetic charge parameters results in shrinkage of the shadow size. However, the effect of GB parameter on the BH shadow size is stronger than the BH magnetic charge one.

5. Conclusion

This work is devoted to study massless, massive neutral tests and magnetized particle dynamics around magnetically charged Bardeen regular BH in 4D EGB theory. First, we have explored properties of horizon structure and found minimum and maximum values of the GB coupling parameter as $\alpha/M \in (-0.1586, 1)$. We have also provided a set of values of the magnetic and GB coupling parameters corresponding to the border of the areas with and without event horizon of the BH. It has been also shown that both magnetic charge and GB parameters cause the decrease of the outer event horizon radius.

We have studied massless-neutrino-like particles and photon motion and shown that the presence of the positive values of α and magnetic charge parameter decreases the impact parameter for the massless particles. We performed the detailed analysis of the impact parameter for photons in circular orbits around rotating Kerr and magnetically charged RN BHs. The mimicking values of Bardeen BH charge g spin of Kerr BH and magnetic charge of RN BH for the different values of the parameter α have been obtained and are shown in Table 1.

We have also investigated intensity of radiation of spherically infalling particles into regular Bardeen BHs in 4D EGB gravity. It is obtained that the increase of the GB parameter forces the particles to radiate with higher intensity. In other words, the effect of the GB parameter on the spacetime geometry around the Bardeen BHs is such that the radiation flux of photons from the radiating particles become more intense relative to the observer at infinity.

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